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Analysis of the age of data in data backup systems

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ABSTRACT

Cloud infrastructures are becoming a common platform for storage and workload operations for industries. With increasing rate of data generation, the cloud storage industry has already grown into a multibillion dollar industry. This industry offers services with very strict service level agreements (SLAs) to insure a high Quality of Service (QoS) for its clients. A breach of these SLAs results in a heavy economic loss for the service provider.

We study a queueing model of data backup systems with a focus on the age of data. The age of data is roughly defined as the time for which data has not been backed up and is therefore a measure of uncertainty for the user. We precisely define the performance measure and compute the generating function of its distribution. It is critical to ensure that the tail probabilities are small so that the system stays within SLAs with a high probability. Therefore, we also analyze the tail distribution of the age of data by performing dominant singularity analysis of its generating function. Our formulas can help the service providers to set the system parameters adequately.

1. Introduction

In the past few years, cloud data services have become one of the important pillars of the Information and Technology (IT) industry. Companies such as Amazon, Microsoft and IBM started offering computing services as IaaS (**I**nfrastructure **A**s **A** **S**ervice) which enables smaller businesses to enter market and generate revenue more quickly (see Deloitte [1]). By outsourcing the requirement of infrastructure setup and management to cloud service providers, the cost and difficulty of setup significantly reduces. Moreover, unlike local storage, these platforms offer remarkable features such as reliability, availability of data, protection from geographical calamities, etc (see Chang and Wills [2] for more details).

Cloud storage industry was worth US$25.171 billion in 2017 and is expected to be worth US$92.488 billion in 2022 [3]. This growth has been driven by the huge volume of data that is being generated every day (Shadroo and Rahmani [4]). Recently, studies have shown that this amount is continuing to grow exponentially. A study by the international data corporation (Reinsel et al. [5]) has estimated that the amount of data generated would reach 163ZB by 2025 which is approximately 10 times the data generated in 2016. Industries have started adopting public clouds for storage and operations. Arul Elumalai and Tandon [6] estimate that about 37% of the companies will be using public IaaS for at least one workload by 2018. Therefore, it is important to study and analyze cloud infrastructure systems and processes.

In this paper, we focus primarily on the data backup process to the cloud. Backup service providers use the infrastructure of data centers offered by companies such as Amazon, Google and provide the services of backup operations. Since cloud service providers charge the user to run these processes (see for example Amazon [7]), some studies have been done to optimize different components of these processes. For example, Gonalves et al. [8] investigate the workload of cloud storage services using Dropbox workloads. Huang et al. [9] study a model to minimize the data redundancy in cloud storage system which reduces the amount of data stored. Boullery et al. [10] optimize the utilization of network bandwidth, and Xia et al. [11] use Markov decision process to decide the storage schedule. In Saxena et al. [12], we model data backup processes as an exhaustive batch service queueing model with vacations. This model helps us to numerically compute the QoS of backup systems. In particular we compute the generating function of backlog size, the probability that a server is busy in a

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random slot, the probability of a new connection at a random slot and the first moment of the age of data.

In this paper, we study a similar discrete-time queueing model described in Saxena et al. [12] with the aim to calculate the distribution of the age of data just before a backup starts (Age_ν). In a real backup system, the backup service provider decides the configuration and schedule of backup operations as well as the communication with the cloud. The backup server alternates between backup-on and backup-off periods where the backup-on periods are much smaller than the backup-off periods. And because the age of data is insensitive to the batch server capacity, in the analysis we will assume that the batch server has an unlimited capacity. The age of the data is defined as the waiting time of the oldest packet in the system for the restart of the backup service. We focus on the age of the data (Age_ν) because it is closely related to the Recovery Point Objective (RPO). RPO is defined as the elapsed time since the closest time point in the history up to which the system is completely recoverable. It is one of the most critical performance measure of backup systems. The distribution of this age is important for various reasons. In addition to the mean value, the higher moments of a performance measure such as variance determine the performance of the system. For instance, a user may prefer lower variance of a performance measure over lower mean value. In Saxena et al. [12] we were successful in computing only the first moment of Age_ν. In this paper, we compute the generating function of the distribution of Age_ν using recursive relations. Using this generating function, one can then compute all moments of Age_ν.

Moreover, the tail probabilities represent the exceptional situations in which Age_ν can attain really large values. Backup service providers guarantee high QoS and even promise very heavy compensation in case of breach of SLA. For instance, Microsoft [13] gives 100% service credits if the RPO is more than 4 hours. Similarly, TEKLINKS [14], an IT solutions provider, gives similar guarantees of RPO between 15mins-24hrs for different service costs. Therefore, the probability that such extreme situations occur is an important performance metric. We analyze the tail of the distribution of Age_ν and scenarios in which a user might see such a breach. In our analysis, we use the recursive relations and the dominant singularity of the generating function of Age_ν to compute the asymptotic behavior of the distribution of the age of data in the backup system.

Some storage systems utilize a time timer mechanism to avoid the backup-off period to become very long. We study a system without timer mechanism but our analysis can be used to measure the performance of such systems in terms of Age of data and the backup frequency. More details about this are presented in Section 8. The analysis of these measures can then be used to find the optimal timer length.

We start with an overview of the results proved in our previous paper and necessary background to explain the analysis of this work. It is followed by computation of generating function of Age_ν and the analysis of its tail distribution. In the last sections we analyze the storage systems with time trigger mechanism and present a numerical example.

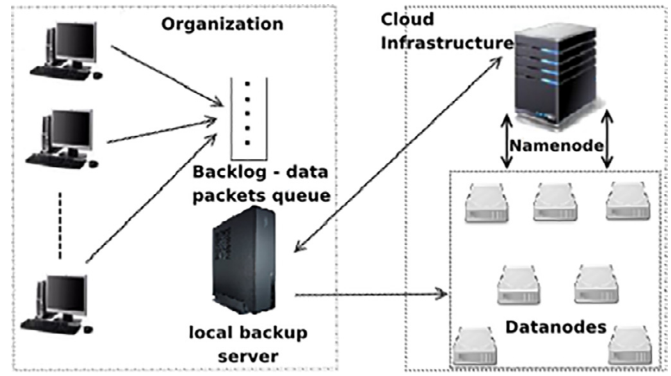

Fig. 1. Components of data backup mechanism.

2. Model description

From the perspective of data backups, cloud infrastructure consists of 3 major components illustrated in Fig. 1. The local backup server provides service to the incoming data packets by uploading them to cloud data storage nodes. The backup service provider manages the operations of the local backup server which performs all the backup operations. To perform these operations, when the backup server needs to start the service, it sends a request to the namenode, the central node of the cloud infrastructure. The namenode then provides the IP addresses of the data nodes where the packets can be uploaded. The backup server then uploads its data to the data nodes.

We model the backup server as a batch server with unlimited service capacity. This server employs an exhaustive service policy i.e. the server continues to serve until its backlog is empty. When the backlog becomes empty, the server goes into a vacation. When the vacation ends, the system checks the backlog of the system and decides whether to restart batch service or go into another vacation. Enough data packets should be present to make back-up operation efficient. Therefore, if the backlog is more than l packets at the end of a vacation, the batch service is resumed. Otherwise, if the backlog is i packets, where $i < l$, the batch service is restarted with probability α_i while with probability $1 - \alpha_i$ it goes into another vacation. The lengths of vacations are independent and identically distributed (i.i.d.) with a common random variable denoted by T.

