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HIGHLIGHTS

- A novel dynamic maintenance strategy is developed which can sequentially plan the system PM schedule based on the actual maintenance history and health information
- Based on the classic rolling horizon approach, both preventive and opportunistic maintenance strategies are integrated into the framework. Some drawbacks of this approach are overcome.
- Less mathematical modeling and computation efforts are required in the proposed strategy, which enables it to address the systems of large scale. An efficient dynamic programming algorithm is developed for optimization.
- The dynamic framework of the proposed strategy is flexible and can be further extended to CBM problems.

Dynamic maintenance strategy with iteratively updated group information

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Abstract

Maintenance grouping methods such as the rolling horizon approach are effective in reducing maintenance costs of multi-component systems. Despite the theoretical advancements of this approach, it still faces three challenges. First, the extensively adopted minimal repair assumption upon failures limits its application. Second, opportunistic maintenance upon corrective maintenance is overlooked, unable to fully take advantage of economic dependence. Third, maintenance plans are not based on actual maintenance history and health information, which may increase failure risks. To address these challenges, this paper formulates a novel dynamic planning framework that captures economic dependence in both preventive and opportunistic replacement. Unlike conventional approaches that restrict all maintenance activities into a finite planning horizon, our proposal focuses on activity-to-activity scheduling without specifying the horizon. As such, the subsequent maintenance schedule is dynamically updated once a system maintenance is executed. A flexible dynamic programming algorithm is developed to optimize the maintenance grouping, and the strategy framework is further extended to condition-based maintenance scenarios. The effectiveness and generality of the proposed maintenance strategy are demonstrated by numerical experiments.

Keywords: dynamic maintenance, opportunistic maintenance, maintenance grouping, multi-component system

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1. Introduction

Diverse industrial systems, such as smart grids, wind farms and high-speed trains are subject to multiple interdependencies existing among components or sub-systems [1]. Typically, there are three categories of dependencies, i.e. economic dependence [2], stochastic dependence [3], and structural dependence [4, 5]. Among them, the economic dependence attracts the most notable attention due to its significant impact on system operations & maintenance costs [6]. Such dependence allows to share set-up and downtime costs when multiple components are maintained simultaneously, so that maintenance resources can be significantly harnessed [7]. Group maintenance [2, 8-10] and opportunistic maintenance (OM) [11-14] are two representative maintenance policies taking advantage of the economic dependence. The former specifies a pre-determined schedule for inspections or preventive maintenance (PM), while the latter provides PM opportunities for other components when a component undergoes preventive or corrective maintenance (CM).

Notably, OM of multi-component systems is generally scheduled based on operational age and/or the reliability level of components. This triggers tremendous operational states and brings difficulties for the analytical modelling [11, 15, 16]. Consequently, many OM policies are optimized via simulations [11, 17, 18], which is trivial and time-consuming. In this regard, recently a few researches employed (deep) reinforcement learning (RL) methods to address this problem [19-22]. On the other hand, some group maintenance approaches, can also alleviate such problems with more mathematically convenient models and lower implementation difficulties [14, 17, 23]. A representative methodology is called "rolling horizon approach" [2], whose core idea is to partition the individual PM activities within the planning horizon into PM groups producing a fixed system maintenance schedule. A distinguished superiority of this approach is that it allows to analytically model the system of any scale but with much less computational resources. Recent advances of this approach include the application to condition-based maintenance (CBM) [24], negative economic dependence [25] and limited maintenance resources [26].

However, the existing rolling horizon approaches in realistic maintenance problems faces several challenges. First, the inherent relationships between two successive maintenance activities have not been taken into consideration, so that the health and grouping information are not dynamically updated. Specifically, let P1 and P2 be two consecutive PM activities of a component. After P1 is advanced to join a maintenance group, the execution time of P2 should be updated accordingly before grouping operation. Such update, however, is not considered in rolling horizon frameworks, which may increase failure hazards since the displacement is not penalized enough so that some PM intervals might be excessively long. This problem would be severer when failure is disastrous requiring immediate replacement, or in CBM problems where degradation information is dynamically updated. Second, most rolling horizon policies assumed a minimal repair upon failures to facilitate model constructions [2, 24-26]. Nevertheless, its application scenarios in realistic industrial systems could be limited due to two reasons: (a) repair effect is usually random and uncontrollable, and (b) a higher repair degree is more recommended in group maintenance. For instance, maintenance of offshore wind turbines is usually high due to the preparation of maintenance materials and the hiring of maintenance teams and vehicles. Thus, it is more cost-effective to achieve a better maintenance effect at each maintenance task. Third, most rolling horizon frameworks ignored the set-up costs in CM tasks and thus the positive economic impact of failure-induced OMs, which leads to an insufficient utilization of economic dependence [6].

This paper addresses the aforementioned challenges by constructing a novel dynamic maintenance framework, which extends the conventional rolling horizon approach from the perspective of both the industrial application and theory improvement. A notable superiority is its capability of maintenance information update, which effectively reduces malfunction risks. Importantly, the proposed maintenance policy always schedules the next maintenance group based on the actual maintenance history and health information once a system maintenance is completed. As a consequence, only the next PM points of all components are considered for grouping rather than all those in the rest of the horizon. Therefore, this policy can be viewed as a dynamic and flexible version of rolling horizon approach, which can be implemented iteratively until the end of the system service life without specifying and rolling the planning horizon.

The flexibility of the proposed approach also lies in its nice compatibility to OM scheduling, which further enhances its cost-effectiveness. Without the restriction of fixed PM schedules, our framework can naturally incorporate OM activities based on the iteratively updated information without establishing extra models. Worth noting that, OMs in this paper are triggered by component failures, which, to the best of our knowledge, have not been addressed in previous rolling horizon policies. Additionally, maintenance upon failure refers to spare part replacement instead of minimal repair, which is closer to actual maintenance scenarios (particularly for systems with diverse components) and easier to track. Last but not the least, the proposed framework can be naturally extended to CBM problems and RUL-based predictive maintenance with fully utilization of historical degradation information, which significantly enhances its application values.

The rest of the paper is organized as follows. Section 2 summarizes state-of-art of group and opportunistic maintenance. Section 3 introduces assumptions and basic concept of the dynamic maintenance strategy. Section

4 elaborates the dynamic grouping methodology and the corresponding optimization algorithms. An extension to condition-based maintenance is addressed in Section 5, and experimental validations are conducted in Section 6. Some final remarks are concluded in Section 7.

2. Literature review

Economic dependence is a crucial concept in maintenance of multiple-component system, which interprets the cost-effectiveness of jointly executing maintenance activities. Recent decades have witnessed numerous successful attempts to exploit economic dependence in the models. Substantially, two categories of policies are addressed the most (1) group maintenance, which groups PM activities producing a pre-determined maintenance schedule; (2) opportunistic maintenance (OM), where PM is performed at unscheduled opportunities.

2.1. Group maintenance

Group maintenance is well studied due to its implementation convenience and mathematical simplification. A classic version is block maintenance [14, 17, 23], where all components are preventively inspected/replaced at the multiple of a basic interval, such that PM of different components can share the same periods. Note that the rolling horizon approach [2, 24-26] also belongs to this category, which groups the PM activities using the cost-saving function concept. However, the pre-determined PM schedules cannot fully exploit the economic dependency since it overlooks extra opportunities of performing maintenance with less cost, e.g. unscheduled failure occurrence.

2.2. Opportunistic maintenance

As an alternative for group maintenance, OM policy performs PM on the survival components when facing system stoppage due to failure [11, 15], operational pattern [27] or other inevitable factors [28, 29]. Unlike group maintenance performing PMs at fixed schedules, OMs are usually executed based on system health conditions [29, 30], which complicates the model due to the randomness of opportunity arrival. Depending on the decision criterions, the OM policies are categorized into two classes:

(1) **Threshold-based OM**. Such policies set thresholds (e.g. age, degradation condition, etc.) to determine whether an OM should be accepted. In [29, 30], OM degradation thresholds were designed for random production interrupts and failures. [11] used ages as OM thresholds, where different thresholds were distinguished for components in failed or survival turbines. [31] employed the average failure rate as the threshold for OM in the multi-stage manufacturing systems. Other related work include: degradation threshold

[32], reliability threshold [15], time window [5], etc.

Despite threshold-based OM presents precise mathematical models, they suffer from the high analytical modeling and computation difficulty in handling complex systems. Therefore, many researches limited itself to two-unit systems [14, 33], employed simulations [5, 18, 32], or approximated OM opportunities arrivals by the Poisson process [16, 29, 30]. In addition, such policies are always a "static" strategy, which is optimal from the "mean" perspective, but not adaptive to the dynamic conditions and the individual information.

(2) **Measure-based OM**. OM decisions are made based on defined measures that reflect the effects of different decisions. [17, 34, 35] adopted the risk-based measure to compare the risks of replacements or not. [36] proposed a "cost-based group improvement factor" (GIF) and selected the OM group with the largest GIF. In [37, 38], the cost-saving/profit functions were defined to measure the effects of advancing PM to the current maintenance point. More related works in this class include [28, 35, 39].

Due to its dynamic nature, measure-based OM can take advantage of the updated and individual information, making a "reasonable decision" though not optimal in global sense. The model complexity is usually not much challenging, which enables it to handle large systems. Nevertheless, related works focused on optimizing the OM decisions without considering system PM plans in the same framework. In addition, some researches, e.g. [28, 37] only utilized the PM opportunities while overlooking failure opportunities. Advancing PM is most considered, which, however, overlooks the possibility of postponing them for life extension.

2.3. Reinforcement learning-based maintenance

With the continuous growth of system scale/complexity, industrial systems are suffering from extremely large health states, bringing difficulties to establishing conventional cost models. To address such challenges, recently a few researches employed a machine learning technology, namely, reinforcement learning (RL) to develop maintenance policies for multi-component systems [19-22].

RL policy is composed of the actions for each state, which is learned/optimized only from a multitude of actions-rewards pairs without complete models of the system [40]. In this manner, [20, 21] adopts the classic RL algorithms, e.g. Sarsa (λ) and Monte Carlo Control, to optimize the maintenance policy. However, for a high-dimensional state space, e.g. large-scale systems, classic RL algorithms may behave unstable producing inferior solutions or requiring intractable computational time [19, 22]. For this reason, deep reinforcement learning (DRL) is developed with the help of deep learning methods. Using this framework, [22] develops a DRL method combining the Q-learning with artificial neutral networks for maintenance of power grids. [19]

presents reviews of some RL/DRL methods and develops a new DRL approach. In comparison, RL/DRL-based methods are not restricted by the strategy framework and the model accuracy. A possible challenge is that, machine learning methods usually require complex parameters fine-tuning and network structure specifications to achieve ideal performance. Nevertheless, as most maintenance decision problems can be described by the state-action framework, RL/DRL-based methods are considered effective and promising.

In this work, the proposed measure-based strategy which can jointly optimize the OM decision and the future system PM schedule using the iteratively updated maintenance and health information. Both the OM and PM scheduling are developed in the same framework, and advancing or postponing PM activities are decided comprehensively to overcome the limitations in this class of strategy. Finally, the proposed strategy is a general framework with limited computational complexity suitable for addressing large systems with many components.

