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Event-Triggered Cooperative Output Regulation for Heterogeneous Multi-Agent Systems With an Uncertain Leader

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ABSTRACT This paper investigates the cooperative output regulation for heterogeneous linear multi-agent systems with an uncertain leader under the event-triggered control. Firstly, a local adaptive observer is designed to estimate the system matrices of the leader. Then, utilizing the estimated matrices, an adaptive estimator is proposed to observe the leader's dynamic behavior and an adaptive regulation law is presented to solve the output regulator equations online. Furthermore, by using the estimated state of the leader and the adaptive solutions of the output regulator equations, a distributed event-triggered controller and a novel self-triggered controller are designed such that the output of each follower can converge the leader's output, and Zeno behavior can be excluded for each agent. Finally, two numerical simulation examples are provided to verify the effectiveness of the proposed control approaches.

INDEX TERMS Event-triggered control, self-triggered control, cooperative output regulation, heterogeneous multi-agent systems.

I. INTRODUCTION

Over the past decade, cooperative control for multi-agent systems (MASs) has been a hot topic due to its significant applications in consensus [1]–[3], flocking [4], [5], formulation [6], [7] and so on. However, the nodes in the most of the previous literatures are assumed to be homogeneous. In fact, the agents have distinct system matrices and even different state dimensions, that is heterogeneous, such as in [8]. Therefore, it is more worthy of studying the heterogeneous MASs. A fundamental problem of the heterogeneous MASs is cooperative output regulation problem whose aim is to make the output of each follower track the reference input or the disturbance generated by the so-called leader. In [9], the cooperative output regulation problem for heterogeneous linear multi-agent systems is investigated in the presence of communication constraints which include intermittent and asynchronous discrete-time information exchange and unknown time-varying delays and possible information losses. Nevertheless, the leader's system matrices R and S are usually unknown to the followers, that is, the followers cannot directly utilize the leader's system matrices R and S. It becomes the first important issue to estimate the leader's system matrices.

To overcome the issue, the authors in [10], [11] study the cooperative output regulation problem for uncertain multi-agent systems subject to an uncertain leader system by combining the adaptive control and robust control techniques. Similarly, a distributed adaptive observer is designed in [12] to estimate the leader's system matrices and the leader's states. Based on the estimated system matrices, the output regulator equations could be solved adaptively. As the extension of [12], the authors in [13] investigate the adaptive output containment control of heterogeneous multi-agent systems with multiple unknown leaders. Local adaptive observers are proposed to estimate the leaders' unknown dynamics and the corresponding output regulator equations are solved adaptively. Although the adaptive observers could estimate

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the leaders' unknown dynamics, information communications and actuation updates are continuous which inevitably increase the computation and cause power wasting.

In order to save power, it is critical to reduce the number of information communications and actuation updates. It has been demonstrated that event-triggered strategy is more beneficial for energy saving than the continuous strategy. Tabuada designed an event-triggered strategy to handle the stabilization problem of a single system in [14], and the strategy could be also applied in the consensus problem of MASs with first order agents [15], [16]. In [17]–[19], the event-triggered strategy is further generalized to the general linear MASs. Recently, the event-triggered strategy is introduced to settle the cooperative output consensus problem for heterogeneous multi-agent systems. In [20], a novel distributed event-triggered control scheme and a novel self-triggered control scheme are developed to solve the cooperative output regulation problem of heterogeneous MASs. Furthermore, a distributed event-triggered control scheme is proposed to solve the cooperative output regulation problem of heterogeneous multi-agent systems with switching communication topologies in [21]. The authors in [22] study the cooperative output regulation problem for discrete-time linear time-delay multi-agent systems subject to jointly connected switching networks. There are many other related results about event-triggered or self-triggered strategies in [23]–[26].

However, to our best knowledge, there are few results on event-triggered cooperative output regulation for heterogeneous multi-agent systems with an unknown leader. In many practical cases, the followers cannot directly achieve the leader's information, that is, the leader is unknown or uncertain to the followers. Owing to the unknown leader, it is necessary to resort to an adaptive observer to estimate the leader's system matrices and the leader's unknown dynamics which makes the event-triggered strategy more complicated and challenging. In addition, there is another difficulty of the event-triggered scheme that is to exclude the Zeno behavior. It has to be proved that there is a strictly positive constant to limit the lower bound of the event interval time. These challenges motivate our research.

In this paper, we propose a distributed event-triggered controller and a novel self-triggered controller for the cooperative output regulation of heterogeneous multi-agent systems. Comparing with the existing literatures, there are three major contributions. Firstly, a local adaptive observer is designed to estimate the leader's unknown system matrices. Then, utilizing the estimated matrices of the leader, an adaptive estimator is proposed to observe the leader's dynamic behavior and an adaptive regulation law is presented to solve the output regulator equations online. Besides, by using the estimated state of the leader and the adaptive solutions of the output regulator equations, a distributed event-triggered controller and a novel self-triggered controller are designed such that the output of each follower can converge the leader's output, and the Zeno behavior can be excluded for each agent.

The rest organization of this paper is outlined as follows. Some preliminaries about Graph Theory and problem formulation are introduced in Section 2. In Section 3, four lemmas are achieved by using of the local adaptive observer. Based upon the lemmas, a distributed event-triggered controller and a novel self-triggered controller are proposed in Section 4. Moreover, it is proved to exclude the Zeno behavior. Finally, an illustrative example is given in Section 5.

II. PRELIMINARIES AND PROBLEM FORMULATION

A. NOTATION AND GRAPH THEORY

Let $\mathbb{R}$ and $\mathbb{R}^n$ be the real numbers and the n-dimensional Euclidean space, respectively. $\|\cdot\|_2$ represents the Euclidean norm for the vectors or the induced 2-norm for matrices. $\mathbf{1}_n \in \mathbb{R}^n$ denotes the n-dimensional column vector with all elements being 1. $I_n \in \mathbb{R}^{n\times n}$ represents the $n \times n$ identity matrix. A^T represents the transpose of A. $\sigma_{\min}(A)$, $\sigma_{\max}(A)$ and $\sigma(A)$ are the minimum and maximum singular values, and the spectum of the matrix A, respectively. $rank(A)$ is the rank of matrix A. $A \otimes B$ denotes the Kronecker product of A and B. $\mathrm{diag}(\cdot)$ represents the diagonal matrix. $vec(X)$ is the column vector generated by all the lolumns of the matrix X.

The connection network of multi-agent systems can be represented by a graph $\bar{\mathscr{G}}(\bar{\mathcal{V}}, \bar{\mathcal{E}}, \bar{\mathcal{A}})$, where $\bar{\mathcal{V}} = \{0, 1, \ldots, N\}$ denotes the vertex indexes of the agents, and $\bar{\mathcal{E}} \subseteq \bar{\mathcal{V}} \times \bar{\mathcal{V}} = \{(i,j)|i,j \in \bar{\mathcal{V}}, i \neq j\}$ represents the connections among the agents, and $\bar{\mathcal{A}} = [a_{ij}] \in \mathbb{R}^{(N+1)\times(N+1)}$ is the adjacency matrix with $a_{ij} = 1$ if $(j,i) \in \bar{\mathcal{E}}$, $a_{ij} = 0$ otherwise. In this paper, node 0 is the leader and all the others are the followers which are in $\mathscr{F} = \{1, 2, \ldots, N\}$. The digraph $\bar{\mathscr{G}}$ contains a spanning tree rooted node 0 if any follower is reachable from node 0. Let $\Delta = \mathrm{diag}(a_{10}, \ldots, a_{N0})$ where $a_{i0} = 1$ for $i = 1, \ldots, N$ denote node i can directly obtain information from the leader, and $a_{i0} = 0$ otherwise. Define $\mathscr{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$ is the subgraph of $\bar{\mathscr{G}}$, where $\mathcal{V} = \{1, 2, \ldots, N\}$ and $\mathcal{E}$ is consisted of the edges between the followers. The neighborhood set of node i in $\mathscr{G}$ is represented by $\mathcal{N}_i = \{j \in \mathcal{V}|(i,j) \in \mathcal{E}\}$ and $|\mathcal{N}_i|$ is the cardinality of $\mathcal{N}_i$. Let $L_{\mathscr{G}} = [l_{ij}] \in \mathbb{R}^{N\times N}$ denote the Laplacian matrix of the subgraph spanned by the N followers. Then, further define $H = L_{\mathscr{G}} + \Delta$.

