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Observer-Based Adaptive Tracking Control of Wheeled Mobile Robots With Unknown Slipping Parameters

MINGYUE CUI
College of Mechanical and Electronical Engineering, Nanyang Normal University, Nanyang 473061, China
e-mail: cuiminyue@sina.com

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ABSTRACT Based on wheeled mobile robots (WMRs) with unknown longitudinal slipping parameters, an adaptive control strategy for a tracked mobile robot is presented, in which the longitudinal slipping of the left and right wheels are described by two unknown parameters. The kinematic model of mobile robot with wheels' slipping is derived from the motion model of mobile robot without wheels' slipping. Employing the Lyapunov direct method, an adaptive nonlinear feedback control law that compensates for the longitudinal slipping is proposed to achieve an objective of tracking a given trajectory. The orientation angle observer is designed to estimate the immeasurable orientation angle of the robot by employing the coordinate information. Asymptotic stability of the closed-loop system is ensured by choosing an appropriate Lyapunov function. Numerical and experimental results show the effectiveness of the proposed control approach.

INDEX TERMS Wheeled mobile robot, longitudinal slipping, adaptive control, backstepping technique, orientation angle observer.

I. INTRODUCTION

In the past two decades, wheeled mobile robots (WMRs) are increasingly presented in the fields of industry, agriculture, national defense and service, accordingly the problem of motion control of WMRs has attracted the interest of the researchers in view of its theoretical challenges result from the nature of the nonholonomic constraints [1], [2]. Many researchers have designed tracking and stabilization controllers for nonholonomic mobile robots using nonlinear control methods, such as sliding mode control[3]–[6], adaptive control[7]–[10], back-stepping control[11], [12], optimal control [13]–[15], intelligent control based on neural network[16]–[18] and fuzzy control[19], [20].

The previous papers mostly assume that the WMR satisfies nonholonomic constraints [1]–[20]. The nonholo- nomic constraints are generated by the assumption that the mobile robots are subject to a 'pure rolling without slipping'. However, since the robotic wheels' slipping can happen in various practical environments such as the on wet or icy roads, rough terrain, or the rapid cornering, the nonholonomic constraint can be disturbed in a few literatures [21]–[28]. Therefore, it is necessary to study the control method of mobile robot considering wheels' slipping. In [21], Wang and Low presented the model of wheeled mobile robot with wheels' longitudinal and lateral slipping, and its controllability was tested according to the maneuverability of the mobile robot. They also proposed a control method for path following and tracking of mobile robots considering slipping [22], [23]. In [25], the trajectory tracking control problem of a mobile robot with longitudinal wheel skidding was studied. The nonlinear model of the mobile robot is transformed into a time-varying linear model, and a trajectory tracking controller is designed by using Linear Matrix Inequality (LMI) method. However, these studies all assume that sensors (such as GPS, photoelectric encoder, etc.) can directly detect the wheels' slipping information of mobile robots in real time, that is, the slipping parameters are known, which is often difficult to achieve in the practical engineering of wheeled mobile robots. Therefore, it is very necessary to study the motion control of mobile robots whose wheels' slipping can not be directly detected. Further, Cui et al. proposed an adaptive tracking controller of the WMRs with unknown wheels' slipping ratios by designing sliding mode observer and Adaptive Unscented

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Kalman Filter (AUKF) respectively [25], [26]. Unfortunately, the design of a separate observer for on-line estimation of wheels' slipping increases the difficulty of controller design, and the stability of closed-loop system based on the principle of independence is not easy to guarantee. Yoo presented a neural network based adaptive control approach for path tracking and obstacle avoidance of a class of mobile robots in the presence of unknown skidding, slipping, and torque saturation [27]. Gao et al. proposed an improved adaptive controller to allow WMRs to track the desired trajectory under unknown longitudinal slipping, where the stabilization of the closed-loop tracking system is guaranteed by the Lyapunov theory [28]. However, adaptive laws of slipping parameter and the orientation angle observer are not designed in the process of controller design in these studies (see [21]–[28]). As a result, difficulty of controller design is increasing, and the scope of application of the designed controller is also limited in practical environment.

Based on the above analysis, in this paper, we mainly study the trajectory tracking control of wheeled mobile robots under the condition that the wheels' longitudinal slipping parameters and orientation angle are all unknown. The main contribution of our work is the design of an adaptive controller with an orientation angle observer for tracking of a class of mobile robots in the presence of unknown longitudinal slipping and orientation angle at the robot kinematic level. Firstly, the kinematic model of the mobile robot is established when the wheels' longitudinal slipping occurs. Subsequently, the adaptive trajectory tracking controller with the adaptive update laws of unknown slipping parameters are designed by employing backstepping technology and Lyapunov direct method. Then the orientation angle observer is designed based on position information, and the exponential convergence of estimation error is proved by Lyapunov method. The stability of the closed-loop system is analyzed by Lyapunov stability theory. Finally we demonstrate the effectiveness of our proposed controllers by simulation and experiment studies.

The rest of this paper is arranged as follows. In Section 2, we present the kinematic model of wheeled mobile robots with longitudinal slipping induced from nonholonomic constraints. In Section 3, an adaptive tracking controller with the adaptive laws of the slipping parameters for mobile robots in the presence of unknown longitudinal slipping is designed by using Lyapunov direct method. In Section 4, an orientation angle observer is designed to estimate the unknown orientation angle. The stability of the proposed control system is analyzed in Section 5. Simulation and experiment results are discussed in Section 6. Finally, Section 7 gives some conclusions.

II. KINEMATIC MODEL OF THE WMR WITH WHEELS' SLIPPING

The model of a differentially steered WMR (Wheeled Mobile Robot) is shown as Fig.1. It has two driving wheels and two universal wheels, where the two driving wheels are powered independently by two direct current motors respectively and have the same wheel radius.

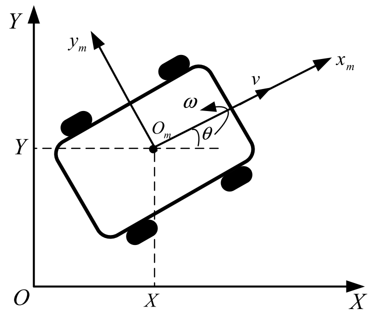


FIGURE 1. Wheeled mobile robot with two independent driving wheels.

To describe the motion characters of tracked mobile robot simply and rigorously in the general plane motion, a fixed reference coordinate frame $F_1(X, Y)$ and a moving coordinate frame $F_2(x_m, y_m)$ are defined which attach to the robot body with origin at the geometric center O_m.

