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New Optimal-Control-Based Advertising Strategies and Coordination of a Supply Chain With Differentiated Products Under Consignment Contract

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ABSTRACT With the aid of optimal control theory, the cooperative advertising problem is discussed for a supply chain under consignment contract in a competitive environment. Consider the case that the market demand of products is affected by advertising effort and goodwill, both the optimal advertising strategies and advertising participation rate in the centralized and decentralized settings are given. The state-dependent contract with the slotting fees is introduced to coordinate the considered supply chain. Our results demonstrate that the participation rate is closely related to the marginal profits of channel members and the competition intensity between products. Besides, the state-dependent contract with the slotting fees can effectively coordinate the supply chain when the contract parameters satisfy certain conditions. Finally, the sensitivity analysis of the competition intensity between products is conducted by using some simulations.

INDEX TERMS Optimal control, consignment contract, brand goodwill, cooperative advertising, coordination contract.

I. INTRODUCTION

In the past two decades, some retailers played increasing role in the supply chain and the consignment contract was adopted accordingly in many retailing industries [1]–[4]. In particular, the retailers prefer the consignment contract so that the inventory risks can be transferred to the supplier. For example, some online retailers (e.g. Amazon.com, JD.com) usually invite other suppliers to sell products through their sale platforms based on the consignment contract, where the retailers would charge a percentage of the sales price according to the item's category when the suppliers' products are sold at the retailer's platform [5]–[7]. In some cases, the retailer will also sell a differentiated product through the sale platform. The two differentiated products with some common functionality often have some levels of substitutability [8], which creates a competition among the retailer and the supplier. Moreover, it should be pointed out that the enterprises have to survive and develop in the competitive market, therefore it is critical yet necessary to execute the marketing tools in order to promote their product goodwill and boost sales. For example, some efficient marketing decisions were given in [9]–[12], where the sales promotion, the product innovation and the trade credit to customers were introduced. Recently, the famous online retailer JD.com in China is commonly using various forms of advertising, promotions or anniversary sales to promote their business goodwill [13]. Generally, it is revealed that the cooperative advertising is a very important way to increase the advertising investment in [14]–[16]. Hence, we aim to investigate the problems of the cooperative advertising strategies and design of coordination mechanism for supply chain with differentiated products under consignment contract, where the product goodwill and advertising efforts can affect the demand.

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As we know, most of the studies in the cooperative advertising literature mainly focus on the decisions on pricing and advertising from a static aspect [17]–[21]. To mention a few, new cooperative advertising strategies in a multiple-retailer's environment were given and the effects of the retailer's multiplicity on the equilibrium strategies were explored in [20]. Moreover, a comprehensive summary of work in cooperative advertising can be found in [21]. Recently, increasing practitioners and researchers studied the cooperative advertising by applying the optimal control theory in a dynamic environment [22]–[25]. The optimal control theory was extensively used to solve different problems in various fields, such as the semiconductor design, the multi-agent persistent monitoring and the analysis of dynamics networks, see e.g. [26]–[29]. The main aim of those problems is to seek an optimal control strategy such that the objective function is minimized with the design of control signal [29], [30]. Particularly, the optimal control methods were applied to the supply chain management and marketing channels as in [31], [32], and a comprehensive survey regarding the applications in the advertising field can be found in [33]. In [34], the dynamic cooperative advertising scheme was given for a supply chain where both players made short and long term advertising efforts, where it is revealed that both players obtained more profits when the cooperative advertising was adopted by the manufacturer. In [35], an optimal control model of cooperative advertising was established in a stochastic environment, and the comparisons between cooperative advertising and non-cooperative advertising show that the cooperative advertising is superior to the non-cooperative one when the manufacturer's margin is higher than the retailer's. Motivated by [35], subsequently, the effects of the cooperative advertising on the pricing and production strategies were discussed in [36] for the deteriorating items by using optimal control theory.

When the competition becomes a concern, the advertising problems for the differentiated products are discussed [37], [38]. For example, by taking the green quality and warranty period of substitutable products into account, the channel coordination problem in a competitive supply chain have been addressed in [38]. They concluded that all channel members are economically better off by coordinating the manufacturer's green quality and competing retailers' warranty periods. In [39], an advertising competition between multiple brands was analyzed by using a modified Vidale-Wolfe model. In [40], the problems of cooperative advertising and pricing were studied in a dynamic channel, where a national brand was competed with a store brand. As mentioned in [40], the cooperation case was not always advantageous for both channel members compared with the non-cooperative case. Furthermore, the advertising' complementary and competitive roles were introduced in [41], and the pricing as well as the advertising strategies were discussed when a national brand was competed with a store brand. In addition, the issue of contract coordination in the dynamic setting was discussed in [42], [43]. However, it is worthwhile to notice that the above mentioned references have considered the traditional channel, and it is additionally necessary to discuss the framework for supply chain under consignment contract with the competition between retailer and manufacturer. So far, there has been little work undertaken on the coordination contract of the supply chain under consignment contract with competition in a dynamic setting, which constitutes the main motivation of current research.

TABLE 1. A comparison of the present work with related previous works.

references	consignment	advertising	dynamic	competition	coordination
[6]	Y	N	N	N	N
[3]	Y	N	N	Y	N
[2]	Y	Y	N	Y	N
[7]	Y	Y	N	N	Y
[13]	Y	Y	Y	N	N
[44]	Y	Y	Y	N	N
[22]	N	Y	Y	Y	N
[41]	N	Y	Y	Y	N
[4]	Y	N	Y	N	Y
our paper	Y	Y	Y	Y	Y

Motivated by the above discussions, we aim to design the optimal control strategies of the advertising for a channel system under consignment contract with the competition. The major contributions of the paper can be listed: 1) For the decentralized and centralized scenarios, the advertising decisions are given by adopting an optimal control theory framework, and the impacts of the degree of product differentiation on the advertising strategies are analyzed; and 2) a novel state-dependent contract with the slotting fees and the economic premium is designed, which could archive the supply chain coordination under the consignment contract in the dynamic competitive environment. In order to show the differences and advantage of this paper, the comparison regarding this paper and existing papers is listed in Table 1.

II. THE PROBLEM DESCRIPTION AND THE MODEL DEVELOPMENT

In this paper, we consider a supply chain consisting of a retailer and a manufacturer, in which the retailer sells product through its own platform and obtains marginal profit π_1. Besides, the manufacturer also sells a horizontally differentiated product through the retailer's platform obeying the consignment contract, in which the retailer's marginal profit is π_2 and the manufacturer's marginal profit is π_3 for each unit sold.

The advertising effort determines the product goodwill. Let the retailer's product goodwill be $G_r(t)$ and the manufacturer's product goodwill be $G_m(t)$. To capture the effect of advertising effort on product goodwill, we assume that the retailer's product goodwill $G_r(t)$ evolves according to the following differential equation:

$$\dot{G}_r(t) = \lambda A_r(t) - \delta G_r(t),\ G_r(0) = G_{r0}, \tag{1}$$

where $A_r(t)$ is the advertising effort at time t, λ represents the goodwill sensitivity to the advertising effort, δ denotes the decay rate, and G_{r0} depicts the initial product goodwill.