B. PROBLEM FORMULATION

As in [28], we will consider the heterogeneous linear multi-agent systems consisting of one leader and N followers. The dynamics of the ith follower can be described as follows:

$$\begin{cases} \dot{x}_i = A_i x_i + B_i u_i, \\ y_i = C_i x_i, \quad i \in \mathscr{F} = \{1, 2, \ldots, N\} \end{cases} \tag{1}$$

where $x_i(t) \in \mathbb{R}^{n_i}$, $u_i(t) \in \mathbb{R}^{m_i}$, and $y_i(t) \in \mathbb{R}^p$ are the system state, the control input and the output of the ith follower, respectively. The dynamics of the leader can be shown as follows:

$$\begin{cases} \dot{x}_0 = Sx_0, \\ y_0 = Rx_0 \end{cases} \tag{2}$$

where $x_0(t) \in \mathbb{R}^q$ and $y_0(t) \in \mathbb{R}^p$ are the system state and the output of the leader, respectively. $S \in \mathbb{R}^{q\times q}$ and $R \in \mathbb{R}^{p\times q}$ are unknown to the followers.

Remark 1: The system matrices of the followers and the leader are various, and even they have different dimensions. Then, (1) and (2) are deemed to be heterogeneous. But, the dimensions of the followers and the leader are identical. In [20], the authors studied the cooperative output regulation of heterogeneous linear multi-agent systems by event-triggered control, but requiring knowledge of the matrix S. In this paper, we consider the case without knowing $S \in \mathbb{R}^{q\times q}$ and $R \in \mathbb{R}^{p\times q}$.

Problem 1: The event-triggered cooperative output control problem is to design an event-triggered controller $u_i(t)$ such that the followers' outputs $y_i(t)$ can converge to the leader's output $y_0(t)$. That is, the output error $e_i(t) = y_i(t) - y_0(t)$ converges to 0 (i.e. $\lim_{t\to\infty} e_i(t) = 0$).

The following assumptions and lemmas are needed to solve the Problem.

Assumption 1: There exists a spanning tree with the leader as the root in the digraph.

Assumption 2: The real parts of the eigenvalues of S are nonnegative.

Assumption 3: For all $i = 1, 2, \ldots, N$, (A_i, B_i) are stabilizable and (A_i, C_i) are detectable.

Assumption 4: For all $i = 1, 2, \ldots, N$,
$$rank\begin{bmatrix} A_i - \lambda I_{n_i} & B_i \\ C_i & 0 \end{bmatrix} = n_i + p, \forall\ \lambda \in \sigma(S).$$

Lemma 1: [27] Under the Assumption 1, all the eigenvalues of the matrix H have positive real parts.

Lemma 2: [29] Under Assumption 4, the following output regulator equations have unique solutions (X_i, U_i),
$$\begin{cases} A_iX_i + B_iU_i = X_iS, \\ C_iX_i = R, \quad i = 1, 2, \ldots, N. \end{cases} \tag{3}$$

Lemma 3: [30] For any vectors $d_1, \ldots, d_n$, the following inequality holds: $\| \sum_{i=1}^n d_i\|^2 \le n\sum_{i=1}^n \|d_i\|^2$.

Lemma 4: [12] Consider the system: $\dot{x} = \varepsilon Fx + F_1(t)x + F_2(t)$, where $x \in \mathbb{R}^n$, $F \in \mathbb{R}^{n\times n}$ is Hurwitz, $\varepsilon > 0$, $F_1(t) \in \mathbb{R}^{n\times n}$ and $F_2(t) \in \mathbb{R}^n$ are bounded and continuous for all $t \ge t_0$. We have, if $F_1(t), F_2(t) \to 0$ as $t \to \infty$ (exponentially), then, for any $x(t_0)$ and any $\varepsilon > 0$, $x(t) \to 0$ as $t \to \infty$ (exponentially).

III. ADAPTIVE DISTRIBUTED OBSERVER FOR OUTPUT REGULATOR EQUATIONS

In Lemma 2, the output regulator equations (3) have unique solutions (X_i, U_i) if Assumption 2 holds. Moreover, the unique solutions (X_i, U_i) of (3) are essential for the output consensus of heterogeneous multi-agent systems. However, it is required to know S and R beforehand to solve the regulator equations (3). At present, there are few research results about the unknown S and R. In this paper, it is investigated the case that S and R are unknown to the followers. Let $\hat{S}_i$ and $\hat{R}_i$ as the observed values of S and R for the ith follower, respectively. Design the distributed observers for S and R as follows:
$$\begin{cases} \dot{\hat{S}}_i = -\iota(\sum_{j=0}^{N} a_{ij}(\hat{S}_i - \hat{S}_j)), \\ \dot{\hat{R}}_i = -\iota(\sum_{j=0}^{N} a_{ij}(\hat{R}_i - \hat{R}_j)), \quad i = 1, 2, \ldots, N, \end{cases} \tag{4}$$
where $\iota > 0$ are the gains to be determined later, $S_0 = S$, $R_0 = R$, $S_i \in \mathbb{R}^{q\times q}$ and $R_i \in \mathbb{R}^{p\times q}$ $(i = 1, \ldots, N)$.

Lemma 5: Under Assumption 1, $\forall\ \iota > 0$, one has
$$\tilde{S}_i = \hat{S}_i - S \to 0, \quad \tilde{R}_i = \hat{R}_i - R \to 0, \ i = 1, 2, \ldots, N. \tag{5}$$

Proof: Define $\tilde{S} = (\tilde{S}_1^T, \tilde{S}_2^T, \ldots, \tilde{S}_N^T)^T$, $\tilde{R} = (\tilde{R}_1^T, \tilde{R}_2^T, \ldots, \tilde{R}_N^T)^T$. Then, the matrix form of (4) is written as follows:
$$\dot{\tilde{S}} = -\iota(H \otimes I_q)\tilde{S}, \quad \dot{\tilde{R}} = -\iota(H \otimes I_q)\tilde{R}. \tag{6}$$
Under Assumption 1, H is positive-definite. Therefore, $\forall\ \iota > 0$, $\tilde{S} \to 0$ and $\tilde{R} \to 0$, that is, $\hat{S}_i \to S$ and $\hat{R}_i \to R$. □

By using of the observers (4), we can estimate the unknown matrix R and S of the leader. Then, the followers can use $\hat{S}_i$ and $\hat{R}_i$ to solve the output regulator equations (3) adaptively. Let $(\hat{X}_i, \hat{U}_i)$ denote the estimator of the unique solutions (X_i, U_i).

Firstly, the output regulator equations (3) can be rewritten as the following form:
$$\begin{bmatrix} A_i & B_i \\ C_i & 0 \end{bmatrix}\begin{bmatrix} X_i \\ U_i \end{bmatrix} - \begin{bmatrix} I_{n_i} & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} X_i \\ U_i \end{bmatrix} S = \begin{bmatrix} 0 \\ R \end{bmatrix}. \tag{7}$$
Then, by using of Theorem 1.9 of [29], (7) can be transformed into the following form
$$\mathcal{Q}_i\mathcal{X}_i = \mathcal{R}, \tag{8}$$
where $\mathcal{Q}_i = I_q \otimes \begin{bmatrix} A_i & B_i \\ C_i & 0 \end{bmatrix} - S^T \otimes \begin{bmatrix} I_{n_i} & 0 \\ 0 & 0 \end{bmatrix}$, $\mathcal{X}_i = \mathrm{vec}(\begin{bmatrix} X_i \\ U_i \end{bmatrix})$, and $\mathcal{R} = \mathrm{vec}(\begin{bmatrix} 0 \\ R \end{bmatrix})$.

Then, by using Lemma 3 of [12], one gets the following lemma.

