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New Formulations for the Orienteering Problem

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Abstract

Problems associated with determining optimal routes from one or several depots (origin, home city) to a set of nodes (vertices, cities, customers, locations) are known as routing problems. The Traveling Salesman Problem (TSP) lies at the heart of routing problems. One of the new variants of the TSP is named as TSP with Profits where the traveler must finish its journey within a predetermined time (cost, distance), by optimizing given objective. In this variant of TSP, all cities ought to not to be visited. The Orienteering Problem (OP) is the most studied case of TSP with Profits which comes from an outdoor sport played on mountains. In OP, traveler gets a gain (profit, reward) from the visited node and the objective is to maximize the total gain that the traveler collects during the predetermined time. The OP is also named as selective TSP. In this paper, we present two polynomial size formulations for OP. The performance of our proposed formulations is tested on benchmark instances. We solved the benchmark problems from the literature via CPLEX 12.5 by using the proposed formulations and existing formulation. The computational experiments demonstrate that; (1) both of the new formulations over estimates the existing one; and (2) the proposed formulations are capable of solving all the benchmark instances that were solved by using special heuristics so far.

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1. Introduction

Travelling Salesman Problem (TSP) has many applications in vehicle routing, scheduling, cellular manufacturing, frequency assignment and etc. The TSP lies at the heart of routing and logistics problems. In recent years, a new

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variant of the TSP is seem to be an attractive research area where the objective focus on maximizing the profit (gains, rewards) obtained from the visited nodes. Those types of the TSP's are called as TSP with Profits (Feillet et al., 2005). The Orienteering Problem (OP) is the most studied case of the TSP with Profits which comes from an outdoor sport played on mountains. This problem with an application is defined by Tsiligirides (1984). Golden et al. (1987) handled home fuel delivery problems as the OP. Tourist tour problems are the most attractive applications of the OP. In OP, traveler gets a gain (profit, reward) from the visited node and the objective is to maximize the total gain that the traveler collects during the predetermined time. The OP is also known as the Selective TSP (Laporte and Martello, 1990). In OP, the journey starts at the depot and ends at a given terminal node and all the nodes may not be visited because of the time or distance restriction. While solving OP, we look for finding a path rather than a circuit between specified two points.

The OP is NP-hard; therefore solution approaches are concentrated on developing exact procedure and/or heuristics (Vansteenwegen et al., 2011). Laporte and Martello (1990) and Ramesh et al. (1992) presented procedures to find optimal solutions within a branch and bound methods. Later, a branch and cut procedure appeared in 1998 for solving OP optimally (Fischetti et al., 1998). Recently Archetti et al. (2013) proposed procedures for optimal solutions for the capacity restricted routing problems with profits. There exist many heuristics for the OP. The first heuristic developed by Tsiligirides (1984) and then by Chao et al. (1996). A brief summary of the heuristics for the OP can be found in Vansteenwegen et al. (2011) and improved algorithms can be seen in Chekuri et al. (2012).

A mathematical formulation is presented in the paper by Laporte and Martello (1990) with the name of the Selective TSP. The subtour elimination constraints of this formulation increases exponentially with respect to the number of the nodes of the underlying graph, so this formulation cannot be used directly by optimization software. Vansteenwegen et al. (2011) present a general OP formulation whose subtour elimination constraints increase polynomially. As far as we are aware, this is the only one formulation that can be used directly by an optimizer.

The remarkable improvement in hardware and software technologies and commercial optimization software allow us to solve many mathematical formulations, especially linear programming formulations, directly; therefore, new and user friendly formulations will facilitate to solve routing problems more easily and rapidly by using any optimizer. The main motivation of this paper is to present new linear programming formulations for solving small and moderate sized OP directly by using any optimizer. Our contributions are two folds: (1) We present new integer linear programming formulations for OP with $O(n^2)$ constraints and $O(n^2)$ binary variables. Both formulations are useable directly by any optimizer. (2) We found the optimal solutions of the benchmark instances and observe that most of the best known solutions obtained by heuristics are far away from the optimal values.

The remainder of the paper is organized as follows: In section 2, problem OP is defined more precisely and general formulation of the OP is given, while two new linear programming formulations for OP are presented in section 3. The performances of the proposed formulations are analyzed in section 4. The conclusion and further remarks appeared in section 5.

2. Problem Identification and General Formulation

Orienteering Problem (OP) is defined on a graph G = (V, A) where $V = \{0, 1, 2, \dots, n\}$ is the set of nodes (vertices), {0} is the depot (origin, home city), $\{n\}$ is the terminating node and the remaining nodes are customer nodes. The set $A = \{(i, j): i, j \in V, i \neq j\}$ is the arc (or edge) set. With each arc (i, j) is associated a travel distance or time or cost c_{ij} (which may be symmetric, asymmetric, Euclidean, deterministic, random, etc.). There is a traveler located at the depot. The traveler gains a profit (reward) p_i if the i^{th} node is visited. The journey start from the depot and terminate at the node n and must end within a predetermined time (or cost or distance) C_{max}. The OP consists of determining a path such that the total reward is maximized, given that the distance traveled (or time spent or cost of traveling) by the traveler cannot exceed a given value C_{max}.

