

Verification of stochastic seismic analysis method and seismic performance evaluation based on multi-indices for high CFRDs



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ABSTRACT

Many high concrete-faced rockfill dams (CFRDs) are located in areas with high earthquake intensity where the ground motions are characterized by randomness; consequently, it is significant to study the seismic responses and evaluate the seismic performance using dynamic time-history analysis by a stochastic vibration method based on failure probability theory. In this paper, a recently developed generalized probability density evolution method (GPDEM) coupled with a spectral representation-random function method is verified to be suitable for strongly nonlinear structures in high CFRDs during earthquakes by comparing the accuracy and efficiency of the GPDEM with those of the Monte Carlo method (MCM). The GPDEM combined with the currently deterministic dam finite element time-history response analysis using a series of simple and common methods, is adopted to analyze the stochastic seismic responses, dynamic probability evaluation and failure probability of high CFRDs subjected to stochastic earthquake excitation. The statistical and probabilistic information of the typical physical quantities are compared between the GPDEM and MCM after a series of deterministic dynamic calculations, and the dynamic nonlinear behavior of rockfills and the random characteristics of ground motions are presented. The strong correspondence between the results obtained using the traditional stochastic probability MCM analysis and the GPDEM analysis demonstrates the accuracy and effectiveness of the newly proposed method despite its significantly lower computational burden. Finally, the failure probabilities of a high CFRD with different failure grades based on three universal evaluation indices are determined by constructing a virtual GPDEM process. The results demonstrate that the GPDEM shows promise as an approach that can reliably analyze strongly nonlinear structures, such as earth-rockfill dams and other geotechnical engineering structures.

1. Introduction

Over the past few decades, the use of hydropower has witnessed a boom in popularity, and a large number of concrete-faced rockfill dams (CFRDs) have been constructed all over the world because of their low cost and rapid construction (Wang et al., 2014; Feng et al., 2018). In addition to the increase in the number of CFRDs being constructed, the height of CFRDs is continually increasing, with many CFRDs exceeding 200 m and even reaching 300 m (Mahinroosta et al., 2015). However, many of these dams are situated in areas with high earthquake intensity, and severe damage to these high dams due to earthquakes could have disastrous consequences (Chen et al., 2014). Therefore, dam safety must be evaluated in regions with seismic activity. In addition, ground motions are characterized by randomness in both time and space; as a result, structural responses behave stochastically (Yazdani and Salimi, 2015). Unfortunately, deterministic seismic response analysis, which is

often employed in seismic safety evaluations of high CFRDs, is limited in its ability to comprehensively reflect rich probability information, and thus, it is more reasonable to adopt stochastic vibration theory to describe ground motions and investigate the seismic responses of high CFRDs.

Because of the randomness of ground motions, stochastic probability analysis has been used to solve earth and rockfill dam engineering problems and has attracted increasing attention from researchers as a consequence. Akköse et al. (2007) investigated the stochastic earthquake responses of the Keban Rockfill Dam; they assumed that the dam materials were linearly elastic, isotropic and homogenous for the investigation and then compared their results with those acquired from deterministic and stochastic analyses. Hacıfendioglu (2009) proposed a linear finite element method in which viscous boundaries were used to study the influence of ice cover on the seismic responses of CFRD-reservoir-foundation interaction

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systems under spatially varying seismic activity and described ground motions with a filtered white noise model. The above description, which is mainly based on linear elastic and nonlinear stochastic earthquake dynamic response analysis, has been adopted in the past decade of years. Alternatively, Wu et al. (2015) introduced a generalized coordinate system into reliability analysis involving correlated random variables and evaluated the seismic slope stability of high rockfill dams.

However, the responses obtained using the power spectral density ground motion model, such as the standard deviation and mean of the values, were second-order statistical parameters, and their structural models were relatively simple. Due to the randomness of earthquake ground motions and the nonlinear characteristics of rockfills, which can lead to the amplification of seismic waves, seismic safety must be investigated based on stochastic vibration theory and probability with dynamic time-history analysis. In addition, the aforementioned studies could not effectively simulate stochastic dynamic systems. As an alternative, the widely accepted traditional Monte Carlo method (MCM) provides a more thorough understanding of the stochastic vibrations of earth and rockfill dam engineering structures, but this method is used mainly for the stochastic simulation of linear structures and cannot be easily applied to the practical engineering of earth and rockfill dams because of its large computational burden (Li and Chen, 2009).

In recent years, Li and Chen (2009) developed the GPDEM and provided a new method for analyzing nonlinear stochastic seismic responses and the probability of complex engineering and large structures, combined with a new spectral representation-random function method that was proposed by Liu et al. (2016) to simulate non-stationary ground motions. Huang and Xiong (2017) evaluated the earthquake liquefaction performance of an earth dam and used it to obtain stochastic dynamic response processes and probability only based on the deformation. Moreover, the stochastic seismic response and reliability of some simple geotechnical structures were analyzed by Huang et al. (2015, 2018) based on this method and obtained good application results. Pang et al. (2018a) also adopted the aforementioned method to determine failure probabilities under stochastic seismic activity based on an elastoplastic analysis. However, the effectiveness and precision of this GPDEM for strongly nonlinear structures, such as earth and rockfill dams, have not been verified.

In this paper, the GPDEM coupled with a spectral representation-random function method is verified to be suitable for strong nonlinear structures high CFRDs during earthquake by comparing the accuracy and efficiency with those of the MCM. Subsequently, the stochastic seismic responses and failure probabilities of high CFRDs subjected to stochastic earthquake excitation are investigated based on multiple evaluation indices. The GPDEM combines the current deterministic finite element method for the time-history response analysis of a dam with a stochastic dynamic analysis technique using high-performance geotechnical dynamic nonlinear analysis software; accordingly, this approach differs dramatically from the traditional stochastic probability analysis method. First, a set of stochastic ground motions are generated by the spectral representation-random function method. Then, numerous deterministic dynamic calculations are performed by the finite element method. Finally, the stochastic seismic responses and the dynamic probability information corresponding to the target physical quantities are obtained based on the GPDEM. This stochastic time-history dynamic method can reveal the random characteristics of earthquakes and the dynamic nonlinear behavior of rockfills subjected to earthquake excitation. The failure probabilities of different evaluation indices are determined by combining a virtual stochastic process with the GPDEM, and the seismic performance of high CFRDs is evaluated based on multi-indices. These results demonstrate that the proposed GPDEM can be used to reliably analyze rockfill dams and other large geotechnical engineering structures; moreover, this method can provide references for the seismic performance design of these structures.

2. GPDEM equation and seismic probability

Generally, the dynamic response control equation of a CFRD under earthquake loading can be established as follows:

$$[M]\{\ddot{u}\}_t + [C]\{\dot{u}\}_t + [K]\{u\}_t = -[M]\{\ddot{u}_g(\Theta)\}_t \quad (1)$$

where $[M]$ is the whole mass matrix, $[C]$ is the damping matrix and $[K]$ is the stiffness matrix of the CFRD; $\{\ddot{u}\}_t$ is the acceleration, $\{\dot{u}\}_t$ is the velocity and $\{u\}_t$ is the displacement of every node of the CFRD at a given time t ; $\{\ddot{u}_g\}_t$, which is the input earthquake acceleration at time t , represents a stochastic earthquake excitation; and Θ is a stochastic vector reflecting the randomness of the stochastic ground motions.