Similarly, the manufacturer's product goodwill $G_m(t)$ evolves according to the following differential equation:

$$\dot{G}_m(t) = \lambda A_m(t) - \delta G_m(t),\ G_m(0) = G_{m0}, \tag{2}$$

where G_{m0} describes the manufacturer's initial product goodwill.

Similar to the way made in [44], [45], the advertising cost functions are assumed to be quadratic in the advertising effort, i.e.,

$$C(A_i(t)) = \frac{1}{2}\mu A_i^2(t), \quad i = r, m.$$

Note that we aim to investigate the advertising strategies in a competitive market environment, and therefore the demand model must adequately reflect the substitutability of the two products. To proceed, let $D_r(t)$ and $D_m(t)$ denote the market demand of the retailer and the manufacturer. According to [46], the product demand is assumed to depend on the product goodwill and the advertising efforts at time t, i.e.,

$$D_r(t) = \alpha_1 G_r(t) - \beta_1 G_m(t) + \gamma_1 A_r(t) + \sigma(A_r(t) - A_m(t)), \tag{3}$$

$$D_m(t) = \alpha_2 G_m(t) - \beta_2 G_r(t) + \gamma_2 A_m(t) + \sigma(A_m(t) - A_r(t)), \tag{4}$$

where α_i and β_i $(i = r, m)$ represent the effects of product goodwill on current demand, γ_i represents the effect of advertising on current demand. In addition, σ represents the degree of product differentiation, the large σ, the weaker degree of product differentiation is, it means that the strength of competition between products is fierce.

In the infinite time horizon with a same discount rate ρ, assume that the retailer's advertising participation rate is ϕ, the retailer's net discounted profit over an infinite planning horizon J_r under the cooperative advertising is expressed as:

$$J_r = \int_0^{+\infty} e^{-\rho t}[\pi_1 D_r(t) + \pi_2 D_m(t) - C(A_r(t)) - \phi C(A_m(t))]dt, \tag{5}$$

where $\pi_1 D_r(t)$ is the retailer's gross profit from selling own product at time t, $\pi_2 D_m(t)$ is the retailer's gross profit from selling manufacturer's product, $C(A_r(t))$ is the advertising cost of the retailer and $\phi C(A_m(t))$ is a part of the manufacturer's advertising cost undertaken by the retailer.

The manufacturer's net discounted profit J_m under the cooperative advertising is expressed as:

$$J_m = \int_0^{+\infty} e^{-\rho t}[\pi_3 D_m(t) - (1-\phi)C(A_m(t))]dt, \tag{6}$$

where $\pi_3 D_m(t)$ is the manufacturer's gross profit from selling own product at time t and $(1-\phi)C(A_m(t))$ is the manufacturer's advertising cost.

Then, the profit of the whole supply chain is the sum of the profits of all players

$$J_c = \int_0^{+\infty} e^{-\rho t}[\pi_1 D_r(t) + (\pi_2+\pi_3) D_m(t) - C(A_r(t)) - C(A_m(t))]dt. \tag{7}$$

Remark 1: In the continuous time interperiod utility function, the discount factor is generally set in the exponential form, which has the advantage on avoiding the time inconsistency. In this paper, we assume that the planning horizon is infinite, that is, $t \in (0, +\infty)$. In fact, although some related studies focus on short-term planning horizons as in [47], [48], the infinite horizon is often considered in the investigations of the marketing in order to discuss the long-run impacts of the marketing dynamics, see e.g [49], [50]. In the present study, an infinite horizon is adopted in order to examine the long-run characteristic of the advertising strategies, where the advertising is utilized to improve the number of the platform users. It should be mentioned that there is no essential difference on the conclusions under the finite and infinite horizons.

III. THE OPTIMAL STRATEGIES IN THE CENTRALIZED SCENARIO

In the centralized scenario, two players as a whole maximize the profit function of whole supply chain by determining advertising efforts A_r and A_m. The optimal control problem with whole supply chain is formulated as

$$\max_{A_r>0, A_i>0} J_c = \int_0^{+\infty} e^{-\rho t}\big[\pi_1 D_r(t) + (\pi_2+\pi_3) \times D_m(t) - C(A_r(t)) - C(A_m(t))\big]dt$$
$$s.t.\ \dot{G}_r(t) = \lambda A_r(t) - \delta G_r(t),\ G_r(0) = G_{r0},$$
$$\dot{G}_m(t) = \lambda A_m(t) - \delta G_m(t), \quad G_m(0) = G_{m0}. \tag{8}$$

For the above optimization problem (8), the optimal solutions are given in the following theorem.

Theorem 1: In the centralized scenario, the optimal advertising strategies are

$$A_r^C = \frac{\pi_1(\gamma_1+\sigma) - \sigma(\pi_2+\pi_3) + \lambda C_1}{\mu}, \tag{9}$$

$$A_m^C = \frac{(\pi_2+\pi_3)(\gamma_2+\sigma) - \pi_1\sigma + \lambda C_2}{\mu}, \tag{10}$$

with

$$C_1 = \frac{\pi_1\alpha_1 - \beta_2(\pi_2+\pi_3)}{\rho+\delta},$$
$$C_2 = \frac{(\pi_2+\pi_3)\alpha_2 - \pi_1\beta_1}{\rho+\delta}.$$

Furthermore, the optimal trajectories of the retailer's product goodwill and the manufacturer's product goodwill are

$$G_r^C(t) = (G_{r0} - G_{rs}^C)e^{-\delta t} + G_{rs}^C, \tag{11}$$
$$G_m^C(t) = (G_{m0} - G_{ms}^C)e^{-\delta t} + G_{ms}^C, \tag{12}$$

with

$$G_{rs}^C = \frac{\lambda[\pi_1(\gamma_1+\sigma) - \sigma(\pi_2+\pi_3) + \lambda C_1]}{\delta\mu},$$
$$G_{ms}^C = \frac{\lambda[(\pi_2+\pi_3)(\gamma_2+\sigma) - \pi_1\sigma + \lambda C_2]}{\delta\mu}.$$

Here, G_{rs}^C and G_{ms}^C correspond to the steady states of the retailer's product goodwill and manufacturer's product goodwill, respectively.

Proof: In the centralized scenario, the current-value Hamiltonian can be given by:

$$\begin{aligned}H_c &= \pi_1(\alpha_1 G_r(t) - \beta_1 G_m(t) + \gamma_1 A_r(t)\\ &\quad +\sigma(A_r(t) - A_m(t))) + (\pi_2+\pi_3)(\alpha_2 G_m(t)\\ &\quad -\beta_2 G_r(t) + \gamma_2 A_m(t) + \sigma(A_m(t) - A_r(t)))\\ &\quad -\frac{1}{2}\mu A_r^2(t) - \frac{1}{2}\mu A_m^2(t) + \mu_c^1(\lambda A_r(t) - \delta G_r(t))\\ &\quad +\mu_c^2(\lambda A_m(t) - \delta G_m(t)),\end{aligned} \tag{13}$$

where μ_c^i $(i = 1, 2)$ are costate variables. By using the optimal control theory, the optimal conditions are described as follows:

$$\frac{\partial H_c}{\partial A_r} = 0, \tag{14}$$

$$\frac{\partial H_c}{\partial A_m} = 0, \tag{15}$$

$$\frac{\partial H_c}{\partial G_r} = \rho\mu_c^1 - \dot{\mu}_c^1, \tag{16}$$

$$\frac{\partial H_c}{\partial G_m} = \rho\mu_c^2 - \dot{\mu}_c^2. \tag{17}$$

It follows from (14)-(15) that

$$A_r = \frac{\pi_1(\gamma_1+\sigma) - \sigma(\pi_2+\pi_3) + \lambda\mu_c^1}{\mu}, \tag{18}$$

$$A_m = \frac{(\pi_2+\pi_3)(\gamma_2+\sigma) - \pi_1\sigma + \lambda\mu_c^2}{\mu}. \tag{19}$$