2.1. Definitions and parameters

In the further sections, we will compute the generating function of the distribution of the age of data. For this analysis, we need the definitions of parameters and terminologies in Table 1.

3. Prior results

We start with a summary of previous results from Saxena et al. [12] which is necessary to continue the analysis in this paper as well as give some background on what we were able to compute earlier.

Table 1
List of notations used in the analysis.

l	the restarting threshold of the backlog size.
T	the random variable which denotes the length of any random vacation, its generating function is denoted by $T(z)$.
$A(z)$	the generating function of the number of arrivals in a single slot.
α_i	the probability of restart of the batch service at the end of a vacation if there are i packets backlogged.

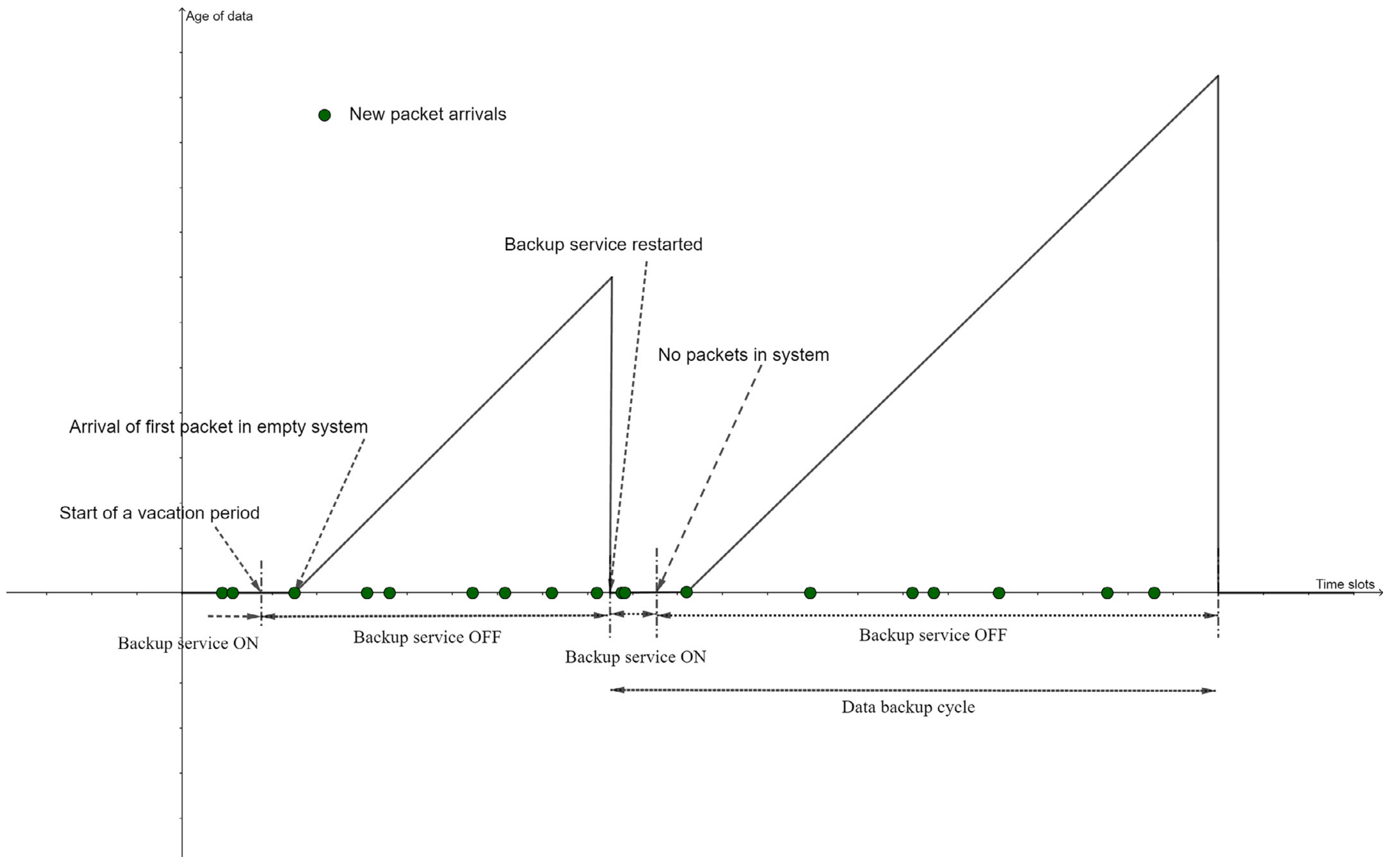

Fig. 2. Illustration of evolution of age of data in a backup system.

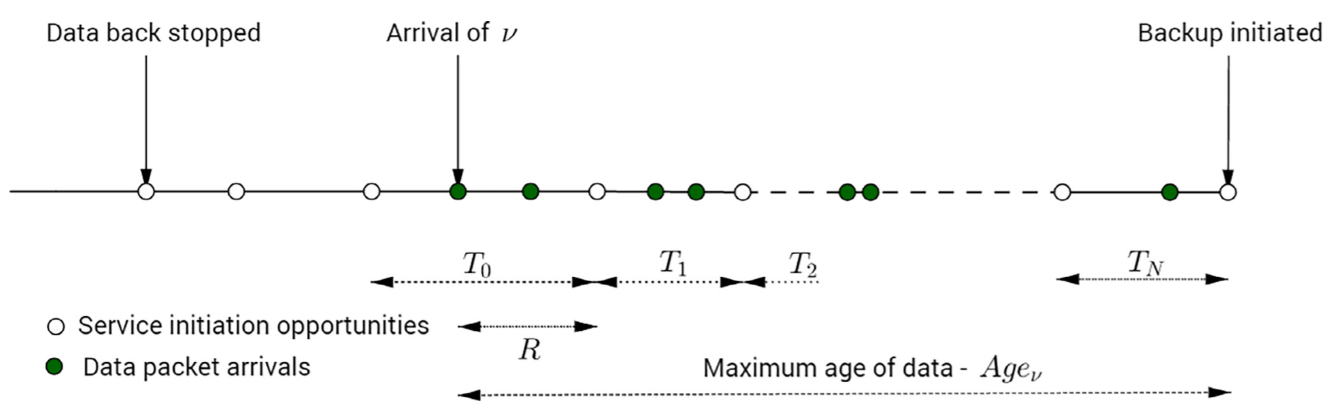

Fig. 3. Illustration of delay of packet ν.

A backup cycle is defined as a vacation period followed by a service period. We defined packet ν as the first packet arriving in a randomly selected backup cycle. Since a backup cycle starts at a backup completion and the batch server follows an exhaustive service policy, the system is empty when packet ν arrives. Further, define Age_ν as the waiting time of packet ν, i.e., the time until the backup service is restarted. Fig. 2 illustrates the evolution of the age of data in a backup system. Therefore,

$$Age_\nu = \sum_{i=1}^{N} T_i + R \tag{1}$$

where N is the number of vacations after the arrival of packet ν and until restart of backup service and R is the number of remaining slots of the vacation in which ν arrived. The length of the vacation in which ν arrives is denoted as T_0. The vacations which follow T_0 until the backup is restarted have lengths T_i (see Fig. 3).

Due to Wald's equation the first moment of Age_ν can be computed as

$$E(Age_\nu) = E(N) \times E(T) + E(R). \tag{2}$$

The joint distribution of R and T_0 is given by

$$P(T_0 = t, R = r) = \frac{A(0)^{t-r-1}[1 - A(0)]}{1 - T(A(0))} P(T = t) \tag{3}$$

which can be used to compute $E(R)$.