Nomenclature

n	Number of components in the system
$F_i(t)$ ($f_i(t)$)	CDF (PDF) of failure time of component i
$R_i(t)$	Reliability function of component i
τ_i^* (ϕ_i^*)	Optimal PR period (minimal cost rate) of component i
t_i^j (t_i^f)	Planned PR time of component i after the j^{th} system PM (after the current failure)
s_i^j (s_i^f)	Age of component i after the j^{th} system PM (after the current system failure)
$t_j^*(G)$	Optimal execution time of the PM group G after the j^{th} system maintenance
$H_i(t \mid s_i)$	Penalty function of component i given its current age s_i
$C_j(t,G)$	Cost-saving function of group G after j^{th} system maintenance
GS (GS^*)	Group structure (the optimal)
$CS_j(GS)$	Cost-saving function of the group structure GS after j^{th} system maintenance
t_f	System failure time
G_{OM}^*	Optimal OM group
S	Set-up cost
C_i^p (C_i^f)	PR (CR) cost of component i
$X_i(t)$	Degradation level of component i at usage time t
L_i	Failure threshold of component i

$g(x,h)$	PDF of the increment of Gamma process over interval length h
$F_i(t;x_0)$	RUL distribution function for component i given the current degradation x_0
τ	Baseline inspection interval
(τ_i, ξ_i)	Coordinated CBM policy for component i with inspection period τ_i and PR threshold ξ_i
x_0, t_0	Degradation level x_0 at the last inspection that occurs t_0 time before
$H_i\left(t \mid x_0, t_0\right)$	Penalty function of component i given the latest degradation information (x_0, t_0)
S_I (S_r)	Set-up cost for inspection-type mission (replacement-type mission)
C_i^I (C_i^r)	Inspection cost (replacement cost) of component i
C_d	Downtime cost per unit time before failure detection
C_f	Penalty cost due to unscheduled replacement mission

3. Problem statement

3.1. System assumptions

Consider a system with n independent components connected in series where each component is either "operational" or "failed". Failures are fatal and self-announcing, subject to an increasing failure rate. CM is performed once found failed, and PM is scheduled based on age. Some specific assumptions are outlined below.

- Both PM and CM on components are perfect, which can restore them back to the "as good as new" state.
- Both PM and CM require the shutdown of the system, and the downtime due to maintenance is negligible. Each maintenance incurs two types of costs: (a) common fixed set-up cost S, such as personnel and logistic cost, etc.; (b) component-specific maintenance cost.
- The component-specific maintenance cost generally includes spare part cost and material cost. The PM cost of component i is C_i^p, and the CR cost is C_i^f. Generally, $C_i^f \gg C_i^p$, $i = 1, 2, \cdots, n$, since failure is disastrous and costly.

Note that in order to distinguish maintenance actions for components and the system, we will call the component maintenance as "replacement", component PM as preventive replacement (PR), and component CM as corrective replacement (CR) in the rest of the paper. PM and CM are reserved for the system maintenance.

3.2. Maintenance strategy

The proposed maintenance policy consists of two phases. **Phase 1**, component-level individual planning, where PR of each component is optimized based on the operational age. If the age of component i,

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$i=1,2,\cdots,n$ reaches a threshold, it undergoes PR; otherwise if failure occurs first, CR is immediate. The optimization result of Phase 1 is then treated as input of **Phase 2**, system-level maintenance grouping, where the individual PR activities are grouped for system maintenance. In this phase, the future components PR time is rescheduled based on the maintenance history. On the other hand, the actual health information, e.g. usage age, is also considered. Two types of system maintenance grouping methods are developed.

(1) PM grouping

If no failure occurs at current maintenance, PM grouping is initiated to decide (1) the next system PM time, and (2) the set of components that should be preventively replaced at the next system PM, i.e. the PM group.

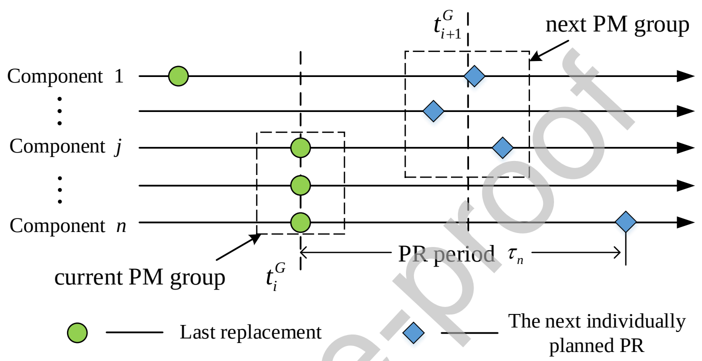

Fig. 1 Illustration of the preventive maintenance grouping

Notably, only the subsequent PR of each component is considered for grouping, which is different from the conventional approach where all PR activities in the planning horizon are grouped at a single time. As illustrated in Fig. 1, let G denote the subsequent group maintenance we intend to schedule. Then all PR activities after G depend on its execution time which is not known yet. Those PR activities should be updated each time the system information is updated when group G is executed. Besides, only the first group is selected for the next system PM with the same reasons. To conclude, all subsequent groups are dynamically updated and the pre-determined "time horizon" in rolling horizon approaches is no longer required.

(2) OM grouping

If a component fails before the scheduled PM time, CM is immediately triggered to replace the failed component and the OM grouping initiates, which provides the opportunity for PR of the surviving components. These components form the OM group. The OM group and the failed component constitute the CM group. Hence, the CM point is also a decision point, which decides (1) the OM group at the current failure time, (2) the next system PM time, and (3) the next PM group.

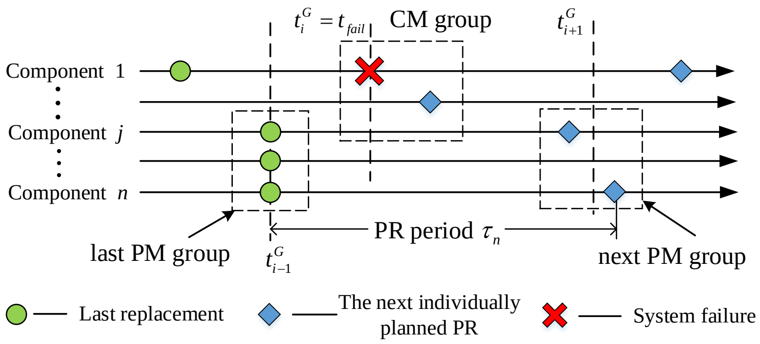

Fig. 2 Illustration of the opportunistic maintenance grouping

As shown in Fig. 2, once the OM group is fixed, the next individually planned PR time is updated accordingly. Then the subsequent system PM group is obtained similar to the "preventive maintenance grouping" case. Finally, these two phases are jointly optimized to determine the OM group, the next PM group and its execution time.

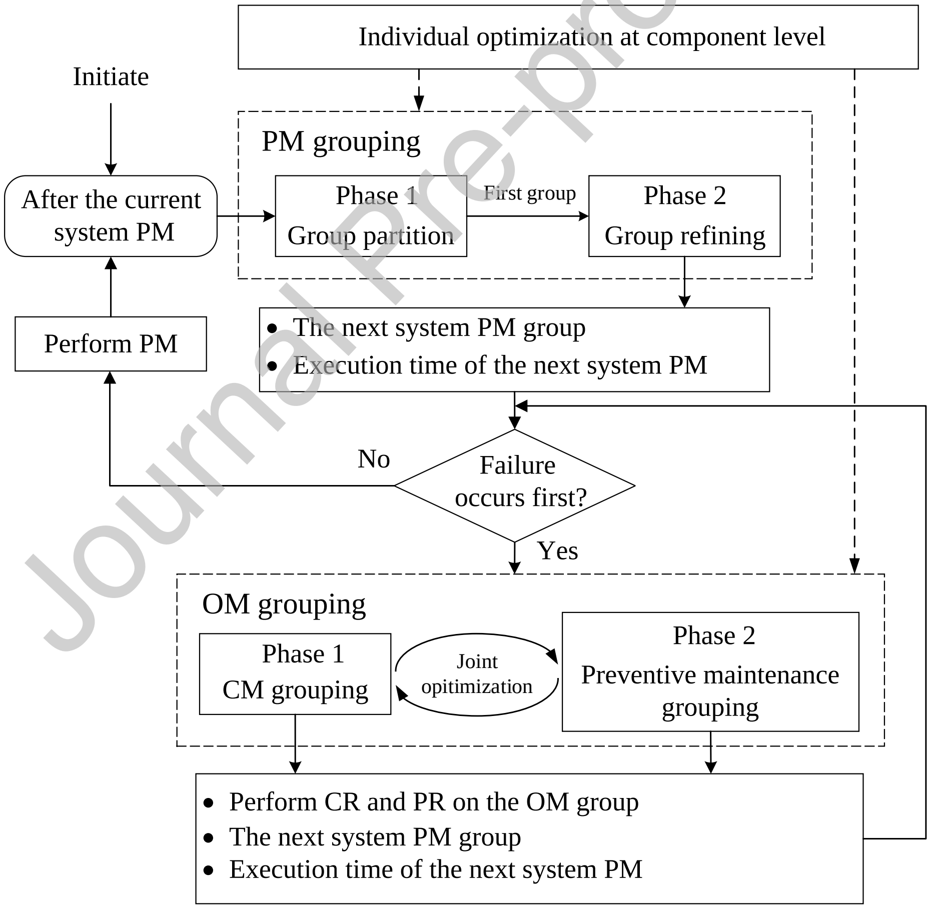

Fig. 3 The flowchart of the proposed system maintenance strategy

Fig. 3 presents the overall framework of the proposed maintenance strategy. The strategy rolls forward whenever the system is in a scheduled PM or a CM. If the system is maintained at the scheduled PM, the "PM grouping" is initiated to decide the subsequent system PM schedule (see Fig. 1); Otherwise if a failure occurs first, the "OM grouping" is triggered to decide the current OM group and the subsequent PM schedule (see Fig. 2). Although it is an age-based maintenance (ABM) framework, it can be easily revised for CBM problems, as we will show in Section 5.

4. Dynamic maintenance grouping approach

4.1. Component-level scheduling

Let τ denote the PR period of component i, C_i^p the PR cost, and C_i^f the CR cost. $F_i(t)$ ($f_i(t)$) is the distribution function (density function). The mean time between replacements ET_i is obtained by

$$ET_i = \int_0^\tau t f_i(t)dt + R_i(\tau)\tau = \int_0^\tau R_i(t)dt \tag{1}$$

where $R_i(t) = 1 - F_i(t)$ is the reliability function. Accordingly, the long-term cost rate $\phi_i(\tau)$ is

$$\phi_i(\tau) = \frac{EC_i}{ET_i} = \frac{C_i^p R_i(\tau) + C_i^f F_i(\tau)}{\int_0^\tau R_i(t)dt} \tag{2}$$

The optimal PR period τ_i^* of component i is obtained by minimizing $\phi_i(\tau)$. If $F_i(t)$ is a Weibull distribution function with an increasing failure rate, it is easy to verify that τ_i^* exists and is uniquely determined by the following equation

$$(C_i^f - C_i^p)\lambda_i(t)\int_0^t R_i(s)ds = C_i^p R_i(t) + C_i^f F_i(t) \tag{3}$$

where $\lambda_i(t) = f_i(t)/R_i(t)$ is the failure rate and the minimal cost rate is $\phi_i^* = \phi_i(\tau_i^*) = (C_i^f - C_i^p)\lambda_i(\tau_i^*)$.

4.2. Penalty function

If PR activities of multiple components are grouped, penalty is incurred for each component since its PR time is no longer optimal. The penalty cost is defined as the averaged additional cost when the PR time is shifted from its optimal point to t.

Suppose component i has age s_i, and the PDF of the failure time T_f is $f_i(t)/R_i(s_i)$, $t > s_i$. For simplicity, the subscript i is omitted in this subsection. In the following, we investigate the penalty function according to whether replacement is postponed or advanced.

(1) Postponement penalty

Depending on the failure time of component i, three sub-scenarios are involved, as shown in Fig. 4 (a).