Lemma 6: Under Assumption 2 and 3, using the distributed observers (4), the following adaptive system
$$\dot{\hat{\mathcal{X}}}_i(t) = -\mu\hat{\mathcal{Q}}_i^T(\hat{\mathcal{Q}}_i\hat{\mathcal{X}}_i(t) - \hat{\mathcal{R}}_i) \tag{9}$$
has a unique bounded solution $\hat{\mathcal{X}}_i(t) = \mathrm{vec}(\begin{bmatrix} \hat{X}_i \\ \hat{U}_i \end{bmatrix})$ for $t \ge t_0$ and $\mu > 0$ such that, $\lim_{t\to\infty}(\hat{\mathcal{X}}_i(t) - \mathcal{X}_i) = 0$ exponentially. That is, $\tilde{X}_i(t) = \hat{X}_i(t) - X_i(t) \to 0$ and $\tilde{U}_i(t) = \hat{U}_i(t) - U_i(t) \to 0$, $i = 1, 2, \ldots, N$.

Define $A_s = I_N \otimes S - (c-1)H \otimes I_q$, $H_s = (I_{Nq} \otimes S) - \iota(H \otimes I_q) \otimes I_q$. Then, one has the following lemma.

Lemma 7: Under Assumption 1, A_s is stable if c is sufficiently large, and H_s is stable if ι is sufficiently large.

Proof: Under Assumption 1, all the eigenvalues of H are positive. The eigenvalues of A_s are

$\lambda_i(S)-(c-1)\lambda_i(H)$, $i=1,\ldots,q$, $j=1,\ldots,N$. Therefore, $\lambda_i(S)-(c-1)\lambda_i(H)$ have positive real parts when c is chosen sufficiently large, that is, A_s is stable. Similarly, H_s can also be stable.

Lemma 8: Under Assumption 3, for sufficiently large $\iota>0$, one gets

$$\tilde{S}_i x_0 \to 0, \quad \tilde{R}_i x_0 \to 0. \tag{10}$$

Proof: The sufficient conditions for (10) are $\tilde{S}\otimes x_0 \to 0$ and $\tilde{R}\otimes x_0 \to 0$. The derivative dynamics of $\tilde{S}\otimes x_0$ and $\tilde{R}\otimes x_0$ are listed as follows:

$$\begin{aligned}\frac{d(\tilde{S}\otimes x_0)}{dt} &= \dot{\tilde{S}}\otimes x_0+\tilde{S}\otimes\dot{x}_0\\ &= -\iota[(H\otimes I_q)\tilde{S}]\otimes(I_q x_0)\\ &\quad+(I_{Nq}\tilde{S})\otimes(Sx_0)\\ &= [(I_{Nq}\otimes S)-\iota(H\otimes I_q)\otimes I_q](\tilde{S}\otimes x_0)\\ &= H_s(\tilde{S}\otimes x_0).\end{aligned} \tag{11}$$

By using of Lemma 7, H_s is stable if $\iota>0$ is chosen sufficiently large. Then, one has $\tilde{S}\otimes x_0\to 0$. Similarly, one gets $\tilde{R}\otimes x_0\to 0$. □

Lemma 9: Under Assumption 3, for sufficiently large $\iota>0$, one gets

$$\tilde{X}_i x_0 \to 0, \quad \tilde{U}_i x_0 \to 0. \tag{12}$$

Proof: Let $\tilde{\mathcal{X}}_i=\hat{\mathcal{X}}_i(t)-\mathcal{X}_i$ and $\tilde{\mathcal{R}}_i=\hat{\mathcal{R}}_i(t)-\mathcal{R}$ represent the estimation error of $\mathcal{X}_i$ and $\mathcal{R}$. Then, one has

$$\begin{aligned}\dot{\tilde{\mathcal{X}}}_i &= \dot{\hat{\mathcal{X}}}_i-\dot{\mathcal{X}}_i=-\mu\hat{\mathcal{Q}}_i^T(\hat{\mathcal{Q}}_i\hat{\mathcal{X}}_i-\hat{\mathcal{R}}_i)\\ &= -\mu\mathcal{Q}_i^T(\mathcal{Q}_i\hat{\mathcal{X}}_i-\mathcal{R})-\mu(\tilde{S}_i^T\\ &\quad\otimes\begin{bmatrix} I_{n_i} & 0\\ 0 & 0\end{bmatrix})^T(\hat{\mathcal{Q}}_i\hat{\mathcal{X}}_i-\mathcal{R})\\ &\quad-\mu\mathcal{Q}_i^T\tilde{S}_i^T\otimes\begin{bmatrix} I_{n_i} & 0\\ 0 & 0\end{bmatrix})\hat{\mathcal{X}}_i+\mu\hat{\mathcal{Q}}_i^T\tilde{\mathcal{R}}_i\\ &= -\mu\mathcal{Q}_i^T\mathcal{Q}_i\tilde{\mathcal{X}}_i-\mu(\tilde{S}_i^T\otimes\begin{bmatrix} I_{n_i} & 0\\ 0 & 0\end{bmatrix})^T\tilde{\mathcal{R}}_i\\ &\quad-\iota\mathcal{Q}_i^T\tilde{S}_i^T\otimes\begin{bmatrix} I_{n_i} & 0\\ 0 & 0\end{bmatrix}\hat{\mathcal{X}}_i+\mu\hat{\mathcal{Q}}_i^T\tilde{\mathcal{R}}_i\\ &= -\mu\mathcal{Q}_i^T\mathcal{Q}_i\tilde{\mathcal{X}}_i-\mu\mathcal{Q}_i^T(\tilde{S}_i^T\otimes\begin{bmatrix} I_{n_i} & 0\\ 0 & 0\end{bmatrix})\tilde{\mathcal{X}}_i\\ &\quad-\mu\mathcal{Q}_i^T\tilde{S}_i^T\otimes\begin{bmatrix} I_{n_i} & 0\\ 0 & 0\end{bmatrix}\mathcal{X}_i+\iota\mathcal{Q}_i^T\tilde{\mathcal{R}}_i\\ &= \mu F_{1i}\tilde{\mathcal{X}}_i+F_{2i}(t)\tilde{\mathcal{X}}_i+F_{3i}(t)\end{aligned} \tag{13}$$

where $F_{1i}=-\mathcal{Q}_i^T\mathcal{Q}_i$ is Hurwitz, $F_{2i}(t)=-\mu\mathcal{Q}_i^T(\tilde{S}_i^T\otimes\begin{bmatrix} I_{n_i} & 0\\ 0 & 0\end{bmatrix})$, $F_{3i}(t)=-\mu\mathcal{Q}_i^T\tilde{S}_i^T\otimes\begin{bmatrix} I_{n_i} & 0\\ 0 & 0\end{bmatrix}\mathcal{X}_i+\mu\mathcal{Q}_i^T\tilde{\mathcal{R}}_i$. Moreover, for any $\iota>0$, there are $f_{2i}>0$ and $f_{3i}>0$ such that

$$\|F_{2i}\|\le f_{2i}e^{-\iota\lambda_1 t}, \quad \|F_{3i}\|\le f_{3i}e^{-\iota\lambda_1 t}, \tag{14}$$

where λ_1 is the minimum eigenvalue of H.

Let $\tilde{\mathscr{X}}_i=[\tilde{X}_i^T \;\; \tilde{U}_i^T]^T\in\mathbb{R}^{(n_i+m_i)\times q}$ of rank r. Then, there exists $\gamma_i>0$ such that

$$\|\tilde{\mathscr{X}}_i(t)\|_2\le\|\tilde{\mathscr{X}}_i(t)\|_F\le\sqrt{r}\|\tilde{\mathcal{X}}_i(t)\|_2\le\sqrt{r}\gamma_i e^{-\iota\lambda_1 t}. \tag{15}$$

In addition, owing to $\dot{x}_0=Sx_0$, one gets $\|x_0(t)\|_2\le e^{\sigma_{\max}(S)t}\|x_0(0)\|_2$ with $\sigma_{\max}(S)$ is the maximum singular values of S. Hence,

$$\begin{aligned}\|\tilde{X}_i x_0\|_2 &\le \sqrt{r}\sum_i^N\gamma_i\|x_0(0)\|_2 e^{-(\iota\lambda_1-\sigma_{\max}(S))t},\\ \|\tilde{U}_i x_0\|_2 &\le \sqrt{r}\sum_i^N\gamma_i\|x_0(0)\|_2 e^{-(\iota\lambda_1-\sigma_{\max}(S))t}.\end{aligned} \tag{16}$$

Therefore, for sufficiently large $\iota>0$, when $t\to\infty$, $\tilde{X}_i x_0\to 0$, $\tilde{U}_i x_0\to 0$. □