We proved that N has a phase-type distribution with initial probability vector ($\boldsymbol{\beta}$, β_l), and with transition probability matrix $\boldsymbol{M}$. Hence the generating function of N, $N(z)$ is given by

$$N(z) = z \cdot \boldsymbol{\beta} \cdot (I - zM)^{-1} \cdot \boldsymbol{M^0} + \beta_l$$

$$\boldsymbol{\beta} = \left(\frac{(1-\alpha_1)t_A(1)}{1-t_A(0)}, \frac{(1-\alpha_2)t_A(2)}{1-t_A(0)}, \frac{(1-\alpha_3)t_A(3)}{1-t_A(0)}, \ldots, \frac{(1-\alpha_{l-1})t_A(l-1)}{1-t_A(0)} \right),$$

$$\beta_l = 1 - \sum_{i=1}^{l-1} \beta_i$$

$$M = \begin{bmatrix} (1-\alpha_1)t_A(0) & (1-\alpha_2)t_A(1) & (1-\alpha_3)t_A(2) & \ldots & (1-\alpha_{l-1})t_A(l-2) \\ 0 & (1-\alpha_2)t_A(0) & (1-\alpha_3)t_A(1) & \ldots & (1-\alpha_{l-1})t_A(l-3) \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \ldots & (1-\alpha_{l-1})t_A(0) \end{bmatrix}$$

$\boldsymbol{M^0}$ can be computed using the fact that row sums of $P = \begin{bmatrix} M & \boldsymbol{M^0} \\ \boldsymbol{0} & 1 \end{bmatrix}$ are equal to 1.

To compute higher moments of Age_ν, Wald's equation is of no use. Therefore, in the following sections, we construct recursive relations that will enable us to compute the probability generating function of Age_ν. From the generating function, higher order moments as well as tail probabilities of Age_ν can be computed.

4. Some useful generating functions

The computation of the generating function of Age_ν requires a few results to be established first. In particular we compute $R(z)$, the generating function of the remaining service time, $S(y, z)$, the joint generating function of the length of remaining vacation time and the number of arrivals in T_0, and $W(y, z)$, the joint generating function of the length of a random vacation and the number of arrivals in it.

Lemma 4.1. *The generating function of the length of the remaining vacation R is given by*

$$R(z) = \frac{\left[1 - A(0)\right]\left[T(A(0)) - T(z)\right]}{\left[1 - T(A(0))\right]\left[A(0) - z\right]}. \tag{4}$$

Proof. Using formula (3), this generating function can be written as the following sum

$$\begin{aligned} R(z) &= \sum_{t=1}^{\infty}\sum_{r=0}^{t-1} z^r Pr(T_0 = t, R = r) \\ &= \sum_{t=1}^{\infty} \frac{A(0)^{t-1}[1 - A(0)]Pr(T = t)}{1 - T(A(0))} \sum_{r=0}^{t-1} \frac{z^r}{A(0)^r} \\ &= \frac{\left[1 - A(0)\right]\left[T(A(0)) - T(z)\right]}{\left[1 - T(A(0))\right]\left[A(0) - z\right]}. \end{aligned}$$

□

Lemma 4.2. *The joint generating function of the length of the remaining vacation R and the number of arrivals in T_0, n_{T_0}, is given by*

$$S(y, z) = E(y^{n_{T_0}} z^R) = \frac{\left[A(y) - A(0)\right]\left[T(zA(y)) - T(A(0))\right]}{\left[zA(y) - A(0)\right]\left[1 - T(A(0))\right]}. \tag{5}$$

Proof. Note that the slot in which ν arrives is not considered as part of R and can have more than one arrival. The generation function $S(y, z)$ can be written as the following sum

$$\begin{aligned} S(y, z) &= \sum_{i=1}^{\infty}\sum_{r=0}^{\infty} y^i z^r Pr(\ i \text{ arrivals in } r+1 \text{ slots } \big| \text{ at} \\ &\qquad \text{least one arrival in slot } 1) Pr(R = r) \\ &= \sum_{r=0}^{\infty} A(y)^r \frac{A(y) - A(0)}{1 - A(0)} z^r Pr(R = r) \\ &= \frac{A(y) - A(0)}{1 - A(0)} R(zA(y)). \end{aligned}$$

Using the results of Lemma 4.1, we arrive at the following expression,

$$S(y, z) = \frac{\left[A(y) - A(0)\right]\left[T(zA(y)) - T(A(0))\right]}{\left[zA(y) - A(0)\right]\left[1 - T(A(0))\right]}.$$

□

We can write the series expansion of $S(y, z)$ in its radius of convergence as

$$S(y, z) = \sum_{i=1}^{\infty} r_i(z) y^i. \tag{6}$$

The coefficients $r_i(z)$ give us the partial generating functions of the length of the remaining vacation slots R with total of i arrivals in the whole vacation T_0. We will use $r_i(z)$ in our analysis in the further sections.

Lemma 4.3. *The joint generating function of the length of the vacation T_i, $i \geq 1$, and the number of arrivals in it n_T is given by*

$$W(y, z) = E(y^{n_T} z^T) = T(zA(y)). \tag{7}$$

Proof. This generating function can be written as the following sum

$$\begin{aligned} W(y, z) &= \sum_{t=1}^{\infty}\sum_{j=0}^{\infty} y^j z^t Pr(\ j \text{ arrivals in } t \text{ slots })Pr(T = t) \\ &= \sum_{t=1}^{\infty} A(y)^t z^t Pr(T = t) = T(zA(y)). \end{aligned}$$

□

We can write the series expansion of $W(y, z)$ in its radius of convergence as

$$W(y, z) = \sum_{i=0}^{\infty} t_i(z) y^i. \tag{8}$$

The coefficients $t_i(z)$ are the partial generating functions of a vacation length with i arrivals.

5. Generating function of $\boldsymbol{Age_\nu}$

In this section we compute the generating function of Age_ν. Therefore, define

- $h(t) = Pr(Age_\nu = t)$: the probability mass function of Age_ν.
- $g_i(t)$: Given the backlog size equals i at the beginning of a vacation, $g_i(t)$ is the probability mass function of the remaining time until the restart of the batch service.

The end of each vacation gives the system a *service initiation opportunity*. The backup server then decides to either restart the batch service or enter a new vacation. To find the distribution of Age_ν, we first condition on the action chosen by the backup server at the end of vacation T_0. That is, we write

$$\begin{aligned} h(t) &= Pr(Age_\nu = t \cap \text{ backup service restarts after vacation } T_0) \\ &\quad + Pr(Age_\nu = t \cap \text{ backup service does not} \\ &\qquad \text{restarts after vacation } T_0) \\ &= \sum_{i=1}^{l-1} \alpha_i Pr(i \text{ arrivals in } t+1 \text{ slots } \big| \text{ at} \\ &\qquad \text{least 1 arrival in slot 1})Pr(R = t) \\ &\quad + \sum_{i=l}^{\infty} Pr(i \text{ arrivals in } t+1 \text{ slots } \big| \text{ at} \\ &\qquad \text{least 1 arrival in slot 1x})Pr(R = t) \end{aligned}$$

$$+\sum_{k=0}^{t-1}\sum_{i=1}^{l-1}(1-\alpha_i)Pr(i \text{ arrivals in } k+1 \text{ slots} \mid \text{at least 1 arrival in slot 1}) \times Pr(R=k)g_i(t-k). \quad (9)$$

