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# The application of dynamic game theory to participant's interaction mechanisms in lean management

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ABSTRACT

This research focuses on the problems that enterprises are affected by multiple stakeholders in the process of lean management. Based on the dynamic game theory, through the construction of a three-stage game model, the article has researched on the interaction mechanism between internal and external stakeholders that affects the implementation of enterprise lean management. Through the research, the following conclusions are drawn: (1) At the beginning of lean management, hiring the third-party consultant experts to train employees, which is conducive to improving management efficiency; (2) the effort level of middle managers has nothing to do with the control of senior managers; (3) senior managers should not be too many constraints on third-party consultant experts; (4) senior managers should strive to coordinate the interests of all parties. The purpose of the paper is aimed to provide a reference of theory and practice of the implementation of lean management for enterprises.

## 1. Introduction

Lean management is derived from lean production, a type of production management based on the prototype of Toyota production system proposed by experts and scholars, and organized in 1990 at the Massachusetts Institute of Technology after more than 90 automobile factories were investigated and a comparative analysis was carried out around the world involving 17 countries in the International Motor Vehicle Program (IMVP). Womack et al., who put in place the guiding principles of lean manufacturing at the corporate strategy level, proposed the concept of lean thinking, and use it as the foundation of lean management, which applies lean thinking to enterprise management activities (Taylor, Taylor, & Mcsweeney, 2013). Lean management takes “full participation, continuous improvement, [eliminating] waste, [creating] value and [stimulating] positive energy” as its core. Its remarkable innovation performance has demonstrated its validity and advantages, and, led by the manufacturing enterprises, has changed the trajectory of the development of enterprises over the past few decades (Jasti & Kodali, 2015). A large number of successful cases of lean management practice show that lean management plays an important role in enhancing corporate governance innovation. Lean management can, that is, help enterprises to improve their levels of management innovation Jing, Niu, & Chang, 2015).

However, currently, the effects of the implementation of lean management are not ideal. Baker (2002) conducts an effect survey of enterprises implementing lean management in the United Kingdom and Australia, and shows that less than 10% of enterprises achieve a higher lean level. In an identical survey conducted in the US, only 26% of those companies implementing lean management obtain better lean implementation effects (Blanchard, 2007; Comm & Mathaisel, 2005). The lean management implementation status of enterprises is even less desirable in China; many enterprises see results after long periods of time, but it is difficult to achieve lean management in the continuing operations of enterprises and some enterprises even appear to exhibit the “retrograde step”, in which many aspects of production and operations return to the state prior to the introduction of the lean management model. There are two primary reasons why the effects of implementing lean management have not been significant: (1) knowledge of lean management is scarce, and (2) not enough staff members participate in lean management implementation.

In terms of lack of knowledge, at present, most enterprises rely on a team of third-party consulting experts to implement lean management. Thus, those participating in the process of lean management typically include not only the enterprises' employees but also a team of third-party consulting experts, in accordance with sub-level participants in lean management implementation including internal senior managers,

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middle managers, junior managers, basic production employees and the team of third-party consulting experts. Although one of the core ideas of lean management is full participation, the benefits obtained by each stakeholder differ when it comes to lean management implementation, and for this reason their participation levels also differ. In other words, both the employees and the team of third-party consulting experts are brokers, and according to the hypothesis of the "economic man", this allows participants to make choices to maximize their own various interests.

Based on this, we believe that internal and external participants have a certain impact on the implementation of lean management. However, the promotion of lean management depends on the synergy effect of stakeholders. According to previous research, most scholars stayed in finding the factors that influence the promotion of lean management and analyzed separately how they affect the promotion of lean management. Few scholars studied the interaction mechanism between these factors (Tortorella & Fogliatto, 2014; Tortorella, Marodin, & Fogliatto, 2015). That is how multiple stakeholders work together to influence the effectiveness of lean management. Therefore, in the process of lean management implementation, designing an effective lean management implementation strategy, which allows different actors to cooperate with one another and actively participate in promoting lean management implementation, effectively solves the problem surrounding the insignificance of implementing lean management. However, a prerequisite for the design of an effective strategy for lean management implementation is clear participant interaction mechanisms. For this reason, this paper analyzes participant's interaction mechanism during the lean management implementation process, laying the foundations for enterprises to effectively design strategies for lean management implementation.

Most enterprises' lean management implementation process participants are senior managers, middle managers, junior managers, basic production employees and a team of third-party consulting experts. In order to more clearly describe the participant's mechanisms during the lean management implementation process, this paper introduces a continuous dynamic game model to describe the implementation process so as to identify the mechanisms by which each participant operates in the process of the lean management implementation. Dynamic game theory is an effective way to explain the multi-stakeholder issue through an analysis of the mechanisms involved (Moffat & Medhurst, 2009; Adlakha, Joharib, & Weintraub, 2015). However, there are many participants involved in the process of lean management implementation. In relation to this concern, Niu, Jing and Yang (2015) shows that the difference in terms of the impact of lean management implementation between enterprise junior managers and basic production employees is small. Therefore, in this paper, we consider junior managers and basic production employees as participants (known hereafter as employees) for our analysis.