Solving above differential equations (16)-(17) and according to transversality condition $\lim_{t\to\infty} e^{-\rho t}\lambda_c^i(t) = 0$ $(i = 1, 2)$, one has

$$\lambda_c^1 = \frac{\pi_1\alpha_1 - \beta(\pi_2+\pi_3)}{(\rho+\delta)^2}, \tag{20}$$

$$\lambda_c^2 = \frac{(\pi_2+\pi_3)\alpha_2 - \pi_1\beta_1}{\rho+\delta}. \tag{21}$$

Substituting (20)-(21) into equations (18)-(19), the optimal advertising strategies of retailer and manufacturer under the centralized channel structure can be given. Furthermore, by using the model (1), the optimal trajectories of the retailer's product goodwill and the manufacturer's product goodwill are obtained, which ends the proof of this theorem.

Subsequently, according to Theorem 1, the profit of supply chain is given as follows:

$$\begin{aligned}J_c^* &= C_1 G_{r0} + C_2 G_{m0} + \frac{\delta}{\rho}(C_1 G_{rs}^C + C_2 G_{ms}^C)\\ &\quad + \frac{\pi_1(\gamma_1+\sigma) - \sigma(\pi_2+\pi_3)}{\rho} A_r^C - \frac{1}{2\rho}\mu(A_r^C)^2\\ &\quad + \frac{(\pi_2+\pi_3)(\gamma_2+\sigma) - \pi_1\sigma}{\rho} A_m^C - \frac{1}{2\rho}\mu(A_m^C)^2.\end{aligned} \tag{22}$$

From Theorem 1, the following property of the optimal advertising strategies can be obtained.

Proposition 1: For the advertising strategies under the centralized scenario, we have

(1) $\dfrac{\partial A_r^C}{\partial\sigma} > 0, \ \dfrac{\partial A_m^C}{\partial\sigma} < 0, \ \text{if} \ \ \pi_1 > \pi_2 + \pi_3.$

(2) $\dfrac{\partial A_r^C}{\partial\sigma} + \dfrac{\partial A_m^C}{\partial\sigma} = 0.$

Remark 2: We can see from the Proposition 1 that the competition intensity between products affect the manufacturer's advertising strategy and the retailer's advertising strategy as well. Besides, when the marginal profits π_1, π_2 and π_3 satisfy $\pi_1 > \pi_2 + \pi_3$, as the competition intensity σ increases, the retailer should increase the advertising effort and the manufacturer should decrease the advertising effort. When the marginal profits π_1, π_2 and π_3 satisfy $\pi_1 < \pi_2+\pi_3$, as the competition intensity σ increases, the retailer should decrease the advertising effort and the manufacturer should increase the advertising effort. The above results are consistent with the conclusions in [51] under the consignment sale environment. On the other hand, from Proposition 1, it should be pointed out that the sum of variation ratio from the retailer's and manufacturer's advertising efforts is equal to 0. It means that the retailer should increase/decrease the advertising effort and the manufacturer should decrease/increase the advertising effort when the competition intensity σ increases, but the total sum of advertising efforts of both channel players doesn't change in order to guarantee optimized channel profit. Compared with [51], this finding is new.

IV. THE OPTIMAL STRATEGIES IN THE DECENTRALIZED SCENARIO

In the decentralized scenario, the retailer as the Stackelberg game leader can declare the advertising effort $A_r(t)$ and the advertising participation rate ϕ firstly, then the manufacturer as the followers decides its own advertising effort $A_m(t)$ based on the retailer's decision. Accordingly, the following optimal advertising strategies within the decentralized scenario can be given.

Theorem 2: In the decentralized scenario, the optimal advertising strategies are

$$A_r^D = \frac{\pi_1(\gamma_1+\sigma) - \pi_2\sigma + \lambda D_1}{\mu}, \tag{23}$$

$$A_m^D = \frac{2[\pi_2(\gamma_2+\sigma) - \pi_1\sigma + \lambda D_3] + L_1}{2\mu}, \tag{24}$$

with

$$\begin{aligned}D_1 &= \frac{\pi_1\alpha_1 - \pi_2\beta_2}{\rho+\delta},\\ D_2 &= \frac{\pi_3\alpha_2}{\rho+\delta},\\ D_3 &= \frac{\pi_2\alpha_2 - \pi_1\beta_1}{\rho+\delta},\\ L_1 &= \pi_3(\gamma_2+\sigma) + \lambda D_2.\end{aligned}$$

The advertising participation rate of retailer is given by

$$\phi^* = \begin{cases} \frac{\Delta - L_1}{\Delta + L_1}, & \Delta > L_1 \\ 0, & \Delta \le L_1 \end{cases} \tag{25}$$

where $\Delta = 2[\pi_2(\gamma_2 + \sigma) - \pi_1\sigma + \lambda D_3]$. Furthermore, the optimal trajectories of the retailer's platform goodwill and the manufacturer's brand goodwill are

$$G_r^D(t) = (G_{r0} - G_{rs}^D)e^{-\delta t} + G_{rs}^D, \tag{26}$$
$$G_m^D(t) = (G_{m0} - G_{ms}^D)e^{-\delta t} + G_{ms}^D, \tag{27}$$

with

$$G_{rs}^D = \frac{\lambda[\pi_1(\gamma_1 + \sigma) - \pi_2\sigma + \lambda D_1]}{\delta\mu},$$
$$G_{ms}^D = \frac{2\lambda[(\pi_2(\gamma_2 + \sigma) - \pi_1\sigma + \lambda D_3) + L_1]}{2\delta\mu}.$$

Here, G_{rs}^D and G_{ms}^D refer to the steady states of the retailer's product goodwill and the manufacturer's product goodwill, respectively.

Proof: To derive the equilibrium in the decentralized scenario, we first solve the optimization problem of the manufacturer:

$$\begin{aligned} \max_{A_m > 0} J_m = &\int_0^{+\infty} e^{-\rho t}(\pi_3 D_m(t) - (1-\phi) \\ &\times C(A_m(t)))dt \\ s.t.\ &\dot{G}_r(t) = \lambda A_r(t) - \delta G_r(t), \quad G_r(0) = G_{r0}, \\ &\dot{G}_m(t) = \lambda A_m(t) - \delta G_m(t), \quad G_m(0) = G_{m0}. \end{aligned}$$

The current-value Hamiltonian for the manufacturer is presented as follows:

$$\begin{aligned} H_m = &\pi_3(\alpha_2 G_m(t) - \beta_2 G_r(t) + \gamma_2 A_m(t) \\ &+\sigma(A_m(t) - A_r(t))) - \frac{1}{2}(1-\phi)\mu A_m^2(t) \\ &+\lambda_m^1(\lambda A_r(t) - \delta G_r(t)) \\ &+\lambda_m^2(\lambda A_m(t) - \delta G_m(t)), \end{aligned} \tag{28}$$

where λ_m^i $(i = 1, 2)$ are the costate variables.