Similarly, $g_i(t)$ can be written recursively as

$$g_i(t)=\sum_{j=i}^{l-1}\alpha_j Pr(j-i \text{ arrivals in } t \text{ slots })Pr(T=t) + \sum_{j=l}^{\infty} Pr(j-i \text{ arrivals in } t \text{ slots })Pr(T=t) + \sum_{k=1}^{t-1}\sum_{j=i}^{l-1}(1-\alpha_j)Pr(j-i \text{ arrivals in } k \text{ slots}) Pr(T=k)g_j(t-k). \quad (10)$$

We will now transform Eqs. (9) and (10) to generating functions. Define therefore

$$H(z)=\sum_{t=0}^{\infty} z^t h(t), \quad G_i(z)=\sum_{t=1}^{\infty} z^t g_i(t), \quad i=1\ldots l-1.$$

the generating functions of the distributions $h(t)$ and $g_i(t)$. We can then use Eq. (9) to write

$$\begin{aligned}H(z) &= \sum_{t=0}^{\infty}\sum_{i=1}^{l-1}(\alpha_i-1)z^t Pr(i \text{ arrivals in } t+1 \text{ slots} | \text{ at least 1 arrival in slot 1})Pr(R=t)+R(z)\\ &+\sum_{t=1}^{\infty}\sum_{k=0}^{t-1}\sum_{i=1}^{l-1} z^t(1-\alpha_i)Pr(i \text{ arrivals in } k+1 \text{ slots}| \text{ at least 1 arrival in slot 1})Pr(R=k)g_i(t-k)\\ &= \sum_{i=1}^{l-1}(\alpha_i-1)r_i(z)+R(z)+\sum_{i=1}^{l-1} \times (1-\alpha_i)\sum_{k=0}^{\infty} Pr(i \text{ arrivals in } k+1 \text{ slots with at least 1 arrival in slot 1}) \times Pr(R=k) \sum_{t=k+1}^{\infty} z^t g_i(t-k)\\ &= \sum_{i=1}^{l-1}(\alpha_i-1)r_i(z)+R(z)+\sum_{i=1}^{l-1}(1-\alpha_i)G_i(z)\sum_{k=0}^{\infty} z^k Pr \times (i \text{ arrivals in } k+1 \text{ slots with at least 1 arrival in slot 1})Pr(R=k)\\ &= \sum_{i=1}^{l-1}(\alpha_i-1)r_i(z)+R(z)+\sum_{i=1}^{l-1}(1-\alpha_i)G_i(z)r_i(z).\end{aligned}$$

Similar calculations using Eq. (10) are possible. We end up with the following recursion:

$$G_i(z)=T(z)+\sum_{j=i}^{l-1}(1-\alpha_j)\big(G_j(z)-1\big)t_{j-i}(z), \quad \forall 1\le i\le l-1. \quad (11)$$

We summarize our results in the following theorem.

Theorem 5.1. *The generating function of Age_ν, $H(z)$, is given by*

$$H(z)=R(z)+\sum_{i=1}^{l-1}(1-\alpha_i)r_i(z)(G_i(z)-1) \quad (12)$$

$$G_i(z) = \frac{1}{1-(1-\alpha_i)t_0(z)}\Big(T(z)-(1-\alpha_i)t_0(z) + \sum_{j=i+1}^{l-1}(1-\alpha_j)t_{j-i}(z)\big(G_j(z)-1\big)\Big), \quad \forall 1\le i\le l-1. \quad (13)$$

where $r_i(z)$ and $t_i(z)$ can be computed from (Eqs. (5)–(8)). Note that the functions $G_i(z)$ can be computed recursively starting from $G_{l-1}(z)$.

6. Mean and variance of Age_ν

Using Eqs. (12) and (13) one can compute moments of Age_ν. First, the mean value is given by

$$E(Age_\nu) = H'(1) = R'(1)+\sum_{i=1}^{l-1}(1-\alpha_i)r_i(1)G_i'(1)$$

$$G_i'(1) = \frac{1}{1-(1-\alpha_i)t_0(1)}\Big(T'(1)+\sum_{j=i+1}^{l-1}(1-\alpha_j)G_j'(1)t_{j-i}(1)\Big), \quad \forall 1\le i\le l-1 \quad (14)$$

We verified that the mean of Age_ν obtained using Eq. (14) is exactly the same as obtained in Saxena et al. [12] where Wald's equation was used to obtain the mean value. Similarly, we can use the second derivatives of $H(z)$ and $G_i(z)$ to compute the variance of Age_ν.

$$\begin{aligned}&H''(1)R''(1)+\sum_{i=1}^{l-1}(1-\alpha_i)(2r_i'(1)G_i'(1)+G_i''(1)r_i(1))\\ &G_i''(1)(1-(1-\alpha_i)t_0(1)) = T''(1)+2G_i'(1)(1-\alpha_i)t_0'(1) + \sum_{j=i+1}^{l-1}(1-\alpha_j)(t_{j-i}(1)G_j''(1)+2G_j'(1)t_{j-i}'(1))\\ &Var(Age_\nu) = H''(1)+H'(1)-H'(1)^2\end{aligned} \quad (15)$$

Computing $r_i^{(n)}(1), t_i^{(n)}(1)$, the n^{th} derivatives of the partial generating functions, involves inversion of derivatives of the joint generating functions in Eq. (6) and Eq. (8). This is a non trivial task for a general distribution function. However, for well behaved functions such as rational functions, we can compute the derivatives to get $r_i^{(n)}(1), t_i^{(n)}(1)$. One can also use a numerical inversion algorithms, such as Fourier series method in Abate and Whitt [15], to compute these values.

For a Poisson arrival process and vacation lengths distribution with support in $[a, b]$ (i.e. $Pr(a\le T\le b)=1$), the coefficients and their derivatives can be directly computed and have a closed form expression given below.

$$t_i(1)=\sum_{t=a}^{b} e^{-\lambda t}(\lambda t)^i\frac{Pr(T=t)}{i!}$$

$$t_i'(1)=\sum_{t=a}^{b} te^{-\lambda t}(\lambda t)^i\frac{Pr(T=t)}{i!}$$

$$r_i(1)=\frac{t_i(1)}{1-T(A(0))}$$

$$r_i'(1)=\frac{t_i'(1)}{1-T(A(0))}-\frac{1}{1-T(A(0))}\sum_{t=a}^{b} e^{-\lambda t}\lambda^i\frac{Pr(T=t)}{i!}\sum_{s=1}^{t}s^i$$

These constants can then be used to compute the mean and variance of Age_ν.

7. Tail asymptotics

As mentioned in Section 1, computing the tail probabilities is of equal importance, since extreme events result in a high financial loss. Therefore, it is important to analyze the cases in which this age can take high values and compute the probability of their occurrence. Therefore, in this section we compute the tail asymptotic of Age_v using the generating function $H(z)$. For this analysis we carry out dominant singularity analysis of $H(z)$. For the ease of presentation we make some naturally applicable assumptions stated below. It is important to note that this analysis is valid even when these assumptions are not met but would complicate the presentation.