Moreover, in order to facilitate the application of dynamic game theory to an analysis of the above-mentioned problems, this study analyzes the following two aspects: (1) we do not consider the team of third-party consulting experts, but instead mainly consider internal participants, including senior managers, middle managers and employees to analyze the mechanisms of the lean management implementation process; (2) introducing external actors (the team of third-party consulting experts), from senior managers, employees and the team of third-party consulting experts to analyze the three participant's lean management mechanisms during the implementation process (specific ideas are shown in Fig. 1). The main innovations of this study are as follows:

(1) Analyzing each participating party's interests and demands in the enterprise's lean management implementation process;
(2) Clearing the mechanism of interactions between the participating parties in the enterprise's lean management implementation process;

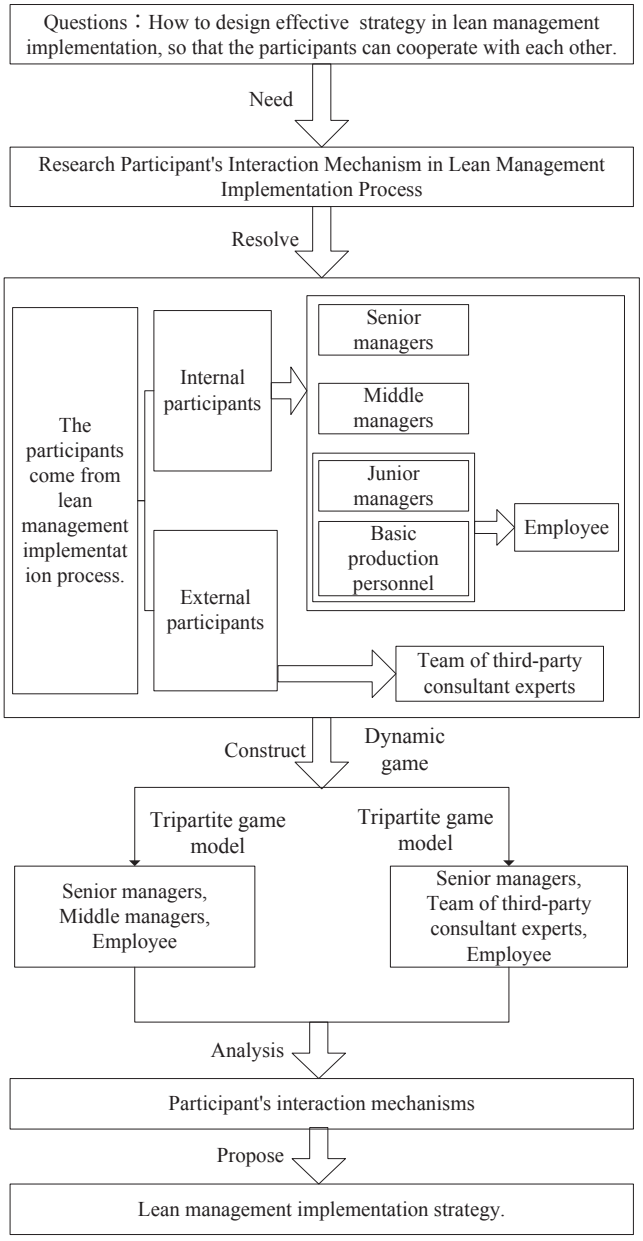


**Fig. 1.** The framework of participant's interaction mechanisms in the lean management implementation process.

(3) Defining the incentive policy for upper level management to the next level of managers in the process of lean management implementation;
(4) Proposing effective lean management implementation strategies.

The overall structure of this article is as follows. Section 2 presents the literature review, which is a major retrospective of lean management implementation, participants involved in the lean management implementation process, and dynamic game method. Section 3 describes the variables and assumptions involved in the research process. Section 4 constructs the game model that considers senior managers, middle managers and employees, as well as the game model that considers senior managers, employees and the team of third-party consulting experts. Section 5 presents the model's solution and analysis. Finally, conclusions and future research directions are presented in Section 6.

## 2. Literature review

### 2.1. The lean management implementation process

LAI (Lean Aerospace Initiative) put forward the concept of lean management as well as a collection of tools, techniques and methods to allocate resources reasonably to acquire maximum utility (Philip, 2004). Furthermore, LAI points out that lean management is not only a production management tool, but is also a series of management techniques and methods including operation options, management thoughts, plans and controls, logistics and supply management, quality management, cost control, inventory management, scene management, work improvement and so on (Darlington, Francis, & Found, 2015; Tillema & Steen, 2015). Numerous managers are now trying to introduce the concept into their enterprises. Various research has been conducted on lean implementation. Mathaisel proposes five steps of lean management implementation, including lean requirement analysis, lean concept design, lean plan making, actual operations and improvement. Wan and Frank (2008) concentrate on actual operations, and identify six steps: strategy, visual management, continual improvement, working standardization, pull-production and customer satisfaction. Others, Hodge, Goforth, and Joines (2011) divide the process into three steps: cultivating the lean culture, implementing lean techniques and tools, and realizing continuous improvement. Although, the processes are different in different cultures and different companies, there always remains something in common. Uwe, Mielke, and SSchulze (2011) puts forward a model of lean implementation including 4 stages and 9 steps. In the first stage, the aim is to establish the overall plan, from forming an intention to arriving at the final plan. Usually, the consultant team and top managers are involved. In the second stage, the aim is to change the organization. More details of the plan will be given at this stage, in which the consultant team, lean improvement team, and middle managers are involved. The third stage is the action stage in which staff from the improvement department, the consultant team and middle managers are involved. In the fourth stage and final stage, lean management operates throughout the entire company as a part of daily management. At this stage all staff members are therefore involved. These studies divide the lean implementation process based on actual operations, but fail to take the human-beings involved, the most significant resource, into consideration. For this point of view, Alagaraja (2014), Tortorella and Fogliatto (2014), Zhang and Niu (2013) analyze the learning organization and build a lean implementation model based on people, teams, organizations, and systems. Combining the previous studies and lean practice in China, Yang, Niu, and Jing (2016) emphasize “top-down” and “bottom-up” principles and “point, line, face, body” as four dimensions that provide direction for Chinese enterprises’ lean practice.

