IEEE Access
Multidisciplinary : Rapid Review : Open Access Journal

Received November 14, 2019, accepted November 28, 2019, date of current version December 17, 2019.

Digital Object Identifier 10.1109/ACCESS.2019.2957402

An Efficient Quadratic Constrained Least Squares Localization Method for Narrow Space With Ranging Measurement

YANJUN HU, XUEMIAO YANG, CHUNYU YANG, (Member, IEEE), AND SHIYIN LI
School of Information and Control Engineering, China University of Mining and Technology, Xuzhou 221116, China

Corresponding author: Shiyin Li (lishiyin@cumt.edu.cn)

This work was supported in part by the Natural Science Foundation of China under Grant 61771474.

ABSTRACT The localization algorithm for mobile robots working in narrow space needs to handle the scenario that the geometric shape of reference nodes tends to a line, which results in the matrix of least squares localization approaches ill-conditioned. Estimator bias becomes an important factor that can degrade the localization performance. In this paper, we present a fast unbiased range-based localization algorithm to resist the ill-conditioned problem. The main strategy is to augment objective function in the resultant optimization formulations via introducing a measurement distance into the locating model, which forms a least squares problem with cone constrained. The proposed model decouples the measurement distances from the matrix of least squares, which avoids the ill-conditioned problem when the target is around the geometric center. The closed-form expression of locating position ensures that the proposed algorithm is unbiased and low computation burden in the presence of zero-mean disturbance. Moreover, the robustness improvement of the augmented objective function is analyzed. Numerical simulations are used to corroborate the analytic results which demonstrate the good performance, robustness, and fastness of the proposed method.

INDEX TERMS Constrained least-square, narrow space, range-based localization.

I. INTRODUCTION

Indoor localization for autonomous robots becomes an attractive subject with the rapid development and application of the autonomous robots technology [1], [2]. The robots need localization systems to provide position information which is critical and fundamental for the control algorithm. Since the distance information can be sourced from various physical signals, such as, laser, ultrasound, ultrawideband (UWB), time-of-arrival (TOA), time-difference-of-arrival (TDOA), received signal strength (RSS), channel state information (CSI) and in various combinations [3]–[9], the robots can flexibly equip the suitable ranging device to adapt working circumstance. Therefore, range-based localization algorithms, which estimate the target position by using distance information, are widely used for robot navigation. To pursue efficient localization performance, methods are proposed, such as using more sensitive ranging sensors, optimizing calculation methods, cooperating localization among the target nodes, etc. Since cooperative localization [10], which utilizes the information among the target nodes, significantly improves the localization accuracy, it has become the current lines of research. However, only a single robot is deployed in some applications, such as inspecting the safety of the underground tunnel. In this paper, we focus on the range-based localization algorithm for a single target node.

Position estimation is not a trivial task because the measurement model constituted by measurement distances is a nonlinear one. The Taylor-series method [11], [12] and the maximum likelihood method [13], [14] are proposed for solving the nonlinear equations directly. An iterative process is needed for those algorithms. Thus, converge and complexity problems are twisted with those algorithms. Recently, circular localization via Euclidean norm elimination (CLENE) [15] explores the way of improving computational efficiency. However, it still needs to guess a proper initiation to achieve the optimum estimation performance. Another idea of solving nonlinear measurement equations is to convert the equations into a set of linear equations so that global convergence is ensured or fast implementation is allowed, which includes subspace [16]–[20] and linear (constrained) least squares [21]–[28] approaches.

The associate editor coordinating the review of this manuscript and approving it for publication was Fatos Xhafa.

Linear least squares (LLS) method [21] constructs a linear localization system by squaring measurement distance equations and subtracting one of them. LLS only uses the information of distance-difference, which is adopted by TDOA localization system. The weighted least squares (WLS) method, which adds the information of measurement distance errors into LLS, is proposed to improve the localization accuracy. LLS and WLS omit the constraint relation of variables during the linearization process. Constrained weighted least squares (CWLS) [23]–[25] restore the omitting information to improve the localization accuracy. Compared with LLS and WLS, CWLS can be considered as an LLS which puts the relationship of the variables back. However, CWLS is an unbiased estimator only on the condition of high signal to noise ratio. Two-step weighted least squares (TWLS) [22], [26] produces an unbiased estimator while it performs poorly in the scenario where the geometry is a uniform circular array, in which case the localization system is ill-conditioned. Separated CWLS (SCWLS) [27], [28] circumvents the ill-conditioned problem by separating the variables to different sides of the linear equation. SCWLS uses iterative quadratic maximum likelihood technique in relaxation procedure, which is not able to guarantee the global optimal solution. To avoid the ill-conditioned problem and reach a global solution efficiently, literature [28] derive a primal-dual interior point algorithm.

Actually, besides the ill-conditioned problem when the target is around the geometric center of reference nodes, the localization system for mobile robots working in narrow space needs to resist the ill-conditioned problem when the reference nodes are almost in a line with high probability. When the geometric shape approaches a line, the matrix condition number of the localization system is larger which amplifies the estimation errors. The localization system is also expected to be low computation burden because of the robots' mobility. In the scenario of narrow space, the aforementioned algorithms do not perform well because those estimators are either biased or iterative, which is demonstrated as numerical simulation in Section IV. In this paper, we propose a fast and robust localization algorithm that can handle the ill-conditioned problems when the target is around the geometric center and the geometric shape approaches a line.

The main contributions of this paper are: a fast unbiased CWLS (UCWLS) for range-based localization in narrow space is modeled and a closed-form estimation algorithm is also proposed. UCWLS introduces the measurement distance equation into the objective function meanwhile keep the system linear, which improves the robustness against the ill-conditioned situation when the geometric shape approaches a line. Also, UCWLS does not have iterative steps in the calculation process, which decreases the computational complexity.

The remained of the paper is organized as follows. Relative methods are briefly introduced in Section II. In Section III, the development and the description of UCWLS are presented in detail. Subsequently, an analytical study of the devised scheme is proposed by comparing with CWLS, TWLS, SCWLS, and CNELE. Numerical simulation and experimental tests results are given in Section IV to corroborate the performance of the proposed method. Finally, conclusions are drawn in Section V.

II. PROBLEM STATEMENT

In this section, symbols are defined. Also, models of CWLS, TWLS, and SCWLS are briefly introduced, which enlightens our method.

Let $\boldsymbol{\theta} = [x, y]^{\mathrm{T}}$ denotes the coordinate of the target node, $\boldsymbol{\theta}_i = [x_i, y_i]^{\mathrm{T}}\,(i = 1, \cdots, m)$ represents the coordinates of i-th anchor nodes, m is the number of reference nodes, $\hat{d}_i$ means the measurement distance between the target node and i-th anchor node, the measurement distances are expressed in matrix form as

$$\underbrace{\begin{bmatrix} -2x_1 & -2y_1 & 1 \\ \vdots & \vdots & \vdots \\ -2x_m & -2y_m & 1 \end{bmatrix}}_{A_m} \underbrace{\begin{bmatrix} x \\ y \\ x^2+y^2 \end{bmatrix}}_{\hat{\boldsymbol{\theta}}_m} = \underbrace{\begin{bmatrix} \hat{d}_1^2 - x_1^2 - y_1^2 \\ \vdots \\ \hat{d}_m^2 - x_m^2 - y_m^2 \end{bmatrix}}_{\hat{\boldsymbol{b}}_m}. \tag{1}$$

Locating target node is to solve Eq.(1).