The linear velocities of the left and right driving wheels of the WMR without wheels' slipping are represented as follows

$$v_L = r\omega_L, \quad v_R = r\omega_R \tag{1}$$

where ω_L and ω_R are the angular velocities of the left and right wheels respectively and $r$is radius of the wheels. Longitudinal slipping ratio of the left and right wheels of a mobile robot are defined as [26]

$$i_L = \frac{r\omega_L - v_L^s}{r\omega_L}, \quad i_R = \frac{r\omega_R - v_R^s}{r\omega_R} \tag{2}$$

where v_L^s and v_R^s are the linear velocities of the left and right wheels of the WMR with wheels' slipping respectively.

Assumption 1: The value ranges of longitudinal slipping ratios i_L and i_R lie in [0, 1).

Remark 1: If $i_R = i_L = 1$, from (2), we know that $v_L^s = v_R^s = 0$, which implies a complete slipping, i.e., the wheels of the mobile robot are rotating, while its forward speed is zero, the mobile robot is uncontrollable. This case is not considered.

In coordinate frame $F_1(X, Y)$, the kinematic mode of the differential WMR without slipping is described as follows

$$\begin{bmatrix} \dot{X} \\ \dot{Y} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} \cos\theta & 0 \\ \sin\theta & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v \\ \omega \end{bmatrix} \tag{3}$$

where v is the linear velocity of the robot, ω is the angular velocity of the robot body around the geometric center, and θ is the direction angle of the robot.

From (1)-(3), in coordinate frame $F_1(X, Y)$, the kinematic mode of the differential WMR with slipping is described as follows [26]

$$\begin{aligned} \dot{X} &= \frac{v_L(1-i_L) + v_R(1-i_R)}{2}\cos\theta \\ \dot{Y} &= \frac{v_L(1-i_L) + v_R(1-i_R)}{2}\sin\theta \\ \dot{\theta} &= \frac{v_R(1-i_R) - v_L(1-i_L)}{b} \end{aligned} \tag{4}$$

where $q = [X, Y, \theta]^{\mathrm{T}}$ is posture vector, b is the distance between two driving wheels of the WMR.

III. DESIGN OF THE ADAPTIVE TRACKING CONTROLLER

We assume that there is a slipping between the wheel and the ground, and that the center of mass coincides with the movement geometric center of mobile robot. The kinematic model of mobile robot can be described by (3), thus, actual posture of robot is decided by equation (3). If the desired posture of robot is defined as $q_r = [X_r, Y_r, \theta_r]^{\mathrm{T}}$, the reference posture satisfies the following equation

$$\begin{bmatrix} \dot{X}_r \\ \dot{Y}_r \\ \dot{\theta}_r \end{bmatrix} = \begin{bmatrix} \cos\theta_r & 0 \\ \sin\theta_r & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v_r \\ \omega_r \end{bmatrix} \tag{5}$$

where v_r and ω_r are reference linear velocity and angular velocity of mobile robot respectively.

Assumption 2: The reference velocities and their derivatives v_r, ω_r, $\dot{v}_r$ and $\dot{\omega}_r$ are all available and bounded. The reference velocities and reference direction angle θ_r can be calculated as follows

$$v_r = \sqrt{\dot{X}_r^2 + \dot{Y}_r^2}, \quad \omega_r = \frac{\dot{X}_r \ddot{Y}_r - \ddot{X}_r \dot{Y}_r}{\dot{X}_r^2 + \dot{Y}_r^2}, \; \theta_r = \arctan\frac{\dot{Y}_r}{\dot{X}_r} \tag{6}$$

Assumption 3: The position coordinates (X, Y) of the robot are the output of measurement, which can be measured by RFID, GPS technology or other methods, and the speeds v and ω are the control input, which are measured by photoelectric encoder, and the direction angle θ is not measurable.

The objective of this research is to design an adaptive trajectory tracking controller for wheeled mobile robot, when the longitudinal slipping occurs between the wheels and the ground, so that the actual and desired pose of the robot can satisfy the following

$$\lim_{t\to\infty}(q - q_r) = 0 \tag{7}$$

In the coordinate frame $F_1(X, Y)$, the trajectory tracking error equation of mobile robot is described as follows

$$\begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X_r - X \\ Y_r - Y \\ \theta_r - \theta \end{bmatrix} \tag{8}$$

where e_1, e_2 and e_3 are tracking errors, they are expressed in the frame of real robot, as shown in Fig. 3.

Tracking errors vector of mobile robot is defined as $e = [e_1, e_2, e_3]^{\mathrm{T}}$, the error dynamic equation of the mobile robot is obtained as [26]

$$\begin{bmatrix} \dot{e}_1 \\ \dot{e}_2 \\ \dot{e}_3 \end{bmatrix} = \begin{bmatrix} \omega e_2 + v_r\cos e_3 - v \\ -\omega e_1 + v_r \sin e_3 \\ \omega_r - \omega \end{bmatrix} \tag{9}$$

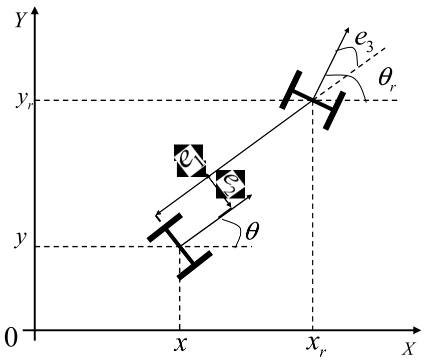


FIGURE 2. Robot pose error coordinates scheme.

In absence of wheels' slipping, by applying backstepping method the auxiliary velocity control laws can be designed as follows [26]

$$\begin{bmatrix} v \\ \omega \end{bmatrix} = \begin{bmatrix} v_r\cos e_3 + k_1 e_1 \\ \omega_r + k_2 v_r e_2 + k_3 \sin e_3 \end{bmatrix} \tag{10}$$

where k_1, k_2, k_3 are adjustable positive numbers.

