

A Fuzzy Receding Horizon Control Strategy for Dynamic Vehicle Routing Problem

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ABSTRACT The receding horizon control (RHC) combining with the various intelligent algorithms is a common method for the dynamic vehicle routing problem (DVRP). However, the traditional RHC only considers the objects within each time window while making route plan, and can't make adjustment according to the situations of the objects near the window. In order to deal with this problem, a fuzzy receding horizon control strategy (FRHC) is proposed. By combining the RHC and the membership function theory, the relationship between objects and time window is redefined. And the travel routes are planned by the genetic algorithm (GA) for each fuzzy time window. Finally, ten instances are selected from the DVRP standard test library to verify the proposed strategy. The experimental results show that when comparing with the RHC strategy, the FRHC can reduce the distance, the waiting time of all customers and the number of waiting customers dramatically. The FRHC combines with the GA (FRHC-GA) method is also reasonable and effective.

INDEX TERMS Dynamic vehicle routing problem, fuzzy control, membership function, receding horizon control.

I. INTRODUCTION

The Vehicle Routing Problem (VRP) is a classical NP-hard problem in the field of operations research, is always a hot topic [1]–[6]. It aims to design an optimal route for a number of vehicles in serving a set of customers. The vehicles serve each customer in an orderly manner to get the plan with the shortest distance or the shortest waiting time under some constraints. The VRP is mainly divided into two categories according to its characteristics: the Static VRP (SVRP) and the Dynamic VRP (DVRP). The main feature of the SVRP is that all the information of the environment such as the customer demands and travel costs is known and unchanged. However, this assumption is rarely true in real life, where the environment is often changing over time, e.g. a new customer request arrives while the vehicles are on their routes. In such a dynamic environment, the theories and the solution methods of the SVRP are no longer applicable.

The DVRP is first proposed by Psaraftis [7], [8]. The main difference between the DVRP and the SVRP is

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that the information of customers (e.g. demand, address, service time, etc.) may change with time. To solve DVRP, many scholars have proposed various optimization algorithms [9]–[25]. These approaches can be roughly divided into three categories. (1) The original travel route is generated at the beginning of the system. The system will modify the original travel route when the dynamic information generates [10]. (2) The original travel route is generated at the beginning of the system. And the system arranges other vehicles for the dynamically changing customers. (3) The system time is divided into a number of time windows which are called time segments. At the beginning of each time window, the system collects the dynamic change information generated in the previous time window, and arranges the vehicles for the customers whose service time is in the current time window [9].

Among the above three approaches, the vehicles are required to be fully loaded at the depot in the first approach. Then the vehicles can serve the newly generated customers in time. Nevertheless for the express delivery and the take-away delivery, the vehicles must return to the depot for replenishment when a new customer generates.

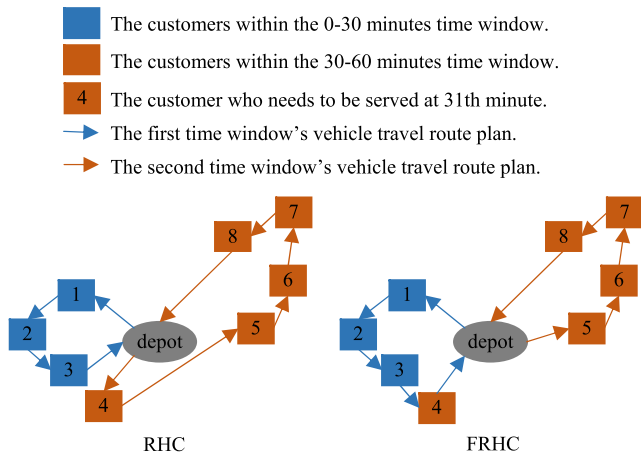


FIGURE 1. The example travel route graph.

Therefore this approach has limitations [12]. In the second approach, the system arranges the vehicles individually for the dynamically changing customers. There is no doubt that this approach increases the cost of distribution. In the third solution, the system time is divided into a number of time windows. The system just need to arrange the vehicles for the customers whose service time is in the current time window. This solution reduces the computational pressure of adjusting the travel plan and restores the dynamic nature of the DVRP, and its optimization effect is verified. It's also a common problem-solving method. This method is often referred to as RHC [26]–[31], it is widely used in the flight scheduling problem [29], [30], the dynamic scheduling problem, the unmanned combat aerial vehicles [26], [27], and the big data research.

The essence of RHC is dividing the whole horizon into a number of time windows. And the system makes each vehicle travel plan for each time window. But RHC generally uses average allocation when dividing time windows, and the setting of the time window is too strict. For example: a system time is divided into the 0-30 minutes and the 30-60 minutes two time windows, and there's a customer who needs to be served at 31th minute. This customer must be divided into 30-60 minutes time window for serving. But this customer is in the same area with customers within the first time window, like Figure 1. The travel route will be better if this customer was traveled in the first time window.

As for the above situation, this paper proposes a FRHC strategy is proposed, which combines the theories of the RHC and the fuzzy set [15], [32]–[36], and defines the membership function [15] of the FRHC. By blurring the end time of the time window, a more efficient and reasonable strategy of the optimizing the DVRP is produced.

The main contributions of this paper have three aspects: 1. this paper proposes the FRHC strategy, and the membership functions of the FRHC is defined; 2. the FRHC optimizes the RHC strategy. The RHC is a common strategy for the dynamic problems. The traditional RHC only considers the objects within current time window, can't make overall plan

according to the situations of the next time window. The FRHC is proposed to optimize this problem. Through experimental comparison, it is verified that results of the FRHC are indeed better than RHC; 3 the DVRP problem is solved by using the FRHC combined with the GA method in this paper, and the experimental results are compared with other algorithms. It is verified that the practicality of the FRHC for solving the DVRP.

II. PROBLEM DESCRIPTION AND MATHEMATICAL MODEL

A. PROBLEM DESCRIPTION

The DVRP is a problem in which a set of dynamically changing customers are served by a number of vehicles. The customers here are generally divided into two categories: the early customers and the late customers [9], [12]. The customers who have been obtained before the start of the system service time are called the early customers. They may not be served during last system service cycle or be obtained during system rest time. The late customers refer to the system receiving new customers continuously during service time, or the early customer changing their demand. As for the early customer changing their demand, it is often called disturb. This paper considers the vehicle travel route coordinated by the early customers and the later customers. To ensure that each customer's need is satisfied, and the customers' demands of each vehicle do not exceed the maximum capacity of the vehicle. The problem aiming is to minimize the vehicle traveled distance and the waiting time of all customers.

DVRP can be defined as follow. There is a depot v_0 with service time $[0-SystemTime]$. $C = \{c_1, c_2, \dots, c_K\}$ is a set of the vehicles. There are K vehicles, and the capacity of vehicle is Q . $V = \{v_1, v_3, \dots, v_I\}$ is a set of the customers. The customers V contains the early customers $VE = \{v_1, v_3, \dots, v_{I'}\}$ and the late customers $VL = \{v_2, v_5, \dots, v_{I''}\}$. Each customer includes the system reception time GT , the expected service time PT , the demand M , the actual arrival time AT , the abscissa X , the ordinate Y , and the waiting time WT . The vehicles start from the depot v_0 and serves the customers in the V . If the customer's AT before the PT , the vehicles need to wait, and the PT of the customer is taken as the departure time of the next customer. Otherwise, the AT of the customer is taken as the departure time of the next customer. Each customer can be served only by one car and once. The aiming is to minimize the vehicle traveled distance and the waiting time of all customers.

B. MATHEMATICAL MODEL

For convenience of description, the symbol or variable is defined as:

- ST : the depot's service time or the system time;
- v_0 : the depot;
- V : a set of the customers;
- I : the number of customers;
- VE : the early customers, $VE \subseteq V$;

VL : the late customers, $VL \subseteq V$;
 v_i : the i th customer, $v_i \in V$;
 X_i : the abscissa of the i th customer;
 Y_i : the ordinate of the i th customer;
 M_i : the demand of the i th customer;
 GT_i : the system reception time of the i th customer;
 PT_i : the expected service time of the i th customer;
 AT_i : the arrival time of the i th customer;
 WT_i : the waiting time of the i th customer;
 C : a set of the vehicles;
 K : the number of vehicles;
 c_k : the k th vehicle;
 Q : the capacity of the vehicle;
 $Speed$: the travel speed;
 L_{ij} : the distance cost from the i th customer to the j th customer;

X_{ijk} : from the i th customer to the j th customer is served by the k th vehicle;

α : adjustment coefficient of the objective function;

According to the above problem descriptions, the model and objective function of minimizing the total cost of the DVRP optimization are as follows:

$$WT_i = \begin{cases} 0, & \text{if } AT_i \leq PT_i \\ AT_i - PT_i, & \text{otherwise} \end{cases} \quad (1)$$

$$F = \sum_{i=0}^I \sum_{j=0}^I \sum_{k=1}^K L_{ij} X_{ijk} + \alpha \sum_{i=1}^I WT_i \quad (2)$$

$$V = VE \cup VL, \quad VE \neq \emptyset, \quad VL \neq \emptyset \quad (3)$$

$$\sum_{i=0}^I M_i \leq KQ \quad (4)$$

$$\sum_{i=0}^I \sum_{k=0}^K X_{ijk} = 1 \quad (5)$$

$$\sum_{j=0}^I \sum_{k=0}^K X_{ijk} = 1 \quad (6)$$

$$\sum_{i=0}^I \sum_{j=0}^I M_i X_{ijk} \leq Q \quad (7)$$

The equation (1) is the calculation method of the waiting time. The formula (2) is the calculation method of the objective function. The equation (3) means that both early and late customers exist. The equation (4) means that the total demands for all customers are less than the total capacities of all vehicles. The equations (5) and (6) ensure that there is only one travel route between any two customers. The equation (7) means that the demands of the customers in one vehicle do not exceed the maximum capacity of the vehicle.