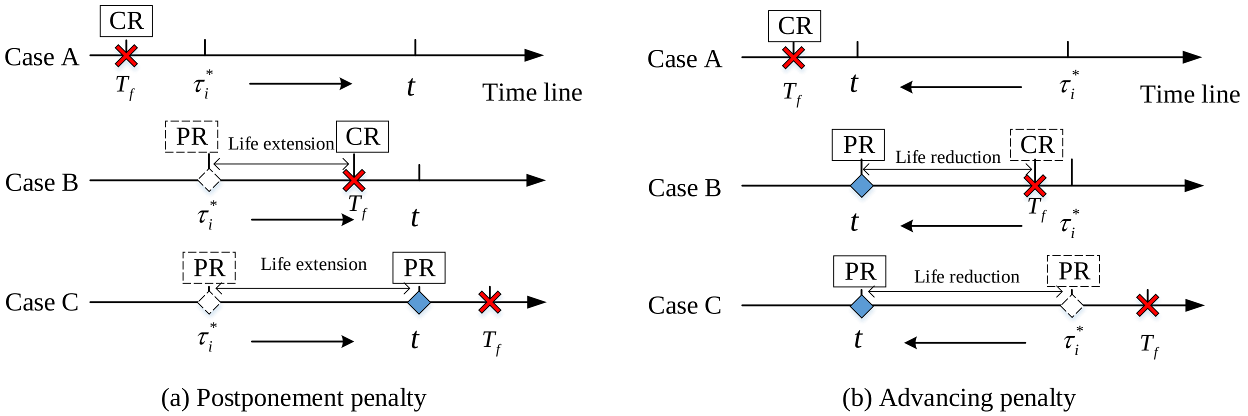

Fig. 4 (a) Penalty scenarios when replacement of component *i* is postponed; (b) Penalty scenarios when replacement of component *i* is advanced.

If T_f occurs before τ^* as in case A, the component fails whether PR is postponed or not. Therefore, the corresponding penalty function $H_i^A(t\,|\,s)$ is

$$H_i^A(t\,|\,s) = 0 \tag{4}$$

If T_f occurs between τ^* and t as in case B, the component would undergo CR, which results in an additional cost $C^f - C^p$ but with the extended lifetime by $T_f - \tau^*$. Therefore, the corresponding penalty function $H_i^B(t\,|\,s)$ is

$$\begin{aligned} H_i^B(t\,|\,s) &= (C^f - C^p)\frac{F(t)-F(\tau^*)}{R(s)} - \phi^* \int_{\tau^*}^{t} (u-\tau^*)\frac{f(u)}{R(s)}du \\ &= \frac{1}{R(s)}\left\{(C^f - C^p)\left[F(t)-F(\tau^*)\right] - \phi^*(t-\tau^*)F(t) + \phi^* \int_{\tau^*}^{t} F(u)du\right\} \end{aligned} \tag{5}$$

If T_f occurs after t as in case C, the component would also undergo PR at t, which extends the lifetime by $t-\tau^*$. Therefore, the corresponding penalty function $H_i^C(t\,|\,s)$ is

$$H_i^C(t\,|\,s) = -\phi^*(t-\tau^*)\frac{R(t)}{R(s)} \tag{6}$$

To sum up, if the PR is postponed to $t > \tau^*$, the penalty function $H_i(t\,|\,s)$ is formulated as

$$\begin{aligned} H_i(t\,|\,s) &= H_i^A(t\,|\,s) + H_i^B(t\,|\,s) + H_i^C(t\,|\,s) \\ &= \frac{1}{R(s)}\left\{(C^f - C^p)\left[F(t)-F(\tau^*)\right] - \phi^*(t-\tau^*)F(t) + \phi^* \int_{\tau^*}^{t} F(u)du - \phi^*(t-\tau^*)R(t)\right\} \\ &= (C^f - C^p)\frac{F(t)-F(\tau^*)}{R(s)} - \phi^* \frac{\int_{\tau^*}^{t} R(u)du}{R(s)} \end{aligned} \tag{7}$$

Substituting Eq. (3) into Eq. (7), $H_i(t\,|\,s)$ can be simplified as

$$H_i(t\,|\,s)=\frac{C^p+(C^f-C^p)F(t)-\phi^*\int_0^t R(u)du}{R(s)}\quad,\qquad t>\tau^* \tag{8}$$

(2) Advancing penalty

Similar to the postponement case, the penalty incurred by advancing replacement also has three sub-scenarios, as shown in Fig. 4 (b). If T_f occurs before t, the shift of PR makes no difference. If T_f occurs between t and τ^*, the component would undergo PR first, resulting in a cost-saving C^f-C^p but also a lifetime reduction. If T_f occurs after τ^*, PR is performed in advance but the lifetime is reduced.

Comparing Fig. 4 (a) and (b), the advancing scenario is the time-reverse version of the postponement one, which gives exact the same penalty form as in (8)

$$\begin{aligned}H_i\left(t\,|\,s\right)&=-(C^f-C^p)\frac{F(\tau^*)-F(t)}{R(s)}+\phi^*\frac{\int_t^{\tau^*}R(u)du}{R(s)}\\&=\frac{C^p+(C^f-C^p)F(t)-\phi^*\int_0^t R(u)du}{R(s)}\quad,\qquad t<\tau^*\end{aligned} \tag{9}$$

In conclusion, the penalty function of shifting PR time of component i to t conditional on age s is

$$H_i\left(t\,|\,s\right)=\frac{C^p+(C^f-C^p)F(t)-\phi^*\int_0^t R(u)du}{R(s)} \tag{10}$$

If no shift is made, $H_i(\tau^*\,|\,s)=0$ and no penalty is caused as desired. Furthermore, $H_i(t\,|\,s)$ is uniquely minimized at τ^* with $H_i(\tau^*\,|\,s)=0$. Hence $H_i(t\,|\,s)\geq 0$. The penalty increases with the shifting distance.

4.3. PM grouping

Suppose after the current (e.g. j^{th}) system maintenance at time t_G^j, the next scheduled PR time for component i is t_i^j, $i=1,2,\cdots,n$. Denote the age of component i after the j^{th} system PM by s_i^j. If component i is replaced, $t_i^j=t_G^j+\tau_i^*$ and $s_i^j=0$; otherwise, $t_i^j=t_i^{j-1}$ with $t_i^0=\tau_i^*$, $t_G^0=0$, and $s_i^j=s_i^{j-1}+t_G^j-t_G^{j-1}$ with $s_i^0=0$.

When a collection of components forms a system PM, the cost saving arises due to the sharing of the common set-up cost S at the price of penalty costs. Then the cost-saving function when simultaneously executing PR for group G at t, denoted as $C(t,G)$, is defined by

$$C(t,G)=\left(|\,G\,|-1\right)S-\sum_{i\in G}H_i(t\,|\,s_i) \tag{11}$$

where $|\,G\,|$ is the size of the group G, and s_i is the age of component i. Accordingly, the cost-saving function $C_j(t,G)$ after the j^{th} PM is given by

$$\begin{aligned} &C_j(t,G) = \left(|G|-1\right)S - \sum_{i\in G} H_i(s_i^j + t - t_G^j \mid s_i^j) \ , \quad t > t_G^j \\ &t_j^*(G) = \arg\max_{t>t_G^j} \ C_j(t,G) \end{aligned} \tag{12}$$

where $t_j^*(G)$ is the optimal execution time and $C_j^*(G) = C_j(t_j^*(G),G)$. On this basis, we can further investigate the optimal group structure for all components. A group structure GS is a partition of the component set $C = \{1,2,\cdots,n\}$ into a collection of mutually exclusive subsets $\{G_1, G_2, \cdots, G_m\}$, such that

$$\bigcup_{i\in C} G_i = C \quad \text{and} \quad \forall i \neq j \in C,\ G_i \bigcap G_j = \varnothing \tag{13}$$

Then the total cost saving of a group structure GS after j^{th} system PM is formulated as

$$CS_j(GS) = \sum_{G\in GS} C_j^*(G) = \sum_{G\in GS} \left[(|G|-1)S - H_i(s_i^j + t_j^*(G) - t_G^j \mid s_i^j)\right] \tag{14}$$

The objective is to determine the optimal group structure GS^* by maximizing the total cost saving, i.e.

$$GS^* = \arg\max_{GS} \ CS_j(GS) \tag{15}$$

The global optimization of group structure can be relaxed by adopting the “consecutive group structure” where each group is composed of a series of consecutive PR activities [2]. Despite the optimal solution of (15) might not be a consecutive structure, it is expected that the latter PR activity tends to join a latter group for less penalty, and thus the adoption is reasonable. A notable advantage of this structure is that it allows the optimization via the dynamic programming which can significantly reduce the optimization complexity. On the other hand, considering that the PM scheduling depends on the previous grouping decisions, finding the global optimal solution at each step would not bring significant improvement from long-term perspective.

For notational convenience, the subscript j is omitted in the following. Let $\{t_1, t_2, \cdots, t_n\}$ be the individually planned PR time of all components sorted in an ascending order corresponding to component index $\{i_1, i_2, \cdots, i_n\}$ and ages $\{a_1, a_2, \cdots, a_n\}$. The dynamic programming algorithm using backward-recursion is provided in Algorithm 1 (For more details please refer to Appendix A).

Algorithm 1: Dynamic programming for PM grouping

INPUT: $\{t_i, t_2, \cdots, t_n\}$, $\{i_1, i_2, \cdots, i_n\}$, and $\{a_1, a_2, \cdots, a_n\}$

OUTPUT: The maximum total cost-saving is $f_1(s_1)$, the first group is grp at time $t^*(grp)$.

Find all the possible states of each stage $k \in \{2,3,\cdots,n\}$;

$f_{n+1}(s_{n+1}) \leftarrow 0$, $\forall\, s_{n+1} \geq 1$;

FOR $k = n$ to 2, **DO**

FOR each state s_k in stage k , **DO**

$A_{s_k} \leftarrow \{ i_{k-s_k}, i_{k-s_k+1}, \cdots, i_k \}$;

Compare $g_0 \leftarrow C_j(t^*(A_{s_k}), A_{s_k}) + f_{k+1}(s_k + 1)$ and $g_1 \leftarrow f_{k+1}(1)$;

If $g_0 \geq g_1$, then $f_k(s_k) \leftarrow g_0$, $x_k(s_k) \leftarrow 0$; else, $f_k(s_k) \leftarrow f_{k+1}(1)$, $x_k(s_k) \leftarrow 1$;

END FOR

END FOR

$f_1(s_1) \leftarrow f_2(1)$

Initiate with $grp \leftarrow i_1$, $x_1(1) \leftarrow 1$ and $k \leftarrow 2$;

WHILE $x_k(k-1) = 0$

$grp \leftarrow grp \cup i_k$, $k \leftarrow k+1$;

END WHILE

The first group in GS^* is selected as a candidate group for system PM. However, to avoid “over grouping”, a further refinement of this group may be required, which is especially the case when PR intervals are small or the set-up cost is high, both leading to excessively advanced PR. Here we still adopt the consecutive structure, i.e. to reserve the former consecutive components in the group and exclude the remaining ones.

Suppose the candidate group is $G_1^* = \{ i_1, i_2, \cdots, i_m \}$ with size m , which is sorted according to the individually planned PR time $\{ t_i, i \in G_1^* \}$ in an ascending order. Let the former $j-1$ components form $A_{j-1} = \{ i_1, i_2, \cdots, i_{j-1} \}$. If t_j is larger than the next PR time of a component that belongs to A_{j-1} , e.g. component i , it would be better to group component j later, as the next PR of component i provides an opportunity with less advancing penalty. Following this criterion, all the components ordered after j are also excluded, suggesting that the exclusion can be conducted by iteratively checking the group backwards as in Algorithm 2.

Algorithm 2: The backwards-exclusion refinement

INPUT: the candidate PM group $G_1^* = \{ i_1, i_2, \cdots, i_m \}$ and $\{ t_i, i \in G_1^* \}$

OUTPUT: The next system PM group is grp and the execution time is $t^*(grp)$

FOR $j = m$ to 2, **DO**

$A_{j-1} \leftarrow \{i_1, i_2, \cdots, i_{j-1}\}$

Compare t_j and $t'(A_{j-1}) \leftarrow \min\{t^*_{A_{j-1}} + \tau_i \ , \ i \in A_{j-1}\}$;

If $t_j \geq t'_{A_{j-1}}$, $grp \leftarrow A_{j-1}$;

END FOR

Note that $t^*(grp)$ and the corresponding cost saving need not to be recalculated, since they were already obtained from Algorithm 1 (see Appendix A). The next system PM schedule is ultimately determined after this refining process.