Associated with each arc, let x_{ij} be a binary variable equal to 1 if the traveler goes from node i to node j and 0 otherwise. Then, a general (i.e., implicit in a sense) two indexed integer linear programming formulation (ILPF) for the OP may be given as follows:

$$\text{Maximize} \quad \sum_{i=0}^{n-1} \sum_{j=1}^{n} p_i x_{ij} \tag{1}$$

Subject to

$$\sum_{i=1}^{n} x_{0i} = 1 \tag{2}$$

$$\sum_{i=0}^{n-1} x_{in} = 1 \tag{3}$$

$$\sum_{i=0}^{n-1} x_{ij} \leq 1 \qquad , \quad j = 1, 2, \ldots, n-1 \tag{4}$$

$$\sum_{j=0}^{n} x_{ij} \leq 1 \qquad , \quad i = 1, 2, \ldots, n-1 \tag{5}$$

$$\sum_{i=0}^{n-1} x_{ij} = \sum_{i=1}^{n-1} x_{ji} \qquad , \quad j = 1, 2, \ldots, n-1 \tag{6}$$

$$\sum_{i=0}^{n-1} \sum_{j=1}^{n} c_{ij} x_{ij} \leq C_{max} \tag{7}$$

+ Subtour elimination constraints (8)

$$x_{ij} \in \{0,1\}, \qquad \forall (i,j) \in A \tag{9}$$

In this formulation, constraints (2) and (3) are used to allow that the path starts from the depot and end at the terminal node, whereas constraints (4) and (5) are the degree constraints ensuring that each customer may be visited at most once. Constraints (6) are the conservation of the flow constraints, constraints (7) is the time or distance or cost restriction that guarantees that the journey must end before the predetermined value C_{max}. Constraints (8) are the implicit form of the subtour elimination constraints. Those constraints must ensure that the solution contains no illegal subtours. Integrality constraints are given in (9).

Existing polynomial size ILPFs of the routing problems in the literature differs with respect to the subtour elimination constraints (SECs) and/or some side constraints. In order to prevent illegal tour, i.e., eliminate subtours, in most of the studies, additional, i.e. auxiliary, decision variables are defined and then SECs of the formulation are developed. A formulation is named as node-based if the additional decision variables defined on the nodes of the graph and arc-based if the additional decision variables are defined on the arcs of the graph. In the following section, we propose a new node based and a new arc based formulations for OP.

3. New Formulations

In subsequent sections we present two new ILPF for OP.

Node Based Formulation for OP

Let us define the auxiliary variables v_i's as:

v_i: is the auxiliary variables defined as the position (visit) number of the node i if it is visited and arbitrary real value otherwise.

Proposition 1: The following inequalities together with the constraints (2) - (7) are valid initializing and bounding constraints for the OP.

$$v_0 = 1 \tag{10}$$

$$v_i - 2x_{0i} \geq 0 \qquad , \qquad i = 1, 2, \ldots, n-1 \tag{11}$$

$$v_i + (n-3)x_{0i} - x_{in} \le n-1, \qquad i = 1, 2, \ldots, n-1 \tag{12}$$

Proposition 2: The following inequalities together with the constraints (2) - (7) and (10) - (12) are valid subtour elimination constraints for the OP.

$$v_i - v_j + nx_{ij} + (n-2)x_{ji} \le n-1 \quad , \qquad i \ne j\ ;\ i, j=1,2,\ldots,n \tag{13}$$

In accordance with the constraints given in (13), the auxiliary variables of the nodes on the path forms a step function with respect to the visit numbers of the nodes which are on the path, so no illegal subtour can be formed.

In view of the proposition 1 and 2, we propose the following ILPF for the OP as;

F1: Maximize $\sum_{i=0}^{n-1}\sum_{j=1}^{n} p_i x_{ij}$

Subject to
Constraints (2) - (7),
Constraints (9) - (13).

where $x_{ij} = 0$ whenever $c_{0i} + c_{jn} + c_{ij} > C_{max}$ and $c_{ii} = 0$ for all i and the distance matrix is Euclidean. As evident, this formulation has O(n^2) binary variables and O(n^2) constraints.

Arc Based Formulation for OP

We define the flow variables y_{ij}'s as,

y_{ij} is the auxiliary variables defined as the position (visit) number of the node j if it is visited just after the node i and zero otherwise.

Note that, if the arc (i, j) is not on the path then y_{ij} must be equal to zero.

