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Non-Weighted *L*₂-Gain Control for Asynchronously Switched Linear Systems With Detectable Switching Instants and Ranged Mode-Identifying Time

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ABSTRACT In this paper, the non-weighted L_2 -gain control problem is addressed for a class of asynchronously switched linear systems, where the asynchronous phenomenon is caused by the mode-identifying process. Unlike the literature concerned with asynchronously switched systems, we construct a new class of clock-dependent Lyapunov function (CDLF), which can be permitted or prohibited to increase when the modes of the controller and system are unmatched. Furthermore, a novel controller design strategy is introduced. The asynchronous and synchronous controllers are designed separately, and are both clock-dependent. By using the CDLF approach, a clock-dependent sufficient condition characterizing the non-weighted L_2 -gain performance is obtained for the asynchronously switched systems. The controller gains can be computed by solving a set of sum of square (SOS) program. At last, the advantages of the results are illustrated within two examples.

INDEX TERMS Clock-dependent Lyapunov function, non-weighted L_2 -gain, asynchronous control, switched systems, sum of square program.

I. INTRODUCTION

In recent decades, switched systems have gotten a lot of attention in virtue of its practical and theoretical values. This class of systems consists of several continuous-time or discrete- subsystems with a switching signal driving them. The feature of switching widely exists in real-world systems, thereby many practical systems can be modeled by switched systems, such as chemical system [1], traffic system [2] and teleoperation robotic system [3].

In practice, a system possessing switching feature may be not stabilized by using any common control inputs, but can be stabilized by using switching control inputs. In other words, one needs to apply different control inputs to different subsystems. Therefore, the switching control problem for switched systems has been researched deeply in

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some literature, e.g., [3]-[12]. Within most of the aforementioned work, it's assumed that the mode of the controller is always consistent with the system's. However, the controller may not switch synchronously with the system in real-world system. Since the mode-identifying process requires some time to complete, the modes of the controller and the system may be unmatched during this period of time. The system which contains unmatched controller is called asynchronously switched system. In the last decade, abundant results have been obtained for asynchronously switched systems with time-controlled switching signal, e.g., [13]–[28]. The aforementioned literature falls into two categories based on how to search the Lyapunov function: in [13]-[21], [26], [28], the increment of the Lyapunov function is permitted when the modes of the controller and system are inconsistent, while in [22]–[25], [27], this is prohibited.

Disturbance inputs are common and inevitable in real-word system. The L_2 -gain can be used to characterize the

disturbance rejection ability of the system. A lot of work has been put in investigating the L_2 -gain performance of switched systems, e.g., [3], [5], [15], [18], [19], [22]–[24], [29]. Among those work, [15], [18], [19], [22]–[24] are concerned with the L_2 -gain performance of asynchronously switched systems. More expressly, in [15], [19], a weighted L_2 -gain is derived, while in [18], [22]–[24], a non-weighted one is obtained. From both points of view of theory and application, the non-weighted one is of more value than the weighted one. However, as far as we know, only a few results on non-weighed L₂-gain control of asynchronously switched systems have been obtained at present, and the research object and methods within these work have some differences: in [18], asynchronously systems with undetectable switching instants are investigated, and the energy function is permitted to increase when the modes of the controller and system are unmatched, while in [22]–[25], the switching instants are assumed to be detected instantaneously online, and the increase of the Lyapunov function is always prohibited. Obviously the results are not abundant and need to be further improved. This motivates our research interests.

Inspired by [4], [30]–[32] concerned with switched systems within the framework of dwell time switching, the CDLF approach is used in the paper to solve out the non-weighted L_2 -gain control problem of a class of asynchronously switched systems. The main contribution of this paper is summarized into two points: (i) A novel class of CDLF is proposed. Unlike [13]–[24], [27], the CDLF can be permitted or prohibited to increase when the mode of the controller is inconsistent with the system's, which makes the results more comprehensive. (ii) A novel controller design strategy is proposed. The asynchronous and synchronous controllers are designed separately, and are both clock-dependent. Compared to [18], a better non-weighted L_2 -gain performance can be guaranteed.

Outlines: The paper is organized as follows: The system descriptions and preliminaries are presented in Section II. The results on L_2 -gain analysis and controller design for asynchronously switched systems are given in Section III. Two numerical examples are given in Section IV, and the conclusion is given in Section V.

Notations: 'T': matrix transposition. '*': transposed elements in the symmetric positions. $\|\cdot\|$: Euclidean vector norm. $\mathbb{N}(\mathbb{N}^+)$: the set of non-negative (positive) integers. \mathbb{R}^n ($\mathbb{R}^{m \times n}$): the set of *n*-dimensional vectors ($m \times n$ -dimensional matrices) with real entries. *I*: identity matrix with appropriate dimension. $L_2[0, \infty)$: space of square sum-able infinite sequence. For $\omega(t) \in L_2[0, \infty)$, its norm is given by $\|\omega(t)\|_2 = \sqrt{\int_0^\infty \omega(t)^T \omega(t) dt}$. A function $\alpha : [0, \infty) \mapsto [0, \infty)$, $\alpha(0) = 0$, is of class \mathcal{K} if it's continuous, strictly increasing, and a function $\beta : [0, \infty) \times [0, \infty) \mapsto [0, \infty)$ is of class \mathcal{KL} if $\beta(\cdot, t)$ is of class \mathcal{K} for fixed $t \geq 0$ and $\beta(s, t)$ decreases to 0 as $t \to \infty$ for fixed $s \geq 0$. P > 0 ($P \geq 0$) means that matrix P is positive (semi-positive) definite.

Some matrix or function expressions will be used in the rest of the work: $\tau(t, s) = t - s$, $\mathcal{N}(A, P, \alpha) = A^T P^T + PA + \alpha P$, $\mathcal{M}(A, P(t), \alpha) = A^T P(t)^T + P(t)A + \dot{P}(t) + \alpha P(t)$.

