



3rd GLOBAL CONFERENCE on BUSINESS, ECONOMICS, MANAGEMENT and TOURISM,
26-28 November 2015, Rome, Italy

Optimal Delays, Safe Floats, or Release Dates? Applications of Simulation Optimization in Stochastic Project Scheduling

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Abstract

In the field of project scheduling, the application of activity floats is often the subject of researches that seek to maximize the net present value (NPV) of project networks. However, NPV-based scheduling in stochastic projects may result in a conflict with the primary objective of scheduling, i.e. minimizing the makespan. The main objective of this study is to improve the financial gain while respecting the makespan (i.e. applying safe floats) in stochastic projects. Next, instead of the traditional use of float which prescribes a fixed delay in the start of activity in any condition, we use release dates as the by-product of above-mentioned method to reduce the variation in time and NPV of the project with less computational effort. The project models are simulated in SAS Simulation Studio. Response Surface Methodology (RSM) is used to design the experiments, interpret the results and predict the solution. The focus is on the temporal analysis of stochastic networks, and no resource constraint is considered.

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Peer-review under responsibility of the Organizing Committee of BEMTUR- 2015

Keywords: Simulation Optimization; Stochastic Project Scheduling; Response Surface Methodology

1. Introduction

Delaying the start of activities postpones the time a cost incurs, and consequently leads to higher net present value of the project which indicates the present value of future money. Other benefits of such delays (e.g. the reduction in holding expenses) may also apply depending on the characteristics of the real life situation. As we will see in the

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third example, the true application of delays reduces the variation in the schedule. Managing a project usually needs coordination among activities, so variation is not very welcomed.

Jorgensen and Wallace (2000) truly discuss that, according to Jensen's inequality, planning methods based on the use of average numbers may lead to underestimation of project costs and durations. So, delaying an activity by a float calculated based on the Pert/CPM procedure may lead to postponement of the project completion. The first two examples, borrowed from the pioneering work of Buss and Rosenblatt (1997) on the subject, will show that NPV-based scheduling may also postpone the project completion. Another issue about NPV-based scheduling is that any activity is a potential candidate for optimizing the objective, even if the activity is critical.

The underlying principle for NPV maximization objective considered here is still the same: delay the expenses as far as possible to capture the time value of money. But we use time-based delays (or safe floats) to remove the possibility of increasing the project duration. In the third example we argue why safe float needs less computational effort. Another issue brought up in that example is how to use safe floats to reduce the variation in the project.

2. Literature Review

The first article on the problem of maximizing the expected present value of Markovian projects-PERT networks with exponentially distributed activities-by Buss and Rosenblatt (1997) brought up an interesting area for the researchers in the field of stochastic project scheduling. The method the authors used (continuous-time Markov decision chain) is still the basis of some recent studies in this field (Creemers, Leus & Lambrecht, 2010; Sobel, Szmerekovsky & Tilson, 2009)

Buss (1995) applied Response Surface Methodology to determine the optimal delays for a stochastic (and not necessarily Markovian) project network. The method we use to calculate safe floats and release dates is the same. The authors are not aware of any other application of designed simulation experiments to optimize project scheduling objectives.

Some researchers studied the float of activities without financial considerations. Tavares, Ferreira and Coelho (1998) set the release date of each activity to its earliest start time plus a fraction of its total float based on the Pert/CPM procedure. Monte Carlo simulation is used to select the optimal fixed fraction for all project activities.

Gong and Rowings (1995) introduce an analytical approach for assessing the change of the expected time of merge events or total project duration with the changes in float use of noncritical activities. They sequentially increase the float of an activity until the disruptive consequence of an activity delay emerges.

Cho and Yum (2004) suggest estimating the criticality index of activity i by means of Monte Carlo simulation, or for a large-sized network, Taguchi orthogonal array experiment. Then, they fit a nonlinear model to predict the criticality index of activity and expected project duration when the mean duration of activity is changed. Delaying an activity has the same effect of change in the mean duration of activity, i.e. it increases the mean completion time of activity while its distribution remains unchanged.

Some address the expected NPV maximization problem for projects with multiple scenarios and their corresponding probabilities for activity durations and cash flows. They suggest an optimal processing time policy for the project activities which prescribes an activity to be started as early as possible in the realized scenario right after (scenario-independent) target processing time (Wiesemann, Kuhn & Rustem, 2010). In this article, the same policy for target release time is advocated, but we do not confine ourselves to finite number of scenarios. There are certainly other valuable researches in stochastic project management, but we are not going to present a thorough literature review on the subject.