4.4. OM grouping

OM grouping is performed as soon as the system fails, which jointly determines the OM group and the subsequent PM group. Suppose the current system failure time is t_f with the failed component h. If there exist components whose originally scheduled PR time is before t_f, they are offered an immediate replacement opportunity, and thus should be included into the OM group without penalty. Let set P_h denote these components, and update their next PR time. Let $C_h = \{1, 2, \cdots, n\} \setminus \{P_h \cup h\}$ denote the remaining components. The scheduled PR time and the current age for component $i \in C_h$ is t_i^f and s_i^f, respectively.

According to (9) and (11), if $G \subseteq C_h$, the cost saving of performing PR on group G at t_f is

$$C_f(G) = |G|S - \sum_{i \in G} H_i(s_i^f + t_i^f - t_f \mid s_i^f) \tag{16}$$

Following the consecutive group structure in Section 4.3, all the possible OM groups would be the collections of former components with positive cost savings. After the OM group is fixed, the ages s_i and the next individually planned PR time t_i, $i = 1, 2, \cdots, n$ are updated. Subsequently, the "PM grouping" is initiated to plan the next system PM schedule. Therefore, the OM group and the subsequent PM schedule are jointly optimized by maximizing the total cost savings, i.e.

$$C_{total}(G) = C_f(G) + CS^*(G) \tag{17}$$

where G is the OM group excluding P_h, and $CS^*(G)$ is the maximum total cost savings of the PM grouping conditional on the OM group G. Finally, the optimal OM group is $G^*_{OM} = P_h \cup G^*_f$, where

$$G^*_f = \arg\max_{G \subseteq C_f^{\max}} \ C_{total}(G) \tag{18}$$

The procedure of OM grouping is presented in Algorithm 3.

Algorithm 3: OM grouping

INPUT: failure time t_f, failed component h, set P, $\{t_i^f, s_i^f, i \in C_h\}$

OUTPUT: The OM group is $P \bigcup G_f^{\max}$, the next PM schedule.

Obtain the component series $\{i_1, i_2, \cdots, i_{n-1}\}$ by sorting $\{t_i^f, i \in C_h\}$ in ascending order;

$G_{\max} \leftarrow \varnothing$;

FOR k = 1 to $n-1$, **DO**

If $i_k \notin h \bigcup P$ and $C_f(i_k) \geq 0$, $G_{\max} \leftarrow G_{\max} \bigcup i_k$; else, break the loop;

END FOR

Initiation with $G_f^{\max} \leftarrow \varnothing$, $C_{total}^{\max} \leftarrow C_{total}(\varnothing)$;

FOR k = 1 to $|G_{\max}|$, **DO**

1. $G_f \leftarrow \{i_1, i_2, \cdots, i_k\}$, OM group $\leftarrow P \bigcup G_f$;
2. Update the ages, and the next individually PR time;
3. Perform the preventive maintenance grouping and calculate $C_{total}(G_f)$;
4. Compare with $C_{total}^{\max}$, update $G_f^{\max}$, $C_{total}^{\max}$ and the next PM schedule;

END FOR

5. Extension to condition-based maintenance

This section explores the application of the dynamic grouping framework in condition-based maintenance (CBM), which characterizes component condition via observable degradation signals/quantities instead of age.

Consider a system composed of n degraded components connected in series. For component i, $i = 1, 2, \cdots, n$, the degradation process is characterized by $X_i(t)$. When $X_i(t)$ reaches the failure threshold L_i, the component is considered as failed. Such degradation-based failure is referred as the soft failure [14, 18], which is usually non-fatal, not self-announcing and revealed only through inspections [14]

Without the loss of generality, degradation process $X_i(t)$, $i = 1, 2, \cdots, n$ is modelled by Gamma process due to its nice physical interpretation and extensive industrial applications [24], satisfying: (1) $X_i(t)$ has independent increments for any disjoint time intervals; (2) for any $t > 0$, $h > 0$, $X_i(t+h) - X_i(t)$ follows Gamma distribution with shape parameter α_i and scale parameter β:

$$g_i(x;h) = \frac{1}{\beta\Gamma(\alpha_i t)}\left(\frac{x}{\beta}\right)^{\alpha_i h-1} e^{-\frac{x}{\beta}} \qquad x > 0 \tag{19}$$

where $g_i(x;h)$ is the PDF of the degradation increment over $(t, t+h)$ and $\Gamma(a) = \int_0^{+\infty} x^{a-1}e^{-x}dx$ is the Gamma function. Therefore, the CDF of the remaining lifetime of component i given the current degradation level x_0 is obtained as

$$F_i(t;x_0) = P\left(X_i(t) > L_i\right) = \frac{\Gamma\left(\alpha_i t,\, \beta(L_i - x_0)\right)}{\Gamma(\alpha_i t)} \qquad t > 0 \tag{20}$$

with $\Gamma\left(u,v\right) = \int_v^{+\infty} x^{u-1}e^{-x}dx$ the incomplete Gamma function.

As the degradation/failure is hidden, inspections and maintenance require different resources, we need to dynamically decide which actions to take based on the collected information. The regarding assumptions are outlined below:

(1) Two types of maintenance missions are considered: inspections and replacements. The inspection (replacement) mission requires a set-up cost S_I (S_r), e.g. logistics of personnel and spare parts. S_I (S_r) can be shared if multiple inspections (replacements) are performed simultaneously. It is assumed $S_r \geq S_I$ since replacements work is normally more complex than inspections. Inspection/replacement duration is considered negligible.

(2) During the inspection-type mission, only inspections are conducted. Each component is periodically inspected with cost C_i^I, $i = 1, 2, \cdots, n$, and the inspection is perfect. When an inspection reveals a failure, a replacement mission is initiated immediately for failure replacement. In such case, the set-up cost S_r for replacements is required for another logistics demand. Besides, a penalty cost C_f due to logistics delay and emergency demands is charged.

(3) During the replacement-type mission, the designated components are replaced and inspections are also conducted opportunistically without charging S_I again. Replacement cost for component i is C_i^r.

(4) During the period of hidden failure before detection, a penalty cost C_d per unit time is incurred due to the system performance loss.

To avoid extra costs due to unscheduled replacement mission (failure replacement) and share set-up costs, the objective is to dynamically determine the maintenance schedule, including the mission type, time and the components for PR for the replacement mission, based on the actual degradation information. The maintenance strategy consists of two parts: (1) the off-line (static) block-inspection policy specifying the inspection interval for each component; (2) the online (dynamic) PM/OM maintenance strategy.

5.1. Block-inspection policy

Due to the economic dependency among inspections, block-inspection policy [41] is employed such that the inspection interval of each component as integer multiple of the smallest one among them, i.e. the baseline interval. As such, multiple components could be inspected in a same maintenance window sharing set-up costs.

Step 1: Individual optimal inspection

For component i, $i = 1, 2, \cdots, n$, the (s, ξ)-policy is adopted: The component is inspected periodically. Once the inspected degradation is over the failure threshold L_i, a replacement mission is immediately triggered with costs $S_r + C_i^r + C_f$; if the inspected degradation is over PR threshold ξ but less than L_i, the replacement mission is scheduled at the next inspection interval.

Denote the inspection interval by s and the failure time of component i by T_i. The probability distribution of inspection intervals number N_i^{PR} before a scheduled replacement mission is

$$p_{i,k}^{PR} = P\left(N_i^{PR} = k\right) = \begin{cases} 0 & \text{if } k = 1 \\ G_i(L_i) - G_i(\xi) & \text{if } k = 2 \\ \int_0^{\xi} \left(G_i(L_i - x) - G_i(\xi - x)\right) g_i^{k-2}(x)dx & \text{if } k \geq 3 \end{cases} \tag{21}$$

where $G_i(x) = \int_0^x g_i(u; s)du$ obtained by Eq. (19), $\bar{G}_i(x) = 1 - G_i(x)$, $s_k = ks$, and $g_i^k(x) = g_i(x; ks)$.

Similarly, the probability distribution of inspection intervals number N_i^{CR} before the unscheduled replacement (failure replacement) mission is

$$p_{i,k}^{CR} = P\left(N_i^{CR} = k\right) = \begin{cases} P\left(X_i(s) \geq L_i\right) = \bar{G}_i(L_i) & \text{if } k = 1 \\ \int_0^{\xi} \bar{G}_i(L_i - x) g_i^{k-1}(x)dx & \text{if } k \geq 2 \end{cases} \tag{22}$$

Meanwhile, the expected downtime T_d before failure detection is

$$\begin{aligned} ET_d &= \sum_{k=1}^{+\infty} E(T_d \mid s_{k-1} < T_i < s_k) P(s_{k-1} < T_i < s_k) \\ &= \sum_{k=1}^{+\infty} \int_{(k-1)s}^{ks} (ks - x) f_i(x)dx \end{aligned} \tag{23}$$

where $f_i(x) = dF_i(t; 0)/dt$ is the PDF of the failure time. Based on Eq. (21)-(23), the long term cost rate of component i is

$$\phi_i(s,\xi)=\frac{E\left\{Replacement\ cycle\ costs\right\}}{E\left\{Replacement\ cycle\ length\right\}}$$
$$=\frac{S_r+C_i^r+C_f\sum_{k=1}^{+\infty}p_{i,k}^{CR}+(C_i^I+S_I)\sum_{k=1}^{+\infty}\left\{(k-1)p_{i,k}^{PR}+kp_{i,k}^{CR}\right\}+\sum_{k=1}^{+\infty}\int_{(k-1)s}^{ks}(ks-x)f_i(x)dx}{s\sum_{k=1}^{+\infty}k\left(p_{i,k}^{PR}+p_{i,k}^{CR}\right)} \tag{24}$$

By minimizing $\phi_i(s,\xi)$, the optimal inspection period s_i^* and PR threshold ξ_i^* of component i is obtained as

$$(s_i^*,\xi_i^*)=\underset{s>0,\ 0<\xi<L_i}{\arg\min}\ \phi_i(s,\xi) \tag{25}$$

Step 2: Inspections synchronization

Here we adopt the method in [41] to optimize the inspection policy. According to the block-inspection policy, the inspection interval of component i, $i=1,2,\cdots,n$ should be $\tau_i=k_i\tau$, $k_i\in N^+$. Thus, it is reasonable to find k_i such that the penalty of moving away from s_i^* is minimal:

$$k_i=\begin{cases}\left\lfloor s_i^*/\tau\right\rfloor & \text{if } G_i\left(\left\lfloor s_i^*/\tau\right\rfloor\right)<G_i\left(\left\lceil s_i^*/\tau\right\rceil\right)\\ \left\lceil s_i^*/\tau\right\rceil & \text{otherwise}\end{cases} \tag{26}$$

Given that the inspection interval is adjusted to $k_i\tau$, the corresponding PR threshold is adjusted to $\xi_i=\xi_i(k_i\tau)=\underset{0<\xi<L_i}{\arg\min}\ \phi_i(k_i\tau,\xi)$. Hence, the total long-term cost rate of the system $\phi_s(\tau)$ and the corresponding optimal policy parameters are respectively given by

$$\begin{aligned}&\phi_s(\tau)=\frac{S_I}{\tau}+\sum_{i=1}^{n}\left[\phi_i(k_i\tau,\xi_i)-\frac{S_I\sum_{j=1}^{+\infty}\left\{(j-1)p_{i,j}^{PR}+jp_{i,j}^{CR}\right\}}{k_i\tau\sum_{j=1}^{+\infty}j\,(p_{i,j}^{PR}+p_{i,j}^{CR})}\right]\\&\tau^*=\underset{\tau>0}{\arg\min}\ \phi_s(\tau)\end{aligned} \tag{27}$$

Based on Eq. (26) and (27), the inspection intervals of all components are determined.

5.2. PM/OM maintenance scheduling

According to Assumptions (1)-(3), grouping replacement missions together is needed for two reasons: (1) to save the high replacement set-up costs; (2) to advance or postpone the scheduled replacement missions based on the degradation information. The latter reason indicates that shifting the scheduled PR away may not incur penalties, because the current degradation can be too close to the failure threshold and advancing the replacement can be beneficial. Based on the grouping results, we dynamically determine the type of the next maintenance mission. The grouping approach is similar to that in ABM except for some key differences, as we will elaborate in the rest of the section.