II. SYSTEM DESCRIPTION AND PRELIMINARIES

Consider the following continuous-time switched linear systems:

$$\begin{cases} \dot{x}(t) = A_{\sigma(t)}x(t) + B_{\sigma(t)}u(t) + E_{\sigma(t)}\omega(t), \\ y(t) = C_{\sigma(t)}x(t) + D_{\sigma(t)}u(t) + F_{\sigma(t)}\omega(t), \end{cases} \quad t \ge t_0, \quad (1)$$

where $t_0 = 0$, $x(t) \in \mathbb{R}^{n_x}$, $u(t) \in \mathbb{R}^{n_u}$, $\omega(t) \in \mathbb{R}^{n_\omega}$, $y(t) \in \mathbb{R}^{n_y}$ represent the states, control inputs, disturbance inputs and outputs, respectively; $\sigma(t)$ represents a switching signal taking value in the finite set $\mathcal{I} = \{1, 2, \dots, N\}$, where $N \in \mathbb{N}^+$ indicates the number of subsystems. t_k indicates the *k*th switching instant, $k \in \mathbb{N}$. Arbitrarily choosing a switching sequence $t_0 < t_1 < \dots < t_k < t_{k+1} < \dots, \sigma(t)$ is continuous from right everywhere.

Several definitions will be used in the rest of the paper:

Definition 1 (See [8]): A system is globally asymptotically stable if there exists a function $\beta(\cdot)$ of class \mathcal{KL} such that $||x(t)|| \leq \beta(||x(t)||)$ holds $\forall t \geq 0$ and $\forall x(t_0) \in \mathbb{R}^{n_x}$.

Definition 2 (See [7]): Given $\overline{\gamma} > 0$, system (1) has a L_2 -gain no greater than $\overline{\gamma}$, if under zero initial condition, the inequality $\int_0^{+\infty} y(s)^T y(s) ds < \int_0^{+\infty} \overline{\gamma}^2 \omega(s)^T \omega(s) ds$ holds for all nonzero $\omega(t) \in L_2[0, \infty)$.

Definition 3 (See [27]): Given a positive scalar T, the set of minimum dwell time switching signal satisfying that $t_k - t_{k-1} \ge T$ holds $\forall k \in \mathbb{N}^+$ is denoted by S_T .

III. MAIN RESULTS

A. NON-WEIGHTED L₂-GAIN ANALYSIS

In practice, the system mode can't be identified instantaneously due to the limitation of devices and influences of environment. However, the detection of switching instants is not complicated and can be done instantaneously. Hence it's reasonable to make the following assumption:

Assumption 1 (See [16], [22]–[24], [27]): The switching instants $t_1, t_2, \dots, t_k, \dots$ can be detected instantaneously online.

The assumption above can be of some use in the design of controller. However, although the information of switching instants has been acquired, the real-time system mode is still unknown. The mode estimator needs some time to achieve the real-time mode of the system online. We call this period of time the mode-identifying time. In [16], the modeidentifying time is assumed to be a constant. However, in practice, the mode-identifying time will be influenced by some inner and outer actions. Hence, it's more practical to assume that the mode-identifying time is within a range. We denote the mode-identifying time of the *k*th switching is Δ_k , then, the following assumption is made:

Assumption 2: Given two positive scalars Δ_{\min} and Δ_{\max} satisfying $0 \leq \Delta_{\min} \leq \Delta_{\max}$, we let Δ_{\min} and Δ_{\max}



FIGURE 1. Controller design strategy.

represent the minimum and maximum mode-identifying time, respectively. Then, the mode-identifying time Δ_k satisfies that $\Delta_{\min} \leq \Delta_k \leq \Delta_{\max}$ holds $\forall k \geq 0$.

In general, it's assumed that the *i*th subsystem is running for $t \in [t_k, t_{k+1})$, and the *j*th subsystem is running in the former interval $[t_{k-1}, t_k)$. We denote $t_k^{\Delta} = t_k + \Delta_k, t_k^{\vartheta} = t_k + \Delta_k + \vartheta, k \ge 0$. The state-feedback control input is constructed as follows:

$$u(t) = \begin{cases} K_j^{\bullet}(\tau(t, t_k))x(t), & t \in [t_k, t_k^{\Delta}), \\ K_i(\tau(t, t_k^{\Delta}))x(t), & t \in [t_k^{\Delta}, t_k^{\vartheta}), \\ K_i(\vartheta)x(t), & t \in [t_k^{\vartheta}, t_{k+1}), \end{cases}$$
(2)

where $\tau(t, t_k)$ and $\tau(t, t_k^{\Delta})$ are timer starting at t_k and t_k^{Δ} , respectively, the matrix-valued functions $K_i^{\bullet}(\tau)$, $K_i(\tau)$ are controller gains to be determined. The control strategy is shown in Fig. 1:

- In [t_k, t^Δ_k), the nearest switching instant has been detected but the system mode is unknown, the clock-dependent asynchronous controller K[•]_j(τ), τ ∈ [0, Δ_k], is applied to the system.
- In [t^Δ_k, t^ϑ_k), the nearest switching instant has been detected, and the system mode has been identified by the mode sensor, the clock-dependent synchronous controller K_i(τ), τ ∈ [0, ϑ], is applied to the system.
- In $[t_k^{\vartheta}, t_{k+1})$, the switching instant has been detected, and the system mode has been known, the synchronous controller $K_i(\vartheta)$ is applied to the system.

Remark 1: Under Assumption 1, an individual clockdependent controller $K_j^{\bullet}(\tau), \tau \in [0, \Delta_k]$ is introduced in the unmatched interval $[t_k, t_k^{\Delta})$. This is different from most literatures concerned with asynchronously switched systems, which continue to use the previous controller. The new controller design strategy provides additional freedom and flexibility to the design of controllers.