### 2.2. Participants in lean management implementation

The core of the lean implementation is total involvement. Only when the participants give their full effort will the implementation of lean management be a success. The participants include all staff members in the company and the external consultant team. In different periods, the primary participants and their effects differ (Maringarcia & Bonavia, 2015). This is because different participants with various demands play different roles in the process (Pei & Wang, 2016), and it is important to meet their needs during the process (Resta, Gaiardelli, and Dotti, 2015). Bortolotti, Boscari, and Danese (2014) find that organizational culture has a great impact on the behavior of participants (Martins, Costa Affonso, and Tamayo, 2015; Chavez et al., 2014). These studies have realized the importance of participants in the lean implementation process, but are less able to clarify their interactions and the ways in which they influence implementation practice.

### 2.3. Dynamic game

In dynamic games, each player acts in turn, and the actors after the pioneers are able to observe the actions of those who come before them prior to making a choice (Tabellini, 1986; Boucekkine, Krawczyk, & Vallée, 2011). Inverse solutions are always used to solve this kind of question (Mu & Guo, 2009; Khademi et al., 2015). Dynamic games can effectively bring together the multi-problems of various stakeholders’ interests and can analyze internal mechanisms. Dynamic game theory has been widely used in research on stakeholders, incentive systems, principal-agents and decision making, etc. (Zhu, Hossain, & Niyato, 2014). In order to increase the interests of the collective, cooperative game has attracted more and more attention (Fei, Li, & Ye, 2018). Therefore, this paper applies this method to clarify the action mechanisms of participants in lean management implementation.

In summary, the existing literature has focus on lean management ideas, lean management promotion methods, tools, etc., and has more in-depth discussion on the factors affecting the implementation of lean management. However, the literature has paid more attention to the single factors, few scholars study the interaction between these influencing factors. Therefore, based on the existing research, this paper uses the dynamic game method to study on the interaction mechanism of stakeholders in the process of lean management. The purpose is to provide a more in-depth understanding of the key influencing factors in the implementation of lean management and provide a theoretical reference for enterprises to continue to implement lean management.

## 3. Variables and assumptions

### 3.1. Variable descriptions

The relevant variables involved in the research process are shown in Table 1.

### 3.2. Assumptions

In order to analyze the mechanisms of internal participants within the enterprise during lean management implementation, without affecting the results of the analysis, for senior managers, middle managers and employees, this study proposes the following hypotheses:

(1) During the lean management implementation process, certain internal employees, such as a senior manager, a middle manager and other employees participate in implementation without consideration being paid to the differences between them.
(2) While there are the same working hours around lean improvement for employees, there are differences in effective working hours.
(3) Price and cost per unit of product are consistent around lean improvement, but the output of products increases after lean improvement without considering layoffs.
(4) Managers are risk-neutral.

In order to analyze the mechanisms of internal participants within the enterprise during lean management implementation, without affecting the results of the analysis, for senior managers, the team of third-party consultant experts and employees, this study proposes the following hypotheses:

(1) During the lean management implementation process, internal employees, including a senior manager and some employees, and external participants, including a team of third-party consultant experts, are considered as a whole without considering the differences between them.
(2) While there are the same working hours around lean improvement for employees, there are differences in effective working hours.
(3) Price and cost per unit of product are consistent around lean

**Table 1**
Relevant variable descriptions.

| Symbol | Variable meaning | Symbol | Variable meaning |
|---|---|---|---|
| $B$ | Value of entrepreneurs | $R$ | Reputation of middle managers |
| $Q$ | The output of products before lean management implementation | $R0$ | The initial reputation of middle managers |
| $Q'$ | The output of products after lean management implementation | $R'0$ | The initial reputation of the team of third-party consultant experts |
| $P$ | Price per unit of product | $R'$ | The reputation of the team of third-party consultant experts |
| $C1$ | Cost per unit of product | $r1$ | Effect of lean management implementation |
| $C2$ | Incentives from senior to middle | $r2$ | Effort of middle managers |
| $C3$ | Incentives to employees | $r3$ | Effort of employees |
| $C4$ | Lean consulting fees to the team of third-party consultant experts | $r4$ | Effort of the team of third-party consultant experts |
| $U1$ | Utility of senior managers | $E$ | Expected corporate layoffs |
| $U2$ | Utility of middle managers | $n0$ | Number of employees before lean management implementation |
| $U3$ | Utility of employees | $t0$ | Effective working hours before lean management implementation |
| $U4$ | Utility of the team of third-party consultant experts | $t$ | Effective working hours after lean management implementation |
| $\pi$ | Net income of senior managers | $B0$ | The initial value of the entrepreneur before lean management implementation |
| $S30$ | Wages of employees before lean management implementation | $K$ | Middle managers' degree of recognition to lean |
| $S20$ | Wages of middle managers before lean management implementation | $\theta$ | Senior managers' leadership to middle managers |
| $n$ | Number of employees after lean management implementation | $m$ | Influence from lean management to reputation of middle managers |
| $C40$ | Fixed costs of lean consulting fees to the team of third-party consultant experts | $\lambda$ | The ability of the team of third-party consultant experts |
| $c4$ | The effort cost of the team of third-party consultant experts | $c2$, $c3$, $b2$, $b3$, $\mu$, $tm$, $\delta$, $B1$, $\alpha$, $\beta$ | The correlation coefficient |

improvement, but the output of products increases after lean improvement without considering layoffs.
(4) Managers are risk-neutral.