IV. MAIN RESULTS

In this section, we investigate the cooperative output regulation problem of heterogeneous linear multi-agent systems by designing event-triggered control scheme and self-triggered control scheme, and analyze the feasibility, that is, it is proved there is no Zeno behavior under the two triggering scheme. By using of $\hat{S}_i$ and $\hat{R}_i$, each follower can estimate the unknown state of the leader. Let $\hat{x}_i(t)\in\mathbb{R}^q$ represent the observer of the ith follower for the unknown leader $x_0(t)$ and $\check{x}_i(t)\in\mathbb{R}^{n_i}$ denote the estimator of $x_i(t)$. We design the triggering scheme depending on the measurement output feedback for the followers are designed as follows:

$$\begin{cases}\dot{\hat{x}}_i(t)=\hat{S}_i\hat{x}_i(t)+c(\sum_{j\in\mathcal{N}_i}(\hat{x}_j(t_i^k)-\hat{x}_i(t_i^k))\\ \qquad +a_{i0}(\hat{x}_i(t_i^k)-x_0(t_i^k))),\\ \dot{\check{x}}_i(t)=A_i\check{x}_i(t)+B_iu_i+L_i(\check{y}_i(t)-\hat{y}_i(t)),\\ u_i=K_{1i}\check{x}_i(t)+K_{2i}\hat{x}_i(t),\ t\in[t_i^k,t_i^{k+1})\end{cases} \tag{17}$$

where $\check{y}_i(t)=C_i\check{x}_i(t)$ and $\hat{y}_i(t)=\hat{R}_i\hat{x}_i(t)$, L_i, K_{1i}, K_{2i} are compatible matrices to be determined later, $c>0$ and t_i^k is the kth triggering time of the ith follower to be designed later.

Remark 2: In [12], [13], each observer estimates the leader's state continuously and the controller can use the followers' states directly. However, it is generally impossible to access the leader's and followers' states. Therefore, the proposed triggering scheme (17) can reduce the transfer times and meanwhile the measurement output feedback control with the distributed observer and estimator is more feasible.

Let $e_i(t)=\hat{x}_i(t)-x_0(t)$ and $\varepsilon_i(t)=\sum_{j\in\mathcal{N}_i}(\hat{x}_i(t)-\hat{x}_j(t))$ denote the local and global observation error, respectively. Define $\bar{e}_i(t)=e_i(t_i^k)-e_i(t)$ and $\bar{\varepsilon}_i(t)=\varepsilon_i(t_i^k)-\varepsilon_i(t)$. Then, one has $e_i(t_i^k)=\bar{e}_i(t)+e_i(t)$ and $\varepsilon_i(t_i^k)=\bar{\varepsilon}_i(t)+\varepsilon_i(t)=\bar{\varepsilon}_i(t)+\sum_{j\in\mathcal{N}_i}(\hat{x}_i(t)-\hat{x}_j(t))=\bar{\varepsilon}_i(t)+\sum_{j\in\mathcal{N}_i}(e_i(t)-e_j(t))$. The observation error system is shown as follows:

$$\begin{aligned}\dot{e}_i(t) &= \dot{\hat{x}}_i(t)-\dot{x}_0(t)\\ &= Se_i(t)+\tilde{S}_i x_0(t)+\tilde{S}_i e_i(t)\\ &\quad -c(\sum_{j\in\mathcal{N}_i}(e_i(t)-e_j(t))+a_{i0}e_i(t))\\ &\quad -c(\bar{\varepsilon}_i(t)+a_{i0}\bar{e}_i(t)).\end{aligned} \tag{18}$$

Let $e(t)=(e_1^T,\ldots,e_N^T)^T$, $\varepsilon(t)=(\varepsilon_1^T,\ldots,\varepsilon_N^T)^T$, $\bar{e}(t)=(\bar{e}_1^T,\ldots,\bar{e}_N^T)^T$, $\bar{\varepsilon}(t)=(\bar{\varepsilon}_1^T,\ldots,\bar{\varepsilon}_N^T)^T$, $\xi_i(t)=\tilde{S}_i x_0(t)$ and

$\xi(t) = (\xi_1^T, \ldots, \xi_N^T)^T$. Then, (18) can be rewritten in the matrix form as follows:

$$\begin{aligned}\dot{e}(t) &= (I_N \otimes S)e(t) + \xi(t) + \mathrm{diag}(\tilde{S}_i)e(t) \\ &\quad - c(H \otimes I_q)e(t) - c(\bar{\varepsilon}(t) + (\Delta \otimes I_q)\bar{e}(t)) \\ &= (I_N \otimes S - cH \otimes I_q)e(t) + \mathrm{diag}(\tilde{S}_i)e(t) \\ &\quad + \xi(t) - c(\bar{\varepsilon}(t) + (\Delta \otimes I_q)\bar{e}(t)).\end{aligned} \tag{19}$$

A. EVENT-TRIGGERED CONTROL

We first consider the event-triggered control scheme. The triggering time instants of agent i with $t_i^0 = 0$ is generated by the following event-triggered mechanism:

$$t_i^{k+1} = \max_t\{t \geq t_i^k | h(p_i(t), q_i(t)) = p_i(t) - \beta_i q_i(t) \leq 0\}, \tag{20}$$

where $p_i(t) = \sqrt{\|\bar{\varepsilon}_i(t)\|^2 + a_{i0}^2\|\bar{e}_i(t)\|^2}$, $q_i(t) = \|\varepsilon_i(t)\| + a_{i0}\|e_i(t)\|$, $\beta_i^2 = \beta^2/(2|\mathcal{N}_i|)$, $\lambda_1 = \min_i\{\lambda_i(H)\}$, $|\mathcal{N}| = \max_i\{|\mathcal{N}_i|\}$, $\beta = \lambda_1/\sqrt{4c(2c+1)(4|\mathcal{N}|+1)}$.

Theorem 1: Under Assumption 1-4, if $\iota, \mu, c > 0$ are chosen sufficiently large, Problem 1 can be solved under the measurement output feedback controller (17) with the triggering mechanism (20) if K_{1i} and L_i are chosen suitably such that $A_i + B_iK_{1i}$ and $A_i + L_iC_i$ ($i = 1, \ldots, N$) are stable and K_{2i} is designed as $K_{2i} = \hat{U}_i - K_{1i}\hat{X}_i$.

Proof: We first prove the observation error could converge to zero. Consider the following Lyapunov function:

$$V(t) = \frac{1}{2}e^T(t)e(t). \tag{21}$$

The derivative of $V(t)$ along (19) is

$$\begin{aligned}\dot{V}(t) &= e^T(t)((I_N \otimes S - cH \otimes I_q)e(t) + \mathrm{diag}(\tilde{S}_i)e(t) \\ &\quad + \xi(t) - c(\bar{\varepsilon}(t) + (\Delta \otimes I_q)\bar{e}(t))) \\ &= e^T(t)(I_N \otimes S - cH \otimes I_q)e(t) \\ &\quad + e^T(t)\mathrm{diag}(\tilde{S}_i)e(t) + e^T(t)\xi(t) - ce^T(t)(\bar{\varepsilon}(t) \\ &\quad + (\Delta \otimes I_q)\bar{e}(t)).\end{aligned} \tag{22}$$

Then by the inequality $2a^Tb \leq \kappa\|a\|^2 + \frac{1}{\kappa}\|b\|^2$ for any $\kappa > 0$, $a, b \in \mathbb{R}^n$, one gets

$$\begin{aligned}\dot{V}(t) &\leq e^T(t)(I_N \otimes S - cH \otimes I_q + \frac{2c+1}{\kappa}I_{Nq})e(t) \\ &\quad + e^T(t)\mathrm{diag}(\tilde{S}_i)e(t) + \kappa\xi^T(t)\xi(t)\end{aligned} \tag{23}$$

By using Lemma 3, one has

$$\begin{aligned}\|\varepsilon_i(t)\|^2 &= \| \textstyle\sum_{j\in\mathcal{N}_i}(e_i(t) - e_j(t))\|^2 \\ &\leq |\mathcal{N}_i| \| \sum_{j\in\mathcal{N}_i}(\|e_i\| + \|e_j(t)\|)^2 \\ &\leq 2|\mathcal{N}_i| \| \sum_{j\in\mathcal{N}_i}(\|e_i\|^2 + \|e_j(t)\|^2).\end{aligned} \tag{24}$$