Substituting (2) into (1), the close-loop system is obtained and given as follows:

$$\begin{cases} \dot{x}(t) = \mathscr{A}_{ij}^{\bullet}(\tau(t,t_k))x(t) + E_i\omega(t), \\ y(t) = \mathscr{C}_{ij}^{\bullet}(\tau(t,t_k))x(t) + F_i\omega(t), \end{cases} \qquad t \in [t_k, t_k^{\Delta}), \\ \begin{cases} \dot{x}(t) = \mathscr{A}_i(\tau(t, t_k^{\Delta}))x(t) + E_i\omega(t), \\ y(t) = \mathscr{C}_i(\tau(t, t_k^{\Delta}))x(t) + F_i\omega(t), \end{cases} \qquad t \in [t_k^{\Delta}, t_k^{\vartheta}), \\ \begin{cases} \dot{x}(t) = \mathscr{A}_i(\vartheta)x(t) + E_i\omega(t), \\ y(t) = \mathscr{C}_i(\vartheta)x(t) + F_i\omega(t), \end{cases} \qquad t \in [t_k^{\vartheta}, t_{k+1}), \end{cases}$$

$$(3)$$

where $\mathscr{A}_{ij}^{\bullet}(\tau(t,t_k)) = A_i + B_i K_j^{\bullet}(\tau(t,t_k)), \mathscr{C}_{ij}^{\bullet}(\tau(t,t_k)) = C_i + D_i K_j^{\bullet}(\tau(t,t_k)), \mathscr{A}_i(\tau(t,t_k^{\Delta}) = A_i + B_i K_i(\tau(t,t_k^{\Delta}), \mathscr{C}_i(\tau(t,t_k^{\Delta}) = C_i + D_i K_i(\tau(t,t_k^{\Delta}).$

The following theorem characterizes the non-weighted L_2 -gain of the close-loop system (3):

Theorem 1: Consider system (3). Given non-negative scalars α and β satisfying

$$\begin{cases} \beta > 0, & \text{if } \alpha > 0, \\ \beta = 0, & \text{otherwise}, \end{cases}$$
(4)

if there exist two scalars $\gamma > 0$ *and* ϑ *satisfying*

$$\begin{cases} \vartheta > \frac{\alpha \Delta_{\max}}{\beta}, & if \ \alpha > 0, \\ \vartheta \ge 0, & otherwise, \end{cases}$$
(5)

and a set of matrix-valued functions $P_i^{\bullet}(\tau) : [0, \Delta_{\max}] \mapsto \mathbb{R}^{n_x \times n_x}$, $P_i^{\bullet}(\tau) > 0$, $P_i(\tau) : [0, \vartheta] \mapsto \mathbb{R}^{n_x \times n_x}$, $P_i(\tau) > 0$, such that $\forall (i \times j) \in \mathcal{I} \times \mathcal{I}$, $i \neq j$, the following conditions hold:

$$\begin{bmatrix} \mathscr{M}\{\mathscr{A}_{ij}^{\bullet}(\tau), P_{j}^{\bullet}(\tau), -\alpha\} & E_{i} & \mathscr{C}_{ij}^{\bullet}(\tau)^{T} \\ * & -\gamma^{2} I & F_{i}^{T} \\ * & * & -I \end{bmatrix}$$

$$< 0, \quad \tau \in [0, \Delta_{\max}], \qquad (6)$$

$$\begin{bmatrix} \mathscr{M}\{\mathscr{A}_{i}(\tau), P_{i}(\tau), \beta\} & E_{i} & \mathscr{C}_{i}(\tau)^{T} \\ * & -\gamma^{2} I & F_{i}^{T} \\ * & * & -I \end{bmatrix}$$

$$< 0, \quad \tau \in [0, \vartheta], \qquad (7)$$

$$\begin{bmatrix} \mathscr{N}\{\mathscr{A}_{i}(\vartheta), P_{i}(\vartheta), \beta\} & E_{i} & \mathscr{C}_{i}^{T} \\ * & -\gamma^{2} I & F_{i}^{T} \\ * & * & -I \end{bmatrix}$$

$$< 0, \qquad (8)$$

$$P_i(0) - P_j^{\bullet}(\tau) \le 0, \quad \tau \in [\Delta_{\min}, \Delta_{\max}], \tag{9}$$

$$P_i^{\bullet}(0) - P_i(\vartheta) \le 0, \tag{10}$$

then system (3) is global asymptotically stable with $\omega(t) = 0$ and any switching signals $\sigma(t) \in S_T$, where $T = \Delta_{\max} + \vartheta$, and has a non-weighted L_2 -gain no larger than $\bar{\gamma} = \sqrt{\frac{T\beta e^{(T-\Delta_{\max})\Theta_{\max}}{T}}{T}} \chi$, where $\Theta_{\max} = -(\alpha + \beta) \Lambda$

 $\sqrt{\frac{I\beta e}{T\beta - \Theta_{\max}}} \gamma, \text{ where } \Theta_{\max} = (\alpha + \beta) \Delta_{\max}.$ *Proof:* The proof is partitioned into two parts based on

Proof: The proof is partitioned into two parts based on whether $\alpha > 0$ or not:

Case 1 ($\alpha > 0$): The stability of system (3) is proved first. We choose the following clock-dependent Lyapunov function

$$V(t) = \begin{cases} x^{T}(t)P_{j}^{\bullet}(\tau(t,t_{k}))x(t), & t \in [t_{k}, t_{k}^{\Delta}), \\ x^{T}(t)P_{i}(\tau(t,t_{k}^{\Delta}))x(t), & t \in [t_{k}^{\Delta}, t_{k}^{\vartheta}), \\ x^{T}(t)P_{i}(\vartheta)x(t), & t \in [t_{k}^{\vartheta}, t_{k+1}), \end{cases}$$
(11)

By virtue of the fact that $P_i(\tau) > 0$ holds $\forall \tau \in [0, \Delta_{\max}]$, and $P_i^{\bullet}(t) > 0$ holds $\forall \tau \in [0, \vartheta]$, one gets that V(t) > 0 holds $\forall x(t) \in \mathbb{R}^{n_x} (x(t) \neq 0)$. Since $\omega(t) = 0, y(t)^T y(t) \ge 0$, from (6)-(8) one has that

$$\dot{V}(t) < \begin{cases} \alpha V(t), & t \in [t_k, t_k^{\Delta}), \\ -\beta V(t), & t \in [t_k^{\Delta}, t_{k+1}), \end{cases}$$
(12)

Meanwhile, from (9) and (10) one has that $\forall k \ge 0$,

$$V(t_k^{\Delta}) \le V(t_k^{\Delta^-}), \quad V(t_{k+1}) \le V(t_{k+1}^-).$$
 (13)

(12) and (13) imply that $V(t_{k+1}) < e^{\alpha \Delta_k - \beta(t_{k+1} - t_k^{\Delta})} V(t_k) \le e^{\alpha \Delta_{\max} - \beta \vartheta} V(t_k)$. Letting $\epsilon = e^{\alpha \Delta_{\max} - \beta \vartheta} < 1$, one has that $V(t_{k+1}) < \epsilon V(t_k) < \epsilon^{k+1} V(t_0)$, which implies that $\lim_{k \to +\infty} V(t_k) = 0$. Then, one can find that V(t) decreases to zero as $t \to +\infty$. The asymptotic stability of system (3) with $\omega(t) = 0$ is proved.