## 4. Proposed model

### 4.1. Model for senior managers, middle managers and employees

(1) The key to a successful implementation of lean management is the joint effort of middle managers and employees, and if either fail to work hard then the enterprise is unable to achieve the effects of lean management implementation $r1$, so the effect of lean management implementation $r1$ and the product of the effort of middle managers $r2$ and the effort of employees $r3$ are positively correlated, that is:

$$r1 = r2r3 \tag{1}$$

(2) The utility of senior managers $U1$ is related to the value of entrepreneurs $B$, net income of senior managers $\pi$, etc. Moreover, the value of entrepreneurs $B$ is related to the initial value of the entrepreneur before lean $B0$ and the effect that lean management implementation bring to entrepreneurs is almost positive. So it is $B = B0 + B1r2r3$; net income $\pi$ is related to the output of products after lean management implementation $Q'$, price per unit of product $P$, cost per unit of product $C1$, incentives from senior to middle $C2$, incentives to employees $C3$, etc., that is $\pi = Q'(P - C1) - C2 - nC3$. So the utility function of senior managers is:

$$U1 = B0 + B1r2r3 + Q'(P - C1) - C2 - nC3 \tag{2}$$

(3) The value of the utility of middle managers $U2$ and the effect of lean management implementation is also captured through the recognition of the effects of lean management implementation on the part of middle managers. This can be expressed as $Kr2r3$, where $K$ refers to the degree of recognition. At the same time, the utility of middle managers $U2$ is also related to the reputation of middle managers $R$, wages of middle managers (this can also be seen in incentives from senior to middle management)$C2$, the cost of the effort made by middle managers, etc. The cost of the effort made by middle managers is usually related to the effort made by middle managers $r2$, and exhibits the law of increasing marginal. So the cost of the effort made by middle managers can be expressed as $\frac{1}{2}b2r2^2$. $r_2$ is influenced by the leadership of senior managers θ; that is, the greater senior managers' leadership in relation to middle managers, the harder middle managers work, and the greater the effort made is. So there is $r_2 = \theta r_{20}$, and $r_{20}$ is the effort made by middle managers in the case of complete freedom (that is, without pressure from senior managers). The reputation of middle managers $R$ can be expressed as $R = R_0 + m(r2r3 - r0)$. $R_0$ is the initial reputation of middle managers. $r_0$ is senior managers' expected lean efforts as targeted at middle managers. $r2r3$ is the effect of lean management implementation. $m$ is the effect of lean management on the reputation of middle managers. Incentives from senior to middle management $C2$, which refers to the wages of middle managers can be expressed as $C2 = S20 + c2r2$. Among them, $S20$ refers to the wages of middle managers before lean management implementation. $c2r2$ refers to the incentive wages based on the effort made by middle managers. $c2$ is an incentive coefficient. So the utility function of middle managers is $U2 = Kr2r3 + C2 + R - \frac{1}{2}b2r2^2$, that is:

$$U2 = Kr2r3 + S20 + c2r2 + R_0 + m(r2r3 - r0) - \frac{1}{2}b2r2^2 \tag{3}$$

$$U2 = K\theta r20r3 + S20 + c2\theta r20 + R_0 + m(\theta r20r3 - r0) - \frac{1}{2}b2\theta^2 r20^2 \tag{4}$$

(4) The utility of employees $U3$ is related to the wages of employees (this can also be seen in the form of incentives passed from middle management to employees) $C3$ and the cost of effort of employees $\frac{1}{2}b3r3^2$. Among them, the cost is usually related to the effort made by employees $r3$, and exhibits the law of increasing marginal. So the cost of the effort made by employees can be expressed as $\frac{1}{2}b3r3^2$. The wages of employees (this can also be seen as incentives passed down from middle management to employees) $C3$ can be expressed as $C3 = S30 + c3r3$. Among them, $S30$ refers to the wages of employees before lean management implementation. $c2r3$ refers to incentive wages that middle managers give to employees based on their level of effort. $c3$ is an incentive coefficient. So the utility function of employees is $U3 = C3 - \frac{1}{2}b3r3^2$, that is:,

$$U3 = C3 - \frac{1}{2}b3r3^2 = S30 + c3r3 - \frac{1}{2}b3r3^2 \tag{5}$$

(5) During the lean management implementation process, as companies gradually eliminate waste and standardize production processes with the application of lean management tools and methods, they are able to increase the effective working time of employees, improve production efficiency and increase the output of products. The effective working time of employees after lean improvement $t$ can be expressed as $t = t_0 + t_m r2r3$. $t_0$ is the effective working time before lean improvement. $t_m$ is the improvement in effective working time resulting from lean management implementation. According to $\frac{Q}{t0} = \frac{Q'}{t}$ (that is, output per effective working hours is unchanged), that is:

$$\frac{Q}{t0} = \frac{Q'}{t_0 + t_m r2r3}, Q' = \frac{t_0 + t_m r2r3}{t0} Q \tag{6}$$

$Q'$ is the output after lean management implementation among them.

(6) In the decision model with the participation of senior managers, employees and the third-party consulting team of experts, the decision variable of employee effort is $r3$ during the process of lean management implementation. The decision variable for the effort of middle managers is $r_{20}$ in cases of complete freedom (without pressure from senior managers). The decision variables for senior managers are the incentive coefficient of employees $c3$, incentive coefficient of middle managers $c2$, and leadership to middle managers θ, etc.

### 4.2. Model for senior managers, employees and the team of third-party consultant experts

(1) At present, due to a lack of lean management implementation personnel, during the implementation of lean management, most enterprises introduce an external team of third-party consultant experts. With the participation of senior managers, employees and the third-party consultant team of experts, the key to successful implementation of lean management is the joint effort of employees and the team of third-party consultant experts. If either party does not work hard enough then the effects of lean management implementation $r1$ are unachievable, so the effect of lean management implementation $r1$ and the product of the effort made by employees $r3$ and the efforts made by the team of third-party consultant experts $r4$ are positively correlated, that is:

