Valuing firm's financial flexibility under default risk and bankruptcy costs: a WACC based approach

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Abstract

Purpose – The purpose of this paper is to present a model to value leveraged firms in the presence of default risk and bankruptcy costs under a flexible firm's debt structure.

Design/methodology/approach – The authors assume that the total debt of the firm is a combination of two debt components. The first component is an active debt component which is assumed to be proportional to the firm's value. The second one is a passive predetermined risk-free debt component. The combination of the two debt categories makes the firm's capital structure more realistic and allows us to include flexibility into the firm's debt structure management. The firm's valuation is performed using the discounted cash flow technique based on the weighted average cost of capital (WACC) method.

Findings – The model can be used to define active debt management strategies that can induce the firm to deviate from its capital structure target in order to preserve debt capacity for future funding needs. The firm's valuation is performed by using the WACC method and a closed form valuation formula is provided. Such a formula can be used to value costs and benefits of financial flexibility.

Research limitations/implications – The proposed approach provides a good compromise between mathematical complexity and model capability of interpreting the various economic and financial aspects involved in the firm's debt structure puzzle.

Practical implications – This model offers a realistic approach to practical applications where real financing decisions are characterized by a simultaneous use of these two debt categories. By comparing costs and benefits deriving from using unused debt capacity for future funding needs, the model provides a quantitative support to investigate if financial flexibility can add value to firms.

Originality/value – To the authors knowledge, the approach the authors propose is the first attempt to build a valuation scheme that accounts for firm's financial flexibility under default risky debt and bankruptcy costs. Including financial flexibility, this model fills an important gap in the literature on this topic.

Keywords Bankruptcy costs, WACC, Financial flexibility, Debt management, Risky debt, Tax shield

Paper type Research paper

1. Introduction

In a seminal paper, Modigliani and Miller (1959) showed that the use of debt financing can increase the market value of a firm as a consequence of the debt tax shield. However, the presence of debt in the capital structure can lead the firm to incur into default and distress costs (Warner, 1977; Altman, 1984; Opler and Titman, 1994). The valuation of distress costs is therefore an important task in financing decisions making (Oded and Michel, 2007; Lahmann *et al.*, 2017).

The impact of default costs on the firm's value has been deeply investigated in the literature. Andrade and Kaplan (1998) estimate losses on the order of 10–23 percent of the predistress firm value. Glover (2016) confirms that the average cost among defaulted firms is about 25 percent of the predistress firm value.

JEL Classification — G30, G31, G32, G33

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Several models have been proposed in the literature in order include in a proper way default costs in the firm's valuation. Almeida and Philippon (2007) try to estimate the present value of distress costs using risk-adjusted default probabilities derived from corporate bond spreads. Koziol (2014) provides a weighted average cost of capital (WACC) formula that accounts for risky debt and bankruptcy costs under a constant debt ratio policy and constant one period default probabilities. Mari and Marra (2018) discuss a mathematically tractable model to value firms under default risk in the presence of bankruptcy costs. In their model the default is treated as an exogenous event which can occur at any time during the lifetime of the firm. Under the hypothesis of a constant debt ratio, the default probability is not constant over time and it is assumed to depend on the entity of the debt ratio.

The literature on capital structure documented the existence of a target leverage and investigated the way in which firms adjust the leverage toward their target debt ratios (Byoun, 2008; Dang and Garrett, 2015; Molnár and Nyborg, 2013). However, some empirical studies indicate that firms have less leverage on average than 1 would expect based on the trade-off between tax shields and bankruptcy costs(Marchica and Mura, 2010; Byoun, 2008). Although Marchica and Mura (2010) underlined that the reasons why it happens is still a puzzle, other Authors tried to understand the nature of this phenomenon. DeAngelo and DeAngelo (2007) observed that uncertainty about earnings, investment opportunities and future security prices give managers incentives to select financial policies that provide the flexibility to respond to unanticipated shocks to these factors. Graham and Harvey (2001) underlined that firms unused debt capacity is often claimed to preserve financial flexibility, to absorb economic bumps or to face future acquisitions. Gamba and Triantis (2008) suggested that firms might prefer financial flexibility in order to access and restructure financing at a low cost. In this context, financially flexible firms are able to avoid financial distress when they face negative shocks, and to readily fund investment when profitable opportunities arise. Arslan-Ayaydin *et al.* (2014) underlined that financial flexibility appears to be an important determinant of investment and performance. Firms with financial flexibility enjoy easier access to external capital markets to meet funding needs arising from unanticipated earnings shortfalls or new growth opportunities and hence, avoid situations that may lead to suboptimal investment and poor performance. De Jong *et al.* (2012) empirically tested if financial flexibility has an effect on investment decisions and they observed that firms with more financial flexibility have higher future investments than firms with less financial flexibility. Yung *et al.* (2015) found that financial flexibility adds value to firms, and it has a positive effect in particular during financial crisis. Dierkes and Schäfer (2017) suggested that financial managers generally pursue capital structure targets but they might add a deterministic debt component in order to reflect firms long term obligations or extraordinary need for liquidity.