According to the triggering mechanism, one achieves

$$\begin{aligned}p_i(t)^2 &\leq \beta_i^2(\|\varepsilon_i(t)\| + a_{i0}\|e_i(t)\|)^2 \\ &\leq \beta^2(2\sum_{j\in\mathcal{N}_i}(\|e_i\|^2 + \|e_j(t)\|^2) + a_{i0}^2\|e_i(t)\|^2).\end{aligned} \tag{25}$$

Then, substituting (25) in (23), we have

$$\begin{aligned}\dot{V}(t) &\leq e^T(t)(I_N \otimes S - cH \otimes I_q + \frac{2c+1}{\kappa}I_{Nq})e(t) \\ &\quad + e^T(t)\mathrm{diag}(\tilde{S}_i)e(t) + \kappa\xi^T(t)\xi(t) \\ &\quad + \kappa c\beta^2\sum_{i=1}^N(4\|\mathcal{N}_i\| + a_{i0}^2)\|\bar{e}_i(t)\|^2 \\ &\leq e^T(t)(I_N \otimes S - cH \otimes I_q + \frac{2c+1}{\kappa}I_{Nq})e(t) \\ &\quad + e^T(t)\mathrm{diag}(\tilde{S}_i)e(t) + \kappa\xi^T(t)\xi(t) \\ &\quad + \kappa c\beta^2(4\|\mathcal{N}\| + 1)\sum_{i=1}^N\|e_i(t)\|^2.\end{aligned} \tag{26}$$

Since κ could be chosen arbitrarily, let $\kappa = \frac{2(2c+1)}{\lambda_1}$. Moreover, owing to $\beta = \lambda_1/\sqrt{4c(2c+1)(4|\mathcal{N}|+1)}$, one has

$$\begin{aligned}\dot{V}(t) &\leq e^T(t)(I_N \otimes S - (c-1)H \otimes I_q)e(t) \\ &\quad + e^T(t)\mathrm{diag}(\tilde{S}_i)e(t) + \kappa\xi^T(t)\xi(t) \\ &= e^T(t)A_se(t) + e^T(t)\mathrm{diag}(\tilde{S}_i)e(t) \\ &\quad + \kappa\xi^T(t)\xi(t).\end{aligned} \tag{27}$$

According to Lemma 5 and Lemma 8, one has $\mathrm{diag}(\tilde{S}_i) \to 0$ and $\xi^T(t)\xi(t) \to 0$. In addition, A_s is Hurwitz if c is chosen sufficiently. Then, by using Lemma 4, one can conclude that $V(t) \to 0$ (exponentially) as $t \to 0$. Therefore, one can have $e(t) \to 0$ as $t \to 0$, that is, $\hat{x}_i(t) \to x_0(t)$. One can further obtain that: $\lim_{t\to\infty}\varepsilon_i(t) \to 0$, $\lim_{t\to\infty}\bar{\varepsilon}_i(t) \to 0$ and $\lim_{t\to\infty}\bar{e}_i(t) \to 0$.

Define the state tracking error and estimation error of the followers as $\delta_i(t) = \check{x}_i(t) - \hat{X}_i\hat{x}_i(t)$ and $\vartheta_i(t) = \check{x}_i(t) - x_i(t)$, and the output error $\zeta_i(t) = y_i(t) - y_0(t)$. Then, under the controller (17), for $i = 1, \ldots, N$, one gets

$$\begin{aligned}\dot{\delta}_i(t) &= \dot{\check{x}}_i(t) - \hat{X}_i\dot{\hat{x}}_i(t) \\ &= (A_i + B_iK_{1i})\check{x}_i(t) + B_i(\hat{U}_i - K_{1i}\hat{X}_i)\hat{x}_i(t) \\ &\quad + L_iC_i(\check{x}_i(t) - \hat{X}_i\hat{x}_i(t)) \\ &\quad - L_i\tilde{R}_i\hat{x}_i - \hat{X}_i\hat{S}_i\hat{x}_i(t) + c\hat{X}_i(\sum_{j\in\mathcal{N}_i}(e_i(t) \\ &\quad - e_j(t)) + a_{i0}e_i(t) + \bar{\varepsilon}_i(t) + a_{i0}\bar{e}_i(t))\end{aligned} \tag{28}$$

where $\psi_i(t) = (L_i\tilde{R}_i + \hat{X}_i\tilde{S}_i - A_i\tilde{X}_i - B_i\tilde{U}_i)(x_0(t) + e_i(t)) + c\hat{X}_i(\sum_{j\in\mathcal{N}_i}(e_i(t) - e_j(t)) + a_{i0}e_i(t) + \bar{\varepsilon}_i(t) + a_{i0}\bar{e}_i(t))$. Combining with Lemma 8 and 9, one can further obtain $\psi_i(t) \to 0$ as $t \to \infty$. Then, one can get $\delta_i(t) \to 0$ as $t \to \infty$.

$$\begin{aligned}\dot{\vartheta}_i(t) &= \dot{\check{x}}_i - \dot{x}_i \\ &= A_i\check{x}_i + B_iu_i + L_i(C_i\check{x}_i - \check{R}_i\hat{x}_i) \\ &\quad - A_ix_i - B_iu_i \\ &= A_i\vartheta_i + L_i(C_i\check{x}_i - \check{R}_i\hat{x}_i) \\ &= (A_i + L_iC_i)\vartheta_i + L_iC_i(\check{x}_i - \hat{X}_i\hat{x}_i) \\ &\quad - L_i\tilde{R}_i(x_0(t) + e_i(t)).\end{aligned} \tag{29}$$

Since $A_i + L_iC_i$ is table and $\check{x}_i - \hat{X}_i\hat{x}_i \to 0$, one has $\vartheta_i(t) \to 0$.

$$\begin{aligned}\zeta_i(t) &= y_i(t) - y_0(t) = C_ix_i(t) - Rx_0(t) \\ &= C_i((x_i(t) - \check{x}_i) + (\check{x}_i - \hat{X}_i\hat{x}_i) \\ &\quad + \hat{X}_i(\hat{x}_i - x_0(t))) \\ &= C_i(-\vartheta_i(t) + \delta_i(t) + \hat{X}_ie_i(t)).\end{aligned} \tag{30}$$

Owing to $\vartheta_i(t) \to 0$, $\delta_i(t) \to 0$ and $e_i(t) \to 0$, one gets $\zeta_i(t) \to 0$. That is, Problem 1 has been solved. □

B. SELF-TRIGGERED CONTROL

Under the triggering mechanism (20), it is required to continuously determine whether trigger conditions are established. To avoid the continuous operation, we design a novel self-triggered control in which the next triggering time t_i^{k+1} only depends on the local information at the time t_i^k. The triggering condition is shown as follows:

$$p_i(t) \le \bar{\beta}_i q_i(t_i^k) = \varrho_i(k), \tag{31}$$

where the right side of inequality only depends on the local information at the time t_i^k.

Lemma 10: If $\bar{\beta}_i = \dfrac{\beta_i}{\sqrt{2+4\beta_i^2}}$, (31) can guarantee $h(p_i(t), q_i(t)) \le 0$.