3. Theories and Methodology

3.1. Optimal Delays

The concept of optimal delays refers to amount of delays in the start of activities which maximizes the net present value (NPV) of a project. All the cash flows (expenses and incomes) that occur during the execution of activities are discounted towards the start time of the project. The present value of cash flows is calculated using a discount factor

$\beta = (1+r)^{-n}$ which denotes the present value of a dollar to be received at the end of period n . The discount rate r is also called the opportunity cost of capital. The following formula calculates the net present value of the project:

$$npv = \sum_{t=1}^{\infty} \frac{C_t}{(1+r)^t} \quad (1)$$

where C_t is the cash flow at the end of period t . If the net present value is a random variable (e.g., in a stochastic project), its expected value shows the profitability of the project and the variance in NPV may be regarded as a measure of financial risk.

3.2. Safe Floats

The concept of safe float is only concerned with merge events in the network. A merge event is an event connected with at least two activities. The float use (delay) in the merging activities should not be higher than the safe range; or otherwise it will change the expected time of the merge event.

According to Gong and Hugsted (1993), the correlation among merging paths is related to the closest cross event. Therefore, separating the preceding paths of the closest cross event, if the mean variance of cross event time is known, should not influence the forward calculation of the network. At the merge event, the earliest finish time and time variances of the merging paths are equal to the earliest finish time and time variances of merging activities. Another important feature of safe float calculation is the Associative law: In a merging processing of n merging paths, the activities can be merged one by one. Since no interaction among activities needs to be considered, it highly reduces the computational requirement of safe float calculation. For more details on the procedure used in the safe floats, one should refer to Gong and Rowings (1995).

3.3. Release Dates

The concept of release date of an activity can be thought of as a predecessor from the dummy start node to the activity. As we mentioned, the earliest finish time and time variances of the merging paths are equal to the earliest finish time and variances of merging activities. Using the notations in Elmaghraby, Ferreira, and Tavares (2000) with some modifications, we have:

$$\pi(k) = \mu_k + \zeta_{\alpha} \sqrt{\text{var}(k)} \quad (2)$$

where $\pi(k)$ denotes the expected finish time of merging activity k to guarantee completion in time with probability α , and ζ_{α} denotes the critical value or α -percentile of the random variable. If $r(k,t)$ denotes the release time for the activity k that satisfies the confidence level of completion by time t , we have:

$$r(k,t) = t - \pi(k) \quad (3)$$

In release date setting problems, there is a desire to schedule the start time of each activity such that the due time is met with the specified confidence α , and the financial gain is maximized (Elmaghraby, Ferreira & Tavares, 2000). But, asking the managers to provide these confidence levels, or setting a given confidence level for all activities in the project is not necessarily optimal. As it is shown in the example 3, optimal release dates are by-product results of the safe float calculations, which obviate the need to ask managers for the confidence levels.

3.4. Simulation Optimization using Response Surface Methodology

3.4.1. Simulating the Stochastic Project Network

Projects are modeled in SAS Simulation Studio to calculate the distribution of time and net present value of the project under different factor settings. In the modeling phase of simulating projects, we respected the usual constraints in project scheduling problems: the start times of activities are subjected to precedence constraints, and the dummy start activity is forced to begin at time zero. For more details on the simulating procedure, refer to SAS Institute (2009).

3.4.2. Designing the Simulation experiments

The simulated model needs a complementary method to design the search points; otherwise the experiment is just a blind guess, with a result that hardly could be interpreted. Response surface methodology, as a popular classic process optimization tool that could also be used in simulation experiments, helped us to analyze the results and find the optimal settings. SAS Simulation Studio was selected as the preferred simulation software, because it easily interacts with JMP Statistical Discovery to receive the experiment points and then pass the simulation results back to the JMP program for analysis (SAS Institute, 2009).

In most simulation-based optimization problems, the form of the relationship between the response and the controllable factors is unknown. RSM usually employs a low-order polynomial in the region of experiment to find a suitable approximation for the relationship between y and the set of factors. The least squares method is used to estimate the parameters in the approximating polynomials. The response surface analysis is then performed using the fitted surface. If the fitted surface is an adequate approximation of the true response function, then analysis of the fitted surface is almost equivalent to analysis of the actual system. The main objective of RSM is to determine the optimum operating conditions for the system. More extensive presentations of RSM can be found in Myers, Montgomery and Anderson-Cook (2011).

In what follows, we discuss how to design RSM experiments to guide the simulation. The *predicted* values of the method are compared with analytical results and *actual* values. By actual values we mean the response of the simulated model at the predicted solution.