According to mathematical logic and the probability conservation principle (Li and Chen, 2009), the GPDEM equation is described as follows:

$$\frac{\partial p_{Z\Theta}(z, \Theta, t)}{\partial t} + Z(\Theta, t) \frac{\partial p_{Z\Theta}(z, \Theta, t)}{\partial z} = 0 \quad (2)$$

The detailed process for solving the stochastic dynamic analysis (Eq. (1)) is provided in Appendix A.

3. Generation of stochastic ground motions

To evaluate the seismic safety of a high CFRD, obtaining the stochastic seismic activity is the first step. A spectral representation-random function method is used to generate the acceleration time series of non-stationary stochastic processes (Liu et al., 2016), expressed as follows:

$$\ddot{X}_g(t) = \sum_{k=1}^N \sqrt{2S_{X_g}^{\cdot\cdot}(t, \omega_k) \Delta\omega} [\cos(\omega_k t) X_k + \sin(\omega_k t) Y_k] \quad (3)$$

where $S_{X_g}^{\cdot\cdot}(t, \omega_k)$ is the evolutionary power spectral density function representing the amplitude and frequency modulation and $\{X_k, Y_k\}$ ($k = 1, 2, \dots, N$) are the standard orthogonal random variables with a frequency interval of $\Delta\omega = 0.15$ rad/s. The brief descriptions of generating processes of stochastic ground motions are shown in Appendix B.

Finally, probabilities that satisfy the same system are assigned to generate the acceleration time series, and the probability summation of the set is equal to 1 based on the abovementioned stochastic function and spectral representation-random function method. The flowchart of the GPDEM is shown in Fig. 1. The acceleration time series is composed of 377 samples, and the time series with second-order statistics between the random seismic spectrum and the stochastic earthquake compared with the target values, confirm that the effectiveness and accuracy of the spectral representation-random function method and its generation of ground motions according to Number-Theoretical method, as illustrated in Fig. 2. In summary, the acceleration time history samples are generated by a set of random variables describing the stochastic function, and the random variables are obtained based on Number-Theoretical method or Monte Carlo method respectively.

4. Numerical case of a high CFRD

4.1. Gushui dam

The Gushui CFRD is situated upstream of the Lancang River in Yunnan Province, China, where strong earthquakes may occur (the seismic fortification intensity is VIII degrees), and is adopted as the numerical case study. The finite element model of the dam is illustrated in Fig. 3. The maximum height of the dam is 242 m, and the length and width of the dam crest are 437 m and 25 m, respectively. The maximum height of impounded water is 220 m, and the thickness of the face slab is 0.36 m at the crest and 1.21 m at the foundation. In this study, the elements have 2×2 integration points and four nodes, and Gaussian

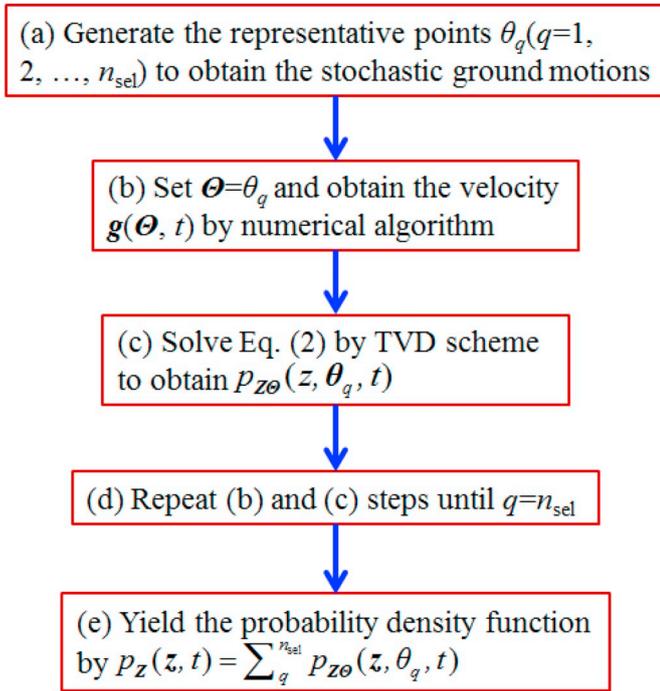


Fig. 1. The flowchart of the GPDEM.

numerical integration is used to calculate the element matrices. Five rockfill zones and concrete face slabs constitute the Gushui dam body from upstream to downstream: Cushion, Transition, Rockfill A, Rockfill B and Rockfill C.

4.2. Numerical methods and parameters

This section mainly introduces the numerical methods utilized in the static and dynamic analyses, permanent deformation calculations and slope stability calculations for CFRDs. In addition, the mechanical behavior and numerical parameters of every material are as follows.

4.2.1. Concrete face slabs

The imperviousness of a high CFRD mainly depends on its concrete face slabs. In this paper, the behavior of concrete face slabs is simulated by a linear elastic model that also represents the effect of hydrodynamic pressure on the face slabs. Assuming that the face slabs are composed of C30 concrete, the properties are as follows: the modulus of elasticity E is 30 GPa, the density ρ is 2.40 g/cm³, the Poisson's ratio ν is 0.167, the damping ratio under dynamic loading is 5%, and the compressive strength of the face slabs is 27.6 MPa. The formulas proposed by Raphael (1984) are used to obtain the face slabs' tensile strengths under static as well as seismic dynamic loads, and this is used in some research to describe the characteristics of concrete (Wang et al., 2015; Pang et al., 2018a). The static tensile strength is 3 MPa. Moreover, the added mass method (Westergaard, 1933), which has also been used for many other dams to characterize the hydrodynamic pressure (Xu et al., 2018b; Chen et al., 2019), is adopted to simulate the hydrodynamic pressure on the face slabs.

4.2.2. Rockfill materials and interfaces

A dynamic calculation is required to determine the initial stress field. Hence, a typical Duncan-Chang E-B model (Duncan and Chang, 1970) with a static calculation is adopted to describe the filling and impounding process. Table 1 lists the model parameters. To determine

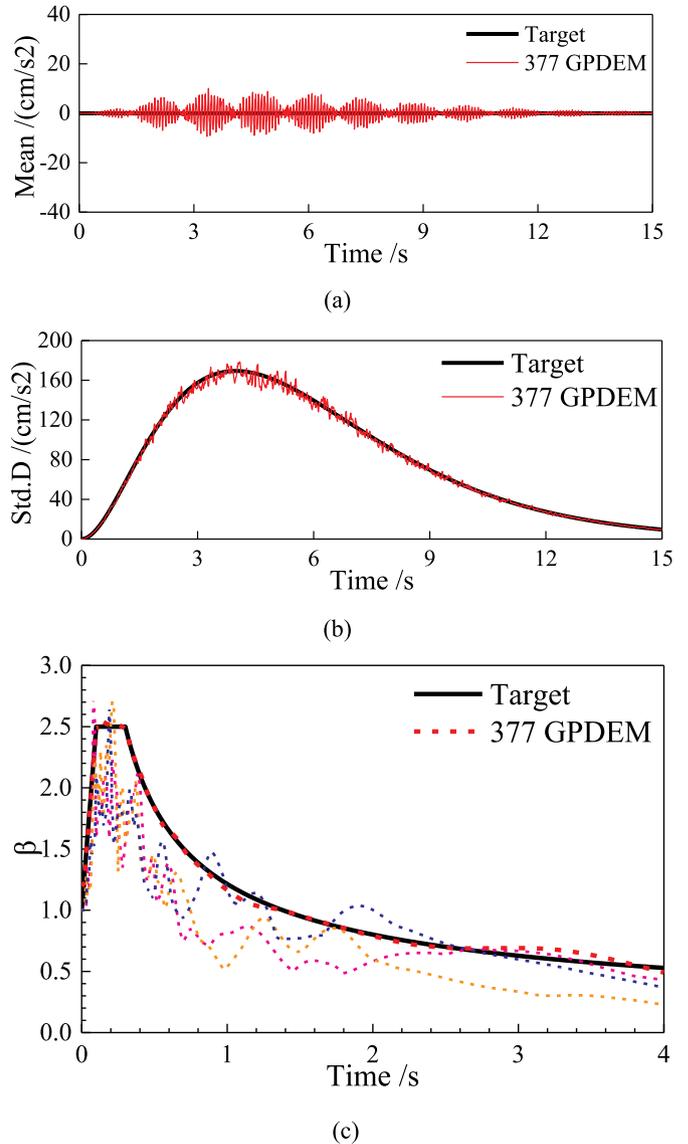


Fig. 2. Comparison of mean, standard deviation and response spectra between GPDEM and target: (a) mean, (b) standard deviation, (c) response spectra.