5.2.1. Penalty function

For simplicity, we consider component i for example but omit the index i when it is not necessary in this subsection. Let the current time be the time origin, prior to which the latest inspection is conducted t_0 time units earlier revealing an degradation x_0. According to Eq. (20), the CDF (PDF) of the remaining useful lifetime since the last inspection is $F(t) = F_i(t; x_0)$ ($f(t)$). Two types of components are defined: (1) Type I: components whose PRs are scheduled at their next inspection points; (2) Type II: components whose PRs are not scheduled yet. Accordingly, two classes of penalty functions are considered:

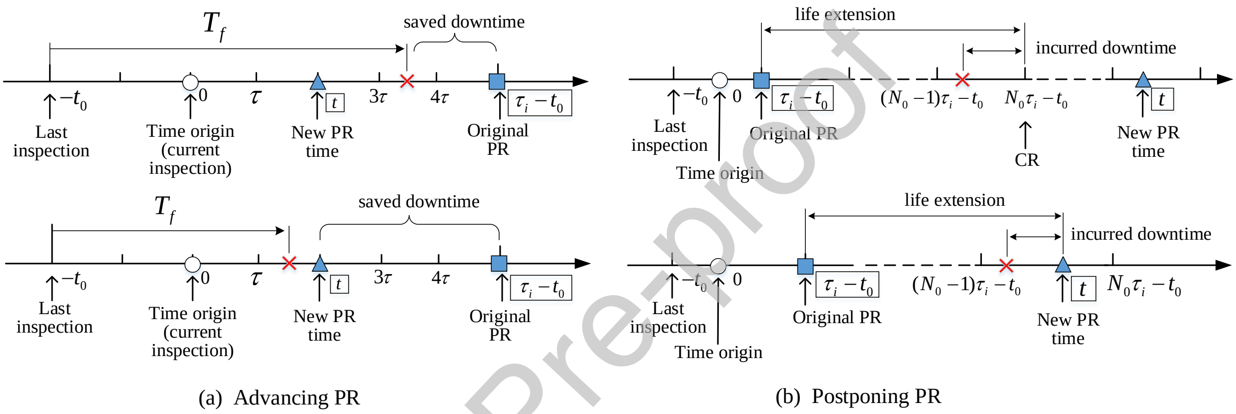

Fig. 5 Penalties for components scheduled for PR at its next inspection interval
(a) advancing PR (b) postponing PR

(1) Penalty function for Type I components

a) Advancing penalty

In this subcase, $t \in [0, \tau_i - t_0)$ during which no inspections are available. Denote by T_f the failure time since the last inspection (conducted at $-t_0$). As shown in Fig. 5 (a), the advancing penalty is formulated as

$$\begin{aligned} H_i(t \mid x_0, t_0) &= E\left[-C_d \min(\tau_i - T_f, \tau_i - t_0 - t) + \phi^*(\tau_i - t_0 - t)\right] \\ &= -C_d\left[\int_{t+t_0}^{\tau_i} (\tau_i - s) f(s) ds + (\tau_i - t_0 - t)\int_{t+t_0}^{\tau_i} f(s) ds\right] + \phi^*(\tau_i - t_0 - t) \\ &= \phi^*(\tau_i - t_0 - t) - C_d \int_{t+t_0}^{\tau_i} F(s) ds \end{aligned} \tag{28}$$

In contrast to the penalty function in ABM, $H_i(t \mid x_0, t_0)$ is not always positive since it is dependent on the inspection result x_0. If x_0 is close to the failure threshold, it would be beneficial to advance the PR. In this manner, the actual degradation information is utilized to develop a more effective maintenance schedule.

b) Postponement penalty

In this subcase, $t \in (\tau_i - t_0, +\infty)$ and inspections could be conducted before the postponed replacement.

Suppose t is located between the $h-1^{th}$ and h^{th} inspection interval since the current time, i.e. $t\in\left((h-1)\tau_i-t_0, h\tau_i-t_0\right)$, $h\geq 2$. As shown in Fig. 5 (b), inspections are available to detect the possible failure, so that CR could be performed first. Therefore, the inspection intervals number N_0 since the current time is introduced to denote the largest delayed time before CR. Clearly,

$$p_m = P(N_0=m) = F(m\tau_i) - F(\,(m-1)\tau_i) \qquad m\geq 1 \tag{29}$$

Thus, the postponement penalty can be formulated as

$$\begin{aligned} H_i\left(t\mid x_0,t_0\right) = E\Big\{ & C_f I_{N_0\leq h-1} - \phi^* \min\{(N_0-1)\tau_i,\, t\} \\ & + C_d\left[I_{N_0\leq h-1}(N_0\tau_i - T_f) + I_{N_0=h}\min\{N_0\tau_i - T_f,\, N_0\tau_i - t_0 - t\}\right]\Big\} \end{aligned} \tag{30}$$

where I_A is the indicator function such that $I_A=1$ if condition A satisfies; $I_A=0$ otherwise. The first term is the penalty cost upon failure; the second term is the cost savings due to life extension; the third term is the downtime time costs dependent on when the failure occurs. Based on Eq. (29), Eq. (30) can be obtained as

$$\begin{aligned} H_i\left(t\mid x_0,t_0\right) = {} & C_f\sum_{m=2}^{h-1} p_m - \phi^*\sum_{m=2}^{+\infty}\min\{(m-1)\tau_i,\, t\}\, p_m + C_d\sum_{m=2}^{h-1}\left[m\tau_i p_m - \int_{(m-1)\tau_i}^{m\,\tau_i} sf(s)ds\right] \\ & + C_d\left(h\tau_i p_h - \int_{t_0+t}^{h\,\tau_i} sf(s)ds\right) + C_d\,(h\tau_i - t_0 - t)\, p_h \end{aligned} \tag{31}$$

where the sum $\sum_{m=a}^{b} = 0$ if $b<a$.

(2) Penalty function for Type II components

For this class of components, PR time is a random variable depending on the collected degradation information, which significantly complicates the problem. By displacing the PR to a specified time t, it could be either an advancing case or a postponement case. Therefore, the penalty function would be the sum of both cases mentioned above. The details for obtaining $H_i\left(t\mid x_0,t_0\right)$ are given in Appendix B.

Finally, the maintenance mission at the original PR time would degrade to an inspection mission due to PR displacement. Therefore, the cost-saving function of a replacement group G at time t is formulated as

$$C(t,G) = (|G|-1)(S_r - S_I) - \sum_{i\in G} H_i(t\mid x_i^0, t_i^0) \tag{32}$$

where x_i^0 is the degradation level of component i revealed at its last inspection t_i^0 before the current time.

5.2.2. PM/OM grouping

At each inspection point $k\tau$, $k=1,2,\cdots$, the degradation information is updated and the maintenance grouping procedures are performed. The inspection points are also known as the "decision points" in Section 3.2. Specifically, if no failure occurs at the current inspection, we perform the "PM grouping"; otherwise the "OM grouping" is performed, as illustrated in Fig. 6.

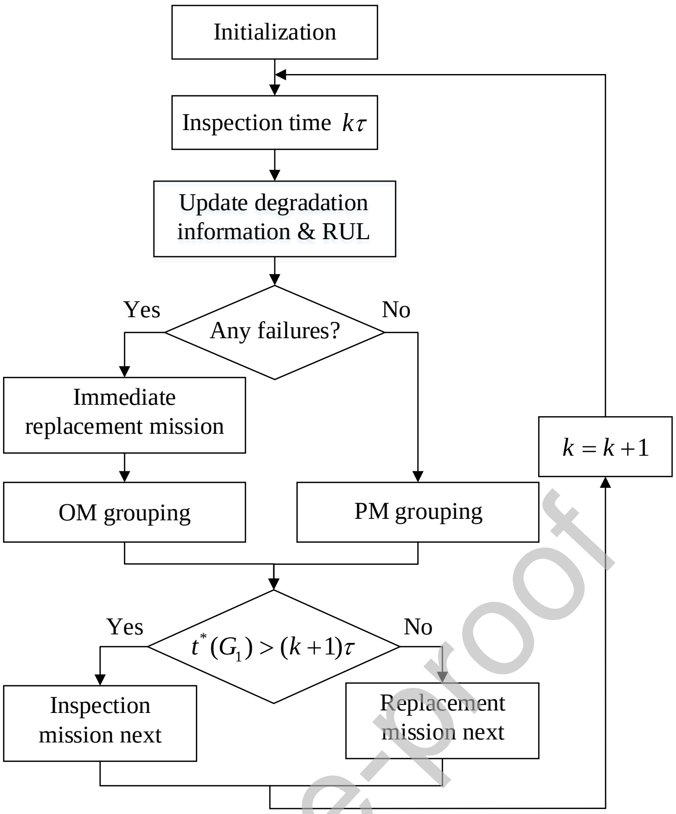

Fig. 6 Flowchart of the dynamic CBM policy based on the proposed maintenance grouping method

The grouping methods in both cases are essentially same to those in Section 4.3 and 4.4, except that the execution time of each group is restricted to an inspection point so that the inspections and replacements share the set-up costs. Based on the grouping results, we also only adopt the first group as the next replacement group. If it is scheduled at the next basic inspection interval (τ time units later), the next maintenance mission is designated as a "replacement mission"; otherwise an "inspection mission". Such a maintenance plan enables us to make the preparation for each type of maintenance in advance so as to reduce maintenance/logistics delay.

6. Experimental validation

6.1. Numerical example

The case considered in this study can refer to the maintenance of offshore windfarms [6, 42]. Since the turbines are installed at least several kilometers away from the shore, the maintenance team must approach the turbines by helicopters or specialized vessels regardless of maintenance types, either preventive or corrective. Under such circumstance, the economic dependence exists in both PM and CM, which incurs an OM problem. Based on this background, our model simplifies this application situation and focuses on the method of maintenance scheduling.

Suppose a series system composed of $n = 8$ components, and the failure time of component i, $i = 1, 2, \cdots, n$ follows Weibull distribution with shape $\beta_i > 1$. The set-up cost is $S = 10$. Table 1 presents the distribution/cost parameters of each component, the optimal PR period τ_i^* and the minimal cost rate ϕ_i^* obtained from Eq. (2) and (3).

Table 1 Parameters of the 8 components and individually optimized PR policy

Component	β_i	η_i	C_i^p	C_i^f	τ_i^*	ϕ_i^*
1	2.7	18	50	1000	5.33	17.98
2	3	30	56	1120	9.44	10.53
3	3	58.5	100	2000	17.98	9.21
4	2.7	30.8	80	1600	8.90	16.14
5	3	48.5	70	1400	15.10	7.98
6	2.75	25	70	1400	7.35	17.18
7	2.5	15	40	800	4.31	19.48
8	2.5	38	60	1200	10.61	11.06

Let the system service life $T = 30$. Using the methods presented in Section 4.3, a series of executed maintenance groups without failures are given in Table 2. According to Table 1, the individually scheduled PR time for all components sorted in ascending order is {4.31, 5.33, 7.35, 8.90, 9.44, 10.61, 15.10, 17.98} corresponding to components {7, 1, 6, 4, 2, 8, 5, 3}. The first two components, i.e. {7, 1}, is close enough for grouping together. Similarly, the second group only contains {6}. As for the third group, the PR time of component {4, 2, 8} is close. However, components {7, 1} have already been replaced in the first group and their PR time is updated at 9.07 and 10.09, respectively. Therefore, {4, 7, 2, 1, 8} are grouped altogether. As an example, Fig. 7 illustrates the implementation of the preventive maintenance planning after the second system PM. Only G_1 is selected as the candidate for the next system PM.