Following this line of research, the aim of the paper is to extend the model introduced by Mari and Marra (2018) in order to include firm's financial flexibility under default risk and bankruptcy costs. Financial flexibility is accounted for in terms of a total debt which is structured as a combination of two components. The first component is an active debt component which is assumed to be proportional to the firm's value. The second one is a passive predetermined risk-free debt component. The combination of the two debt categories makes the debt structure more realistic (DeAngelo *et al.*, 2011) and allows to define active debt management strategies that can induce the firm to deviate from its capital structure target thus preserving debt capacity for future funding needs. The simultaneous use of an active debt component proportional to the firm's market value as well as a passive predetermined debt component can be interpreted as a combination of the financing strategies introduced, respectively, by Modigliani and Miller (1959, 1963) and Miles and Ezzel (1980, 1985).

In this paper, we extend the model in order to include both debt components. The default risk and distress costs are modeled as follows. The default event is treated as an exogenous event which can occur at any time during the lifetime of the firm and the default probability is assumed to depend on the level of the active debt component. To model distress costs we assume that they are proportional to the predistress firm's value (Koziol, 2014). This assumption is motivated by the fact that data available on distress costs are expressed as a given percentage of the predistress firm value (Andrade and Kaplan, 1998; Glover, 2016). The firm's valuation is performed using the discounted cash flow (DCF) technique based on the WACC method. A WACC formula taking into account both debt components is derived. We will show that such a formula is a powerful tool of analysis for valuing flexible levered firms under default risk and bankruptcy costs and provides an extension of the WACC formula derived by Mari and Marra (2018).

The proposed model can be used to define active debt management strategies. The model, in fact, allows to quantify the difference in terms of the firm's market value between the optimal firm's value, obtained in correspondence of the target debt ratio (without passive debt), and the firm's value computed for a given level of unused debt capacity. By comparing costs and benefits deriving from using such unused debt capacity for future funding needs, the model provides a quantitative support to investigate if financial flexibility can add value to firms. To our knowledge, the approach we propose is the first attempt to build a valuation scheme that accounts for financial flexibility under default risky debt and bankruptcy costs. Including financial flexibility, this model fills an important gap in the literature on this topic.

The paper is organized as follows. The model is presented and discussed in Section 2. We first derive a generalization of the Proposition II of the Modigliani and Miller (1959) theorem and then we provide a WACC formula properly accounting for the two debt categories and for distress costs. Finally, the model is used to value levered flexible firms. A numerical simulation is presented in Section 3.

2. A general approach for valuing firm's financial flexibility under default risk and bankruptcy costs

In this Section, we propose a general methodology to value leveraged firms under default risky debt and bankruptcy costs. We assume that the firm's debt is a combination of two debt components. The first one is an active stochastic debt component. The second one is a passive predetermined risk-free debt component.

Let us denote by $\{F_t^X\}_{t=1}^m$ a collection of measurable random variables with respect a given filtration of a given probability space (Duffie, 1998), describing a stream of stochastic payments, namely, a cash flow, and by V_t^X the present value at time t ($t = 1, 2, \ldots, m$) of the cash flow. The superscript X stands for U (unlevered), S (levered), D_s (stochastic debt), D_p (predetermined risk-free debt), TS_s (tax shields due to stochastic debt), TS_p (tax shield due to predetermined debt) and DC (default costs)[1]. By definition, the levered cash flow (or flow to equity), F_t^S, can be expressed as:

$$F_t^S = F_t^U - F_t^{D_s} - F_t^{D_p} + F_t^{TS_s} + F_t^{TS_p} - F_t^{DC}, \tag{1}$$

obtained adding to the unlevered cash flow F_t^U, i.e. the cash flow accounting for the activities of the firm, the tax shield contributions $F_t^{TS_s}$ and $F_t^{TS_p}$, and subtracting the debt repayments $F_t^{D_s}$ for the active stochastic debt and $F_t^{D_p}$ for the passive predetermined debt and distress costs F_t^{DC}. From Equation (1) and using a no-arbitrage argument, we can express the value at time t of a leveraged firm V_t in the following way:

$$V_t \equiv V_t^S + V_t^{D_s} + V_t^{D_p} = V_t^U + V_t^{TS_s} + V_t^{TS_p} - V_t^{DC}. \tag{2}$$