Proof: From (31), after simple calculation, one has

$$\begin{aligned} p_i^2(t) &\le \bar{\beta}_i^2 q_i^2(t_i^k) = \bar{\beta}_i^2(\|\varepsilon_i(t_i^k)\| + a_{i0}\|e_i(t_i^k)\|)^2 \\ &= \bar{\beta}_i^2(\|\bar{\varepsilon}_i(t) + \varepsilon_i(t)\| + a_{i0}\|\bar{e}_i(t) + e_i(t)\|)^2 \\ &\le \bar{\beta}_i^2(\|\bar{\varepsilon}_i(t)\| + \|\varepsilon_i(t)\| + a_{i0}\|\bar{e}_i(t)\| \\ &\quad + a_{i0}\|e_i(t)\|)^2 \\ &= \bar{\beta}_i^2(q_i(t) + (\|\bar{\varepsilon}_i(t)\| + a_{i0}\|\bar{e}_i(t)\|))^2 \\ &\le 2\bar{\beta}_i^2(q_i^2(t) + (\|\bar{\varepsilon}_i(t)\| + a_{i0}\|\bar{e}_i(t)\|)^2) \\ &\le 2\bar{\beta}_i^2(q_i^2(t) + 2(\|\bar{\varepsilon}_i(t)\|^2 + a_{i0}^2\|\bar{e}_i(t)\|^2)) \\ &= 2\bar{\beta}_i^2(q_i^2(t) + 2p_i^2(t)), \end{aligned} \tag{32}$$

which yields $p_i^2(t) \le \dfrac{2\bar{\beta}_i^2}{1-4\bar{\beta}_i^2} q_i^2(t) = \beta_i^2 q_i^2(t)$, that is, $\bar{\beta}_i = \dfrac{\beta_i}{\sqrt{2+4\beta_i^2}}$. □

Next, we will calculate the derivatives of $\|\bar{\varepsilon}_i(t)\|$ and $\|\bar{e}_i(t)\|$.

$$\begin{aligned} \frac{d}{dt}\|\bar{\varepsilon}_i(t)\| &\le \|\dot{\varepsilon}_i(t)\| = \|\sum_{j\in\mathcal{N}_i}(\dot{\hat{x}}_i(t) - \dot{\hat{x}}_j(t))\| \\ &\le \|S\|\|\bar{\varepsilon}_i(t)\| + \|\sum_{j\in\mathcal{N}_i}(\tilde{S}_i\hat{x}_i - \tilde{S}_j\hat{x}_j)\| \\ &\quad + \|S\varepsilon_i(t_i^k) - c(\varepsilon_i(t_i^k) - \varepsilon_i(t_j^{k_j'}) \\ &\quad + a_{i0}(e_i(t_i^k) - e_j(t_j^{k_j'})))]\| \\ &= \|S\|\|\bar{\varepsilon}_i(t)\| + \omega_i(t) + \theta_i^k(t). \end{aligned} \tag{33}$$

where $k_j' = \arg\max_{k\in\mathbb{Z}}\{t_j^k | t_j^k \le t\}$, $\omega_i(t) = \|\sum_{j\in\mathcal{N}_i}(\tilde{S}_i\hat{x}_i - \tilde{S}_j\hat{x}_j)\| \to 0$ as $t \to \infty$, $\theta_i^k(t) = \|S\varepsilon_i(t_i^k) - c(\varepsilon_i(t_i^k) - \varepsilon_i(t_j^{k_j'}) + a_{i0}(e_i(t_i^k) - e_j(t_j^{k_j'})))]\|$ is a constant as $t \in [t_i^k, t_i^{k+1})$. Similarly,

$$\begin{aligned} \frac{d}{dt}\|\bar{e}_i(t)\| &\le \|\dot{e}_i(t)\| = \|\hat{S}_i\hat{x}_i(t) - Sx_0(t) \\ &\quad - c(\varepsilon_i(t_i^k) + a_{i0}e_i(t_i^k))\| \\ &\le \|S\|\|\bar{e}_i(t)\| + \|\tilde{S}_ix_0(t) + \tilde{S}_ie_i(t)\| \\ &\quad + \|Se_i(t_i^k) - c(\varepsilon_i(t_i^k) + a_{i0}e_i(t_i^k))\| \\ &= \|S\|\|\bar{e}_i(t)\| + \phi_i(t) + \varphi_i^k(t). \end{aligned} \tag{34}$$

where $\phi_i(t) = \|\tilde{S}_ix_0(t) + \tilde{S}_ie_i(t)\| \to 0$ as $t \to \infty$, $\varphi_i^k(t) = \|Se_i(t_i^k) - c(\varepsilon_i(t_i^k) + a_{i0}e_i(t_i^k))\|$ is a constant as $t \in [t_i^k, t_i^{k+1})$. Then, one can calculate the derivative of $p_i^2(t)$ as $t \to \infty$:

$$\begin{aligned} \frac{d}{dt}p_i^2(t) &= \frac{d}{dt}(\|\bar{\varepsilon}_i(t)\|^2 + a_{i0}^2\|\bar{e}_i(t)\|^2) \\ &= 2\|\bar{\varepsilon}_i(t)\|\frac{d}{dt}\|\bar{\varepsilon}_i(t)\| + 2a_{i0}^2\|\bar{e}_i(t)\|\frac{d}{dt}\|\bar{e}_i(t)\| \\ &\le 2\|\bar{\varepsilon}_i(t)\|(\|S\|\|\bar{\varepsilon}_i(t)\| + \theta_i^k(t)) \\ &\quad + 2a_{i0}^2\|\bar{e}_i(t)\|(\|S\|\|\bar{e}_i(t)\| + \varphi_i^k(t)) \\ &= 2\|S\|(\|\bar{\varepsilon}_i(t)\|^2 + a_{i0}^2\|\bar{e}_i(t)\|^2) \\ &\quad + 2\theta_i^k(t)\|\bar{\varepsilon}_i(t)\| + 2a_{i0}^2\varphi_i^k(t)\|\bar{e}_i(t)\| \\ &\le 2\|S\|p_i^2(t) + 2\sqrt{2}\alpha_i^k p_i(t) \\ &\le 2\|S\|\varrho_i^2(k) + 2\sqrt{2}\alpha_i^k(t)\varrho_i(k), \end{aligned} \tag{35}$$

where $\alpha_i^k(t) = \max\{\theta_i^k(t), a_{i0}\varphi_i^k(t)\}$ only depends on the time t_i^k as $t \in [t_i^k, t_i^{k+1})$. Let $2\|S\|\varrho_i^2(k) + 2\sqrt{2}\alpha_i^k\varrho_i(k) = \Psi_i^k(t)$ as $t \in [t_i^k, t_i^{k+1})$ and the right side of (35) only depends on the time t_i^k as $t \in [t_i^k, t_i^{k+1})$.

Then, one can has the following result.

Theorem 2: Suppose the conditions of Theorem 4.1 are satisfied. The triggering condition is $p_i(t) \le \bar{\beta}_i q_i(t_i^k) = \varrho_i(k)$ where $\bar{\beta}_i = \beta_i/\sqrt{2+4\beta_i^2}$, and the triggering time series $\{t_i^k\}$ are generated by the following self-triggering scheme:

Step 1: Initialization: $t_i^0 = 0$ for all $i = 1, \ldots, N$;

Step 2: For node i, set $t_0' = t_i^k$ and calculate $t' = t_0' + \varrho_i^2(k)/\Psi_i^k(t_0')$, if there is no neighbor triggered before t', then set $t_i^{k+1} = t'$, otherwise, turn to Step 3;

Step 3: Let t_1' be the first triggered time before t', then recalculate $t' = t_1' + (\varrho_i^2(k) - \Psi_i^k(t_0')(t_1' - t_0'))/\Psi_i^k(t_1')$; if there is another neighbor triggered at t_2' before t', then recalculate $t' = t_2' + (\varrho_i^2(k) - \Psi_i^k(t_0')(t_1' - t_0') - \Psi_i^k(t_1')(t_2' - t_1'))/\Psi_i^k(t_2')$; this iterative updating process lasts until there is no triggering before the latest t', then set $t_i^{k+1} = t'$.

Then, the cooperative output regulation problem could be solved.

Proof: According to Lemma 10, when $\bar{\beta}_i = \beta_i/\sqrt{2+4\beta_i^2}$, the self-triggering condition $p_i(t) \le \bar{\beta}_i q_i(t_i^k) = \varrho_i(k)$ can guarantee the event-triggering condition $h(p_i(t), q_i(t)) \le 0$. The triggering time series $\{t_i^k\}$ are generated based on the self-triggering condition. Therefore, the output of node i can track the output of the leader if the information of its neighbors and the leader is sampled at the triggering time t_i^k. The proof is similar to Theorem 1, thus it is omitted. □

Remark 3: For node i, suppose $t_1', \ldots, t_l'$ with $t_1' < t_2' < \ldots < t_l'$ are the triggering times between t_i^k and t_i^{k+1} of the neighbors, and it is assumed there is only one node triggered at each triggering time. By means of the above scheme, one can have the triggering time series $\{t_i^k\}$ which are only depending on the latest triggering times of their neighbors. The right side of the self-triggering condition is only dependent on the self-triggering time t_i^k, which can avoid the continuous verification about the triggering condition.