However, in real life, the model can be modified according to different problems. For example, the model only consider one vehicle can server several customers at a time, without considering the customer's demand and the vehicle capacity.

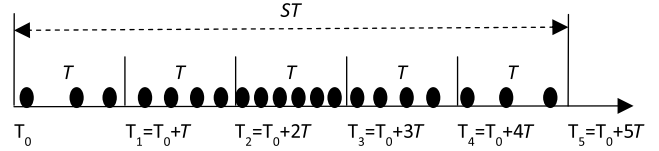


FIGURE 2. The RHC example graph.

III. METHODOLOGY

This chapter introduces the specific ideas of the FRHC to solve the DVRP. The theoretical basis and model definition of the FRHC are introduced in detail. And two examples are given to illustrate the advantages of the FRHC. The framework and method steps of the FRHC combines the GA (FRHC-GA) for solving the DVRP are introduced, and the GA is modified according to the model of the problem.

A. FRHC METHOD

1) RHC METHOD

The system time ST is divided into N time windows, the time window size is T . The start time and the end time of the system service time are T_0 and $T_0 + NT$. The start time and the end time of the n th ($n > 0$ and $n \leq N$) time window are $TS_n = T_0 + (n - 1)T$ and $TE_n = T_0 + nT$.

As shown in Figure 2. It is an example for the RHC optimizing the DVRP. For the processing of the n th time window, all objects in current time window are optimally sorted by one intelligent algorithm, such as the genetic algorithm. Generally, one time window is served by one car, and the unprocessed objects are put into next time window. Those objects can be prioritized in next time window. The DVRP is optimized by scrolling the time window and repeating the above steps.

2) FRHC STRATEGY ANALYSIS AND MODEL DEFINITION

Under the traditional set theory, customers in the same time window are called a set. The customers are called the elements. The relationship between the elements and a set is only “belonging” and “not belonging”.

Definition: The set A is the customers in the n th time window, for any customer v_i in customer V , v_i only has two cases of “belonging to A ” or “not belonging to A ”. This feature can be expressed as a function:

$$V_A(v_i) = \begin{cases} 1, & \text{if } v_i \in A \\ 0, & \text{otherwise} \end{cases} \quad (8)$$

Or

$$V_A(v_i) = \begin{cases} 1, & \text{if } TS_n \leq PT_i < TE_n \\ 0, & \text{otherwise} \end{cases} \quad (9)$$

$V_A(v_i)$ is the feature function of the set A .

The customers who are not in the set A will be served in the other time windows. In fact, some customers do not belong to the set A , but their PT_i are very close to the end time of the n th time window. If these customers can be served together with the customers in set A , the entire travel route will be optimized.

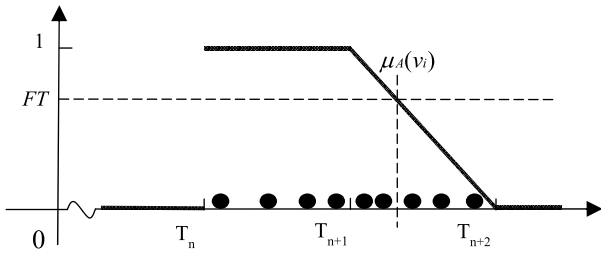


FIGURE 3. The membership function graph.

For example, the size of the time window is 30 minutes, there is a customer’s *PT* is 31th minute. At this time, this customer should be served in the second time window, but 31th minute is also very close to 0-30 minutes time window. So it is said that there is 29/30 similarity degree between this customer with the customers in the first time window, and 1/30 not similar. This 29/30 is just a degree of similarity. In a fuzzy set, the similarity degree of an element belonging to a fuzzy set is called membership degree.

In the classical set method, $V_A(v_i)$ only has values 0 and 1, which are extended to intervals $[0, 1]$ in fuzzy set method. Let the fuzzy set \tilde{A} be a mapping from the domain V to $[0, 1]$:

$$\tilde{A} \rightarrow [0, 1] \tag{10}$$

$$v_i \mapsto \tilde{A}(v_i) \tag{11}$$

\tilde{A} is the fuzzy set on domain V , \tilde{A} is the membership function of the fuzzy set \tilde{A} . The membership function can be defined by functions such as the trigonometric functions, the trapezoidal functions, and the Gaussian functions.

For the convenience of calculation, the membership function in this paper is defined by a piecewise function, and the membership function $\mu_A(v_i)$ is defined as follows:

$$\mu_A(v_i) = \begin{cases} 1, & \text{if } TS_n \leq PT_i \leq TE_n \\ 1 - \frac{PT_i - TS_n}{T}, & \text{if } PT_i > TS_{(n+1)} \text{ and } PT_i < TE_{(n+1)} \\ 0, & \text{otherwise} \end{cases} \tag{12}$$

$\mu_A(v_i)$ indicates the degree of ambiguity of customers in adjacent time windows. But how to distinguish if the customer can join the current time window, an important parameter is also required: the decision-making threshold value $FT \in [0, 1]$, $\mu_A(v_i) \geq FT$ can extract some customers form next time window into the fuzzy time window for service.

B. THE FRHC-GA FRAMEWORK AND STEPS OF FRHC

Step1: Initialize the FRHC. According to the service time of depot, the system set the value of the time window size T , the membership function and the value of FT .

Step2: Get the customers information. If it is the first time window: (1) get all the early customers information, (2) get the late customers information in the current time window, (3) use the membership function and the FT value, extract all the customers that can be served in the current time window and arrange the service for them. If it is not the first time window:

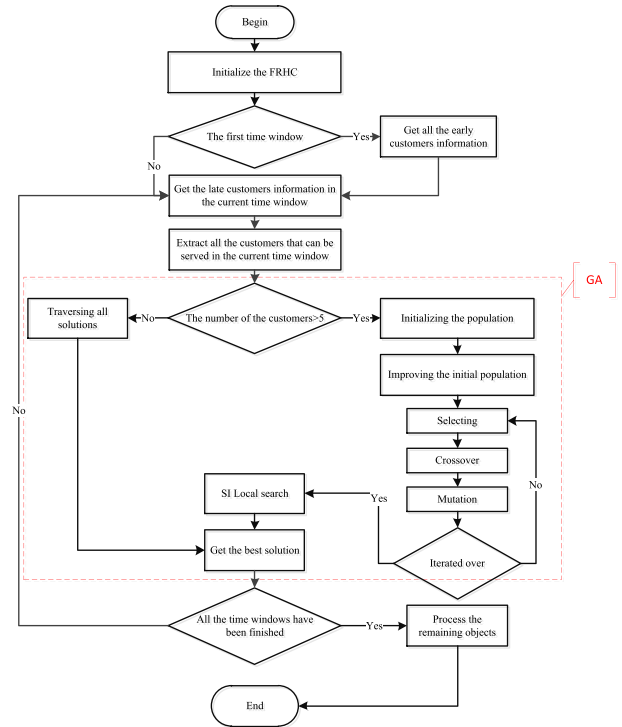


FIGURE 4. The framework flow chart.

(1) get the late customers information in current time window, (2) use the membership function and the FT value, extract all the customers that can be served in current time window and arrange the services for them.

Step3: Make the travel route. The extracted customers will be optimized the travel route by the GA, with the manner of priority serving the previous time window customers. The customers not served in this time window are handed over to the next time window.

Step4: Scroll the time window. Go to Step2 to continue in the next window, until all the windows have been finished.

Step5: Process the remaining objects. For the customers who has not yet served, the system will arrange a vehicle for service (regardless of various restrictions).

C. GENETIC ALGORITHM

The FRHC can dynamically dispatch vehicles to serve customers. As for customers whose *PT* is in current fuzzy time window, the system use the GA algorithm to generate the vehicle route. According to the structure and characteristics of the DVRP model, the traditional GA has been partially adjusted.

The GA mainly includes “initializing the population”, “improving the initial population”, “selecting”, “crossover” and “mutation”. It combines the local search to optimize the solution. In this paper, for the case where the number of customers does not exceed 5, the optimal solution is found by traversing all individuals. The specific design of each part will be carefully described below.

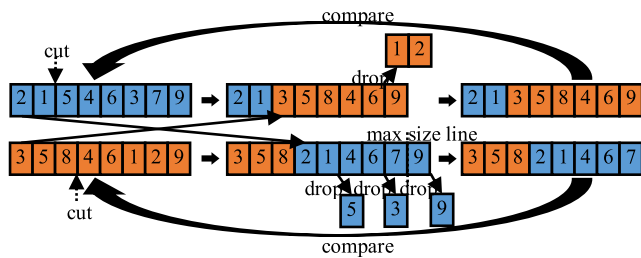


FIGURE 5. Crossover operator graph.

(1) Initialize the population. The algorithm uses the customer sorting number to encode. According to the customer number in each fuzzy time window, many random customer number sequences are generated. The algorithm traverse each sequence, reads each customer’s information orderly from the beginning to the end, and judging if the total number of served customers exceeds the limit. If the limit is not exceeded, the sequence will be retained; if it is exceeded, the previously read customer number will be intercepted as a new sequence and retained. A sequence here is called an individual, and a group of sequences is called a population.

(2) Evaluation function: $P = 1000/F$. The P is the value of the evaluation function, and the F is the value of the objective function in DVRP. The smaller the F value, the larger the P value is, and the better the individual is.