By using the results from the optimal control theory, the optimal conditions can be got as follows:

$$\frac{\partial H_m}{\partial A_m} = 0, \tag{29}$$
$$\frac{\partial H_m}{\partial G_r} = \rho\mu_m^1 - \dot{\mu}_m^1, \tag{30}$$
$$\frac{\partial H_m}{\partial G_m} = \rho\mu_m^2 - \dot{\mu}_m^2. \tag{31}$$

Equation (29) implies

$$A_m = \frac{\pi_3(\gamma_2 + \sigma) + \lambda\mu_m^2}{\mu(1-\phi)}. \tag{32}$$

Solving above differential equation (30) and according to transversality condition $\lim_{t\to\infty} e^{-\rho t}\mu_m^2 = 0$, we arrive at

$$\mu_m^2 = \frac{\pi_3\alpha_2}{\rho + \delta}. \tag{33}$$

Therefore,

$$A_m = \frac{\pi_3(\gamma_2 + \sigma) + \lambda\mu_m^2}{\mu(1-\phi)}. \tag{34}$$

On the other hand, considering the manufacturer's response, the retailer's optimization problem can be described by

$$\begin{aligned} \max_{A_r > 0} J_r = &\int_0^{+\infty} e^{-\rho t}[\pi_1 D_r(t) + \pi_2 D_m(t) \\ &- \phi C(A_m(t)) - C(A_r(t))]dt \\ s.t.\ &\dot{G}_r(t) = \lambda A_r(t) - \delta G_r(t),\ G_r(0) = G_{r0}, \\ &\dot{G}_m(t) = \lambda A_m(t) - \delta G_m(t),\ G_m(0) = G_{m0}. \end{aligned}$$

Accordingly, the current-value Hamiltonian for the retailer is given by

$$\begin{aligned} H_r = &\pi_1(\alpha_1 G_r(t) - \beta_1 G_m(t) + \gamma_1 A_r(t) \\ &+\sigma(A_r(t) - A_m(t))) + \pi_2(\alpha_2 G_m(t) \\ &-\beta_2 G_r(t) + \gamma_2 A_m(t) + \sigma(A_m(t) - A_r(t))) \\ &-\frac{1}{2}\mu A_r^2(t) - \frac{1}{2}\phi\mu A_m^2(t) + \mu_r^1(\lambda A_r(t) - \delta G_r(t)) \\ &+\mu_r^2(\lambda A_m(t) - \delta G_m(t)), \end{aligned} \tag{35}$$

where λ_r^i $(i = 1, 2)$ are the costate variables. Substituting (34) into (35), applying the necessary conditions of maximum principle, we obtain

$$\frac{\partial H_r}{\partial A_r} = 0, \tag{36}$$
$$\frac{\partial H_r}{\partial \phi} = 0, \tag{37}$$
$$\frac{\partial H_r}{\partial G_r} = \rho\mu_r^1 - \dot{\mu}_r^1, \tag{38}$$
$$\frac{\partial H_r}{\partial G_m} = \rho\mu_r^2 - \dot{\mu}_r^2. \tag{39}$$

Equations (36)-(37) imply

$$A_r = \frac{\pi_1(\gamma_1 + \sigma) - \pi_2\sigma + \lambda\mu_r^1}{\mu}, \tag{40}$$
$$\phi = \frac{2[\pi_2(\gamma_2 + \sigma) - \pi_1\sigma + \lambda\mu_r^2] - L_1}{2[\pi_2(\gamma_2 + \sigma) - \pi_1\sigma + \lambda\mu_r^2] + L_1}. \tag{41}$$

By solving the above differential equations (38)-(39) and according to transversality condition $\lim_{t\to\infty} e^{-\rho t}\mu_r^i = 0$ $(i = 1, 2$ one has

$$\mu_r^1 = \frac{\pi_1\alpha_1 - \pi_2\beta_2}{\rho + \delta}, \tag{42}$$
$$\mu_r^2 = \frac{\pi_2\alpha_2 - \pi_1\beta_1}{\rho + \delta}. \tag{43}$$

Together with (34) and (40)-(43), the optimal advertising effort of retailer under the decentralized decision can be given. Finally, by using the model (1), the optimal trajectories of the retailer's platform goodwill and the manufacturer's brand goodwill are also obtained. Then, the proof of this theorem is complete.

Based on the Theorem 2, the profits of retailer and manufacturer under the cooperative advertising are expressed as follows:

$$\begin{aligned} J_m^{D*} &= D_2G_{m0} + \frac{\delta}{\rho}D_2G_{ms}^D - \frac{\beta_2\pi_3}{\rho+\delta}G_{r0} \\ &\quad - \frac{\beta_2\pi_3\delta}{\rho(\rho+\delta)}G_{rs}^D + \frac{\pi_3(\gamma_2+\sigma)}{\rho}A_m^D \\ &\quad - \frac{\pi_3\sigma}{\rho}A_r^D - \frac{1-\phi^*}{2\rho}\mu(A_m^D)^2, \end{aligned} \tag{44}$$

$$\begin{aligned} J_r^{D*} &= D_1G_{r0} + D_3G_{m0} + \frac{\delta}{\rho}(D_1G_{rs}^D + D_3G_{ms}^D) \\ &\quad + \frac{\pi_1(\gamma_1+\sigma)-\sigma\pi_2}{\rho}A_r^D - \frac{1}{2\rho}\mu(A_r^D)^2 \\ &\quad + \frac{\pi_2(\gamma_2+\sigma)-\pi_1\sigma}{\rho}A_m^D - \frac{\phi^*}{2\rho}\mu(A_m^D)^2. \end{aligned} \tag{45}$$

Similarly, the optimal advertising strategies are obtained under the non-cooperative advertising: $A_r^{ND}=\frac{\pi_1(\gamma_1+\sigma)-\pi_2\sigma+\lambda D_1}{\mu}$, $A_m^{ND} = \frac{\pi_1(\gamma_1+\sigma)-\pi_2\sigma+\lambda D_3}{\mu}$. Furthermore, we have $A_r^{ND} = A_r^D$, $A_m^D > A_m^{ND}$ when $\phi^* > 0$, which mean the cooperative advertising program lead to higher manufacturer's investment in advertising. This is consistent with the literature about cooperative advertising programs in [16], [36].