It is notable that (a) the equations are nonlinear because of the term $x^2 + y^2$ in $\hat{\boldsymbol{\theta}}_m$, (b) the measurement distance $\hat{d}_i$ is imprecise because of the inevitable measurement noise.

CWLS [23]–[25], which is a biased estimator, models the localization problem as

$$\begin{aligned} \texttt{CWLS:} \quad & \underset{\hat{\boldsymbol{\theta}}_m}{\arg\min} \left\| W_{cw}(A_m\hat{\boldsymbol{\theta}}_m - \hat{\boldsymbol{b}}_m) \right\|_2^2 \\ & \texttt{s.t.}\ \boldsymbol{p}^{\mathrm{T}}\hat{\boldsymbol{\theta}}_m + \hat{\boldsymbol{\theta}}_m^{\mathrm{T}} P_m \hat{\boldsymbol{\theta}}_m = 0 \end{aligned} \tag{2}$$

where

$$\boldsymbol{p} = \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix} \quad \texttt{and} \quad P_m = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}. \tag{3}$$

As discussed in [24], this algorithm derives a biased estimator. And the bias is

$$\begin{aligned} E(\hat{\boldsymbol{\theta}}_m) - \boldsymbol{\theta} = -\frac{1}{2}G_m\boldsymbol{p} + \sum\nolimits_{n=1}^{\infty} (-\lambda G_m P_m)^n \boldsymbol{\theta}_m \\ - \frac{\lambda}{2}\sum\nolimits_{n=1}^{\infty} (-\lambda G_m P_m)^n G_m \boldsymbol{p}, \end{aligned} \tag{4}$$

where $E(\cdot)$ is the expectation operator, $G_m = (A_m^{\mathrm{T}} W_m^{-1} A_m)^{-1}$, λ is a root of an equation which minimizes the Lagrangian function sourced from Eq.(4).

To eliminate the bias of CWLS, TWLS [26] transforms the measurement equations into

$$A_{tw}\hat{\boldsymbol{\theta}}_{tw} = \hat{\boldsymbol{b}}_{tw} \tag{5}$$

where

$$
\begin{aligned}
r_1 &= \sqrt{(x-x_1)^2+(y-y_1)^2}\,, \\
r_{i,1} &= \hat{d}_i - \hat{d}_1 \; (i=2,3,\cdots,m)\,, \\
A_{tw} &= \begin{bmatrix} x_2-x_1 & y_2-y_1 & r_{2,1} \\ \vdots & \vdots & \vdots \\ x_m-x_1 & y_m-y_1 & r_{m,1} \end{bmatrix}, \\
\hat{\boldsymbol{\theta}}_{tw} &= \begin{bmatrix} x-x_1 \\ y-y_1 \\ r_1 \end{bmatrix}, \\
\hat{\boldsymbol{b}}_{tw} &= \frac{1}{2}\begin{bmatrix} (x_2-x_1)^2+(y_2-y_1)^2-r_{2,1}^2 \\ \vdots \\ (x_m-x_1)^2+(y_m-y_1)^2-r_{m,1}^2 \end{bmatrix}.
\end{aligned} \tag{6}
$$

Then, TWLS models the localization problem as

$$
\begin{aligned}
\texttt{TWLS:} \quad & \arg\min_{\hat{\boldsymbol{\theta}}_{tw}} \left\| W_{tw}(A_{tw}\hat{\boldsymbol{\theta}}_{tw} - \hat{\boldsymbol{b}}_{tw}) \right\|_2^2 \\
& \texttt{s.t.}\ \hat{\boldsymbol{\theta}}_{tw}^{\mathrm{T}} P_{tw} \hat{\boldsymbol{\theta}}_{tw} = 0
\end{aligned} \tag{7}
$$

where

$$
P_{tw} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}. \tag{8}
$$

Two steps are carried out to solve Eq.(7). The first step is to obtain a unconstrained solution of the object function. A second weighted least squares step utilizes the relationship of variables. Since only the differences of measurement distances are used, the size of matrix A_{tw} is $m-1$ by $m-1$. The rank of A_{tw} is less than the rank of A_{cw}.

The matrix A_{tw} is singular or ill-conditioned when the target node is around the geometric center. To fix this problem, SCWLS [27], [28] reforms the Eq.(7) as

$$
\begin{aligned}
\texttt{SCWLS:} \quad & \arg\min_{\boldsymbol{\eta}} \left\| W_{sw}(G\boldsymbol{\eta} - \hat{\boldsymbol{b}}_{tw} + \boldsymbol{g}R) \right\|_2^2 \\
& \texttt{s.t.}\ \hat{\boldsymbol{\theta}}_{tw}^{\mathrm{T}} P_{tw} \hat{\boldsymbol{\theta}}_{tw} = 0
\end{aligned} \tag{9}
$$

where $G = \begin{bmatrix} [A_{tw}]_{:,1} & [A_{tw}]_{:,2} \end{bmatrix}$ and $\boldsymbol{g} = [A_{tw}]_{:,3}$ with $[A_{tw}]_{:,i}$ being the i-th column of the matrix A_{tw}; $\boldsymbol{\eta} = [[\hat{\boldsymbol{\theta}}_{tw}]_1\ [\hat{\boldsymbol{\theta}}_{tw}]_2]^{\mathrm{T}}$ and $R = [\hat{\boldsymbol{\theta}}_{tw}]_3$ with $[\hat{\boldsymbol{\theta}}_{tw}]_i$ being the i-th element of the vector $\hat{\boldsymbol{\theta}}_{tw}$.

In summary, CWLS uses all measurement distances information while it produces a bias. TWLS trade off distance information for eliminating the biased estimator. SCWLS only amends the performance of TWLS when the target node is around the geometric center of reference nodes. Those algorithms either produce a biased estimator or insufficiently use the measurement distances.

III. MODEL AND ALGORITHM

A. UCWLS MODEL

The idea of our model is to restore the subtracted measurement distance information while keeping the constraint relation of variables.

Without loss of generality, assume the target node is closest to the first anchor node, we firstly move the origin coordinate to the position of first anchor node. It results that the measurement system, Eq.(1), becomes

$$
\begin{cases}
x'^2+y'^2-2x_1'x'-2y_1'y'+x_1'^2+y_1'^2=\hat{d}_1^2 \\
x'^2+y'^2-2x_2'x'-2y_2'y'+x_2'^2+y_2'^2=\hat{d}_2^2 \\
\quad\vdots \\
x'^2+y'^2-2x_m'x'-2y_m'y'+x_m'^2+y_m'^2=\hat{d}_m^2,
\end{cases} \tag{10}
$$

where $x' = x - x_1$, $y' = y - y_1$, $x_i' = x_i - x_1$, $y_i' = y_i - y_1$, respectively. $\begin{bmatrix} x' & y' \end{bmatrix}^{\mathrm{T}}$ is the position of the target node in the moved coordinate system. To compact the symbols in the following discussion, we still use $\begin{bmatrix} x & y \end{bmatrix}^{\mathrm{T}}$ to denote the coordination of target node in the moved coordinate system. Subtracting the first sub-equation in Eq.(10), which can be reduced to $x^2+y^2=\hat{d}_1^2$, from the other sub-equations gives that

$$
\begin{cases}
x^2+y^2=\hat{d}_1^2 \\
2x_2'x+2y_2'y=\hat{d}_1^2-\hat{d}_2^2+x_2'^2+y_2'^2 \\
\quad\vdots \\
2x_m'x+2y_m'y=\hat{d}_1^2-\hat{d}_m^2+x_m'^2+y_m'^2/
\end{cases} \tag{11}
$$

