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End-to-end Performance of Hybrid DF/AF (HDAF) Relayed Underlay Cognitive Radio Networks

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Abstract

In the presented work, the performance analysis of an underlay cognitive relay network with a hybrid-decode/amplify-and-forward relaying protocol is undertaken. Since the system is an underlay system, the secondary network will communicate simultaneously with the primary network. Hence interference temperature will act as a constraint to keep the secondary communication from interfering with the primary transmission. To ensure this, secondary transmit powers need to be kept below a pre-defined threshold level. Severe multi-path fading and shadowing or a long distance between the secondary source and the destination make a direct link between the two infeasible. These nodes can only be linked by a relay between the two. All the radio links in the system are characterized by the extended generalized-*K* fading distribution. The closed form expressions for the outage probability and ergodic channel capacity of the system are derived. The system outage performance and capacity with hybrid-decode/amplify-and-forward relay is compared to the performance with amplify-and-forward and decode-and-forward relays. Optimum relay position for best outage performance is also investigated. Further, the results of the performance analysis of the system are also illustrated by the numerical plots.

keywords: Cognitive relay networks, extended generalized-K distribution, fading and shadowing, hybrid-decode/amplify-and-forward relay, interference temperature, underlay system.

1. Introduction

Due to the increased data applications and services worldwide, global data traffic and volume has increased exponentially, giving rise to spectrum scarcity. This scarcity is more due to the under utilization of the available spectrum than due to actual lack of it. As a remedy to this problem, the cognitive radio networks came into existence in late twentieth century [1],[2]. These networks provided a means to alleviate the problem of spectrum scarcity. The said networks constitute multiple user networks that enable unlicensed (secondary) users to access a channel belonging to a licensed (primary) user. The access may be through one of the three laid down methods viz. overlay, underlay or interweave [3]. The channel has to be free before the secondary user (SU) can access it in overlay system, [4]. The SU has to vacate the channel if the primary user (PU) needs access to the channel as the PU will always have a higher priority. Underlay systems can give both PU and SU access to the channel simultaneously as in [5], provided the SU does not interfere with the PU signal while communicating. For this the powers transmitted by the secondary nodes need to be kept below the pre-defined threshold values. The secondary user in the interweave systems, employs some

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spectrum awareness methods to explore and then opportunistically utilize the unused spectrum, also called the spectrum holes as explained in [6]. In this work, the ultra wideband antenna configuration is used for spectrum sensing and the reconfigurable antenna is used for communication for secondary transmission on the radio channel.

A pointed out in [7], underlay cognitive radio networks are faced with certain fundamental performance issues. It was found that many of these issues can be resolved by relay assisted communication in these networks. Certain other issues like imperfect channel state information (CSI) [8] or hardware impairments [9], can also be addressed by introducing relays in cognitive radio networks, thus giving rise to cognitive relayed networks (CRNs). To counter the effects of multi-path fading and shadowing in radio frequency links [10], [11], the relay-assisted communication has been proven to be very effective. The relays used may be decode-and-forward (DF), amplify-and-forward (AF) or hybrid-decode/amplify-and-forward (HDAF) relays. Of all the relaying protocols, more commonly used are DF and AF protocols. While AF relay systems are simpler to implement, DF relayed systems have a better performance though the system complexity is much higher. As proposed in [12], [13], HDAF relaying cooperative protocol is by far a more efficient protocol compared to AF and DF relaying protocols. These relays combine the benefits of both AF and DF relaying and thereby improve the overall system performance. The underlying feature of this relaying system is that it performs like a DF relaying system when the instantaneous signal-to-noise ratio (SNR) is above the predefined threshold value and switches to the AF mode when the instantaneous SNR falls below that value. The system may also go into AF mode, in spite of the instantaneous SNR being above the threshold level, if there is a problem with decoding the input data bits, for example, when decoded cyclic redundancy check bits are incorrect. In such a case the relay does not stop the data flow during that period but continues forwarding the data to the destination node using AF relaying mode. In [12], the error probability of HDAF, AF and DF relays over a three node configuration in RF channel is evaluated. It concludes that the HDAF relay outperforms the other two relays when it is not placed too close to the source. Optical wireless communication systems with an HDAF relay have been investigated in [14] for outage performance. The system considers the influence of path attenuation, atmospheric turbulence and misalignment for the analysis. Outage and error performances, along with capacity, of a mixed RF-FSO system using an HDAF relay have been thoroughly investigated in [15]. The paper clearly illustrates the performance enhancement in the system with HDAF relay for outage probability. Multiple-relay systems using HDAF relaying protocol are investigated in [16], over RF channels having flat-fading and independent and non identically distributed (i.n.i.d.) characteristics.