The profits of retailer and manufacturer under the non-cooperative advertising are expressed as follows:

$$\begin{aligned} J_m^{ND*} &= D_2G_{m0} + \frac{\delta}{\rho}D_2G_{ms}^{ND} - \frac{\beta_2\pi_3}{\rho+\delta}G_{r0} \\ &\quad - \frac{\beta_2\pi_3\delta}{\rho(\rho+\delta)}G_{rs}^{ND} + \frac{\pi_3(\gamma_2+\sigma)}{\rho}A_m^{ND} \\ &\quad - \frac{\pi_3\sigma}{\rho}A_r^{ND} - \frac{1}{2\rho}\mu(A_m^{ND})^2, \end{aligned} \tag{46}$$

$$\begin{aligned} J_r^{ND*} &= D_1G_{r0} + D_3G_{m0} + \frac{\delta}{\rho}(D_1G_{rs}^{ND} + D_3G_{ms}^{ND}) \\ &\quad + \frac{\pi_1(\gamma_1+\sigma)-\sigma\pi_2}{\rho}A_r^{ND} - \frac{1}{2\rho}\mu(A_r^{ND})^2 \\ &\quad + \frac{\pi_2(\gamma_2+\sigma)-\pi_1\sigma}{\rho}A_m^{ND}, \end{aligned} \tag{47}$$

where

$$G_{rs}^{ND} = \frac{\lambda[\pi_1(\gamma_1+\sigma)-\pi_2\sigma+\lambda D_1]}{\delta\mu},$$
$$G_{ms}^{ND} = \frac{\lambda[\pi_1(\gamma_1+\sigma)-\pi_2\sigma+\lambda D_3]}{\delta\mu}.$$

Because of the complexity of the profit functions, the effects of cooperative advertising programs on the profits will be discussed in the numerical simulations section.

Based on the Theorem 2, it is not difficult to obtain the following results.

Proposition 2: For the advertising participation rate, we have

$$\frac{\partial\phi^*}{\partial\pi_1} < 0, \quad \frac{\partial\phi^*}{\partial\pi_2} > 0.$$

Remark 3: Proposition 2 indicates that the advertising participation rate of retailer decreases with the retailer's product marginal profit and increases with the retailer's product marginal profit from the manufacturer's product.

Proposition 3: For the advertising strategies under the decentralized scenario, we have

$$(1)\ \textit{if}\ \pi_2 < \pi_1,\ \frac{\partial A_r^D}{\partial\sigma} > 0,\ \frac{\partial A_m^D}{\partial\sigma} > 0;$$
$$(2)\ \textit{if}\ \pi_1 < \pi_2 < \pi_1 + \frac{\pi_3}{2},\ \frac{\partial A_r^D}{\partial\sigma} < 0,\ \frac{\partial A_m^D}{\partial\sigma} > 0;$$
$$(3)\ \textit{if}\ \pi_1 + \frac{\pi_3}{2} < \pi_2,\ \frac{\partial A_r^D}{\partial\sigma} < 0,\ \frac{\partial A_m^D}{\partial\sigma} < 0.$$

Proposition 3 indicates that the specific relationship between the advertising strategies of players and the competition intensity on a great degree of the relative sizes of players' marginal profits (i.e. π_1, π_2, π_3). In particular, it can be observed that the following three facts.

(i) When the marginal profits π_1 and π_2 satisfy $\pi_2 < \pi_1$, the higher competition intensity σ between the products is beneficial to increase the advertising investment. Thus, both the retailer and the manufacturer should increase the advertising investment in order to obtain more profits.

(ii) When the marginal profits π_1, π_2 and π_3 satisfy $\pi_1 < \pi_2 < \pi_1+\frac{\pi_3}{2}$, accompanying with the increasing competition intensity between the products, the retailer should reduce the advertising investment and the manufacturer should increase the advertising investment.

(iii) When the marginal profits π_1, π_2 and π_3 satisfy $\pi_1 + \frac{\pi_3}{2} < \pi_2$, accompanying with the increasing competition intensity between the products, both the retailer and the manufacturer should reduce the advertising investment.

V. THE COORDINATION CONTRACT

As discussed in [50], both the control-dependent incentive scheme and the state-dependent incentive scheme are used to improve the performance of supply chain under the decentralized decision structure, where it is shown that the cooperative advertising contract is an essential control-dependent incentive scheme. Hence, in this section, we aim to design a novel state-dependent contract with the slotting fees (denoted by the superscript SS). Subsequently, we study the optimal advertising strategies under the SS contract and utilize this contract to coordinate the supply chain system. Under the SS contract, the retailer not only obtains the revenue generated from each unit sold, but also charges the platform use fees from the manufacturer simultaneously, i.e., the slotting fees. A similar phenomenon can be observed in practice, for example, Amazon.com deducts a commission of 0.99 plus 15% of the sales price when the items are sold [6], and JD.com also charges the commission and the platform use fees according to the item's category [13].

In this paper, the slotting fees $f(G_r, G_m)$ linearly depend on the difference between the retailer's product goodwill G_r and the manufacturer's product goodwill G_m, and also linearly depend on the manufacturer's product goodwill G_m, i.e.,

$$f(G_r, G_m) = M(G_r(t) - G_m(t)) + NG_m(t) + F,$$

where M, N and F are constants. In this scenario, the retailer firstly announces the slotting fees $f(G_r, G_m)$. Secondly, the retailer decides the advertising effort $A_r(t)$ subsequently by maximizing its own profit, and then the manufacturer decides the advertising effort $A_m(t)$ by maximizing its own profit. Next, the optimal advertising strategies under the SS contract can be obtained. Moreover, the supply chain can be coordinated by the SS contract if the contract parameters satisfy certain conditions.

Theorem 3: Under the SS contract, the optimal advertising strategies are given by

$$A_r^{SS} = \frac{\pi_1(\gamma_1+\sigma) - \pi_2\sigma + \lambda E_1}{\mu}, \quad (48)$$
$$A_m^{SS} = \frac{\pi_3(\gamma_2+\sigma) + \lambda E_2}{2\mu}, \quad (49)$$

where

$$E_1 = \frac{\pi_1\alpha_1 - \pi_2\beta_2 - M}{\rho+\delta},$$
$$E_2 = \frac{\pi_3\alpha_2 + M + N}{\rho+\delta}.$$

Proof: Under the SS contract, the objective functional of the retailer and the manufacturer are expressed as:

$$J_r^{SS} = \int_0^{+\infty} e^{-\rho t}[\pi_1 D_r(t) + \pi_2 D_m(t) - C(A_r(t)) + F + M(G_r(t) - G_m(t)) - NG_m(t)]dt, \quad (50)$$
$$J_m^{SS} = \int_0^{+\infty} e^{-\rho t}[\pi_3 D_m(t) - C(A_m(t)) - F - M(G_r(t) - G_m(t)) + NG_m(t)]dt. \quad (51)$$

Similar to the proof of Theorem 2, the optimal strategies under the SS contract can be obtained readily. Hence, the detailed proof is omitted for brevity.

Theorem 4: The supply chain can be coordinated by the SS contract, if the contract parameters M and N satisfy following conditions

$$M = \frac{[\sigma(\rho+\delta) + \lambda\beta_2]\pi_3}{\lambda}, \quad (52)$$
$$N = \frac{(\rho+\delta)[\pi_2(\gamma_2+\sigma) - \pi_1\sigma] + \lambda(\pi_2\alpha_2 - \pi_1\beta_1)}{\lambda} - M. \quad (53)$$

Proof: When the supply chain is coordinated by the SS contract, the optimal advertising efforts of the retailer and the manufacturer should satisfy:

$$A_r^C = A_r^{SS},\ A_m^C = A_m^{SS}.$$

From (9), (48) and using $A_r^C = A_r^{SS}$, we have

$$M = \frac{[\sigma(\rho+\delta) + \lambda\beta_2]\pi_3}{\lambda}.$$

Subsequently, it follows from (10), (49) and $A_m^C = A_m^{SS}$, one has

$$N = \frac{(\rho+\delta)[\pi_2(\gamma_2+\sigma) - \pi_1\sigma] + \lambda(\pi_2\alpha_2 - \pi_1\beta_1)}{\lambda} - M.$$

Then, the proof is complete.

Accordingly, the profits of the manufacturer and the retailer are given by

$$\begin{aligned} J_m^{SS*} &= \frac{\pi_3\alpha_2 + M + N}{\rho+\delta}G_{m0} + \frac{\delta(\pi_3\alpha_2 + M + N)}{\rho(\rho+\delta)}G_{ms}^C \\ &\quad - \frac{M + \beta_2\pi_3}{\rho+\delta}G_{r0} - \frac{\delta(M + \beta_2\pi_3)}{\rho(\rho+\delta)}G_{rs}^C \\ &\quad + \frac{\pi_3(\gamma_2+\sigma)}{\rho}A_m^C - \frac{\sigma}{\rho}A_r^C \\ &\quad - \frac{1}{2\rho}\mu(A_m^C)^2 - \frac{F}{\rho}, \\ &= J_1 - \frac{F}{\rho}, \end{aligned} \quad (54)$$

$$\begin{aligned} J_r^{SS*} &= \frac{\pi_1\alpha_1 - \pi_2\alpha_2 + M}{\rho+\delta}G_{r0} \\ &\quad + \frac{\delta(\pi_1\alpha_1 - \pi_2\alpha_2 + M)}{\rho(\rho+\delta)}G_{rs}^C \\ &\quad + \frac{\pi_2\alpha_2 - \pi_1\alpha_1 - M - N}{\rho+\delta}G_{m0} \\ &\quad + \frac{\delta(\pi_2\alpha_2 - \pi_1\alpha_1 - M - N)}{\rho(\rho+\delta)}G_{ms}^C \\ &\quad + \frac{\pi_2(\gamma_2+\sigma) - \pi_1\sigma}{\rho}A_m^C \\ &\quad + \frac{\pi_1(\gamma_1+\sigma) - \pi_2\sigma}{\rho}A_r^C \\ &\quad - \frac{1}{2\rho}\mu(A_r^C)^2 + \frac{F}{\rho} \\ &= J_2 + \frac{F}{\rho}. \end{aligned} \quad (55)$$

Remark 4: From Theorem 4, the contract parameters M and N can be determined according to the equations (52)-(53) involved the system parameters. As long as the system parameters are fixed, then the contract parameters M and N are obtained easily. Thus, the supply chain can be coordinated under the selected M and N. On the other hand, it follows from Theorems 3-4 that the optimal advertising strategies under the SS contract are independent of parameter F and the profits of the retailer and the manufacturer depend on the parameter F. Therefore, the retailer and the manufacturer may allocate the profits through the parameter F such that all channel members will participate the implementation of this contract. Specifically, according to conditions $J_r^{SS*} > J_r^{D*}$ and $J_m^{SS*} > J_m^{D*}$, we can get that both the retailer and manufacturer are economically better off if $F_1 < F < F_2$, where $F_1 = \rho(J_1 - J_m^{D*})$ and $F_2 = \rho(J_r^{D*} - J_2)$. During the implementation, the value of parameter F can be determined by the Nash bargaining model in [52] based on the information about the risk preferences of two players.

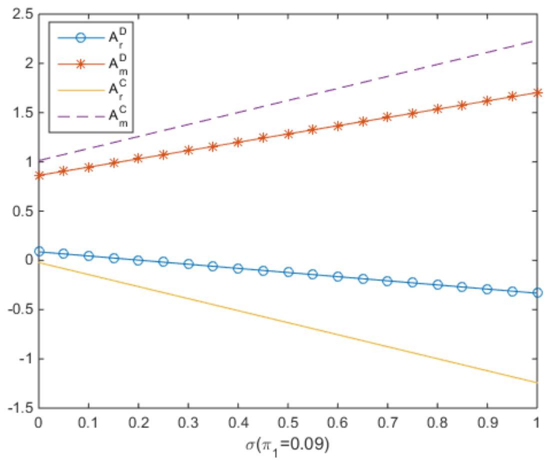

FIGURE 1. The advertising efforts as σ changes when $\pi_1 = 0.09$.

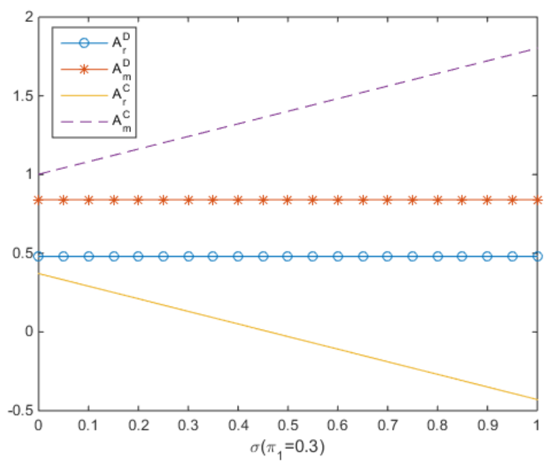

FIGURE 2. The advertising efforts as σ changes when $\pi_1 = 0.3$.

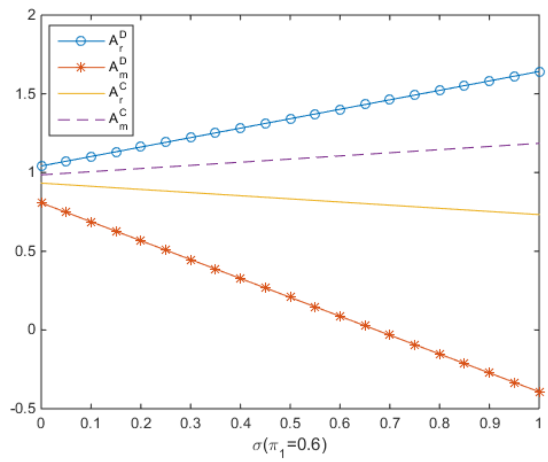

FIGURE 3. The advertising efforts as σ changes when $\pi_1 = 0.6$.

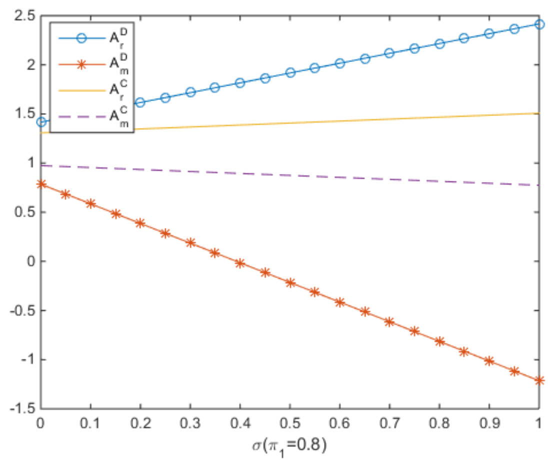

FIGURE 4. The advertising efforts as σ changes when $\pi_1 = 0.8$.

VI. NUMERICAL ANALYSIS

As made in [51], [52], the following simulation example is used to show the applicability of proposed main results. In particular, some simulations are given to discuss the effects from the competition intensity between products on the optimal advertising strategies and the profits under both decentralized and centralized scenarios.