Table 2 The executed maintenance groups without failures

Number	Type	Group	execution time	Cost savings
1	PM	{7, 1}	4.76	63.64
2	PM	{6}	7.35	0
3	PM	{4, 7, 2, 1, 8}	9.50	62.76
4	PM	{7}	13.81	0
5	PM	{6, 1, 5, 3, 7, 4, 2, 8}	16.81	25.79
6	PM	{7, 1}	21.57	55.74
7	PM	{6, 4, 7, 2, 1, 8}	25.91	39.23

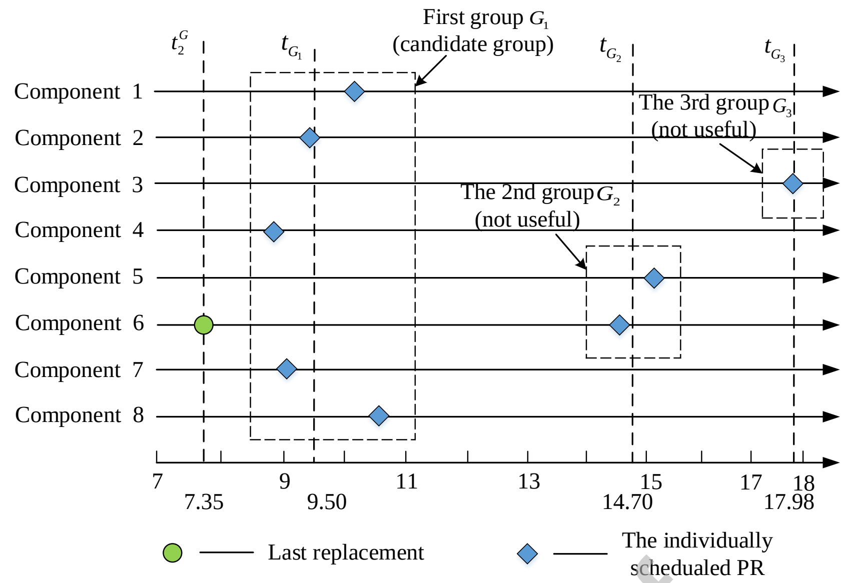

Fig. 7 Illustrations of preventive maintenance planning after the 2nd system PM

In another case, if component 1 fails at time 15.45 after the execution of the 4th group, the originally planned 5th group {6, 1, 5, 3, 7, 4, 2, 8} would not be useful, and opportunistic maintenance planning should be performed at current failure time to decide (1) the current system OM group, (2) the next system PM time, and (3) the subsequent PM group. The grouping results are given in Table 3.

Table 3 The executed maintenance groups when component 1 fails at time 15.4514

Number	Type	Group	execution time	Cost saving
1	PM	{7, 1}	4.76	63.64
2	PM	{6}	7.35	0
3	PM	{4, 7, 2, 1, 8}	9.50	62.76
4	PM	{7}	13.81	0
5	**CM/OM**	{1, 6, 5}, where 1 is failed	15.45	20
6	PM	{3, 7, 4, 2, 8, 1}	19.09	37.31
7	PM	{6, 7, 1}	23.58	45.17
8	PM	{7, 4, 2, 1, 8}	28.41	36.92

As presented in Table 3, the 5th group is an OM group with components {6, 5} are preventively replaced with the failed component 1. And the next planned system PM is scheduled with the group {3, 7, 4, 2, 8, 1} at time 19.0852. The corresponding planning process is illustrated in Fig. 8. In addition, by comparing Table 2 and Table 3, we can find that the maintenance execution after the system failure is also different, where the latter case has more groups. Besides, component 6 also shifts from the 7th to the 8th group. Therefore, it can be concluded that CM would indeed have a nonnegligible effect on the maintenance execution, which is not reflected by the traditional rolling horizon approach.

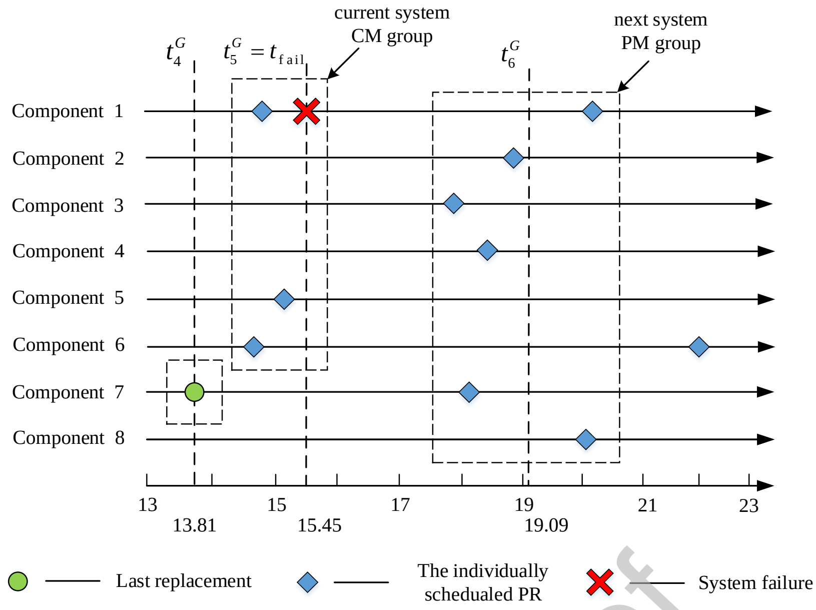

Fig. 8 Illustrations of opportunistic planning when component 1 fails

As the theoretical mean cost is not available in the proposed maintenance strategy, Monte-Carlo simulations are carried out to demonstrate its effectiveness. We set $S=10$, $T=20$ and perform 1000 simulations. The mean total cost of the proposed strategy is 1.6902 while that of the no-grouping strategy is 1.8747, which indicates a cost saving ratio of $(1.8747-1.6902)/1.8747=9.84\%$. Our experiments show that the variation of the results after 1000 simulations is insignificant enough to prove the effectiveness of our strategy.

6.2. Strategies comparison

In this subsection we will compare and analyze the performance of three maintenance strategies: The basic strategy without any grouping, strategy based on the traditional rolling horizon approach, and the proposed strategy. For convenience, we refer to them in turn as Strategy A, Strategy B, and Strategy C.

- Strategy A is the simplest maintenance strategy where each component of the system is maintained individually according to its PR schedule. No grouping is considered.
- Strategy B is formulated based on the traditional rolling horizon approach, where the PR activities in the given horizon are optimally grouped by maximizing the total cost saving. All the failures during the PR intervals are immediately correctively replaced.
- Strategy C is the proposed maintenance strategy elaborated in the previous sections.

Since the horizon in Strategy B must exceed the largest PR period, we let $T=20$ and consider only one horizon. Considering that the PM schedule in strategy A or B is actually predetermined and fixed, the mean costs incurred by the given PM schedule GS is given by

$$\text{ECost} = \sum_{G \in GS} (S + \sum_{i \in G} C_i^p) + \sum_{i \in G} (S + C_i^f) \sum_{t_i^j \in \Omega_i} M_i(t_i^j) \tag{33}$$

where GS contains groups of components at each PM time; t_i^j is the j^{th} PR interval of component i given the system PM schedule; $M_i(t) = \sum_{k=1}^{+\infty} F_i^{(k)}(t)$ is the renewal function measuring the expected number of CRs within the interval t; $F_i^{(k)}(t)$ is the k^{th} convolution of $F_i(t)$. On the other hand, the mean costs of Strategy C are obtained by Monte-Carlo simulations. Based on Eq. (33), the total costs of all strategies with respect to set-up costs are given in Table 4.

Table 4 Total cost ($\times 10^3$) of each strategy with respect to set-up cost in $T = 20$

Strategy	5	10	15	20	30	40
A	1.7658	1.8747	1.9885	2.1070	2.2632	2.4010
B	1.7310	1.7675	1.8281	1.8925	1.9576	1.9674
C	1.7283	1.6902	1.6484	1.6700	1.6801	1.8170

Table 4 indicates that Strategy C generally outperforms both Strategy A and B. The total costs of different strategies are similar when the set-up cost is small ($S = 5$ in the example). Taking the randomness of the Monte-Carlo simulations into consideration, we cannot conclude that Strategy C outperforms the others with certainty. Hence, significance tests are required for justification.

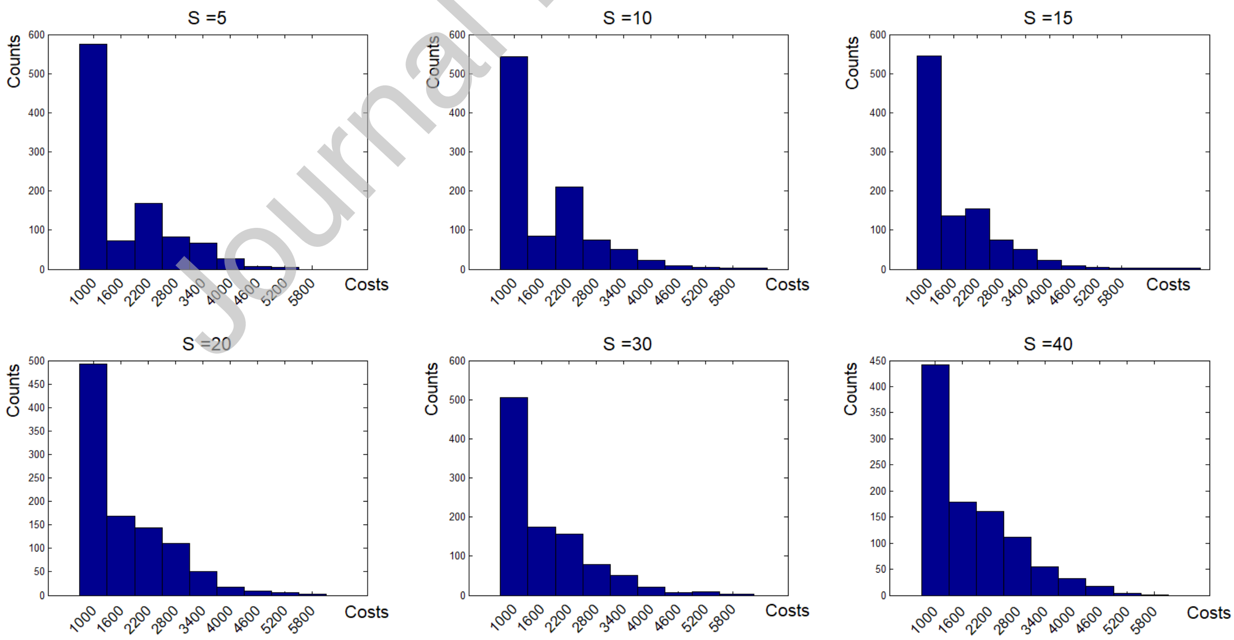

Fig. 9 Simulation results of total costs in $T = 20$ (1000 simulations)

Fig. 9 presents the frequency distribution histogram of costs of Strategy C for 1000 simulations. From Fig. 9 the costs samples cannot be modelled by a classic distribution (Normal, Weibull, etc.). Nevertheless, we can still test whether the cost expectation of Strategy C is lower than those of the others. Denote the total costs of Strategy C by the random variable X with the expectation θ, and the simulation samples by $\{X_1, X_2, \cdots, X_n\}$, $n = 1000$. The estimate of θ is $\hat{\theta} = \sum_{i=1}^{n} X_i / n$, which asymptotically follows the normal distribution $N(\theta, \mathrm{Var}(X)/n)$ based on the central limit theory. Hence, given the significance level $\alpha = 0.05$, the significance test is formulated as follows:

$$H_0 : \theta < \theta_0 ; \qquad H_1 : \theta \geq \theta_0$$

$$p = P(\theta \geq \theta_0) = P(\sqrt{n}(\hat{\theta} - \theta)/SD \leq \sqrt{n}(\hat{\theta} - \theta_0)/SD) = t_{n-1}(\sqrt{n}(\hat{\theta} - \theta_0)/SD)$$

where θ_0 is the mean cost for comparison; $SD = \sqrt{\sum_{i=1}^{n}(X_i - \bar{X})/(n-1)}$ is the standard deviation; $t_{n-1}(x)$ is the CDF of the Student-t distribution with degrees of freedom $n-1$. The test results are presented in Table 5, from which Strategy C outperforms the others only except when $S = 5$ in the example. In such case the set-up cost is too small to reflect the advantage of the grouping maintenance strategy.