When the driving wheels occur longitudinal slipping, the relationship between the auxiliary control input $[v, \omega]^{\mathrm{T}}$ and the actual control input $[\omega_L, \omega_R]^{\mathrm{T}}$ is as following

$$\begin{bmatrix} v \\ \omega \end{bmatrix} = \begin{bmatrix} \dfrac{r(1-i_L)\omega_L + r(1-i_R)\omega_R}{2} \\ \dfrac{-r(1-i_L)\omega_L + r(1-i_R)\omega_R}{b} \end{bmatrix} = T\begin{bmatrix} \omega_L \\ \omega_R \end{bmatrix} \tag{11}$$

where the matrix $T = r\begin{bmatrix} \frac{1-i_L}{2} & \frac{1-i_R}{2} \\ \frac{-(1-i_L)}{b} & \frac{1-i_R}{b} \end{bmatrix}$. We can verify that the matrix T is nonsingular. From (11), we obtain the actual control input as following

$$\begin{bmatrix} \omega_L \\ \omega_R \end{bmatrix} = T^{-1}\begin{bmatrix} v \\ \omega \end{bmatrix} = \frac{1}{r}\begin{bmatrix} \dfrac{1}{1-i_L} & -\dfrac{b}{2(1-i_L)} \\ \dfrac{1}{1-i_R} & \dfrac{b}{2(1-i_R)} \end{bmatrix}\begin{bmatrix} v \\ \omega \end{bmatrix} \tag{12}$$

Now, if the slipping parameters i_L and i_R that appear in (12) are unknown, the actual control input ω_L and ω_R cannot be calculated directly by the equations (10) and (12). Hence, we design the adaptive updating laws to attain the control objective using estimation of i_L and i_R. If $\hat{i}_L$ and $\hat{i}_R$ denote the estimations of i_L and i_R respectively, from equation (12), we can obtain actual velocity control input as following

$$\begin{bmatrix} \omega_L \\ \omega_R \end{bmatrix} = \frac{1}{r}\begin{bmatrix} \dfrac{1}{1-\hat{i}_L} & -\dfrac{b}{2(1-\hat{i}_L)} \\ \dfrac{1}{1-\hat{i}_R} & \dfrac{b}{2(1-\hat{i}_R)} \end{bmatrix}\begin{bmatrix} v \\ \omega \end{bmatrix} \tag{13}$$

The estimation errors of slipping ratios for the left and right driving wheels are defined as follows: $\tilde{i}_L = \hat{i}_L - i_L$, $\tilde{i}_R = \hat{i}_R - i_R$.

To facilitate design controller, the slipping parameters are redefined as following

$$a_k = \frac{1}{1-i_k}, \quad k = L, R \tag{14}$$

We define that $\hat{a}_k(k = L, R)$ are the estimations of $a_k(k = L, R)$. Since $\tilde{a}_k = \frac{1}{1-\tilde{i}_k}$, $\hat{a}_k = \frac{1}{1-\hat{i}_k}$, $\tilde{a}_k = \hat{a}_k - a_k, k = L, R$, so (13) can be rewritten as following

$$\begin{bmatrix} \omega_L \\ \omega_R \end{bmatrix} = \frac{1}{r}\begin{bmatrix} \hat{a}_L & -\frac{b\hat{a}_L}{2} \\ \hat{a}_R & \frac{b\hat{a}_R}{2} \end{bmatrix}\begin{bmatrix} v \\ \omega \end{bmatrix} \tag{15}$$

In order to derive the adaptive law of unknown slipping parameters conveniently, according to formulas (9), (10) and (14), the dynamic error equation (9) of mobile robot can be expressed as follows

$$\begin{aligned} \dot{e}_1 &= \frac{a_L+\tilde{a}_L}{a_L}[-(\frac{e_2 v}{b}+\frac{v}{2})+(\frac{e_2\omega}{2}+\frac{b\omega}{4})] \\ &\quad +\frac{a_R+\tilde{a}_R}{a_R}[(\frac{e_2 v}{b}-\frac{v}{2})+(\frac{e_2\omega}{2}-\frac{b\omega}{4})]+v_r\cos e_3 \\ \dot{e}_2 &= \frac{a_L+\tilde{a}_L}{a_L}(\frac{e_1 v}{b}-\frac{e_1\omega}{2})-\frac{\hat{a}_R+\tilde{a}_R}{a_R}(\frac{e_1 v}{b}+\frac{e_1\omega}{2}) \\ &\quad +v_r\sin e_3 \\ \dot{e}_3 &= \omega_r+\frac{a_L+\tilde{a}_L}{a_L}(\frac{v}{b}-\frac{\omega}{2})-\frac{\hat{a}_R+\tilde{a}_R}{a_R}(\frac{v}{b}+\frac{\omega}{2}) \end{aligned} \tag{16}$$

In order to design the adaptive update laws for two unknown slipping parameters, Lyapunov function candidate is chosen as following

$$V(t) = \frac{1}{2}(e_1^2+e_2^2)+\frac{1-\cos e_3}{k_2}+\frac{\tilde{a}_L^2}{2\rho_1 a_L}+\frac{\tilde{a}_R^2}{2\rho_2 a_R} \tag{17}$$

where k_2, ρ_1 and ρ_2 are positive constants. The equation (17) is taken a derivative with respect to time t, we can obtain

$$\dot{V}(t) = e_1\dot{e}_1+e_2\dot{e}_2+\dot{e}_3\frac{\sin e_3}{k_2}+\frac{\tilde{a}_L\dot{\hat{a}}_L}{\rho_1 a_L}+\frac{\tilde{a}_R\dot{\hat{a}}_R}{\rho_2 a_R} \tag{18}$$

Substituting (10) and (16) into (18), we can obtain

$$\begin{aligned} \dot{V}(t) &= -k_1 e_1^2-\frac{k_3}{k_2}\sin^2 e_3 \\ &\quad +\frac{\tilde{a}_L}{a_L}[\frac{\dot{\hat{a}}_L}{\rho_1}+\frac{be_1\omega}{4}-\frac{e_1 v}{2}+\frac{\sin e_3}{k_2}(\frac{v}{b}-\frac{\omega}{2})] \\ &\quad +\frac{\tilde{a}_R}{a_R}[\frac{\dot{\hat{a}}_R}{\rho_2}-\frac{be_1\omega}{4}-\frac{e_1 v}{2}-\frac{\sin e_3}{k_2}(\frac{v}{b}+\frac{\omega}{2})] \end{aligned} \tag{19}$$

From (19), the adaptive update laws of the slipping parameters a_L and a_R can be designed as follows:

$$\begin{aligned} \dot{\hat{a}}_L &= \rho_1[(\frac{e_2}{b}+\frac{1}{2})e_1 v-(\frac{e_2}{2}+\frac{b}{4})e_1\omega-\frac{\sin e_3}{k_2}(\frac{v}{b}-\frac{\omega}{2})] \\ \dot{\hat{a}}_R &= \rho_2[-(\frac{e_2}{b}-\frac{1}{2})e_1 v-(\frac{e_2}{2}-\frac{b}{4})e_1\omega+\frac{\sin e_3}{k_2}(\frac{v}{b}+\frac{\omega}{2})] \end{aligned} \tag{20}$$

Substituting (20) into (19), we can obtain

$$\dot{V}(t) = -k_1 e_1^2-\frac{k_3}{k_2}\sin^2 e_3 \le 0 \tag{21}$$

IV. DESIGN OF ORIENTATION ANGLE OBSERVER

A. DESIGN OF OBSERVER

The reference input v_r and ω_r can be calculated directly by equation (6) from the desired trajectory. However the error coordinates e_1 and e_2 are not readily available as they must be calculated with (8) from measurement coordinates X and Y, reference coordinates X_r and Y_r and orientation angle θ, the latter of which is not available, so the orientation angle error e_3 is not available, too.