(3) Improving the initial population. 20 individuals are selected randomly, and sorts by the customer’s PT from small to large.

(4) Selecting. The P value of each individual is calculated according to the evaluation function. Two individuals are selected for crossover using the roulette algorithm.

(5) Crossover. For the selected individuals, a random number is generated as the dividing point respectively. This random number is less than half of the length of the sequence. The sequence of the previous part after cutting is retained. The customers are added orderly according to another individual original sequence. The repeated customer will be dropped, and others will be retained. After a new sequence generated, it is necessary to determine if the customers’ demands exceeds the limit of the vehicle capacity. If it is not exceeded, the sequence will be retained, otherwise the sequence will be cut. Then two new individuals are generated. The original individual is compared with the new individual who retains the previous part. If the new individual’s P value is smaller, then the new individual will replace the original individual, otherwise the original individual will be retained.

(6) Mutation. Mutation 1: a customer in the individual is selected randomly, the customer is inserted into all positions of the sequence sequentially, and the P values of all inserted individuals are compared, and the individual with the smallest P value are retained. Mutation 2: the customer with the longest waiting time is found, this customer is inserted into all positions of the sequence sequentially, and the P values of all inserted individuals are compared, and the individuals with the smallest P value are retained.

(7) Local search. After the iteration is over, all individuals are optimized by the Single Insertion (SI) local search method. The SI is a commonly used local search operator for solving the VRP. For a individual, one customer is extracted in turn, and this customer is inserted into all positions of the sequence sequentially, and the P values of all inserted individuals are compared, and the individual with the smallest P value are retained.

IV. EXPERIMENTAL STUDIES

The experimental environment of this paper is Intel Core i5-7500 CPU with main frequency of 3.4 GHz and 8G memory hardware platform. The algorithm is written in JAVA language and simulated for the vehicle travel route of the DVRP with the FRHC strategy. Multiple instances in the DVRP standard test library are verified by the FRHC-GA method. The experimental results are compared with the results obtained by RHC, the first come first served algorithm (FCFS), the PT sorting method (FAST), the nearest neighborhood search algorithm (NN), and the objective function nearest neighborhood method (NNF).

A. INSTANCE DESIGN AND PARAMETER SETTING

The instances are from the VRP standard test library, they can be downloaded from the URL “<http://neo.lcc.uma.es/vrp/vrp-instances/>”. The instance includes some necessary information (e.g. the depot, the abscissa X , the ordinate Y , and the demand M). In order to reflect the dynamics of the DVRP, this experiment adds the GT and the PT information based on those instances. In this paper, 10 instances are selected for simulation, such as A-n32-k5(n32k5), A-n45-k6(n45k6) and so on. The number of customers in those instances ranges from 32 to 65. Those instances can verify the effectiveness of the FRHC in the different customer group size.

Each customer’s PT is assigned in a random manner and distributed within the ST evenly. The ratio of early customers to late customers is 4:1. In this proportion, the customers are selected as the late customers randomly, and the late customers’ GT s are assigned (0, PT). Table 1 selects the A-n61-k9 instance for data display.

Because of each customer’s PT taking a random way to assign values, here is a statistical analysis of the data in the above table. There are 60 customers in addition to the depot v_0 , the PT s of these 60 customers are counted and a histogram is drawn as shown in Figure 6. The customer’s PT line is tortuous. Some customers have small customer numbers, but their PT is big. The PT s of these 60 customers are sorted from small to large, and a histogram is drawn as shown in Figure 7. It can be seen that the customer’s PT line can be regarded as a straight line approximately. From this it can be verified that the PT assigned randomly obeys the average distribution approximately.

This experiment uses multiple instances to simulate DVRP. For the convenience of experiment, the number of vehicles K is not limited, each vehicle can serve up to 8 customers, and the travel speed is $Speed = 20\text{km/h}$. The service start time

TABLE 1. n61k9 instance data display.

| No. | X | Y | M | GT | PT | No. | X | Y | M | GT | PT | No. | X | Y | M | GT | PT |
|-----|------|------|----|-------|-------|-----|------|------|----|-------|-------|-----|------|------|----|-------|-------|
| / | (km) | (km) | / | (min) | (min) | / | (km) | (km) | / | (min) | (min) | / | (km) | (km) | / | (min) | (min) |
| 0 | 61 | 37 | 0 | 0 | 0 | 21 | 49 | 99 | 10 | 0 | 64 | 41 | 27 | 73 | 14 | 0 | 167 |
| 1 | 93 | 57 | 23 | 1 | 7 | 22 | 63 | 9 | 19 | 0 | 119 | 42 | 31 | 21 | 11 | 51 | 174 |
| 2 | 15 | 67 | 17 | 0 | 28 | 23 | 47 | 37 | 22 | 4 | 93 | 43 | 47 | 9 | 19 | 14 | 189 |
| 3 | 23 | 43 | 12 | 0 | 1 | 24 | 33 | 47 | 19 | 0 | 98 | 44 | 87 | 45 | 16 | 0 | 204 |
| 4 | 53 | 5 | 6 | 0 | 7 | 25 | 39 | 69 | 9 | 8 | 111 | 45 | 1 | 49 | 19 | 0 | 198 |
| 5 | 13 | 75 | 22 | 0 | 9 | 26 | 49 | 3 | 18 | 0 | 118 | 46 | 1 | 77 | 3 | 0 | 186 |
| 6 | 29 | 73 | 3 | 0 | 29 | 27 | 49 | 87 | 2 | 68 | 106 | 47 | 63 | 73 | 12 | 0 | 192 |
| 7 | 47 | 37 | 24 | 0 | 21 | 28 | 87 | 39 | 18 | 107 | 116 | 48 | 79 | 71 | 10 | 0 | 206 |
| 8 | 23 | 71 | 24 | 0 | 30 | 29 | 37 | 91 | 11 | 0 | 140 | 49 | 21 | 55 | 20 | 0 | 204 |
| 9 | 67 | 45 | 11 | 30 | 50 | 30 | 19 | 33 | 19 | 0 | 144 | 50 | 65 | 23 | 7 | 0 | 235 |
| 10 | 21 | 49 | 7 | 0 | 42 | 31 | 97 | 35 | 18 | 0 | 123 | 51 | 65 | 47 | 13 | 0 | 216 |
| 11 | 93 | 43 | 12 | 30 | 54 | 32 | 31 | 5 | 15 | 0 | 120 | 52 | 97 | 23 | 16 | 132 | 233 |
| 12 | 67 | 13 | 8 | 0 | 45 | 33 | 35 | 25 | 4 | 0 | 148 | 53 | 23 | 71 | 23 | 0 | 213 |
| 13 | 69 | 25 | 14 | 0 | 36 | 34 | 79 | 61 | 12 | 0 | 149 | 54 | 5 | 81 | 22 | 0 | 239 |
| 14 | 53 | 35 | 20 | 0 | 45 | 35 | 73 | 73 | 8 | 0 | 120 | 55 | 53 | 27 | 18 | 0 | 236 |
| 15 | 25 | 39 | 16 | 0 | 85 | 36 | 35 | 95 | 18 | 0 | 178 | 56 | 57 | 85 | 6 | 83 | 229 |
| 16 | 85 | 69 | 16 | 10 | 70 | 37 | 5 | 43 | 12 | 149 | 171 | 57 | 89 | 23 | 12 | 0 | 232 |
| 17 | 81 | 27 | 4 | 47 | 81 | 38 | 19 | 45 | 72 | 0 | 162 | 58 | 51 | 65 | 27 | 216 | 232 |
| 18 | 77 | 79 | 9 | 0 | 88 | 39 | 71 | 39 | 2 | 0 | 160 | 59 | 13 | 49 | 9 | 129 | 221 |
| 19 | 45 | 43 | 18 | 25 | 86 | 40 | 35 | 63 | 5 | 0 | 165 | 60 | 91 | 41 | 15 | 0 | 222 |
| 20 | 31 | 75 | 14 | 0 | 71 | | | | | | | | | | | | |

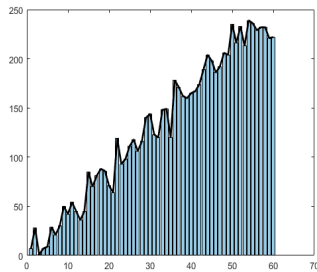


FIGURE 6. The PT histogram.

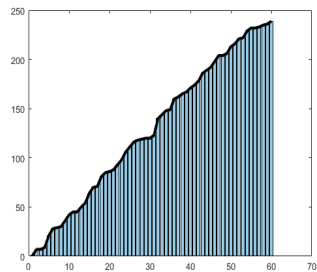


FIGURE 7. The PT histogram2.

is $T_0 = 0h$, the system service time is $ST = 4h$, and a time window size is $T = 0.5h$.

The objective function is a weighted sum of the travel distance and the customers waiting time. In order to balance the weight of the travel distance and the customers waiting time, the adjustment coefficient of the objective function α is set to 20. In the GA, the number of initialized populations

is 200, and 180 individuals are selected for crossover during each iteration (the selection rate is 0.9), and all individuals are mutated. The algorithm run 300 generations to end the iteration.

B. FRHC ANALYSIS

1) MEMBERSHIP FUNCTION AND THRESHOLD VALIDITY ANALYSIS

In order to explore the validity of the membership function in FRHC, five values are selected for the FT ($FT = \{29/30, 27/30, 25/30, 23/30, 21/30\}$). A set of experiments use these five values to simulate all instances. The results compare the optimal solution obtained by RHC to discuss the validity of membership function and the values of FT .