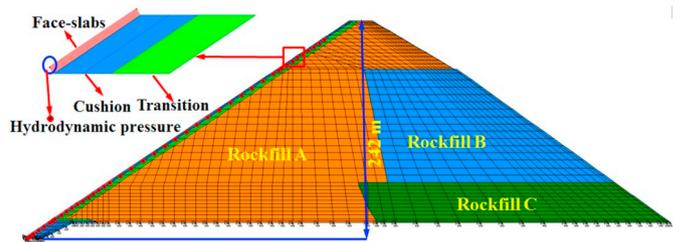


Fig. 3. 2-D Finite element mesh of the Gushui CFRD.

Table 1
Parameters for Duncan E-B model.

Materials	ρ /(kg/m ³)	K	n	R_f	K_b	m	ϕ_0 (°)	$\Delta\phi$ (°)
Rockfill A	2214	1305	0.28	0.80	780	0.18	55.5	11.3
Rockfill B	2100	1000	0.26	0.79	700	0.16	53.0	11.0
Rockfill C	2170	980	0.26	0.79	400	0.31	50	8.2
Transition	2222	1250	0.31	0.78	500	0.16	53.5	10.7
Cushion	2258	1200	0.30	0.75	680	0.15	54.4	10.6

Table 2
Parameters for Hardin-Drnevich model.

Materials	K	n	ν
Rockfill A	2660	0.444	0.33
Rockfill B	3115	0.396	0.33
Rockfill C	4997	0.298	0.33
Transition	3223	0.455	0.40
Cushion	3828	0.345	0.40

the dynamic nonlinear behavior of the rockfill, the equivalent linear viscoelastic model proposed by [Hardin and Drnevich \(1972\)](#) is introduced, and the maximum dynamic shear modulus of a damming rockfill is expressed as follows:

$$G_{max} = K \cdot p_a \cdot (p/p_a)^n \tag{4}$$

where p_a is the technical atmospheric pressure, K and n represent experimental parameters, and $p = (\sigma_1 + \sigma_2)/2$ is the average effective stress, where σ_1 and σ_2 are the effective principal stresses acting on the specimen. The model parameters are listed in [Table 2](#).

High CFRDs will produce permanent deformation irretrievably under the action of an earthquake. That is, the seismic dynamic response analysis methods currently used for earth or rockfill dams based on an equivalent linear viscoelastic model can obtain only the time-history response of the dam's acceleration, dynamic stress or dynamic strain, whereas they cannot directly calculate the permanent deformation of the dam body. Therefore, the method of applying the strain potential to permanent deformation analysis proposed by [Serff and Seed \(1976\)](#) is used in this study to obtain the permanent deformation, and the relationships among the residual volumetric strain increment $\Delta\epsilon_{vr}$, the residual shear strain increment $\Delta\gamma_r$, the stress state, and the vibration duration initially proposed by [Shen and Xu \(1996\)](#) and further refined by [Zou et al. \(2008\)](#) through numerous large-scale cycle triaxial experiments are employed and expressed as follows:

$$\Delta\epsilon_{vr} = c_1 \gamma_d^{c_2} \exp(-c_3 S_1^2) \frac{\Delta N}{1 + N} \tag{5}$$

$$\Delta\gamma_r = c_4 \gamma_d^{c_5} S_1^n \frac{\Delta N}{1 + N} \tag{6}$$

where γ_d is the dynamic strain amplitude; N and ΔN are the total vibration times and time increment, respectively; S_1 is the stress level; n is the stress level index and is typically 0.9–1.0; and $c_1, c_2, c_3, c_4,$ and c_5 are the experimental parameters. The permanent deformation model parameters of the rockfill materials are shown in [Table 3](#).

In addition, to obtain more realistic results, Goodman zero-thickness contact elements ([Goodman et al., 1968](#)) are applied between the face slabs and the cushion in this study. A Clough-Duncan hyperbolic model ([Clough and Duncan, 1971](#)) is used to simulate the interfaces, consistent with the static analysis, and the dynamic hyperbolic model proposed by [Wu and Jiang \(1992\)](#) is applied to describe the dynamic behavior of the interfaces. The parameters of the interface elements are summarized in [Tables 4 and 5](#).

Table 3
Parameters for Residual deformation calculations.

Materials	c_1 /(%)	c_2	c_3	c_4 /(%)	c_5
Rockfill A	1.36	0.85	0.00	23.26	0.74
Rockfill B	3.46	1.03	0.00	31.46	0.83
Rockfill C	0.84	0.81	0.00	10.82	0.62
Transition	1.45	0.97	0.00	4.91	0.51
Cushion	1.45	0.97	0.00	4.91	0.51

Table 4
Static parameters for interface element model.

Materials	K	n	φ	R _f
Interface	4800	0.56	36.6	0.74

Table 5
Dynamic parameters for interface element model.

Materials	C	M	δ	λ _{max}
Interface	4800	0.56	36.6	0.74

Finally, the linear elastic model of the concrete face slabs, the Dunchan-Chang E-B static analysis model, the Hardin-Drnevich dynamic analysis model, the rockfill permanent deformation model and the interface models are incorporated into the high-performance geotechnical dynamic nonlinear analysis software GEODYNA with parallel CPU + GPU computing; this approach was originally developed by [Zou and Kong \(2013\)](#) to simulate static responses, dynamic responses and permanent deformation. These models were successfully applied to the Zipingpu CFRD during the Wenchuan earthquake ([Zou et al., 2012, 2013](#)).