Assumptions:

1. Intuitively, an efficient data backup policy should restart the backup service with higher probability if there are more packets in the backlog. This is because when more packets are waiting for service, there are more packets at risk. Therefore, we assume that α_i is non decreasing, i.e., $i < j$ implies $\alpha_i \le \alpha_j$.
2. The first s restarting probabilities are equal, i.e., $\alpha_1 = \alpha_2 = \ldots = \alpha_s$. This is not a restriction as any value of s between 1 and $l-1$ is allowed. However, the value of s has an impact on tail probabilities.
3. $T(z)$, the generating function of the vacation period is defined by the user. Since it is a user defined function, it is reasonable to expect it to be analytic everywhere. Therefore, we assume that $T(z)$ does not have any singularities. This implies that $R(z)$, $t_i(z)$ and $r_i(z)$ also do not have any singularities.

A function $f(z)$ is said to have a singularity at a point z_0 if it is not analytic at this point. Further, a dominant singularity of a function is a singularity with smallest absolute value among all singularities. In this work, we make extensive use of the following result (see Flajolet and Sedgewick [16] for details).

Theorem 7.1 (Darboux theorem)**.** *Suppose the power series $X(z) = \sum_{n=0}^{\infty} x(n)z^n$ with positive real coefficients $x(n)$ is analytic near 0 and has only algebraic singularities σ_k on its circle of convergence $|z| = \Re_X$, in other words, in a neighborhood of σ_k we have*

$$X(z) \sim \left(1 - \frac{z}{\sigma_k}\right)^{-w_k} Y_k(z),$$

where $w_k \in \mathbb{C}\backslash\{0, -1, -2, \ldots\}$ and $Y_k(z)$ denotes a nonzero analytic function near σ_k. Let $w = max_k Re(w_k)$ denote the maximum of the real parts of the w_k. Then we have

$$x(n) = \sum_k \frac{Y_k(\sigma_k)}{\Gamma(w_k)} n^{w_k - 1} \sigma_k^{-n} + o(n^{w-1}\Re^{-n})$$

with the sum taken over all k with $Re(w_k) = w$ and $\Gamma(w)$ the Gamma-function of w (with $\Gamma(n) = (n-1)!$ for n discrete).

Therefore, if the dominant singularities, $\sigma_k \in \mathbb{C}$, of the generating function $X(z)$ and the behavior of the generating function in the neighborhood of these dominant singularities (w_k and $Y_k(\sigma_k)$) are identified, this theorem expresses an approximation for the tail of the corresponding distribution ($x(n)$ for large n).

From Eq. (12), $H(z)$ contains the generating functions $G_i(z)$. Therefore, we start with the analysis of $G_i(z)$ to compute the singularities of $H(z)$.

Lemma 7.2. *The singularities of $G_i(z)$ are given by*

$$\left\{ z : \prod_{j=i}^{l-1} \Big(1 - (1-\alpha_j)t_0(z)\Big) = 0 \right\}.$$

Proof. From Eq. (13), we can compute

$$G_{l-1}(z) = \frac{T(z) - (1-\alpha_{l-1})t_0(z)}{1 - (1-\alpha_{l-1})t_0(z)}$$

Clearly, $G_{l-1}(z)$ has a singularity of order 1 at the zeros of $1 - (1-\alpha_{l-1})t_0(z)$, which proves the lemma for $i = l-1$. For $i = 1, \ldots, l-2$ we will prove this result by induction, i.e., for all $m > i$ assume $G_m(z)$ has singularities at

$$\left\{ z : \prod_{j=m}^{l-1} \Big(1 - (1-\alpha_j)t_0(z)\Big) = 0 \right\},$$

we will show that the result holds for $G_i(z)$. We can rewrite Eq. (13) as,

$$\begin{aligned} G_i(z) = {} & \frac{1}{1-(1-\alpha_i)t_0(z)} \Bigg(T(z) - (1-\alpha_i)t_0(z) - (1-\alpha_{i+1})t_1(z) \\ & + \sum_{j=i+2}^{l-1} (1-\alpha_j)t_{j-i}(z)\big(G_j(z) - 1\big) \Bigg) \\ & + (1-\alpha_{i+1})t_1(z) \frac{G_{i+1}(z)}{1-(1-\alpha_i)t_0(z)}. \end{aligned}$$

Hence, the singularities of $G_i(z)$ are the singularities of $G_{i+1}(z)$ and the zeros of $1 - (1-\alpha_i)t_0(z)$. Therefore, these singularities are given by

$$\left\{ z : \prod_{j=i}^{l-1} \Big(1 - (1-\alpha_j)t_0(z)\Big) = 0 \right\}.$$

□

Lemma 7.3. *The singularities of $H(z)$ are given by zeros of*

$$\prod_{i=1}^{l-1} \Big(1 - (1-\alpha_i)t_0(z)\Big) = 0 \tag{16}$$

where the dominant singularities are the solutions of

$$t_0(z) = \frac{1}{1-\alpha_1}. \tag{17}$$

Moreover, the order of these singularities is equal to s.

Proof. From Eqs. (12) and (13), $H(z)$ is a linear combination of $G_1(z), \ldots, G_{l-1}(z)$. Therefore, $H(z)$ has a singularity at every singularity of $G_i(z)$, $1 \le i < l$. Note that there are no additional singularities of $H(z)$ which come from $T(z)$ and $R(z)$ because of assumption 3. This proves the first part of the lemma.

From Eqs. (7) and (8), we find $t_0(z) = T(zA(0))$, i.e. $t_0(z)$ is the partial generating function of the length of a vacation with 0 arrivals.

From Pringsheims Theorem, Flajolet and Sedgewick [16], we know that for probability generating functions, at least one dominant singularity lies on the real axis. Moreover, for $x \in \mathbb{R}$, $T(x)$ is an increasing function of x as it is a probability generating function. Since α_i are non decreasing in i,

$$\min_i \left(\frac{1}{1-\alpha_i}\right) = \frac{1}{1-\alpha_1}$$

and dominant singularities on the real axis are given by

$$\begin{aligned} & \min\left\{ z : t_0(z) = \frac{1}{1-\alpha_i}, 0 < i < l, z \in \mathbb{R} \right\} \\ & = \min\left\{ z : t_0(z) = \frac{1}{1-\alpha_1}, z \in \mathbb{R} \right\} \end{aligned}$$

Moreover, since all the dominant singularities have the same modulus (equal to the radius of convergence), all the dominant singularities are given by the solutions of $(1-\alpha_1)t_0(z) = 1$.

From assumption 2, we know that

$$\alpha_1 = \alpha_2 = \ldots = \alpha_s < \alpha_{s+1} \le \alpha_{s+2} \le \ldots \alpha_{l-2} \le \alpha_{l-1}$$

Using this relation, the singularities of $H(z)$ can be rewritten as

$$\big(1-(1-\alpha_1)t_0(z)\big)^s \prod_{i=s+1}^{l-1}\big(1-(1-\alpha_i)t_0(z)\big)=0$$

Therefore, the order of each dominant singularity equals s. □

Note that if we did not have assumption 1, Eq. (17) would still be valid with α_1 replaced by $\alpha_{\min}=\min_i \alpha_i$.

Lemma 7.4. *For $1\le i\le s$, $G_i(z)$ has a singularity at σ_k of the order of $s-i+1$ where σ_k is the k^{th} dominant singularity of $H(z)$.*

Proof. From Lemma 7.2, singularities of $G_i(z)$ are solutions of

$$\prod_{j=i}^{l-1}\big(1-(1-\alpha_j)t_0(z)\big)=0$$

which can be rewritten as

$$\prod_{j=i}^{s}\big(1-(1-\alpha_j)t_0(z)\big)\prod_{j=s+1}^{l-1}\big(1-(1-\alpha_j)t_0(z)\big)=0 \tag{18}$$

We know that $\alpha_1=\alpha_2\ldots=\alpha_s$. Therefore Eq. (18) can be rewritten as

$$\big(1-(1-\alpha_1)t_0(z)\big)^{s-i+1}\prod_{j=s+1}^{l-1}\big(1-(1-\alpha_j)t_0(z)\big)=0$$

Moreover, from Lemma 7.3, if σ_k is a dominant singularity of $H(z)$, $1-(1-\alpha_1)t_0(\sigma_k)=0$. Therefore, $G_i(z)$ also has a singularity at σ_k of the order of $s-i+1$. □

We now compute the coefficient $Y_k(\sigma_k)$ which defines the behavior of the generating function $H(z)$ around the k^{th} dominant singularity σ_k.