Table 5 Significance test results ($\alpha = 0.05$)

Set-up cost S	5	10	15	20	30	40
$\hat{\theta}$ ($\times 10^3$)	1.7283	1.6902	1.6484	1.6700	1.6801	1.8170
SD	27.70	28.79	29.46	29.35	29.45	30.61
p value (against Strategy A)	0.0883	1.13×10^{-10}	$<10^{-20}$	$<10^{-20}$	$<10^{-20}$	$<10^{-20}$
Test results	Reject	Accept	Accept	Accept	Accept	Accept
p value (against Strategy B)	0.4617	0.0037	7.61×10^{-10}	3.92×10^{-14}	1.48×10^{-20}	5.27×10^{-7}
Test results	Reject	Accept	Accept	Accept	Accept	Accept

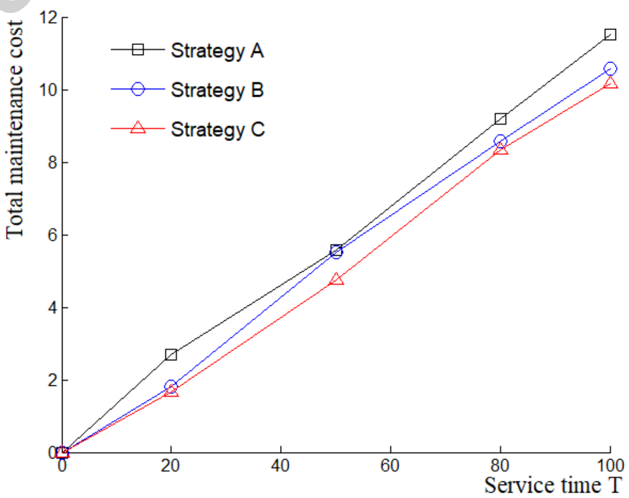

Fig. 10 Total maintenance cost of different strategies with respect to the service time ($S = 20$)

When the service life increases, the total maintenance costs of different strategies are plotted in Fig. 10. The cost of Strategy C is the lowest among all the strategies but the differences are relatively small. Taking the randomness of simulation results into consideration, we may conclude that the proposed strategy at least has a similar performance as the traditional rolling horizon approach, but outperforms the individual maintenances.

6.3. Sensitivity analysis

In this subsection, sensitivity analysis is conducted and the objective is to investigate the impacts of the set-up cost and the life information of components when applying the proposed maintenance strategy.

The impact of the set-up cost: The impact of the set-up cost S is analyzed by varying S from 0 to 40, and the other parameters remain as in Table 1 with $T = 20$. For each S value, the corresponding mean total costs are obtained by conducting 1000 simulations. Obviously, when $S = 0$, the cost savings of grouping actions must be negative and no group structure is obtained. Fig. 11 plots the total maintenance costs with respect to different set-up costs, which exhibits a concave trend with the minimum around $S = 15$. It indicates that the excessive low and high set-up cost would increase the costs. If the set-up cost is low, less maintenance activities are grouped together, and thus the benefits of grouping are not sufficiently reflected. However, if the set-up cost is high, more maintenance activities are grouped together and thus they would be advanced or postponed by a large amount time, which requires more replacements or incurs more failures.

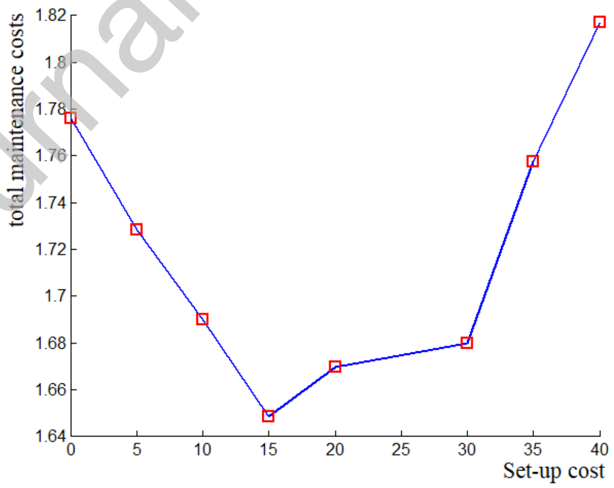

Fig. 11 The total maintenance costs with respect to the set-up cost (T=20)

Impact of the life information of components: We are also interested in investigating the impact of the inaccuracy of components life information on the maintenance strategy. For Weibull distribution, the scale parameter is essential in determining the mean life of a component. Hence, we intend to investigate the impact of the estimation error by varying the scale parameters of all components. For simplicity, a common estimation error factor $\omega > 0$ is employed. If the real value of the scale parameter of component i, $i = 1, 2, \cdots, n$ is η_i, the value adopted in scheduling maintenance strategy is $\omega\eta_i$. If $\omega > 1$, it implies the component life is overestimated; if $\omega < 1$, it implies the component life is underestimated. Therefore, in simulations, $\omega\eta_i$ is used to schedule the maintenance strategy while η_i is used in generating failure time.

For demonstration, the set-up cost $S = 20$, and the service time $T = 50$. The other parameters except $\eta_i, i = 1, 2, \cdots, n$ remain as in Table 1. ω varies from 0.6 to 1.4. For each ω, the corresponding mean total costs are obtained by applying the proposed strategy.

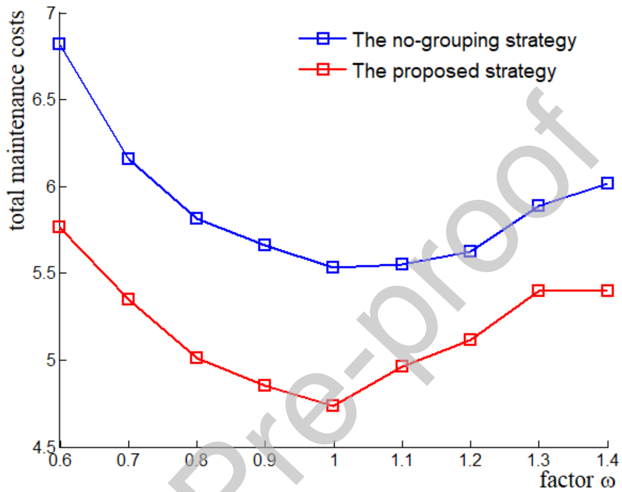

Fig. 12 The maintenance costs with respect to the estimation error factor ω under two strategies

Fig. 12 plots the costs with respect to different life estimation error factor ω under the proposed maintenance strategy and the no-grouping strategy. The concavity of the curve implies that the maintenance strategy achieves its best performance when life parameters are precisely estimated. Both the overestimation or the underestimation would cause the strategy to be less effective with similar additional costs. More importantly, the proposed strategy always outperforms the no-grouping one at the same estimation error level, which demonstrates the superiority of the proposed maintenance strategy.

7. Conclusions

We proposed a dynamic maintenance grouping framework for multi-component systems, successfully employed to both age-based and condition-based maintenance problems. Our framework improves conventional rolling horizon approaches from three perspectives. First, novel cost models considering spare part replacement upon failures are formulated and optimized, which is one step closer to the realistic problems. Second, opportunistic maintenance upon failure is integrated into the maintenance framework, which ensures a sufficient utilization of economic dependence. Third, fixed group information is substituted by iteratively updated grouping and health information, which successfully reduces failure risks. The superior performance of the proposed policy is demonstrated via numerical experiments.

The future research includes the extension of the proposed policy to systems with: (a) multiple failure modes, (b) complex and dependent structures. Moreover, imperfect maintenance can be integrated into the strategy where the decision-making between imperfect repairs and replacements are captured.

Author statement:

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Li Yang	Conceptualization, Methodology, Investigation, Formal Analysis, Writing - Original Draft, Writing - Review & Editing
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Zihan Zhang	Methodology, Investigation, Visualization
Yu Zhao	Project administration, Funding acquisition

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Appendix A

Let $\{t_1, t_2, \cdots, t_n\}$ be the individually planned PR time of all components sorted in ascending order with the corresponding component index $\{i_1, i_2, \cdots, i_n\}$ and ages $\{a_1, a_2, \cdots, a_n\}$. Under the consecutive group structure, the partition process of components $\{1, 2, \cdots, n\}$ can be regarded as successively checking $\{i_1, i_2, \cdots, i_n\}$ and make a decision whether the current component joins the current group or starts a new one.

Therefore, the process can be divided into n stages, and in each stage we consider the arrangement of component i_k. Define the state s_k as the size of the group that component i_{k-1} belongs to. Let $x_k(s_k)$ be the decision variable with respect to state s_k in stage k, $k = 1, 2, \cdots, n$, If component i_k joins the current group, $x_k(s_k) = 0$; otherwise $x_k(s_k) = 1$ and it starts a new group. $s_1 = 0$ and $x_1(s_1) = x_1(0) = 1$. Then we have the recursion equation of the state as

$$\begin{cases} s_{k+1} = s_k + 1 \ , & x_k(s_k) = 0 \\ s_{k+1} = 1 \ , & x_k(s_k) = 1 \end{cases} \tag{34}$$

which has the Markov property and thus the dynamic programming is applicable.

In stage k, if $x_k(s_k) = 0$, the cost saving in stage k is $g_k(s_k, x_k(s_k)) = C_j\left(t^*\left(A_{s_k}\right), A_{s_k}\right)$ based on (12), where $A_{s_k} = \{i_{k-s_k}, i_{k-s_k+1}, \cdots, i_k\}$ is the current group when i_k is grouped with the former s_k components; if $x_k(s_k) = 1$, i_k starts a new group only containing itself, and thus $g_k(s_k, x_k(s_k)) = 0$.

Denote by $f_k(s_k)$, $k = 1, 2, \cdots, n$, the maximum cost saving among consecutive group structures of components $\{i_k, i_{k+1}, \cdots, i_n\}$ when the state is $s_k \in \{1, 2, \cdots, k-1\}$. Then $f_1(s_1) = f_1(0)$ is the desired total maximum cost saving. Let $f_{n+1}(s_{n+1}) = 0$, $\forall s_{n+1} \in \mathrm{N}$.

Based on the Bellman's principle of optimality, the backward dynamic optimization equation is given by

$$\begin{cases} f_k(s_k) = \max\limits_{x_k \in \{0,1\}} \{g_k(s_k, x_k(s_k)) + f_{k+1}(s_{k+1})\}, \quad k = 1, 2, \cdots, n \\ f_{n+1}(s_{n+1}) = 0, \ s_1 = 1, \ x_1(s_1) = 1 \end{cases} \tag{35}$$

and $f_1(0)$ is the maximum cost saving of the partition decided by $x_k(s_k)$, $k = 1, 2, \cdots, n$. The optimal first group is obtained by following the recursion equation (34) with $x_k(s_k) = 0$.

In conclusion, the detailed dynamic programming of group optimization is given as in Algorithm 1. To make the program more efficient, we can refine all possible states in stage k. If a group G is optimal, all the components in G must have a non-negative cost saving. Hence, for each component k, $k = 1, 2, \cdots, n$, there is a feasible time interval $I_k = [I_{k-}, I_{k+}]$ obtained by letting $C_j(t_j^*(k), k) \geq 0$ containing all the reasonable time for PR. Therefore, all the possible states s_k can be determined by finding all the consecutive components including i_k whose feasible intervals have a common intersection.

Appendix B

Following the notations used in Section 5.2.1, we consider a component i which is not scheduled to replacement at its next inspection interval. Unlike the case Section 4.2, the actual replacement time is unknown, we first obtain the probability distribution of the predicted replacement time. Denote the inspection interval number after the time origin before a PR by N^{PR}, similar to Eq. (21), we have

$$P(N^{PR} = k) = \begin{cases} 0 & \text{if } k = 1 \\ G(L - x_0) - G(\xi - x_0) & \text{if } k = 2 \\ \int_0^{\xi - x_0} \left(G(L - x_0 - x) - G(\xi - x_0 - x)\right) g^{k-2}(x) dx & \text{if } k \geq 3 \end{cases} \tag{36}$$

where $G(x) = \int_0^x g_i(u; \tau_i) du$ given in Eq. (19), $\bar{G}(x) = 1 - G(x)$, $s_k = ks$, and $g^k(x) = g_i(x; k\tau_i)$.