The firm value, i.e. the sum of its equity, V_t^S, and of the outstanding debt $V_t^{D_s} + V_t^{D_p}$, can be obtained adding to the value of the unlevered firm V_t^U, the present value of tax shields $V_t^{TS_s} + V_t^{TS_p}$, and subtracting the present value of bankruptcy costs V_t^{DC}. We calculated expected present values by discounting expected future cash flows at risk-adjusted discount rates. If we denote k_t^X as the cost of capital (i.e. the discount rate) accounting for the risk of the cash flow F_t^X, the following recursive relation holds:

$$\mathbb{E}_0\left[V_t^X\right] = \frac{\mathbb{E}_0\left[F_{t+1}^X + V_{t+1}^X\right]}{1+k_t^X}, \tag{3}$$

where $\mathbb{E}_0$ is the conditional expectation operator under the information available at the present time (i.e. time 0 in our analysis). The iterative application of Equation (3) is straightforward and allows to express the expected value at time t of a risky cash flow MCMM as:

$$\mathbb{E}_0\left[V_t^X\right] = \sum_{j=t+1}^{m} \frac{\mathbb{E}_0\left[F_j^X\right]}{\prod_{i=t}^{j-1}\left(1+k_i^X\right)}. \tag{4}$$

2.1 *Generalizing Proposition II of the Modigliani-Miller theorem*

Equity rates k_t^S are related to unlevered rates k_t^U, debt rates k_t^D, and to the risk-free rate r_f, by the linear combination:

$$k_t^S = k_t^U + \left(k_t^U - k_t^{D_s}\right)\frac{\mathbb{E}_0\left[V_t^{D_s}\right]}{\mathbb{E}_0\left[V_t^S\right]} + \left(k_t^U - r_f\right)\frac{\mathbb{E}_0\left[V_t^{D_p}\right]}{\mathbb{E}_0\left[V_t^S\right]} - \left(k_t^U - k_t^{TS_s}\right)\frac{\mathbb{E}_0\left[V_t^{TS_s}\right]}{\mathbb{E}_0\left[V_t^S\right]} +$$

$$-\left(k_t^U - r_f\right)\frac{\mathbb{E}_0\left[V_t^{TS_p}\right]}{\mathbb{E}_0\left[V_t^S\right]} + \left(k_t^U - k_t^{DC}\right)\frac{\mathbb{E}_0\left[V_t^{DC}\right]}{\mathbb{E}_0\left[V_t^S\right]}, \tag{5}$$

in which tax shields rates $k_t^{TS_s}$ of the stochastic debt, and default costs rates k_t^{DC}, explicitly appear. Appendix contains a detailed proof of the above result.

Equation (5) provides a generalization of Proposition II of the Modigliani-Miller theorem and it accounts for the risk of the levered cash flow due to the presence of risk-free debt and risky debt, tax shields and distress costs. The entity of the equity risk depends on several variables. Among the others, the ratio between the expected value of the outstanding debt and the expected equity value is the most important one. Also, the ratio between the expected value of tax shields and the expected equity value plays an important role. The last term accounts for default costs. As shown in Mari and Marra (2018), discount rates for the bankruptcy costs k_t^{DC} depend on the firm's market risk. Since distress costs are high when the firm does poorly, the β of distress costs will have an opposite sign with respect to that of the firm. If the firm has a positive β, distress costs have a negative β and the relative discount rate is lower than the risk-free rate. Due to the magnitude of the distress cost β, the discount rate may assume negative values.

2.2 *The weighted average cost of capital (WACC)*

We can determine the market value of a firm by discounting unlevered cash flows at WACC rates. In such a case, tax shields and distress costs effects are explicitly included in the WACC formula. Under default risk and bankruptcy costs, WACC rates k_t^W can be introduced in the following way. By definition, the expected value of a leveraged firm can be determined discounting the expected unlevered cash flow at WACC rates. They can be, therefore, introduced by the following recursive relation:

$$\mathbb{E}_0[V_t] = \frac{\mathbb{E}_0\left[F^U_{t+1} + V_{t+1}\right]}{1+k^W_t}. \tag{6}$$