Proposition 3: The following inequalities together with the constraints (2) - (7) are valid initializing and bounding constraints for the OP.

$$y_{0i} - x_{0i} = 0, \qquad i = 1, 2, \ldots, n \tag{14}$$

$$y_{ij} - (n-1)x_{ij} \le 0, \qquad i = 0, 1, 2, \ldots, n-1\ ,\ \ j = 1, 2, \ldots, n \tag{15}$$

$$y_{ij} \ge 0, \qquad \forall (i,j) \in A \tag{16}$$

Proposition 4: The following inequalities together with the constraints (2) - (7) and (14) - (16) are valid subtour elimination constraints for the OP.

$$\sum_{j=1}^{n} y_{ij} - \sum_{j=0}^{n-1} y_{ji} - \sum_{j=0}^{n-1} x_{ji} = 0, \qquad i=1, 2, \ldots, n-1 \tag{17}$$

In view of the proposition 3 and 4, we propose the following arc based formulation for the OP as;

F2: Maximize $\sum_{i=0}^{n-1}\sum_{j=1}^{n} p_i x_{ij}$

Subject to
Constraints (2) - (7),
Constraints (9),
Constraints (14) - (17).

where $x_{ij} = 0$ whenever $c_{0i} + c_{j0} + c_{ij} > C_{max}$ and $c_{ii} = 0$ for all i and the distance matrix is Euclidean. In this formulation, constraint (17) is the subtour elimination constraint, and it guarantees that the solution contains no illegal subtours. As evident, the formulations given above has O(n^2) binary variables and O(n^2) constraints.

4. Computational Analysis

We analyze the performance of our proposed node-based (F1) and arc-based (F2) formulations by solving test problems with the linear programming solver, CPLEX 12.5. We look at linear programming relaxations and especially, CPU for each instance. These formulations are compared to the formulation (VSO) of Vansteenwegen et al. (2011) from the literature.

The experiment is based on two available data sets of benchmark instances taken from the literature (http://www.mech.kuleuven.be/en/cib/op). Benchmark instances are as follows:

- Data set 1: The most used OP instances by Tsiligrides (1984) consist of 49 instances as three data set named TS21, TS32 and TS33. 21, 32 and 33 indicate the number of nodes.
- Data set 2: The OP instances by Chao et al. (1996) consist of 40 instances as two data set named CH64 and CH66. 64 and 66 indicate the number of nodes.

All OP test instances were solved separately by VSO, F1 and F2 formulations using CPLEX 12.5 solver on Intel Core2 Duo processors of 2.66 Ghz clock speed with 2.00 GB of RAM computer. For each OP-instance, a time limit of 7200 seconds was imposed.

Optimal solutions for all OP-instances proposed by Tsiligrides (1984) are achieved by VSO, F1 and F2 formulations in the given time limit of 2 hours. In table 1, for Tsiligrides's benchmark instances, a summary of the mean (Avg.) and standard deviation (Std. D.) of the CPU times required to find the optimal solution is presented.

Table 1. Comparison of the mean and standart deviation of the CPU times for data set 1

	VSO		F1		F2	
Problem	Avg.	Std. D.	Avg.	Std. D.	Avg.	Std. D.
TS21	2,25	0,63	0,73	0,46	1,15	0,39
TS32	9,05	8,63	3,54	3,78	1,40	0,67
TS33	9,12	5,44	2,89	2,12	3,26	1,95

According to the results in table 1, F1 and F2 formulations sufficiently and capably find optimal solutions for problems with up to 33 customers within seconds, and our proposed formulations are considerably faster than the existing formulation (VSO).

Table 2 gives the average results for the data set 2. The results are shown in comparison with VSO, F2 formulations. While VSO formulation solves 18 of 40 Chao-Golden-Wasil's instances to optimality in the given time limit of 2 hours, F2 formulation finds optimal solutions in 40 cases within a short period of time.

Table 2. Comparison of the mean and standart deviation of the CPU times for data set 2

	VSO		F1		F2	
Problem	Avg.	Std. D.	Avg.	Std. D.	Avg.	Std. D.
CH64	177,97	175,22	76,91	87,63	40,88	55,46
CH66	-	-	-	-	107,34	138,37

Tables 2 reveal that F2 formulation clearly outperforms VSO formulation in the experiment, and F2 formulation is able to achieve the optimal solution within a short amount of CPU time.

5. Conclusions

In this study, we address the orienteering problem, where the objective function is to maximize the total gain that the traveler collects from the visited nodes during the predetermined time. For this case, two integer linear programming formulation having O(n^2) binary variables and O(n^2) constraints are developed to solve OP-instances optimally by a standard optimization solver.

According to CPU times and optimal solution values obtained from the Tsiligrides problems, we observe that, F1's and F2's performances are close to each other while they are better than VSO's. Besides, according to the benchmark instances given by Chao et al. (1996), it is seen that F2 is better than F1 and F2's performance is clearly higher than VSO's. F2 solved all OP-instances optimally and we see that known best values of the most instances are far away from the optimal values. As a consequence of these computational analyses, we conclude that F2 may be used for solving OP directly by any optimizers.

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