4.2.3. Dynamic stability of the dam slope

The dynamic stability of the dam slope is another essential characteristic to consider when evaluating the dynamic safety of high CFRDs. In recent years, a modified Newmark method ([Zou et al., 2012; Zhou et al., 2016](#)) was used to determine the time-history responses of safety factors and the dynamic slippage of dam slopes according to the superimposed results of the static and dynamic stresses considering the softening characteristics of rockfill ([Zhou et al., 2016](#)). This analytical method was further incorporated into FEMSTABLE 2.0 and applied to the seismic stability or dislocation analysis of various CFRDs ([Pang et al., 2018b, 2018c; Zou et al., 2012; Zhou et al., 2016](#)). The safety factor is expressed as follows:

$$F_s = \frac{\sum_{i=1}^n (c_i + \sigma_{ni} \operatorname{tg}\varphi_i) l_i}{\sum_{i=1}^n \tau_i l_i} \tag{7}$$

The maximum cumulative slippage of the dam slope is the maximum value of the cumulative slippage, which can be expressed as follows:

$$D_{max} = \operatorname{MAX}(D_1, D^2, \dots, D_k \dots D^m) \tag{8}$$

4.3. Numerical analysis

In this paper, a set of deterministic finite element calculations with static, dynamic, and permanent deformation and then slope stability analysis based on the Gushui CFRD with a height of 242 m are performed for the two-dimensional stochastic numerical calculation. The static calculations of filling and impounding based on the Duncan-Chang EB model for the rockfill and the linear elastic model for the face slabs are first performed to obtain the initial stress field for the dynamic calculation. Then, the dynamic analysis using the Hardin-Drnevich model for the rockfill and the linear elastic model for the faced-slabs are introduced to determine the response time histories of the acceleration, dynamic stress and dynamic strain of the CFRD subjected to the different stochastic ground motions. Finally, after the static and a series of dynamic analyses, a permanent deformation analysis based on strain

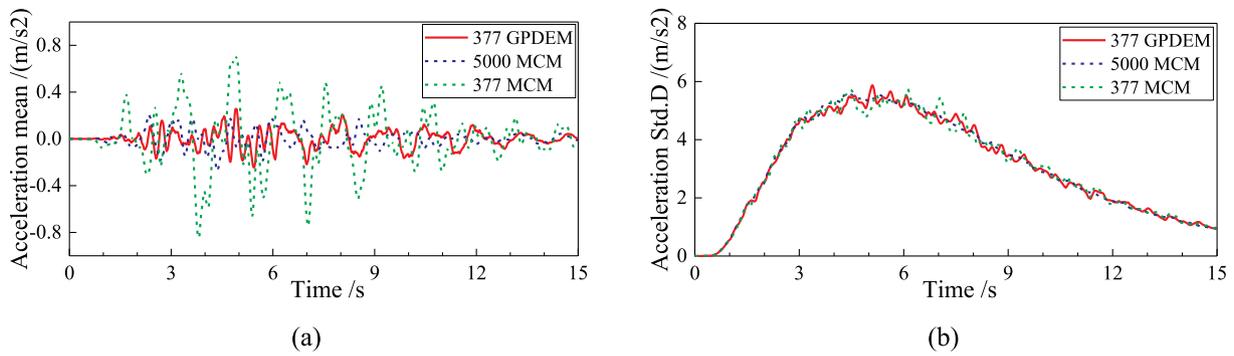


Fig. 4. Comparison of mean and standard deviation of dynamic acceleration between GPDEM and MCM: (a) Mean, (b) Standard deviation.

potential is used to obtain the permanent deformation after the earthquake, and a dynamic stability analysis based on finite element time history method for the dam slope is implemented to determine the safety factor time history and the slippage.

5. Verification of the stochastic seismic analysis method

The dynamic time histories of acceleration, stress and safety factors are the primary aspects to consider when analyzing the seismic responses of high CFRDs (Pang et al., 2018c; Zou et al., 2013; Zhou et al., 2016). In this study, the stochastic response time history of the acceleration in the dam crest, the face-slab stress and the safety factor on the dam slope are obtained under numerous deterministic earthquake acceleration time history samples with PGA = 0.4 g (selected based on an appropriate value of peak ground acceleration) based on the GPDEM. In addition, a Monte Carlo method, which appears to be the only universal

method with adequate precision for arbitrary-dimension nonlinear stochastic dynamic systems (Spanos and Evangelatos, 2010), is used for a comparison with the dam stochastic response results based on the GPDEM and thereby demonstrate the accuracy and effectiveness of the GPDEM calculations. After the above analysis, the velocity responses are obtained by substituting the dynamic acceleration, dynamic stress and the safety factor into the GPDEM equation, and the finite difference method (Li and Chen, 2009) is used to solve the GPDEM equation. Then, the stochastic responses and the dynamic probability information of the three desired physical quantities, namely, the dynamic acceleration, dynamic stress and dynamic safety factor, can be acquired.

The means and standard deviations of the time histories of the dynamic acceleration, the dynamic stress and the dynamic safety factor are illustrated in Figs. 4, 5 and 6, respectively, in which the results evaluated by the GPDEM and the MCM are compared. The comparison is based on 377 GPDEM simulations and 5000 MCM, 377 MCM

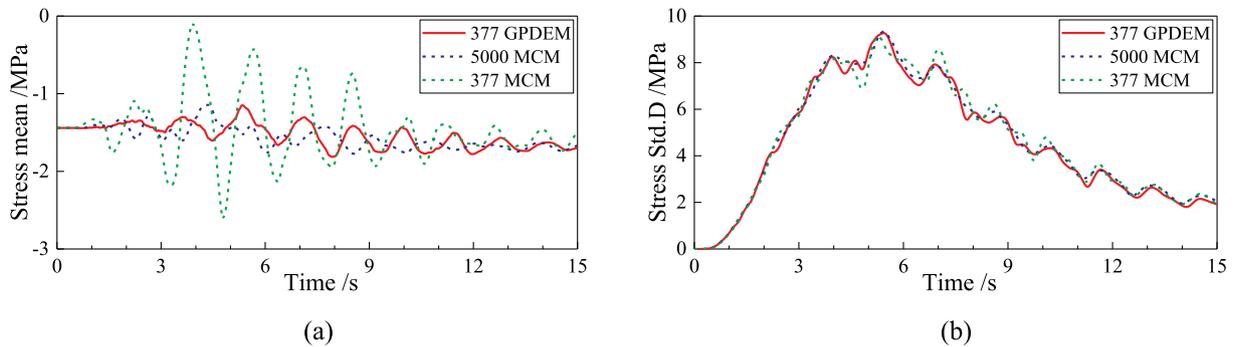


Fig. 5. Comparison of mean and standard deviation of dynamic stress between GPDEM and MCM: (a) Mean, (b) Standard deviation.

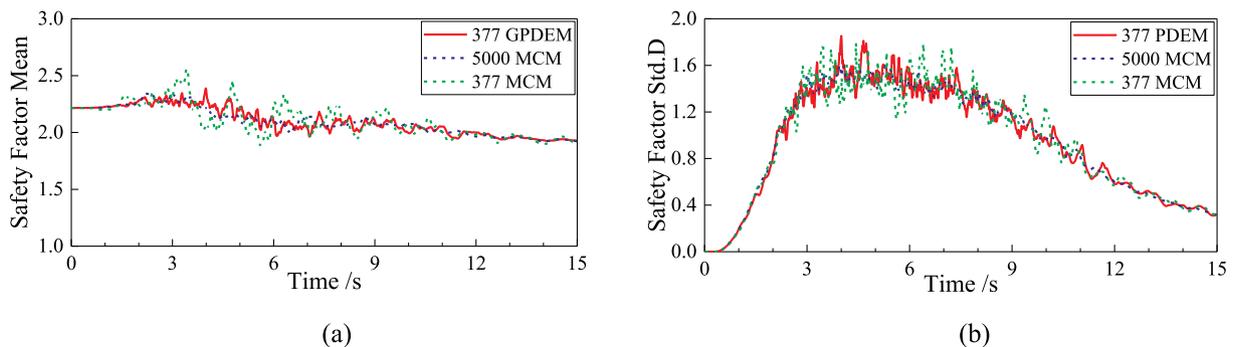


Fig. 6. Comparison of mean and standard deviation of safety factor between GPDEM and MCM: (a) Mean, (b) Standard deviation.

simulations, the root mean squared errors are 0.1102 and 0.2761 for the acceleration mean time histories, 0.1418 and 0.4634 for the stress, and 0.0356 and 0.0948 for the safety factor, so the results demonstrate the efficiency and accuracy of the proposed method despite the lower quantity of computational samples and lower time requirements. In addition, its standard deviations and mean are on the same order of magnitude as those used in the 5000 MCM. However, the 377 MCM results have a more difference with the 5000 MCM results. These further indicate the high precision of the GPDEM for stochastic seismic analysis with dynamic time-series calculation. The variations of mean and standard deviations time-histories also indicate that the seismic response physical quantities exhibit significant variation because of the randomness of the seismic ground motions. Moreover, the dynamic responses can vary greatly when the seismic ground motions are different. However, it should be noted that there is still some large errors between 377 GPDEM and 5000 MCM.