According to different instances, the algorithm runs 30 times to get the best solution in the cases of the RHC and the different FT values. From the two aspects of the average value and the minimum value, the objective function F , the travel distance L , the customers waiting time WT and the number of waiting customers WN (The customers whose AT are later than PT .) are counted. The specific data are shown in Tables 2 and 3.

Table 2 shows the minimum values of the F based on each instance running 30 times. Compared with the solution of the RHC: there are 46 solutions (92%) with the F values less than the RHC's F values, and 4 solutions (8%) are flat; there are 35 solutions (70%) with the L values less than the RHC's L values, and 5 solutions (10%) are flat; and there are 46 solutions (92%) with the WT values less than the

TABLE 2. The minimum value of the solution at different FT values.

| | F | | | | | L | | | | | WT | | | | | WN | | | | | | | | |
|--------|------|-------|-------|-------|-------|-------|------|-------|-------|-------|-------|-------|-----|-------|-------|-------|-------|-------|-----|-------|-------|-------|-------|-------|
| | RHC | 29/30 | 27/30 | 25/30 | 23/30 | 21/30 | RHC | 29/30 | 27/30 | 25/30 | 23/30 | 21/30 | RHC | 29/30 | 27/30 | 25/30 | 23/30 | 21/30 | RHC | 29/30 | 27/30 | 25/30 | 23/30 | 21/30 |
| n32k5 | 3455 | 3455 | 3455 | 2751↓ | 2751↓ | 2890↓ | 2342 | 2342 | 2342 | 2206↓ | 2206↓ | 2325↓ | 56 | 56 | 56 | 27↓ | 27↓ | 28↓ | 10 | 10 | 10 | 9↓ | 9↓ | 10 |
| n45k7 | 4043 | 3934↓ | 3463↓ | 3359↓ | 3359↓ | 3302↓ | 2642 | 2721↑ | 2827↑ | 2722↑ | 2722↑ | 2666↑ | 70 | 61↓ | 32↓ | 32↓ | 32↓ | 32↓ | 18 | 15↓ | 15↓ | 15↓ | 15↓ | 15↓ |
| n46k7 | 3448 | 3151↓ | 3151↓ | 3040↓ | 2837↓ | 2859↓ | 2541 | 2485↓ | 2485↓ | 2367↓ | 2216↓ | 2217↓ | 45 | 33↓ | 33↓ | 34↓ | 31↓ | 32↓ | 18 | 15↓ | 15↓ | 14↓ | 14↓ | 15↓ |
| n53k7 | 4865 | 4049↓ | 3931↓ | 3867↓ | 3920↓ | 3765↓ | 3010 | 2920↓ | 2839↓ | 2929↓ | 2792↓ | 2813↓ | 93 | 56↓ | 55↓ | 47↓ | 56↓ | 48↓ | 26 | 25↓ | 23↓ | 22↓ | 20↓ | 22↓ |
| n54k7 | 5624 | 5624 | 5600↓ | 4934↓ | 4364↓ | 4225↓ | 3271 | 3271 | 3272↑ | 3394↑ | 3325↑ | 3094↓ | 118 | 118 | 116↓ | 77↓ | 52↓ | 57↓ | 19 | 19 | 18↓ | 16↓ | 16↓ | 17↓ |
| n55k9 | 5540 | 5540 | 4585↓ | 4585↓ | 4498↓ | 4148↓ | 2948 | 2948 | 2720↓ | 2720↓ | 2637↓ | 2661↓ | 130 | 130 | 93↓ | 93↓ | 93↓ | 74↓ | 25 | 25 | 22↓ | 22↓ | 22↓ | 22↓ |
| n61k9 | 4973 | 4646↓ | 4646↓ | 4578↓ | 4276↓ | 4404↓ | 3031 | 2882↓ | 2882↓ | 2964↓ | 2821↓ | 2862↓ | 97 | 88↓ | 88↓ | 81↓ | 73↓ | 77↓ | 24 | 22↓ | 22↓ | 22↓ | 20↓ | 20↓ |
| n63k10 | 6499 | 5693↓ | 5332↓ | 5422↓ | 5249↓ | 5309↓ | 3256 | 3265 | 3143↓ | 3111↓ | 3074↓ | 2895↓ | 162 | 121↓ | 109↓ | 116↓ | 109↓ | 121↓ | 36 | 36 | 32↓ | 31↓ | 28↓ | 27↓ |
| n63k9 | 7675 | 7457↓ | 7011↓ | 5939↓ | 5910↓ | 5766↓ | 3647 | 3644↓ | 3585↓ | 3447↓ | 3469↓ | 3540↓ | 201 | 191↓ | 171↓ | 125↓ | 122↓ | 111↓ | 31 | 30↓ | 29↓ | 26↓ | 25↓ | 22↓ |
| n64k9 | 6950 | 6268↓ | 6207↓ | 5819↓ | 5139↓ | 5180↓ | 3226 | 3261↑ | 3150↓ | 3184↓ | 3238↑ | 3154↓ | 186 | 150↓ | 153↓ | 132↓ | 95↓ | 101↓ | 35 | 33↓ | 27↓ | 28↓ | 27↓ | 25↓ |

TABLE 3. The average value of the solution at different FT values.

| | F | | | | | L | | | | | WT | | | | | WN | | | | | | | | |
|--------|------|-------|-------|-------|-------|-------|------|-------|-------|-------|-------|-------|-----|-------|-------|-------|-------|-------|-----|-------|-------|-------|-------|-------|
| | RHC | 29/30 | 27/30 | 25/30 | 23/30 | 21/30 | RHC | 29/30 | 27/30 | 25/30 | 23/30 | 21/30 | RHC | 29/30 | 27/30 | 25/30 | 23/30 | 21/30 | RHC | 29/30 | 27/30 | 25/30 | 23/30 | 21/30 |
| n32k5 | 3455 | 3455 | 3455 | 2751↓ | 2751↓ | 2890↓ | 2342 | 2342 | 2342 | 2206↓ | 2206↓ | 2325↓ | 56 | 56 | 56 | 27↓ | 27↓ | 28↓ | 10 | 10 | 10 | 9↓ | 9↓ | 10 |
| n45k7 | 4043 | 3934↓ | 3463↓ | 3359↓ | 3359↓ | 3302↓ | 2642 | 2721↑ | 2827↑ | 2722↑ | 2722↑ | 2666↑ | 70 | 61↓ | 32↓ | 32↓ | 32↓ | 32↓ | 18 | 15↓ | 15↓ | 15↓ | 15↓ | 15↓ |
| n46k7 | 3448 | 3151↓ | 3151↓ | 3040↓ | 2837↓ | 2859↓ | 2541 | 2485↓ | 2485↓ | 2367↓ | 2216↓ | 2217↓ | 45 | 33↓ | 33↓ | 34↓ | 31↓ | 32↓ | 18 | 15↓ | 15↓ | 14↓ | 14↓ | 15↓ |
| n53k7 | 4865 | 4049↓ | 3931↓ | 3867↓ | 3968↓ | 3916↓ | 3010 | 2920↓ | 2839↓ | 2929↓ | 2810↓ | 2817↓ | 93 | 56↓ | 55↓ | 47↓ | 58↓ | 55↓ | 26 | 25↓ | 23↓ | 22↓ | 20↓ | 21↓ |
| n54k7 | 5624 | 5624 | 5600↓ | 4934↓ | 4364↓ | 4225↓ | 3271 | 3271 | 3272↑ | 3394↑ | 3325↑ | 3094↓ | 118 | 118 | 116↓ | 77↓ | 52↓ | 57↓ | 19 | 19 | 18↓ | 16↓ | 16↓ | 17↓ |
| n55k9 | 5540 | 5540 | 4589↓ | 4591↓ | 4498↓ | 4159↓ | 2948 | 2948 | 2731↓ | 2737↓ | 2637↓ | 2661↓ | 130 | 130 | 93↓ | 93↓ | 93↓ | 75↓ | 25 | 25 | 22↓ | 22↓ | 22↓ | 22↓ |
| n61k9 | 4973 | 4646↓ | 4646↓ | 4578↓ | 4305↓ | 4437↓ | 3031 | 2882↓ | 2882↓ | 2964↓ | 2843↓ | 2887↓ | 97 | 88↓ | 88↓ | 81↓ | 73↓ | 77↓ | 24 | 22↓ | 22↓ | 22↓ | 20↓ | 20↓ |
| n63k10 | 6499 | 5693↓ | 5332↓ | 5613↓ | 5414↓ | 5391↓ | 3256 | 3265↑ | 3143↓ | 3030↓ | 2947↓ | 2937↓ | 162 | 121↓ | 109↓ | 129↓ | 123↓ | 123↓ | 36 | 36 | 32↓ | 31↓ | 28↓ | 27↓ |
| n63k9 | 7675 | 7457↓ | 7011↓ | 6089↓ | 6023↓ | 6535↓ | 3647 | 3644↓ | 3585↓ | 3436↓ | 3492↓ | 3590↓ | 201 | 191↓ | 171↓ | 133↓ | 127↓ | 147↓ | 31 | 30↓ | 29↓ | 26↓ | 24↓ | 23↓ |
| n64k9 | 6950 | 6268↓ | 6207↓ | 5819↓ | 5611↓ | 5204↓ | 3226 | 3261↑ | 3149↓ | 3184↓ | 3222↓ | 3168↓ | 186 | 150↓ | 153↓ | 132↓ | 119↓ | 102↓ | 35 | 33↓ | 27↓ | 28↓ | 28↓ | 24↓ |
| AVG | 5307 | 4982↓ | 4739↓ | 4464↓ | 4313↓ | 4292↓ | 2991 | 2974↓ | 2926↓ | 2897↓ | 2842↓ | 2836↓ | 116 | 100↓ | 91↓ | 79↓ | 74↓ | 73↓ | 24 | 23↓ | 21↓ | 21↓ | 20↓ | 19↓ |

RHC’s WT values, and 4 solutions (8%) are flat; and there are 44 solutions (88%) with the WN values less than the RHC’s WN values, and 6 solutions (12%) are flat.