Since the first sub-equation isolates from the other sub-equations in Eq.(11), we can replace the first sub-equation from $x^2+y^2=\hat{d}_1^2$ to $\sqrt{x^2+y^2}=\hat{d}_1$. Thus, we transform the measurement system into

$$
A_{uw}\hat{\boldsymbol{\theta}}_{uw} = \hat{\boldsymbol{b}}_{uw} \tag{12}
$$

where

$$
\begin{aligned}
A_{uw} &= \begin{bmatrix} x_2-x_1 & y_2-y_1 & 0 \\ \vdots & \vdots & \vdots \\ x_m-x_1 & y_m-y_1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \\
\hat{\boldsymbol{\theta}}_{uw} &= \begin{bmatrix} x \\ y \\ \sqrt{x^2+y^2} \end{bmatrix}, \\
\hat{\boldsymbol{b}}_{uw} &= \begin{bmatrix} (\hat{d}_1^2+x_1^2+y_1^2-\hat{d}_2^2+x_2^2+y_2^2-2x_1x_2-2y_1y_2)/2 \\ \vdots \\ (\hat{d}_1^2+x_1^2+y_1^2-\hat{d}_m^2+x_m^2+y_m^2-2x_1x_m-2y_1y_m)/2 \\ \hat{d}_1 \end{bmatrix}.
\end{aligned} \tag{13}
$$

Remark: (a) Compared with Eq.(1), Eq.(12) shows that UCWLS introduces $\sqrt{x^2+y^2}$ *as an intermediate variable while CWLS adopts* x^2+y^2*. (b) Compared with Eq.(5), Eq.(12) indicates that UCWLS uses all measurement distances information while TWLS and SCWLS do not.*

Base on this measurement system, we form the localization problem as

$$\begin{aligned} \texttt{UCWLS:} \quad & \arg\min_{\hat{\boldsymbol{\theta}}_{uw}} \ \| W_{uw}^{1/2}(A_{uw}\hat{\boldsymbol{\theta}}_{uw} - \hat{\boldsymbol{b}}_{uw}) \|_2^2 \\ & \texttt{s.t.}\ \hat{\boldsymbol{\theta}}_{uw}^{\mathrm{T}} Q \hat{\boldsymbol{\theta}}_{uw} = 0 \end{aligned} \tag{14}$$

where

$$Q = \begin{bmatrix} \boldsymbol{I}_{(k-1)\times(k-1)} & \boldsymbol{0}_{(k-1)\times 1} \\ \boldsymbol{0}_{1\times(k-1)} & -1 \end{bmatrix}. \tag{15}$$

$k = 3$, W_{uw} is a weight matrix that is used to improve performance.

It is clear that the proposed method is a quadratic constrained weighted least squares method, which naturally has low computational complexity. In addition, the structure of A_{uw} suggests that UCWLS uses not only the difference of measurement distances but also a measurement distance $\hat{d}_1$. It augments the information used in the objective function which leads to better performance.

B. ALGORITHM DESCRIPTION

To solve Eq.(14), we convert it to an unconstrained normal equation by constructing a Lagrange function

$$\mathcal{L}(\hat{\boldsymbol{\theta}}_{uw}, \lambda) = \| W_{uw}^{1/2}(A_{uw}\hat{\boldsymbol{\theta}}_{uw} - \hat{\boldsymbol{b}}_{uw}) \|_2^2 + \lambda \hat{\boldsymbol{\theta}}_{uw}^{\mathrm{T}} Q \hat{\boldsymbol{\theta}}_{uw}. \tag{16}$$

Differentiate $\mathcal{L}(\hat{\boldsymbol{\theta}}_{uw}, \lambda)$ with respect to $\hat{\boldsymbol{\theta}}_{uw}$,

$$\frac{\partial \mathcal{L}(\hat{\boldsymbol{\theta}}_{uw}, \lambda)}{\partial \hat{\boldsymbol{\theta}}_{uw}} = 2(A_{uw}^{\mathrm{T}} W_{uw} A_{uw} + \lambda Q)\hat{\boldsymbol{\theta}}_{uw} - 2A_{uw}^{\mathrm{T}} W_{uw} \hat{\boldsymbol{b}}_{uw}. \tag{17}$$

Equate the Eq.(17) to zero, the minimum of Lagrange function is obtained as

$$\hat{\boldsymbol{\theta}}_{uw} = (A_{uw}^{\mathrm{T}} W_{uw} A_{uw} + \lambda Q)^{-1} A_{uw}^{\mathrm{T}} W_{uw} \hat{\boldsymbol{b}}_{uw} \tag{18}$$

where multiplier λ has yet to be determined.

To find λ, we substitute Eq.(18) into the equality constraint of Eq.(14), which is

$$\begin{aligned} &[(A_{uw}^{\mathrm{T}} W_{uw} A_{uw} + \lambda Q)^{-1} A_{uw}^{\mathrm{T}} W_{uw} \hat{\boldsymbol{b}}_{uw}]^{\mathrm{T}} \\ &\quad Q (A_{uw}^{\mathrm{T}} W_{uw} A_{uw} + \lambda Q)^{-1} A_{uw}^{\mathrm{T}} W_{uw} \hat{\boldsymbol{b}}_{uw} = 0. \end{aligned} \tag{19}$$

Note that $A_{uw}^{\mathrm{T}} W_{uw} A_{uw}$ and Q are both real symmetric matrices, $(A_{uw}^{\mathrm{T}} W_{uw} A_{uw})^{-1}$ can be diagonalized as

$$(A_{uw}^{\mathrm{T}} W_{uw} A_{uw})^{-1} = U \Lambda U^{-1} \tag{20}$$

where $D = \texttt{diag}(d_1, d_2, d_3)$ and $d_i (i = 1, 2, 3)$ are the eigenvalues of the matrix $(A_{uw}^{\mathrm{T}} W_{uw} A_{uw})^{-1}$. Putting Eq.(20) into Eq.(19) gives

$$\begin{aligned} 0 &= [U \Lambda U^{-1}(A_{uw}^{\mathrm{T}} W_{uw} A_{uw})]^{\mathrm{T}} Q \\ &\quad \times [U \Lambda U^{-1}(A_{uw}^{\mathrm{T}} W_{uw} A_{uw})] \\ &= (U^{-1} A_{uw}^{\mathrm{T}} W_{uw} A_{uw})^{\mathrm{T}} \\ &\quad \times \texttt{diag}\left((d_1+\lambda)^{-2}, (d_2+\lambda)^{-2}, -(d_3-\lambda)^{-2}\right) \\ &\quad \times (U^{-1} A_{uw}^{\mathrm{T}} W_{uw} A_{uw}). \end{aligned} \tag{21}$$

To clarify the equation, a vector $\boldsymbol{c} = U^{-1} A_{uw}^{\mathrm{T}} W_{uw} A_{uw}$ is introduced. The polynomial of λ is

$$\frac{c_1^2}{(d_1+\lambda)^2} + \frac{c_2^2}{(d_2+\lambda)^2} - \frac{c_3^2}{(d_3-\lambda)^2} = 0 \tag{22}$$

where

$$\boldsymbol{c} = U^{-1} A_{uw}^{\mathrm{T}} W_{uw} A_{uw} = [c_1,\ c_2,\ c_3]^{\mathrm{T}}. \tag{23}$$

Among the four roots of Eq.(22), we use a positive real one to minimize the effect of ill-conditioned problem.