Then, the L_2 -gain performance of system (3) is considered. Let $\Gamma(t) = y(t)^T y(t) - \gamma^2 \omega(t)^T \omega(t)$, $\mathcal{T}^{\alpha}(s, t)$ represent the time from *s* to *t* during which the modes of controller and system are unmatched, and $\mathcal{T}^{\beta}(s, t)$ represent the time from *s* to *t* during which the mode of system and controller are matched. Similarly to the proof of the bounded real lemma, from (6)–(8) one can obtain the following results

$$\dot{V}(t) < \begin{cases} \alpha V(t) - \Gamma(t), & t \in [t_k, t_k^{\Delta}), \\ -\beta V(t) - \Gamma(t), & t \in [t_k^{\Delta}, t_{k+1}), \end{cases}$$
(14)

which combined with (13) gives that

$$V(t) \le e^{\alpha \mathcal{T}^{\alpha}(t_0, t) - \beta \mathcal{T}^{\beta}(t_0, t)} V(t_0) + \int_{t_0}^t e^{\alpha \mathcal{T}^{\alpha}(s, t) - \beta \mathcal{T}^{\beta}(s, t)} \Gamma(s) ds$$

Since $V(t_0) = 0$ and $V(t) \ge 0$, we get

a t

$$\int_{t_0}^{t} e^{\alpha \mathcal{T}^{\alpha}(s,t) - \beta \mathcal{T}^{\beta}(s,t)} y(s)^T y(s) ds$$

$$\leq \int_{t_0}^{t} e^{\alpha \mathcal{T}^{\alpha}(s,t) - \beta \mathcal{T}^{\beta}(s,t)} \gamma^2 \omega(s)^T \omega(s) ds,$$

where the left side satisfies $\int_{t_0}^t e^{-\beta(t-s)} y(s)^T y(s) ds \leq \int_{t_0}^t e^{\alpha T^{\alpha}(s,t) - \beta T^{\beta}(s,t)} y(s)^T y(s) ds$, and the right side satisfies that $\int_{t_0}^t e^{\alpha T^{\alpha}(s,t) - \beta T^{\beta}(s,t)} \gamma^2 \omega(s)^T \omega(s) ds \leq \int_{t_0}^t e^{(\alpha+\beta)T^{\alpha}(s,t) - \beta(t-s)} \gamma^2 \omega(s)^T \omega(s) ds$. Next, since $T^{\alpha}(s,t)$ satisfies that $\forall n \in \mathbb{N}^+$, $T^{\alpha}(s,t) \leq (1 + \frac{t-s - \Delta_{\max}}{T}) \Delta_{\max}$, we have that

$$\int_{t_0}^t e^{\Theta_{\max}(1+\frac{t-s-\Delta_{\max}}{T})-\beta(t-s)} \gamma^2 \omega(s)^T \omega(s) ds$$

$$\leq \int_{t_0}^t e^{\Theta_{\max}(1-\frac{\Delta_{\max}}{T})+\frac{\Theta_{\max}-\beta T}{T}(t-s)} \gamma^2 \omega(s)^T \omega(s) ds,$$

which implies that

$$\int_{t_0}^t e^{-\beta(t-s)} y(s)^T y(s) ds$$

<
$$\int_{t_0}^t e^{\Theta_{\max}(1-\frac{\Delta_{\max}}{T})+\frac{\Theta_{\max}-\beta T}{T}(t-s)} \gamma^2 \omega(s)^T \omega(s) ds.$$

Integrating the inequality above from $t_0 = 0$ to $+\infty$ and exchanging the order of integration arrives that $\int_0^{+\infty} y(s)^T y(s) ds < \int_0^{+\infty} \frac{T\beta e^{\frac{(T-\Delta_{\max})\Theta_{\max}}{T}} \gamma^2 \omega(s)^T \omega(s) ds}{T\beta - \Theta_{\max}} \gamma^2 \omega(s)^T \omega(s) ds}$. Then one can conclude that system (3) has a L_2 -gain no larger than $\overline{\gamma} = \sqrt{\frac{T\beta e^{\frac{(T-\Delta_{\max})\Theta_{\max}}{T}} \gamma}{T\beta - \Theta_{\max}}} \gamma$.

Case 2 ($\alpha = 0$): One has that $\sqrt{\frac{T\beta e^{\frac{(T-\Delta_{\text{max}})\Theta_{\text{max}}}{T\beta - \Theta_{\text{max}}}} = 1}$ since $\alpha = \beta = 0$. Hence one needs to prove that system (3) has a L_2 -gain no larger than γ . The stability of system (1) is established since $\dot{V}(t) < 0$ holds $\forall t \ge 0$. Then, the L_2 -gain performance of (1) is considered. From (6)-(8) one has that

$$\dot{V}(t) < \begin{cases} -\Gamma(t), & t \in [t_k, t_k^{\Delta}), \\ -\beta V(t) - \Gamma(t) \le -\Gamma(t), & t \in [t_k^{\Delta}, t_{k+1}), \end{cases}$$

which means that $\forall t \in [t_k, t_{k+1}), \dot{V}(t) + \Gamma(t) < 0$. Integrating the inequality from t_0 to t gives that $\int_0^{+\infty} \Gamma(t) < -V(t) < 0$, which implies that $\int_0^{+\infty} y(s)^T y(s) ds < \int_0^{+\infty} \gamma^2 \omega(s)^T \omega(s) ds$ as $t \to +\infty$. Hence one can conclude that system (3) has a L_2 -gain no larger than γ . The proof is completed. \Box

Remark 2: In Theorem 1, it can be seen that an individual CDLF $P_i^{\bullet}(\tau)$ is introduced during the unmatched interval $[t_k, t_k^{\Delta})$, which is different to the CDLF for matched interval, while in [18], the Lyapunov function for the unmatched interval $[t_k, t_k^{\Delta})$ and the former matched interval $[t_{k-1}^{\vartheta}, t_k)$ are the same, and in [22]–[24], [27], the Lyapunov function for the unmatched interval $[t_k, t_k^{\Delta})$ and the next matched interval $[t_k^{\Delta}, t_k^{\vartheta})$ are the same. This implies that under Assumption 1, less conservative results are gotten by Theorem 1.