Iterative applications, Equation (6) provides the expected value of a leveraged firm at time t in term of the present value of the expected future unlevered cash flow, namely:

$$\mathbb{E}_0[V_t] = \sum_{j=t+1}^{m} \frac{\mathbb{E}_0\left[F^U_j\right]}{\prod_{i=t}^{j-1}\left(1+k^W_i\right)}, \tag{7}$$

where WACC rates are related to unlevered rates by the linear combination:

$$\begin{aligned} k^W_t &= k^U_t - \left(k^U_t - k^{TS_s}_t\right)\frac{\mathbb{E}_0\left[V^{TS_s}_t\right]}{\mathbb{E}_0[V_t]} - \left(k^U_t - r_f\right)\frac{\mathbb{E}_0\left[V^{TS_p}_t\right]}{\mathbb{E}_0[V_t]} + \\ &\quad + \left(k^U_t - k^{DC}_t\right)\frac{\mathbb{E}_0\left[V^{DC}_t\right]}{\mathbb{E}_0[V_t]} - \frac{\mathbb{E}_0\left[F^{TS_s}_{t+1}\right]}{\mathbb{E}_0[V_t]} - \frac{\mathbb{E}_0\left[F^{TS_p}_{t+1}\right]}{\mathbb{E}_0[V_t]} + \frac{\mathbb{E}_0\left[F^{DC}_{t+1}\right]}{\mathbb{E}_0[V_t]}. \end{aligned} \tag{8}$$

WACC rates can be also expressed in terms of equity and debt rates:

$$k^W_t = \frac{\mathbb{E}_0\left[V^S_t\right]}{\mathbb{E}_0[V_t]}k^S_t + \frac{\mathbb{E}_0\left[V^{D_s}_t\right]}{\mathbb{E}_0[V_t]}k^{D_s}_t + \frac{E_0\left[V^{D_p}_t\right]}{E_0[V_t]}r_f - \frac{\mathbb{E}_0\left[F^{TS_s}_{t+1}\right]}{\mathbb{E}_0[V_t]} - \frac{E_0\left[F^{TS_p}_{t+1}\right]}{E_0[V_t]} + \frac{\mathbb{E}_0\left[F^{DC}_{t+1}\right]}{\mathbb{E}_0[V_t]}. \tag{9}$$

Appendix contains a detailed proof of both Equations (8) and (9).

2.3 Firm's valuation

Starting from the general methodology discussed in the previous subsections, we derive in this subsection a firm's valuation formula that accounts for a well-defined financial structure under default risk and bankruptcy costs. We assume that the total debt is a combination of two debt components. The first component is an active debt component which is proportional to the firm's value. We denote by L the active debt ratio which is constant over time. The second one is a passive predetermined risk-free debt component. Distress costs are assumed to be proportional to the predistress firm's value (Koziol, 2014), i.e. $F^{DC}_{t+1} = \alpha V_t$.

According to Equations (6) and (8), we can cast the value of the levered firm in the following form:

$$\mathbb{E}_0[V_t] = \frac{\mathbb{E}_0\left[F^U_{t+1} + V_{t+1}\right] + \left(k^U_t - r_f\right)E_0\left[V^{TS_p}_t\right] + E_0\left[F^{TS_p}_{t+1}\right]}{1+\tilde{k}^W_t}, \tag{10}$$

where:

$$\tilde{k}^W_t = k^U_t - \left(k^U_t - k^{TS_s}_t\right)\frac{\mathbb{E}_0\left[V^{TS_s}_t\right]}{\mathbb{E}_0[V_t]} + \left(k^U_t - k^{DC}_t\right)\frac{\mathbb{E}_0\left[V^{DC}_t\right]}{\mathbb{E}_0[V_t]} - \frac{\mathbb{E}_0\left[F^{TS_s}_{t+1}\right]}{\mathbb{E}_0[V_t]} + \frac{\mathbb{E}_0\left[F^{DC}_{t+1}\right]}{\mathbb{E}_0[V_t]}. \tag{11}$$

Mari and Marra (2018) showed that under suitable assumptions WACC rates $\tilde{k}_t^W$ can be expressed as follows:

$$\tilde{k}_t^W = \left[1+\alpha \sum_{k=t+1}^{m} 1-\frac{p(k)}{p(k-1)}\right] k_t^U - T_c k^N \frac{p(t+1)}{p(t)} L + \alpha\left(1-\frac{p(t+1)}{p(t)}\right), \tag{12}$$

where $p(t)$ is the unconditional firm's survivorship probability at time t, T_c is the corporate tax, rate and k_N the nominal active debt rate[2]. For practical purposes it may be convenient to use the following formula:

$$\tilde{k}_t^W = \left[1+\alpha \ln\left(\frac{p(t)}{p(m)}\right)\right] k_t^U - T_c k^N \frac{p(t+1)}{p(t)} L + \alpha\left(1-\frac{p(t+1)}{p(t)}\right), \tag{13}$$