The physical quantities dynamic acceleration, stress and safety factor of the time histories are substituted into the GPDEM equation to be the response velocities after a series of deterministic seismic response time-series analyses. Then the stochastic dynamic probabilities (cumulative distribution functions, CDFs) at 2 typical time instants of the desired physical quantities are obtained by solving the GPDEM equation based on the finite difference method, as shown in Fig. 7. The 377 GPDEM and 5000 MCM CDFs at different typical time instants agree relatively well with each other, which shows the accuracy of the GPDEM. To better demonstrate the accuracy and efficiency of GPDEM, the 377 MCM CDFs are also presented to compare with the 377 GPDEM and 5000 MCM CDFs, which agrees relatively weak with them. Furthermore, as shown in Fig. 7, the GPDEM illustrates the changes that occur in the CDF over time, indicating that the seismic ground motions significantly influence the seismic responses, and the seismic responses of high CFRDs must be analyzed from a stochastic perspective because of the coupling effect of the nonlinearity of the rockfill and the randomness of ground motions. In addition, Fig. 8 shows the CDFs of the maximum acceleration, maximum stress and minimum safety factor by establishing an equivalent extreme-value event that can be solved based on a virtual stochastic process and the GPDEM (Li and Chen, 2009). The comparison based on CDFs with the equivalent extreme-value event of 377 GPDEM simulations with 5000 and 377 MCM simulations further demonstrates the accuracy and efficiency of the proposed GPDEM. Moreover, the accuracy of GPDEM is raised, possibly due to an effect of the GPDEM evolutionary process.

6. Seismic performance evaluation based on multi-indices

Generally, vertical deformation, cumulative slippage of the dam slope, and face-slab damage index are commonly used to be the indices for evaluating the seismic performance of high CFRDs (Pang et al., 2018a, 2018c, 2018d). In addition, for the sake of evaluating the seismic safety of the face-slabs, a new face-slab damage index that simultaneously considers the stress amplitude exceeding the tensile strength (the demand capacity ratio, $DCR = \sigma_t/f_s$, where σ_t is the calculated tensile stress and f_s is the tensile strength) and the cumulative overstress duration (COD, the cumulative time that the calculated tensile stress exceeds the tensile strength) based on stress with a linear elasticity analysis of the face-slabs is presented (Pang et al., 2018a).

To evaluate seismic performance and safety, three failure grades were established based on the aforementioned three evaluating indices

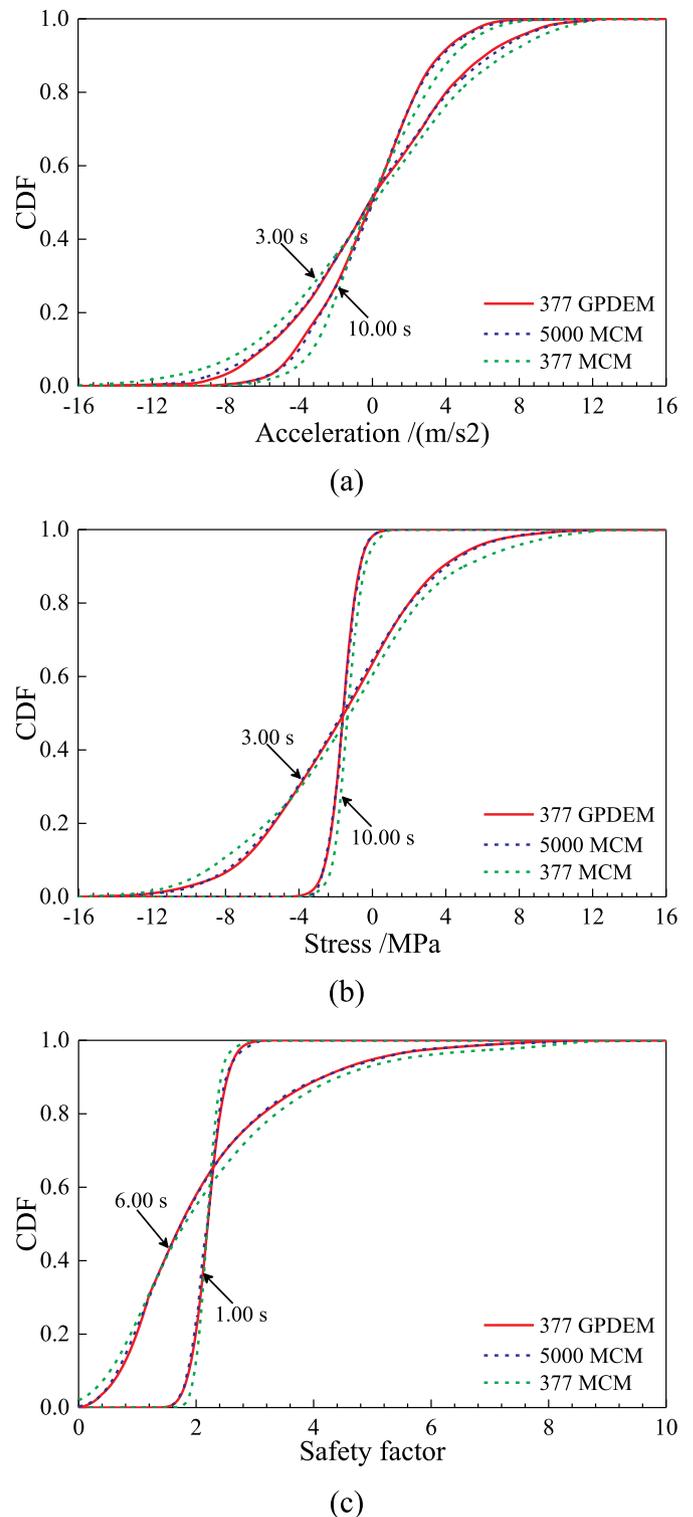
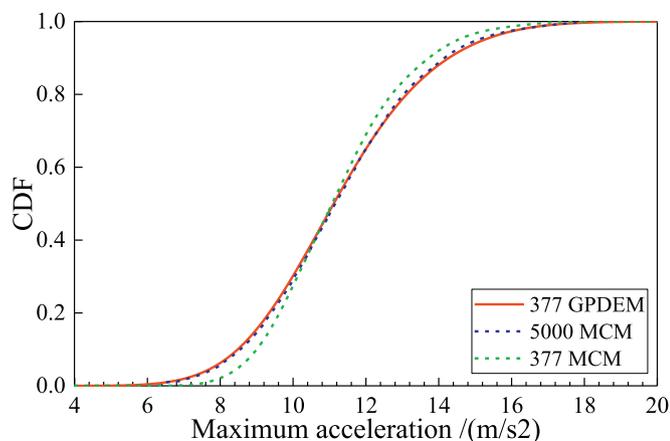
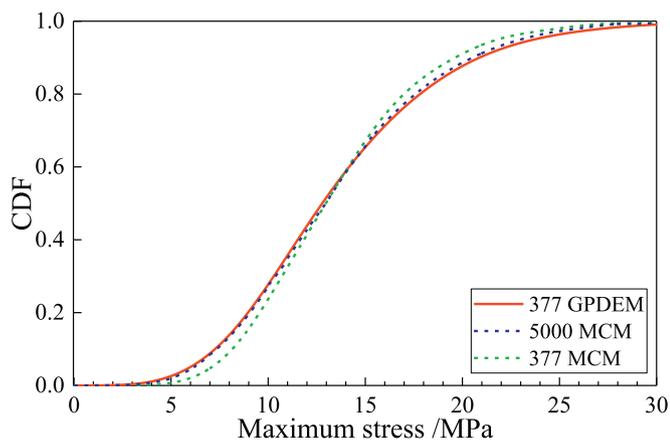