Remark 3: In [18], the increasing of the Lyapunov function is permitted during the asynchronous interval $[t_k, t_k^{\Delta})$, while in [22]–[24], [27], the increasing of the energy function is always prohibited. The two cases are unified in this paper. In Theorem 1, $\alpha > 0$ and $\alpha = 0$ separately imply that the CDLF is allowed and prohibited to increase during the unmatched interval. Since $\alpha > 0$ implies that $\sqrt{\frac{T\beta e^{(T-\Delta \max)\Theta \max}}{T\beta - \Theta \max}} > 1$, while $\alpha = 0$ implies that $\sqrt{\frac{T\beta e^{(T-\Delta \max)\Theta \max}}{T\beta - \Theta \max}} = 1$, one can set $\alpha = 0$ first to get a smaller $\overline{\gamma}$. If no feasible solutions can be found for $\alpha = 0$, one can set $\alpha > 0$ to relax the limitation on the Lyapunov function, and make the problem feasible.

Remark 4: This case that $\alpha < 0$ and $\beta > 0$ must exist because the system may be still stable even if the modes of the controller and the subsystems are unmatched. But for this case, we can let $\alpha = 0$, $\beta = 0$ directly. Hence we do not consider this case in our paper.

B. NON-WEIGHTED L₂-GAIN CONTROLLER DESIGN

The following result checks the existence of controller gains:

Theorem 2: Consider system (3). Given non-negative scalars α and β satisfying (4), if there exist two scalars $\gamma > 0$, ϑ satisfying (5) and a set of matrix-valued functions $Q_i^{\bullet}(\tau)$: $[0, \Delta_{\max}] \mapsto \mathbb{R}^{n_x \times n_x}, Q_i^{\bullet}(\tau) > 0, Q_i(\tau) : [0, \vartheta] \mapsto \mathbb{R}^{n_x \times n_x}, Q_i(\tau) > 0$, such that $\forall (i \times j) \in \mathcal{I} \times \mathcal{I}, i \neq j$, the following conditions hold:

$$\begin{bmatrix} \Phi_{ij}^{\bullet}(\tau) & E_i & \Psi_{ij}^{\bullet}(\tau) \\ * & -\gamma^2 I & F_i^T \\ * & * & -I \end{bmatrix} < 0, \quad \tau \in [0, \Delta_{\max}], \quad (15)$$

$$\begin{bmatrix} \Phi_i(\tau) & E_i & \Psi_i(\tau) \\ * & -\gamma^2 I & F_i^T \\ * & * & -I \end{bmatrix} < 0, \quad \tau \in [0, \vartheta], \quad (16)$$

$$\begin{bmatrix} \Phi_i^* & E_i & \Psi_i^* \\ * & -\gamma^2 I & F_i^T \\ * & * & -I \end{bmatrix} < 0, \tag{17}$$

$$-Q_i(0) + Q_j^{\bullet}(\tau) \le 0, \quad \tau \in [\Delta_{\min}, \Delta_{\max}],$$

$$-Q_i^{\bullet}(0) + Q_i(\vartheta) \le 0, \tag{19}$$

where $\Phi_{ij}^{\bullet}(\tau) = Q_j^{\bullet}(\tau)A_i^T + A_iQ_j^{\bullet}(\tau) + U_j^{\bullet}(\tau)^T B_i^T + B_iU_j^{\bullet}(\tau) - \dot{Q}_j^{\bullet}(\tau) - \alpha Q_j^{\bullet}(\tau), \quad \Psi_{ij}^{\bullet}(\tau) = Q_j^{\bullet}(\tau)C_i^T + U_j^{\bullet}(\tau)^T D_i^T, \quad \Phi_i(\tau) = Q_i(\tau)A_i^T + A_iQ_i(\tau) + U_i(\tau)^T B_i^T + B_iU_i(\tau) - \dot{Q}_i(\tau) + \beta Q_i(\tau), \quad \Psi_i(\tau) = Q_i(\tau)C_i^T + U_i(\tau)^T D_i^T, \quad \Phi_i^* = Q_i(\vartheta)A_i^T + A_iQ_i(\vartheta) + U_i(\vartheta)^T B_i^T + B_iU_i(\vartheta) + \beta Q_i(\vartheta), \quad \Psi_i^* = Q_i(\vartheta)C_i^T + U_i(\vartheta)^T D_i^T.$ Then, system (3) is global asymptotically stable with any switching signals $\sigma(t) \in S_T$, where $T \geq \Delta_{\max} + \vartheta$, and has a non-weighted L_2 -gain no larger than $\bar{\gamma} = \sqrt{T_B e^{(T-\Delta_{\max})\Theta_{\max}}}$

$$\sqrt{\frac{T\beta e^{-T}}{T\beta - \Theta_{\max}}}\gamma$$
, where $\Theta_{\max} = (\alpha + \beta)\Delta_{\max}$.