in which the sum has been approximated with its continuous limit. We notice that WACC Formulas (12) and (13) explicitly depend on the unconditional survivorship probability and in this way they account for credit spreads of corporate debt. Such formulas can be used to value levered firms by specifying the functional form of the unconditional survivorship probability. From this point of view, the model provides a flexible and a powerful tool of analysis for investigating the impact on the firm's value of different debt policies under default risk and bankruptcy costs. The model can be used, in fact, to compare different debt management strategies. Equation (10) allows us to compute firm's values for any combination of active and passive debt. Then, the model can be used to quantify the difference in terms of the firm's market value between the optimal firm's value, obtained in correspondence of the target debt ratio (without passive debt), and the optimal firm's value computed for a given level of unused debt capacity. By comparing costs and benefits deriving from using such unused debt capacity for future funding needs, the model provides a quantitative support to investigate under what conditions financial flexibility can add value to firms.

3. A simulation analysis

In this section, we provide a simulation analysis to illustrate the main features of the model. In particular, we show that the model can be used to investigate the optimal capital structure problem when the firm's debt is a combination of a passive deterministic debt component and an active debt component proportional to the firm's value. In the specific, we discuss the firm's valuation problem on an infinite time horizon in the case in which the passive predetermined component is constant over time $V_t^{D_p} = V^{D_p}$, and the active component is $V_t^{D_s} = LV_t$. Both debt components are perpetuities. We assume that the survivorship probability $p(t)$ is modeled according to the following functional form (see Mari and Marra, 2018):

$$p(t) = 1-a\left(1-e^{-bt}\right), \tag{14}$$

where:

$$a = c \max[L-L\text{th}, 0]. \tag{15}$$

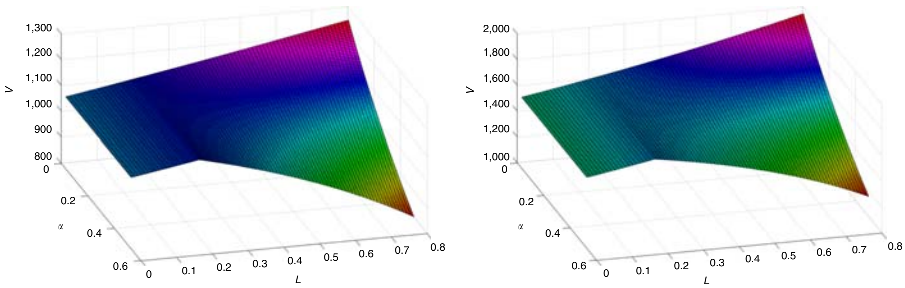
where b and c are constant parameters and Lth is a threshold value such that, for active debt ratios L lower than Lth, the debt is risk free (in such a case $a=0$ and $p(t)=1$). This functional form of the survivorship probability has two main features: first, it depends on the debt ratio to account for the fact that higher debt ratios induce greater values of the default probability; second, the probability of survivorship at time t, in the limit $t\to+\infty$ is strictly positive, i.e. $\lim_{t\to+\infty} p(t) = 1-a$. Moreover, we assume that unlevered rates are constant, i.e. $k_t^U = k^U$. The firm valuation is performed by using Equations (10) and (13) for different values of the debt ratio, threshold values and distress costs parameters. Table I reports the numerical values of the parameters used in the simulation analysis[3].

Two cases are considered. In the first one, the firm's expected cash flow is assumed to be constant over time, i.e. $\mathbb{E}_0[F_t^U] = \mathbb{E}_0[F^U]$, at the value reported in Table I. In the second case, the valuation of a growing firm at a constant rate is provided. In such a case, the expected cash flow value reported in Table I refers to the first value of the expected cash flow stream. We will show that in both cases the optimal financial structure of the firm arises in a quite natural way as a consequence of the trade-off between the tax shields contribution to the firm's value and the distress costs contribution. It is determined in correspondence of the active debt component that maximizes the firm's value (see Figures 1 and 2), plus the predetermined passive debt component.

As expected, the empirical analysis reveals that for low values of the distress costs parameter α and for high values of the active debt threshold Lth, the optimal capital structure is characterized by very high values of the active debt ratio. For increasing values of the distress costs level and for decreasing values of threshold, the optimal active debt ratio reduces.