For this work, the RF channel is considered to be having an extended generalized-K (EGK) fading distribution. This distribution is considered by many to be an extension of the generalized-K fading distribution that can be used to obtain many other fading distributions like Rayleigh, Nakagami-m and Weibull etc. This is demonstrated in [17], where it is shown how other distributions can be obtained as limiting or special case of EGK distribution. Also since this distribution can characterize accurately, both fading and shadowing in mobile radio channels, it is hence more appropriate for wireless communication. The probability of error for EGK fading channel has been computed for different modulation schemes in [18] where DF relaying has been used in the cognitive radio network. The EGK distribution has been used in [19] for spectrum sensing, using energy detection, and also for the derivation of the equations for the probability of detection of the CRN. The paper also demonstrates that for different detection schemes, the average probability of detection can be derived as a special case of the EGK distribution, for the various other multipath fading/shadowing distributions. The capacity and average symbol error rate for coherent and non-coherent modulation schemes of a relayed network having i.n.i.d EGK fading characteristics, is computed in [20]. The computation of these results involves calculation of the n^{th} moment of end-to-end SNR for the network. The outage probability of an underlay CRN with a DF relay, modeled on EGK fading characteristics, has been analyzed in [21]. The severity of fading and shadowing has also been considered in the analysis that also takes into account the influence of fading and shadowing in the RF channel.

The above mentioned facts served as the motivation to investigate the performance of a dual-hop HDAF relay-assisted cognitive relay network over an EGK fading channel. To the best of our knowledge, the performance of a dual-hop cognitive relay system, employing an HDAF relay in the secondary network and working under the constraint of interference temperature, has not been analyzed so far. Since it can approximate many fading distributions, under limiting conditions, as proven in [22], the EGK distribution is chosen for modeling the RF channel. Also, since this distribution can nearly approximate different fading and shadowing scenarios encountered over wireless channels [23] and is known to adapt to the changing channel conditions extremely well, it appears to be most suitable to model RF environment for the considered CRN. The main contributions of this paper can be summarised as follows:

- The hybrid decode/amplify-and-forward HDAF relaying protocol is used in an underlay cognitive radio system to enhance its performance, and we do its comprehensive performance analysis here.
- We evaluate the probability of outage and the ergodic capacity of the cognitive radio network for the considered system model taking into consideration the effects of multi-path fading and shadowing on the channel. Also, we assume the source and destination are far apart for a direct link to exist between these nodes and that the communication from source to destination can be established only through a relay. This assumption can also help us in analyzing the system performance in practical situations where line of sight (LOS) communication may not be possible.
- The analysis of system performance is done under the constraint of interference temperature, which in turn puts a limit of the transmit powers of the source and the relay. This is to avoid interference in the primary network due to communication in the secondary network, in the considered underlay system. Being an underlay system, both the primary and secondary networks will communicate simultaneously.
- The exact closed form expressions for the system parameters like outage probability and channel capacity of the considered system model are derived.
- The comparison of outage performance of the three relay types, AF, DF and HDAF, clearly illustrates the better performance of the hybrid relay as compared to the other two types.
- The ergodic capacity comparison for the three relay types is also performed in this work to showcase how the hybrid relay out performs the other two types of relays.
- The optimum relay position for a better system performance is also illustrated in this work.

The following gives the outline of the rest of the paper - A comprehensive description of the system model is given in section 2. Section 3 examines the equivalent SNR for the system using HDAF relaying protocol. Detailed performance analysis of the proposed system is undertaken in section 4. This includes the evaluation of exact outage probability and ergodic channel capacity in respective subsections. The obtained results are discussed in numerical analysis in section 5 and the paper is concluded in section 6.

2. System Model

Fig. 1. depicts a dual-hop CRN that consists of a primary network and a secondary network. The system under consideration being an underlay system, both the primary and the secondary networks will communicate simultaneously. The primary network comprises a primary source (S_p) and a primary destination (D_p). We assume that S_p is far removed from the secondary network to cause any interference to the signals of the secondary network and hence we show only the primary destination as the primary user (PU) in the system model and throughout this work. The secondary network consists of a secondary source (S_s), a secondary destination (D_s) with a secondary relay (R_s), connecting the source and the destination nodes. A direct link between S_s and D_s is assumed non-existent, either due to long distance or severe multipath fading/shadowing in the link between the two nodes. Hence the transmission of data in the secondary network takes place only through R_s in two hops. In the first hop, data is transmitted from S_s to R_s and then in the second hop from R_s to D_s, with powers P_s and P_r respectively. For the primary and the secondary networks to communicate simultaneously, the secondary transmission should not interference with the primary communication. This brings in the constraint of interference temperature which will in turn constrain the transmit powers of the secondary source and the relay. Due to these constraints, the powers that S_s and R_s can transmit, need to be kept below P, the maximum power available at these nodes. The channel coefficients for the links, $S_s - PU$, $R_s - PU$, $S_s - R_s$ and $R_s - D_s$, are modeled on EGK fading statistics and are denoted by h_{sp}, h_{rp}, h_{sr} and h_{rd} respectively. In the presented work, the HDAF relaying protocol is used for re-transmission of data from the relay to the destination. Further, the first-hop $S_s - R_s$ link and second-hop $R_s - D_s$ link, are taken to be having i.n.i.d fading characteristics, in addition to being memoryless and ergodic in nature. It is also assumed that the data flow is in one-direction only.