$$r1 = r3r4 \tag{7}$$

(2) The utility of senior managers $U1$ is related to the value of entrepreneurs $B$, net income $\pi$, etc. Moreover, the value of entrepreneurs $B$ is related to the initial value of the entrepreneur before lean management implementation $B0$ and the effect that lean management implementation bring to entrepreneurs, which is positive. So there is $B = B0 + B1r3r4$. Net income $\pi$ is related to the output of products after lean management implementation $Q'$, price per unit of product $P$, cost per unit of product $C1$, incentives to employees $C3$, lean consulting fees to the team of third-party consultant experts, etc.; that is, $\pi = Q'(P - C1) - nC3 - C4$. So the utility function of the senior managers is:

$$U1 = B0 + B1r3r4 + Q'(P - C1) - nC3 - C4 \tag{8}$$

(3) The utility of the team of third-party consultant experts $U4$ is primarily related to the lean consulting fees paid to the team of third-party consultant experts $C4$, the reputation of the team of third-party consultant experts $R'$ and the costs of the effort made by the team of third-party consultant experts $c4$. Lean consulting fees paid to the team of third-party consultant experts $C4$ is decided by fixed income $C40$ and variable income according to the effect of lean management implementation $\mu r3r4$; that is, $C4 = C40 + \mu r3r4$. The reputation of $R'$ is related to initial reputation $R'0$, the efforts made by the team of third-party consultant experts $r4$ and the expected level of effort that senior managers hope the consulting team can achieve $r40$, such that $R' = R'_0 + \alpha(r4 - r_{40})$. $\alpha$ is a coefficient. The cost of the effort made by the team of third-party consultant experts $c4$ is usually related to the degree of effort $r4$, and exhibits the law of diminishing marginal utility, that is $c4 = \frac{1}{2} b r_4^2$. $b$ is an influence coefficient. So the utility of the team of third-party consultant experts $U4$ can be expressed as:

$$U4 = C4 + \beta R' - c4$$

$$U4 = C40 + \mu r3r4 + \beta R'_0 + \alpha\beta(r4 - r_{40}) - \frac{1}{2} b r_4^2 \tag{9}$$

(4) The utility of employees $U_3$ is related to the wages of employees $C3$ and the cost of the efforts made by employees $\frac{1}{2} b3r3^2$. Among them, this cost is usually related to the effort made by employees $r3$, and exhibits the law of increasing marginal. So the cost of the effort made by employees can be expressed as $\frac{1}{2} b3r3^2$. With the participation of the team of third-party consultant experts, the effort made by employees $r_3$ is often influenced by knowledge, such as the effort made by the team of third-party consultant experts $r_4$, the lean management implementation program proposed by the consulting team, lean management tools and methods, the introduced lean culture, management system, etc. So the effort made by employees $r_3$ can be expressed as $r_3 = r_{30} r_4^{\lambda}$. Among them, $r_{30}$ is the degree of effort made by employees in the absence of a team of third-party consultant experts. $\lambda$ refers to the knowledge or ability of the team of third-party consultant experts. The wages of employees $C_3$ can be expressed as $C_3 = S_{30} + c_3 r_3$. Among them, $S_{30}$ refers to the wages of employees before lean management implementation. $C_2 r_3$ refers to the incentive wages given to employees based on their degree of effort made $r_3$ during the process of lean management implementation. $c_3$ is the incentive coefficient of employees. So the utility function of employees is $U3 = C3 - \frac{1}{2} b3r3^2$; that is:

$$U3 = C3 - \frac{1}{2} b3r3^2 = S30 + c3r3 - \frac{1}{2} b3r3^2$$

$$U3 = S30 + c3r30r4^{\lambda} - \frac{1}{2} b3r30^2 r4^{2\lambda} \tag{10}$$

(5) During the lean management implementation process, as companies gradually eliminate waste and standardize their production processes by applying lean management tools and methods, they are able to increase the effective working hours of employees, improve production efficiency and increase the output of products produced. The effective working time of employees after lean improvement $t$ can be expressed as $t = t_0 + t_m r3r4$. $t_0$ refers to effective working hours before lean improvement. $t_m$ is the improvement factor of the effect of lean management implementation on effective working time. According to $\frac{Q}{t0} = \frac{Q'}{t}$(that is, output per effective working hours is unchanged), there are:

$$\frac{Q}{t0} = \frac{Q'}{t_0 + t_m r3r4}$$

$$Q' = \frac{t_0 + t_m r3r4}{t0} Q \tag{11}$$

where $Q'$ is the output after lean management implementation.

(6) In terms of the decision model constructed with senior managers, employees and the team of third-party consultant experts, the decision variable of employees is the effort made by themselves $r30$ in the absence of the team of third-party consultant experts. The decision variable for the team of third-party consultant experts is the effort made by themselves $r4$ during the process of lean management implementation. The decision variables of senior managers

are the incentive coefficient of employees $c3$, the variable income incentive coefficient of the team of third-party consultant experts $\mu$, etc..

## 5. Solution and analysis

### 5.1. Calculation and analysis for three participant model made up of senior managers, middle managers and employees

#### 5.1.1. Solution

The model is a three-stage dynamic game model, and using the reverse method, we are able to obtain a sub-game perfect Nash equilibrium.

Step 1: Solve the enterprise employee decision.

$\frac{\partial U3}{\partial r3} = c3 - b3r3$, make $\frac{\partial U3}{\partial r3} = 0$, obtain an outcome:

$$r3^* = \frac{c3}{b3} \tag{12}$$

$\frac{\partial^2 U3}{\partial^2 r3} = -b3 < 0$ can be multiplied, so $r3^*$ is the best decision-making requirement for enterprise employees.

Step 2: Solve the decision of middle managers.

In the utility function, the optimal decision of enterprise staff $r3^* = \frac{c3}{b3}$ is brought into the utility function of middle management $U2$ in order to arrive at a solution.