Proof: Letting $P_i^{\bullet}(\tau) = Q_i^{\bullet}(\tau)^{-1}$, multiplying both sides of (15)-(17) implies that (6)-(8) hold. Meanwhile, (18) and (19) imply that (9) and (10) hold. The proof is completed. \Box

C. SOS FORMULATION

The SOS program, which is an approximation of the conditions in Theorem 2, is presented below:

Proposition 1: Consider the conditions in Theorem 2. Given non-negative scalars α and β satisfying (4), and $d \in \mathbb{N}^+$, the conditions in Theorem 2 hold if there exists a set of scalars $\epsilon > 0$, $\gamma > 0$, ϑ satisfying (5), and a set of SOS matrix polynomials $Q_i^{\bullet}(\tau) : [0, \Delta_{\max}] \mapsto \mathbb{R}^{n_x \times n_x}$, $Q_i^{\bullet}(\tau) : [0, \vartheta] \mapsto \mathbb{R}^{n_x \times n_x}$, $\Xi_{ij}^{\bullet}(\tau) : [0, \Delta_{\max}] \mapsto \mathbb{R}^{n_x \times n_x}$, $\Xi_i(\tau) : [0, \vartheta] \mapsto \mathbb{R}^{n_x \times n_x}$ and $\Sigma_{ij}^{\bullet}(\tau) : [\Delta_{\min}, \Delta_{\max}] \mapsto \mathbb{R}^{n_x \times n_x}$ $\mathbb{R}^{n_x \times n_x}$ of degree 2d such that $\forall (i \times j) \in \mathcal{I} \times \mathcal{I}$, $i \neq j$, the following conditions hold:

$$-\begin{bmatrix} \Phi_{ij}^{\bullet}(\tau) & E_{i} & \Psi_{ij}^{\bullet}(\tau) \\ * & -\gamma^{2} I & F_{i}^{T} \\ * & * & -I \end{bmatrix}$$

$$-\tau(\Delta_{\max} - \tau)\Xi_{ij}^{\bullet}(\tau) - \epsilon I \text{ is SOS},$$

$$-\begin{bmatrix} \Phi_{i}(\tau) & E_{i} & \Psi_{i}(\tau) \\ * & -\gamma^{2} I & F_{i}^{T} \\ * & * & -I \end{bmatrix}$$

$$-\tau(\vartheta - \tau)\Xi_{i}(\tau) - \epsilon I \text{ is SOS},$$

$$\begin{bmatrix} \Phi_{i}^{*} & E_{i} & \Psi_{i}^{*} \\ * & -\gamma^{2} I & F_{i}^{T} \\ * & * & -I \end{bmatrix} < 0, \quad -Q_{i}^{\bullet}(0) + Q_{i}(\vartheta) \leq 0,$$

$$\begin{bmatrix} -Q_{i}(0) + Q_{i}^{\bullet}(\tau) - (\tau - \Delta_{\min})(\Delta_{\max} - \tau)\Sigma_{ij}^{\bullet}(\tau) \text{ is SOS}. \end{bmatrix}$$

Moreover, if a feasible solution can be found for the conditions above, the asynchronous and synchronous controller gains can be computed by $K_i^{\bullet}(\tau) = U_i^{\bullet}(\tau)Q_i^{\bullet}(\tau)^{-1}, \tau \in$ $[0, \Delta_{\max}], K_i(\tau) = U_i(\tau)Q_i(\tau)^{-1}, \tau \in [0, \vartheta]$, respectively. *Proof:* It's similar to the proof of Proposition 8 \implies Theorem 5 in [33] and omitted.

The optimized L_2 -gain can be computed by two steps: firstly, one can get γ_{min} by solving out the optimization problem as follows:

$$\gamma_{\min}$$
 s.t. The conditions in Proposition 1 hold. (20)

Secondly, the optimized L_2 -gain is obtained by $\bar{\gamma}_{\min} = \sqrt{\frac{T\beta e^{\frac{(T-\Delta_{\max})\Theta_{\max}}{T}}}{T\beta-\Theta_{\max}}}\gamma_{\min}$. Meanwhile, the optimized L_2 -gain controller can also be achieved.

Remark 5: The clock-dependent conditions in Theorem 2 can also be relaxed into computable ones by using the discretisation approach, see [3], [18], [34]–[36] for details. It's omitted here for brevity.

IV. SIMULATION

This section provides two examples to verify the validity of the results. The SOS program is solved out with the help of the package SOSTOOLs [37] and SDP solver SeDuMi [38].

Example 1 (A Case of $\alpha = 0$): Consider the following switched linear systems appearing in [18]:

$$A_{1} = \begin{bmatrix} 0.9 & -5.8\\ 2.75 & 0.9 \end{bmatrix}, \quad A_{2} = \begin{bmatrix} -2 & 2\\ 2.12 & -1.3 \end{bmatrix}, \quad B_{1} = \begin{bmatrix} 1.5\\ 2.2 \end{bmatrix}, \\ B_{2} = \begin{bmatrix} 1.85\\ 1.75 \end{bmatrix}, \quad C_{1} = \begin{bmatrix} 1\\ 0 \end{bmatrix}, \quad C_{2} = \begin{bmatrix} 0.45\\ 0 \end{bmatrix}, \quad E_{1} = \begin{bmatrix} 0.1\\ 0.5 \end{bmatrix}, \\ E_{2} = \begin{bmatrix} 0.2\\ 0.6 \end{bmatrix}, \quad D_{1} = D_{2} = 1.5, \quad F_{1} = F_{2} = 0.65.$$
(21)

Similarly to Example 1 in [18], we choose $\Delta_{\min} = 0$, $\Delta_{\max} = 2$, $\vartheta = 3.6$, $\epsilon = 0.001$, and let $\alpha = \beta = 0$. Through solving the optimization problem (20), we get the optimized L_2 -gain index, which is $\overline{\gamma}_{\min} = \gamma_{\min} = 0.66$. Compared to the optimized L_2 -gain $\overline{\gamma}_{\min} = 4.7865$ obtained in [18], our results are obviously less conservative.

Next, we choose the same initial condition, disturbance input and switching signal as in [18]. In practice, before the system is activated, there must exist sufficient time for the mode sensor to identify the modes of the controller and the system. Hence we assume that we can get the modes of the system and the controller at the initial time. The simulated results are shown in Figure 2, from which it's noted that the trajectory of the states vanished to zero more rapidly compared to Fig. 7 in [18]. This implies that the design controller guarantees an better L_2 -gain performance compared to [18]. The evolution of the energy function for the closedloop system with $\omega(t) = 0$ is shown in Figure 2 (d), from which we can note that the Lyapunov function decreases to zero rapidly.

Remark 6: This example shows that through our approach, a non-increasing Lyapunov function still may be found even if asynchronous phenomenon exists. However, the nonincreasing Lyapunov function may be hardly found for asynchronously switched linear systems with a large number of modes. Then we need to relax the limitation on the Lyapunov function. This will be shown in the next example.



FIGURE 2. Simulated results for closed-loop system (21).