Since we carry out a coordinate transformation in the first step, an inverse coordinate transformation is done on the $\hat{\boldsymbol{\theta}}_{uw}$ to yield $\tilde{\boldsymbol{\theta}}_{ru}$. The position of the target node which consists of the first and second elements of $\tilde{\boldsymbol{\theta}}_{ru}$ is finally obtained.

To summarize, the position of the target node is calculated by Algorithm 1.

Algorithm 1 UCWLS Algorithm

input : Reference nodes coordinates $(x_i, y_i)(i = 1, \cdots, m)$ and measurement data $\hat{d}_i (i = 1, \cdots, m)$.

output: The position of the target node (x, y)

Find the reference node which is closest to the target node.

Move the coordinate origin to the found reference node.

Construct the localization system $A_{uw}\hat{\boldsymbol{\theta}}_{uw} = \hat{\boldsymbol{b}}_{uw}$ using Eq.(12).

Decompose $(A_{uw}^{\mathrm{T}} W_{uw} A_{uw})^{-1}$ to obtain U and Λ using Eq.(20).

Generate the polynomial of λ using Eq.(22).

Find the maximum real root $\lambda_{\texttt{max}}$ among the four roots of Eq.(22).

Calculate $\hat{\boldsymbol{\theta}}_{ru,\lambda_{\texttt{max}}}$ and $\hat{\boldsymbol{\theta}}_{ru,\lambda_0}$ with $\lambda = \lambda_{\texttt{max}}$ and $\lambda = 0$ using Eq.(18), respectively.

Let $\hat{\boldsymbol{\theta}}_{uw}$ is the one being smaller of residual errors,

$$\begin{aligned} \hat{\boldsymbol{\theta}}_{uw} = \arg\min\nolimits_{\hat{\boldsymbol{\theta}}_{ru,\lambda_{\texttt{max}}}, \hat{\boldsymbol{\theta}}_{ru,\lambda_0}} \{ &\| A_{uw}\hat{\boldsymbol{\theta}}_{ru,\lambda_{\texttt{max}}} - \hat{\boldsymbol{b}}_{uw} \|_2, \\ &\| A_{uw}\hat{\boldsymbol{\theta}}_{ru,\lambda_0} - \hat{\boldsymbol{b}}_{uw} \|_2 \}. \end{aligned}$$

Return the position $[1\ 1\ 0]\hat{\boldsymbol{\theta}}_{uw}$ carried on an inverse coordinate transformation.

C. ALGORITHMS ANALYSIS

In briefly, compared with TWLS, CWLS, SCWLS, and CLENE, the performance of UCWLS is expected in Table 1.

1) ESTIMATOR BIAS

Proposition 1: On conditions that (a) the mean of measurement noise is zero, and (b) the measurement noise is small compared with the physical distance, the UCWLS is an unbiased estimator.

For the measurement error is sufficiently small, which is the practical situation, the squared value $\hat{d}_i$ can be

TABLE 1. Performance comparison.

	UCWLS	[15]	[27]	[26]	[25]
Estimation bias	Unbias	*	*	*	Bias
Iterative method	No	Yes	Yes	Yes	Yes
Complexity$^+$	$O(m)$	$O(mn^{1.3})$	$O(mn^{1.5})$	$O(mn)$	$O(mn^2)$
Robustness†	High	High	Medium	Low	Low
Robustness‡	High	High	Medium	Low	Low

Remark:
(1) [15]-CLENE; [27]-SCWLS; [26]-TWLS; [25]-CWLS
(2) *: uncertain because the solution is not in closed-form
(3) Complexity$^+$: m and n are the number of anchor nodes and iteration rounds respectively
(4) Robustness†: the scenario that target is around the geometric center
(5) Robustness‡: the scenario that the geometric shape approaches a line

approximated as

$$\hat{d}_i^2 = (d_i + \varepsilon_i)^2 \approx d_i^2 + 2d_i\varepsilon_i \tag{24}$$

where ε_i is measurement noise. Since the mean of noise is zero, which means $E(\hat{d}_i - d_i) = 0$. It conducts that $E(\Delta \boldsymbol{b}) = 0$. Based on this fact, the mean of localization bias $\Delta\hat{\boldsymbol{\theta}}$ is

$$\begin{aligned} E[\Delta\hat{\boldsymbol{\theta}}] &= E[(A^{\mathrm{T}}WA + \lambda Q)^{-1}A^{\mathrm{T}}W\Delta\boldsymbol{b}] \\ &= (A^{\mathrm{T}}WA + \lambda Q)^{-1}A^{\mathrm{T}}W \cdot E[\Delta\boldsymbol{b}] \\ &= 0. \end{aligned} \tag{25}$$

2) COMPUTATION TIME

Actually, all algorithms, CWLS, TWLS, SCWLS and UCWLS have the same time complexities in each iteration if we use Big-O to describe the algorithms' computation complexity. It is because that a least squares process is carried out in each iteration. Conceptually, in each iteration, the computational complexity of these algorithms is almost the same because all of them estimate the position by solving a system of equations like $A\boldsymbol{\theta} = \boldsymbol{b}$. Therefore, iterative times dominate the computation burden.

Inspect the whole process of Algorithm 1, each step is determinable with a closed-form expression. The only doubt might be calculating λ which is the solution of Eq.(22). Actually, Eq.(22) is a polynomial of degree four also known as quartic polynomial which has a closed-form solution formula proposed by Ferrari's algorithm. A fast and highly accurate algorithm for solving quartic equations is introduced in literature [29], which declares that it is more than six times as fast than the quasi-standard companion matrix eigenvalue quartic solver.

The SCWLS has the same way of calculating λ as UCWLS. However, SCWLS needs an iterative process to counter solving difficulty, which is caused by intermediate variables coupling with measurement distances. Therefore, we consider that SCWLS is medium complexity.

CWLS obtains a λ by solving a fifth degree polynomial known as quintic polynomial. It is well known that there are no general solutions to polynomials of degree higher than four. The solutions only are calculated by iteration or evolutionary algorithm. Therefore, we consider that the computation burden of CWLS is high. Also, TWLS has a second step of weighted least squares step beside the first weighted least squares step. Although TWLS also needs iteration, it does not need to update λ, so the complexity is not as high as CWLS and SCWLS.

The initial value, the power of the noise, and the algorithm itself have different effects on the convergence of the iterative process. Due to the uncertainty of the number of iteration rounds, we use numerical simulation results to illustrate the computational complexity, as shown in Table 1. As can be seen from Table 1, UCWLS has the lowest computational complexity. Although TWLS, CWLS, SCWLS, and UCWLS all have a closed-form expression, UCWLS only needs to solve the closed expression once, while other algorithms, including CLENE without closed-form expression, need to update the target node location in an iteration manner, so the computational complexity of UCWLS is relatively low.

3) ROBUSTNESS DISCUSSION

The localization system for mobile robots working in narrow space needs to resist two ill-conditioned problems: (i) the distances from the target node to reference nodes are approximately the same, (ii) the reference nodes are almost in a line.

As it is discussed in [27], TWLS and CWLS are unable to handle the scenario where the distances from the target node to reference nodes are approximately the same with the geometry of reference nodes is likely a uniform circular array. The issue is that the coefficients matrix A_{tw} couples with the measurement distances in that case. Therefore, SCWLS separates the additional variable, which is relative with measurement distances, from the coefficients matrix G to overcome matrix ill-conditioned problem.

Proposition 2: When the target node is close to the geometry center of reference nodes, UCWLS avoids being ill-conditioned.