Assumption 4: the linear velocity v of the robot is bounded, it can be expressed as follows: $0 < v_{\min} \le v \le v_{\max}$ ($v_{\min}$ and $v_{\max}$ are the minimum and maximum linear velocity of robot respectively).

Due to e_3 is not directly available, so $\sin e_3$ and $\cos e_3$ are all unknown, the auxiliary velocity control law (10) is not implemented. To mend this situation, an orientation angle observer is designed to estimate the orientation angle θ. From the kinematic mode (3) of the mobile robot, we introduce two observer state variables

$$z_1 = x, \quad z_2 = y, \; z_3 = \cos\theta, \; z_4 = \sin\theta \tag{22}$$

The following reduced dimensional observer is designed as

$$\begin{cases} \dot{\alpha} = -\hat{z}_4\omega - L\hat{z}_3 v \\ \dot{\beta} = \hat{z}_3\omega - L\hat{z}_4 v \\ \hat{z}_3 = \alpha + Lz_1 \\ \hat{z}_4 = \beta + Lz_2 \end{cases} \tag{23}$$

where L is a positive constant, α and β are the state variables of the observer, $\hat{z}_3$ and $\hat{z}_4$ are the estimation values of z_3 and z_4 respectively.

B. CONVERGENCE ANALYSIS OF OBSERVER ERRORS

In the following, the stability of the orientation angle observer is discussed.

Theorem 1: Consider orientation angle observer (23) and kinematic model (3) of mobile robot with unknown the orientation angle θ, under the condition of satisfying the assumption 3 and assumption 4, the dynamic errors in the orientation angle observer system will asymptotically converge exponentially to zeroes in finite time.

Proof: From (3) and (22) the differential equations of the observer state can be obtained as follows

$$\begin{aligned} \dot{z}_1 &= z_3 v, \quad \dot{z}_2 = z_4 v \\ \dot{z}_3 &= -z_4\omega, \quad \dot{z}_4 = z_3\omega \end{aligned} \tag{24}$$

The observer errors are defined as $\tilde{z}_3 = z_3 - \hat{z}_3$ and $\tilde{z}_4 = z_4 - \hat{z}_4$. From (21) and (22) the observer error dynamic equations can be obtained as

$$\begin{cases} \dot{\tilde{z}}_3 = \dot{z}_3 - \dot{\hat{z}}_3 = -\tilde{z}_4\omega - L\tilde{z}_3 v \\ \dot{\tilde{z}}_4 = \dot{z}_4 - \dot{\hat{z}}_4 = \tilde{z}_3\omega - L\tilde{z}_4 v \end{cases} \tag{25}$$

Lyapunov function candidate is chosen as follow

$$V_o = \frac{1}{2}\tilde{z}_3^2 + \frac{1}{2}\tilde{z}_4^2 \tag{26}$$

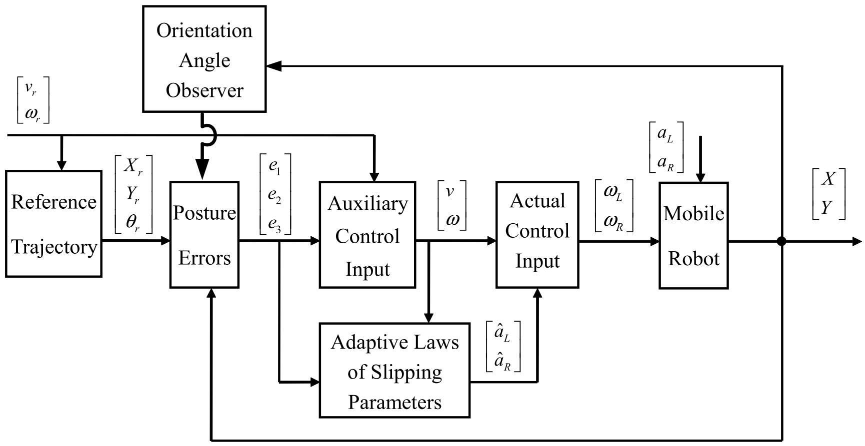


FIGURE 3. Trajectory tracking control principle of mobile robot.

From (24) and (25), we can obtain

$$\dot{V}_o = -L(\tilde{z}_3^2 + \tilde{z}_4^2)v \leq -2v_{\min} L V_o \tag{27}$$

where $v_{\min}$ is the minimum linear velocity of robot, then

$$V_o \leq V_o(0) e^{-2v_{\min} L t} \tag{28}$$

From (25) and (27), we can obtain

$$\|(\tilde{z}_3, \tilde{z}_4)\|_2 \leq \|(\tilde{z}_3(0), \tilde{z}_4(0))\|_2 \, e^{-v_{\min} L t} \tag{29}$$

Due to $L > 0, v_{\min} > 0$, the observer errors $\tilde{z}_3$ and $\tilde{z}_4$ asymptotically converge exponentially to zeroes in finite time. This completes proof of the Theorem 1. □

Remark 2: From (28), we know that the observer errors $\tilde{z}_3$ and $\tilde{z}_4$ exponentially approach $\sin\theta$ and $\cos\theta$, so the estimation of the orientation angle θ in the interval with $(-\pi, \pi]$ can be directly derived as

$$\hat{\theta} = \arctan 2(\hat{z}_4, \hat{z}_3) \tag{30}$$

The tracking error of the orientation angle $e_3$can be calculated as$e_3 = \hat{\theta} - \theta_r$. The implementation of the auxiliary velocity control law (10) is therefore readily possible.

V. CONVERGENCE ANALYSES OF TRACKING ERRORS

Lemma 1: (Barbalat lemma [30]) If $f(x)$ is uniformly continuous and if the limit of the integral $\lim_{t\to\infty} \int_0^t f(x)dx$ exists and is finite then $\lim_{t\to\infty} f(t) = 0$.