Table 3 shows the average values of the solutions, based on each instance running 30 times. This experiment aiming is to minimize the vehicle traveled distance and the waiting time of all customers. It can be clearly seen that the solutions obtained by the FRHC are generally smaller than the solutions obtained by RHC. This experiment has 5 different values of FT and 10 instances. Compared with the solution of RHC: there are 46 solutions (92%) with the F values less than the RHC’s F values, and 4 solutions (8%) are flat; there are 36 solutions (72%) with the L values less than the RHC’s L values, and 4 solutions (8%) are flat; there are 46 solutions (92%) with the WT values less than the RHC’s W values, and 4 solutions (8%) are flat; and there are 44 solutions (88%) with the WN values less than the RHC’s WN values, and 6 solutions (12%) are flat.

In some case, the L values increase slightly, such as the n45k7 instances in Table 2 and 3. This is a normal situation, because of the objective function $F = L + \alpha WT$. When the magnitude of WT decreases is significantly greater than the magnitude of L increase, the value of F is also smaller.

The AVG data in Table 3 is the average of all instances. In order to discuss which value is more reasonable from the five values of $FT = \{29/30, 27/30, 25/30, 23/30, 21/30\}$, the AVG data fold-line graph is drawn, Figure 8. The abscissa represents RHC, $FT = 29/30$, $FT = 27/30$, $FT = 25/30$, $FT = 23/30$, $FT = 21/30$ from left to right. The solid line represents F. The dotted line represents the WN.

From this fold-line graph, it can be seen that the lines show a downward trend with the decrease of the FT value. It can be seen from the line of F and WN that the lines decreases with a certain slope (the slope is a number less 0), but the slope of the fall at $FT = 23/30$ becomes larger; the line

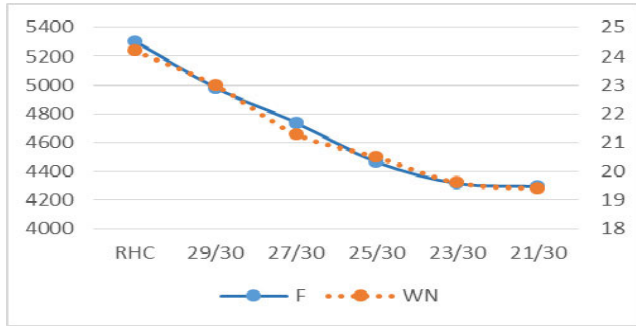


FIGURE 8. The AVG data's *F* and *WN* fold-line graph.

of *WT* has the same situation at $FT = 23/30$. Therefore, it is inferred that the $FT = 23/30$ of FRHC can get the best effect.

From the data from Table 2 and Table 3, it can be seen that all solutions obtained by the FRHC are better than the solutions obtained by the RHC. Therefore, it can be said that the model of the FRHC and membership function designed in this experiment are effective. By comparing the results of the values of the *FT*, it is determined that this FRHC model has the best optimization effect when *FT* is 23/30. The FRHC mentioned in the experiments below uses this parameter by default.

2) FRHC ADVANTAGE ANALYSIS

The traditional RHC has too strict rules for setting the time window. It only serves the customers in current time window. The FRHC uses the membership function and the decision-making threshold value to blur the time window. It can extract some customers in other time windows, and join them in the fuzzy time window for service. And it optimizes the travel route compared to the RHC. Here are two simulation scenarios to analyze the advantages of FRHC.

Scenario 1, the customers are concentrated in some time windows. This has caused some customers can't be serviced quickly. But the vehicle still has service capability in other time windows. The FRHC extract some customers in other time windows and, join them in the fuzzy time window for service. In this way, it can reduce the concentration of customers and optimize the vehicle travel route, as shown in Figure 9.

Scenario 2, the customers in the DVRP are randomly appearing, so the locations of customers within a time window are random. One or two more faraway customers may appear in each time window. If the vehicle serve those customer in current time window, it will cause other customers waiting. And these customers locations are close to the customers' area in the next time window. At the same time, the next time window has one or two customers locations are close to the customers' area in the current time window. If these two time windows can exchange these faraway customers, it will reduce customers' average waiting time and vehicle's travel distance.

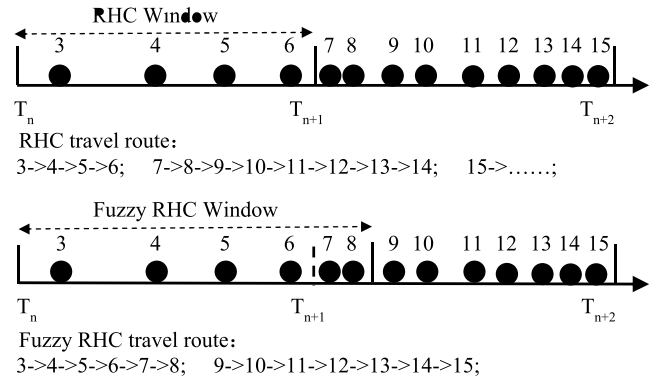


FIGURE 9. The FRHC optimization effect example for scenario 1 graph.

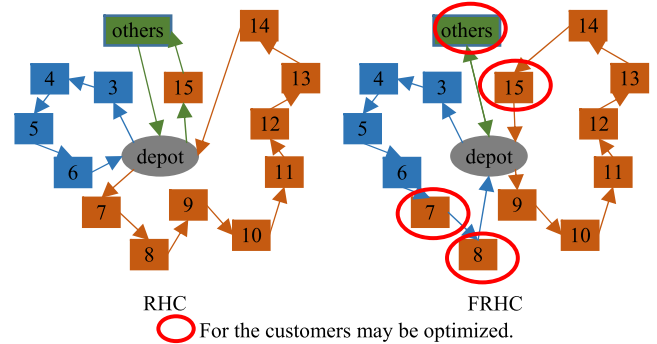


FIGURE 10. The FRHC optimization routing example for scenario 1 graph.

C. COMPARISON ALGORITHM

The first come first served algorithm (FCFS) and the nearest neighborhood search algorithm (NN) are also classic algorithms for the DVRP besides GA. Here lists four algorithm: the FCFS, the NN, the *PT* sorting method (FAST) and the objective function nearest neighborhood method (NNF).

FCFS, the first come first served algorithm. The customers are sorted according to the order of the customers' *GT*s from small to large. The vehicle travels to the sorted customers in turn. If the customer's *AT* before the *PT*, the vehicles need to wait, and the *PT* of customer is taken as the departure time of the next customer. Otherwise, the *AT* of customer is taken as the departure time of the next customer.

FAST, the *PT* sorting method. The customers are sorted according to the order of the customers' *PT*s from small to large. The vehicle travels to the sorted customers in turn. If the customer's *AT* before the *PT*, the vehicles need to wait, and the *PT* of customer is taken as the departure time of the next customer. Otherwise, the *AT* of customer is taken as the departure time of the next customer.

NN, the nearest neighborhood search algorithm. The starting position of vehicle is the depot. The system calculates the distances from all unserved customers to the current location, and finds the nearest customer as the next customer. If the customer's *AT* before the *PT*, the vehicles need to wait, and the *PT* of customer is taken as the departure time of the

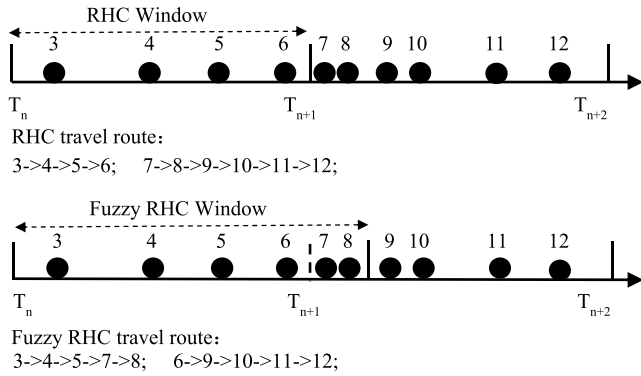


FIGURE 11. The FRHC optimization effect example for scenario 2 graph.

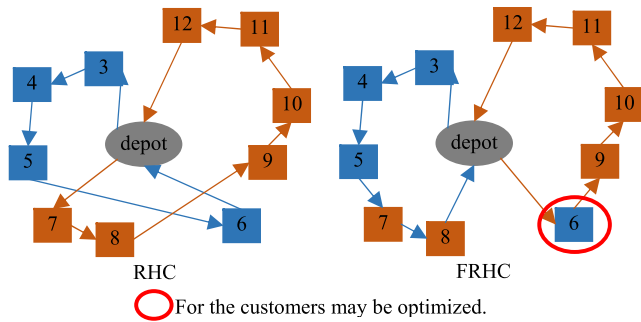


FIGURE 12. The FRHC optimization routing example for scenario 2 graph.

next customer. Otherwise, the *AT* of customer is taken as the departure time of the next customer.

NNF, the objective function nearest neighborhood method. The starting position of the vehicle is the depot. The system calculates the objective function values from all unserved customers to the current location, and finds the customer whose objective function value is nearest as the next customer. If the customer's *AT* before the *PT*, the vehicles need to wait, and the *PT* of customer is taken as the departure time of the next customer. Otherwise, the *AT* of customer is taken as the departure time of the next customer.