Note the relationship of A_{uw} and G, which is

$$A_{uw} = \begin{bmatrix} G & \mathbf{0}_{1\times(m-1)} \\ \mathbf{0}_{2\times 1} & 1 \end{bmatrix} \tag{26}$$

and express W_{uw} and $\hat{\boldsymbol{b}}_{uw}$ as block matrix form whose size match the A_{uw}

$$W_{uw} = \begin{bmatrix} W_G & \mathbf{0} \\ \mathbf{0} & w_m \end{bmatrix} \quad \texttt{and} \quad \boldsymbol{b} = \begin{bmatrix} \boldsymbol{b}_G \\ b_m \end{bmatrix}, \tag{27}$$

it is easily to verify that the position of target node is

$$\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \hat{\boldsymbol{\theta}}_{uw} = (G^{\mathrm{T}}W_G G + \lambda I)^{-1} G^{\mathrm{T}} W_G \boldsymbol{b}_G. \tag{28}$$

Eq.(28) shows that UCWLS is same way adapted by SCWLS to avoids ill-conditioned situation.

Furthermore, we declare that UCWLS is more robust than SCWLS which is the state-of-the-art of quadratic constrained weighted least squares method.

Proposition 3: The localization variance of UCWLS is small than the one of SCWLS.

Proof: SCWLS is equivalent to a generic unconstrained estimation problem as follows:

$$\hat{\boldsymbol{\eta}}_{\mathrm{sc}} = \arg\min_{\boldsymbol{\eta}_{\mathrm{sc}}} J_{\mathrm{sc}}(\boldsymbol{\eta}) \tag{29}$$

where

$$J(\boldsymbol{\eta})_{\mathrm{sc}} = (G\boldsymbol{\eta} - \hat{\boldsymbol{b}}_{tw} + \boldsymbol{g}R)^{\mathrm{T}} W_{sw} (G\boldsymbol{\eta} - \hat{\boldsymbol{b}}_{tw} + \boldsymbol{g}R). \tag{30}$$

The covariance matrix C_{sc} associated with $J(\boldsymbol{\eta})_{\mathrm{sc}}$ can be expressed as

$$C_{\mathrm{sc}} \approx E\left[\frac{\partial^2 J(\boldsymbol{\eta})_{\mathrm{sc}}}{\partial\boldsymbol{\eta}\partial\boldsymbol{\eta}^{\mathrm{T}}}\right]^{-1} E\left[\left(\frac{\partial J(\boldsymbol{\eta})_{\mathrm{sc}}}{\partial\boldsymbol{\eta}}\right)\left(\frac{\partial J(\boldsymbol{\eta})_{\mathrm{sc}}}{\partial\boldsymbol{\eta}}\right)^{\mathrm{T}}\right] \times E\left[\frac{\partial^2 J(\boldsymbol{\eta})_{\mathrm{sc}}}{\partial\boldsymbol{\eta}\partial\boldsymbol{\eta}^{\mathrm{T}}}\right]^{-1}\Bigg|_{\boldsymbol{\eta}=\boldsymbol{\eta}_{\mathrm{sc}}}. \tag{31}$$

And the localization variance of SCWLS, denoted as v_{sc}, is

$$v_{\mathrm{sc}} = \mathtt{trace}\{C_{\mathrm{sc}}\}. \tag{32}$$

Based on Eq.(14), Eq.(26) and Eq.(27), UCWLS is equivalent to

$$\hat{\boldsymbol{\theta}}_{uw} = \arg\min_{\boldsymbol{\theta}} J(\boldsymbol{\theta}) \tag{33}$$

where

$$J(\boldsymbol{\theta}) = J_{\mathrm{sc}}(\boldsymbol{\eta}) + w_m(\hat{d}_1 - \hat{\boldsymbol{\eta}}_{\mathrm{sc}}^{\mathrm{T}}\hat{\boldsymbol{\eta}}_{\mathrm{sc}})^2. \tag{34}$$

Therefore, it is easy to verify that the localization variance of UCWLS is small than v_{sc}. □

IV. NUMERICAL SIMULATIONS AND EXPERIMENTAL TESTS

Simulations are done to assess the performance of UCWLS, SCWLS, TWLS, CWLS and CLENE. It is worthy notable that CLENE is a nonlinear least squares method, which trade localization with computation time. Therefore, CLENE is used as representative of localization accuracy of nonlinear least squares. Since the performance of CLENE depends on the initial value, two types of setting initiations are used in the simulations. The first way is to set the initiation as the output of LS algorithm, named CLENE-1. The second way is simply to set $[0 \quad 0]^{\mathrm{T}}$, named CLENE-2.

The performance measure uses mean squared error (MSE) as a function of average noise power $p = 10\log_{10}(\sigma^2)$ dB, where σ^2 is the variance of the noise. The MSE is computed by

$$MSE = 10\log_{10}\left(\frac{\sum_{l=1}^{M} \|\boldsymbol{\theta}(l) - \boldsymbol{\theta}\|_2^2}{M}\right) \mathrm{dB} \tag{35}$$

where $\boldsymbol{\theta}(l)$ is the estimate of $\boldsymbol{\theta}$ at the l-th trial, M is trials times. In addition, Cramér-Rao lower bound (CRLB) [22] is used to indicate the optimality of localization results.

The ideal weighting matrix is that $W_{uw} = E(\hat{\boldsymbol{b}}_{uw}\hat{\boldsymbol{b}}_{uw}^{\mathrm{T}})^{-1}$, which involves the accurate distances between the target node and reference nodes. The error of $\hat{\boldsymbol{b}}_{uw}$ can be modeled as zero-mean Gausssian process [27]. Ignoring the second-order error term, the weighting matrix is approximated as

$$W_{uw} \approx \mathtt{diag}(\hat{d}_2^2, \hat{d}_m^2, \cdots, \hat{d}_1^2)^{-1} \tag{36}$$

Eq.(36) is used as a weighting matrix in simulation. It is worth noting that CWLS, SCWLS, and CLENE use an iterative process to obtain an optimal weighting matrix. UCWLS can also adopt a process of optimizing weighting matrix to improve locating accuracy. However, UCWLS trades off a small MSE for a low computation complexity, which is discussed and illustrated in Section IV-A and Section IV-B. If the measurement errors are large, the weighting matrix heavily betrays ideal weighting, UCWLS will produce a low localization accuracy.

A. SCENARIO 1

Let us consider the same localization geometry used in [27], where the four reference nodes configuration are at coordinates $[0 \quad 0]^{\mathrm{T}}$, $[0 \quad 10]^{\mathrm{T}}$, $[10 \quad 10]^{\mathrm{T}}$, $[10 \quad 0]^{\mathrm{T}}$, and the target node is at $[5.1 \quad 4.9]^{\mathrm{T}}$, as shown in Fig.1. This scenario simulates the ill-conditioned problem that the reference nodes geometry is a uniform circular array and the source is close to the array center [27], [28].

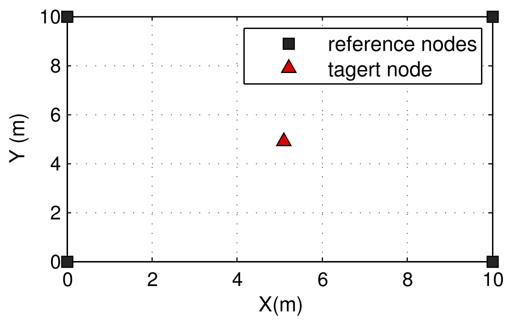

FIGURE 1. Location deployment of the nodes in Scenario 1.