Similarly, for the inspection interval number N^{CR} after the time origin before a CR, we have

$$P(N^{CR}=k)=\begin{cases}\bar{G}(L-x_0) & \text{if } k=1\\ \int_0^{\xi-x_0}\bar{G}(L-x_0-x)g^{k-1}(x)dx & \text{if } k\ge 2\end{cases}\tag{37}$$

For simplicity, let $L_0=L-x_0$, $\xi_0=\xi-x_0$ and $X(k\tau_i)=X_k$. If the component is scheduled for replacement at time $t\in[(h-1)\tau_i-t_0,h\tau_i-t_0]$, two cases are considered to obtain the penalty function:

1. t is before the predicted replacement —— advancing penalty

As the actual original replacement time is unknown, the advancing penalty is the expected value of additional costs with respect to the original replacement time.

(1) Advancing penalty with respect to a scheduled PR

Denote the failure time by T_f (since the last inspection) and the unavailable time before the failure detection by T_d. As shown in Fig. 13, the penalty $H^{-PR}(t\,|\,x_0,t_0)$ is given by

$$\begin{aligned}H^{-PR}(t\,|\,x_0,t_0)&=E\left[\phi^*(N^{PR}\tau_i-t_0-t)-C_dT_d\right]\\&=-C_dE(T_d)+\sum_{k=h}^{+\infty}\phi^*(k\tau_i-t_0-t)P(N^{PR}=k)\end{aligned}\tag{38}$$

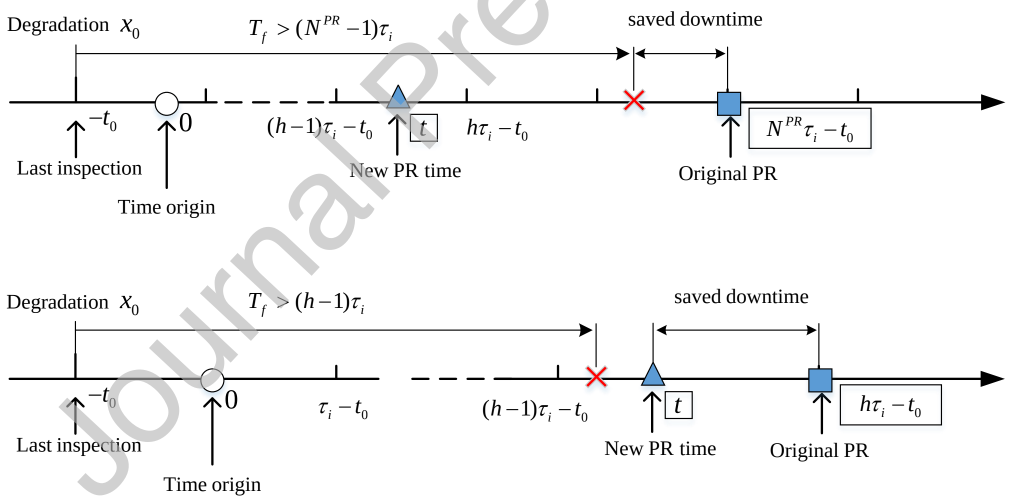

Fig. 13 Advancing penalty with respect to a scheduled PR

To obtain $E(T_d)$, the probability function $P(T_f<s,\,N^{PR}=k)$ is first derived, where $(k-1)\tau_i<s<k\tau_i$, $k\ge h$, which is

$$\begin{aligned}P(T_f<s,\,N^{PR}=k)&=P\{X_{k-2}<\xi_0,\,\xi_0<X_{k-1}<L_0,\,X(s)>L_0\}\\&=\int_0^{\xi_0}g^{k-2}(x)dx\int_{\xi_0-x}^{L_0-x}\bar{G}(L_0-x-y,\,s)g(y)dy\end{aligned}\tag{39}$$

with $\bar{G}(x,s)=1-\int_0^x g_i(u;s)du$. Let $f_k^{PR}(s)=\partial P(T_f<s,\,N^{PR}=k)/\partial s$ be the density function. Thus, the expectation of T_d is obtained as

$$ET_d = \int_{(h-1)\tau_i}^{h\tau_i} \min(h\tau_i - s, h\tau_i - t) f_k^{PR}(s)ds + \sum_{k=h+1}^{+\infty} \int_{(k-1)\tau_i}^{k\tau_i} (k\tau_i - s) f_k^{PR}(s)ds \quad (40)$$

where the first term refers to the case when the PR is scheduled at the next nearest inspection interval.

(2) Advancing penalty with respect to an unscheduled CR

Similar to (1), the advancing penalty with respect to CR has the same form as in Eq. (38)

$$H^{-CR}(t \mid x_0, t_0) = -C_d E(T_d) + \sum_{k=h}^{+\infty} \left[\phi^*(k\tau_i - t_0 - t) \right] P(N^{CR} = k) \quad (41)$$

On the other hand, the probability function $P(T_f < s, N^{CR} = k)$, where $(k-1)\tau_i < s < k\tau_i$, $k \ge h$ is

$$\begin{aligned} P(T_f < s, N^{CR} = k) &= P\left\{ X_{k-1} < \xi_0, X(s) > L_0 \right\} \\ &= \int_0^{\xi_0 - x} \bar{G}(L_0 - x, s) g^{k-1}(x) dx \end{aligned} \quad (42)$$

Let $f_k^{CR}(s) = \partial P(T_f < s, N^{CR} = k) / \partial s$ be the density function. Thus, the expectation of T_d is obtained as

$$ET_d = \int_{(h-1)\tau_i}^{h\tau_i} \min(h\tau_i - s, h\tau_i - t) f_k^{CR}(s)ds + \sum_{k=h+1}^{+\infty} \int_{(k-1)\tau_i}^{k\tau_i} (k\tau_i - s) f_k^{CR}(s)ds \quad (43)$$

2. t is after the predicted replacement —— postponement penalty

Note that if the original replacement is CR, no penalty would be incurred since a new component is put into use right after CR. The considered replacement at t would not be actually conducted. Hence, we only need to consider the penalty incurred by postponing the PR.

$$H^{+CR}(t \mid x_0, t_0) = 0 \quad (44)$$

As shown in Fig. 14, suppose we postpone the scheduled PR to the new time point t, two effects would be incurred: a) risks of failures and unavailable time before the rescheduled PR; b) cost savings due to the extension of lifetime until the rescheduled PR or unscheduled CR.

Denote by N_0 the inspection intervals number before the failure replacement. The penalty is given by

$$H^{+PR}(t \mid x_0, t_0) = E\left[C_f I_{N_0 \le h-1} + C_d T_d - \phi^* \min(N_0\tau_i, t + t_0 - N^{PR}\tau_i) \right] \quad (45)$$

where I_A is the indicator function such that $I_A = 1$ if condition A satisfies; $I_A = 0$ otherwise.

To calculate Eq. (45), we first derive the joint probability distribution of (N_0, N^{PR}), which is

$$\begin{aligned} p_{m,k} &= P(N_0 = m, N^{PR} = k) \\ &= P\left\{ X_{k-2} < \xi_0, \xi_0 < X_{k-1} < L_0, X_{k+m-1} < L_0, X_{k+m} > L_0 \right\} \\ &= \int_0^{\xi_0} g^{k-2}(x)dx \int_{\xi_0 - x}^{L_0 - x} g(y)dy \int_0^{L_0 - x - y} \bar{G}(L_0 - x - y - z) g^m(z)dz \end{aligned} \quad (46)$$

where $2 \le k \le h-1$, $m \ge 1$. Note that, when $m = 0$, the originally scheduled PR is actually a failure replacement but no penalty is incurred with the same reason as the CR case.

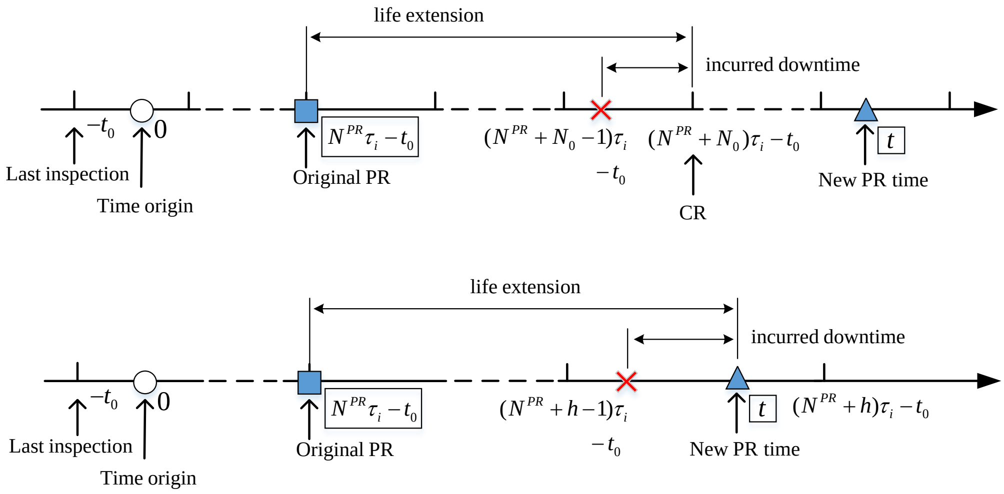

Fig. 14 Postponement penalty with respect to a scheduled PR

Furthermore, the failure time distribution $P(T_f < s, N_0 = m, N^{CR} = k)$, where $k+(m-1)\tau_i < s < k + m\tau_i$ is

$$\begin{aligned} & P(T_f < s, N_0 = m, N^{CR} = k) \\ & = P\left\{X_{k-2} < \xi_0, \xi_0 < X_{k-1} < L_0, X_{k+m-1} < L_0, X(s) > L_0\right\} \\ & = \int_0^{\xi_0} g^{k-2}(x)dx \int_{\xi_0-x}^{L_0-x} g(y)dy \int_0^{L_0-x-y} \bar{G}(L_0 - x - y - z, s) g^m(z)dz \end{aligned} \tag{47}$$

Let $f_{m,k}(s) = \partial P(T_f < s, N_0 = m, N^{CR} = k) / \partial s$ be the density function. Based on Eq. (45)-(47), the penalty is obtained by

$$\begin{aligned} & H^{+PR}(t \mid x_0, t_0) \\ & = C_f P(N_0 \le h-1) - \phi^* E\left[\min(N_0\tau_i, t + t_0 - N^{PR}\tau_i)\right] + C_d ET_d \\ & = C_f \sum_{m=1}^{h-1} p_{m,k} - \phi^* \sum_{k=2}^{h-1} \sum_{m=1}^{+\infty} \min(m\tau_i, t + t_0 - k\tau_i) p_{m,k} \\ & + C_d \sum_{k=2}^{h-1} \left\{ \sum_{m=1}^{h-1} \int_{k+(m-1)\tau_i}^{k+m\tau_i} (k + m\tau_i - s) f_{m,k}(s) ds + \int_{k+(h-1)\tau_i}^{k+h\tau_i} \min(k + h\tau_i - s, k + h\tau_i - t_0 - t) f_{h,k}(s) ds \right\} \end{aligned} \tag{48}$$

In summary, by summing up Eq. (38)(41)(44)(48), the final penalty function when reschedule the PR to time t is given by

$$H(t \mid x_0, t_0) = H^{-PR}(t \mid x_0, t_0) + H^{-CR}(t \mid x_0, t_0) + H^{+PR}(t \mid x_0, t_0) + H^{+CR}(t \mid x_0, t_0) \tag{49}$$

Declaration of interests

☒ The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

☐The authors declare the following financial interests/personal relationships which may be considered as potential competing interests: