

Received November 5, 2019, accepted November 15, 2019, date of publication November 19, 2019, date of current version December 2, 2019.

Digital Object Identifier 10.1109/ACCESS.2019.2954377

A Method of Multi-Attribute Decision Making With Double-Reference Points and Its Application in Location of Agricultural Products Logistics Center

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This work was supported by the Natural Science Foundation of Guangdong Province under Grant 2018A030313317.

ABSTRACT A single reference point (SRP) is typically employed in traditional studies on the multi-attribute decision making (MADM) method. However, SRP lacks the advantage of the multiplicity of reference points and is thus unable to adequately describe loss aversion actions. With the goal of determining the loss aversion characteristics of decision makers, this work proposes an MADM method that is based on double reference points (DRPs) to solve the MADM problem using crisp and interval fuzzy numbers. First, this work describes the universality of decision-making problems with DRPs and then evaluates the characteristics of DRPs and their effects on decision making. Second, attitude and utility functions are established on the basis of the requirements of loss aversion. Third, considering the “one-vote veto” characteristic of a “dissatisfaction” attitude, a binary utility value is created by integrating the occurrence probability p , and the utility value u . The fourth, the main characteristics of and the aggregation method for the binary utility value are analyzed and established. Finally, a new decision-making method is applied to resolve the location decision of an agricultural product logistics center. Results indicate that the relationship between attribute and reference point significantly influences decision behavior. Compared with the traditional decision-making method, the proposed decision-making method can effectively identify the feasibility levels of alternatives. The ranking of the proposed decision-making method for feasible alternatives is basically the same as that of the traditional decision-making method. These results can effectively help solve agricultural engineering problems and broaden the coverage of and provide references for MADM research.

INDEX TERMS Double reference points (DRPs), loss aversion, multi-attribute decision-making (MADM), utility, agricultural product logistic center.

I. INTRODUCTION

Multi-attribute decision-making (MADM) problem refers to the ranking and selection of finite alternatives with multiple attributes [1], [2]. Owing to the MADM problems generally occurring in daily production and activities [1]–[5], the academic community has carried out comprehensive analyses.

Today, studies on MADM problems come in two forms: those focused on perfect rational decision making and those focused on bounded rational decision making. The former is

typically based on expected utility theory and is frequently adopted to solve MADM problems. The recent studies on the theory and method of MADM suggest that the research on MADM based on complete rationality has achieved substantial results [6], [7], [8]. The latter type is based on loss aversion theory. According to this theory, loss, relative to a reference point, produces greater psychological utility than the same amount of gain does [9], [10]. Loss aversion is a common phenomenon that is rooted in people’s decision-making actions and occurs in the areas of politics, economics, athletics, and so on [10]–[12]. Tom et al. even published a thesis on the *science* that proves loss aversion as a basic

The associate editor coordinating the review of this manuscript and approving it for publication was Alba Amato¹.

mechanism of human beings or animals [13]. The bounded rational decision-making method based on loss aversion has recently gained research attention, and it includes the common methods of prospect theory, cumulative prospect theory, regret theory, and so on. For example, Wang *et al.* [14] and Qin *et al.* [15] respectively integrated prospect theory with the Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS) method and VIKOR (Vlsekriterijumska Optimizacija I Kompromisno Resenje) method to solve hesitant fuzzy and interval fuzzy MADM problems, respectively. Wang *et al.* [16] and Tian *et al.* [17] utilized cumulative prospect theory to solve the design and selection of product concepts, travel modes, and other kinds of MADM problems. Wang *et al.* [18], Hang *et al.* [19], and Zhang *et al.* [20], [21] utilized regret theory to construct a tri-level MADM method, multiple attribute case decision-making method, and risk-type MADM method, respectively.

Loss and gain are relative to reference points. With a reference point, thinking of loss aversion can effectively explain irrational decision-making actions and decision-making biases, including risk attitude difference, framing effect, and endowment effect [22]. According to current studies, prospect theory, cumulative prospect theory, and regret theory are all based on loss aversion thinking, and all of them calculate loss/gain values through a single reference point. However, single or multiple reference points can exist in objective reality. Numerous scholars have emphasized the multiplicity of reference points in theory and have the idea of using multiple reference points to describe the psychological underpinnings of decision makers' loss aversion [23], [24], [25]. Double reference points (DRPs) represent the fundamental type of multiple reference points. Decision-making problems have been addressed with DRPs. For example, Lu *et al.* investigated risk-type decision-making problems concerning the DRPs of social and financial matters [26]. Zhu *et al.* developed a risk decision-making method with two or three reference points [27]. Pei *et al.* studied dynamic hybrid multi-attribute group decision-making problems with DRPs [28]. Luo *et al.* proposed a decision-making method with DRPs to solve the two-sided matching problem with [29].

The literature review reveals an evolving trend in the study of MADM problems from the perspective of loss avoidance. In particular, investigating MADM problems from the perspective of DRPs carries important scientific meaning, but relevant research remains at its infancy. Double-reference points decision-making problems conform to the psychological underpinnings of decision-makers more carefully, which is the basis of other multiple reference point decision-making problems. Therefore, we seek to study MADM problems with DRPs from a loss aversion viewpoint. Compared with the traditional decision-making method, the proposed decision-making method can conform to the psychological underpinnings of loss aversion by constructing a new attitude function and utility function. The new decision-making method will also effectively promote the resolution

of other multiple reference point decision-making problems. Compared with the traditional decision-making method, the proposed decision-making method can not only effectively identify the feasibility levels of alternatives, but also the ranking results for feasible alternatives is basically the same as that of the traditional decision-making method.

II. THEORETICAL ANALYSIS OF DRPs PROBLEMS

People's actual decision making usually involves two reference points, namely, the bottom line and the target. In a commodity exchange, for example, the buyer and the seller maintain their own bottom line prices and target prices. When the offer from the buyer exceeds the seller's ideal price, the seller is satisfied and becomes willing to engage in the transaction. When the offer from the buyer is less than the seller's bottom line price, the seller is not satisfied and is unwilling to engage in the transaction. When the offer from the buyer falls between the seller's bottom line price and target price, the seller shows hesitation and may or may not be willing to engage in the transaction. Theoretical studies have emphasized the important meaning behind animals' risk-foraging behaviors in foraging decision making [23], [30]. Wang and Johnson found that decision makers usually consider the target and bottom line during their decision making [31]. Liu *et al.* [32] and Zhou *et al.* [33] also highlighted an existing threshold interval in people's decision making. The upper limit value of this threshold interval is similar to the target value, and the lower limit value is similar to the bottom line value. A reasonable assumption based on the objective reality and existing theoretical studies is that DRPs exist in people's decision making.

Here, we take B as the bottom line reference point and G as the target reference point. When attribute value x is superior to target reference point G, the decision maker is satisfied; when attribute value x is inferior to bottom line reference point B, the decision maker is dissatisfied [32]; when attribute value x falls between the bottom line reference point B and the target reference point G, the decision maker normally experiences contradicting emotions [22], [23]. On the basis of the abovementioned analysis and the literature [23], [30], [32], Assumption 1 can be reasonably put forward.

Assumption 1: On the basis of the relationship among attribute value x , bottom line reference point B, and target reference point G, the universe of attribute value x is divided into three regions: satisfaction (x is superior to G), contradiction (x is between B and G), and dissatisfaction (x is inferior to B) (refer to Figure 1).

With regard to the priority level of reference points, Wang and Johnson proved through an experiment that ensuring the bottom line is more important than realizing the target [23], [31]. As for the types of attitudes, "satisfaction" can be considered better than "contradiction," and "contradiction" is better than "dissatisfaction." "satisfaction" is the ideal status for a decision maker and can be deemed as a "gain". "Contradiction" and "dissatisfaction" are not ideal status and can be deemed as a "loss". Owing to the fact that

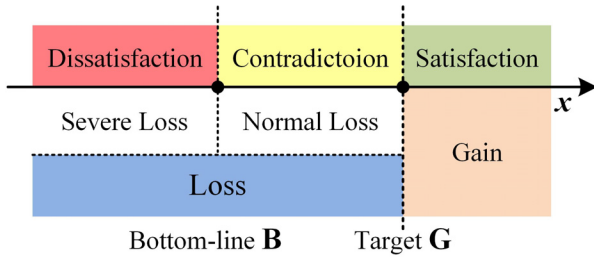


FIGURE 1. DRPs and universe division.

the negative influence brought by a “dissatisfaction” status is far greater than the negative influence brought by a “contradiction” status, loss can thus be further subdivided into “normal loss” and “severe loss.” “Normal loss” corresponds to a “contradiction” status, and “severe loss” corresponds to a “dissatisfaction” status. Figure 1 shows the relationship between different attitudes and loss/gain. With regard to loss aversion, decision makers, in general, constantly seek alternatives that would minimize loss. Therefore, on the basis of Assumption 1, Assumption 2 can be reasonably put forward.

Assumption 2: First, decision makers initially evade “dissatisfaction” in the decision-making process. Next, they evade “contradiction”. Finally, they pursue “satisfaction”.

III. DECISION-MAKING METHOD

A. PROBLEM DESCRIPTION

In MADM problems, $A = \{a_1, a_2, \dots, a_m\}$ denotes the set of alternatives containing m pieces of alternatives, and $C = \{c_1, c_2, \dots, c_n\}$ denotes the set of attributes containing n pieces of attributes. $x_{i,j}$ represents the measured value of alternative a_i on attribute c_j . C_1 and C_2 denote the sets of attributes whose measured values are crisp and interval numbers, respectively; that is, $C_1 \cap C_2 = \emptyset, C_1 \cup C_2 = C$. When attribute $c_j \in C_1$, attribute value $x_{i,j} \in R$, R is a set of real numbers; when attribute $c_j \in C_2$, attribute value $x_{i,j} = [\underline{x}_{i,j}, \bar{x}_{i,j}]$, in which $\underline{x}_{i,j} \leq \bar{x}_{i,j}, \underline{x}_{i,j}, \bar{x}_{i,j} \in R$. C_3 is the set of benefit type attributes, and C_4 is the set of cost type attributes; that is, $C_3 \cap C_4 = \emptyset, C_3 \cup C_4 = C$. The subscript sets of elements in sets $A, C, C_1, C_2, C_3,$ and C_4 are expressed as M, N, N_1, N_2, N_3, N_4 , respectively. The attribute weight vector is expressed as $W = \{w_1, w_2, \dots, w_n\}$. The decision maker adopts DRPs comprising bottom line and target points. The bottom line reference point is $B = (b_1, b_2, \dots, b_n)$, with b_j being the bottom line value of attribute c_j . The target reference point is $G = (g_1, g_2, \dots, g_n)$, with g_j being the target value of attribute c_j .

B. DECISION-MAKING MODEL

1) CONSTRUCTION OF ATTITUDE FUNCTION

In accordance with Assumption 1 and on the basis of the relationship among attribute value x , bottom line reference point B , and target reference point G , the attitudes of a decision maker can be categorized as “satisfaction,” “contradiction,” and “dissatisfaction.” Figure 1 shows that attitude is reflects loss/gain status. The quantitative description of

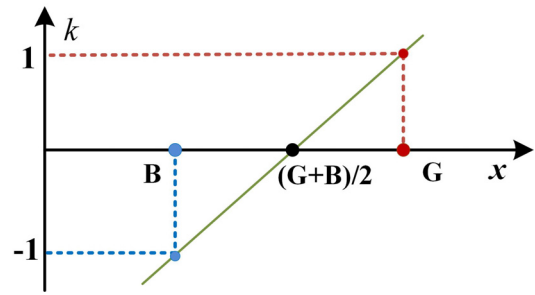


FIGURE 2. Attitude function curve.

attitude is essentially the quantitative description of loss/gain status. Here, a value greater than or equal to 1 refers to “satisfaction,” with a large number indicating a large gain and a high satisfaction level. A value less than or equal to -1 refers to “dissatisfaction,” with a small number indicating severe damage and high dissatisfaction level. A value within $(-1, 1)$ refers to “contradiction,” with numbers close to 1 indicating that the attitude is close to “satisfaction,” numbers close to -1 indicating that the attitude is close to “dissatisfaction,” and 0 means half of “satisfaction” and “dissatisfaction” attitudes. Given this representation, a straight-line function is constructed (Formula 1) to describe the attitude characteristics of a decision maker. Figure 2 shows the straight-line function.

$$k(x) = \frac{2x - (G + B)}{G - B} \tag{1}$$

Formula (1) shows that when the attribute value $x_{i,j}$ is a crisp number ($x_{i,j} \in R$), the attitude value $k_{i,j}$ that corresponds to attribute value $x_{i,j}$ can be expressed as Formula (2) regardless of whether attribute c_j is of benefit type or cost type.

$$k_{i,j} = \frac{2x_{i,j} - (g_j + b_j)}{g_j - b_j}, \quad i \in M, j \in N_1 \tag{2}$$

When the attribute value $x_{i,j}$ is an interval number ($x_{i,j} = [\underline{x}_{i,j}, \bar{x}_{i,j}]$), the attitude value $k_{i,j}$ that corresponds to the attribute value $x_{i,j}$ can also be expressed in the form of interval number. For the benefit-type attribute, the attitude value $k_{i,j}$ can be expressed as Formula (3). For the cost-type attribute, the attitude value $k_{i,j}$ can be expressed as Formula (4).

$$k_{i,j} = \left[\frac{2\underline{x}_{i,j} - (g_j + b_j)}{g_j - b_j}, \frac{2\bar{x}_{i,j} - (g_j + b_j)}{g_j - b_j} \right], \tag{3}$$

$i \in M, j \in N_2 \cup N_3$

$$k_{i,j} = \left[\frac{2\bar{x}_{i,j} - (g_j + b_j)}{g_j - b_j}, \frac{2\underline{x}_{i,j} - (g_j + b_j)}{g_j - b_j} \right], \tag{4}$$

$i \in M, j \in N_2 \cup N_4$

2) CONSTRUCTION OF UTILITY FUNCTION

In loss aversion, the negative utility caused by loss exceeds the positive utility caused by the same quantity of gain. Assumptions 1 and 2 indicate that under the environment of DRPs, loss aversion can be expressed as follows: the negative utility caused by severe loss exceeds the negative

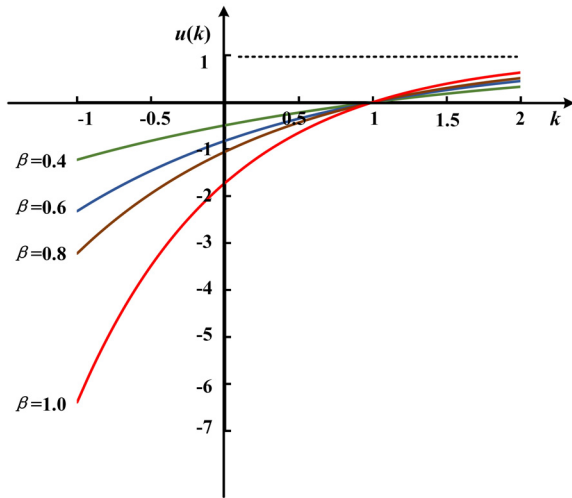


FIGURE 3. Utility function.

utility caused by the same quantity of normal loss, and the negative utility caused by normal loss exceeds the positive utility caused by the same quantity of gain. According to the relationship between attitude and loss/gain shown in Figure 1, loss aversion can also be expressed as follows: the negative utility caused by a “dissatisfaction” status exceeds the negative utility caused by the same quantity of “contradiction” status, and the negative utility caused by a “contradiction” status exceeds the positive utility caused by the same quantity of “satisfaction” status. In this work, we describe the utility of different attitudes for a decision maker by establishing a utility function. To conform to the psychological underpinnings of loss aversion, we should establish the utility function as a monotone increasing convex function. We used an exponential function as the utility function of attitude, as presented in Formula (5).

$$u(k) = 1 - \exp(\beta(1 - k)), \tag{5}$$

where β is the loss aversion coefficient, $\beta > 0$. A large β value indicates that the decision-maker’s degree of loss aversion is great. The value of β can be obtained through multiple tests and investigations. Figure 3 illustrates the utility function $u(k)$ when β takes different values; here, k denotes the attitude value, and $u(k)$ denotes the utility value.

Formula (5) and Figure 3 show that utility function $u(k)$ has the following characteristics:

- (1) $u'(k) > 0$, $u''(k) < 0$, and $u(k)$ conform to the requirement of a monotone increasing convex function.
- (2) When $k = 1$, utility value $u(k) = 0$; when $k > 1$, utility value $u(k) > 0$; when $k < 1$, utility value $u(k) < 0$.
- (3) When k tends to be positive infinity, $u(k)$ approaches 1 (Formula (6)); when k tends to be negative infinity, $u(k)$ approaches negative infinity (Formula (7)).

$$\lim_{k \rightarrow +\infty} u(k) = 1 \tag{6}$$

$$\lim_{k \rightarrow -\infty} u(k) = -\infty. \tag{7}$$

According to the established utility function and the data type of attribute values, the following cases determine the utility values corresponding to different attitude values.

(1) When $j \in N_1$, attitude value $k_{i,j}$ is a crisp number, $k_{i,j} \in \mathbb{R}$. Utility value $u_{i,j}$ corresponding to $k_{i,j}$ can be expressed as Formula (8).

$$u_{i,j} = 1 - \exp(\beta(1 - k_{i,j})), \quad j \in N_1 \tag{8}$$

(2) When $j \in N_2$, attitude value $k_{i,j}$ is an interval number, $k_{i,j} = [k_{i,j}, \bar{k}_{i,j}]$. According to the characteristics of an interval number, $k_{i,j}$ can be deemed as a continuous random variable subject to a uniform distribution within interval $[k_{i,j}, \bar{k}_{i,j}]$, and probability density function can be expressed as Formula (9).

$$f_{ij}(k) = \begin{cases} \frac{1}{\bar{k}_{ij} - k_{ij}}, & k_{ij} \leq k \leq \bar{k}_{ij} \\ 0, & \text{others} \end{cases} \tag{9}$$

In this case, utility value $u_{i,j}$ corresponding to $k_{i,j}$ can be expressed as Formula (10). When Formula (5) is substituted into Formula (10), Formula (11) is obtained.

$$u_{i,j} = \int_{k_{i,j}}^{\bar{k}_{i,j}} u(k)f_{ij}(k), \quad j \in N_2 \tag{10}$$

$$u_{i,j} = \int_{k_{i,j}}^{\bar{k}_{i,j}} (1 - \exp(\beta(1 - k_{i,j})))f_{ij}(k), \quad j \in N_2 \tag{11}$$

3) CONFIRMATION OF BINARY UTILITY VALUE

Utility value u obtained through a utility function quantitatively reflects the effectiveness of attitude value k for the decision maker. In a decision-making process involving DRPs, decision makers normally hold the bottom line as a fundamental principle. That is, these decision makers would not adopt an alternative that breaks the bottom line. Figure 1 shows that holding the “bottom line” equates to evidently avoiding a “dissatisfaction” status. Thus, the “dissatisfaction” attitude has a “one-vote veto” during the decision-making process. Utilizing the utility value alone cannot completely express the psychological behavior of a decision maker. This requires a combination of utility values and information that reflects the occurrence probability of “dissatisfaction.” This concept paves the way for the proposal of the concept of binary utility value based on a utility value.

Definition 1: The binary pairs (p, u) containing the occurrence probability of “dissatisfaction” p ($0 < p < 1$) and utility value u are called the binary utility value.

The “dissatisfaction” status does exhibit “one-vote veto” characteristics, but such is true only when the occurrence probability of “dissatisfaction” p (hereinafter referred to as occurrence probability p) reaches the level that could adequately lead to the decision maker’s awareness. If occurrence probability p is relatively small, then it could easily cause the awareness of the decision maker. Thus, the decision maker’s psychological characteristics will not qualitatively change.

At this point, “dissatisfaction” would not appear as a “one-vote veto” characteristic. Previous research results indicate that the critical point that can cause a qualitative change in psychological characteristics is called the psychological threshold [35]. Here, we suppose that the psychological threshold value of the “dissatisfaction” occurrence probability by a decision maker is p_0 . p_0 is obtained through actual investigation or consultation with relevant decision makers. When $p \geq p_0$, binary utility value (p, u) has a “one-vote veto” characteristic; otherwise, it does not.

By setting any two binary utility values (p_i, u_i) and (p_j, u_j) according to the “one-vote veto” of “dissatisfaction” and the size of the utility value, we can identify a few characteristics of the binary utility value.

- (1) Equality: If $p_i = p_j$ and $u_i = u_j$, then $(p_i, u_i) = (p_j, u_j)$.
- (2) Ordering: If $p_i < p_j$, then $(p_i, u_i) > (p_j, u_j)$; if $p_i = p_j$ and $u_i > u_j$, then $(p_i, u_i) > (p_j, u_j)$.
- (3) Negativity: If $p_i > p_0$, then the alternative (or attribute value) corresponding to (p_i, u_i) will not conform to the bottom line requirement of the decision maker.

For specific attribute value $x_{i,j}$, its corresponding binary utility value can be expressed as

$$v_{i,j} = (p_{i,j}, u_{i,j}), \tag{12}$$

where the value of $p_{i,j}$ and the data type of attribute value $x_{i,j}$ are closely associated. When $j \in N_1$, attitude value $k_{i,j}$ is a crisp number. If $k_{i,j} \leq -1$ and the decision maker has a “dissatisfaction” attitude, then $p_{i,j} = 1$. Conversely, if $k_{i,j} > -1$ and the decision maker has no “dissatisfaction” attitude, then $p_{i,j} = 0$. Obviously, $p_{i,j}$ can be expressed as Formula (13).

$$p_{i,j} = \begin{cases} 1, & k_{i,j} \leq -1 \\ 0, & k_{i,j} > -1, \end{cases} \quad j \in N_1 \tag{13}$$

When $j \in N_2$, attitude value $k_{i,j}$ is an interval number, $k_{i,j} = [\underline{k}_{i,j}, \bar{k}_{i,j}]$. If $\bar{k}_{i,j} \leq -1$, then the decision maker will show a “dissatisfaction” attitude, and $p_{i,j} = 1$. If $\underline{k}_{i,j} \geq -1$, then the decision maker will show no “dissatisfaction” attitude, and $p_{i,j} = 0$. If $\underline{k}_{i,j} < -1 < \bar{k}_{i,j}$, then the decision maker has a certain “dissatisfaction” attitude, and $p_{i,j} = \int_{\underline{k}_{i,j}}^{-1} f_{i,j}(k)$. At this point, $p_{i,j}$ can be expressed as Formula (14).

$$p_{i,j} = \begin{cases} 1, & \bar{k}_{i,j} \leq -1 \\ \int_{\underline{k}_{i,j}}^{-1} f_{i,j}(k), & \underline{k}_{i,j} < -1 < \bar{k}_{i,j} \\ 0, & \underline{k}_{i,j} \geq -1, \end{cases} \quad j \in N_2 \tag{14}$$

4) AGGREGATION OF BINARY UTILITY VALUES

Once the binary utility values of the alternatives on each attribute are determined, we should aggregate them to rank and select the alternatives. The aggregation concerns occurrence probability p and utility value u .

The aggregation of p can be expressed with a weighted arithmetic average method, as in Formula (15).

$$p_i^{(e)} = \sum_{j=1}^n w_j p_{i,j}, \quad i \in M \tag{15}$$

Given the “one-vote veto” characteristic of the “dissatisfaction” attitude, a decision maker may be extremely sensitive to the occurrence probability p of the binary utility value. In such a case, the decision maker focuses on the largest p of each binary utility value’s “dissatisfaction” aspect. Therefore, a pessimistic rule can be used to aggregate occurrence probability p (Formula (16)). Obviously, $p_i^{(e)} \leq p_i^{(p)}$.

$$p_i^{(p)} = \max\{p_{ij} | j \in N\}, \quad i \in M \tag{16}$$

The aggregation of u can be expressed with the weighted arithmetic average method from the linear superposition perspective (Formula (17)).

$$u_i = \sum_{j=1}^n w_j u_{i,j}, \quad i \in M \tag{17}$$

In accordance with the different aggregation methods of occurrence probability p , we can categorize the aggregation methods of binary utility values as follows:

(1) Average expectation method. This method uses the weighted arithmetic average technique in aggregating occurrence probability p and utility value u . The comprehensive binary utility value after aggregation is expressed as $v_i^{(e)} = (p_i^{(e)}, u_i)$, where

$$v_i^{(e)} = (\sum_{j=1}^n w_j p_{i,j}, \sum_{j=1}^n w_j u_{i,j}), \quad i \in M. \tag{18}$$

(2) Pessimistic expectation method. This method adopts a pessimistic rule in aggregating occurrence probability p and uses the weighted arithmetic average method in aggregating u . The comprehensive binary utility value after aggregation is expressed as $v_i^{(p)} = (p_i^{(p)}, u_i)$, where

$$v_i^{(p)} = (\max\{p_{ij} | j \in N\}, \sum_{j=1}^n w_j u_{i,j}), \quad i \in M. \tag{19}$$

These two aggregation methods have their corresponding characteristics. To maximize their advantages and those of the two types of information, namely, $p_i^{(e)}$ and $p_i^{(p)}$, we utilize the combination method in aggregating occurrence probability p . Such method is expressed as

$$p_i = \delta p_i^{(e)} + (1 - \delta) p_i^{(p)}. \tag{20}$$

where δ is the combination coefficient, $0 \leq \delta \leq 1$, and $i \in M$. δ reflects the decision maker’s preference determined by the decision makers.

At this point, we can express the comprehensive binary utility value as $v_i = (p_i, u_i)$, where

$$v_i = (\delta p_i^{(e)} + (1 - \delta) p_i^{(p)}, u_i), \quad i \in M. \tag{21}$$

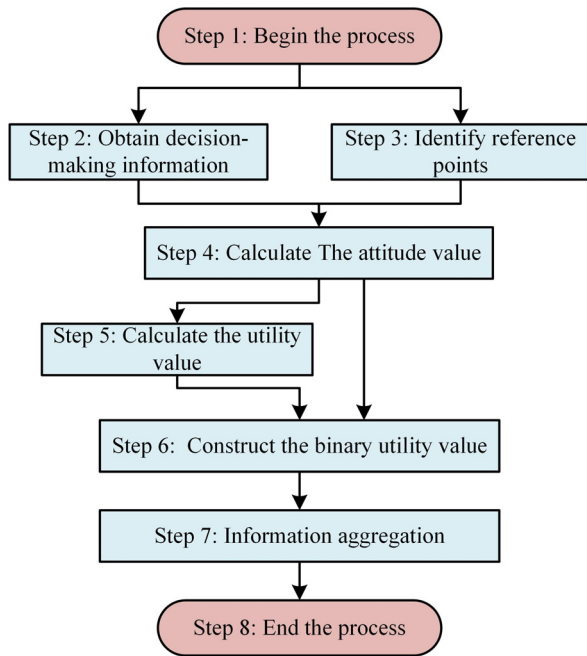


FIGURE 4. Step of decision-making method.

Obviously, when $\delta = 1$, Formula (21) is changed into Formula (18). Conversely, when $\delta = 0$, Formula (21) is changed into Formula (19). Thus, the average expectation method and pessimistic expectation method are two special cases of the combined aggregation method.

For the comprehensive binary utility value $v_i = (p_i, u_i)$, if $p_i \geq p_0$ according to the negativity of the binary utility value, then alternative a_i is categorized as an infeasible alternative. Conversely, if $0 < p_i < p_0$, then a “dissatisfaction” attitude could emerge, but the occurrence probability is relatively small and insufficient to be a concern for the decision maker. At such moment, a_i is classified as a weakly feasible alternative. If $p_i = 0$, then a “dissatisfaction” attitude would not occur, and a_i is classified as a feasible alternative at such moment. The ordering of the binary utility value shows that a feasible alternative is obviously better than a weakly feasible alternative and that a weakly feasible alternative is obviously better than an infeasible alternative. Under normal conditions, the number of feasible alternatives must exceed 1, and the ranking and selection of feasible alternatives should be based on the ordering of the binary utility values. Given existing feasible alternatives, the best alternative can only be selected from feasible alternatives. If no feasible alternative exists, then the best alternative out of the weakly feasible alternatives must be selected as the second choice. Infeasible alternatives should be removed from the list of candidates.

C. STEPS OF DECISION-MAKING METHOD

The main steps of the MADM method with DRPs under a loss aversion viewpoint are as follows.

Step 1: Begin the process.

Step 2: Obtain decision-making information. Obtain the relevant decision-making information through investigation and analysis.

Step 3: Identify reference points. Identify B and G of the decision-making problems using various methods, including onsite investigation, interviews, and document perusal.

Step 4: Calculate the attitude value. Calculate the corresponding attitude value by using Formulas (2) and (3) according to the data type of the attribute value.

Step 5: Calculate the utility value. Calculate the utility value when an attitude value is a crisp number by using Formula (8), and compute the utility value when the attitude value is an interval number by using Formula (11).

Step 6: Construct the binary utility value. Integrate the occurrence probability p of the “dissatisfaction” attitude and utility value u to construct the binary utility value.

Step 7: Information aggregation. According to the aggregation method of occurrence probability p and utility value u , aggregate the binary utility value with Formula (21) on the basis of the confirmed combination coefficient δ . Then, perform the classification, ranking, and selection of alternatives in line with the aggregation result.

Step 8: End the process.

IV. APPLICATION OF DECISION-MAKING METHOD TO THE LOCATION OF AN AGRICULTURAL PRODUCT LOGISTICS CENTER

A. DESCRIPTION OF THE LOCATION PROBLEM FOR AN AGRICULTURAL PRODUCT LOGISTICS CENTER

An agricultural product logistics center connects upstream producers and downstream agricultural product suppliers and is thus the core of the agricultural product logistics system [36]. A reasonable location benefits the optimal allocation of agricultural product logistics resources and helps cut costs. The location of a logistics center normally entails several considerations, including transportation infrastructure conditions, investment costs, socioeconomic levels, regional populations, and environmental influence. According to previous research [37], [38], [39], [40], the features of agricultural product logistics, and the principles behind reflecting an objective reality and maximal simplification, the following seven attributes are configured in this work.

(1) Transportation infrastructure (c_1): The ratio of the overall length of public roads and standard gauge railway lines of a candidate region to the total area of the candidate region (in km/100 km²).

(2) Construction cost (c_2): Total costs of land expropriation, demolition, compensation, and leveling (in million USD).

(3) Economic environment (c_3): Local regional GDP per capita (in 1,000 USD/person).

(4) Population size (c_4): Total population of the local region (in million). Population size affects the demand quantity of agricultural products directly.

TABLE 1. Decision-making matrix.

Region	c_1	c_2	c_3	c_4	c_5	c_6	c_7
a_1	238	[20,22]	23	10.8	[1,2,2.7]	90	95
a_2	248	[21,25]	24	11.2	[1.4,2.5]	92	94
a_3	219	[19,22]	14	9.8	[4.1,5.8]	78	69
a_4	95	[20,23]	16	6.1	[6.6,7.8]	65	75
a_5	169	[18,20]	20	8.2	[4.2,4.7]	82	85

(5) Decay ratio (c_5): The expected average level of the ratio of the decayed quantity of agricultural products to the total transportation quantity (in %).

(6) Environmental protection factor (c_6): The situation of carbon emissions, noise and dust pollution during the operation of the logistics center. This attribute is evaluated by experts using a percentile system (in points).

(7) Government support strength (c_7): Strength of local government support for the construction of the logistics center in the candidate region. This attribute is also scored with a percentile system that is based on government-issued policies or methods (in points).

As a result of its rapid economic development, D province in the southern area of nation C faces a growing demand for agricultural products. To relieve the logistics pressure in relation to agricultural products and to satisfy social requirements, D province is preparing to construct a new agricultural product logistics center. Assume the five candidate regions represented by (a_1, a_2, a_3, a_4, a_5) are initially chosen as newly-built logistics center for agricultural product after collecting site plan. Assume that the decision matrix has been obtained, as shown in Table 1. The decision maker’s psychological threshold value of “dissatisfaction” occurrence probability is $p_0 = 18\%$. The optimal region for the new agricultural product logistics center can be ascertained accordingly.

B. DECISION-MAKING ANALYSIS

To solve the problems mentioned above, the bottom-line value and target value of each attribute, which are the reference points, are defined in Table 2. We obtained the weight of each attribute with the AHP method in accordance with the experts’ opinions. In addition, the types of attributes were confirmed according to the definition of attributes. The data types of the attributes were ascertained through the information provided by the decision-making matrix. This information is presented in Table 2. The attitude value matrix obtained through Formulas (2)–(4) on the basis of the decision-making matrix is shown in Table 3.

The utility curve in Figure 3 indicates that if the loss aversion coefficient β is too small in the utility function, then the loss aversion characteristic of the decision maker is not obvious. Conversely, if the loss aversion coefficient β is too large, then the loss aversion characteristic of the decision maker is excessive. Therefore, an appropriate value of β should be utilized. After multiple trials, we set $\beta = 0.72$. Then, we obtained the utility matrix with Formulas (8) and (11).

Next, occurrence probability p was calculated through Formulas (13) and (14). Subsequently, we integrated p and u to obtain the binary utility matrix (Table 4).

Finally, we aggregated the binary utility value by using Formulas (15)–(17) and (21) (with the combination coefficient $\delta = 0.5$). Table 5 shows the following aggregation results:

(1) The occurrence probability in the comprehensive binary utility value of region a_3 is $p_3 = 58.5\%$, $p_3 > p_0$. Region a_3 is an infeasible alternative and is inappropriate for a newly constructed agricultural product logistics center.

(2) The occurrence probability in the comprehensive binary utility value of region a_2 is $p_2 = 14.1\%$, $0 < p_2 < p_0$. Region a_2 is a weakly feasible alternative. Hence, it is not recommended for a newly constructed agricultural product logistics center.

(3) The occurrence probability in the comprehensive binary utility value of regions $a_1, a_4,$ and a_5 is equal to 0. Thus, all of these three regions are feasible alternatives and appropriate for a newly constructed agricultural product logistics center. In terms of ranking, region a_1 shall be considered the top priority, followed by region a_5 and region a_4 .

C. COMPARISON OF METHODS

We further explain the differences between the proposed decision-making method and the traditional methods by assessing the decision-making problem using three representative methods: the expected utility method, the TOPSIS method, and regret theory. The interval numbers in the decision-making matrix are converted into crisp numbers through the averaging technique to achieve a convenient calculation.

(1) Expected utility method. This method is the simplest and most basic information aggregation method. Specifically, it uses a weighted arithmetic average operator for aggregation. Table 6 presents the aggregation results.

(2) TOPSIS method. With this method, G is the positive ideal solution, and B is the negative ideal solution. For other specific steps, please refer to [38] and [40]. Table 6 shows the relative closeness and ranking result of TOPSIS.

(3) Regret theory. For the fundamental principle of regret theory, please refer to [20] and [41]. Herein, we perform our calculation according to G and B as reference points. Table 6 shows the results.

The comparison of the results in Table 6 reveals that:

(1) The ranking results for each candidate region via the traditional methods are similar.

(2) We note an obvious difference in the analysis results between the proposed and traditional decision-making methods. This difference is mainly reflected in the outcomes wherein a_3 is an infeasible alternative and a_2 is a weakly feasible alternative, as categorized by the proposed method. Thus, the proposed method considers region a_3 as inappropriate for the construction of a new agricultural product logistics center. Moreover, region a_2 is not recommended as a location for such center.

TABLE 2. Reference point, weight and type of each attribute.

Attribute	Weight	Attribute type	Bottom-line value B	Target value G	Data type of attribute value
c_1	0.14	Benefit type	90	250	Crisp number
c_2	0.13	Cost type	24	18	fuzzy interval number
c_3	0.17	Benefit type	15	25	Crisp number
c_4	0.19	Benefit type	4	12	Crisp number
c_5	0.15	Cost type	8	1	fuzzy interval number
c_6	0.10	Benefit type	60	100	Crisp number
c_7	0.12	Benefit type	60	100	Crisp number

TABLE 3. Attitude value matrix.

Region	c_1	c_2	c_3	c_4	c_5	c_6	c_7
a_1	0.850	[-0.333,0.333]	0.600	0.700	[0.514,0.943]	0.500	0.750
a_2	0.975	[-1.333,0.000]	0.800	0.800	[0.571,0.886]	0.600	0.700
a_3	0.613	[-0.333,0.667]	-1.200	0.450	[-0.371,0.114]	-0.100	-0.550
a_4	-0.938	[-0.667,0.333]	-0.800	-0.475	[-0.943,-0.600]	-0.750	-0.250
a_5	-0.013	[0.333,1.000]	0.000	0.050	[-0.057,0.086]	0.100	0.250

TABLE 4. Binary utility value matrix.

Region	c_1	c_2	c_3	c_4	c_5	c_6	c_7
a_1	(0,-0.114)	(0,-1.073)	(0,-0.334)	(0,-0.241)	(0,-0.221)	(0,-0.433)	(0,-0.197)
a_2	(0,-0.018)	(0.25,-2.449)	(0,-0.155)	(0,-0.155)	(0,-0.219)	(0,-0.334)	(0,-0.241)
a_3	(0,-0.322)	(0,-0.861)	(1,-3.874)	(0,-0.486)	(0,-1.265)	(0,-1.208)	(0,-2.053)
a_4	(0,-3.035)	(0,-1.367)	(-2.655)	(0,-1.892)	(0,-2.589)	(0,-2.525)	(0,-1.460)
a_5	(0,-1.073)	(0,-0.284)	(-1.054)	(0,-0.982)	(0,-1.032)	(0,-0.912)	(0,-0.716)

TABLE 5. Aggregation and ranking of binary utility values.

Region	Occurrence probability p		Utility value u	Comprehensive binary utility value v	Ranking	Characteristic
	$p^{(e)}$	$p^{(p)}$				
a_1	0	0	-0.358	(0.0%,-0.358)	1	feasible alternative
a_2	3.3%	25%	-0.472	(14.1%,-0.472)	4	weakly feasible alternative
a_3	17%	100%	-1.465	(58.5%,-1.465)	5	infeasible alternative
a_4	0	0	-2.229	(0.0%,-2.229)	3	feasible alternative
a_5	0	0	-0.885	(0.0%,-0.885)	2	feasible alternative

TABLE 6. Comparison of decision-making methods.

Region	Expected utility method		TOPSIS method		Regret theory				New method	
	Expected value	Ranking	relative closeness	Ranking	B as reference point		G as reference point		Ranking	Characteristic
					Comprehensive perception utility	Ranking	Comprehensive perception utility	Ranking		
a_1	0.853	2	0.675	2	0.971	2	0.827	2	1	feasible alternative
a_2	0.863	1	0.685	1	0.981	1	0.838	1	4	weakly feasible alternative
a_3	0.678	4	0.440	3	0.763	4	0.610	4	5	infeasible alternative
a_4	0.543	5	0.170	5	0.601	5	0.441	5	3	feasible alternative
a_5	0.701	3	0.406	4	0.792	3	0.640	3	2	feasible alternative

(3) The ranking results for the feasible alternatives (a_1 , a_4 , and a_5) from the proposed and traditional methods are similar, with all methods considering $a_1 > a_5 > a_2$.

V. CONCLUSION

Most traditional MADM methods adopt a single or no reference point. The universality of decision-making problems

with DRPs and loss aversion behavior generates a new requirement for the research of decision-making methods. From the viewpoint of loss aversion, this study proposes an MADM method that is based on DRPs. The main research results are follows.

(1) The DRPs of the bottom line and target are used to reflect the psychological behavior of a decision maker. These DRPs are also utilized to divide the universe of attribute values into three intervals of “dissatisfaction,” “contradiction,” and “satisfaction.”

(2) Distance ratio is taken as an attitude function while the exponential function is taken as the utility function.

(3) The binary utility value is constructed by utilizing occurrence probability p and utility value u . We assess the main characteristics of the binary utility value and perform the aggregation of the binary utility value.

Using an example, we compare the proposed decision-making method with the expected utility method, the TOPSIS method, and regret theory. The comparison results show the following advantages of the proposed decision-making method: (1) The new decision-making method can classify alternatives as feasible, weakly feasible, and infeasible alternatives. Thus, it can effectively determine the feasibility of alternatives. (2) The ranking of feasible alternatives by the proposed decision-making method remains essentially similar to those obtained by traditional methods.

This research will provide reference for the research of other multi-attribute decision-making problems with multiple reference points, and further enrich the theory and method of multi-attribute decision-making. We should note that given the limitations of time and space, we only focused on MADM problems with DRPs and decision-making information that entails crisp and interval numbers. For future work, we are interested in: MADM problems with three or more reference points; MADM problems comprising multiple time intervals and reference points; MADM problems with multiple reference points, in which decision-making information contains linguistic variables, intuitionistic fuzzy numbers, and hesitant fuzzy numbers for increasingly complicated conditions.

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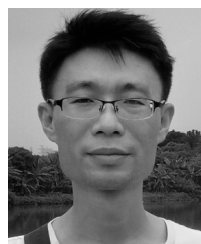
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