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Hybridized fuzzy analytic hierarchy process and fuzzy weighted average for identifying optimal design concept

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ABSTRACT

In this article, a novel hybridized Multi-Attribute Decision Model (MADM) is developed to identify an optimal design of a Reconfigurable Assembly Fixture (RAF) from a set of alternative design concepts. The model combines the comparative advantage of Fuzzy Analytic Hierarchy Process (FAHP) and the computational strength of the Fuzzy Weighted Average (FWA) based on left and right scores in order to obtain aggregates for the design alternatives considering the relative importance of the design criteria as needed in the optimal design. The model was applied to evaluate four design concepts of a RAF with six design features having numerous sub-features. Results obtained from the evaluation process shows that there are differences in final values of the design alternatives. However, a close variation exists between these values. These differences can be accrued to the interrelationships between the design features and sub-features obtained from the Fuzzy Synthetic Extent (FSE) of the FAHP and an unambiguity judgment of the FWA when aggregating availability of the design features and sub-features in the design alternatives.

1. Introduction

Robust design of Products and industrial machineries is important from conceptualization to manufacturing and up till usage in order for manufacturers to obtain a share of the competitive market flooded with changeable designs (Olabanji, 2018). The need for these robust designs calls for development of different design concepts of a particular product or machine before a detail design analysis can be done (Song et al., 2013). The availability of alternative design concepts necessitates the need for selection of optimal design concept (Wei and Chang, 2008). Concept selection in engineering design has attracted importance in recent times because it has a direct implication on the quality of the final design. Problems that arises when it is not properly done includes; repetitive alterations and modifications of drafted designs, prolonged developmental time and amplified cost of actualization (Tiwari et al., 2017). In order to arrive at robust design of a new product or engineering component, identification of design attributes and sub features viz a viz the numerous functional requirements from the customers or intended end users becomes the first task (Ayag and Ozdem, 2007; Brackea et al., 2017).

The engineering design process attempts to give a holistic approach to identification of the design attributes, sub features and functional requirements. It follows an established design standard by proposing four phases (product planning and clarification of task, conceptual design, embodiment design and detail design). These phases are usually applied to arrive at a detail design of the new product (Yeo et al., 2004; Olabanji and Mpofu, 2014). Also, the engineering design process can be imagined to have a set of eleven steps as described by (Ayag and Ozdem, 2007). The relationship between these two analogies is described in Figure 1 where these four phases are disintegrated into the eleven steps. It may be assumed that the information needed in each step and phase will also follow the same manner. However, since the concept selection step is a decision-making process, adequate information is needed for successful selection process. Concept selection in engineering design can be modelled as a multicriteria decision-making (MCDM) problem since it involves multiple design attributes that are having different sub features. Considering the steps in the engineering design process as an all-inclusive approach, it is possible to develop a relationship between the constraints, design attributes and sub features for determining optimal design concept using the multi-criteria analysis as presented in Figure 2. In essence, the task of evaluating alternative designs in order to select the optimal design is usually difficult because of the multiple design attributes, ambiguity and dimensionless nature of the sub features,

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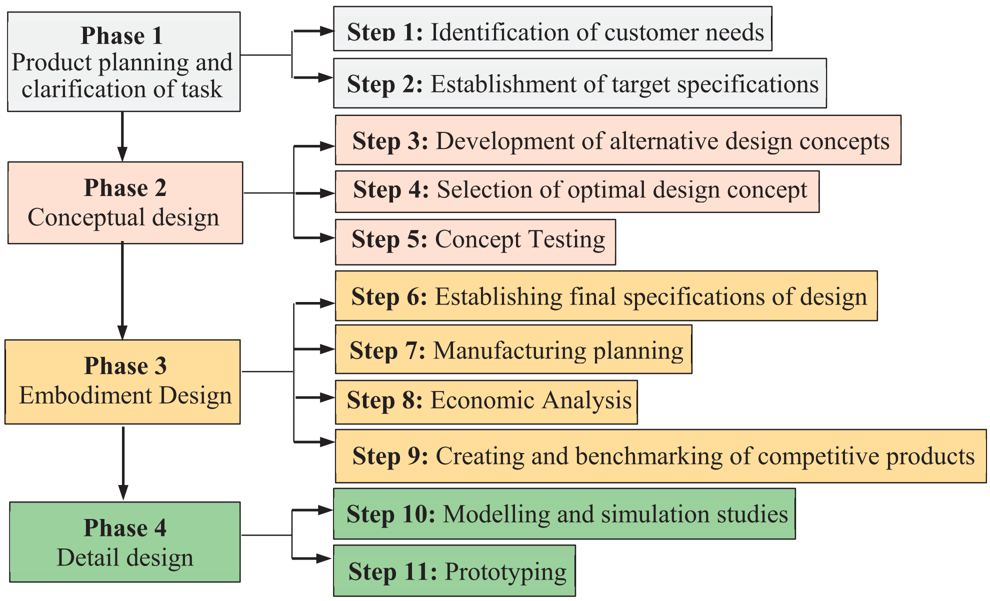

Figure 1. Phases and steps in Engineering Design Process.

manufacturing constraints, interrelationships and dependencies between the design attributes and sub features (Wan and Dong, 2014; Bae et al., 2017). The main features of the MCDM techniques identified by Abdel-malak et al. (2017) can be applied to the engineering design process in order to achieve a robust decision process. In the context of conceptual design, these features are; the design alternatives, the design attributes and sub-features. These features provide scores that reflect the performance of a design alternative with respect to a particular design attribute and the scores that measure the relative importance of the design attributes as required in the optimal design (Hagman et al., 2015).

The classification of MCDM model into Multiple Attribute Decision Making (MADM) and multiple objective decision making MODM (Yeo et al., 2004) has been achieved by disintegrating the design attributes into different sub features. However, considering the relative weights of the design attributes and importance of the weight of sub features in the optimal design is still an area of keen interest. The identification and grouping of the sub features in each design attributes is usually carried out at the early stage of the design task before any conceptual design is developed. Also, the determination of the relative weights of the design attributes is a task that must be given peak attention because it has a role to play in evaluating the alternative concepts towards selecting the optimal design concept.

Various multicriteria decision model has been developed for solving MODM and MADM problems in the fields of management, science and engineering. Examples of these models are the conventional decision models such as Analytic Hierarchy Process (AHP), Weighted Sum Model (WSM), Weighted average model (WA), Technique for Order Preference by Similarity to Ideal Solution (TOPSIS), Elimination and Choice Translating Reality (ELECTRE) and VIseKriterijumska Optimizacija I Kompromisno Resenje (VIKOR) among others (Olabanji and Mpofu, 2014). The vagueness and dimensionless nature of the design attributes and the enormous sub features calls for the optimization of these conventional decision models using different optimization theories. Also, when it is required to consider a combination of the design attributes and sub features as it impacts the decision on optimal design an incoherent scenario is developed. This scenario may also be solved by fuzzifying the conventional decision models. Examples of the fuzzy set decision models are fuzzy AHP (FAHP), fuzzy weighted average (FWA), fuzzy TOPSIS among others. Although the fuzzified decision models have been used extensively in various fields and has yielded results. In order to achieve an optimized decision-making model, it is possible to hybridized two or more of these decision models. The hybridized model will assist in harnessing the advantages of the separate models in order to achieve at an optimal solution. A reason for selecting FAHP in this article is the fuzzified pairwise comparison matrix that is usually developed for the design attributes, sub-features and relative importance of the design attributes in the optimal selection. Considering this advantage, it is important to determine the relative weights of the design attributes and the sub features as required in the optimal selection. This advantage is not available in the FWA model. In FWA, relative importance of design attributes and sub features in the decision process are done based on the experience of the managerial decision makers and heuristic information. However, a germane strength of the FWA is the aggregation of the design alternatives viz a viz the availability of design attributes in them alongside the weights of the sub features and over all weights of the attributes in making decisions (Mokhtarian, 2011).

The application of AHP and FAHP has gained attention in the fields of science, management and engineering due to its simplicity and ability to provide analysis to complex situations by virtue of its comparison matrix approach and hierarchical structure layout. Selection of most suitable academic staff (Rouyendegh and Erkan, 2012), optimal solution to design phase of a building (Szűts and István, 2015), success factors of E-commerce (Kong and Liu, 2005), six sigma implementation in electronics industry (Somsuk and Simcharoen, 2011), lead free equipment selection (Tang and Lin, 2011), service evaluation (Mikhailov and Tsvetinov, 2004), capital investment study (Tang and Beynon, 2005), safety and national identity card management (Dagdeviren; Yuksel, 2008; Catak et al., 2012), selection of notebook computer products (Srichetta and Thurachon, 2012), risk assessment to assembly of satellites (Tian and Yan, 2013), and selection waste water facilities (Anagnostopoulos et al., 2007) have been achieved with FAHP using different criteria and dimensionless sub-criteria. Although the impression and subjectiveness in the pairwise comparison process has been improved in the FAHP model but the problem of accessing the availability of sub-criteria in the alternatives and aggregating their Triangular Fuzzy Numbers (TFNs) under each criterion have not been addressed. Also, the FAHP was able to generate range of values to incorporate the decision makers uncertainty because of the fuzzification of the crisp values and the fuzzy synthetic extent estimations from the decision matrices but the decomposition of the criteria and sub-criteria in order to analyze the interrelationships between them needs to be addressed.

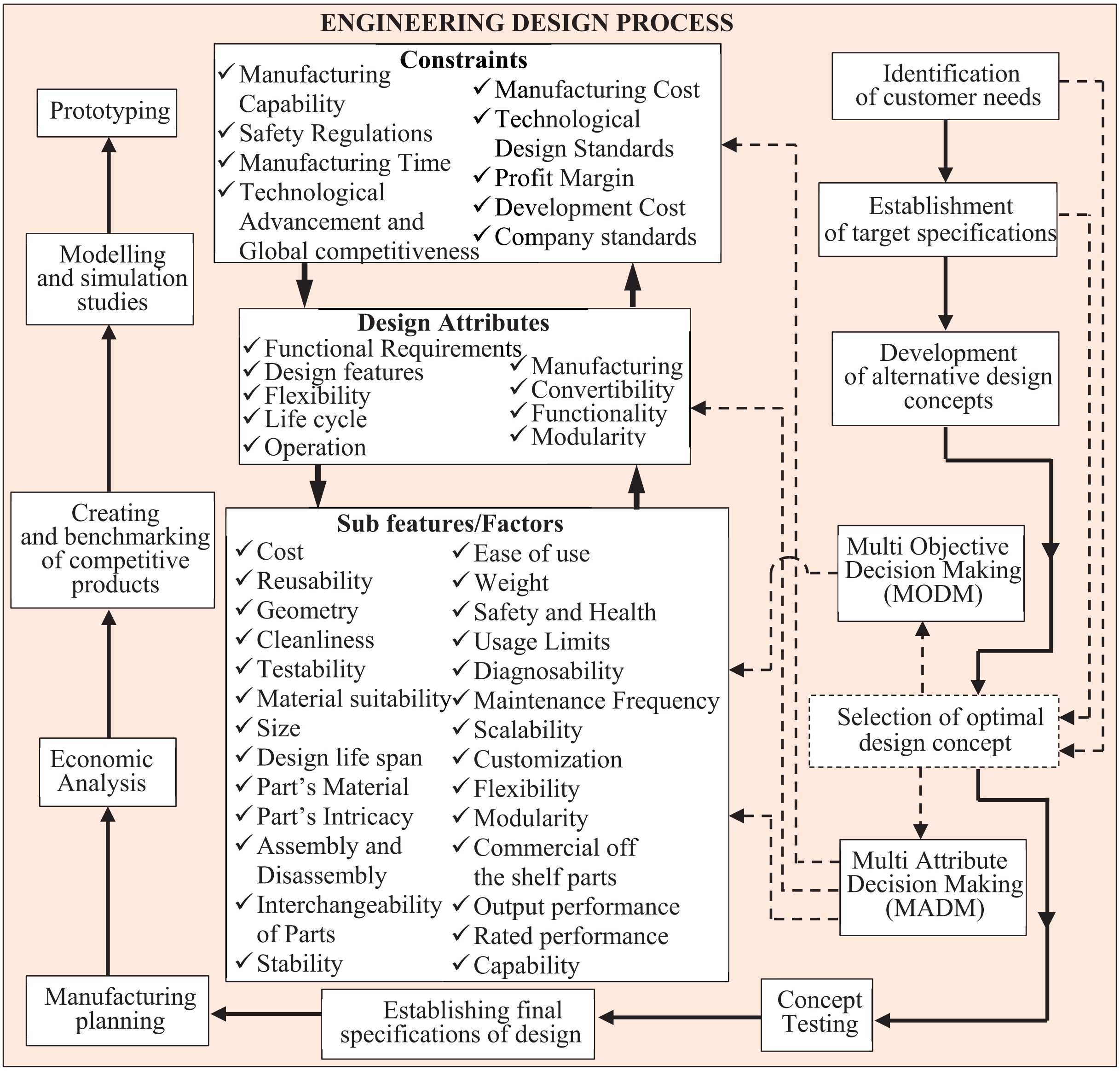

Figure 2. Framework for selection of optimal design concept in engineering design process.

Furthermore, AHP and FAHP have been applied in the engineering design process in order to arrive at an optimal design. Yeo et al. (2004) compared the application of Self-Explicated approach (SE), AHP and FAHP for selecting optimal design of a precision fixture from four conceptual designs. The criteria used in the decision-making process are rigidity and stiffness, accuracy of alignment, cost and ease of use. The result of the three decision making processes yielded different results but selected the same design concept as the optimal design. Despite the fact that the Simple Additive Weighting model (SAW) computes the overall score of a design alternative as a weighted sum of the attributes value, the explicit judgement of the decision makers in assigning attributes value to alternatives in the SE method has not been addressed. The AHP and its fuzzy counterpart (FAHP) disintegrate the problems into levels with the help of the hierarchical structure approach using pairwise comparison and fuzzified comparison matrices respectively. The exclusion of sub-criteria in the comparison process limits the hierarchical computational advantage of the FAHP method because the weights of the criteria is only subjected to the fuzzy synthetic extent value of individual comparison matrix of the criteria. However, the aspect of decision maker's policy in the assigning of attributes have not been addressed by the FAHP method. In essence, a rule that is usually observed in order to address bias in computation is ensuring that factors are mutually exclusive by selecting non-related attributes that will not mislead the decision makers.

Also, the FWA model have been applied in different fields of research (Pavlačka and Talašová, 2006). Decision models involving hierarchical evaluation problems including fuzzy consideration for operations of scoring, weighting and aggregating are usually achieved by applying the FWA method (Guha et al., 2008). The FWA are suitable for computations when the criteria of comparison are further disintegrated into various sub-features. The flexibility in using the FWA model has called for the development of different approaches towards optimizing the model. Some of these approaches are left and right scores (Mokhtarian, 2011), aggregated index intervals (Guha et al., 2008), optimistic approach (Galichet; Boukezzoula, 2009), end point approach (Hu et al., 2010) and intuitionistic fuzzy ordered approach (Zeng, 2012). Despite the fact that the FWA model apportion fuzzified values to alternatives considering their cumulative performance in each of the attributes and sub-factors, the dominance effect of the weights of the criteria (which is subjected to the decision-making policy) still plays a role in the final decision (Guha et al., 2008; Yeo et al., 2004).

Hybridizing two MCDM models have been attempted by researchers in order to optimize decision-making process in management and science. These attempts have yielded optimal results when compared to the use of a single model (Balin et al., 2016; Zeynali et al., 2012). The selection of two models to hybridized depends on the methods of operation of the models and where possible integration can be achieved. It may also be a function of the nature of the attributes and sub-features used in the comparison process and the importance of the outcome of the selection process. In actual fact, selection of models to hybridized has been done by seeking for ways to combine the merits of two different models in order to arrive at a robust decision making (Hu et al., 2017; Awasthi and Chauhan, 2012). The integration of fuzzy AHP and Fuzzy TOPSIS have been applied in different decision-making problems. These applications include design of products (Chakraborty et al., 2017), energy storage (Gumus et al., 2013), gas turbine (Balin et al., 2016), material selection and performance evaluation (Zeynali et al., 2012; Sun, 2010), construction site selection (Turskis et al., 2015), supply chain management, project selection and ship main engine selection (Nazam et al., 2015; Alarcin et al., 2014; Mahmoodzadeh et al., 2007). An attempt presented by Olabanji and Mpofu (2019) was hybridizing of Fuzzified Weighted Decision Matrix (FWDM) and Fuzzy TOPSIS to determine the optimal design of a reconfigurable assembly fixture using design for X tools but the article did not determine the weights of the design features and sub-features. Values in the form of TFN were apportioned to the design features and sub-features which may involve bias scoring and there is no means of expressing interrelationships between design features and sub-features. Also, it is necessary to consider design features and sub-features that are pertinent to the design under consideration rather than the general design for X tools. In all these applications, the relative importance of the sub features is not given priority in the decision-making process. In essence, this article aims at achieving a robust design concept selection process by proposing a novel hybridized model. This is achieved by hybridizing the FAHP and FWA based on left and right scores. The reason for hybridizing these models is to ascertain that the relative importance of the design attributes is considered in the comparison process by obtaining their weights from a fuzzified comparison matrix. The interrelationships and dependencies between the design attributes and their sub features are captured in the process of determining the Fuzzy Synthetic Extent (FSE). Also, fuzzified comparison matrices of the design alternatives and the sub features will provide a basis for obtaining the availability of the sub features in the alternative design conceptsIn design selection process, the relative importance of the sub features is necessary because it plays a role in the optimal design. In essence, one of the novelties in the integration of fuzzy AHP and fuzzy weighted average used in this article can be attributed to the generation of weights for design features and subfeatures from fuzzy synthetic extent. The other uniqueness is the involvement of the weights of sub features in the comparison and computation using the fuzzified weighted average based on left and right scores.

2. Methodology

In order to clearly describe the hybridized model, it is necessary to describe expressions and definitions for fuzzy sets and numbers that are applicable to it. A framework is developed and applied to a set of design alternatives or concepts with design attributes and sub features. Consider i number of design alternatives (D_{C_i}) that are selected for decision making with a mixed scenario of factors and constraints which can be expressed by n number of design attributes. These features can be expressed as criteria (C_n). Each of the design concepts are further analyzed by sub features (C_s) of different dimensions according to the criteria under consideration. In order to compensate for variable dimensions of the sub features, it is necessary to assign relative importance of the attributes with fuzzy number M using a triangular fuzzy number (TFN) which membership function $\mu_m(x)$ is contained in [0 1] and defined as (Mokhtarian, 2011).

$$\mu_m(x) = \begin{cases} \dfrac{1}{m-l}x - \dfrac{l}{m-l} & x \in [l \quad m], \\ \dfrac{1}{m-u}x - \dfrac{u}{m-u} & x \in [m \quad u], \\ 0 & \text{Otherwise} \end{cases} \tag{1}$$

Where $l \leq m \leq u$ and l, m and u represent the lower, modal and upper values of the fuzzy number M respectively. The identified design attributes can be rated according to their level of importance as needed in the optimal design. The conventional concept selection methods assign crisp values to all the ratings (Olabanji, and Mpofu, 2014). In order to fuzzify the level of each criteria and sub-criteria in the optimal design, membership functions are allocated to the criteria. The TFNs used are tabulated in Table 1 below. The ranking of the design attributes and sub features will be analyzed using the linguistic terms for rating their significance in the optimal design while the availability of the sub features in the design alternatives will be assessed using the linguistic terms for ranking features in the design concepts. The inclusion of sub features membership functions for all the design alternatives will provide a robust selection process because an analysis of the design characteristics of all the concepts would have been considered during the pairwise comparison process.

2.1. Fuzzy analytical hierarchy process (FAHP)

The motive for applying the FAHP is to assist in determining the weights of the design attributes and sub features in respect to their relative importance in the optimal design. As stated earlier, the n number of design attributes expressed as criteria (C_n) and having sub features (C_s) of different dimensions can be rated in TFNs and represented as judgement matrices that will be in the form of fuzzy pairwise comparisons. An example of the judgement matrix $\tilde{B} = \{\tilde{b}^j_{gi}\}$ can be presented as;

Table 1. TFN for Rating and Ranking Design Criteria and Sub-features respectively.

Fuzzy AHP			Fuzzy Weighted Average		
Linguistic Terms for Rating of Relative Significance of design attributes in the Optimal Design	Triangular Fuzzy Scale Membership Function	Crisp Value of Ranking	Linguistic Terms for Ranking of availability of Sub-features in the Design concepts	Triangular Fuzzy Scale Membership Function	Crisp Value of Rating
Equally Important	1 1 1	1	Very high	5/2 3 7/2	5
Weakly Important	1 3/2 1	2	High	2 5/2 3	4
Essentially Important	3/2 2 5/2	3	Medium	3/2 2 5/2	3
Very Strong Important	2 5/2 3	4	Low	1 3/2 1	2
Absolutely Important	5/2 3 7/2	5	Very low	1/2 1 3/2	1

$$\tilde{B} = \begin{pmatrix} \tilde{b}^1_{g1} & \tilde{b}^2_{g1} & \cdots\cdots & \tilde{b}^s_{g1} \\ \tilde{b}^1_{g2} & \tilde{b}^2_{g2} & \cdots\cdots & \tilde{b}^s_{g2} \\ \vdots & \vdots & & \vdots \\ \tilde{b}^1_{gk} & \tilde{b}^2_{gk} & \cdots\cdots & \tilde{b}^s_{gk} \end{pmatrix} \tag{2}$$

Where $\tilde{b}_{ij}$is a TFN that can be represented by $(l_{ij} \quad m_{ij} \quad u_{ij})$ as presented in Eq. (1). For *i=1, 2, 3.... k, j=1, 2, 3......s*, such that, when *i=j*, then $\tilde{b}^j_{gi} = \{1\ 1\ 1\}$.

The value of the fuzzy synthetic extent is required from the fuzzy pairwise comparison matrices that have been obtained for all the design attributes and the sub features. It is important to know that the value of the fuzzy synthetic extent will represent the weights of individual object in the comparison matrix. In essence, the weights of the relative importance of the design attributes ($W_{d\,s}$) and sub features ($W_{s\,c}$) will be obtained from the value of the fuzzy synthetic extent (S_i) which can be defined as;

$$S_i = \sum_{j=1}^{s} b^j_{gi} \otimes \left[\sum_{i=1}^{k} \sum_{j=1}^{s} b^j_{gi} \right]^{-1} \tag{3}$$

2.2. Fuzzy weighted average (FWA) based on left and right scores

Assigning TFNs to the availability of the sub features in the alternative design concepts based on the parts analysis of each concept will produce a comparison matrix which aggregate will form a basis for the relative importance of the design attributes. It is important to know that this aggregate will be a weight function of the significance of the sub criteria ($W_{s\,c}$). In essence, the ranking of the design alternative with reference to some sub features considering a particular design attribute will be of the form of a triangular fuzzy matrix whose judgment matrix $\tilde{B} = \{\tilde{b}^s_{sfk}\}$ of n define set of design attributes can be presented as;

$$\tilde{B}_{sf} = \begin{pmatrix} \tilde{b}^1_{sf1} & \tilde{b}^2_{sf1} & \cdots\cdots & \tilde{b}^s_{sf1} \\ \tilde{b}^1_{sf2} & \tilde{b}^2_{sf2} & \cdots\cdots & \tilde{b}^s_{sf2} \\ & & & \\ \tilde{b}^1_{sfk} & \tilde{b}^2_{sfk} & \cdots\cdots & \tilde{b}^s_{sfk} \end{pmatrix} \tag{4}$$

Where $\tilde{b}_{sf}$is a TFN that can be represented by that can be represented by $(l_{ij} \quad m_{ij} \quad u_{ij})$ as presented in Eq. (1). For *i=1, 2, 3.... k, j=1, 2, 3......s*. The cumulative weight of the design concepts with reference to the sub features under each design attribute is necessary to provide a basis for comparison using the weights of the relative importance of the design attribute. A matrix of the aggregate TFNs from all the sub-features $\tilde{W} = \{W^j_{sfi}\}$ for n number of design attributes can be represented by;

$$W_{Sf_n} = \begin{pmatrix} \tilde{B}^1_{sf1} & \tilde{B}^2_{sf1} & \cdots\cdots & \tilde{B}^j_{sf1} \\ \tilde{B}^1_{sf2} & \tilde{B}^2_{sf2} & \cdots\cdots & \tilde{B}^j_{sf2} \\ & & & \\ \tilde{B}^1_{sfi} & \tilde{B}^2_{sfi} & \cdots\cdots & \tilde{B}^j_{sfi} \end{pmatrix} \tag{5}$$

Where $\tilde{B}_{sf}$is a TFN that is equal to the cumulative aggregate of the design concepts considering all the sub features in a design attribute. In order to normalize the fuzzy matrix, consider a fuzzy number $y_{ij} = (l_{ij} \quad m_{ij} \quad u_{ij})$ for $(i = 1.....n \;\; j = 1.....m)$the normalization process can be represented as; (Mokhtarian, 2011; Aryanezhad et al., 2011; Mokhtarian, and Vencheh, 2012)

$$(y_{ij})_N = [(l_{ij})_N \quad (m_{ij})_N \quad (u_{ij})_N] \tag{6}$$

$$(y_{ij})_N = \left[\frac{l_{ij} - l_j^{\mathrm{Min}}}{\Delta^{\mathrm{Max}}_{\mathrm{Min}}}, \quad \frac{m_{ij} - l_j^{\mathrm{Min}}}{\Delta^{\mathrm{Max}}_{\mathrm{Min}}}, \quad \frac{u_{ij} - l_j^{\mathrm{Min}}}{\Delta^{\mathrm{Max}}_{\mathrm{Min}}} \right], \quad i = 1, ...n; \;\; j \in \Omega_b \tag{7}$$

$$(y_{ij})_N = \left[\frac{u_{ij} - u_j^{\mathrm{Max}}}{\Delta^{\mathrm{Max}}_{\mathrm{Min}}}, \quad \frac{m_{ij} - u_j^{\mathrm{Max}}}{\Delta^{\mathrm{Max}}_{\mathrm{Min}}}, \quad \frac{l_{ij} - u_j^{\mathrm{Max}}}{\Delta^{\mathrm{Max}}_{\mathrm{Min}}} \right], \quad i = 1, ...n; \;\; j \in \Omega_c \tag{8}$$

Where $l_j^{\mathrm{Min}} = \mathrm{Min}\, l_{ij}$ and $u_j^{\mathrm{Max}} = \mathrm{Max}\, u_{ij}$ for $i = 1,\ ......n;\ \Delta^{\mathrm{Max}}_{\mathrm{Min}} = u_j^{\mathrm{Max}} - l_j^{\mathrm{Min}}$. Also, Ω_b and Ω_c are sets of benefit and cost attributes respectively. In order to simplify the analysis, the normalized performance value of the *ith* alternative in terms of the *nth* design attribute in a TFN that can be represented as;

$$((W_{1l})_N (W_{1m})_N (W_{1u})_N)((W_{2l})_N (W_{2m})_N (W_{2u})_N)((W_{3l})_N (W_{3a})_N (W_{3u})_N) \\ ((W_{nl})_N (W_{nm})_N (W_{nu})_N)$$

$$\tilde{W}_{dA} * W_{ds_n} = \begin{pmatrix} (\tilde{B}^1_{sf_1})_N & (\tilde{B}^2_{sf_1})_N & (\tilde{B}^3_{sf_1})_N & \cdots & (\tilde{B}^n_{sf_1})_N \\ (\tilde{B}^1_{sf_2})_N & (\tilde{B}^2_{sf_2})_N & (\tilde{B}^3_{sf_2})_N & & (\tilde{B}^n_{sf_2})_N \\ (\tilde{B}^1_{sf_3})_N & (\tilde{B}^2_{sf_3})_N & (\tilde{B}^3_{sf_3})_N & & (\tilde{B}^n_{sf_3})_N \\ \vdots & \vdots & \vdots & & \vdots \\ (\tilde{B}^1_{sf_j})_N & (\tilde{B}^2_{sf_j})_N & (\tilde{B}^3_{sf_j})_N & & (\tilde{B}^n_{sf_j})_N \end{pmatrix} \tag{9}$$

The left and right scores of the normalized decision matrix and the weighted priority are important for computations of the fuzzy weighted average. It is necessary to present an analysis on the determination of left and right score from the TFNs which can be obtained from Eqs. (9) and (10).

$$(L_S)_{ij} = \frac{(m_{ij})_N}{1 + (m_{ij})_N - (l_{ij})_N} \tag{10}$$

$$(R_S)_{ij} = \frac{(u_{ij})_N}{1 + (u_{ij})_N - (m_{ij})_N} \tag{11}$$

Considering Eqs. (10) and (11), two matrices that includes intervals of the left and right score can be constructed for the normalized fuzzy decision matrix and the fuzzy weights of the design attributes. The value for the weighted average of each design alternative can also be obtained in form of the intervals of the left and right scores.

For ease of analysis, let $d_{ij} = [(L_S), (R_S)]_{ij} = [(L_S)_{ij},\ (R_S)_{ij}]$ and $w_j = [(L_S), (R_S)]_j = [(L_S)_j, (R_S)_j]$, then the FWA (θ_i) for design alternative D_{C_i} can be obtained from Eq. (16).

$$(W_{Sf_n})_N = \begin{pmatrix} (\tilde{B}^1_{sf_1})_N & (\tilde{B}^2_{sf_1})_N & (\tilde{B}^3_{sf_1})_N & \cdots & (\tilde{B}^n_{sf_1})_N \\ (\tilde{B}^1_{sf_2})_N & (\tilde{B}^2_{sf_2})_N & (\tilde{B}^3_{sf_2})_N & \cdots & (\tilde{B}^n_{sf_2})_N \\ (\tilde{B}^1_{sf_3})_N & (\tilde{B}^2_{sf_3})_N & (\tilde{B}^3_{sf_3})_N & \cdots & (\tilde{B}^n_{sf_3})_N \\ \vdots & \vdots & \vdots & & \vdots \\ (\tilde{B}^1_{sf_j})_N & (\tilde{B}^2_{sf_j})_N & (\tilde{B}^3_{sf_j})_N & \cdots & (\tilde{B}^i_{sf_j})_N \end{pmatrix} \tag{12}$$

$$(\tilde{W}_{dA})_N = [(\tilde{W}_1)_N \quad (\tilde{W}_2)_N \quad (\tilde{W}_3)_N \quad \cdots \quad (\tilde{W}_n)_N] \tag{13}$$

$$[(L_S),(R_S)]_{(\tilde{W}_{sf})_N} = \begin{bmatrix} [(L_S),(R_S)]_{11} & \cdots & [(L_S),(R_S)]_{12} & \cdots & [(L_S),(R_S)]_{1n} \\ \vdots & & \vdots & & \vdots \\ [(L_S),(R_S)]_{21} & \cdots & [(L_S),(R_S)]_{22} & \cdots & [(L_S),(R_S)]_{2n} \\ \vdots & & \vdots & & \vdots \\ [(L_S),(R_S)]_{j1} & \cdots & [(L_S),(R_S)]_{j2} & \cdots & [(L_S),(R_S)]_{jn} \end{bmatrix} \quad (14)$$

$$[(L_S),\ (R_S)]_{(\tilde{W}_{dA})_N} = [\,[(L_S),\ (R_S)]_1 \quad \cdots \quad [(L_S),\ (R_S)]_2 \quad \cdots \quad [(L_S),\ (R_S)]_n] \quad (15)$$

$$\theta_i = \frac{\sum_{j=1}^{n}(w_j * d_{ij})}{\sum_{j=1}^{n} w_j} = \frac{w_1 d_{i1} + w_2 d_{i2} + \ldots\ldots + w_n d_{in}}{w_1 + w_2 + \ldots\ldots + w_n}\ ;\ i = 1, \ldots m \quad (16)$$

Eq. (16) is subject to $\begin{matrix}(L_s)_j \le w_j \le (R_s)_j, & j = 1\ldots..n \\ (L_s)_j \le d_{ij} \le (R_s)_j, & j = 1\ldots..n\end{matrix}$ (17).

The FWA can be considered as lower and upper bound of a fractional programming model since its components for each of the design alternative obtained in Eq. (16) is a function of the intervals of the left and right scores. In addition, since the FWA is a monotonically increasing function of d_{ij} which reaches it minimum and maximum at $d_{ij} = (L_S)_{ij}$and $d_{ij} = (R_S)_{ij}$respectively then the pair of fractional programming model can be presented as;

$$\theta_i^L = \text{Min}\frac{\sum_{j=1}^{n}(w_j * (L_S)_{ij})}{\sum_{j=1}^{n} w_j}\ \text{subject to}\ (L_s)_j \le w_j \le (R_s)_j, j = 1\ldots..n \quad (17)$$

$$\theta_i^U = \text{Max}\frac{\sum_{j=1}^{n}(w_j * (R_S)_{ij})}{\sum_{j=1}^{n} w_j}\ \text{subject to}\ (L_s)_j \le w_j \le (R_s)_j, j = 1\ldots..n \quad (18)$$

Transportation equations are needed to transform the fractional programming model presented in Eqs. (17) and (18) intoa linear programming model. In essence, Eqs. (17) and (18) can be defined as;

$$z = \frac{1}{\sum_{j=1}^{n} w_j} \quad (19)$$

$$t_j = z * w_j\ j = 1\ldots\ldots n \quad (20)$$

$$(\theta_i)^L = \text{Min}\sum_{j=1}^{n}(t_j * (L_S)_{ij})\quad \text{subject to}\ \sum_{j=1}^{n} t_j = 1 \\ (z * (L_S)_{ij}) \le t_j \le (z * (L_S)_{ij}),\quad j = 1\ldots..n \quad (21)$$

$$(\theta_i)^U = \text{Max}\sum_{j=1}^{n}(t_j * (R_S)_{ij})\quad \text{subject to}\ \sum_{j=1}^{n} t_j = 1 \\ (z * (L_S)_{ij}) \le t_j \le (z * (L_S)_{ij}),\quad j = 1\ldots..n \quad (22)$$

Furthermore, Eqs. (21) and (22) will create an interval $[(\theta_i)^L, (\theta_i)^U]$for each design alternative whose average value ($\theta_{i\ \text{average}}$) will provide the weight for each design alternative as presented in Eq. (23).

$$(\theta_i)_{\text{average}} = \frac{(\theta_i)^L +\ (\theta_i)^U}{2} \quad (23)$$

2.3. Framework for the hybridized model of FAHP and FWA

In order to establish the mode of operation and interpretation of the model, it is necessary to create a framework for better understanding of the procedure. The framework will ease the application of the model to various decision problem. The framework provided in this article concentrates on how the model can be applied to decision making in engineering design. This does not imply that it is not suitable for other decision problems. Figure 3 presents the framework of the hybridized model of FAHP and FWA based on left and right scores.

3. Application of the model to design of RAF

In order to evaluate the developed model, four conceptual designs of a Reconfigurable Assembly Fixture (RAF) are compared using six design attributes with various sub features under each design attribute as shown in the hierarchy diagram in Figure 4. RAFs are enabling equipment in a reconfigurable assembly system which are used to uniquely locate and support varying work-pieces during the assembly process. RAFs assemble a range of workpiece in as much as the variation in dimensions of the work-piece is within the reconfigurable limits of the RAF. They are precision equipment requiring effective design and planning because of its long-term use and functional requirements (Olabanji et al., 2016). Considering Figure 4 and the framework developed in Figure 3, it is necessary to develop fuzzy pairwise comparison matrices for the design attributes and sub features. A fuzzy pairwise comparison matrix for the design attributes is presented in Table 2. Furthermore, fuzzy pairwise comparison matrices for the sub features are presented in Tables 3, 4, 5, 6, 7, and 8. In order to simplify the application, the value of the fuzzy synthetic extent (FSE) for all the design attributes and sub features present in the pairwise comparison has been added as the last row in each of the matrices. This will represent the weights of the design attributes and sub features in the form of TFNs.

On obtaining the FSE for the design attributes and sub features to represent their priority weights, it is important to assess the design concepts based on these sub features considering parts analysis and morphology of the component parts in each of the design alternatives. An assessment of the design alternatives with respect to the sub features is presented in Tables 9, 10, 11, 12, 13, and 14.

The cumulative TFNs obtained from the assessments will generate the fuzzified decision matrix for the design alternatives alongside the FSE value obtained from the fuzzy pairwise comparison matrix for the design attributes in Table 2. The fuzzified decision matrix is presented in Table 15.

In order to ensure that the values of the TFNs in the fuzzified decision matrix are in the range of [0 1], the decision matrix will be normalized applying equations 6 -8. The normalized decision matrix is presented in Table 16. Similarly, in order to ensure that the summation of all the weights do not exceed unity, the weight component of the design attributes will be normalized. However, this will be achieved by the transportation models provided in Eqs. (19) and (20) after which the left and right scores of the weight might have been obtained. In order to arrive at the fuzzy weighted average for each of the design alternative concepts, it is necessary to obtain their weighted intervals. These intervals are obtained from Eqs. (21) and (22). Applying Eq. (23) to the intervals produces the weighted average for the design alternatives. Table 17 presents the left and right scores of the normalized fuzzy decision matrix for alternative design concepts, normalized left and right scores of weights for the design attributes, weighted interval, weighted average and ranking of the design alternatives.

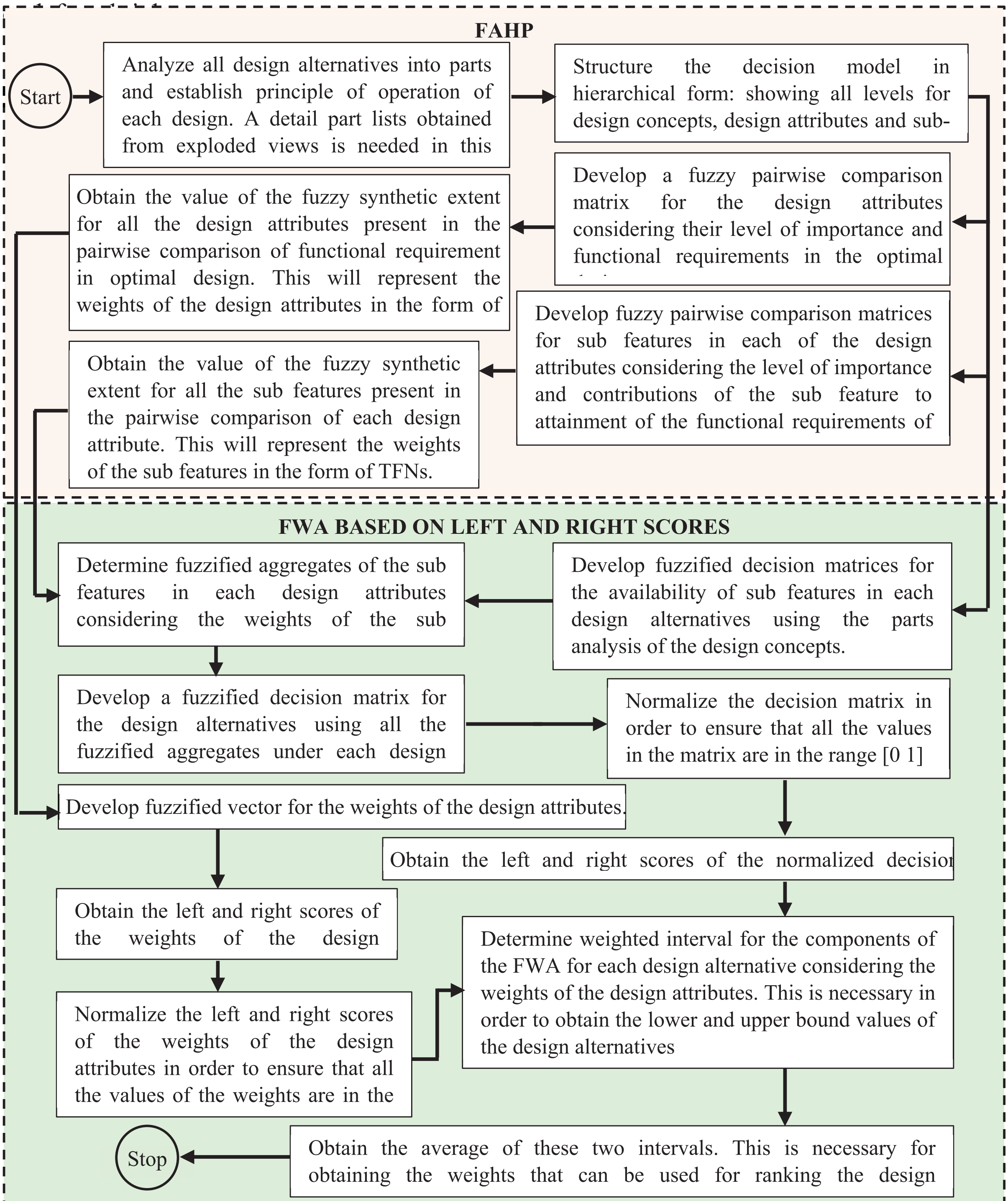

Figure 3. Framework for Hybridized FAHP and FWA based on left and right scores.

4. Results and discussion

Obtaining the weights of sub features and design attributes from the fuzzy synthetic extent (FSE) derived from the fuzzy pairwise comparison matrix provided a platform for expressing the interrelationship between the sub features. This eliminates ambiguity in the final values of the weights rather than apportioning TFNs directly to the sub features based on intuition or decision of the design engineer (Olabanji and Mpofu, 2014; Olabanji, 2018). Although, the computational stress is high but the priority weights obtained from the outcome will eliminate the doubt of over scoring a design concept over others. This is justifiable from the closeness of the final values of the design concepts. The weights of the sub features and design attributes plays a significant role in the weighted average computations. Furthermore, it can be observed from FSE values Tables 2, 3, 4, 5, 6, 7, and 8 that the fuzzy pairwise comparison ensured that the fuzzy synthetic extents representing the weights of the design attributes and sub features have marginal differences rather than using the conventional TFNs obtained from conversion of the crisp value that is usually common in the multiattribute decision making models. These marginal differences also contribute to the closeness of the final values of the design concepts. The relevance of the pairwise comparisons achieved for the design

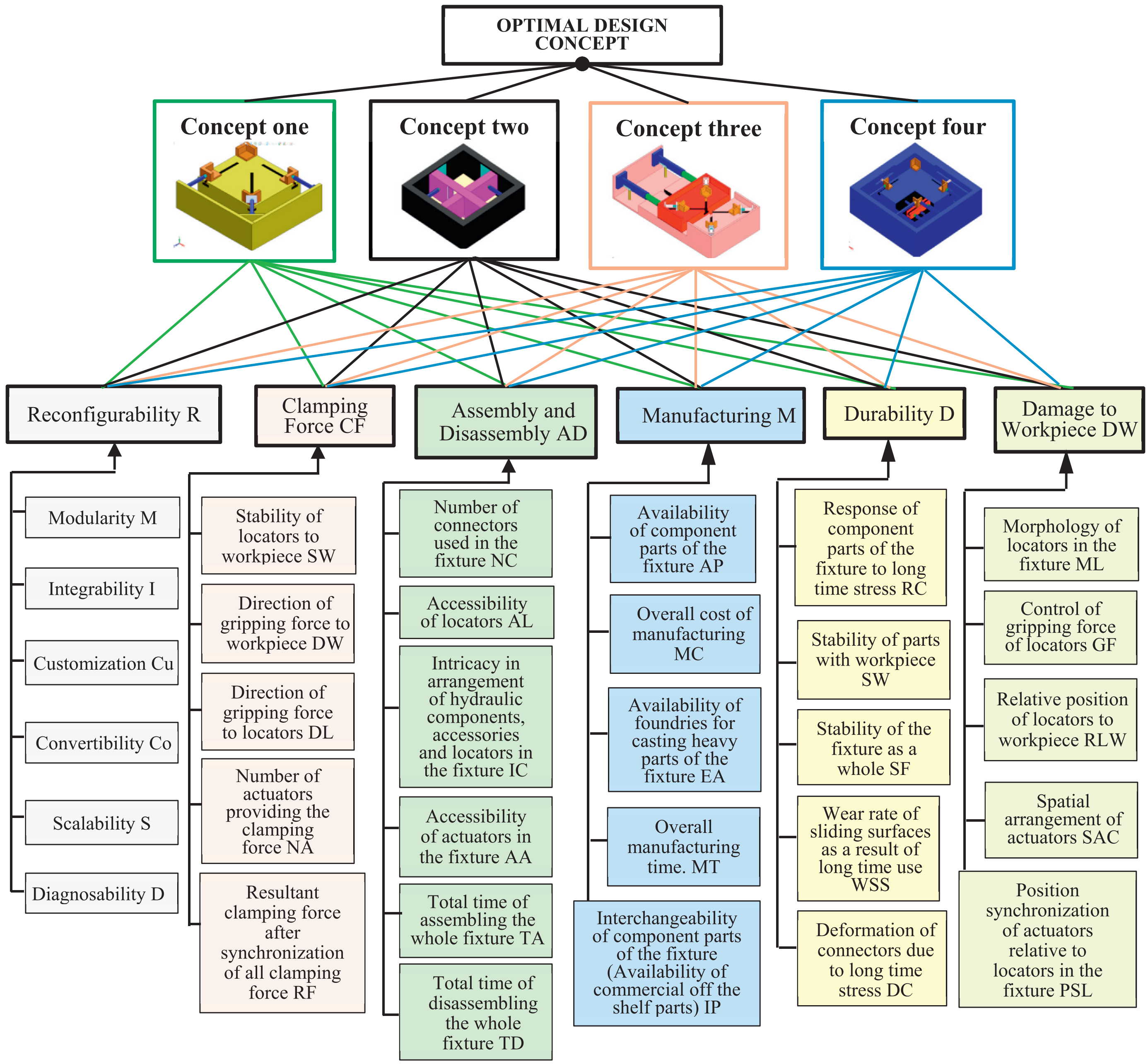

Figure 4. Application of the Hybridized model to Design concepts of a reconfigurable assembly fixture.

attributes and sub features is much appreciated in the fuzzy weighted average computations where an aggregating operation was performed to estimate the availability of the numerous sub features used in the comparison process as opposed to Yeo et al. (2004) where sub features are not considered in the decision process. The fuzzy weighted average provided a means of aggregating the availability of the sub features in

Table 2. Fuzzy pairwise comparison matrix for the design attributes.

Design Attributes	R	CF	AD	M	D	DW
R	1 1 1	3/2 2 5/2	2 3/2 3	2 5/2 3	2 5/2 3	3/2 2 5/2
CF	2/5 1/2 2/3	1 1 1	3/2 2 5/2	3/2 2 5/2	3/2 2 5/2	1/2 1 3/2
AD	1/3 2/5 1/2	2/5 1/2 2/3	1 1 1	1/2 1 3/2	2/3 1 2	2/5 1/2 2/3
M	1/3 2/5 1/2	2/5 1/2 2/3	2/3 1 2	1 1 1	1/2 1 3/2	2/5 1/2 2/3
D	1/3 2/5 1/2	2/5 1/2 2/3	1/2 1 3/2	2/3 1 2	1 1 1	2/5 1/2 2/3
DW	2/5 1/2 2/3	2/3 1 2	3/2 2 5/2	3/2 2 5/2	3/2 2 5/2	1 1 1
FSE	$\frac{12}{67}$ $\frac{12}{41}$ $\frac{21}{46}$	$\frac{11}{96}$ $\frac{1}{5}$ $\frac{12}{37}$	$\frac{1}{17}$ $\frac{10}{97}$ $\frac{16}{83}$	$\frac{1}{17}$ $\frac{10}{97}$ $\frac{16}{83}$	$\frac{1}{17}$ $\frac{10}{97}$ $\frac{16}{83}$	$\frac{2}{17}$ $\frac{1}{5}$ $\frac{18}{53}$

Table 3. Fuzzy Pairwise Comparison Matrix for Sub features of Reconfigurability.

Reconfigurability R

	M	I	CU	CO	S	D
M	1 1 1	3/2 2 5/2	2 5/2 3	1 3/2 2	5/2 3 7/2	1 3/2 2
I	2/5 1/2 2/3	1 1 1	1 3/2 2	3/2 2 5/2	1 3/2 2	2 5/2 3
CU	1/3 2/5 1/2	1/2 2/3 1	1 1 1	1 3/2 2	3/2 2 5/2	2 5/2 3
CO	1 3/2 2	2/5 1/2 2/3	1/2 2/3 1	1 1 1	2 5/2 3	3/2 2 5/2
S	2/7 1/3 2/5	1/2 2/3 1	2/5 1/2 2/3	1/3 2/5 1/2	1 1 1	5/2 3 7/2
D	1/2 2/3 1	1/3 2/5 1/2	1/3 2/5 1/2	2/5 1/2 2/3	2/7 1/3 2/5	1 1 1
FSE	$\frac{11}{69}$ $\frac{1}{4}$ $\frac{28}{73}$	$\frac{11}{90}$ $\frac{19}{97}$ $\frac{26}{85}$	$\frac{11}{98}$ $\frac{13}{74}$ $\frac{20}{73}$	$\frac{11}{97}$ $\frac{8}{45}$ $\frac{22}{79}$	$\frac{4}{45}$ $\frac{9}{70}$ $\frac{6}{31}$	$\frac{5}{99}$ $\frac{1}{14}$ $\frac{1}{9}$

Table 4. Fuzzy Pairwise Comparison Matrix for Sub features of Clamping force.

Clamping Force CF

	SW	DW	DL	NA	RF
SW	1 1 1	3/2 2 5/2	1 3/2 2	2 5/2 3	1 3/2 2
DW	2/5 1/2 2/3	1 1 1	1 3/2 2	1 3/2 2	2 5/2 3
DL	1/2 2/3 1	1/2 2/3 1	1 1 1	3/2 2 5/2	3/2 2 5/2
NA	1/3 2/5 1/2	1/2 2/3 1	2/5 1/2 2/3	1 1 1	5/2 3 7/2
RF	1/2 2/3 1	1/3 2/5 1/2	2/5 1/2 2/3	2/7 1/3 2/5	1 1 1
FSE	$\frac{4}{23}$ $\frac{23}{82}$ $\frac{10}{23}$	$\frac{14}{97}$ $\frac{3}{13}$ $\frac{14}{39}$	$\frac{2}{15}$ $\frac{14}{67}$ $\frac{1}{3}$	$\frac{10}{79}$ $\frac{9}{49}$ $\frac{8}{29}$	$\frac{6}{89}$ $\frac{9}{94}$ $\frac{13}{88}$

Table 5. Fuzzy Pairwise Comparison Matrix for Sub features of Assembly and Disassembly.

Assembly and Disassembly AD

	NC	AL	IC	AA	TA	TD
NC	1 1 1	1 3/2 2	3/2 2 5/2	1 3/2 2	5/2 3 7/2	2 5/2 3
AL	1/2 2/3 1	1 1 1	1 3/2 2	1 3/2 2	5/2 3 7/2	2 5/2 3
IC	2/5 1/2 2/3	1/2 2/3 1	1 1 1	1 3/2 2	3/2 2 5/2	3/2 2 5/2
AA	1/2 2/3 1	1/2 2/3 1	1/2 2/3 1	1 1 1	2 5/2 3	3/2 2 5/2
TA	2/7 1/3 2/5	2/7 1/3 2/5	2/5 1/2 2/3	1/3 2/5 1/2	1 1 1	1 3/2 2
TD	1/3 2/5 1/2	1/3 2/5 1/2	2/5 1/2 2/3	2/5 1/2 2/3	1/2 2/3 1	1 1 1
FSE	$\frac{9}{55}$ $\frac{7}{27}$ $\frac{39}{98}$	$\frac{8}{55}$ $\frac{11}{48}$ $\frac{16}{45}$	$\frac{3}{28}$ $\frac{14}{81}$ $\frac{11}{40}$	$\frac{6}{55}$ $\frac{12}{71}$ $\frac{10}{37}$	$\frac{3}{50}$ $\frac{1}{11}$ $\frac{12}{85}$	$\frac{2}{37}$ $\frac{5}{64}$ $\frac{9}{73}$

Table 6. Fuzzy Pairwise Comparison Matrix for Sub features of Manufacturing.

Manufacturing M

	AP	MC	EA	MT	IP
AP	1 1 1	2 5/2 3	1 3/2 2	1 3/2 2	1 3/2 2
MC	1/3 2/5 1/2	1 1 1	1 3/2 2	3/2 2 5/2	5/2 3 7/2
EA	1/2 2/3 1	1/2 2/3 1	1 1 1	1 3/2 2	2 5/2 3
MT	1/2 2/3 1	2/5 1/2 2/3	1/2 2/3 1	1 1 1	3/2 2 5/2
IP	1/2 2/3 1	2/7 1/3 2/5	1/3 2/5 1/2	2/5 1/2 2/3	1 1 1
FSE	$\frac{5}{31}$ $\frac{4}{15}$ $\frac{8}{19}$	$\frac{8}{47}$ $\frac{5}{19}$ $\frac{2}{5}$	$\frac{9}{67}$ $\frac{15}{71}$ $\frac{32}{95}$	$\frac{9}{86}$ $\frac{5}{31}$ $\frac{20}{77}$	$\frac{5}{74}$ $\frac{3}{31}$ $\frac{3}{20}$

Table 7. Fuzzy Pairwise Comparison Matrix for Sub features of Durability.

Durability D

	RP	SW	SF	WSS	DC
RP	1 1 1	1 3/2 2	5/2 3 7/2	3/2 2 5/2	2 5/2 3
SW	1/2 2/3 1	1 1 1	2 5/2 3	1 3/2 2	3/2 2 5/2
SF	2/7 1/3 2/5	1/3 2/5 1/2	1 1 1	3/2 2 5/2	1 3/2 2
WSS	2/5 1/2 2/3	1/2 2/3 1	2/5 1/2 2/3	1 1 1	1 3/2 2
DC	1/3 2/5 1/2	2/5 1/2 2/3	1/2 2/3 1	1/2 2/3 1	1 1 1
FSE	$\frac{3}{14}$ $\frac{1}{3}$ $\frac{1}{2}$	$\frac{13}{81}$ $\frac{21}{83}$ $\frac{24}{61}$	$\frac{1}{9}$ $\frac{5}{29}$ $\frac{22}{83}$	$\frac{3}{34}$ $\frac{11}{80}$ $\frac{17}{77}$	$\frac{3}{41}$ $\frac{8}{75}$ $\frac{5}{29}$

Table 8. Fuzzy Pairwise Comparison Matrix for Sub features of Damage to workpiece.

Damage to Workpiece DW					
	ML	GF	RLW	SAC	PSL
ML	1 1 1	1 3/2 2	3/2 2 5/2	2 5/2 3	3/2 2 5/2
GF	1/2 2/3 1	1 1 1	3/2 2 5/2	3/2 2 5/2	1 3/2 2
RLW	2/5 1/2 2/3	2/5 1/2 2/3	1 1 1	5/2 3 7/2	2 5/2 3
SAC	1/3 2/5 1/2	2/5 1/2 2/3	2/7 1/3 2/5	1 1 1	1 3/2 2
PSL	2/5 1/2 2/3	1/2 2/3 1	1/3 2/5 1/2	1/2 2/3 1	1 1 1
FSE	$\frac{11}{59}$ $\frac{5}{17}$ $\frac{13}{29}$	$\frac{6}{41}$ $\frac{11}{47}$ $\frac{11}{30}$	$\frac{1}{6}$ $\frac{12}{49}$ $\frac{9}{25}$	$\frac{7}{87}$ $\frac{5}{41}$ $\frac{8}{43}$	$\frac{4}{55}$ $\frac{2}{19}$ $\frac{9}{53}$

Table 9. Assessing design concepts based on Sub features of Reconfigurability.

Reconfigurability (R)	Design Alternatives			
	Concept 1	Concept 2	Concept 3	Concept 4
M (11/69 1/4 28/73)	3/2 2 5/2	1 3/2 2	2 5/2 3	1 3/2 2
I (11/90 19/97 26/85)	2 5/2 3	2 5/2 3	2 5/2 3	2 5/2 3
Cu (11/90 13/74 20/73)	3/2 2 5/2	2 5/2 3	3/2 2 5/2	3/2 2 5/2
Co (11/97 8/45 22/79)	3/2 2 5/2	3/2 2 5/2	2 5/2 3	2 5/2 3
S (4/45 9/70 6/31)	3/2 2 5/2	3/2 2 5/2	2 5/2 3	3/2 2 5/2
D (5/99 1/14 1/9)	2 5/2 3	2 5/2 3	3/2 2 5/2	2 5/2 3
Cumulative TFN	$\frac{15}{14}$ $\frac{145}{68}$ $\frac{163}{40}$	$\frac{20}{19}$ $\frac{44}{21}$ $\frac{201}{50}$	$\frac{27}{22}$ $\frac{19}{8}$ $\frac{378}{85}$	$\frac{65}{62}$ $\frac{109}{52}$ $\frac{181}{45}$

Table 10. Assessing design concepts based on Sub features of Clamping Force.

Clamping Force (CF)	Design Alternatives			
	Concept 1	Concept 2	Concept 3	Concept 4
SW (4/23 23/82 10/23)	3/2 2 5/2	1 3/2 2	2 5/2 3	2 5/2 3
DW (14/97 3/13 14/39)	3/2 2 5/2	1 3/2 2	3/2 2 5/2	3/2 2 5/2
DL (2/15 14/67 1/3)	3/2 2 5/2	1 3/2 2	3/2 2 5/2	2 5/2 3
NA (10/79 9/49 8/29)	3/2 2 5/2	1 3/2 2	2 5/2 3	5/2 3 7/2
RF (6/89 9/94 13/88)	3/2 2 5/2	1 3/2 2	2 5/2 3	5/2 3 7/2
Cumulative TFN	$\frac{92}{95}$ $\frac{8}{4}$ $\frac{277}{66}$	$\frac{51}{79}$ $\frac{3}{2}$ $\frac{47}{14}$	$\frac{53}{46}$ $\frac{98}{43}$ $\frac{197}{42}$	$\frac{25}{19}$ $\frac{53}{21}$ $\frac{77}{15}$

Table 11. Assessing design concepts based on Sub features of Assembly and Disassembly.

Assembly/Disassembly (AD)	Design Alternatives			
	Concept 1	Concept 2	Concept 3	Concept 4
NC (9/55 7/27 39/98)	2 5/2 3	1 3/2 2	3/2 2 5/2	1 3/2 2
AL (8/55 11/48 16/45)	2 5/2 3	1/2 1 3/2	2 5/2 3	3/2 2 5/2
IC (3/28 14/81 11/40)	3/2 2 5/2	5/2 3 7/2	3/2 2 5/2	3/2 2 5/2
AA (6/55 12/71 10/37)	2 5/2 3	2 5/2 3	2 5/2 3	3/2 2 5/2
TA (3/50 1/11 12/85)	3/2 2 5/2	5/2 3 7/2	3/2 2 5/2	3/2 2 5/2
TD (2/37 5/64 9/73)	3/2 2 5/2	2 5/2 3	2 5/2 3	2 5/2 3
Cumulative TFN	$\frac{7}{6}$ $\frac{128}{55}$ $\frac{221}{50}$	$\frac{50}{51}$ $\frac{75}{37}$ $\frac{119}{30}$	$\frac{108}{97}$ $\frac{85}{38}$ $\frac{197}{46}$	$\frac{85}{94}$ $\frac{166}{87}$ $\frac{181}{48}$

the design alternatives in terms of TFNs considering the priority weights in order to further eliminate the ambiguity of over scoring the design alternatives contrary to the conventional AHP where crisp values are used a priority vector (Hambali et al., 2008; Kalashetty et al., 2012). The introduction of the left and right scores in the fuzzy weighted average process expressed the defuzzification process based on the intervals of the fuzzy ratings for the design alternatives and fuzzy weights of the design criteria compared to the alpha level set method where the degree of accuracy depends on the number of the alpha levels of fuzzy numbers. Furthermore, the fuzzy weighted average based on left and right scores further create a relationship between the design features and make the computing process robust because normalizing the weights of the design features constrains the value to be in a particular range. Considering the result obtained from the application of the developed model, it can be stated that design concept three can be assumed to be the optimal design followed by other design concepts as shown in Table 17. In essence, the hybridized model can be applied to making decision on optimal design concept provide that the design features and sub-features are specified alongside detail information on the design alternatives.

Table 12. Assessing design concepts based on Sub features of Manufacturing.

Manufacturing (M)	Design Alternatives			
	Concept 1	Concept 2	Concept 3	Concept 4
AP (5/31 4/15 8/19)	2 5/2 3	2 5/2 3	2 5/2 3	2 5/2 3
MC (8/47 5/19 2/5)	3/2 2 5/2	2 5/2 3	1 3/2 2	2 5/2 3
EA (9/67 15/71 32/95)	3/2 2 5/2	5/2 3 7/2	3/2 2 5/2	3/2 2 5/2
MT (9/86 5/31 20/77)	2 5/2 3	5/2 3 7/2	3/2 2 5/2	2 5/2 3
IP (5/74 3/31 3/20)	3/2 2 5/2	3/2 2 5/2	2 5/2 3	1 3/2 2
Cumulative TFN	12/11 73/33 115/27	64/47 29/11 133/27	73/74 127/62 16/4	113/99 101/44 57/13

Table 13. Assessing design concepts based on Sub features of Durability.

Durability (D)	Design Alternatives			
	Concept 1	Concept 2	Concept 3	Concept 4
RP (3/14 1/3 1/2)	3/2 2 5/2	1 3/2 2	5/2 3 7/2	2 5/2 3
SW (13/81 21/83 24/61)	3/2 2 5/2	1 3/2 2	2 5/2 3	3/2 2 5/2
SF (1/9 5/29 22/83)	2 5/2 3	3/2 2 5/2	5/2 3 7/2	2 5/2 3
WSS (3/34 11/80 17/77)	2 5/2 3	5/2 3 7/2	2 5/2 3	2 5/2 3
DC (3/41 8/75 5/29)	2 5/2 3	3/2 2 5/2	2 5/2 3	2 5/2 3
Cumulative TFN	31/28 31/14 101/24	34/39 37/20 274/75	51/35 69/25 267/53	17/14 50/21 107/24

Table 14. Assessing design concepts based on Sub features of Damage to Workpiece.

Damage to Workpiece (DW)	Design Alternatives			
	Concept 1	Concept 2	Concept 3	Concept 4
ML (11/59 5/17 13/29)	3/2 2 5/2	1 3/2 2	2 5/2 3	3/2 2 5/2
GF (6/41 11/47 11/30)	3/2 2 5/2	3/2 2 5/2	2 5/2 3	3/2 2 5/2
RLW (1/6 12/49 9/25)	2 5/2 3	3/2 2 5/2	5/2 3 7/2	2 5/2 3
SAC (7/87 5/41 8/43)	3/2 2 5/2	1 3/2 2	3/2 2 5/2	3/2 2 5/2
PSL (4/55 2/19 9/53)	3/2 2 5/2	2 5/2 3	2 5/2 3	3/2 2 5/2
Cumulative TFN	17/16 138/65 16/4	15/17 131/71 133/37	120/89 41/16 248/53	17/16 138/65 16/4

Table 15. Fuzzified decision matrix for the design alternatives.

Design Alternatives	Design Attributes					
	R	CF	AD	M	D	DW
	12/67 12/41 21/46	11/96 1/5 12/37	1/17 10/97 16/83	1/17 10/97 16/83	1/17 10/97 16/83	2/17 1/5 18/53
Concept 1	15/14 145/68 163/40	92/95 8/4 277/66	7/6 128/55 221/50	12/11 73/33 115/27	31/28 31/14 101/24	17/16 138/65 16/4
Concept 2	20/19 44/21 201/50	51/79 3/2 47/14	50/51 75/37 119/30	64/47 29/11 133/27	34/39 37/20 274/75	15/17 131/71 133/37
Concept 3	27/22 19/8 378/85	53/46 98/43 197/42	108/97 85/38 197/46	73/74 127/62 16/4	51/35 69/25 267/53	120/89 41/16 248/53
Concept 4	65/62 109/52 181/45	25/19 53/21 77/15	85/94 166/87 181/48	113/99 101/44 57/13	17/14 50/21 107/24	17/16 138/65 16/4

Table 16. Normalized fuzzy decision matrix for the design alternatives.

Design Alternatives	Design Attributes					
	R	CF	AD	M	D	DW
	12/67 12/41 21/46	11/96 1/5 12/37	1/17 10/97 16/83	1/17 10/97 16/83	1/17 10/97 16/83	2/17 1/5 18/53
Concept 1	2/295 96/301 724/813	10/139 67/222 732/925	5/67 17/42 5/5	11/415 14/45 54/65	10/177 223/692 177/221	13/274 83/254 817/995
Concept 2	1/801 73/237 167/191	0/10 107/562 258/427	17/785 99/310 635/729	2/21 49/117 5/5	0/10 143/609 655/981	0/5 107/422 5/7
Concept 3	1/19 105/269 5/5	106/939 281/772 137/152	21/353 47/124 418/435	0/5 255/946 179/234	94/669 286/631 10/10	101/823 431/974 5/5
Concept 4	0/5 279/905 7/8	148/991 244/583 10/10	0/5 203/711 393/482	7/178 209/629 113/131	55/669 359/991 749/870	13/274 83/254 817/995

Table 17. Weighted Average and Ranking for each Design Concept.

Left and Right Scores for weights of Design Attributes	Normalized Left and Right Scores for weights of Design Attributes	Left and Right Scores of Design Alternatives			
		Concept 1	Concept 2	Concept 3	Concept 4
R $\left[\frac{5}{19}\quad\frac{20}{51}\right]$	$\left[\frac{17}{60}\quad\frac{13}{50}\right]$	$\left[\frac{9}{37}\quad\frac{17}{30}\right]$	$\left[\frac{4}{17}\quad\frac{24}{43}\right]$	$\left[\frac{7}{24}\quad\frac{41}{66}\right]$	$\left[\frac{4}{17}\quad\frac{43}{77}\right]$
CF $\left[\frac{7}{38}\quad\frac{15}{52}\right]$	$\left[\frac{1}{5}\quad\frac{13}{68}\right]$	$\left[\frac{13}{53}\quad\frac{17}{32}\right]$	$\left[\frac{4}{25}\quad\frac{3}{7}\right]$	$\left[\frac{16}{55}\quad\frac{17}{29}\right]$	$\left[\frac{31}{94}\quad\frac{43}{68}\right]$
AD $\left[\frac{8}{81}\quad\frac{3}{17}\right]$	$\left[\frac{5}{47}\quad\frac{2}{17}\right]$	$\left[\frac{7}{23}\quad\frac{42}{67}\right]$	$\left[\frac{16}{65}\quad\frac{32}{57}\right]$	$\left[\frac{27}{94}\quad\frac{17}{28}\right]$	$\left[\frac{2}{9}\quad\frac{8}{15}\right]$
M $\left[\frac{8}{81}\quad\frac{3}{17}\right]$	$\left[\frac{5}{47}\quad\frac{2}{17}\right]$	$\left[\frac{23}{95}\quad\frac{41}{75}\right]$	$\left[\frac{25}{79}\quad\frac{43}{68}\right]$	$\left[\frac{7}{33}\quad\frac{22}{43}\right]$	$\left[\frac{9}{35}\quad\frac{31}{55}\right]$
D $\left[\frac{8}{81}\quad\frac{3}{17}\right]$	$\left[\frac{5}{47}\quad\frac{2}{17}\right]$	$\left[\frac{14}{55}\quad\frac{13}{24}\right]$	$\left[\frac{4}{21}\quad\frac{41}{88}\right]$	$\left[\frac{29}{84}\quad\frac{64}{99}\right]$	$\left[\frac{15}{53}\quad\frac{27}{47}\right]$
DW $\left[\frac{17}{92}\quad\frac{14}{47}\right]$	$\left[\frac{1}{5}\quad\frac{15}{76}\right]$	$\left[\frac{12}{47}\quad\frac{50}{91}\right]$	$\left[\frac{18}{89}\quad\frac{22}{45}\right]$	$\left[\frac{1}{3}\quad\frac{61}{95}\right]$	$\left[\frac{12}{47}\quad\frac{50}{91}\right]$
Weighted Interval $[(\theta_i)^L, (\theta_i)^U]$		$\left[\frac{15}{59}\quad\frac{52}{93}\right]$	$\left[\frac{16}{73}\quad\frac{14}{27}\right]$	$\left[\frac{11}{37}\quad\frac{31}{51}\right]$	$\left[\frac{9}{34}\quad\frac{4}{7}\right]$
Weighted Average $\left[\frac{(\theta_i)^L + (\theta_i)^U}{2}\right]$		$\left[\frac{24}{59}\right]$	$\left[\frac{31}{84}\right]$	$\left[\frac{43}{95}\right]$	$\left[\frac{28}{67}\right]$
Ranking		3	4	1	2

5. Conclusion

The measurements of design attributes and sub-features for making decision on optimal design concept is logically indeterminate and imprecise because of the nature of information and interrelationships between them. In view of this, it is necessary to develop a structured decision-making model in order to cater for the imprecision or vagueness that are intrinsic in the linguistic assessments and interrelationships between the design features and sub-features. The conventional multi-objective or multi-attribute decision making models tend to be less effective particularly when numerous sub-features appear to have inherent relationships. To some extent, the multi-attribute models can deal with the ambiguity nature of the linguistic assessment through the use of fuzzy numbers. However, when it is required to consider the effects of design criteria and its sub-features on ranking the design alternatives and consider their relationships, it is possible to harness the strengths of two or more of these multi-attribute models. Hybridizing multi-attribute decision making models is an innovative research that requires attention in order to take care of the cumbersome data involved in decision process particularly when several features and sub factor are considered. This is necessary in order to have a robust selection process. In this article, the pairwise comparison strength of the fuzzy analytic hierarchy process was harnessed to examine the interrelationships between the design features and sub-features and the fuzzy synthetic extent analysis generates the fuzzy numbers that will represents the priority weights of the design features and sub-features. This expanded the solution because the design concepts need to assessed based on all the design features and sub-features. In order to solve the expanded solution, the computational strength of the fuzzy weighted average based on left and right scores was introduced to consider the involvement of the design features and sub-features in aggregating the TFNs for each design alternative and defuzzify the solution based on the intervals of the fuzzy ratings of the design alternatives and fuzzy weights of the design criteria.

Declarations

Author contribution statement

Olayinka Mohammed Olabanji & Khumbulani Mpofu: Conceived and designed the experiments; Performed the experiments; Analyzed and interpreted the data; Contributed reagents, materials, analysis tools or data; Wrote the paper.

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Competing interest statement

The authors declare no conflict of interest.

Additional information

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