As a benchmark, the following parameters values are set: $\rho = 0.05$, $\alpha_1 = 0.25$, $\mu = 0.5$, $\beta_1 = 0.05$, $\lambda = 0.3$, $\delta = 0.5$, $\gamma_1 = 0.8$, $\alpha_2 = 0.05$, $\beta_2 = 0.25$, $\gamma_2 = 0.7$, $\pi_2 = 0.4$, $\pi_3 = 0.3$, $G_{r0} = 5$ and $G_{m0} = 5$. According to Propositions 1 and 3, we provide different values for π_1 and study the results of advertising efforts comparisons. Figs. 1-4 plot the impacts of the competition intensity between products on the advertising efforts of the manufacturer and the retailer under different π_1 by changing the competition intensity σ. Fig. 5 plots the impacts of the competition intensity on the retailer's advertising participation rate. Figs. 6-13 plot the impacts of the competition intensity on the profits of the channel members and the total profits of supply chain.

We observe from Figs. 1-2 that, for a small retailer's marginal profit from own product π_1 regardless of the centralization and decentralization, the retailer's advertising effort decreases with the increasing of σ and the manufacturer's advertising effort increases with the increasing of σ. Fig. 3 shows that, for a large retailer's marginal profit from own product π_1, the retailer's advertising effort increases with the increasing of σ and the manufacturer's advertising effort decreases with the increasing of σ in decentralized channel, but it will be a contrary result in centralized channel. Fig. 4 reflects that, for a very large retailer's marginal profit from own product π_1 regardless of the centralization and decentralization, the retailer's the advertising effort increases with the increasing of σ and the manufacturer's advertising effort decreases with the increasing of σ. In addition, Figs. 1-4 show that, compared with the decentralized equilibria, the manufacturer's advertising effort in the centralized structure are relatively higher, but the retailer's advertising effort in the centralized structure are relatively lower.

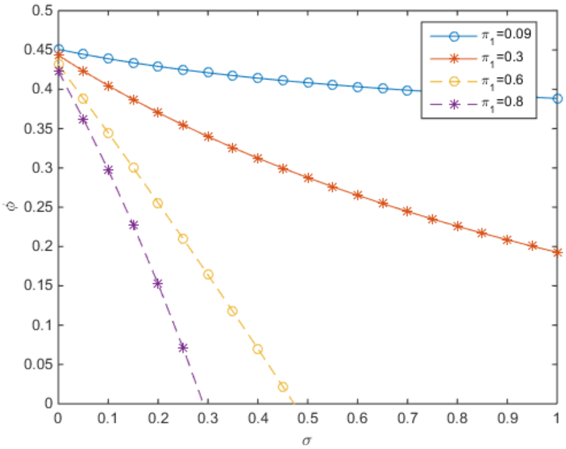

FIGURE 5. The impact σ on the advertising participation rate of retailer.

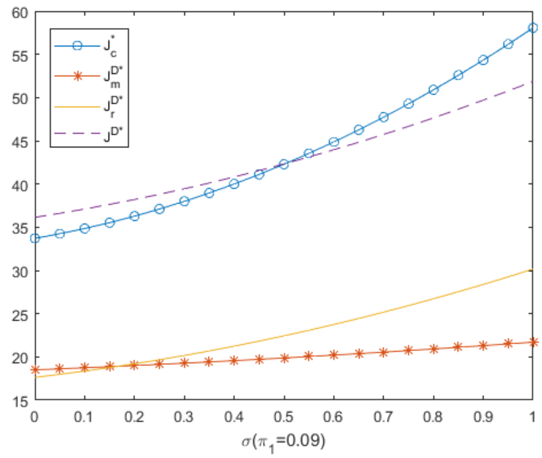

FIGURE 6. The impact σ on the profits when $\pi_1 = 0.09$.

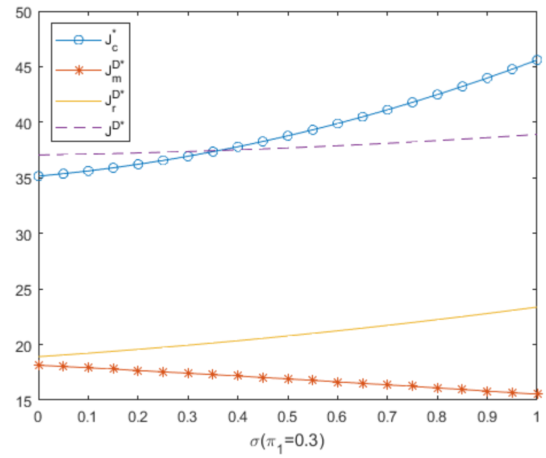

FIGURE 7. The impact σ on the profits when $\pi_1 = 0.3$.

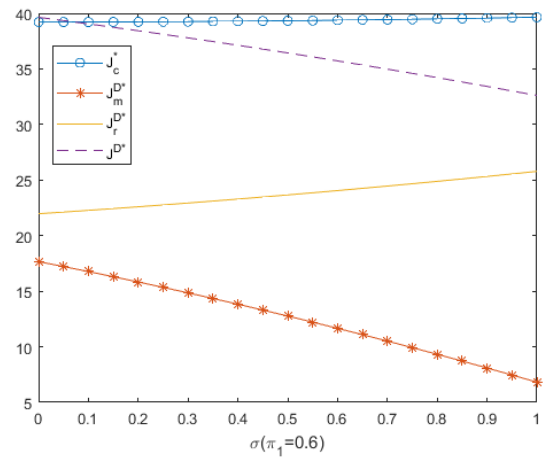

FIGURE 8. The impact σ on the profits when $\pi_1 = 0.6$.

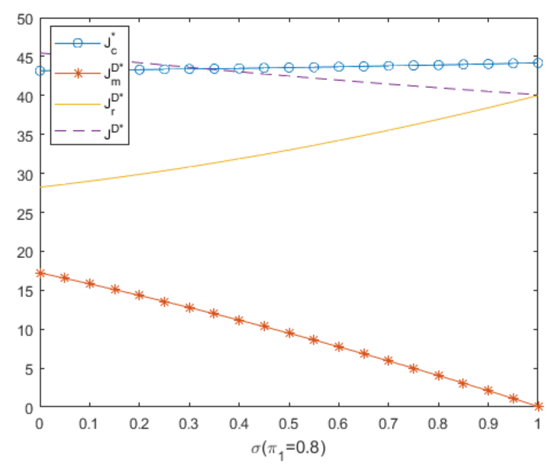

FIGURE 9. The impact σ on the profits when $\pi_1 = 0.8$.

Fig. 5 shows that the retailer's advertising participation rate decreases with the competition intensity σ. Moreover, the retailer's advertising participation rate decreases with the increasing retailer's marginal profit from own product π_1. It means that the retailer will share a less proportion of advertising cost to stimulate the manufacturer on investing more in advertising effort when the retailer's product marginal profit π_1 increases.