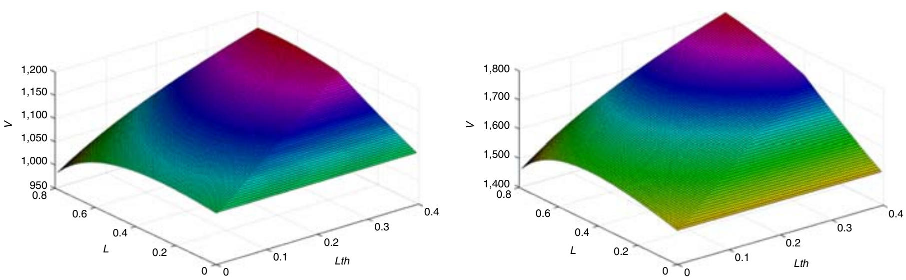
Table I. Parameters assumptions

$\mathbb{E}_0[F^U]$	k^U	V^{D_p}	T_c	k^N	b	c
100	10%	160	35%	6%	0.1	1



Notes: Left panel: the constant expected cash flow case; right panel: the case of a growing firm at a constant rate $g = 3$ percent

Figure 1. Firm's value vs active debt ratio and distress costs level



Notes: Left panel: the constant expected cash flow case; right panel: the case of a growing firm at a constant rate $g = 3$ percent

Figure 2. Firm value vs active debt ratio and active debt ratio threshold

Figure 1 shows the behavior of the firm's value as a function of the active debt ratio and the distress costs level. The active debt ratio threshold Lth has been fixed at 20 percent.

We note that for low values of the distress costs parameter α, the optimal capital structure is characterized by very high values of the active debt ratio. The left panel of Figure 1 shows the case of the constant expected cash flow stream. If $\alpha = 0.10$ the optimal capital structure is obtained when the active debt ratio is about 75 percent. For increasing values of the distress costs level, the optimal active debt ratio reduces: to about 51 percent for $\alpha = 0.15$, to 28 percent for $\alpha = 0.20$ and to 20 percent (at the level of the riskless debt threshold) in the case $\alpha = 0.30$. The right panel of Figure 1 shows the results of the numerical analysis in the case of a growing firm at constant rate $g = 3$ percent. For $\alpha = 0.10$ the optimal capital structure is obtained when the active debt ratio is about 80 percent. For increasing values of the distress costs, the optimal active debt ratio reduces: to about 70 percent in the case $\alpha = 0.15$, to 53 percent for $\alpha = 0.20$ and to 20 percent in the case $\alpha = 0.30$.

Figure 2 shows the behavior of the firm's value as a function of the active debt ratio and the active debt threshold Lth. Distress costs are kept at a constant level $\alpha = 0.15$.

We note that for high values of the threshold Lth, the optimal capital structure is characterized by very high values of the active debt ratio. In the case of the constant expected cash flow stream depicted in the left panel of Figure 2, if $L\text{th} = 0.40$ the optimal capital structure is obtained for a debt ratio of about 73 percent. For decreasing values of the active debt ratio threshold, the optimum reduces: to about 61 percent for $L\text{th} = 0.30$ and about 41 percent in the case $l\text{th}= 0.10$. The right panel of Figure 2 shows the results of the numerical analysis in the case of a growing firm at constant rate $g = 3$ percent. For $L\text{th} = 0.30$ the optimal capital structure is obtained when the debt ratio is about 80 percent. The optimum reduces to about 59 percent in the case $l\text{th}= 0.10$.

Varying the amount of the passive debt component, this model allows to compare the optimal capital structure for different values of the deterministic debt component. By considering costs and benefits of unused debt capacity, the model provides a quantitative support to investigate under what conditions financial flexibility can add value to firms.

4. Concluding remarks

The main contribution of this paper is to provide a model to value flexible levered firms under default risk and bankruptcy costs within the context of the DCF valuation technique. Based on the WACC method, this model provides a firm's valuation scheme that accounts for composite capital structures given by a combination of an active debt component proportional to the firm's market value, and a passive predetermined debt component. This model offers a realistic approach to practical applications where real financing decisions are characterized by a simultaneous use of these two debt categories. Finally, let us underline that the proposed approach provides a good compromise between mathematical complexity and model capability of interpreting the various economic and financial aspects involved in the firm's debt structure puzzle.

Notes

1. We remark that in the case $X = D_p$, and $X = TS_p$, the quantities F_t^X and V_t^X are non-stochastic and are described by degenerate random variables.
2. Referring to the paper by Mari and Marra (2018), Equation (12) which accounts only for the active debt component, corresponds to Equation (24) of the cited work.
3. The risk-free rate value is not necessary in this simulation analysis. Since for a deterministic debt structured as a perpetuity $V_t^{TS_p} = T_c V^{D_p}$ and $F_{t+1}^{TS_p} = r_f T_c V^{D_p}$, the term proportional to r_f cancels the last term in the numerator of the r.h.s. of Equation (10).