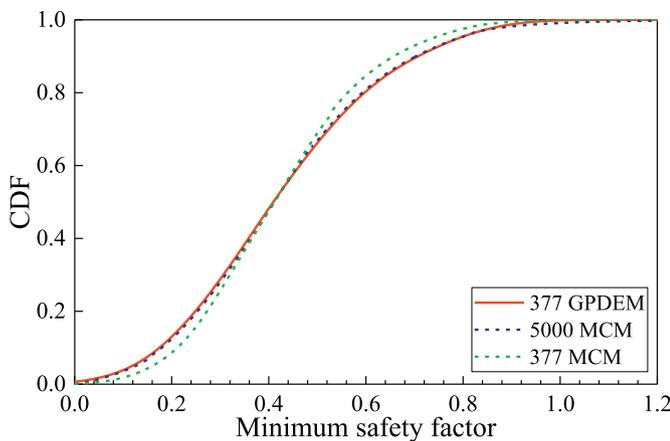
Fig. 7. Dynamic CDF of (a) acceleration, (b) stress and (c) safety factor in 2 typical time instants. (The 6 s means the CDF values corresponding to different values at 6 s time points.)



(a)



(b)



(c)

Fig. 8. CDF with the equivalent extreme-value event of (a) maximum acceleration, (b) maximum stress and (c) minimum safety factor.

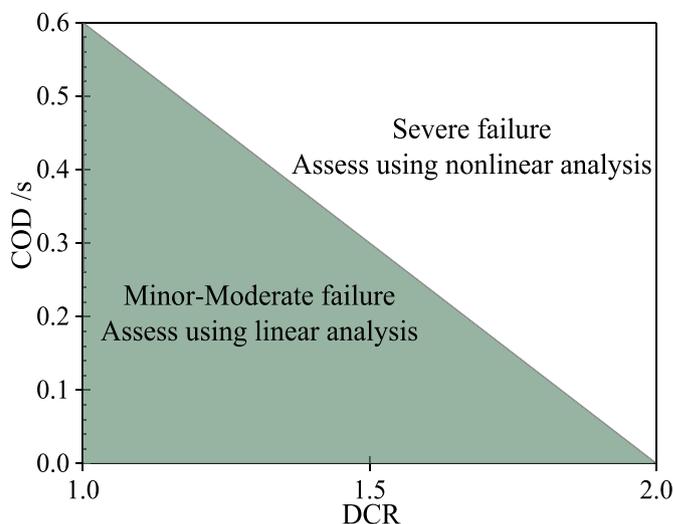


Fig. 9. Face-slab damage index and recommended failure grades (DCR, ratio of calculated tensile stress to tensile strength; COD, cumulative time of calculated tensile stress exceeding tensile strength).

for high CFRDs according to the proposal of Pang et al. (2018a). Currently, according to these three limit failure states, the relative settlement ratios (the ratio of the vertical deformation of the dam crest to the dam height) are 0.4% (0.968 m), 0.7% (1.694 m) and 1.0% (2.420 m) for dam crest ratios corresponding to minor, moderate and severe failure grades, respectively. The cumulative slippage of the dam slope is a new evaluating index applied to judge the stability of the dam slope based on a Newmark method and has been proposed in previously developed codes and research (NB 35047–2015, 2015). Pang et al. (2018c) presented three limit failure states based on cumulative slippages of 5, 50 and 100 cm corresponding to the minor, moderate and severe failure grades, respectively. For the face-slab damage index, the classification of the failure grades is as follows: (1) no or minor failure: the face slab response is within the linear elastic range of behavior, with little or no possibility of destruction if $DCR \leq 1$; (2) minor to moderate failure: the shaded section of Fig. 9 covers the DCR and COD, and cracking of the face-slabs is apparent; and (3) severe failure: the COD is outside the shaded section or $DCR > 2$. A nonlinear time-history analysis of the face-slabs is required in this third situation.

To evaluate the seismic failure probabilities based on the three assessment indices in high CFRDs, a virtual GPDEM is constructed to obtain the PDFs for vertical deformation (Fig. 10(a)) and cumulative slippage (Fig. 11(a)) and the PDFs (Fig. 12(a)) of the CODs with different DCRs ($DCR = 1$ indicates that the COD of $DCR > 1$). Then, the CDFs of the vertical deformation, the cumulative slippage and CODs with different DCRs are calculated by integrating the PDFs, as illustrated in Figs. 10(b), 11(b), and 12(b). The seismic failure probabilities are obtained on the basis of the aforementioned evaluation criteria for failure grades and CODs with different DCRs. The seismic failure probabilities based on the vertical deformation and the cumulative slippage are listed in Table 6. Fig. 12 illustrates the probability changes in different DCRs with a COD according to the face-slab damage index. In accordance with the criteria used to determine the failure grade

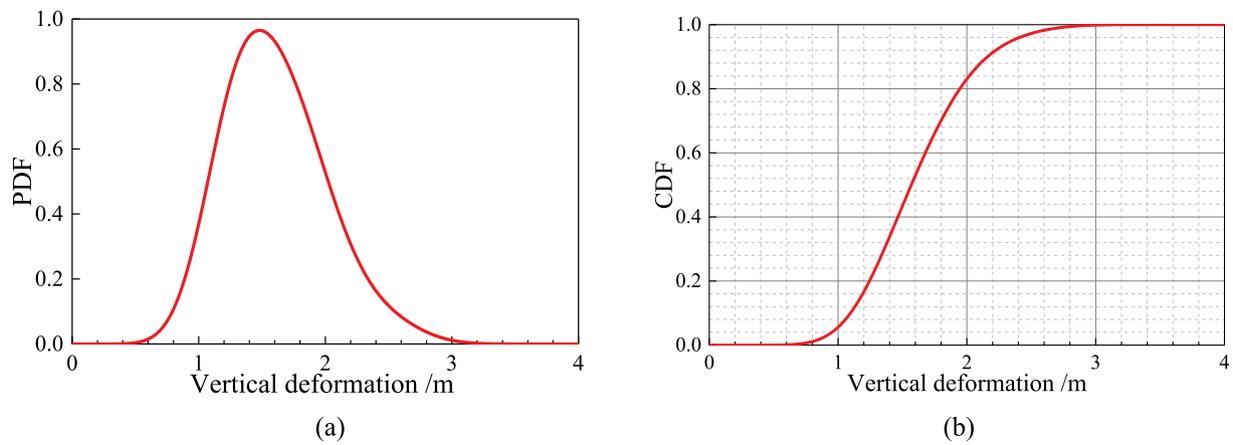


Fig. 10. (a) PDF and (b) CDF of the vertical deformation.