$\frac{\partial U2}{\partial r20} = \frac{c3}{b3}K\theta + c2\theta + m\theta\frac{c3}{b3} - b2\theta^2 r20$, make $\frac{\partial U2}{\partial r20} = 0$, obtain an outcome:

$$r20^* = \frac{(K+m)c3 + b3c2}{\theta b2b3} \tag{13}$$

$\frac{\partial^2 U2}{\partial^2 r20} = -b2^2\theta^2 < 0$ can be multiplied, so $r20^*$ is the best decision-making requirement for middle managers.

Step 3: It is now necessary to bring the optimal decision $r3^*$ of employees and the optimal decision $r20^*$ of middle management into the utility function $U1$ of senior management.

$\frac{\partial U1}{\partial c3} = \frac{B1(K+m)}{b2b3} + \frac{2tm(K+m)c3 + tmb3c2}{t0b2b3^2}Q(P-C1) - \frac{(K+m)c2}{b2b3} - \frac{2nc3}{b3}$, make $\frac{\partial U1}{\partial c3} = 0$, obtain an outcome:

$$c3 = \frac{[B1t0(K+m) + tmc2Q(P-C1) - t0c2(K+m)]b3}{2nt0b2b3 - 2tmQ(K+m)(P-C1)} \tag{14}$$

$\frac{\partial U1}{\partial c2} = \frac{B1}{b2} + \frac{tmc3}{t0b2b3}Q(P-C1) - \frac{(K+m)c3}{b2b3} - \frac{2c2}{b2}$, make $\frac{\partial U1}{\partial c2} = 0$, obtain an outcome

$$c2 = \frac{1}{2}[B1 + \frac{tmc3}{t0b3}Q(P-C1) - \frac{(K+m)c3}{b3}] \tag{15}$$

Step 4: Applying formulae (14) and (15) simultaneously, and considering $c2$ and $c3$ is

$$c2^* = \frac{2nB1t0^2b2b3 - B1t0^2(K+m)^2 - B1t0tmQ(K+m)(P-C1)}{4nt0^2b2b3 - 2t0tmQ(K+m)(P-C1) - tm^2Q^2(P-C1)^2 - t0^2(K+m)^2} \tag{16}$$

$$c3^* = \frac{B1t0(K+m)b3}{2nt0b2b3 - 2tmQ(K+m)(P-C1)} + \frac{[tmQ(P-C1) - t0(K+m)]b3}{2nt0b2b3 - 2tmQ(K+m)(P-C1)} * [\frac{2nB1t0^2b2b3 - B1t0^2(K+m)^2 - B1t0tmQ(K+m)(P-C1)}{4nt0^2b2b3 - 2t0tmQ(K+m)(P-C1) - tm^2Q^2(P-C1)^2 - t0^2(K+m)^2}] \tag{17}$$

#### 5.1.2. Equilibrium analysis

(1) During lean management implementation, we learn that the degree of effort made by enterprise staff is proportional to the incentive coefficient $c3$ of the employees' salaries, and is inversely proportional to the cost coefficient $b3$ of the enterprise employee's participation in lean management from $r3^* = \frac{c3}{b3}$. Since the lean cost coefficient $b3$ is primarily related to the understanding on the part of staff of lean management, the higher the degree of recognition, the smaller the lean cost coefficient. Therefore, in the initial stage of lean management implementation, one needs to improve staff awareness of lean management, and reduce the cost coefficient through training; or in cases where the level of awareness of employees is not high, enterprise managers need to improve employees' wage incentive coefficient $c3$ in order to promote employees to participate in lean management implementation.

(2) From the formula $r20^* = \frac{(K+m)c3 + b3c2}{\theta b2b3}$ it can be seen that the level of initiative of middle managers $r20$ is negatively related to the leadership of senior managers $\theta$. It can also be seen that the stronger the control of senior managers over middle managers, the lower the level of spontaneous effort made by middle managers in the lean management implementation process; on the other hand, the lower the control of top management over middle managers, the more freedom middle managers have and the more spontaneous is the effort made by middle managers. However, in the process of lean management implementation, the effort made by middle managers $r2 = r20^*\theta = \frac{(K+m)c3 + b3c2}{b2b3}$ has no relationship with the leadership $\theta$ of senior managers. This means that in the course of lean management implementation, the leadership of senior managers has only a very small effect on lean management and can be ignored during actual lean management implementation. Moreover, in terms of recognition levels of the effects of lean management on the part of middle managers $K$, the incentive coefficient of senior managers to middle managers $c2$, the influence $m$ of middle managers reputation and middle managers' efforts are positively correlated. The effort-cost coefficient $b2$ of middle managers is relative to the knowledge held by middle managers; the higher the degree of recognition of lean management, the smaller the effort-cost coefficient involved in lean management. Therefore, in the process of lean management implementation, especially in the early stages of implementation, we need to enhance the understanding of middle managers through training in order to reduce the cost factor. At the same time, in the early stages of lean management implementation, in order to improve the efforts made by middle managers, middle managers need to recognize the effects of lean management as early as possible, as this gives middle managers a higher incentive coefficient $c2$ to promote participation in lean management and thereby improve the recognition of its effects. At the same time, from $r2 = \frac{(K+m)c3 + b3c2}{b2b3} = \frac{(K+m)r3}{b2} + \frac{c2}{b2}$ we are able to see that the effort made by middle managers is positively correlated with the effort made by employees. Therefore, in the lean management implementation process, improving the level of effort made by enterprise staff and thereby improving the effort of middle managers, ultimately promotes lean management implementation and promotes the achievement of a good lean effect.