The SNR of the link from source to relay is taken as Z_1 and that from relay to destination as Z_2. These can be expressed, from [24], as

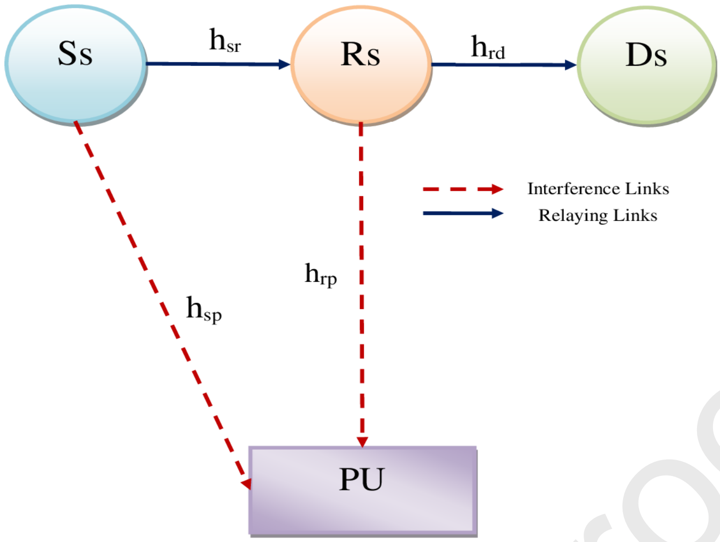

Figure 1: System model of a dual-hop HDAF relayed CRN system.

$$Z_1 = \frac{I}{|h_{sp}|^2}\frac{|h_{sr}|^2}{N_0}, Z_2 = \frac{I}{|h_{rp}|^2}\frac{|h_{rd}|^2}{N_0} \tag{1}$$

where I depicts the peak interference power that PU can tolerate and N_0 denotes the noise variance or noise power, [24].

The probability density function (PDF) $f_{Z_1}(\gamma)$ of Z_1, for the EGK distributed link between S_s and R_s in terms of Fox H-Function [25, Eq. (8.3.1)], is given from [26] as

$$f_{Z_1}(\gamma) = A_1\gamma^{-1}H^{2,4}_{4,4}\left(\frac{C_{sr}N_0}{C_{sp}I}\gamma \left| \begin{matrix} (0,1),(1,1),(\phi_1) \\ (\phi_2),(0,1),(1,1) \end{matrix} \right.\right) \tag{2}$$

where $A_1 = \frac{1}{\Gamma(m_{sr})\Gamma(m_{s_{sr}})\Gamma(m_{sp})\Gamma(m_{s_{sp}})}$, $\phi_1 = \left[\left(1-m_{sp}, \frac{1}{\zeta_{sp}}\right), \left(1-m_{s_{sp}}, \frac{1}{\zeta_{s_{sp}}}\right)\right]$, $C_{sp} = \frac{\beta_{sp}\beta_{s_{sp}}}{\bar{\gamma}_{sp}}$, $\phi_2 = \left[\left(m_{sr}, \frac{1}{\zeta_{sr}}\right), \left(m_{s_{sr}}, \frac{1}{\zeta_{s_{sr}}}\right)\right]$ and $C_{sr} = \frac{\beta_{sr}\beta_{s_{sr}}}{\bar{\gamma}_{sr}}$. The severity of fading is depicted by the terms m_{sp} and m_{sr} while shaping factors of fading for the respective links are defined by ζ_{sp} and ζ_{sr}. Likewise $m_{s_{sp}}$, $m_{s_{sr}}$, $\zeta_{s_{sp}}$ and $\zeta_{s_{sr}}$ represent the severity and shaping factors for shadowing in the channel. Less severe fading and shadowing in the channel are indicated by higher values of these parameters while smaller values indicate more intense fading and shadowing. Gamma function $\Gamma(.)$ is the defined as $\Gamma(n) = \int_0^\infty t^{n-1}\exp(-t)dt, R(t) > 0$ from [27], $H^{m,n}_{p,q}[.]$ denotes the Fox H-Function [28] and $\bar{\gamma}_{sp}$ and $\bar{\gamma}sr$ show the average received SNR for each link. Terms β_{sp}, $\beta_{s_{sp}}$, β_{sr} and $\beta_{s_{sr}}$ may be given, from [29] as $\beta_{sp} = \frac{\Gamma(m_{sp}+\frac{1}{\zeta_{sp}})}{\Gamma(m_{sp})}$, $\beta_{s_{sp}} = \frac{\Gamma(m_{s_{sp}}+\frac{1}{\zeta_{s_{sp}}})}{\Gamma(m_{s_{sp}})}$, $\beta_{sr} = \frac{\Gamma(m_{sr}+\frac{1}{\zeta_{sr}})}{\Gamma(m_{sr})}$ and $\beta_{s_{sr}} = \frac{\Gamma(m_{s_{sr}}+\frac{1}{\zeta_{s_{sr}}})}{\Gamma(m_{s_{sr}})}$.

The cumulative distribution function (CDF) of Z_1, $F_{Z_1}(\gamma)$, can be obtained by integrating (2) and is given by

$$F_{Z_1}(\gamma) = A_1 H^{2,3}_{3,3}\left(\frac{C_{sr}}{C_{sp}}\frac{N_0}{I}\gamma \left| \begin{matrix} (1,1),(\phi_1) \\ (\phi_2),(0,1) \end{matrix} \right.\right) \tag{3}$$

Similarly we get the PDF and CDF of Z_2, the link from relay to destination, as

$$f_{Z_2}(\gamma) = A_2\gamma^{-1}H^{2,4}_{4,4}\left(\frac{C_{rd}}{C_{rp}}\frac{N_0}{I}\gamma \left| \begin{matrix} (0,1),(1,1),(\phi_3) \\ (\phi_4),(0,1),(1,1) \end{matrix} \right.\right) \tag{4}$$

$$F_{Z_2}(\gamma) = A_2 H^{2,3}_{3,3}\left(\frac{C_{rd}}{C_{rp}}\frac{N_0}{I}\gamma \middle| \begin{matrix}(1,1),(\phi_3)\\(\phi_4),(0,1)\end{matrix}\right) \tag{5}$$

where $A_2 = \frac{1}{\Gamma(m_{rd})\Gamma(m_{s_{rd}})\Gamma(m_{rp})\Gamma(m_{s_{rp}})}$, $\phi_3 = \left[\left(1-m_{rp}, \frac{1}{\zeta_{rp}}\right), \left(1-m_{s_{rp}}, \frac{1}{\zeta_{s_{rp}}}\right)\right]$, $C_{rp} = \frac{\beta_{rp}\beta_{s_{rp}}}{\bar{\gamma}_{rp}}$, $\phi_4 = \left[\left(m_{rd}, \frac{1}{\zeta_{rd}}\right), \left(m_{s_{rd}}, \frac{1}{\zeta_{s_{rd}}}\right)\right]$ and $C_{rd} = \frac{\beta_{rd}\beta_{s_{rd}}}{\bar{\gamma}_{rd}}$. Again the terms m_{rp}, m_{rd}, $m_{s_{rp}}$, $m_{s_{rd}}$, ζ_{rp}, ζ_{rd}, $\zeta_{s_{rp}}$ and $\zeta_{s_{rd}}$ define the severity and shaping factors of fading and shadowing respectively. Terms $\beta_{rp} = \frac{\Gamma(m_{rp}+\frac{1}{\zeta_{rp}})}{\Gamma(m_{rp})}$, $\beta_{s_{rp}} = \frac{\Gamma(m_{s_{rp}}+\frac{1}{\zeta_{s_{rp}}})}{\Gamma(m_{s_{rp}})}$, $\beta_{rd} = \frac{\Gamma(m_{rd}+\frac{1}{\zeta_{rd}})}{\Gamma(m_{rd})}$ and $\beta_{s_{rd}} = \frac{\Gamma(m_{s_{rd}}+\frac{1}{\zeta_{s_{rd}}})}{\Gamma(m_{s_{rd}})}$. The average received SNR for the respective link is denoted by $\bar{\gamma}_{rp}$ and $\bar{\gamma}_{rd}$.

3. End-to-end SNR for HDAF relaying

To compute the end-to-end equivalent SNR for HDAF relaying, it is necessary to revisit the DF and AF relaying protocols first. For a dual-hop CRN system with a DF relay, the hop with the minimum SNR determines instantaneous equivalent SNR. This equivalent end-to-end SNR, γ_{DF}, from [26], can be written as

$$\gamma_{eq} = \gamma_{DF} = \min(Z_1, Z_2) \tag{6}$$

For a dual-hop variable-gain AF relayed CRN system, the harmonic mean of the two instantaneous SNRs, Z_1 and Z_1 determines the equivalent instantaneous SNR γ_{AF} of the system [30], and can be expressed as

$$\gamma_{AF} = \left(\frac{1}{Z_1} + \frac{1}{Z_2}\right)^{-1} = \frac{Z_1 Z_2}{Z_1 + Z_2} \tag{7}$$

For a CRN system using HDAF relaying, the relay R_s may choose either AF or DF mode for relaying, depending upon the received signal SNR. Determination of the equivalent SNR involves the following:

If the SNR received at the relay is higher than γ_{th}, i.e $Z_1 > \gamma_{th}$, the relay goes into the DF mode. The relay will perform the decoding of the data and then re-transmit it to the destination. The equivalent SNR γ_{HDAF} will then be same as equivalent SNR for DF relaying γ_{DF}. If, on the other hand, the SNR received at the relay is less than γ_{th}, i.e. $Z_1 < \gamma_{th}$, the relay will adopt the AF protocol for operating. In such a case the relay will simply amplify the received signal and forward it to the destination node. The equivalent SNR of the AF relayed system γ_{AF}, will determine the equivalent SNR of the HDAF relaying system. The above two conditions can be merged to define a deciding criterion for hybrid relaying as

$$Z_1 \underset{AF}{\overset{DF}{\gtrless}} \gamma_{th}. \tag{8}$$

The hybrid relaying can also be summarised as

$$\gamma_{HDAF} = (1-\xi)\gamma_{DF} + \xi\gamma_{AF} \tag{9}$$

where the random variable ξ is defined as $\xi = 1$ if $Z_1 < \gamma_{th}$, $\xi = 0$ if $Z_1 > \gamma_{th}$, and γ_{DF} and γ_{AF} are the SNRs in DF and AF relaying modes respectively.

4. Performance Analysis

4.1. Outage Probability

This subsection evaluates the outage performance for the considered CRN system, under the constraint of interference temperature I. The analytic expression for the probability of outage P_{out} is derived with the help of the decision criterion defined by (8).

As already discussed, when Z_1 exceeds the threshold SNR γ_{th}, the outage probability of HDAF relayed CRN becomes equal to the outage probability of the DF relayed CRN, after correct decoding of the signal by the relay. On the other hand, when Z_1 is lower than γ_{th}, the outage probability will be defined by the probability of an AF relayed CRN. Hence the average outage probability of the system may be given by

$$P_{out-HDAF} = P(Z_1 > \gamma_{th})P^1_{out-DF} + P(Z_1 < \gamma_{th}) \times P_{out-AF} \tag{10}$$

where P^1_{out-DF} depicts the outage probability in the DF mode after the correct decoding of the received signal by the relay while P_{out-AF} denotes the outage probability when AF relaying protocol is used by the CRN system. Using [15, Eq. (A2)] for P^1_{out-DF} and converting it to fit in the considered CRN, we get

$$P^1_{out-DF} = P(Z_2 < \gamma_{th}) = F_{Z_2}(\gamma_{th}) \tag{11}$$

Further, $P(Z_1 > \gamma_{th}) = 1 - P(Z_1 < \gamma_{th}) = 1 - F_{Z_1}(\gamma_{th})$, where $F_{Z_1}(\gamma_{th})$ can be substituted from (3), replacing γ by γ_{th}. Re-writing (10), we get

$$P_{out-HDAF} = [1 - F_{Z_1}(\gamma_{th})]P^1_{out-DF} + F_{Z_1}(\gamma_{th}) \times P_{out-AF} \tag{12}$$

The outage probability of a CRN with a variable-gain AF relay, P_{out-AF}, is derived in [31], and is given as

$$P_{out-AF} = 1 - A_1 A_2 H \left[\begin{array}{c} \left[\begin{array}{cc} 0 & 0 \\ 1 & 0 \end{array}\right] \\ \left(\begin{array}{cc} 3 & 4 \\ 4 & 5 \end{array}\right) \\ \left(\begin{array}{cc} 3 & 4 \\ 4 & 5 \end{array}\right) \end{array} \left| \begin{array}{c} (0,1);\ - \\ (0,1),(1,1),(\phi_1);(\phi_2),(0,1),(0,1),(1,1) \\ (0,1),(1,1),(\phi_3);(\phi_4),(0,1),(0,1),(1,1) \end{array} \right| \begin{array}{c} \\ X_1\gamma_{th} \\ X_2\gamma_{th} \end{array} \right] \tag{13}$$