Theorem 7.5. *The tail distribution of Age_ν is given by*

$$Pr(Age_\nu=n)\sim\sum_{k=1}^{M}Y_k(\sigma_k)\frac{n^{s-1}}{(s-1)!\sigma_k^n} \tag{19}$$

where M is the total number of zeros of $t_0(z)=\frac{1}{1-\alpha_1}$ with smallest norm and

$$Y_k(\sigma_k)=\frac{r_1(\sigma_k)[t_1(\sigma_k)]^{s-1}}{[T'(\sigma_k A(0))A(0)]^s}\left(T(\sigma_k)-1-\sum_{j=s+1}^{l-1}(1-\alpha_j)t_{j-s}(\sigma_k)\right.$$
$$\left.+\sum_{j=s+1}^{l-1}(1-\alpha_j)t_{j-s}(\sigma_k)G_j(\sigma_k)\right). \tag{20}$$

Proof. Since σ_k is a dominant singularity of $H(z)$ of order s, $Y_k(\sigma_k)$ (see Theorem 7.1) is given by

$$Y_k(\sigma_k)=\lim_{z\to\sigma_k}(\sigma_k-z)^sH(z)$$
$$=\lim_{z\to\sigma_k}(\sigma_k-z)^s\left(R(z)+\sum_{i=1}^{l-1}(1-\alpha_i)r_i(z)(G_i(z)-1)\right)$$

From Lemma 7.4, we know that for $1\le i\le s$, $G_i(z)$ has a singularity of order $s-i+1$ at σ_k. Moreover, $G_i(z)$, $i>s$, does not have a singularity at σ_k. Therefore, the expression for $Y_k(\sigma_k)$ simplifies to

$$Y_k(\sigma_k)=\lim_{z\to\sigma_k}(\sigma_k-z)^s(1-\alpha_1)r_1(z)G_1(z)$$
$$=(1-\alpha_1)r_1(\sigma_k)\lim_{z\to\sigma_k}(\sigma_k-z)^sG_1(z) \tag{21}$$

Therefore, to compute $Y_k(\sigma_k)$, we need to compute the c_{G_1}, the constant which defines the behavior of $G_1(z)$ around the dominant singularity σ_k.

$$c_{G_1}=\lim_{z\to\sigma_k}(\sigma_k-z)^sG_1(z)$$

Using Eq. (13), we can write

$$G_i(z)=\frac{T(z)-\sum_{j=i}^{l-1}(1-\alpha_j)t_{j-1}(z)+\sum_{j=s+1}^{l-1}(1-\alpha_j)t_{j-1}(z)G_j(z)}{1-(1-\alpha_s)t_0(z)}$$
$$+\frac{(1-\alpha_s)\sum_{j=i+1}^{s}t_{j-1}(z)G_j(z)}{1-(1-\alpha_s)t_0(z)}. \tag{22}$$

To compute c_{G_1}, we first compute the coefficient of $\frac{1}{[1-(1-\alpha_s)t_0(z)]^s}$ in Eq. (22) recursively with result

$$(1-\alpha_s)^{s-1}t_1(z)^{s-1}\left(T(z)-\sum_{j=s}^{l-1}(1-\alpha_j)t_{j-s}(z)\right.$$
$$\left.+\sum_{j=s+1}^{l-1}(1-\alpha_j)t_{j-s}(z)G_j(z)\right)$$

Note that while computing c_{G_1} the terms other than the coefficient of $\frac{1}{[1-(1-\alpha_s)t_0(z)]^s}$ will become 0 under the limits $z\to\sigma_k$. Therefore, the coefficient c_{G_1} is given by

$$c_{G_1}=\lim_{z\to\sigma_k}(\sigma_k-z)^sG_1(z)$$
$$=\lim_{z\to\sigma_k}(\sigma_k-z)^s\frac{(1-\alpha_s)^{s-1}[t_1(z)]^{s-1}}{[1-(1-\alpha_s)t_0(z)]^s}\left(T(z)\right.$$
$$\left.-\sum_{j=s}^{l-1}(1-\alpha_j)t_{j-s}(z)+\sum_{j=s+1}^{l-1}(1-\alpha_j)t_{j-s}(z)G_j(z)\right)$$

Using L'Hôpital's rule s times in the above expression and using the relation $(1-\alpha_s)t_0(\sigma_k)=1$ we get

$$c_{G_1}=\frac{[t_1(\sigma_k)]^{s-1}}{(1-\alpha_s)[T'(\sigma_kA(0))A(0)]^s}$$
$$\times\left(T(\sigma_k)-1+\sum_{j=s+1}^{l-1}(1-\alpha_j)t_{j-s}(\sigma_k)\big(G_k(\sigma_k)-1\big)\right), \tag{23}$$

where we also used $t_0(z)=T(zA(0))$. Therefore, from Eq. (21)

$$Y_k(\sigma_k)=(1-\alpha_1)r_1(\sigma_k)c_{G_1}$$
$$=\frac{r_1(\sigma_k)[t_1(\sigma_k)]^{s-1}}{[T'(\sigma_kA(0))A(0)]^s}$$
$$\times\left(T(\sigma_k)-1+\sum_{j=s+1}^{l-1}(1-\alpha_j)t_{j-s}(\sigma_k)\big(G_j(\sigma_k)-1\big)\right).$$

Therefore, from Theorem 7.1, for large n, we can approximate the tail of Age_ν as

$$Pr(Age_\nu=n)\sim\sum_{k=1}^{M}Y_k(\sigma_k)\frac{n^{s-1}}{(s-1)!\sigma_k^n}$$

□

Note that, to compute the tail distribution, we need to compute $r_1(\sigma_k)$, $t_1(\sigma_k)$, $\ldots t_{l-1-s}(\sigma_k)$, $G_{s+1}(\sigma_k),\ldots,G_{l-1}(\sigma_k)$. Using Eq. (13), one can recursively compute $G_{s+1}(\sigma_k),\ \ldots,G_{l-1}(\sigma_k)$ starting from $G_{l-1}(\sigma_k)$. Moreover, from Eq. (6) we have,

$$r_1(\sigma_k)=\left.\frac{\partial S(y,z)}{\partial y}\right|_{\{y=0,z=\sigma_k\}}$$
$$=\frac{A'(0)\big(T(\sigma_kA(0))-T(A(0))\big)}{1-T(A(0))}.$$

Similarly, to compute $t_1(\sigma_k),\ldots,t_{l-1-s}(\sigma_k)$ use Eq. (8), for example,

$$t_1(\sigma_k)=\left.\frac{\partial W(y,z)}{\partial y}\right|_{\{y=0,z=\sigma_k\}}$$
$$=T'(\sigma_kA(0))\sigma_kA'(0).$$