D. EXPERIMENTAL RESULTS AND ANALYSIS

As the 3th solution method introduced in the Introduction, the method is generally divided into two parts for solving the DVRP. The first step is dividing the system time into some time windows. The second step is optimizing the travel routes for each time window by the algorithms, like the GA, the NN, or the FCFS. The FRHC is a new strategy proposed for the first step, and the FRHC-GA method is used to solve the DVRP in this paper. In order to verify the performance of the FRHC and the effectiveness of the FRHC-GA, several sets of experiments are designed.

1) COMPARISON ANALYSIS OF THE FRHC AND THE RHC

In order to further verify the versatility and strategy performance of the FRHC, the FRHC strategy is also

TABLE 4. The average values of the FRHC-GA and the RHC-GA.

| | <i>F</i> | | <i>L</i> | | <i>WT</i> | | <i>WN</i> | | <i>RT</i> (ms) | |
|--------|----------|------|----------|------|-----------|------|-----------|------|----------------|--------|
| | RHC | FRHC | RHC | FRHC | RHC | FRHC | RHC | FRHC | RHC | FRHC |
| n32k5 | 3455 | 2751 | 2342 | 2206 | 56 | 27 | 10 | 9 | 404 | 363 |
| n45k7 | 4043 | 3359 | 2642 | 2722 | 70 | 32 | 18 | 15 | 389 | 1394 |
| n46k7 | 3448 | 2837 | 2541 | 2216 | 45 | 31 | 18 | 14 | 372 | 906 |
| n53k7 | 4865 | 3968 | 3010 | 2810 | 93 | 58 | 26 | 20 | 2141 | 1880 |
| n54k7 | 5624 | 4364 | 3271 | 3325 | 118 | 52 | 19 | 16 | 2127 | 2079 |
| n55k9 | 5540 | 4498 | 2948 | 2637 | 130 | 93 | 25 | 22 | 2125 | 2056 |
| n61k9 | 4973 | 4305 | 3031 | 2843 | 97 | 73 | 24 | 20 | 2496 | 2474 |
| n63k10 | 6499 | 5414 | 3256 | 2947 | 162 | 123 | 36 | 28 | 2489 | 2496 |
| n63k9 | 7675 | 6023 | 3647 | 3492 | 201 | 127 | 31 | 24 | 2478 | 2725 |
| n64k9 | 6950 | 5611 | 3226 | 3222 | 186 | 119 | 35 | 28 | 2726 | 2673 |
| AVG | 5307 | 4313 | 2991 | 2842 | 116 | 74 | 24 | 20 | 1774.7 | 1904.6 |

TABLE 5. The results of the FRHC-FCFS and the RHC-FCFS.

| | <i>F</i> | | <i>L</i> | | <i>WT</i> | | <i>WN</i> | | <i>RT</i> (ms) | |
|--------|----------|-------|----------|------|-----------|------|-----------|------|----------------|------|
| | RHC | FRHC | RHC | FRHC | RHC | FRHC | RHC | FRHC | RHC | FRHC |
| n32k5 | 9122 | 8869 | 2464 | 2494 | 333 | 319 | 20 | 20 | 1 | 2 |
| n45k7 | 16711 | 12132 | 2803 | 2544 | 695 | 479 | 33 | 31 | 1 | 1 |
| n46k7 | 11217 | 8644 | 2861 | 2878 | 418 | 288 | 33 | 29 | 2 | 2 |
| n53k7 | 13720 | 10437 | 3183 | 3099 | 527 | 367 | 38 | 34 | 2 | 2 |
| n54k7 | 17077 | 12113 | 3555 | 3360 | 676 | 438 | 38 | 33 | 1 | 2 |
| n55k9 | 19110 | 13565 | 2798 | 2951 | 816 | 531 | 44 | 40 | 1 | 2 |
| n61k9 | 20202 | 17765 | 2930 | 3192 | 864 | 729 | 48 | 46 | 1 | 1 |
| n63k10 | 21069 | 15320 | 3521 | 3487 | 877 | 592 | 50 | 48 | 1 | 2 |
| n63k9 | 25519 | 18493 | 4254 | 4084 | 1063 | 720 | 49 | 47 | 2 | 2 |
| n64k9 | 21538 | 15197 | 3784 | 3910 | 888 | 564 | 50 | 49 | 2 | 2 |
| AVG | 17528 | 13253 | 3215 | 3200 | 716 | 503 | 40 | 38 | 1.4 | 1.8 |

combined with the FCFS, the FAST, the NN, and the NNF algorithms to solve the DVRP. In addition, the RCH is also combined with these algorithms. Comparing the results under different strategies and algorithms, the objective function *F*, the travel distance *L*, the customers waiting time *WT* and the number of waiting customers *WN*, the calculation time (algorithm runtime) *RT* are counted.

As can be seen from Table 4-8, the FRHC greatly reduces the number of waiting customers. From the AVG data of each table, it can see that and the values of FRHC are smaller than the values of RHC. At the same time, the calculation times of the algorithm between FRHC and RHC are basically equal. It can be seen that the FRHC is universal and its performance is better than that of the RHC.

2) COMPARISON ANALYSIS OF THE FRHC-GA AND OTHER ALGORITHMS

In this paper, the FRHC combined with the GA is proposed to solve the DVRP. In order to further verify the effectiveness

TABLE 6. The results of the FRHC-FAST and the RHC-FAST.

| | <i>F</i> | | <i>L</i> | | <i>WT</i> | | <i>WN</i> | | <i>RT</i> (ms) | |
|--------|----------|-------------|-------------|-------------|-----------|-----------|-----------|-----------|----------------|------|
| | RHC | FRHC | RHC | FRHC | RHC | FRHC | RHC | FRHC | RHC | FRHC |
| n32k5 | 3137 | 2837 | 2366 | 2362 | 39 | 24 | 11 | 10 | 1 | 1 |
| n45k7 | 4572 | 3749 | 2870 | 2912 | 85 | 42 | 23 | 19 | 1 | 2 |
| n46k7 | 3647 | 3211 | 2700 | 2366 | 47 | 42 | 19 | 17 | 2 | 1 |
| n53k7 | 5273 | 4192 | 3176 | 2998 | 105 | 60 | 33 | 26 | 1 | 3 |
| n54k7 | 5997 | 4666 | 3595 | 3575 | 120 | 55 | 28 | 19 | 1 | 2 |
| n55k9 | 4795 | 5364 | 3057 | 3062 | 87 | 115 | 29 | 27 | 2 | 1 |
| n61k9 | 4749 | 4558 | 3240 | 3102 | 75 | 73 | 23 | 25 | 1 | 1 |
| n63k10 | 5711 | 5150 | 3446 | 3390 | 113 | 88 | 39 | 33 | 1 | 1 |
| n63k9 | 8023 | 5599 | 4271 | 4019 | 188 | 79 | 37 | 28 | 2 | 1 |
| n64k9 | 6616 | 5361 | 3525 | 3598 | 155 | 88 | 38 | 32 | 1 | 2 |
| AVG | 5252 | 4469 | 3225 | 3138 | 101 | 67 | 28 | 24 | 1.3 | 1.5 |

TABLE 7. The results of the FRHC-NN and the RHC-NN.

| | <i>F</i> | | <i>L</i> | | <i>WT</i> | | <i>WN</i> | | <i>RT</i> (ms) | |
|--------|--------------|--------------|-------------|-------------|------------|------------|-----------|-----------|----------------|------|
| | RHC | FRHC | RHC | FRHC | RHC | FRHC | RHC | FRHC | RHC | FRHC |
| n32k5 | 7388 | 6306 | 2004 | 1983 | 269 | 216 | 18 | 16 | 1 | 1 |
| n45k7 | 14268 | 10813 | 2043 | 1941 | 611 | 444 | 32 | 29 | 1 | 1 |
| n46k7 | 9952 | 8636 | 2136 | 2023 | 391 | 331 | 30 | 28 | 1 | 1 |
| n53k7 | 11248 | 12517 | 2173 | 2444 | 454 | 504 | 33 | 33 | 2 | 1 |
| n54k7 | 14813 | 14885 | 2419 | 2143 | 620 | 637 | 39 | 36 | 3 | 1 |
| n55k9 | 13153 | 13035 | 2156 | 2134 | 550 | 545 | 38 | 37 | 1 | 1 |
| n61k9 | 14621 | 15698 | 2133 | 1982 | 624 | 686 | 40 | 42 | 2 | 1 |
| n63k10 | 13604 | 16238 | 2284 | 2329 | 566 | 695 | 44 | 46 | 2 | 1 |
| n63k9 | 18924 | 15688 | 2862 | 2649 | 803 | 652 | 47 | 41 | 1 | 2 |
| n64k9 | 15355 | 16955 | 2580 | 2689 | 639 | 713 | 48 | 49 | 1 | 1 |
| AVG | 13333 | 13077 | 2279 | 2232 | 553 | 542 | 37 | 36 | 1.5 | 1.1 |

TABLE 8. The results of the FRHC-NNF and the RHC-NNF.

| | <i>F</i> | | <i>L</i> | | <i>WT</i> | | <i>WN</i> | | <i>RT</i> (ms) | |
|--------|--------------|--------------|-------------|-------------|------------|------------|-----------|-----------|----------------|------|
| | RHC | FRHC | RHC | FRHC | RHC | FRHC | RHC | FRHC | RHC | FRHC |
| n32k5 | 8376 | 6776 | 2190 | 2028 | 309 | 237 | 18 | 16 | 1 | 1 |
| n45k7 | 16065 | 13583 | 2390 | 2366 | 684 | 561 | 32 | 29 | 1 | 1 |
| n46k7 | 11730 | 11653 | 2353 | 2128 | 469 | 476 | 30 | 27 | 1 | 1 |
| n53k7 | 14529 | 15286 | 2727 | 2668 | 590 | 631 | 31 | 31 | 1 | 1 |
| n54k7 | 19314 | 17850 | 3068 | 2814 | 812 | 752 | 38 | 34 | 3 | 2 |
| n55k9 | 16933 | 17188 | 2674 | 2602 | 713 | 729 | 37 | 36 | 1 | 1 |
| n61k9 | 18654 | 19639 | 2948 | 2662 | 785 | 849 | 40 | 42 | 2 | 1 |
| n63k10 | 18796 | 20329 | 2846 | 2772 | 797 | 878 | 45 | 44 | 1 | 1 |
| n63k9 | 22867 | 19936 | 3341 | 3212 | 976 | 836 | 47 | 40 | 2 | 2 |
| n64k9 | 20706 | 20747 | 2984 | 3068 | 886 | 884 | 47 | 45 | 1 | 2 |
| AVG | 16797 | 16299 | 2752 | 2632 | 702 | 683 | 37 | 34 | 1.4 | 1.3 |

of this method, the experimental results of the other four algorithms under the FRCH strategy are compared.