The expression for P^1_{out-DF} can be obtained from (11) by substituting for $F_{Z_2}(\gamma_{th})$ from (5) with γ_{th} replacing γ. Further, by substituting for $F_{Z_1}(\gamma_{th})$ from (3), again with γ_{th} replacing γ, P^1_{out-DF} from (11) and P_{out-AF} from (13) in (12), and further mathematical manipulation using [32], the final closed form expression for the probability of outage $P_{out-HDAF}$ for an underlay CRN using an HDAF relay, can be obtained as

$$\begin{aligned} P_{out-HDAF} = {} & A_2 H^{3,3}_{2,3}\left(X_2\gamma_{th} \left| \begin{array}{c} (1,1),(\phi_3) \\ (\phi_4),(0,1) \end{array} \right.\right) - A_1 A_2 H \left[\begin{array}{c} \left[\begin{array}{cc} 0 & 0 \\ 0 & 0 \end{array}\right] \\ \left(\begin{array}{cc} 2 & 3 \\ 3 & 3 \end{array}\right) \\ \left(\begin{array}{cc} 2 & 3 \\ 3 & 3 \end{array}\right) \end{array} \left| \begin{array}{c} -;\ - \\ (1,1),(\phi_1);\ (\phi_2),(0,1) \\ (1,1),(\phi_3);\ (\phi_4),(0,1) \end{array} \right| \begin{array}{c} \\ X_1\gamma_{th} \\ X_2\gamma_{th} \end{array} \right] \\ & + \left[A_1 H^{2,3}_{3,3}\left(X_1\gamma_{th} \left| \begin{array}{c} (1,1),(\phi_1) \\ (\phi_2),(0,1) \end{array} \right.\right)\right] \left(1 - A_1 A_2 H \left[\begin{array}{c} \left[\begin{array}{cc} 0 & 0 \\ 1 & 0 \end{array}\right] \\ \left(\begin{array}{cc} 3 & 4 \\ 4 & 5 \end{array}\right) \\ \left(\begin{array}{cc} 3 & 4 \\ 4 & 5 \end{array}\right) \end{array} \left| \begin{array}{c} (0,1);\ - \\ (0,1),(1,1),(\phi_1);\ (\phi_2),(0,1),(0,1),(1,1) \\ (0,1),(1,1),(\phi_3);\ (\phi_4),(0,1),(0,1),(1,1) \end{array} \right| \begin{array}{c} \\ X_1\gamma_{th} \\ X_2\gamma_{th} \end{array} \right]\right). \end{aligned} \tag{14}$$

4.2. *Average Channel Capacity*

In this subsection, the analytical expression for the average channel capacity of the HDAF relayed underlay CRN is derived. For dual-hop underlay CRN with an HDAF relay operating over EGK fading channel, the channel capacity is obtained with the help of the decision criterion given by (8) which summarises the basic principle HDAF relaying. As per that criterion, the DF relaying protocol determines the channel capacity of the CRN system when Z_1 exceeds γ_{th} and for Z_1 less than γ_{th}, the channel capacity follows an AF relaying protocol. Hence the average channel capacity can be written as

$$C_{HDAF} = P(Z_1 > \gamma_{th})C_{DF} + P(Z_1 < \gamma_{th})C_{AF}, \tag{15}$$

where the respective channel capacities for the AF and DF relaying modes are denoted by C_{AF} and C_{DF}. For the HDAF relayed system, the channel capacity C_{HDAF} from [15] can be expressed in terms of $F_{Z_1}(\gamma_{th})$, the CDF of the end-to-end SNR for the $S_s - R_s$ link. This can be written as

$$C_{HDAF} = [1 - F_{Z_1}(\gamma_{th})]C_{DF} + F_{Z_1}(\gamma_{th})C_{AF}, \tag{16}$$

where $F_{Z_1}(\gamma_{th})$ can be found from (3), with γ_{th} replacing γ.