(3) By $c2^*$ and $c3^*$ we are able to see that the incentive coefficient that the senior managers have over middle managers bears a certain relationship to the incentive coefficient of senior managers over staff. Therefore we must take into consideration the relationship between the two coefficients before high level managers make specific incentive plans with middle managers and employees in order to enable lean management implementation to achieve optimal levels. If the incentive coefficient offered to middle managers and employee does not satisfy the relationship described in Eq. (15), then middle managers and employees will not put in their best effort and the lean promotion effect will not reach peak levels. At the same time, in the lean management implementation process, $c2^*$ and $c3^*$ will change with changes to the lean propulsive effect of middle managers' acceptance $K$, the effort-cost coefficient of employee participation in lean management $b3$, the effort cost coefficient of middle managers' participation in lean management $b2$, the lean promotion effect on the effective working time improvement factor $t_m$, the number of employees $n$, and the continuous advancement of the process of lean management implementation.

Therefore, during this process, senior managers ought to adjust their incentive coefficient in relation to management and employees.

### 5.2. Solving and analyzing the three participant model including senior management, employees and the team of third party consulting experts

#### 5.2.1. Solution

The model is a continuous game model, so the inverse solution method is used to obtain the Nash equilibrium of the sub-game.

Step 1: Solve the enterprise employees' decision.

$\frac{\partial U3}{\partial r30} = c3r4^{\lambda} - b3r4^{2\lambda}r30$, make $\frac{\partial U3}{\partial r30} = 0$, obtain an outcome:

$$r30^{*} = \frac{c3}{b3r4^{\lambda}} \tag{18}$$

Once $\frac{\partial^2 U3}{\partial^2 r30} = -b3r4^{2\lambda} < 0$ is established, $r30^{*}$ is the best decision-making requirement for enterprise employees.

Step 2: Solve the decision of the team of third party consulting experts.

The optimal decision of employees $r30^{*} = \frac{c3}{b3r4^{\lambda}}$ is brought into the utility function $U4$ of the team of third party consulting experts.

$\frac{\partial U4}{\partial r4} = \frac{\mu c3}{b3} + \alpha\beta - br4$, make $\frac{\partial U4}{\partial r4} = 0$, obtain an outcome:

$$r4^{*} = \frac{\mu c3 + \alpha\beta b3}{bb3} \tag{19}$$

Once $\frac{\partial^2 U4}{\partial^2 r4} = -b < 0$ is established, $r4^{*}$ is the best decision-making requirement for the third party advisory team.

Step 3: It is now necessary to bring the optimal decision $r30^{*}$ for employees and the optimal decision of the third party consultant team $r4^{*}$ into the benefit function $U1$ of senior managers.

$\frac{\partial U1}{\partial c3} = \frac{2\mu B1c3}{bb3^2} + \frac{2tm\mu c3 + tmb3\alpha\beta}{t0bb3^2}Q(P - C1) - \frac{\mu^2 c3 + \mu\alpha\beta b3}{bb3^2} - \frac{2nc3}{b3}$, make $\frac{\partial U1}{\partial c3} = 0$, obtain the outcome:

$$c3 = \frac{\mu\alpha\beta b3t0 - tm\alpha\beta b3Q(P - C1)}{2B1\mu t0 + tm\mu Q(P - C1) + 2nbb3t0 - \mu^2 t0} \tag{20}$$

$\frac{\partial U1}{\partial \mu} = \frac{B1c3^2}{bb3^2} + \frac{tmc3^2}{t0bb3^2}Q(P - C1) - \frac{\mu c3^2}{bb3^2} - \frac{\alpha\beta c3b3}{bb3^2}$, 令$\frac{\partial U1}{\partial \mu} = 0$, obtain an outcome:

$$\mu = B1 + \frac{tm}{t0}Q(P - C1) - \frac{\alpha\beta b3}{c3} \tag{21}$$

Step 4: Applying formulae (20) and (21) simultaneously, we are able to acquire the solution that

$$\mu^{*} = \frac{1}{2}(B1 + 1) + \frac{1}{2t0}\sqrt{(B1 + 1)^2 t0^2 + 8nb2b3t0^2 + 4B1t0tmQ(P - C1) + tm^2Q^2(P - C1)} \tag{22}$$

$$c3^{*} = \frac{2\alpha\beta b3t0}{B1t0 + 2tmQ(P - C1) - t0 - \frac{1}{t0}\sqrt{(B1 + 1)^2 t0^2 + 8nb2b3t0^2 + 4B1t0tmQ(P - C1) + tm^2Q^2(P - C1)}} \tag{23}$$

#### 5.2.2. Equilibrium analysis

(1) From $r30^{*} = \frac{c3}{b3r4^{\lambda}}$, we are able to see that the lean management implementation process that the third-party advisory team of experts participate in and the employees' willingness to participate in the level of lean management are not only proportional to the employee's incentive coefficient $c3$ but is in inverse proportion to the effort-cost factor $b3$ of the enterprise staff involved in the process, as well as by the effects of efforts and ability of the team of third party consulting experts. As usual, the higher the degree and ability of the third party advisory team's effort, the lower the degree of self-involvement in the process on the part of staff, revealing a phenomenon of mutual prevaricate between the team of third party consulting experts and employees during the process of lean management implementation. However, in the process of lean management implementation the final effort level of the enterprise's staff $r3 = r30r4^{\lambda} = \frac{c3}{b3}$ is a fixed value, which is not related to the degree of effort made by the team of third party consulting experts. Therefore, it is still necessary to improve the incentive coefficient $c3$ of staff and reduce the effort-cost factor $b3$ of enterprise employees involved in the process in order to enhance $r3$. Moreover, the lean cost coefficient $b3$ which refers to employees' participation is primarily related to employees' understanding of lean management. Therefore, if the awareness level of employees of lean management is not great, then it is first necessary to improve the employee incentive coefficient to promote employees to begin to participate in lean management implementation and to change employees' understanding through lean management implementation.