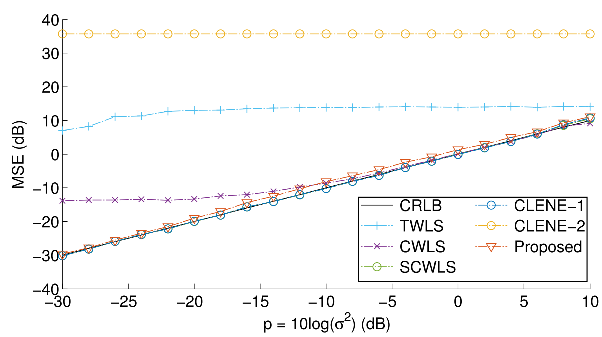

FIGURE 2. Average MSE versus noise power at $\theta = [5.1 \quad 4.9]^{\mathrm{T}}$ with 500 trials.

Show as Fig.2, for $p < -5$ dB, the proposed algorithm avoids the ill-conditioned as same as SCWLS, which is expected as proposition 2. UCWLS does not improve localization accuracy, meanwhile, it is the fastest one shown as Fig.3. The different performance of CLENE-1 and CLENE-2

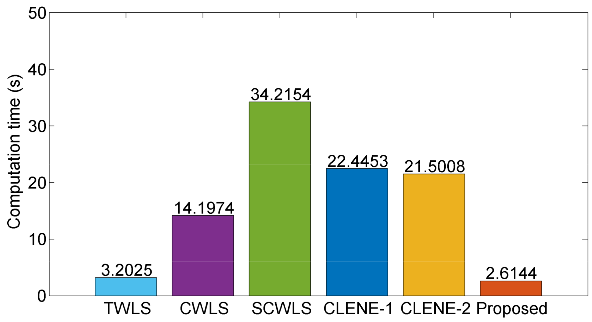

FIGURE 3. Corresponding computation time of the algorithms in Fig.2.

illuminates the importance of proper guess to a nonlinear least squares method. For $p > -5$ dB, except TWLS and CLENE-2, all algorithms have MSEs that are close to the CRLB. It is because the higher noise power reduces the sensitivity of the algorithms to data errors.

Both SCWLS and CLENE have MSEs exactly on the CRLB, which benefits from the iterative process. In contrast, UCWLS can not stick to the CRLB when the $p > -20$ dB. It is because that UCWLS constructs the weighting matrix directly from the measurement distance.

B. SCENARIO 2

As shown in Fig.4, four reference nodes are configured at coordinates $[0 \quad 0]^T$, $[0 \quad 2.4]^T$, $[8 \quad 2.4]^T$, $[8 \quad 0]^T$, and the target node is node 1 in the figure, whose coordinate is $[0.4 \quad 1.2]^T$.

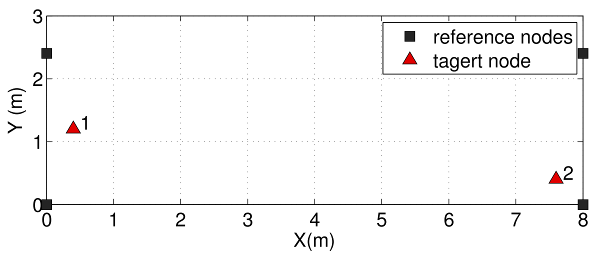

FIGURE 4. Location deployment of the nodes in Scenario 2 and Scenario 3.

Observe Fig.5, only UCWLS and CLENE-1 are close to CRLB. Meanwhile, In addition to TWLS, UCWLS is the fastest algorithm of several other algorithms shown as Fig.6. The localization error of SCWLS climbs up with an increase in the noise power because the condition number of the localization system amplifies the bias in this scenario.

In this scenario, UCWLS is slightly above not on the CRLB when $p > 0$ dB. It is still caused by the weighting matrix. Based on Eq. (24) and Eq. (36), in this case, the smallest distance, which has the smallest error in measurement error, dominates the localization error.

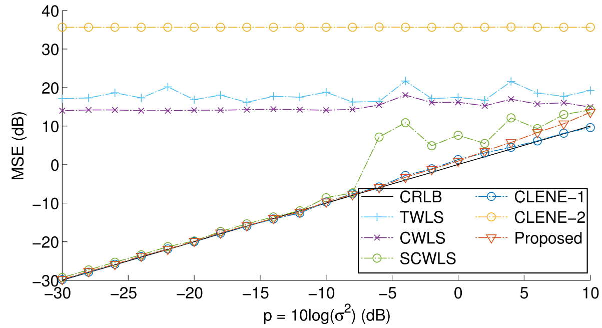

FIGURE 5. Average MSE versus noise power at $\theta = [0.4 \quad 1.2]^T$ with 500 trials.

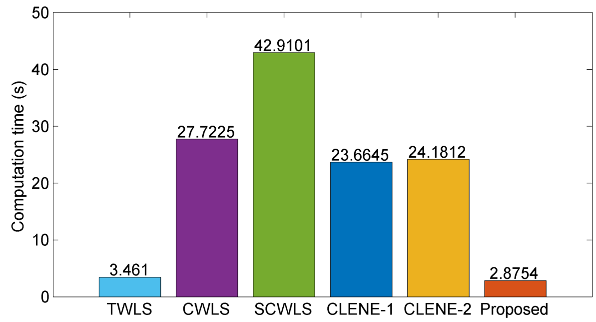

FIGURE 6. Corresponding computation time of the algorithms in Fig.5.

C. SCENARIO 3

Keep the configuration of four reference nodes, which are at $[0 \quad 0]^T$, $[0 \quad 2.4]^T$, $[8 \quad 2.4]^T$, $[8 \quad 0]^T$, and the target node is set at $[7.6 \quad 0.4]^T$,which is node 2 in Fig.4.

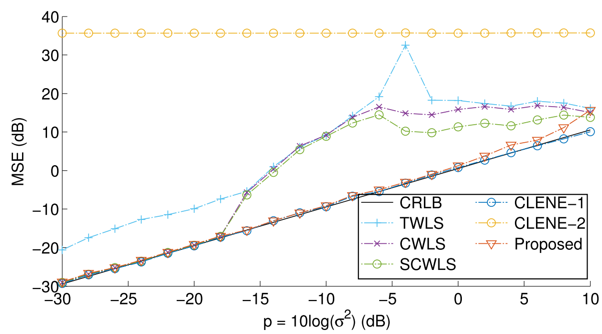

FIGURE 7. Average MSE versus noise power at $\theta = [7.6 \quad 0.4]^T$ with 500 trials.

Compare Fig.7 with Fig.5, SCWLS is no longer close to CRLB with a smaller error, which is -18 dB. However, UCWLS still approaches on the CRLB with low computation burden shown as Fig.7 and Fig.8, which is expected as proposition 2.

In this scenario, UCWLS shows its advantage that it can achieve a good localization accuracy with low computation complexity, at a cost of a small deviation from CRLB under large noise.

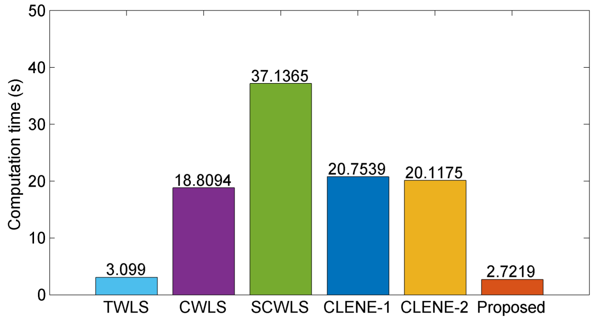

FIGURE 8. Corresponding computation time of the algorithms in Fig.7.