For the underlay CRN system with a DF relay, the channel capacity is given by [26] as

$$\begin{aligned}
C_{DF} = & \frac{A_1}{\ln(2)} H^{4,5}_{6,6}\left(X_1 \left| \begin{matrix} (0,1),(1,1)(\phi_1),(0,1),(1,1) \\ (\phi_2),(0,1),(0,1),(0,1),(1,1) \end{matrix}\right.\right) \\
& + \frac{A_2}{\ln(2)} H^{4,5}_{6,6}\left(X_2 \left| \begin{matrix} (0,1),(1,1),(\phi_3),(0,1),(1,1) \\ (\phi_4),(0,1),(0,1),(0,1)(1,1) \end{matrix}\right.\right) \\
& - \frac{A_1 A_2}{\ln(2)} H^{2,1:2,4:2,3}_{2,2:4,4:3,3}\left(\begin{matrix}\gamma X_1 \\ \gamma X_2\end{matrix} \left| \begin{matrix} (0,1),(1,1);(0,1),(1,1),(\phi_1);(1,1),(\phi_3) \\ (0,1),(0,1);(\phi_2),(0,1),(1,1);(\phi_4),(0,1) \end{matrix}\right.\right) \\
& - \frac{A_1 A_2}{\ln(2)} H^{2,1:2,4:2,3}_{2,2:4,4:3,3}\left(\begin{matrix}\gamma X_1 \\ \gamma X_2\end{matrix} \left| \begin{matrix} (0,1),(1,1);(0,1),(1,1),(\phi_3);(1,1),(\phi_1) \\ (0,1),(0,1);(\phi_4),(0,1),(1,1);(\phi_2),(0,1) \end{matrix}\right.\right)
\end{aligned} \tag{17}$$

The channel capacity of an AF relayed, dual-hop underlay CRN system can be obtained from [31] and is given as

$$\begin{aligned}
C_{AF} = & \frac{A_1 A_2}{\ln(2)} \left(H^{7,7}_{9,9}\left(\frac{X_2}{X_1} \left| \begin{matrix} (0,1),(1,1),(\phi_3),(\phi_5),(1,1),(1,1),(0,1) \\ (\phi_4),(0,1),(1,1),(0,1),(\phi_6),(0,1),(1,1) \end{matrix}\right.\right) \right. \\
& \left. - H\left[\left.\begin{matrix} \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \\ \begin{pmatrix} 3 & 4 \\ 4 & 5 \end{pmatrix} \\ \begin{pmatrix} 3 & 4 \\ 4 & 5 \end{pmatrix} \end{matrix} \right| \begin{matrix} (0,1);- \\ (0,1),(1,1),(\phi_1);\ (\phi_2),(0,1),(0,1),(1,1) \\ (0,1),(1,1),(\phi_3);\ (\phi_4),(0,1),(0,1),(1,1) \end{matrix} \left| \begin{matrix} \\ X_1 \\ X_2 \end{matrix} \right.\right] \right)
\end{aligned} \tag{18}$$

Substituting for C_{DF} and C_{AF} from (17) and (18) in (16), the closed form analytic expression for the average channel capacity of the CRN system operating with an HDAF relay under the interference temperature constraint can be obtained and is as expressed in (19).

$$\begin{aligned}
C_{HDAF} = & \frac{1}{\ln(2)} \left[1 - F_{Z_1}(\gamma)\right] \left[A_1 H^{4,5}_{6,6}\left(X_1 \left| \begin{matrix} (0,1),(1,1),(\phi_1),(0,1),(1,1) \\ (\phi_2),(0,1),(0,1),(0,1),(1,1) \end{matrix}\right.\right) + A_2 H^{4,5}_{6,6}\left(X_2 \left| \begin{matrix} (0,1),(1,1),(\phi_3),(0,1),(1,1) \\ (\phi_4),(0,1),(0,1),(0,1)(1,1) \end{matrix}\right.\right) \right. \\
& - A_1 A_2 H^{2,1:2,4:2,3}_{2,2:4,4:3,3}\left(\begin{matrix}\gamma X_1 \\ \gamma X_2\end{matrix} \left| \begin{matrix} (0,1),(1,1);(0,1),(1,1),(\phi_1);(1,1),(\phi_3) \\ (0,1),(0,1);(\phi_2),(0,1),(1,1);(\phi_4),(0,1) \end{matrix}\right.\right) \\
& \left. - A_1 A_2 H^{2,1:2,4:2,3}_{2,2:4,4:3,3}\left(\begin{matrix}\gamma X_1 \\ \gamma X_2\end{matrix} \left| \begin{matrix} (0,1),(1,1);(0,1),(1,1),(\phi_3);(1,1),(\phi_1) \\ (0,1),(0,1);(\phi_4),(0,1),(1,1);(\phi_2),(0,1) \end{matrix}\right.\right) \right] \\
& + F_{Z_1}(\gamma) \left[\frac{A_1 A_2}{\ln(2)} H^{7,7}_{9,9}\left(\frac{X_2}{X_1} \left| \begin{matrix} (0,1),(1,1),(\phi_3),(\phi_5),(1,1),(1,1),(0,1) \\ (\phi_4),(0,1),(1,1),(0,1),(\phi_6),(0,1),(1,1) \end{matrix}\right.\right) \right. \\
& \left. - H\left[\left.\begin{matrix} \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \\ \begin{pmatrix} 3 & 4 \\ 4 & 5 \end{pmatrix} \\ \begin{pmatrix} 3 & 4 \\ 4 & 5 \end{pmatrix} \end{matrix} \right| \begin{matrix} (0,1);- \\ (0,1),(1,1),(\phi_1);\ (\phi_2),(0,1),(0,1),(1,1) \\ (0,1),(1,1),(\phi_3);\ (\phi_4),(0,1),(0,1),(1,1) \end{matrix} \left| \begin{matrix} \\ X_1 \\ X_2 \end{matrix} \right.\right] \right]
\end{aligned} \tag{19}$$