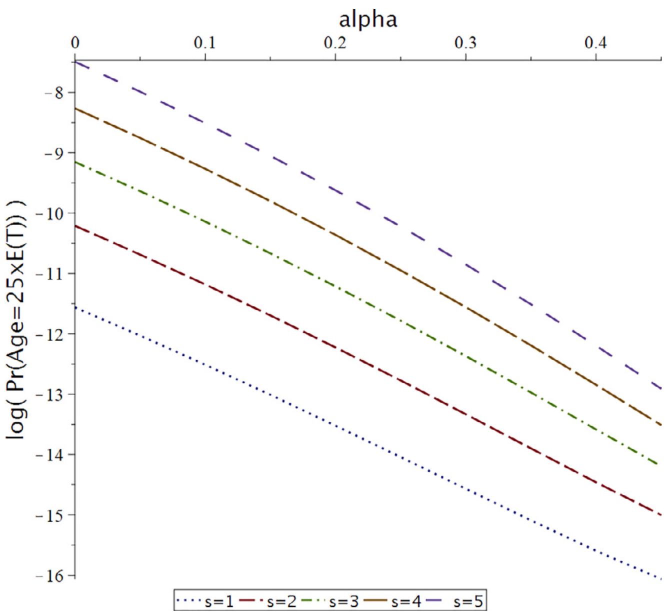

Fig. 4. Impact of restarting probability α_1 on tail of Age_ν, Arrival rate = 0.1, $l = 20$.

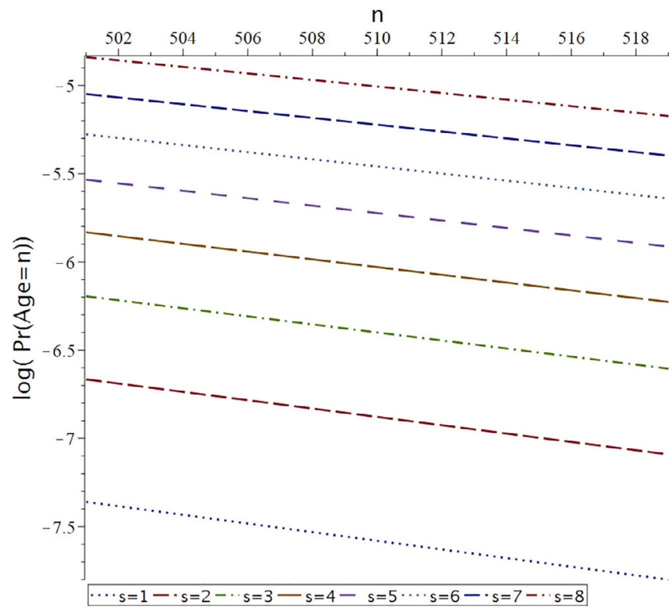

Fig. 5. Tail of Age_ν with increasing n, Arrival rate = 0.1, $l = 20$.

8. Backup systems with time trigger mechanism

Some backup systems have a time trigger mechanism to initiate a backup period if the backup-off period becomes too long. Let us assume this timer is of length ψ, then the maximum age of data is given by

$$Age = \min(Age_\nu, \psi) \tag{24}$$

Therefore, the age of data for such backup systems can be computed from our analysis using Eq. (24).

Note: We assume that the timer starts when the first packet arrives during the vacation period. It can be defined from beginning of the off-period or start of the vacation in which the packet arrived. In those cases, Eq. (24) would have to be modified accordingly. Similarly, we assume that ψ is a constant; however, it can be a stochastic variable.

It is important to note that, for such backup systems to work efficiently, ψ has to be chosen wisely. On one hand, a small value of ψ would result in backup operations being triggered too often. While on the other hand, a large value of ψ defeats the whole purpose of a time trigger mechanism, which is to avoid long backup off periods. Therefore, it is important for such backup systems to choose an optimal timer length ψ. To measure this efficiency, we define a new performance measure, the frequency of backups.

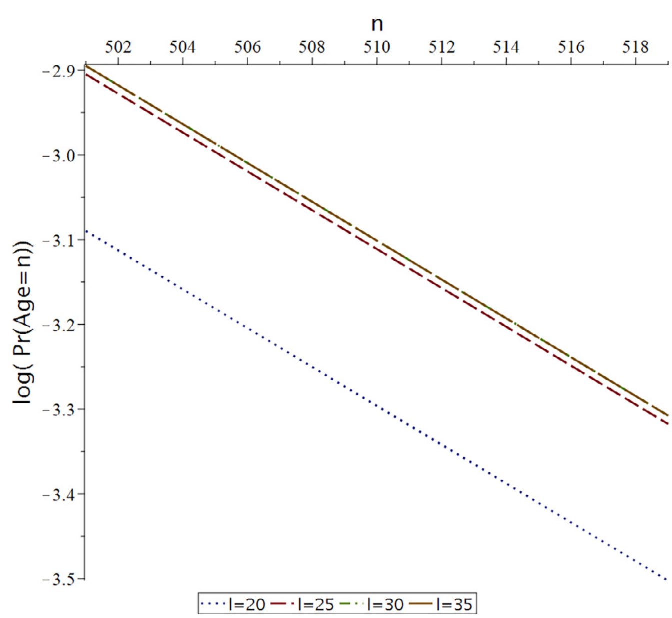

Fig. 6. Tail of Age_ν with increasing n, Arrival rate = 0.1, $s = 1$.

8.1. Frequency of backups

For any general event repeating at a period of P, its frequency is defined as $F = \frac{1}{P}$. Since the backup time itself is typically very short as compared to backup-off periods, we define the frequency of backup operations as

$$Freq = \frac{1}{E(Age)} = \frac{1}{E(\min(Age_\nu, \psi))} \tag{25}$$

To be able to select the optimal trigger parameter, it is important to study the effect of ψ on both the frequency and Age of data. The QoS of backup processes is defined in terms of the RPO. That is, a service provider would guarantee an RPO to its user, such as 15min-24hrs guaranteed by TEKLINKS [14] for different costs, as mentioned in Section 1. Using Eqs. (24) and (25), we can compute the moments of Age and the frequency for such backup mechanisms.

$$\begin{aligned} E(Age) &= E(min(Age_\nu, \psi)) \\ &= \psi \times Pr(Age_\nu > \psi) + E(Age_\nu | Age_\nu \le \psi) Pr(Age_\nu \le \psi) \\ &= \psi Pr(Age_\nu > \psi) + \sum_{n=1}^{\psi} n Pr(Age_\nu = n) \\ &= \psi Pr(Age_\nu > \psi) + E(Age_\nu) - \sum_{n=\psi+1}^{\infty} n Pr(Age_\nu = n) \\ &= E(Age_\nu) + \sum_{n=\psi+1}^{\infty} (\psi - n) Pr(Age_\nu = n) \end{aligned}$$