Example 2 (A Case of $\alpha > 0$): Consider the following switched linear systems with three modes:

$$A_{1} = \begin{bmatrix} 0.9 & -5.8\\ 2.75 & 0.9 \end{bmatrix}, \quad A_{2} = \begin{bmatrix} -2 & 2\\ 2.12 & -1.3 \end{bmatrix}, \\A_{3} = \begin{bmatrix} 1.9 & 0.8\\ 1.75 & 0.4 \end{bmatrix}, \quad B_{1} = \begin{bmatrix} 1.5\\ 2.2 \end{bmatrix}, \quad B_{2} = \begin{bmatrix} 1.85\\ 1.75 \end{bmatrix} \\B_{3} = \begin{bmatrix} 1.0\\ 1.2 \end{bmatrix}, \quad C_{1} = \begin{bmatrix} 1\\ 0 \end{bmatrix}, \\C_{2} = \begin{bmatrix} 0.45\\ 0 \end{bmatrix}, \quad C_{3} = \begin{bmatrix} 1\\ 0 \end{bmatrix}, \quad E_{1} = \begin{bmatrix} 0.1\\ 0.34 \end{bmatrix},$$



(d) Evolution of the Lyapunov function

FIGURE 3. Simulated results for closed-loop system (22).

$$E_{2} = \begin{bmatrix} 0.2\\ 0.45 \end{bmatrix}, \quad E_{3} = \begin{bmatrix} 0.1\\ 0.24 \end{bmatrix}, \quad D_{1} = D_{2} = D_{3} = 1.5,$$

$$F_{1} = F_{2} = 0.25, \quad F_{3} = 0.15.$$
(22)

First, we solve the conditions in Proposition 1 with $\alpha = \beta = 0$, $\Delta_{\min} = 0$, $\Delta_{\max} = 2$, $\vartheta = 3.6$, d = 2, $\epsilon = 0.001$. However, no feasible solutions can be found for any γ . This implies that a non-increasing Lyapunov can't be easily found in this case.

Then, according to Remark 3, we set $\alpha = \beta = 0.5$, and let other parameters remain unchanged. Through permitting the increase of the Lyapunov function when the modes of

the controller and system are unmatched, the optimization problem (20) becomes feasible, and $\bar{\gamma}_{min} = 5.27$ is obtained.

Finally, we let $t_{k+1} - t_k = T = 5.6$, $k \ge 0$, and apply the same initial condition and disturbance input as in [18] and Example 1. Some simulated results are shown in Figure 3, from which we can see that although the state trajectory diverges at the first unmatched interval, it converges to zero rapidly during the rest time. Similarly, the energy function increases during the unmatched interval, but it still decreases to zero quickly during the matched interval. This shows the L_2 -gain performance is guaranteed for the system.

V. CONCLUSION

The non-weighted L_2 -gain control problem has been investigated in this paper for asynchronously switched systems with detectable switching instants and ranged mode-identifying time. A clock-dependent sufficient condition has been obtained to verify the existence of non-weighted L_2 -gain controller for the concerned systems. The condition is relaxed into computable conditions via SOS approximation approach. By solving a set of SOS program, a couple of synchronous and asynchronous controller gains is obtained. Two examples are provided in the end to illustrate the results of the paper.

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REFERENCES

- Y. Mao, H. Zhang, and Z. Zhang, "Finite-time stabilization of discrete-time switched nonlinear systems without stable subsystems via optimal switching signal design," *IEEE Trans. Fuzzy Syst.*, vol. 25, no. 1, pp. 172–180, Mar. 2017.
- [2] W. Xiang, J. Lam, and J. Shen, "Stability analysis and L₁-gain characterization for switched positive systems under dwell-time constraint," *Automatica*, vol. 85, pp. 1–8, Nov. 2017.
- [3] L. I. Allerhand and U. Shaked, "Robust stability and stabilization of linear switched systems with dwell time," *IEEE Trans. Autom. Control*, vol. 56, no. 2, pp. 381–386, Feb. 2011.
- [4] C. Briat, "Convex conditions for robust stabilization of uncertain switched systems with guaranteed minimum and mode-dependent dwell-time," *Syst. Control Lett.*, vol. 78, pp. 63–72, Apr. 2015.
- [5] J. Zhao and D. J. Hill, "On stability, L₂-gain and H∞ control for switched systems," *Automatica*, vol. 44, no. 5, pp. 1220–1232, 2008.
- [6] Y. Li and H. Zhang, "Dwell time stability and stabilization of interval discrete-time switched positive linear systems," *Nonlinear Anal., Hybrid Syst.*, vol. 33, pp. 116–129, Aug. 2019.
- [7] L. Zhang, S. Zhuang, and P. Shi, "Non-weighted quasi-time-dependent H∞ filtering for switched linear systems with persistent dwell-time," *Automatica*, vol. 54, pp. 201–209, Apr. 2015.
- [8] D. Liberzon, Switching in Systems and Control. Boston, MA, USA: Birkhäuser, 2003.
- [9] X.-Q. Zhao, D. Ye, and G.-X. Zhong, "Passivity and passification of switched systems with the persistent dwell time switching," *Nonlinear Anal.*, *Hybrid Syst.*, vol. 34, pp. 18–29, Nov. 2019.
- [10] G.-X. Zhong and G.-H. Yang, "Dynamic output feedback control of saturated switched delay systems under the PDT switching," *Int. J. Robust Nonlinear Control*, vol. 27, no. 15, pp. 2567–2588, Oct. 2017.
- [11] Z. Wang, G. Chen, and H. Ba, "Stability analysis of nonlinear switched systems with sampled-data controllers," *Appl. Math. Comput.*, vol. 357, pp. 297–309, Sep. 2019.