Theorem 2: The posture tracking error $e = [e_1, e_2, e_3]^T$ of the complete kinematic equation (4) of the WMR with unknown slipping parameters will asymptotically converge to zero vector $[0, 0, 0]^T$, if the kinematic controller input as (10) and (15), and the adaptive update laws (20) of the slipping parameters are applied.

Proof: Lyapunov function candidate is chosen as (17), we let domain D be given by $D = \left\{e \in R^3 \,|-\pi < e_3 < \pi\right\}$, then the Lyapunov function $V(t) > 0$ given in (17) is positive definite in domain

$$D' = \left\{e \in R^3 \,|-\pi < e_3 < \pi, e_3 \neq 0\right\}$$

that is $V(t) > 0$.

From the previous analysis, it can be seen that under the condition of satisfying the error dynamic equation (16) and the adaptive law (20) of the slipping parameters a_L and a_R, in the domain D, we can obtain as (21). As $t \in [0, \infty)$, $V(t)$ is a monotone and nonincreasing function, we obtain

$$V(t) \leq V(0), \quad \forall t \geq 0 \tag{31}$$

This implies V is bounded as $t \in [0, \infty)$. From (17), we know as $t \in [0, \infty)$, e_1, e_2, $\tilde{a}_L$, $\tilde{a}_R$ are bounded. From assumption 2 and equation (10), we know the auxiliary control input $[v, \omega]^T$ is bounded. From equation (9), we can obtain $\dot{e} = [\dot{e}_1, \dot{e}_2, \dot{e}_3]^T$ is bounded. Take the second order derivatives of the Lyapunov function V is given by

$$\ddot{V} = -2k_1 e_1 \dot{e}_1 - \frac{2k_3}{k_2} \dot{e}_3 \sin e_3 \cos e_3 \tag{32}$$

From (32), we can know $\ddot{V}$ is bounded, so $\dot{V}$ is uniformly continuous, Barbalat's Lemma [30] shows that $\dot{V} \to 0$ as $t \to \infty$. From equation (21), we know $e_1 \to 0$ and $e_3 \to 0$ as $t \to \infty$ in the domain $D = \left\{e \in R^3 \,|-\pi < e_3 < \pi\right\}$.

From equations (9) and (10), we have

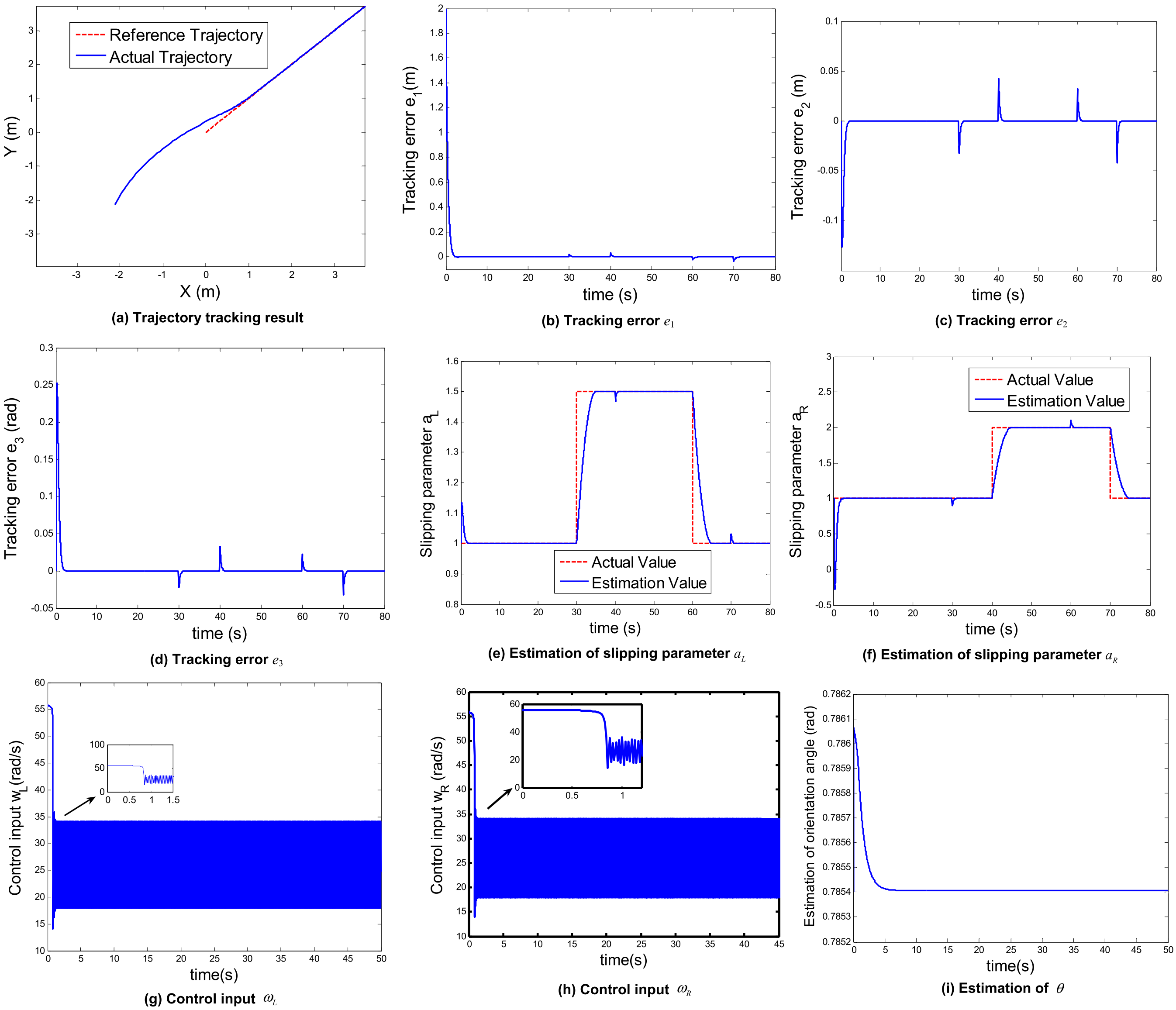
$$\dot{e}_3 = -k_2 v_r e_2 - k_3 \sin e_3 \tag{33}$$

Thus

$$\ddot{e}_3 = -k_2 v_r \dot{e}_2 - k_3 \dot{e}_3 \cos e_3 \tag{34}$$

From (13), we know that $\dot{e}_2$ and $\dot{e}_3$ are bounded. Since robotic reference line velocity v_r is a finite value, $\ddot{e}_3$ is bounded, that is $\ddot{e}_3 \in L_\infty$, so $\dot{e}_3$ is uniformly continuous. Barbalat's Lemma [30] shows that $\dot{e}_3 \to 0$ as $t \to \infty$. Since $e_3, \dot{e}_3 \to 0$ as $t \to \infty$, from (33) we have $-k_2 v_r e_2 \to 0$ as $t \to \infty$. If the reference linear velocity v_r does not go to zero as $t \to \infty$, then $e_2 \to 0$ as $t \to \infty$. The proof of the Theorem 2 is completed. □