Fig. 6 shows that, regardless of the centralization or decentralization, an increasing competition intensity between products leads to higher total profits of supply chain when $\pi_1 = 0.09$. Besides, it can be testified that the condition $\pi_2 > \pi_1 + \frac{\pi_3}{2}$ is satisfied when $\pi_1 = 0.09$. According to the results in the Proposition 3, the retailer and the manufacturer decrease the advertising investments accompanying with increasing competition intensity between products, which can achieve more profits. Fig. 7 shows that, the profit of the retailer under the decentralized setting increases accompanying with increasing competition intensity between products when $\pi_1 = 0.3$, but the profit of the manufacturer decreases accompanying with increasing competition intensity between products. The major reasons lie in the following two aspects. On the one hand, the condition $\pi_1 < \pi_2 < \pi_1 + \frac{\pi_3}{2}$ is satisfied when $\pi_1 = 0.3$. According to the results in the Proposition 3, although

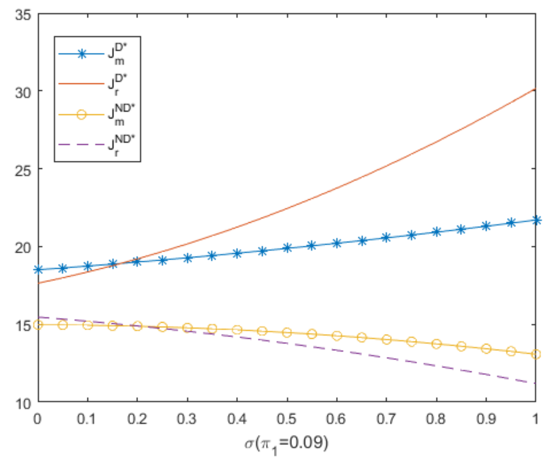

FIGURE 10. The impact σ on the profits under different advertising program when $\pi_1 = 0.09$.

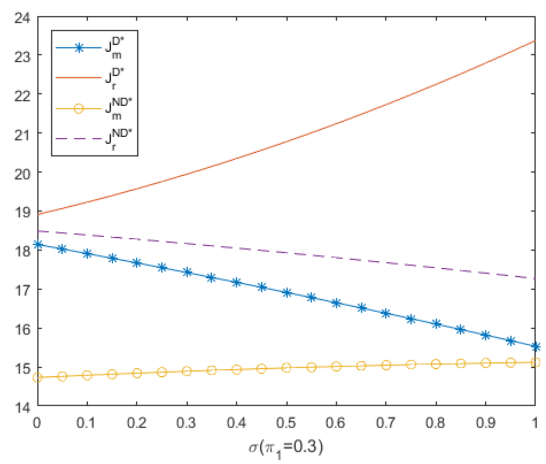

FIGURE 11. The impact σ on the profits under different advertising program when $\pi_1 = 0.3$.

the retailer decreases the advertising investment, the manufacturer increases the advertising investment. On the other hand, the retailer's advertising participation rate decreases accompanying with the competition intensity from Fig. 5. Hence, the profit of the retailer increases and the profit of the manufacturer decreases. Figs. 8-9 show that, an increasing competition intensity between products leads to higher retailer's profit and lower manufacturer's profit when the values of π_1 are 0.6 and 0.8, and also leads to lower total profit of supply chain under the decentralized channel. The main reason is that the condition $\pi_2 < \pi_1$ is satisfied when π_1 are 0.6 and 0.8, which leads to the increasing advertising investments of the retailer and the manufacturer with the competition intensity. Besides, the retailer's advertising participation rate decreases accompanying with the competition intensity. Furthermore, we observe from Figs. 6-9 that the profit of supply chain under the decentralized channel is higher than the one under the centralized channel when the competition intensity between products is greater than some thresholds.

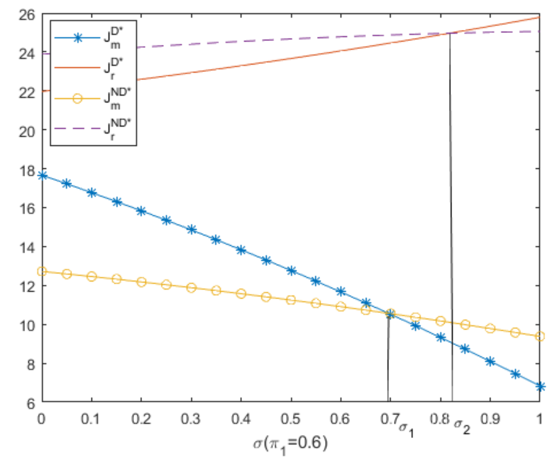

FIGURE 12. The impact σ on the profits under different advertising program when $\pi_1 = 0.6$.

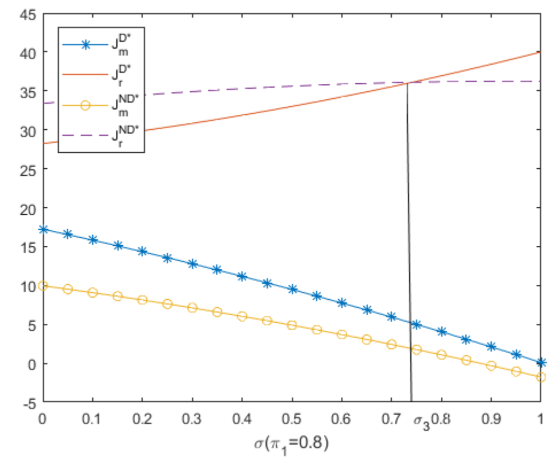

FIGURE 13. The impact σ on the profits under different advertising program when $\pi_1 = 0.8$.

Figs. 10-11 show that, the profits of the channel members under cooperative adverting program are always higher than the one under the non-cooperative adverting program with the increasing of σ when the values of π_1 are 0.09 and 0.3. Fig. 12 shows that, the profit of the retailer under the non-cooperative adverting program is higher than the one under the cooperative adverting program when $\sigma > \sigma_1$, and the profit of the manufacturer under the non-cooperative adverting program is higher than the one under the cooperative adverting program when $\sigma < \sigma_2$. Fig. 13 shows that, the profit of retailer under the non-cooperative adverting program is lower than the one under the cooperative adverting program, and the profit of manufacturer under the non-cooperative adverting program is higher than the one under the cooperative adverting program when $\sigma < \sigma_3$. In a word, the channel players not always prefer to the cooperation when the retailer sells the differentiated product under the consignment mode. Actually, as mentioned in [44], we can find the fact that it is very difficult to implement the cooperative advertising program in the presence of a VMI policy complemented by a consignment contract with revenue sharing. Hence, the finding in this paper is in line with the result in [44].

VII. CONCLUSION

This paper discusses the problems of advertising strategies and coordination of supply chain in the competitive setting. By applying the optimal control theory, the optimal advertising decisions of channel member are presented under the decentralized and centralized settings. Furthermore, the novel state-dependent incentive scheme is proposed to coordinate the decentralized supply chain. The main features lie in the following two aspects: i) the internal competition from platform are considered; ii) by constructing the optimal control model, the advertising efforts of channel member are given under the centralized and decentralized scenarios, and a state-dependent contract with the slotting fees is proposed to coordinate the decentralized supply chain. Finally, a numerical analysis is conducted to illustrate the effects from the degree of product differentiation on the equilibria. In particular, the following results are obtained: (1) the relationship between the advertising strategies of players and the the competition intensity between products is dependent on a great degree on the relative sizes of players's marginal profits; and (2) the state-dependent contract with the slotting fees can effectively coordinate the supply chain when the contract parameters satisfy some conditions.

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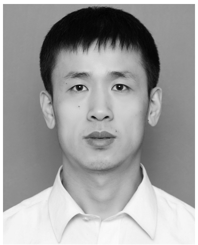

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