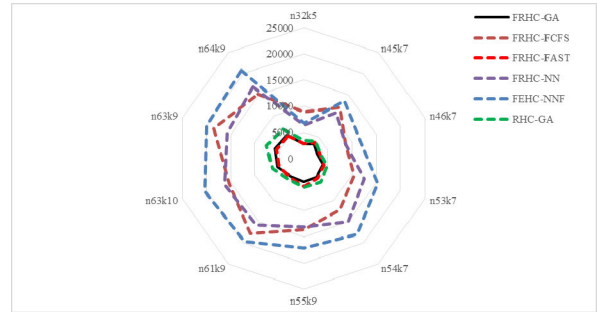


FIGURE 13. The *F* values radar graphs of various algorithms.

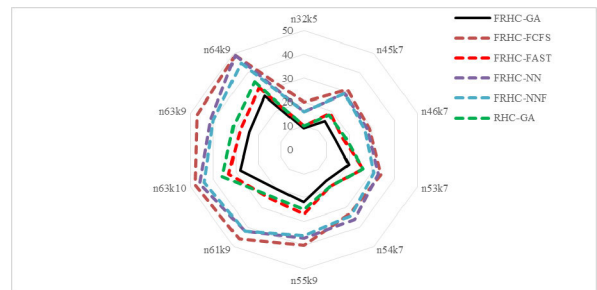


FIGURE 14. The *WN* values radar graphs of various algorithms.

TABLE 9. The average results from Table 4-8.

| | FRHC-GA | RHC-GA | FRHC-FCFS | FRHC-FAST | FRHC-NN | FRHC-NNF |
|-----------|-------------|--------|-----------|-----------|-------------|----------|
| <i>F</i> | 4313 | 5307 | 13253 | 4469 | 13077 | 16299 |
| <i>L</i> | 2842 | 2991 | 3200 | 3138 | 2232 | 2632 |
| <i>WT</i> | 74 | 116 | 503 | 67 | 542 | 683 |
| <i>WN</i> | 20 | 24 | 38 | 24 | 36 | 34 |

The DVRP’s aiming in this paper is to minimize the vehicle traveled distance and the waiting time of all customers, so the objective function $F = L + \alpha WT$.

From the Figure 13 and 14, it is shown that the FRHC-GA’s *F* values and the FRHC-FAST’s *F* values are closed. But the FRHC-GA’s *WN* values are smaller than the FRHC-FAST’s *WN* values.

It can be seen from Table 9. The *L* values obtained by the FRHC-NN method are the shortest, but *WT* values are the longest, the *WT* values are about 7 times longer than the FRHC-GA, and the *WN* values are bigger than the FRHC-GA. The *WT* values obtained by the FRHC-FAST method are the smallest, but the *L*, the *F* and the *WN* values are bigger than FRHC-GA. The FRHC-GA is better than the RHC-GA, FRHC-FCFS, FRHC-NNF and RHC-GA in every points.

From Table 4-8, the *RT*s are less than 3s, and the different are very small. The GA algorithm is a metaheuristic algorithm. Compared with the other four algorithms, the GA will spent more time, but can get more better solution. The calculation times will increases of all algorithms with the number of customer, but the GA always can get the better solution than others. To sum up all the above points, the FRHC-GA method for the DVRP is rational.

TABLE 10. The results of the n61k9 instance running 30 times under the FRHC-GA method.

| NO. | F | L | WT | RT | WN | Travel route |
|-------|------|------|----|----|----|---|
| 1 | 4276 | 2821 | 73 | 2 | 20 | 3,4,1,7,2,6,8,13;5,10,14,12,9,11,21;... |
| 2 | 4276 | 2821 | 73 | 2 | 20 | 3,4,1,7,2,6,8,13;5,10,14,12,9,11,21;... |
| 3 | 4276 | 2821 | 73 | 2 | 20 | 3,4,1,7,2,6,8,13;5,10,14,12,9,11,21;... |
| 4 | 4276 | 2821 | 73 | 2 | 20 | 3,4,1,7,2,6,8,13;5,10,14,12,9,11,21;... |
| 5 | 4276 | 2821 | 73 | 2 | 20 | 3,4,1,7,2,6,8,13;5,10,14,12,9,11,21;... |
| 6 | 4276 | 2821 | 73 | 2 | 20 | 3,4,1,7,2,6,8,13;5,10,14,12,9,11,21;... |
| 7 | 4276 | 2821 | 73 | 2 | 20 | 3,4,1,7,2,6,8,13;5,10,14,12,9,11,21;... |
| 8 | 4293 | 2813 | 74 | 2 | 21 | 3,4,1,7,2,8,6,13;5,10,14,12,9,11,21;... |
| 9 | 4303 | 2861 | 72 | 2 | 20 | 3,1,5,7,2,6,8,13;4,10,14,12,9,11,21;... |
| 10 | 4303 | 2861 | 72 | 2 | 20 | 3,1,5,7,2,6,8,13;4,10,14,12,9,11,21;... |
| 11 | 4305 | 2859 | 72 | 2 | 20 | 3,4,5,7,2,6,8,13;1,10,14,12,9,11,21;... |
| 12 | 4305 | 2859 | 72 | 2 | 20 | 3,4,5,7,2,6,8,13;1,10,14,12,9,11,21;... |
| 13 | 4305 | 2859 | 72 | 2 | 20 | 3,4,5,7,2,6,8,13;1,10,14,12,9,11,21;... |
| 14 | 4305 | 2859 | 72 | 2 | 20 | 3,4,5,7,2,6,8,13;1,10,14,12,9,11,21;... |
| 15 | 4305 | 2859 | 72 | 2 | 20 | 3,4,5,7,2,6,8,13;1,10,14,12,9,11,21;... |
| 16 | 4305 | 2859 | 72 | 2 | 20 | 3,4,5,7,2,6,8,13;1,10,14,12,9,11,21;... |
| 17 | 4305 | 2859 | 72 | 2 | 20 | 3,4,5,7,2,6,8,13;1,10,14,12,9,11,21;... |
| 18 | 4305 | 2859 | 72 | 2 | 20 | 3,4,5,7,2,6,8,13;1,10,14,12,9,11,21;... |
| 19 | 4305 | 2859 | 72 | 2 | 20 | 3,4,5,7,2,6,8,13;1,10,14,12,9,11,21;... |
| 20 | 4305 | 2859 | 72 | 2 | 20 | 3,4,5,7,2,6,8,13;1,10,14,12,9,11,21;... |
| 21 | 4305 | 2859 | 72 | 2 | 20 | 3,4,5,7,2,6,8,13;1,10,14,12,9,11,21;... |
| 22 | 4305 | 2859 | 72 | 2 | 20 | 3,4,5,7,2,6,8,13;1,10,14,12,9,11,21;... |
| 23 | 4305 | 2859 | 72 | 2 | 20 | 3,4,5,7,2,6,8,13;1,10,14,12,9,11,21;... |
| 24 | 4305 | 2859 | 72 | 2 | 20 | 3,4,5,7,2,6,8,13;1,10,14,12,9,11,21;... |
| 25 | 4305 | 2859 | 72 | 2 | 20 | 3,4,5,7,2,6,8,13;1,10,14,12,9,11,21;... |
| 26 | 4345 | 2822 | 76 | 2 | 20 | 3,5,1,7,2,6,8,13;4,10,14,12,9,11,21;... |
| 27 | 4345 | 2822 | 76 | 2 | 20 | 3,5,1,7,2,6,8,13;4,10,14,12,9,11,21;... |
| 28 | 4345 | 2822 | 76 | 2 | 20 | 3,5,1,7,2,6,8,13;4,10,14,12,9,11,21;... |
| 29 | 4345 | 2822 | 76 | 2 | 20 | 3,5,1,7,2,6,8,13;4,10,14,12,9,11,21;... |
| 30 | 4352 | 2826 | 76 | 2 | 20 | 3,5,4,7,2,6,8,13;1,10,14,12,9,11,21;... |
| STDEV | | | | | | 22.36 19.24 1.44 0.00 0.18 |

3) ANALYSIS OF THE FRHC-GA RESULT

In order to verify the convergence and validity of the FRHC-GA. This section lists the results of the FRHC-GA for the n61k9 instance, counts the number of best solutions, and analyses the travel route of the best solutions.