Remark 3: According to reference [30], the "Separation Principle" [30] allow us to separate the design into two tasks: design of the auxiliary velocity control laws (10) and the adaptive update laws (20) of the slipping parameters the orientation angle observer(23) is design. The observer errors



(a) Trajectory tracking result
(b) Tracking error e_1
(c) Tracking error e_2
(d) Tracking error e_3
(e) Estimation of slipping parameter a_L
(f) Estimation of slipping parameter a_R
(g) Control input ω_L
(h) Control input ω_R
(i) Estimation of θ

FIGURE 4. Straight line trajectory tracking result.

do not have effect on asymptotic stability of the closed-loop controlled system.

In conclusion, the equilibrium point $[e_1, e_2, e_3]^{\mathrm{T}} = [0, 0, 0]^{\mathrm{T}}$ is uniformly asymptotically stable. This implies the mobile robot can converge to the reference pose $q_r = [X_r, Y_r, \theta_r]^{\mathrm{T}}$ asymptotically from the arbitrary initial pose $q(0) = [X(0), Y(0), \theta(0)]^{\mathrm{T}}$. Thus, the tracking control objective of the mobile robot is able to be implemented accordingly.

From the above analysis, the principle of closed-loop control system for trajectory tracking of mobile robot can be shown as following (See Fig. 3.).

VI. SMULATIONS AND EXPERIMENTS

A. SIMULATIONS

In this section, we will carry out some simulations on the kinematic model of the tracked wheeled mobile robot with slipping, using the proposed control method in previous sections. In the simulation, the angle velocities of the two driving wheels are considered as input variables. To observe and compare the simulation results more easily, we choose two kinds of reference trajectories for the simulations: one is a straight line trajectory, and the other is a circle one. The physical parameters of the WMR are chosen as follows: $b = 0.5$m, $r = 0.125$m, $\rho_1 = 20$, $\rho_2 = 20$, $\xi = 0.8$, $\lambda = 60$; initial estimates of slipping parameters are set as follows: $\hat{a}_L(0) = \hat{a}_R(0) = 1$; the controller gains are chosen as follows: $k_1 = 2$, $k_2 = 6$, $k_3 = 3$; parameter of the orientation angle observer is chosen as $L = 1.2$m.

To verify the robustness of the system, it is assumed that the slipping parameters vary as follows:

(*a*) The actual initial value of the left wheel's slipping parameter of the mobile robot is set at $a_L(0) = 1$, which suddenly changes to 1.5 at $t = 30s$ and 1 at $t = 60s$.

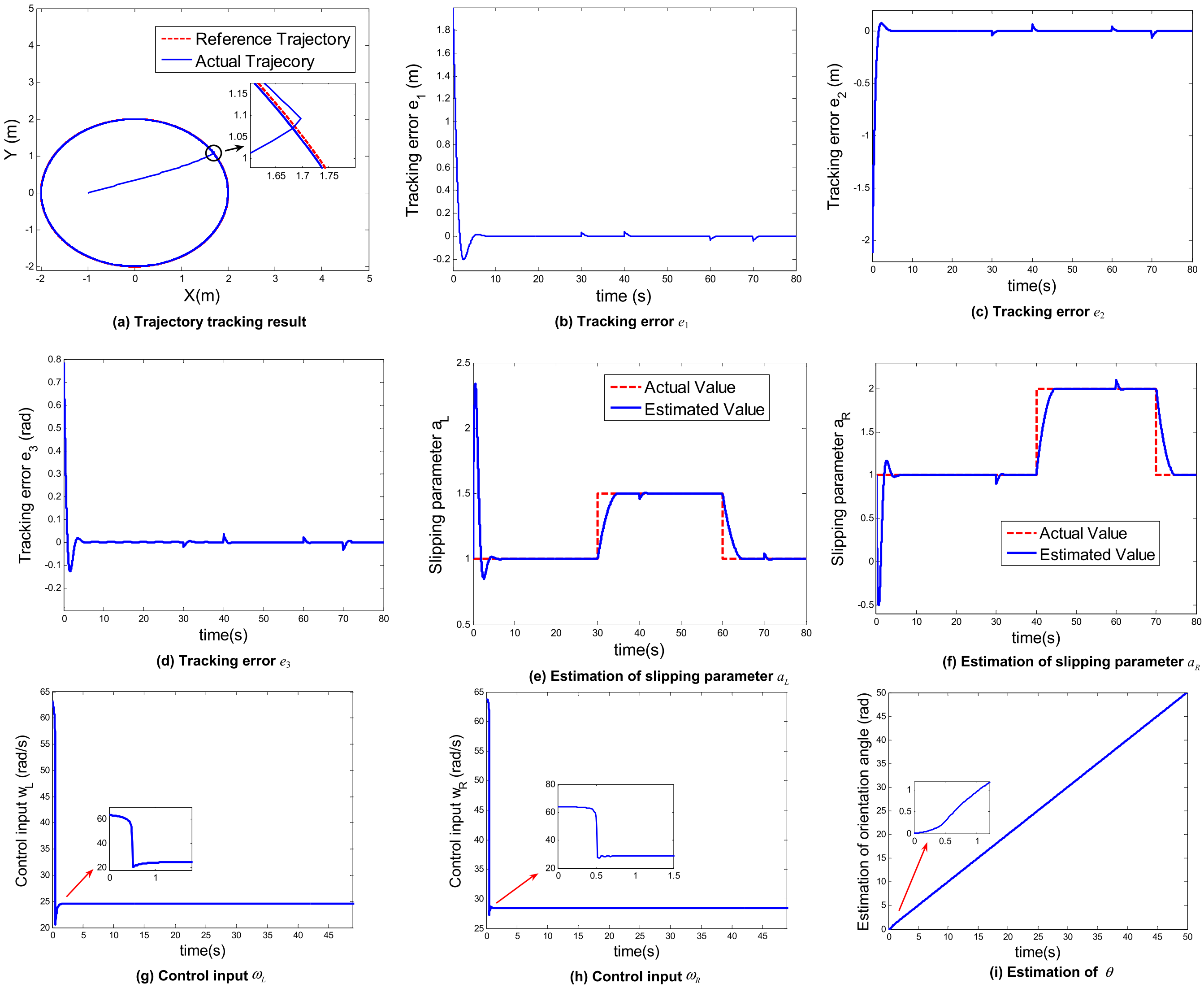


FIGURE 5. **Circle trajectory tracking result.**

(*b*) The initial value of the right wheel's slipping parameter of the mobile robot is set at $a_R(0) = 1$, which suddenly changes to 2 at $t = 40s$ and 1 at $t = 70s$.