D. SCENARIO 4

Keep the configuration of four reference nodes, which are at $[0 \quad 0]^T$, $[0 \quad 2.4]^T$, $[8 \quad 2.4]^T$, $[8 \quad 0]^T$, and the target node is within the square area formed by four receivers but is at a random position for each of the 500 trials.

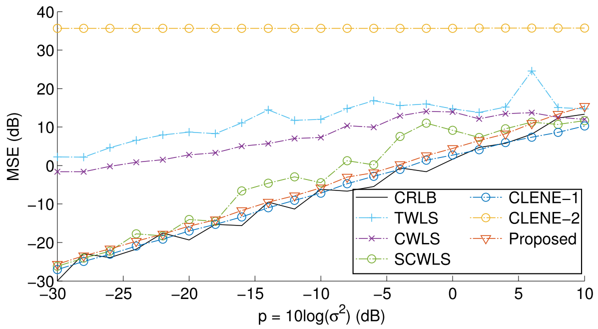

FIGURE 9. Average MSE versus noise power at random target node position with 500 trials.

As can be seen from Fig.9, the curve of CRLB shows jitter because the target node position of each trial is different, and the CRLB is no longer proportional to $10 \log 10(\sigma^2)$. Fig.9 also shows that UCWLS has a smaller MSE than TWLS, CWLS, and SCWLS. Compared with CLENE-1, UCWLS approaches it in the case of small noise, with a difference of about 1.3dB and when the noise is large, the gap of their MSEs gradually increases. It is because that CLENE-1 benefits from iteration and the weighting matrix of UCWLS heavily betrays ideal weighting when the measurement errors are large, UCWLS produces a low localization accuracy. Fig.9 and Fig.10 indicate that UCWLS has higher positioning accuracy and the lowest complexity once again.

E. EXPERIMENTAL TESTS

Experimental tests are carried out. The experimental scenario is shown in Fig.11. Four reference nodes are configured as the scenarios of IV-B and IV-C. The geometry of reference nodes is a rectangle with 8 meters by 2.4 meters. The nodes combine a DW1000 UWB module and a STM 32F407 microcontroller, which are powered via batteries.

In the first experiment, the target node was placed at $[0.4 \quad 1.2]^T$. As can be seen from Fig.12, the CLENE-1 has

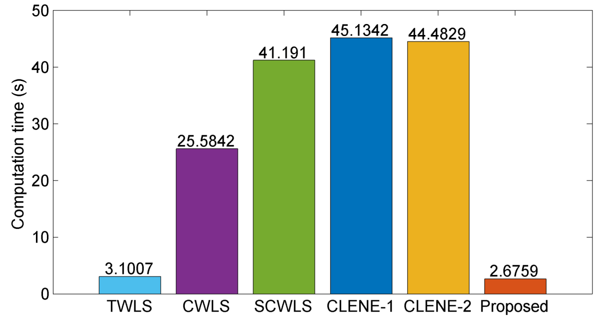

FIGURE 10. Corresponding computation time of the algorithms in Fig.9.

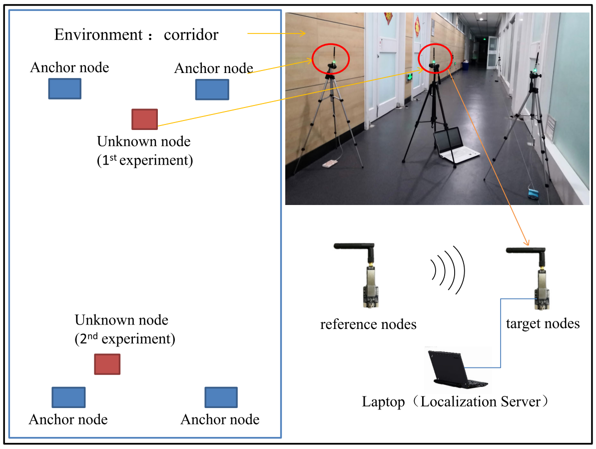

FIGURE 11. Experimental scenario.

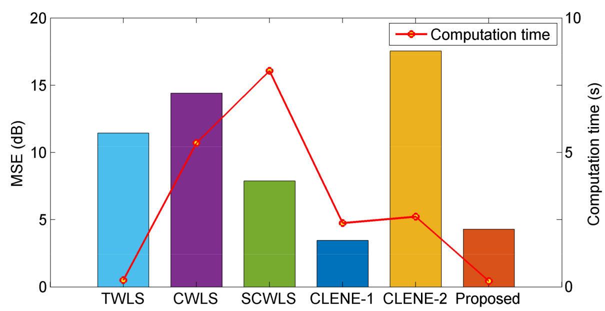

FIGURE 12. MSE and computation time of the algorithms at $\theta = [0.4 \quad 1.2]^T$ with 500 trials.

the highest localization accuracy. The MSE of UCWLS is about 1dB higher than CLENE-1, however, its computation time is much lower. In addition, in contrast to TWLS, UCWLS has smaller MSE but similar computation time. The experimental results are basically matched to the numerical simulations of Section IV-B. It is verified that UCWLS can achieve a good localization accuracy with low computation complexity.

The target node was placed at $[7.6 \quad 0.4]^T$ in the second test. It is also basically consistent with the numerical simulation of Section IV-C. Show as Fig.13, the difference in localization accuracy between CLENE-1 and UCWLS is

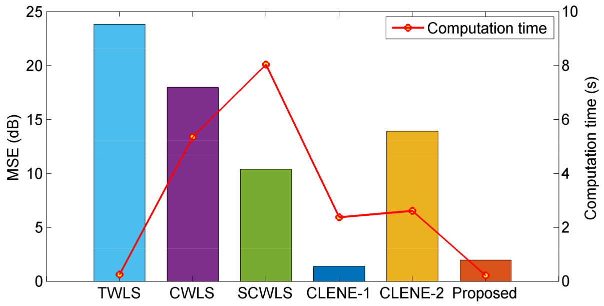

FIGURE 13. MSE and computation time of the algorithms at $\theta = [7.6 \quad 0.4]^{\mathrm{T}}$ with 500 trials.

small, but the MSE of UCWLS is significantly lower than other algorithms, and UCWLS has the lowest computational complexity.

V. CONCLUSION

In this paper, an efficient range-based localization algorithm for mobile robots acting in narrow space was explored. The proposed algorithm augmented the information used in objective function to resist the ill-conditioned problems caused by nodes' geometry. It utilized a measurement distance besides all of the difference of measurement distance. Compared with CLENE, SCWLS, TWLS, and CWLS, the performance was analysed. Simulation results have confirmed our analytical results that the proposed method can efficiently avoid the ill-conditioned problems.