Table 10 show the results of the n61k9 instance running 30 times under the FRHC-GA method. This table lists first two vehicles' travel routes. There are 7 best results in 30 results. And the standard deviations are very small. So the results of FRHC-GA method solving the DVRP are convergent.

Table 11 lists the customer service information of the best travel route in Table 9.

In Table 1, the *PT* value of the No.5 customer is 9, and the *PT* value of the No.1 customer is 7. The distance between the No.1 customer and the No.5 customer is too far. If the No.5 customer is served after the No.7 customer, it will cause

TABLE 11. The best travel route information of the n61k9 instance.

| No. | GT | PT | AT | WT | Car No. | No. | GT | PT | AT | WT | Car No. | No. | GT | PT | AT | WT | Car No. |
|-----|----|----|----|----|---------|-----|-----|-----|-----|----|---------|-----|-----|-----|-----|----|---------|
| 3 | 0 | 1 | 2 | 1 | 1 | 18 | 0 | 88 | 88 | 0 | 3 | 42 | 51 | 174 | 173 | 0 | 6 |
| 4 | 0 | 7 | 4 | 0 | 1 | 23 | 4 | 93 | 91 | 0 | 3 | 36 | 0 | 178 | 178 | 0 | 6 |
| 1 | 1 | 7 | 10 | 3 | 1 | 24 | 0 | 98 | 91 | 0 | 4 | 46 | 0 | 186 | 180 | 0 | 6 |
| 7 | 0 | 21 | 13 | 0 | 1 | 27 | 68 | 106 | 100 | 0 | 4 | 43 | 14 | 189 | 182 | 0 | 7 |
| 2 | 0 | 28 | 23 | 0 | 1 | 25 | 8 | 111 | 107 | 0 | 4 | 47 | 0 | 192 | 192 | 0 | 7 |
| 6 | 0 | 29 | 29 | 0 | 1 | 28 | 107 | 116 | 114 | 0 | 4 | 45 | 0 | 198 | 196 | 0 | 7 |
| 8 | 0 | 30 | 29 | 0 | 1 | 22 | 0 | 119 | 118 | 0 | 4 | 49 | 0 | 204 | 199 | 0 | 7 |
| 13 | 0 | 36 | 33 | 0 | 1 | 26 | 0 | 118 | 120 | 2 | 4 | 48 | 0 | 206 | 207 | 1 | 7 |
| 5 | 0 | 9 | 33 | 24 | 2 | 32 | 0 | 120 | 121 | 1 | 4 | 44 | 0 | 204 | 208 | 4 | 7 |
| 10 | 0 | 42 | 34 | 0 | 2 | 31 | 0 | 123 | 124 | 1 | 4 | 53 | 0 | 213 | 212 | 0 | 7 |
| 14 | 0 | 45 | 44 | 0 | 2 | 35 | 0 | 120 | 122 | 2 | 5 | 51 | 0 | 216 | 215 | 0 | 7 |
| 12 | 0 | 45 | 46 | 1 | 2 | 29 | 0 | 140 | 124 | 0 | 5 | 59 | 129 | 221 | 212 | 0 | 8 |
| 9 | 30 | 50 | 48 | 0 | 2 | 30 | 0 | 144 | 143 | 0 | 5 | 56 | 83 | 229 | 224 | 0 | 8 |
| 11 | 30 | 54 | 51 | 0 | 2 | 33 | 0 | 148 | 145 | 0 | 5 | 57 | 0 | 232 | 232 | 0 | 8 |
| 21 | 0 | 64 | 58 | 0 | 2 | 34 | 0 | 149 | 151 | 2 | 5 | 52 | 132 | 233 | 233 | 0 | 8 |
| 16 | 10 | 70 | 62 | 0 | 3 | 39 | 0 | 160 | 151 | 0 | 6 | 50 | 0 | 235 | 235 | 0 | 8 |
| 20 | 0 | 71 | 73 | 2 | 3 | 38 | 0 | 162 | 163 | 1 | 6 | 55 | 0 | 236 | 236 | 0 | 8 |
| 17 | 47 | 81 | 76 | 0 | 3 | 40 | 0 | 165 | 164 | 0 | 6 | 58 | 216 | 232 | 238 | 6 | 8 |
| 15 | 0 | 85 | 84 | 0 | 3 | 41 | 0 | 167 | 166 | 0 | 6 | 54 | 0 | 239 | 240 | 1 | 8 |
| 19 | 25 | 86 | 86 | 0 | 3 | 37 | 149 | 171 | 169 | 0 | 6 | 60 | 0 | 222 | 242 | 20 | 9 |

the other customers to wait. Therefore, the No.5 customer is deferred from service. The No.13 customer is in the 2th time windows in the traditional RHC strategy, but in the 1th time windows in the FRHC. The No.13 customer is near to the No.8 customer, so this customer is served after No.8. Because of each vehicle can serve up to 8 customers, the No.5 customer is placed in a second fuzzy time window for processing.

The FRHC exchanges the No.5 customer and No.13 customer compared with the traditional RHC strategy. This is the situation of the Scenario2 introduced in the section III.A.3).

Combine all the above conclusions, this set of experiments verifies the rationality of the FRHC strategy, and the effectiveness of the FRHC-GA method in solving DVRP.

E. HYPOTHESEIS TESTING

According to the experimental results, the significance level of 5% Nemenyi test is used to compare the *FT* values, the FRHC and other algorithms. If the average position between two algorithm in all data set is less than or equal to the critical difference, we believe that there is no significant difference between the two algorithms. Otherwise there is significant existed.

For the average accuracy evaluation indexes, as shown in Figure 15-22: the CD values of different *FT* values are less than the values of RHC, and the values of *FT* = 21/30 and *FT* = 23/30 are closed to. It verifies the FRHC strategy is better than RHC.

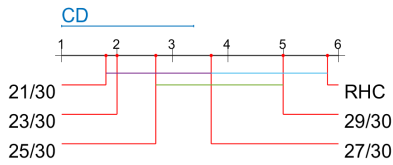


FIGURE 15. The *F* average result data's CD graph.

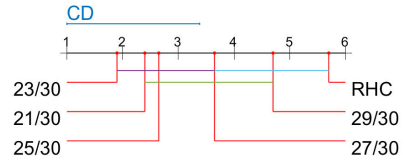


FIGURE 22. The *WN* minimum result data's CD graph.

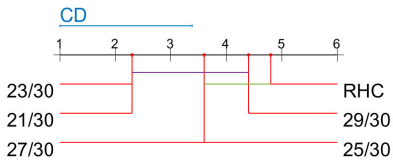


FIGURE 16. The *L* average result data's CD graph.

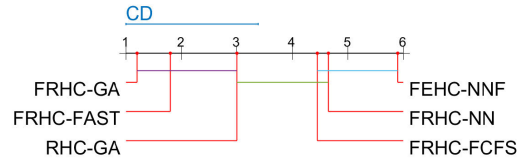


FIGURE 23. The algorithms average results' CD graph.

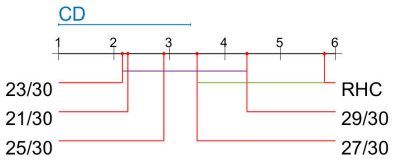


FIGURE 17. The *WT* average result data's CD graph.

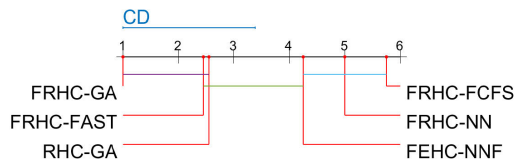


FIGURE 24. The algorithm minimum results' CD graph.

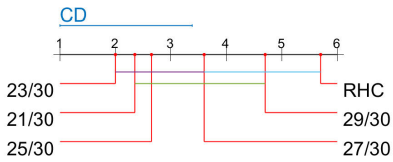


FIGURE 18. The *WN* average result data's CD graph.

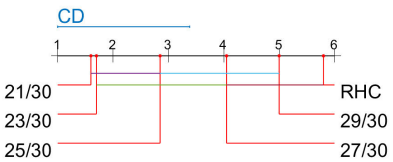


FIGURE 19. The *F* minimum result data's CD graph.

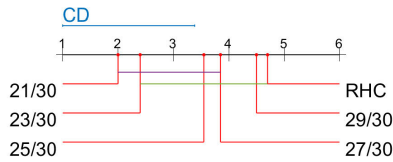


FIGURE 20. The *L* minimum result data's CD graph.

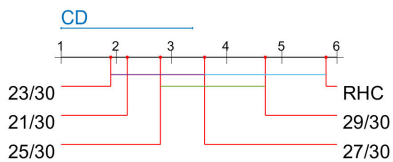


FIGURE 21. The *WT* minimum result data's CD graph.

From Figure 22-24, by comparing the CD values between the different algorithms, the FRHC-GA is the smallest. So the FRHC-GA method for the DVRP provided in this paper is effective.

V. CONCLUSION

The RHC is an important strategy of solving the DVRP. The traditional RHC only considers the objects in the current time window, and can't make overall plan according to the situation of the objects near the window. This paper proposed a FRHC strategy to optimize this question. By combining RHC and membership function theory, the time windows of the RHC is blurred. The decision-making threshold value helps to redefine the fuzzy time windows. The membership function of the FRHC is defined in this paper. This strategy increases communication between adjacent time windows. The advantage and the effective of the FRHC are verified by comparing with the RHC. The proposed method for solving the DVRP, the FRHC combined with the GA, can reduce the travel distance, the customers waiting time and the number of waiting customer compared with FRHC-NN, FRHC-FCFS. ... The FRHC-GA method is also reasonable and effective.

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