4. Applications of Response Surface Methodology

4.1. Determining Optimal Delays by RSM

At first, we applied our method to the two-activity project network from Buss and Rosenblatt (1997). Activity durations are exponentially distributed. Upon completion of activity i , a fixed cost of v_i is incurred, and upon completion of the project, revenue R is received. The objective is to maximize the expected present value of the project, with a discount rate of $r = 0.01$ per time unit (in this case, a month). The expected durations and respective costs of activities are shown in Figure 1. The authors argue that EPV for early start schedule (with no delay) is \$1908.26. When activity 1 is delayed by optimal amount of $d_1 = 12$, EPV increases to \$2536.61.

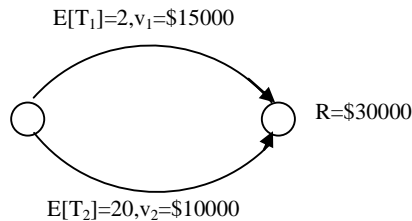


Fig. 1. Two-activity project network, adapted from Buss and Rosenblatt (1997)

The first step in RSM is to find an approximation of the functional relation between NPV and the only independent variable in this project (delay of activity 1).

The solution of the fitted model is $d_1 = 12.33$, which is very close to the optimal amount of 12. The predicted expected values of project duration and NPV at the solution are 23.98 and \$2573.06 respectively. After running the simulation with $d_1 = 12.33$, we earn the actual values of 24.16 and \$2542.79, indicating that the predicted values sound like a good approximation of the actual amounts. According to simulation results, six months (the difference between PERT float and optimal delay) is not a safe lead time for noncritical activity 1, since the project makespan is increased by about four months. In the next example we try to compromise between the two objectives of maximizing the financial gain and minimizing the duration of project.

4.1.1. Balancing the Two Conflicting Objectives

The five-activity project network from Buss and Rosenblatt (1997) is shown in Figure 2. Authors argue that expected NPV for early start schedule (using the discount rate of $r = 0.01$) is \$4181.31. Optimal delays are $d_1 = 1.729$ and $d_3 = 5.911$, which result in an expected NPV of \$5253.24.

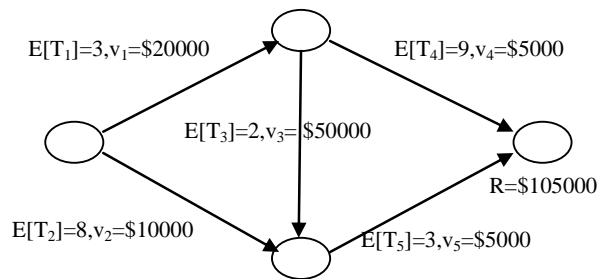


Fig. 2. Five-activity project network, adapted from Buss and Rosenblatt (1997)

As the authors suggest, activity 1 and activity 3 are candidates to be delayed. To fit the second order function for two control factors, an eight point design is implemented to fit a second order polynomial which results in the solution $d_1=1.99$ and $d_3=5.89$, very close to the optimal delays $d_1 = 1.729$ and $d_3 = 5.911$. The predicted values of project duration and NPV at solution are 20.2 and \$5341.88, and their actual values are 20.22 and \$5317.83 respectively. Again, the project duration increases by four months.

JMP is capable of setting the desirability for each response that works like a weighting function. Setting the same desirability for Time and NPV objectives results in the solution $d_1 = 0$, and $d_3 = 5.01$ with predicted expected NPV=\$5232.87 and Time = 18.22. Actual values are NPV=\$5217.55 and Time = 18.2 months.

It is worth mentioning that the risk associated with any random variable can be expressed in terms of the dispersion of the variable around the expected value. Using the expected values for project selection and budgeting is suited for risk neutral managers. A known alternative to expected values are utility functions which guide managers to choose the best projects among the alternatives, according to their risk preferences. These functions also have the capability to combine the profitability and financial risk measure with other decision making criteria in the form of multi-attribute utility functions; which are research fields for future studies.

4.2. Determining the Safe Floats by RSM

In this example, we use designed simulation experiments as an alternative to the analytical approaches for calculating the safe floats of merging activities. The third example is adapted from Gong and Rowings Jr. (1994). Activity durations are normally distributed with means and standard deviations placed below each arc in Figure 3.