REFERENCES

[1] A. Yassin, Y. Nasser, M. Awad, A. Al-Dubai, R. Liu, C. Yuen, R. Raulefs, and E. Aboutanios, "Recent advances in indoor localization: A survey on theoretical approaches and applications," *IEEE Commun. Surveys Tutr.*, vol. 19, no. 2, pp. 1327–1346, 2nd Quart., 2016.
[2] F. Ingrand and M. Ghallab, "Deliberation for autonomous robots: A survey," *Artif. Intell.*, vol. 247, pp. 10–44, 2017.
[3] J. Guivant, E. Nebot, and S. Baiker, "Localization and map building using laser range sensors in outdoor applications," *J. Robot. Syst.*, vol. 17, no. 10, pp. 565–583, 2000.
[4] S. Zhao, B. Huang, and F. Liu, "Localization of indoor mobile robot using minimum variance unbiased FIR filter," *IEEE Trans. Autom. Sci. Eng.*, vol. 15, no. 2, pp. 410–419, Apr. 2016.
[5] J. M. Pak, C. K. Ahn, P. Shi, Y. S. Shmaliy, and M. T. Lim, "Distributed hybrid particle/FIR filtering for mitigating NLOS effects in TOA-based localization using wireless sensor networks," *IEEE Trans. Ind. Electron.*, vol. 64, no. 6, pp. 5182–5191, Jun. 2017.
[6] A. Abdelhadi, F. Rechia, A. Narayanan, T. Teixeira, R. Lent, D. Benhaddou, H. Lee, and T. C. Clancy, "Position estimation of robotic mobile nodes in wireless testbed using GENI," in *Proc. Annu. IEEE Syst. Conf. (SysCon)*, Apr. 2016, pp. 1–6.
[7] U. Larsson, J. Forsberg, and A. Wernersson, "Mobile robot localization: Integrating measurements from a time-of-flight laser," *IEEE Trans. Ind. Electron.*, vol. 43, no. 3, pp. 422–431, Jun. 1996.
[8] X. Wang, L. Gao, S. Mao, and S. Pandey, "CSI-based fingerprinting for indoor localization: A deep learning approach," *IEEE Trans. Veh. Technol.*, vol. 66, no. 1, pp. 763–776, Jan. 2017.
[9] U. Yayan, H. Yucel, and A. Yazıcı, "A low cost ultrasonic based positioning system for the indoor navigation of mobile robots," *J. Intell. Robot. Syst.*, vol. 78, nos. 3–4, pp. 541–552, Jun. 2015.
[10] H. Wymeersch, J. Lien, and M. Z. Win, "Cooperative localization in wireless networks," *Proc. IEEE*, vol. 97, no. 2, pp. 427–450, Feb. 2009.
[11] L. Kovavisaruch and K. C. Ho, "Modified Taylor-series method for source and receiver localization using TDOA measurements with erroneous receiver positions," in *Proc. IEEE Int. Symp. Circuits Syst.*, May 2005, pp. 2295–2298.
[12] H. Godrich, A. M. Haimovich, and R. S. Blum, "Target localization accuracy gain in MIMO radar-based systems," *IEEE Trans. Inf. Theory*, vol. 56, no. 6, pp. 2783–2803, Jun. 2010.
[13] D. J. Torrieri, "Statistical theory of passive location systems," *IEEE Trans. Aerosp. Electron. Syst.*, vol. AES-20, no. 2, pp. 183–198, Mar. 1984.
[14] L. Rui and K. C. Ho, "Elliptic localization: Performance study and optimum receiver placement," *IEEE Trans. Signal Process.*, vol. 62, no. 18, pp. 4673–4688, Sep. 2014.
[15] J. Liang, Y. Chen, H. C. So, and Y. Jing, "Circular/hyperbolic/elliptic localization via Euclidean norm elimination," *Signal Process.*, vol. 148, pp. 102–113, 2018.
[16] R. N. Challa and S. Shamsunder, "High-order subspace-based algorithms for passive localization of near-field sources," in *Proc. Conf. Rec. 29th Asilomar Conf. Signals, Syst. Comput.*, vol. 2, Oct./Nov. 1995, pp. 777–781.
[17] H. C. So and F. K. W. Chan, "A generalized subspace approach for mobile positioning with time-of-arrival measurements," *IEEE Trans. Signal Process.*, vol. 55, no. 10, pp. 5103–5107, Oct. 2007.
[18] A. Zoubir, Y. Wang, and P. Chargé, "Efficient subspace-based estimator for localization of multiple incoherently distributed sources," *IEEE Trans. Signal Process.*, vol. 56, no. 2, pp. 532–542, Feb. 2008.
[19] Y. Wang and K. C. Ho, "TDOA source localization in the presence of synchronization clock bias and sensor position errors," *IEEE Trans. Signal Process.*, vol. 61, no. 18, pp. 4532–4544, Sep. 2013.
[20] W. Jiang, C. Xu, L. Pei, and W. Yu, "Multidimensional scaling-based TDOA localization scheme using an auxiliary line," *IEEE Signal Process Lett.*, vol. 23, no. 4, pp. 546–550, Apr. 2016.
[21] Y. Huang, J. Benesty, G. W. Elko, and R. M. Mersereati, "Real-time passive source localization: A practical linear-correction least-squares approach," *IEEE Trans. Speech Audio Process.*, vol. 9, no. 8, pp. 943–956, Nov. 2001.
[22] Y. T. Chan and K. C. Ho, "A simple and efficient estimator for hyperbolic location," *IEEE Trans. Signal Process.*, vol. 42, no. 8, pp. 1905–1915, Aug. 1994.
[23] K. W. Cheung, H.-C. So, W.-K. Ma, and Y.-T. Chan, "Received signal strength based mobile positioning via constrained weighted least squares," in *Proc. IEEE Int. Conf. Acoust., Speech, Signal Process.*, vol. 5, Apr. 2003, p. 137.
[24] K. W. Cheung, H. C. So, W.-K. Ma, and Y. T. Chan, "Least squares algorithms for time-of-arrival-based mobile location," *IEEE Trans. Signal Process.*, vol. 52, no. 4, pp. 1121–1130, Apr. 2004.
[25] K. W. Cheung, H.-C. So, W.-K. Ma, and Y.-T. Chan, "A constrained least squares approach to mobile positioning: Algorithms and optimality," *EURASIP J. Adv. Signal Process.*, vol. 2006, no. 1, pp. 1–22, 2006.
[26] H. C. So and L. Lin, "Linear least squares approach for accurate received signal strength based source localization," *IEEE Trans. Signal Process.*, vol. 59, no. 8, pp. 4035–4040, Aug. 2011.
[27] L. Lin, H. C. So, F. K. W. Chan, Y. T. Chan, and K. C. Ho, "A new constrained weighted least squares algorithm for TDOA-based localization," *Signal Process.*, vol. 93, no. 11, pp. 2872–2878, 2013.
[28] X. Qu and L. Xie, "An efficient convex constrained weighted least squares source localization algorithm based on TDOA measurements," *Signal Process.*, vol. 119, pp. 142–152, Feb. 2016.
[29] P. Strobach, "The fast quartic solver," *J. Comput. Appl. Math.*, vol. 234, no. 10, pp. 3007–3024, 2010.

YANJUN HU received the Ph.D. degree in signal and information processing from the China University of Mining and Technology (CUMT), in 2008. He is currently an Associate Professor with the School of Information and Control Engineering, CUMT. His research interests include device-based and device-free localization based on wireless sensors, wireless sensor networks, and their application in coal mines.

XUEMIAO YANG received the B.S. degree from the North China Institute of Science and Technology, Langfang, China, in 2018. She is currently pursuing the M.E. degree in information and communication engineering with the School of Information and Control Engineering, China University of Mining and Technology, Xuzhou, China. Her research interests include wireless sensor networks and localization algorithm.

CHUNYU YANG received the B.S. degree in applied mathematics and the Ph.D. degree in control theory and control engineering from Northeastern University, China, in 2002 and 2009, respectively. He is currently a Professor with the China University of Mining and Technology, Xuzhou, China. His research interests include the analysis and design of singularly perturbed systems, robust control, and fuzzy control.

SHIYIN LI received the Ph.D. degree in information and communication engineering from the China University of Mining and Technology, Xuzhou, China, in 2010. Since 2010, he has been a Professor with the School of Information and Control Engineering, China University of Mining and Technology. He is the Head of the Department of Information Engineering. His research interests include wireless communication and network congestion control.