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Further reading

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Appendix

This Appendix contains detailed proofs of the main results stated in Section 2.

Proof of Equation (5)

Let us explicitly rewrite Equation (3) in the following different specifications:

$$\left(1+k_t^S\right)\mathbb{E}_0\left[V_t^S\right] = \mathbb{E}_0\left[F_{t+1}^S + V_{t+1}^S\right],$$

$$\left(1+k_t^{D_s}\right)\mathbb{E}_0\left[V_t^{D_s}\right] = \mathbb{E}_0\left[F_{t+1}^{D_s} + V_{t+1}^{D_s}\right],$$

$$(1+r_f)\mathbb{E}_0\left[V_t^{D_p}\right] = \mathbb{E}_0\left[F_{t+1}^{D_p} + V_{t+1}^{D_p}\right],$$

$$\left(1+k_t^U\right)\mathbb{E}_0\left[V_t^U\right] = \mathbb{E}_0\left[F_{t+1}^U + V_{t+1}^U\right],$$

$$\left(1+k_t^{TS_s}\right)\mathbb{E}_0\left[V_t^{TS_s}\right] = \mathbb{E}_0\left[F_{t+1}^{TS_s} + V_{t+1}^{TS_s}\right],$$

$$(1+r_f)\mathbb{E}_0\left[V_t^{TS_p}\right] = \mathbb{E}_0\left[F_{t+1}^{TS_p} + V_{t+1}^{TS_p}\right],$$

$$\left(1+k_t^{DC}\right)\mathbb{E}_0\left[V_t^{DC}\right] = \mathbb{E}_0\left[F_{t+1}^{DC} + V_{t+1}^{DC}\right].$$

Let us sum the first, the second and the third equation, then subtract the fourth, the fifth and the sixth one, and finally, sum the last equation, after some algebraic manipulations (in which Equations (2) and (1) have been used), we easily get:

$$k_t^S\mathbb{E}_0\left[V_t^S\right] + k_t^{D_s}\mathbb{E}_0\left[V_t^{D_s}\right] + r_f\mathbb{E}_0\left[V_t^{D_p}\right] - k_t^U\mathbb{E}_0\left[V_t^U\right] - k_t^{TS_s}\mathbb{E}_0\left[V_t^{TS_s}\right] - r_f\mathbb{E}_0\left[V_t^{TS_p}\right] + k_t^{DC}\mathbb{E}_0\left[V_t^{DC}\right] = 0.$$

Since from Equation (2) it follows that $V_t^U = V_t^S + V_t^{D_s} + V^{D_p} - V_t^{TS_s} - V_t^{TS_p} + V_t^{DC}$, the above equation can be rewritten in the following way:

$$k_t^S = k_t^U + \left(k_t^U - k_t^{D_s}\right)\frac{\mathbb{E}_0\left[V_t^{D_s}\right]}{\mathbb{E}_0\left[V_t^S\right]} + \left(k_t^U - r_f\right)\frac{\mathbb{E}_0\left[V_t^{D_p}\right]}{\mathbb{E}_0\left[V_t^S\right]} - \left(k_t^U - k_t^{TS_s}\right)\frac{\mathbb{E}_0\left[V_t^{TS_s}\right]}{\mathbb{E}_0\left[V_t^S\right]} +$$
$$- \left(k_t^U - r_f\right)\frac{\mathbb{E}_0\left[V_t^{TS_p}\right]}{\mathbb{E}_0\left[V_t^S\right]} + \left(k_t^U - k_t^{DC}\right)\frac{\mathbb{E}_0\left[V_t^{DC}\right]}{\mathbb{E}_0\left[V_t^S\right]}. \tag{A1}$$

Proof of Equation (8)

Let us consider the following set of recursive equations:

$$\left(1+k_t^W\right)\mathbb{E}_0[V_t] = \mathbb{E}_0\left[F_{t+1}^U + V_{t+1}\right],$$

$$\left(1+k_t^U\right)\mathbb{E}_0\left[V_t^U\right] = \mathbb{E}_0\left[F_{t+1}^U + V_{t+1}^U\right],$$

$$\left(1+k_t^{TS_s}\right)\mathbb{E}_0\left[V_t^{TS_s}\right] = \mathbb{E}_0\left[F_{t+1}^{TS_s} + V_{t+1}^{TS_s}\right],$$

$$(1+r_f)\mathbb{E}_0\left[V_t^{TS_p}\right] = \mathbb{E}_0\left[F_{t+1}^{TS_p} + V_{t+1}^{TS_p}\right],$$

$$\left(1+k_t^{DC}\right)\mathbb{E}_0\left[V_t^{DC}\right] = \mathbb{E}_0\left[F_{t+1}^{DC} + V_{t+1}^{DC}\right].$$