1) THE STRAIGHT LINE REFERENCE TRAJECTORY

A straight reference trajectory is considered in this example. The equation of the straight line reference trajectory is given as $X = t, Y = t(t \geq 0)$, where $t \geq 0$ is simulation time. The initial posture of the reference trajectory of the mobile robot is set at $[X_r(0), Y_r(0), \theta_r(0)]^{\mathrm{T}} = [0, 0, \pi/4]^{\mathrm{T}}$. The actual initial posture of the WMR is set at $[X(0), Y(0), \theta(0)]^{\mathrm{T}} = [-2, -2, \pi/4]^{\mathrm{T}}$, reference velocity $v_r = 0.5$m/s and $\omega_r = 0$rad/s.

Fig. 4 (a)∼(f) are the tracking results for straight-line trajectory of WMR. From Fig. 4, we can see that the mobile robot can keep up with and track the straight line reference trajectory quickly from any initial position, the posture error of the mobile robot can converge to zero asymptotically in a short time. Moreover, estimated values of the two slipping parameters can track the actual value asymptotically under the action of the adaptive laws, and it is to say that the estimation algorithm of the slipping parameters has good performance. We also observe that the proposed adaptive tracking controller has excellent ability to suppress the slipping perturbation.

2) THE CIRCLE REFERENCE TRAJECTORY

The equation of the circle reference trajectory is given as $x = 2\cos t, y = 2\sin t(t \geq 0)$, where $t \geq 0$ is simulation time. The initial posture of the reference trajectory is set at $[X_r(0), Y_r(0), \theta_r(0)]^{\mathrm{T}} = [2, 0, \pi/2]^{\mathrm{T}}$. The actual initial posture of the WMR is set at $[X(0), Y(0), \theta(0)]^{\mathrm{T}} = [-1, 0, \pi/4]^{\mathrm{T}}$, reference velocity $v_r = 0.5$m/s and $\omega_r = 0.25$rad/s.

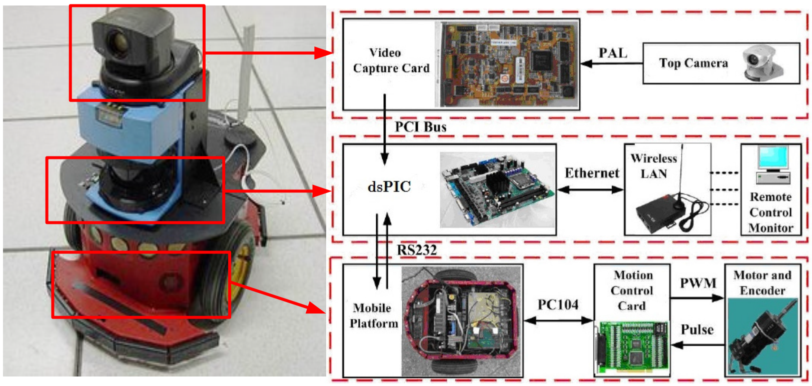


FIGURE 6. Hardware configuration of the mobile robot control system.

TABLE 1. Convergence time of tracking errors.

Reference trajectory	RMS value of tracking error	Convergence time t_s (s)
Straight trajectory	0.030	2.234
Circular trajectory	0.030	4.132

Fig. 5 (a) $\sim$ (f) are tracking results of the circle trajectory for wheeled mobile robot. From Fig. 5, we can see that the mobile robot can track to the circular reference trajectory quickly from the initial position, and the posture error of the mobile robot can converge to zero asymptotically in a relatively short time. Moreover, we can also observe that estimated values of the two slipping parameters (a_L, a_R) can track the actual value asymptotically under the action of the adaptive laws. We also observe that the proposed adaptive tracking controller has excellent ability to suppress the slipping perturbation.

To compare the control effect of tracking two trajectories, we define the RMS (Root Mean Square) value of the tracking error as following

$$\|e\| = \sqrt{\frac{e_1^2 + e_2^2 + e_3^2}{3}}$$

The convergence time t_s, is defined as the minimum time required for the RMS value of tracking errors to reach and remain within $\pm 3\%$ of the final value. The RMS values ($\pm 3\%$ of the final value) and convergence time for both trajectories are shown in Table 1.

We can see from Table 1, the convergence time of robot tracking circular trajectory is longer than that of linear trajectory, because the curvature of circular trajectory is variable and that of straight trajectory is constant.

B. REAL EXPERIMENTS

In order to demonstrate the effectiveness, superiority and applicability of the proposed control method, a real-time control system is implemented for the wheeled mobile robot self-developed based on the Pioneer 2DX mobile robot platform (See Fig.6). The hardware configuration of the mobile robot control system is shown in Fig.6. The motion of the robot is controlled by adjusting the velocities of the left and right wheels by way of a motion control card. The Industrial Personal Computer (IPC) only needs to send the velocity commands to the motion control card, which manages the velocity servo control. In experiment, the control parameters are all same as the simulations. The maximum angular velocities of two driving wheels are all 30rad/s, and the control input is limited to this range.