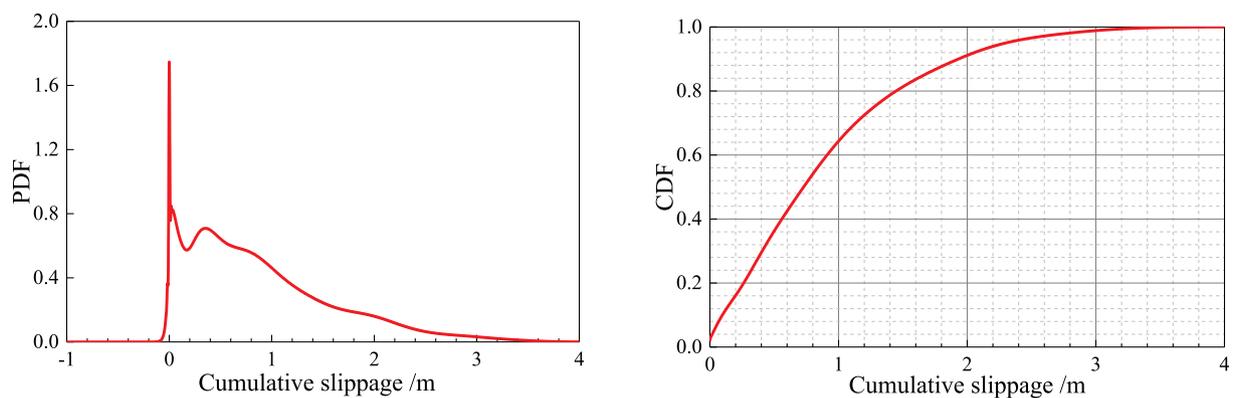


Fig. 11. (a) PDF and (b) CDF of the cumulative slippage.

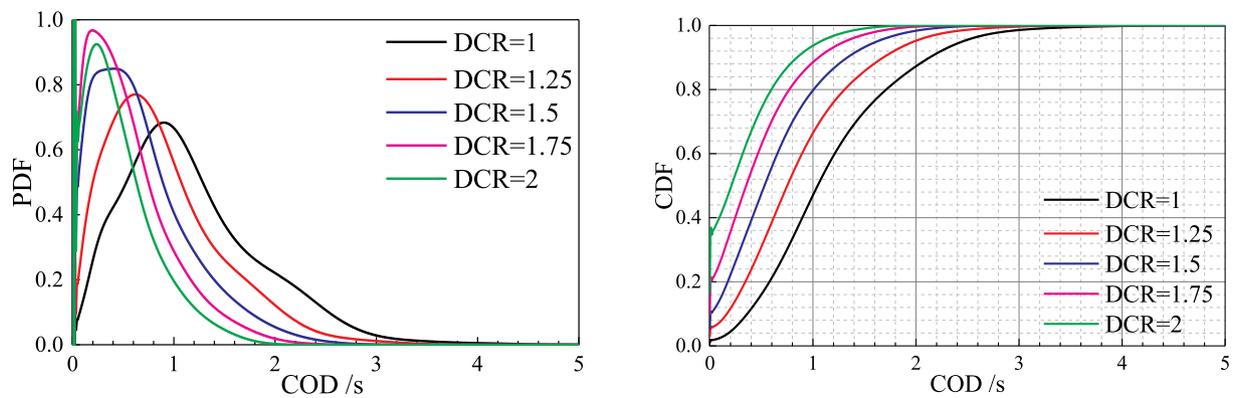


Fig. 12. (a) PDFs and (b) CDFs of the cumulative overstress duration (COD) in different DCRs.

Table 6
Failure probabilities of different failure grades based on different evaluating indices.

Evaluating indices	No	Minor	Moderate	Severe
Vertical deformation	100%	95.5%	38.6%	3.8%
Cumulative slippage	100%	93.4%	63.7%	35.7%
Face-slab damage index	100%	100%	72.0%	Assess using nonlinear analysis

(Fig. 13), the minor dynamic failure probability reaches to nearly 98%, and the moderate dynamic failure probability is 72% subjected to the earthquake with PGA = 0.4 g, as shown in Table 6. A nonlinear analysis is required for moderate-severe and severe failures. The failure probabilities show that the dam slope and face slabs are more easily damaged under earthquake activity.

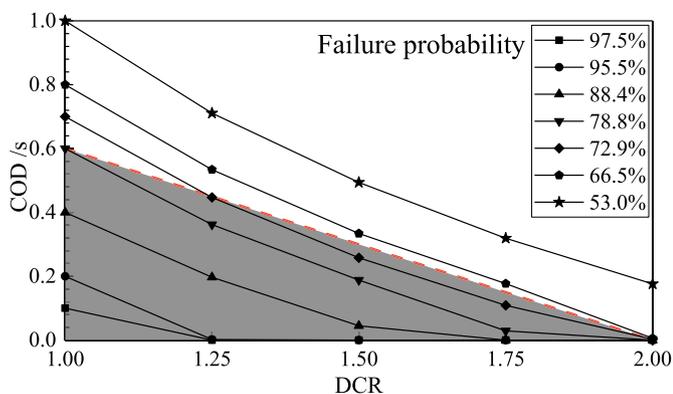


Fig. 13. Failure curves for the face-slabs using linear elastic analysis in different failure probability.

7. Conclusions

Seismic performance evaluation of high CFRDs subjected to stochastic earthquake excitation is a complex problem that will lead to dynamic responses with random time histories. The conclusions of the above studies are as follows:

- (1) In this paper, the proposed GPDEM was utilized to acquire the stochastic dynamic response time histories and probability information for high CFRDs subjected to stochastic ground motion excitation. The stochastic ground motions were obtained by the spectral representation-random function method and were coupled with the GPDEM.
- (2) The statistical information and failure probabilities of the typical physical quantities, including the dynamic acceleration, stress and safety factor, were obtained according to the GPDEM and the commonly employed MCM after numerous deterministic dynamic calculations based on a 242-m-high CFRD. Obviously, the 377 GPDEM simulations yielded sufficient results, the difference of which from the results obtained from the 5000 MCM simulations was very small. This finding demonstrates that the GPDEM is much more efficient (> 10 times faster) than the MCM with an acceptable

Appendix A. GPDEM equation

Generally, the dynamic response control equation of the CFRD under earthquake loading can be established as follows:

$$[M]\{\ddot{u}\}_t + [C]\{\dot{u}\}_t + [K]\{u\}_t = -[M]\{\ddot{u}_g(\Theta)\}_t \tag{A-1}$$

Under stochastic seismic activity, the solution to system Eq. (A-1) depends on the random parameter Θ , denoted as follows:

$$\{u\}_t = X(t) = G(\Theta, t) \tag{A-2}$$

Analogously, the velocity process is also described by the function Θ and can be expressed as follows:

$$\{\dot{u}\}_t = \dot{X}(t) = g(\Theta, t) \tag{A-3}$$

Obviously, there exists $g(\Theta, t) = \partial G(\Theta, t) / \partial t$.

More generally, noting the physical parameters required for $Z = (Z_1, Z_2, \dots, Z_m)^T$ with a coupled system,

$$\dot{Z}(t) = \Psi \left[X(t), \dot{X}(t) \right] \tag{A-4}$$

$\Psi(\cdot)$ is a conversion operator from the state vector to the required physical parameters.