Subtracting from the first equation the second, the third and the fourth one, and then summing the fifth one, after some algebraic manipulations (in which Equation (2) has been used), we easily get:

$$k_t^W\mathbb{E}_0[V_t] - k_t^U\mathbb{E}_0\left[V_t^U\right] - k_t^{TS_s}\mathbb{E}_0\left[V_t^{TS_s}\right] - r_f\mathbb{E}_0\left[V_t^{TS_p}\right] + k_t^{DC}\mathbb{E}_0\left[V_t^{DC}\right] + \mathbb{E}_0\left[F_{t+1}^{TS_s}\right] + \mathbb{E}_0\left[F_{t+1}^{TS_p}\right] - E_0\left[F_{t+1}^{DC}\right] = 0.$$

Since from Equation (2) it follows that $V_t^U = V_t - V_t^{TS_s} - V_t^{TS_p} + V_t^{DC}$, the above equation can be rewritten as follows:

$$k_t^W = k_t^U - \left(k_t^U - k_t^{TS_s}\right)\frac{\mathbb{E}_0\left[V_t^{TS_s}\right]}{\mathbb{E}_0[V_t]} - \left(k_t^U - r_f\right)\frac{\mathbb{E}_0\left[V_t^{TS_p}\right]}{\mathbb{E}_0[V_t]}$$
$$+ \left(k_t^U - k_t^{DC}\right)\frac{\mathbb{E}_0\left[V_t^{DC}\right]}{\mathbb{E}_0[V_t]} - \frac{\mathbb{E}_0\left[F_{t+1}^{TS_s}\right]}{\mathbb{E}_0[V_t]} - \frac{\mathbb{E}_0\left[F_{t+1}^{TS_p}\right]}{\mathbb{E}_0[V_t]} + \frac{\mathbb{E}_0\left[F_{t+1}^{DC}\right]}{\mathbb{E}_0[V_t]}. \quad \text{(A2)}$$

Proof of Equation (9)

Let us consider the following set of recursive equations:

$$\left(1+k_t^W\right)\mathbb{E}_0[V_t] = \mathbb{E}_0\left[F_{t+1}^U + V_{t+1}\right],$$

$$\left(1+k_t^S\right)\mathbb{E}_0\left[V_t^S\right] = \mathbb{E}_0\left[F_{t+1}^S + V_{t+1}^S\right],$$

$$\left(1+k_t^{D_s}\right)\mathbb{E}_0\left[V_t^{D_s}\right] = \mathbb{E}_0\left[F_{t+1}^{D_s} + V_{t+1}^{D_s}\right].$$

$$(1+r_f)\mathbb{E}_0\left[V_t^{D_p}\right] = \mathbb{E}_0\left[F_{t+1}^{D_p} + V_{t+1}^{D_p}\right].$$

Subtracting from the first equation the second, the third and the fourth one, after some algebraic manipulations (in which Equations (2) and (1) have been used), we easily get:

$$k_t^W \mathbb{E}_0[V_t] - k_t^S \mathbb{E}_0\left[V_t^S\right] - k_t^{D_s} \mathbb{E}_0\left[V_t^{D_s}\right] - r_f \mathbb{E}_0\left[V_t^{D_p}\right] + \mathbb{E}_0\left[F_{t+1}^{TS_s}\right] + \mathbb{E}_0\left[F_{t+1}^{TS_p}\right] - \mathbb{E}_0\left[F_{t+1}^{DC}\right] = 0.$$

The above equation can be rewritten, therefore, as follows:

$$k_t^W = \frac{\mathbb{E}_0\left[V_t^S\right]}{\mathbb{E}_0[V_t]} k_t^S + \frac{\mathbb{E}_0\left[V_t^{D_s}\right]}{\mathbb{E}_0[V_t]} k_t^{D_s} + \frac{\mathbb{E}_0\left[V_t^{D_p}\right]}{\mathbb{E}_0[V_t]} r_f - \frac{\mathbb{E}_0\left[F_{t+1}^{TS_s}\right]}{\mathbb{E}_0[V_t]} - \frac{\mathbb{E}_0\left[F_{t+1}^{TS_p}\right]}{\mathbb{E}_0[V_t]} + \frac{\mathbb{E}_0\left[F_{t+1}^{DC}\right]}{\mathbb{E}_0[V_t]}. \tag{A3}$$

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