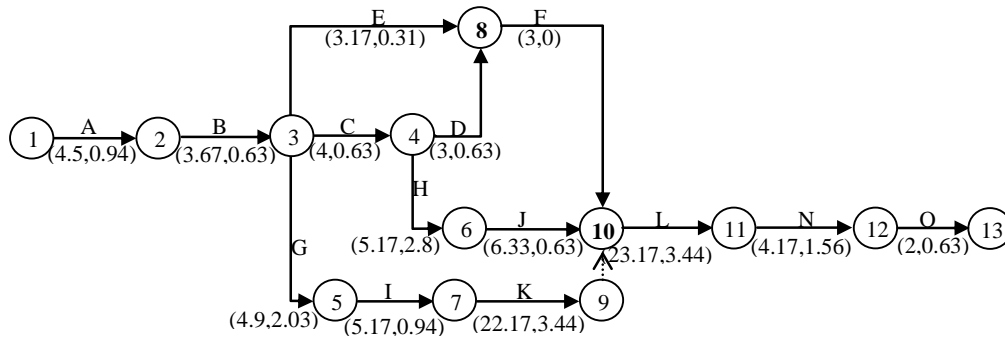


Fig. 3. The project network, adapted from Gong and Rowings (1994)

There are two merge events in the network: merge event 8 with an expected time of 15.17, and merge event 10 with an expected time of 40.41. Delays in merging activities should not lead to a critical change in the expected time of the merge event. The Pert/CPM based float times for activities F, J, and E are 22.24, 16.74, and 3.83 respectively. According to the authors, the safe floats for activities F, J, and E are 18, 10, and 2 weeks. The simulation results show that expected times of merge events as the result of safe float use (delay) change to 15.19 for merge event 8, and 40.9 for merge event 10.

According to Gong and Hugsted (1993), instead of all possible interactions in an NPV maximization problem, safe float calculation is only concerned with merge events and their respective merging activities. No interaction among factors needs to be considered, so the safe floats of activities are calculated in isolation from the levels of other factors. RSM results match the safe floats of the analytical approach, and the predicted values of fitted Time function almost perfectly match the actual ones.

4.3. Optimal Delays or Safe Floats?

To apply the RSM for finding NPV-based delays in the sample project of Gong and Rowings Jr. (1994), hypothetical costs of \$10,000 upon completion for each activity, a payment of \$300,000 at the end of project, and a discount rate $r = 0.01$ per unit time are added to project.

Direct application of RSM resulted in highly variable predicted results. Optimal delays of NPV-based procedure ($dE = 0.43$, $dD = 2.8$, and $dH = 2.02$) result in actual expected values $NPV = \$42,023$ and duration = 69.64, far away from predicted values.

For a deterministic problem, the recursive procedure of Vanhoucke, Demeulemeester, and Herroelen (2001) determines how to use activity floats to maximize the NPV of a project. Applying the safe floats we acquired in the prior example to the recursive procedure, and treating the problem as a deterministic one leads to delays $dE = 20$, $dC = 10$, $dD = 8$, and the consequent expected values of \$47439 for NPV and 70.02 for duration. Less computational effort, better results, and respecting the makespan are the advantages of safe float based delays in this example.

4.3.1. Release Dates or Fixed Delays?

In safe float based delays, we degrade the stochastic problem to a deterministic project scheduling problem. Optimal delays and safe floats are static approaches; they suggest delaying the start of succeeding activities irrespective of the realization time of preceding events, which is not very sensible. To take the occurrence time of preceding events into account, we set the latest start times (latest finish time minus the expected duration) of activities determined by safe float procedure as the release date of activities. So, release date of non-critical activities E, C, D, H, J, and F are 28.17, 18.17, 30.17, 22.17, 27.34, and 33.17.

Release date minus the realization time of the preceding event gives the amount of delay for each activity (zero, if negative). For example, according to simulation results, the realization time of event 3 ranges from 3.97 to 12.68. In the realization time 3.97, release date of E delays its start time for 24.2, and release date of C delays its start time for 14.2 time units. If the ready time of an activity is close to the release date, the start of the activity delays less, and vice versa.

There's no need for recursive procedure of Vanhoucke, Demeulemeester, and Herroelen (2001) in the application of safe float based release date. Substituting delays with release dates significantly reduces the variation of non-critical activities. This feature is more appreciated in situations with earliness and tardiness penalties. But, improvements in the whole project are not very tangible in this example; release date policy leads to expected project duration of 70.24 and expected project NPV of \$47,227. This is because the mean and variation of a merge event does not change in cases where there is a dominant critical merging activity.

5. Conclusion

The purpose of this study is a critical review of setting optimal delay in project scheduling. We suggest using release dates, instead of a fixed amount of delay in the start of activity in any condition, not only to lower the variation of time and NPV of the project, but also to reduce the computational demands and eliminate the possibility of due date violations.

We are not prescribing RSM for all projects. This method also has its drawbacks. For example, safe float technique is recognized as a technique for the bias correction, and because of the limited magnitude of the bias, may not have very significant results in practice. Undertaking computationally demanding designed simulation experiments to set release dates seems to be economically justifiable only when deviation from schedule costs high.

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