Substituting Eq. (A-2) and Eq. (A-3) into Eq. (A-4),

$$\dot{Z}(t) = \Psi[G(\Theta, t), g(\Theta, t)] = h(\Theta, t) \tag{A-5}$$

Clearly, a stochastic dynamic system can be represented by Eq. (A-4). Thus, the augmented system is regarded as a probability conservative system consisting of (Z, Θ) ; the PDF of (Z, Θ) can be written to be $p_{Z\Theta}(z, \theta, t)$. According to mathematical processing and the probability conservation principle (Li and Chen, 2009), the GPDEM equation is described as follows:

degree of precision. Hence, the GPDEM is a cost-effective, efficient, and feasible analytical method for the stochastic seismic analysis of high CFRDs. These results also demonstrate that the proposed method could be applied in practice to evaluate the seismic performance of high CFRDs based on stochastic theory.

- (3) The probability information and statistical values of the three physical quantities presented the random characteristics of the ground motions, and the dynamic nonlinear rockfill behavior under stochastic earthquake excitation conditions was also shown. Finally, seismic performance was evaluated considering the failure probabilities based on three typical evaluating indices: vertical deformation, cumulative slippage, and face-slab damage.
- (4) The novel and efficient method presented in this paper could also be used for stochastic vibration and reliability analyses performed by dam and geotechnical engineers. Generally, the randomness and uncertainties mainly include two aspects for stochastic seismic responses and reliability analysis, the randomness of ground motions and the material parameters. In this paper, we only considered the uncertainties of the earthquake ground motion inputs and obtained the time histories of PDFs for three physical parameters based on GPDEM. This proposed method is also efficient for analyzing the stochastic seismic responses and reliability in consideration of the variability of the model parameters. Considering the variability of the material parameters of dam, slope or other geo-structures will be the further research work by the authors.

Declaration of Competing Interest

None

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$$\frac{\partial p_{Z\Theta}(z, \theta, t)}{\partial t} + \dot{Z}(\theta, t) \frac{\partial p_{Z\Theta}(z, \theta, t)}{\partial z} = 0 \quad (\text{A-6})$$

The initial condition is

$$p_{Z\Theta}(z, \theta, t)|_{t=t_0} = \delta(z - z_0) p_{\Theta}(\theta) \quad (\text{A-7})$$

and the PDF $p_Z(z, t)$ of $\mathbf{Z}(t)$ is

$$p_Z(z, t) = \int_{\Omega_{\Theta}} p_{Z\Theta}(z, \theta, t) d\theta \quad (\text{A-8})$$

A series of discrete representative points θ_q ($q = 1, 2, \dots, n_{sel}$) in the distribution space Ω_{Θ} of basic random variable space Θ are selected to solve the GPDEM equation, where n_{sel} is the total number of discrete points according to the Number-Theoretical method proposed by Hua-Wang (Hua and Wang, 1981). The earthquake ground motion is the random variable in this research. Then, for a given $\Theta = \theta_q$, a series of finite element calculations is performed to solve Eq. (A-1) and provide the time derivative of the required quantities $\dot{Z}(\theta_q, t)$. Finally, the finite difference method is used to obtain Eq. (A-6) in light of the TVD scheme (Li and Chen, 2009). Adding the abovementioned $p_{Z\Theta}(z, \theta_q, t)$ and the numerical solution $p_Z(z, t)$ yields the following:

$$p_Z(z, t) = \sum_q^{n_{sel}} p_{Z\Theta}(z, \theta_q, t) \quad (\text{A-9})$$

The steps aforementioned show how the GPDEM can convert the stochastic equation into a series of deterministic analysis. After solving the GPDEM equations, the finite difference method is used to obtain a set of abundant probability information. Finally, the data of the finite element calculation are combined with a virtual GPDEM process (Li and Chen, 2009) to determine the seismic failure probability.

Appendix B. Generation of stochastic ground motions

A spectral representation-random function method is used to generate the acceleration time series of nonstationary stochastic processes (Liu et al., 2016) as follows:

$$\ddot{X}_g(t) = \sum_{k=1}^N \sqrt{2S_{\ddot{X}_g}(t, \omega_k) \Delta\omega} [\cos(\omega_k t) X_k + \sin(\omega_k t) Y_k] \quad (\text{B-1})$$

- Constructing two sets of orthonormal random variables \bar{X}_n and \bar{Y}_n ($n = 1, 2, \dots, N$) provides the following expressions:

$$\bar{X}_n = \text{cas}(n\theta_1), \quad \bar{Y}_n = \text{cas}(n\theta_2) \quad (\text{B-2})$$

where $\text{cas}(x) = \cos(x) + \sin(x)$ is the Hartley orthogonal function and θ_1 and θ_2 are two independent random variables. By performing a deterministic mapping, the standard orthogonal random variables $\{X_k, Y_k\}$ can be obtained.

A generalized Clough and Penzien evolutionary power spectrum model for earthquake ground motions from the study by Cacciola and Deodatis (2011) is introduced by the following power spectral density function:

$$S_{\ddot{X}_g}(t, \omega) = A^2(t) \frac{\omega_g^4(t) + 4\xi_g^2(t)\omega_g^2(t)\omega^2}{[\omega^2 - \omega_g^2(t)]^2 + 4\xi_g^2(t)\omega_g^2(t)\omega^2} \cdot \frac{\omega^4}{[\omega^2 - \omega_f^2(t)]^2 + 4\xi_f^2(t)\omega_f^2(t)\omega^2} \cdot S_0(t) \quad (\text{B-3})$$

where $A(t)$ is the intensity modulation function and is expressed as follows:

$$A(t) = \left[\frac{t}{c} \exp\left(1 - \frac{t}{c}\right) \right]^d \quad (\text{B-4})$$

where $c = 4$ s is the average time of the PGA and $d = 2$ is the parameter controlling the shape of $A(t)$. These parameters can effectively regulate the strength of stochastic earthquake processes.

The model's nonstationarity can be reflected by the following parameters:

$$\omega_g(t) = \omega_0 - a \frac{t}{T}, \quad \xi_g(t) = \xi_0 + b \frac{t}{T} \quad (\text{B-5})$$

$$\omega_f(t) = 0.1\omega_g(t), \quad \xi_f(t) = \xi_g(t) \quad (\text{B-6})$$

where $\omega_0 = 25 \text{ s}^{-1}$ and $\xi_0 = 0.45$ are the initial values of $\omega_g(t)$ and $\xi_g(t)$, respectively, $a = 3.5 \text{ s}^{-1}$ and $b = 0.3$ are the variance ratios of the site parameters, ω_0 , ξ_0 , a and b can be determined according to the seismic environment categories and field classification, and $T = 15$ s is the duration of the nonstationary acceleration time series. These are described in detail in Pang et al. (2018a), and the justification is shown in Xu et al. (2018a).

$S_0(t)$, which is the spectrum parameter, represents the earthquake intensity and can be described as follows:

$$S_0(t) = \frac{\bar{a}_{\max}^2}{\gamma^2 \pi \omega_g(t) [2\xi_g^2(t) + 1/(2\xi_g^2(t))]} \quad (\text{B-7})$$

where $\bar{a}_{\max} = 3.92 \text{ m/s}^2$ is the average value of the PGA and $\gamma = 2.6$ is the equivalent peak factor.

To strongly correlate the code spectrum and average response spectrum (NB 35047–2015), the evolutionary power spectrum is corrected to obtain a series of nonstationary ground motions (Pang et al., 2018a).

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