For the event-triggered control and the self-triggered control, the main difficulty is to prove Zeno behavior can be excluded. From the self-triggering scheme, $t_i^{k+1} - t_i^k$ is shorter under the self-triggering control than under the event-triggering control. Therefore, it is only proved there is no Zeno behavior under the self-triggered control, that is, the interevent interval of every node is strictly positive and is lower bounded by a common constant under the self-triggered control.

C. FEASIBILITY ANALYSIS

From Lemma 10, the self-triggering condition $p_i(t) \leq \bar{\beta}_i q_i(t_i^k) = \varrho_i(k)$ can guarantee the event-triggering condition $h(p_i(t), q_i(t)) \leq 0$ if $\bar{\beta}_i = \beta_i/\sqrt{2+4\beta_i^2}$. Moreover, according to the self-triggering scheme, each node is triggered more frequently, that is, $t_i^{k+1} - t_i^k$ is shorter than under the event-triggering control. Therefore, it is just proved Zeno behavior can be excluded under the self-triggering scheme.

Theorem 3: Under the self-triggering condition $p_i(t) \leq \bar{\beta}_i q_i(t_i^k) = \varrho_i(k)$ with $\bar{\beta}_i = \beta_i/\sqrt{2+4\beta_i^2}$, Zeno behavior can be excluded under the self-triggering scheme.

Proof: Suppose t_i^k is the kth triggering time of node i. To prove there is no Zeno behavior, we consider two cases $q_i(t_i^k) > 0$ and $q_i(t_i^k) = 0$.

Case (1): $q_i(t_i^k) > 0, t_i^{k+1} < \infty$. Then, one has $\varrho_i(k) = \bar{\beta}_i q_i(t_i^k) > 0$. It can be proven $q_i(t) > 0$ for $t \in [t_i^k, t_i^{k+1}]$ from the following equations:

$$\begin{aligned} |q_i(t_i^k) - q_i(t)|^2 &= |\|\varepsilon_i(t_i^k)\| + a_{i0}\|e_i(t_i^k)\| \\ &\quad - (\|\varepsilon_i(t)\| + a_{i0}\|e_i(t)\|)|^2 \\ &\leq |\|\varepsilon_i(t_i^k) - \varepsilon_i(t)\| \\ &\quad + a_{i0}\|e_i(t_i^k) - e_i(t)\||^2 \\ &= |\|\bar{\varepsilon}_i(t)\| + a_{i0}\|\bar{e}_i(t)\||^2 \\ &\leq 2(\|\bar{\varepsilon}_i(t)\|^2 + a_{i0}^2\|\bar{e}_i(t)\|^2) \\ &= 2p_i^2(t) \leq 2\bar{\beta}_i^2 q_i^2(t_i^k), \end{aligned} \tag{36}$$

which results in

$$(1 - \sqrt{2}\bar{\beta}_i)q_i(t_i^k) \leq q_i(t) \leq (1 + \sqrt{2}\bar{\beta}_i)q_i(t_i^k). \tag{37}$$

where $1 - \sqrt{2}\bar{\beta}_i > 0$ since $\bar{\beta}_i = \beta_i/\sqrt{2+4\beta_i^2} < 1/2 < 1/\sqrt{2}$. Then, $q_i(t) \geq (1 - \sqrt{2}\beta_i)q_i(t_i^k) > 0$ for $t \in [t_i^k, t_i^{k+1}]$.

Thus, $q_i(t_i^k) > 0$ could deduce that $\varrho_i(k)$ is strictly positive. By using of (37), one has

$$\frac{d}{dt}p_i^2(t) \leq 2\|S\|\varrho_i^2(k) + 2\sqrt{2}\alpha_i^k(t)\varrho_i(k), \tag{38}$$

where $\alpha_i^k(t) = \max\{\theta_i^k(t), a_{i0}\varphi_i^k(t)\}$ only depends on the time t_i^k as $t \in [t_i^k, t_i^{k+1})$.

It can be calculated by using of (35) that

$$t_i^{k+1} - t_i^k \geq \frac{\varrho_i^2(k)}{2\|S\|\varrho_i^2(k) + 2\sqrt{2}\alpha_i^k(t)\varrho_i(k)} > 0. \tag{39}$$

Case (2): $\lim_{k\to\infty} q_i(t_i^k) = 0$. According to (37), one can obtain $\lim_{t\to\infty} q_i(t) = 0$ and $0 < 1 - \sqrt{2}\bar{\beta}_i \leq \frac{q_i(t)}{q_i(t_i^k)} \leq 1 + \sqrt{2}\bar{\beta}_i$. Furthermore, one has $\lim_{t\to\infty} \varepsilon_i(t) = 0$ and $\lim_{t\to\infty} e_i(t) = 0$ if $a_{i0} \neq 0$ which can lead to $\lim_{t\to\infty} \dot{\varepsilon}_i(t) = 0$ and $\lim_{t\to\infty} \dot{e}_i(t) = 0$. Then, one can get (40), as shown at the bottom of this page.

And

$$\begin{aligned} t_i^{k+1} - t_i^k &\geq \frac{\bar{\beta}_i^2 q_i^2(t_i^k)}{2\bar{\beta}_i^2\|S\|q_i^2(t_i^k) + 2\sqrt{2}\bar{\beta}_i\alpha_i^k q_i(t_i^k)} \\ &\geq \frac{\bar{\beta}_i}{2\bar{\beta}_i\|S\| + 2\sqrt{2}(2+\sqrt{2}\bar{\beta}_i)\|S\|} \\ &= \frac{\bar{\beta}_i}{2\|S\|(4\bar{\beta}_i + 2\sqrt{2})} > 0. \end{aligned} \tag{41}$$

It can be concluded that the interevent internal is strictly positive and is lower bounded by a common constant. □

V. AN ILLUSTRATIVE EXAMPLE

In this section, some numerical simulation examples are provided to verify the effectiveness of the proposed triggering approaches.

Consider a directed network concluding a class of four followers and a leader. The communication network topology is depicted in Figure 1, where the node 0 is regarded as the leader and the others are the followers. It is obvious that Assumption 1 is satisfied. The minimum eigenvalue λ_1 of H is 0.3820.

The followers' systems are listed as follows:

$$\begin{cases} \dot{x}_i = A_i x_i + B_i u_i, \\ y_i = C_i x_i, \end{cases} \tag{42}$$

$$\begin{aligned} \lim_{t\to\infty} \frac{\alpha_i^k(t)}{q_i(t_i^k)} &\leq \lim_{t\to\infty} \frac{\theta_i^k(t) + a_{i0}\varphi_i^k(t)}{q_i(t_i^k)} \\ &\leq \lim_{t\to\infty} \frac{\|S\varepsilon_i(t_i^k) - S\varepsilon_i(t)\| + a_{i0}\|Se_i(t_i^k) - Se_i(t)\|}{q_i(t_i^k)} \\ &\leq \lim_{t\to\infty} \frac{\|S\|(\|\varepsilon_i(t_i^k)\| + \|\varepsilon_i(t)\| + a_{i0}(\|e_i(t_i^k)\| + \|e_i(t)\|))}{q_i(t_i^k)} \\ &= \lim_{t\to\infty} \frac{\|S\|(q_i(t_i^k) + q_i(t))}{q_i(t_i^k)} \leq \lim_{t\to\infty} \|S\|(1 + \frac{q_i(t)}{q_i(t_i^k)}) \leq (2 + \sqrt{2}\bar{\beta}_i)\|S\| \end{aligned} \tag{40}$$

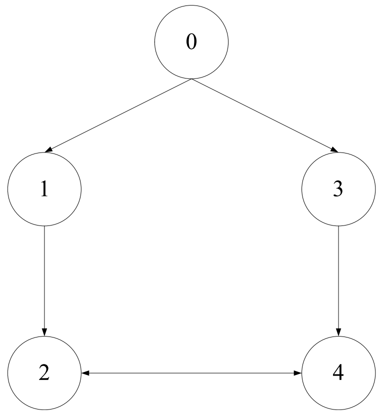

FIGURE 1. Communication network topology.