(2) From $r4^{*} = \frac{\mu c3 + \alpha\beta b3}{bb3}$ we are able to see that the degree of effort of the team of third party consultants experts is related to the degree of influence on the effectiveness of the team $\beta$. If the third party consulting team believes that the impact of their reputation on effectiveness is greater, then more effort will be required during lean management implementation. This situation reveals the reason that the third party consulting team enhances the effects of lean management through a doubling of efforts in cases where employee effort is not high during the process of lean management implementation. The reason for this is that the team of third party consulting experts is concerned with the impact of its reputation. At the same time, in order to improve the effort level of the team of third party consultation experts, senior managers need to improve the revenue sharing coefficient of the team of third party consultation experts $\mu$ as well as the efforts of employees $r3$.

(3) From $\mu^{*}$ and $c3^{*}$, as the revenue sharing coefficient $\mu$ of senior managers to the team of third party consulting experts increases, the incentive coefficient $c3$ of the need for enterprises to participate in lean management increases; there are various relations between them. Therefore, in the process of lean management implementation, in order to enable lean management implementation to achieve optimal levels, senior managers need to give overall consideration to the relationship between the two coefficients before making the decision to share revenue coefficients with the team of third party consultant experts and employees, and should make the two meet in the form of the relationship described in (21). Otherwise, the effect of lean management implementation is not optimal, and the effectiveness of senior managers is not at its maximum.

(4) From $\frac{\partial \mu^{*}}{\partial tm} > 0$, the greater the lean coefficient effect in terms of promoting the improvement of effective working hours $t_m$, the greater the revenue sharing coefficient of the third party consultation expert team in relation to senior managers. The team of third party consultant experts will make an effort to improve the effective working hours of employees by providing intellectual knowledge, such as high efficiency lean implementation plans, and lean management tools and methods to obtain higher returns on coefficients. Therefore, in the lean management implementation process, senior managers need not directly supervise and restrain the team of third party consultant experts, and should promote the efforts of the team of third party consultant experts by stimulating employees' efforts $r3$ indirectly.

## 6. Conclusion

In this paper, the dynamic game method is used to conduct an equilibrium analysis of three partners: including senior managers, middle managers and employees, and senior managers, middle managers and a team of third-party consultant experts. Some strategies are adopted for enterprise managers to enhance the effects of lean management implementation based on the above equilibrium analysis. Strategies are shown as follows:

(1) A team of third-party consultant experts should be engaged to undertake training activities on lean management for middle managers and employees, including junior managers and front-line staff, in order to enhance their levels of understanding of lean management, reduce effort costs, improve their level of effort, and promote the effects of lean management implementation especially during the preliminary stages of lean management implementation. (2) Although, less control exerted by senior managers on middle managers translates into higher effort levels for middle managers, the effort levels of middle managers has nothing to do with the control of senior managers. Given this, senior managers could improve the motivation of middle managers to mobilize their enthusiasm for participating in the promotion of lean management in the process of lean management implementation. (3) The effort levels of middle managers and the team of third-party consultant experts is improved by motivating the effort levels of employees, including junior managers and front-line staff, during the process of lean management implementation. At the same time, senior managers should not overly supervise and limit the team of third-party consultant experts, but should determine the appropriate income distribution coefficient to the team of third-party consultant experts. (4) When senior managers determine the income distribution coefficient for the team of third-party consultant experts and the motivating coefficient for middle managers and employees, including junior managers and front-line staff, they should co-ordinate relations between each coefficient to meet the relationships described as formulas including (16), (17), (22) and (23). At this point, the effects of lean management implementation will reach their best state. Furthermore, motivating coefficients should be adjusted adaptively to respond to relevant changing parameters in the promotion of lean management.

According to provide lean management consults for the companies, we find that the above alternatives have been taken effect in many companies implementing lean management. These companies include engine production enterprise founded in 1964, the First Automobile Group, China International Marine Containers Co., Ltd, Junlebao Dairy and so on. They can improve operational efficiency and increase more benefit by use of the above strategies in the process of lean management implementation.

In this study, the mechanisms of interaction for participants are defined in the process of lean management implementation. Based on this, strategies for enhancing enterprise lean management implementation effects are proposed, and significant breakthroughs are obtained. However, it is hard to implement simultaneously many alternatives for a company. The company need make an optimal decision by evaluating these alternatives. While, intuitionistic fuzzy sets, trapezoidal fuzzy numbers, intervals and real numbers are always used to represent multiple types of assessment information of alternatives' attributes given by the decision making (Wan & Li, 2013). Thus, the key is that how to deal the heterogeneous indicators so that making an ideal decision. In the future, we could focus on solving heterogeneous MADM based on the existing research (Yu, Li, & Fei, 2018; Yu, Fei, & Li, 2019). In future work we also consider: (1) an application of a case study in order to verify our conclusions subsequent to using dynamic game theory to analyze the mechanisms of interaction for participants. (2) Five participants from the process of enterprise lean management implementation could be synthetically analyzed in the future.

However, this paper also has some shortcomings. On the one hand, the research hypothesis itself has some limits such as ignored some individual influence factors. On the other hand, besides the third-party consultant experts, there are many other external participants that can make some impact to the implementation of lean management in enterprises. In the future, we can conduct more in-depth research on the above issues. As for the application of method that paper put forward, the authors have carried out some research and conducted empirical analysis with the companies such as FAW (First Auto Works) Car and CIMC (China International Marine Containers) to make the results more convincing. Because of the limit of length, further verification will be explained in the future article.

## CRediT authorship contribution statement

**Shuwei Jing:** Conceptualization, Methodology, Writing - original draft, Investigation. **Rui Li:** Validation, Formal analysis, Writing - review & editing. **Zhanwen Niu:** Supervision, Funding acquisition, Data curation. **Junai Yan:** Supervision, Project administration, Data curation.

## Declaration of Competing Interest

The author declare that there is no conflict of interest.

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