- [12] Z. Wang, J. Sun, G. Chen, and J. Chen, "Stability analysis of switched nonlinear delay systems with sampled-data inputs," *Int. J. Robust Nonlinear Control*, vol. 29, no. 14, pp. 4700–4715, 2019.
- [13] L. Etienne, K. M. D. Motchon, and S. Lecoeuche, "Stability analysis for switched uncertain nonlinear systems with dwell time and delay in the active mode detection," *IEEE Control Syst. Lett.*, vol. 3, no. 2, pp. 362–367, Apr. 2019.
- [14] L. Zhang and H. Gao, "Asynchronously switched control of switched linear systems with average dwell time," *Automatica*, vol. 46, no. 5, pp. 953–958, 2010.
- [15] L. Zhang and P. Shi, "Stability, l₂-gain and asynchronous H∞ control of discrete-time switched systems with average dwell time," *IEEE Trans. Autom. Control*, vol. 54, no. 9, pp. 2192–2199, Sep. 2009.
- [16] L. Zhang and W. Xiang, "Mode-identifying time estimation and switchingdelay tolerant control for switched systems: An elementary time unit approach," *Automatica*, vol. 64, pp. 174–181, Feb. 2016.
- [17] H. Zhang, B. Wang, D. Xie, S. Xu, and C. Dang, "Stability, L₂-gain and asynchronous H∞ control for continuous-time switched systems," *Int. J. Robust Nonlinear Control*, vol. 25, no. 4, pp. 575–587, Mar. 2015.
- [18] S. Yuan, L. Zhang, B. De Schutter, and S. Baldi, "A novel Lyapunov function for a non-weighted L₂ gain of asynchronously switched linear systems," *Automatica*, vol. 87, pp. 310–317, Jan. 2018.
- [19] P. Marannino and F. Zanellini, "Discussion on 'stability, l₂-gain and asynchronous H∞ control of discrete-time switched systems with average dwell time"'*IEEE Trans. Power Syst.*, vol. 19, no. 1, pp. 692–693, Feb. 2004.
- [20] X. Zhao, P. Shi, and L. Zhang, "Asynchronously switched control of a class of slowly switched linear systems," *Syst. Control Lett.*, vol. 61, no. 12, pp. 1151–1156, 2012.
- [21] Y. Mao, H. Zhang, and S. Xu, "The exponential stability and asynchronous stabilization of a class of switched nonlinear system via the T–S fuzzy model," *IEEE Trans. Fuzzy Syst.*, vol. 22, no. 4, pp. 817–828, Aug. 2014.
- [22] J. Xiao and W. Xiang, "New results on asynchronous H∞ control for switched discrete-time linear systems under dwell time constraint," *Appl. Math. Comput.*, vol. 242, pp. 601–611, Sep. 2014.
- [23] W. Xiang and J. Xiao, "Stabilization of switched continuous-time systems with all modes unstable via dwell time switching," *Automatica*, vol. 50, no. 3, pp. 940–945, 2014.
- [24] Y. Li, H. Zhang, D. Zheng, and Y. Wang, "Asynchronous H∞ control of discrete-time switched T–S fuzzy systems with dwell time," *Int. J. Fuzzy Syst.*, vol. 20, no. 4, pp. 1098–1114, 2018.
- [25] Y. Li and H. Zhang, "Asynchronous L₁-gain control of uncertain switched positive linear systems with dwell time," *ISA Trans.*, vol. 75, pp. 25–37, Apr. 2018.
- [26] Y. Li and H. Zhang, "Asynchronous H8 control of switched uncertain discrete-time fuzzy systems via basis-dependent multiple Lyapunov functions approach," *Circuits, Syst., Signal Process.*, vol. 37, no. 1, pp. 135–162, 2018.
- [27] Y. Li and H. Zhang, "Stability, L₁-gain analysis and asynchronous L₁-gain control of uncertain discrete-time switched positive linear systems with dwell time," *J. Franklin Inst.*, vol. 356, no. 1, pp. 382–406, 2019.
- [28] G.-X. Zhong and G.-H. Yang, "Asynchronous fault detection and robust control for switched systems with state reset strategy," *J. Franklin Inst.*, vol. 355, no. 1, pp. 250–272, 2018.
- [29] G. Zhai, B. Hu, K. Yasuda, and A. N. Michel, "Disturbance attenuation properties of time-controlled switched systems," *J. Franklin Inst.*, vol. 338, no. 7, pp. 765–779, 2001.
- [30] C. Briat, "Stability analysis and stabilization of stochastic linear impulsive, switched and sampled-data systems under dwell-time constraints," *Automatica*, vol. 74, pp. 279–287, Dec. 2016.
- [31] Y. Li, H. Zhang, and L. Zhang, "Equivalence of several stability conditions for switched linear systems with dwell time," *Int. J. Robust Nonlinear Control*, vol. 29, no. 1, pp. 306–331, 2018.
- [32] A. Koru, A. Delibaşi, and H. Özbay, "Dwell time-based stabilisation of switched linear delay systems using clock-dependent Lyapunov–Krasovskii functionals," *Int. J. Control*, to be published. doi: 10.1080/00207179.2018.1500036.
- [33] C. Briat, "Stability analysis and state-feedback control of LPV systems with piecewise constant parameters subject to spontaneous Poissonian jumps," *IEEE Control Syst. Lett.*, vol. 2, no. 2, pp. 230–235, Apr. 2018.
- [34] Y. Li, H. Zhang, and Q. Zheng, "Robust stability and L₁-gain analysis of interval positive switched T-S fuzzy systems with mode-dependent dwell time," *Neurocomputing*, vol. 235, pp. 90–97, Apr. 2017.

- [35] W. Xiang, "Necessary and sufficient condition for stability of switched uncertain linear systems under dwell-time constraint," *IEEE Trans. Autom. Control*, vol. 61, no. 11, pp. 3619–3624, Nov. 2016.
- [36] Y. Li, H. Zhang, and W. Xiang, "New alternative convex conditions on exponential stability and stabilisation of switched positive linear systems with dwell time," *IET Control Theory Appl.*, vol. 13, no. 5, pp. 620–631, Mar. 2019.
- [37] A. Papachristodoulou, J. Anderson, G. Valmorbida, S. Prajna, P. Seiler, and P. Parrilo, "SOSTOOLS Version 3.00 sum of squares optimization toolbox for MATLAB," 2013, arXiv:1310.4716. [Online]. Available: https://arxiv.org/abs/1310.4716
- [38] J. F. Sturm, "Using SeDuMi 1.02, a MATLAB toolbox for optimization over symmetric cones," *Optim. Methods Softw.*, vol. 11, nos. 1–4, pp. 625–653, 1999.



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