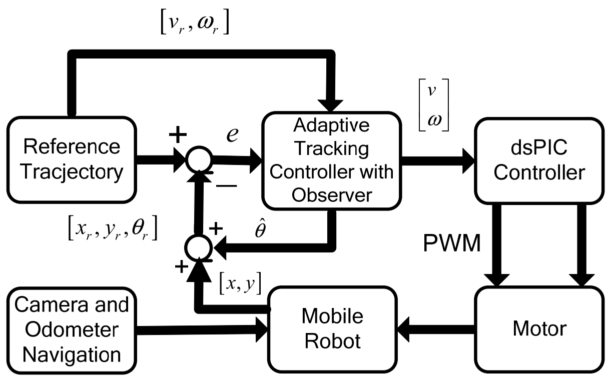
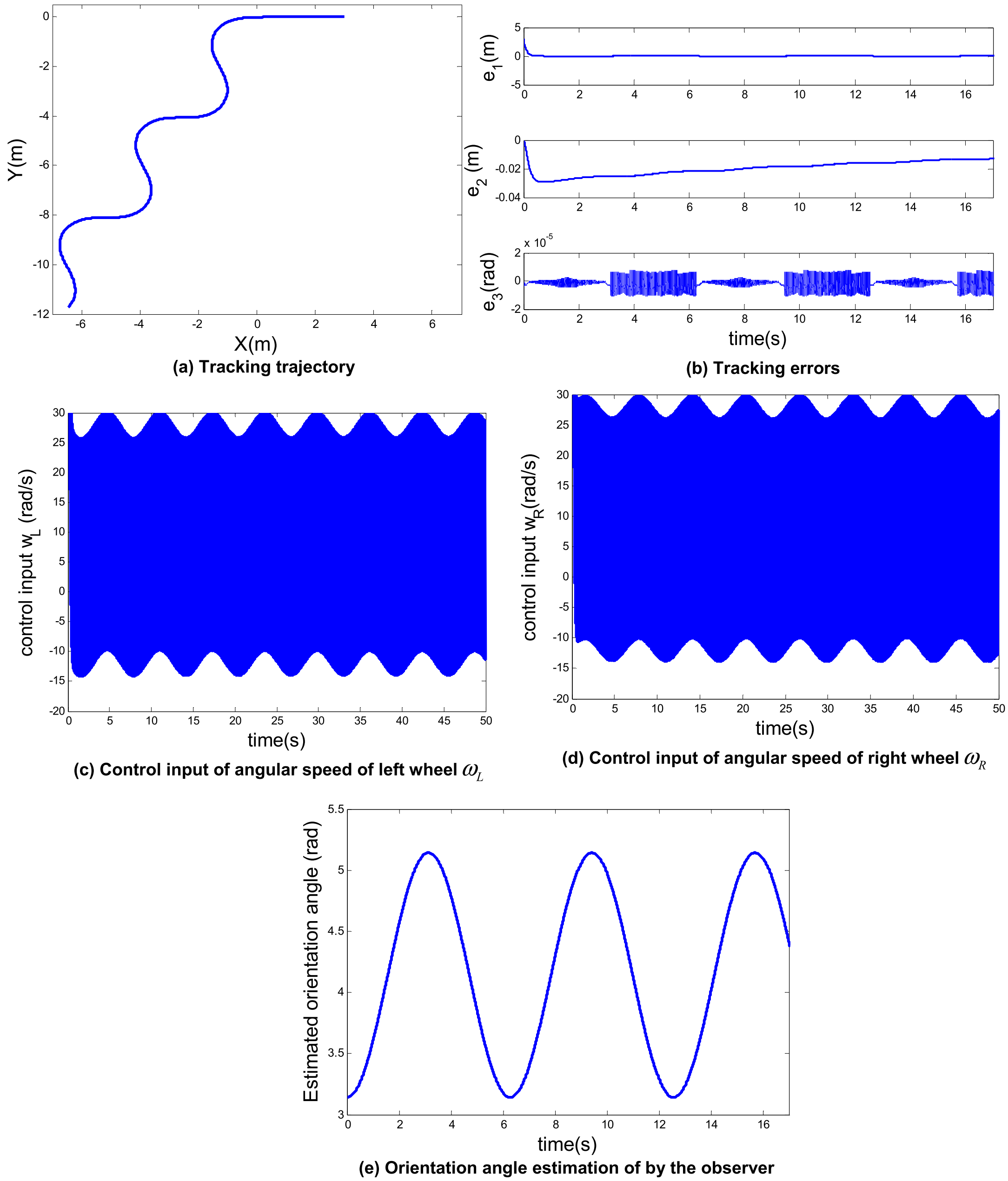


FIGURE 7. Schematic diagram of the experiment control system.

Fig.7 shows the whole schematic diagram of the trajectory tracking control system for the mobile robot. Because of the complexity of the calculation process, the proposed adaptive tracking control method (See equations (10, 15, 20, 23, 30)) is carried out in the main computer running. The arbitrary external wheels' slipping disturbance is fed into the robotic system by laying sand on the ground in desired path of the robot every 3m. "**S**" shape reference trajectory is chosen as Fig. 8(a).

The experiment results are shown as Fig.8. According to the experimental results, we can see that desired trajectory tracking is implemented by using the proposed control method, which can overcome the influence of initial errors and wheels' slipping disturbance. Meanwhile, Fig.8 (e)



(a) Tracking trajectory
(b) Tracking errors
(c) Control input of angular speed of left wheel ω_L
(d) Control input of angular speed of right wheel ω_R
(e) Orientation angle estimation of by the observer

FIGURE 8. Experimental results of tracking control.

shows that the orientation angle observer can achieve rather good estimation performance.

To demonstrate the superiority of the proposed adaptive tracking control approach, comparison experiment is implemented under the same experimental conditions. The adaptive control method proposed in this paper is compared with the control method proposed in reference [25]. In real experiments, comparative experimental results of the RMS values and convergence time for "S" shape trajectory are shown in Table 2.

After a period of adjustment, all tracking errors converge to be within required compact sets, which are related to the posture information between two controllers. However, the tracking errors controlled by proposed controller in this paper take less time than the errors controlled by proposed controller in reference [25], and the regulating processes based on proposed controller in this paper appear gentler that is more conducive to perform in practice. As a result,

TABLE 2. Convergence time of tracking errors.

Control method	RMS value of tracking errors	Convergence time t_s (s)
Control method in this paper	0.030	0.613
Control method in Ref.[25]	0.030	3.486

the previous experiment results tell that the proposed controller in this paper possesses much more favorable control performance compared with the proposed controller in reference [25].

VII. CONCLUSION

In this paper, backstepping technology and Lyapunov direct method are applied to design an adaptive tracking controller with adaptive law of the slipping parameters for the WMR with unknown wheels' longitudinal slipping. Based on the kinematic mode of the WMR, the orientation angle observer is designed to estimate the immeasurable orientation angle by the available coordinate information of WMR. At the same time, it is proved that the posture errors of the mobile robot can converge asymptotically to zeroes by Lyapunov stability theory. The simulation and experiment results show that the controlled mobile robot system has excellent trajectory tracking performance. Even if the slipping parameters of the WMR suddenly change, the adaptive control system of the robot still has good tracking control performance and robustness.

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MINGYUE CUI received the B.S. degree in automation engineering from the Luoyang University of Science and Technology, Luoyang, China, in 1997, the M.S. degree in control theory and control engineering from the Lanzhou University of Technology, Lanzhou, China, in 2009, and the Ph.D. degree in control theory and control engineering from Chonqging University, Chongqing, China, in 2012. He is currently an Associate Professor of control science and engineering with Nanyang Normal University, Nanyang, China. His current research interests include nonlinear control and mobile robot control.