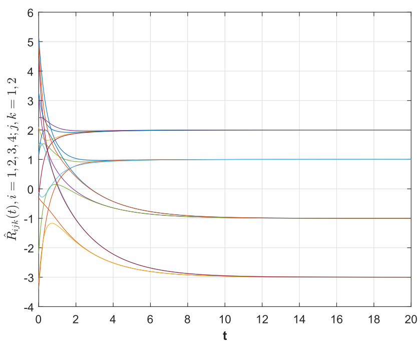

FIGURE 2. Estimated elements of leader's unknown R.

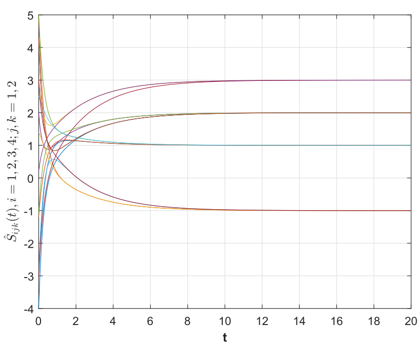

FIGURE 3. Estimated elements of leader's unknown S.

where

$$A_1 = A_3 = \begin{bmatrix} 0.3 & -2 \\ 0.1 & -0.2 \end{bmatrix}, \quad A_2 = A_4 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & -2 \end{bmatrix},$$

$$B_1 = B_3 = \begin{bmatrix} 1.8 & -0.8 \\ 0.9 & 1.6 \end{bmatrix}, \quad B_2 = B_4 = \begin{bmatrix} 6 & 0 \\ 0 & 1 \\ 1 & 0 \end{bmatrix},$$

$$C_1 = C_3 = \begin{bmatrix} -0.1 & 1.2 \\ 0.4 & 1.4 \end{bmatrix}, \quad C_2 = C_4 = \begin{bmatrix} 1 & -1 & 1 \\ -1 & -1 & 1 \end{bmatrix}.$$

The leader's system is written as follows:

$$\begin{cases} \dot{x}_0 = Sx_0, \\ y_0 = Rx_0 \end{cases} \tag{43}$$

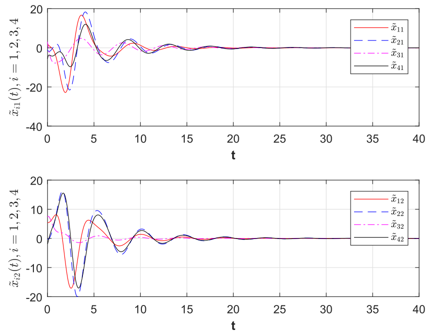

FIGURE 4. The leader's estimated errors for the followers.

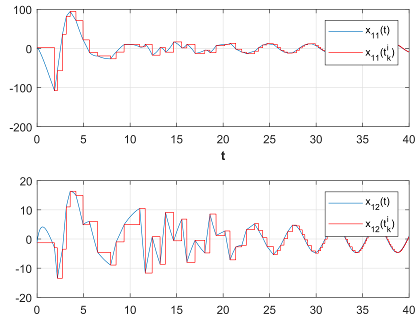

FIGURE 5. The trajectories of $x_1(t)$ and $x_1(t_k^i)$.

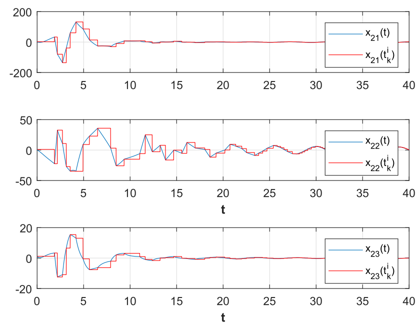

FIGURE 6. The trajectories of $x_2(t)$ and $x_2(t_k^i)$.

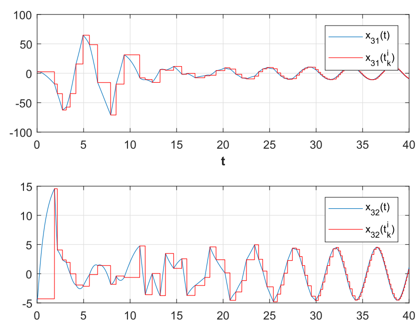

FIGURE 7. The trajectories of $x_3(t)$ and $x_3(t_k^i)$.

where

$$R = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad S = \begin{bmatrix} 1 & -3 \\ 1 & -1 \end{bmatrix}.$$

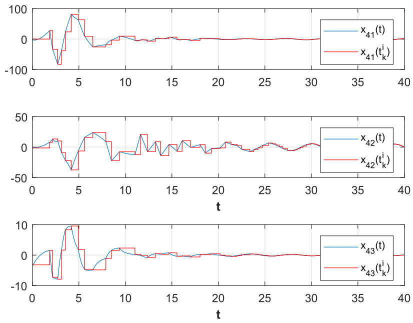

FIGURE 8. The trajectories of $x_4(t)$ and $x_4(t_k^i)$.

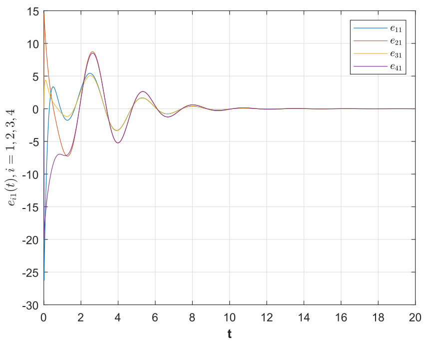

FIGURE 9. The first components of the output errors.

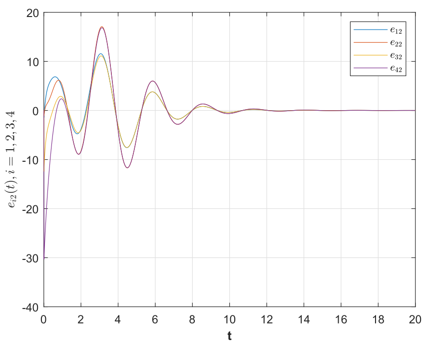

FIGURE 10. The second components of the output errors.

Let

$$K_{11} = K_{13} = \begin{bmatrix} -0.6 & 0.27 \\ 0.28 & -1.9 \end{bmatrix},$$

$$K_{12} = K_{14} = \begin{bmatrix} -0.21 & -0.05 & 0 \\ 0.70 & -2.72 & -0.97 \end{bmatrix},$$

$$L_1 = L_3 = \begin{bmatrix} -0.16 & -6.017 \\ 0.08 & -0.9 \end{bmatrix},$$

$$L_2 = L_4 = \begin{bmatrix} -0.78 & 1.69 \\ 0.14 & 1.16 \\ -0.02 & -0.21 \end{bmatrix}.$$

Then, it is easily verified that $A_i + B_i K_{1i}$ and $A_i + L_i C_i$ $(i = 1, \ldots, N)$ are stable. Figure 2 and Figure 3 show the estimated elements of the leader's unknown matrices R and S which could construct the estimated matrices $\tilde{R}_i$ and $\tilde{S}_i$. Figure 4 shows the leader's estimated errors under the estimated matrices $\tilde{R}_i$ and $\tilde{S}_i$. Figure 5-8 illustrate the trajectories of the followers $x_i(t)$, $i = 1, \ldots, 4$ and their sampled states $x_i(t_k^i)$ under the event-triggered controller (17) with $K_1 = K_3 = \begin{bmatrix} -0.6 & 0.27 \\ 0.28 & -1.9 \end{bmatrix}$, $K_2 = K_4 = \begin{bmatrix} -0.21 & -0.05 & 0 \\ 0.70 & -2.72 & -0.97 \end{bmatrix}$. The output error $e_i(t) = y_i(t) - y_0(t) = [e_{i1}(t) e_{i2}(t)]'$ under the sampled feedback-state controller (17) is shown in Figure 9-10.

VI. CONCLUSION

In this paper, we have studied the cooperative output regulation for heterogeneous linear multi-agent systems with an unknown leader. A local adaptive observer and an adaptive estimator are proposed to estimate the unknown matrices and dynamic behavior of the leader. Two event-triggered controllers which can reduce the number of information transmission are designed to assure the output error can converge to zero. Some numerical simulation examples can verify the effectiveness of the obtained theoretical results. For future work, we will focus on the cooperative output regulation for heterogeneous nonlinear multi-agent systems under event-triggered control.

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