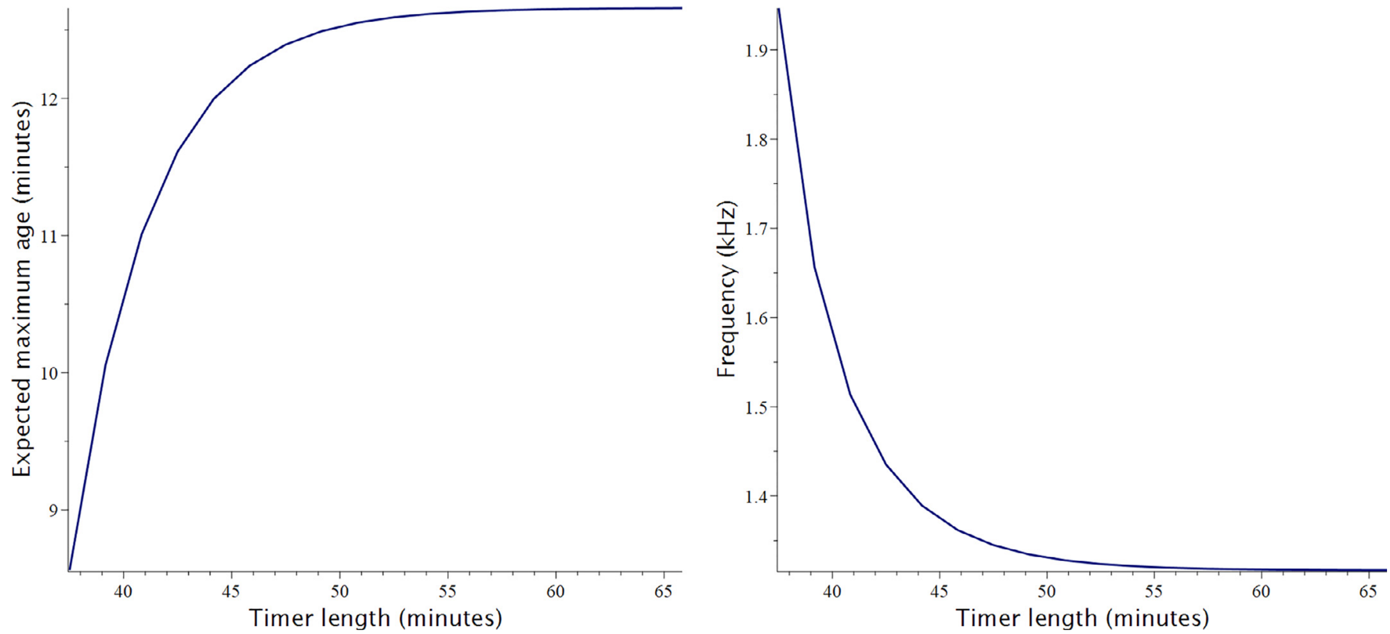

Fig. 7. Maximum age of data and frequency of backup with change in timer length. $\lambda = 0.07$, slot length = 5 second, $\alpha_i = \frac{i}{30}$, and $l = 20$.

$E(Age_\nu)$ can be computed from Eq. (14). Assuming ψ is reasonably large, using the Theorem 7.5, we can approximate the $E(Age)$ as

$$E(Age) \approx E(Age_\nu) + \sum_{n=\psi+1}^{\infty} (\psi - n) \sum_{k=1}^{M} Y_k(\sigma_k) \frac{n^{s-1}}{(s-1)!\sigma_k^n}$$
$$= E(Age_\nu) + \sum_{k=1}^{M} Y_k(\sigma_k) \sum_{n=\psi+1}^{\infty} (\psi - n) \frac{n^{s-1}}{(s-1)!\sigma_k^n}$$

Using Eq. (25), the frequency is therefore given by

$$F \approx \left[E(Age_\nu) + \sum_{k=1}^{M} Y_k(\sigma_k) \sum_{n=\psi+1}^{\infty} (\psi - n) \frac{n^{s-1}}{(s-1)!\sigma_k^n} \right]^{-1} \qquad (26)$$

9. Numerical evaluation

In this section, we analyze the performance of the system using the numerical example used in Saxena et al. [12]. Since it is well known that the arrival distribution can have a heavy tail (see eg. Liebeherr et al. [17]) we assume that the number of arrivals in a slot follows a Mixed distribution of Poisson and Power Law. The vacation lengths have a discrete uniform distribution.

$$A(z) = pe^{\lambda(z-1)} + (1-p)\frac{L_\gamma(z)}{L_\gamma(1)}, \qquad p = 0.999$$

$$L_\gamma(z) = \sum_{k=a}^{\infty} \frac{z^\gamma}{k^\gamma}, \qquad a = 25, \quad \gamma = 2.5$$

$$T(z) = \frac{1}{5}\left(z^{\nu-2} + z^{\nu-1} + z^{\nu} + z^{\nu+1} + z^{\nu+2}\right) \qquad (27)$$

Using **Maple**, we find the solutions of the Eq. (17). We find that the generating function $H(z)$ has only 1 dominant singularity and plot $Age_\nu(n) = Pr(Age_\nu = n)$ with varying system parameters[1].

Using the tail distribution of Age_ν, F and $E(Age)$ can be computed easily using Eq. (26). Therefore, analyzing the tail distribution of Age_ν is sufficient. To do this analysis with different restarting probabilities, we vary α_s between 0 and 0.5, for values of s between 1 and 5, while keeping the remaining α_i for $i > s$ fixed. That is,

$$\alpha_1 = \alpha_2 = \ldots = \alpha_s = \alpha$$
$$\alpha_i = 0.6 + 0.4 \times \frac{i}{l}, \quad \forall i > s.$$

9.1. Observations and key insights

A system administrator would desire to keep the age of data as low as possible. However, increasing the frequency of backups would come with an increased economic cost. We are able to exactly compute the generating function of Age_ν as well as the tail distribution. We observe that the tail distribution is sensitive to the change in model parameters.

In the Figs. 4–6, we plot the tail probabilities of Age_ν with respect to the change in the model parameters α_1, s and l. In Fig. 7, we compute the expected maximum age, $E(Age)$ and frequency (F) for backup operations with time trigger mechanism.

- Choice of the smallest restarting probability α_1 needs to be smart. A smaller value of α_1 would give a dominant singularity of smaller modulus from Eq. (17). Since σ_k^n is in the denominator of Eq. (19), it would lead to larger tail probabilities. This is also observed in Fig. 4.
- The order of dominant singularities s is directly determined by the choice of restarting probabilities α_j, $0 < j < l$. Since in the Eq. (19) of tail probability distribution, n^s appears in the numerator, a smaller value of s leads to smaller tail probabilities. This is also observed in Fig. 4 and 5.
- A larger restarting threshold l results in a higher value of $Y_k(\sigma_k)$ from Eq. (20). Therefore, tail probabilities are higher for higher restarting threshold. This is also observed in Fig. 6.
- For backup operations with time trigger mechanism, increasing the timer length drastically decreases the backup frequency. As shown in example in Fig. 7, selecting timer length of 45min instead of 40min reduces the backup frequency by approximately 15%. Users can define their cost functions based on $E(Age)$, F and other performance measures computed in Saxena et al. [12]. Minimizing this cost function would give the optimal parameters, including the optimal trigger length.

[1] The code used for tail distribution analysis is available at https://github.com/saxe405/tail_distribution.

10. Conclusions

In our previous paper, we modeled data backup systems as a batch service queueing model with vacations and restarting probabilities. We obtained various performance measures such as the moments of the backlog size and the mean value of the age of data at the beginning of a backup. The age of data is one of the most important quantities in backup systems because it captures the risk of data loss in case of a disaster. In this paper we have computed the generating function of the age of data at the beginning of the backup. As a consequence, we can now compute higher order moments and analyze the tail of this distribution by computing the dominant singularities of the generating function. The choice of restarting probabilities determines the value of these singularities as well as their order. We have analyzed the behavior of the tail of the distribution of Age_v for different values of model parameters. We are able to precisely compute the tail distribution characterized by the model parameters. We also use this analysis to compute and analyze the performance measures of backup systems with time trigger mechanism.

Declaration of interests

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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