5. Numerical results

We present the simulation results for the outage probability and channel capacity of the considered HDAF relay-assisted underlay CRN system in this section. It is considered that a direct link between the secondary source transmitter and destination receiver cannot be realized owing to the long distance between these nodes or excessive fading and shadowing in the connecting radio link. The RF channel, linking the different nodes of the system, is considered to have an *EGK* fading distribution, for the advantages already enumerated. Due to the proximity of *PU* with the secondary network, interference temperature will present a constraint in the system. This imposes a constraint on the powers transmitted by S_s and R_s. This constraint ensures that no interference is caused at the primary user destination *PU* when the primary and the secondary networks are communicating simultaneously, as happens in an underlay system. All the radio channels are considered to have i. n. i. d. characteristics. After normalizing all the distances, the propagation constant α is allotted a value of 4 for a typical non-line-of-sight propagation model described in [33]. Initially S_s, R_s and D_s from the secondary network are positioned at (0, 0), (0.5, 0) and (1, 0) respectively while the primary user destination, *PU* is considered to be at (0.5, 0.5). The terms depicting severity of fading and shadowing, $m_{sp}, m_{sr}, m_{rp}, m_{rd}, m_{s_{sp}}, m_{s_{sr}}, m_{s_{rp}}$ and $m_{s_{rd}}$ are all assumed to be equal and represented by a general term m_k while the shaping factors of fading and shadowing, $\zeta_{sp}, \zeta_{sr}, \zeta_{rp}, \zeta_{rd}, \zeta_{s_{sp}}, \zeta_{s_{sr}}, \zeta_{s_{rp}}$ and $\zeta_{s_{rd}}$, are also assumed to be equal and are represented by a common term ζ_k. These quantities, when assigned different values, demonstrate their effect on the considered performance parameters of the system under the given constraints.

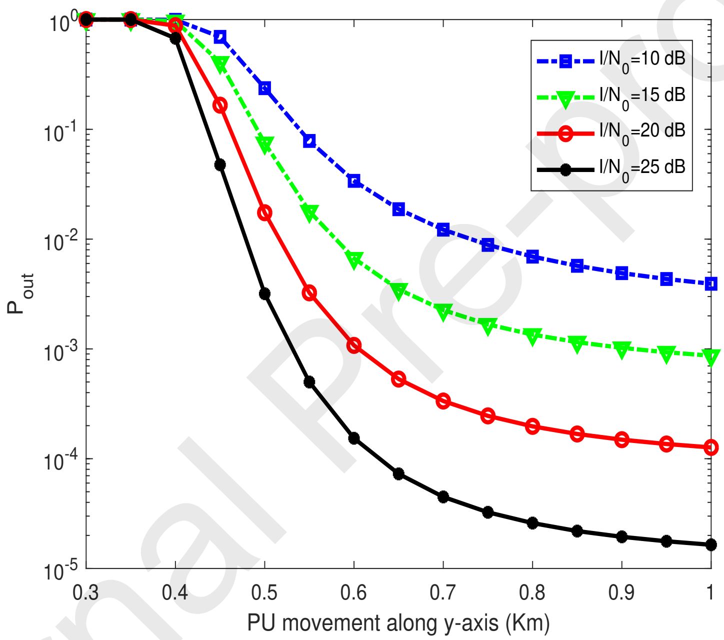

Figure 2: Outage probability vs. primary movement along the vertical axis for different values of interference to noise power ratio I/N_0, considering S_s at (0,0), R_s at (0.5, 0), D_s at (1,0), m_k =2, ζ_k =1 and γ_{th} = 5 dB.

Fig. 2 depicts the variation of outage probability of the HDAF relayed CRN against *PU* movement from its initial position of (0.5, 0. 5), along y-axis. The plots are taken for different values of interference to noise power ratio I/N_0. All other nodes are at their initial positions while m_k =2, ζ_k =1 and γ_{th} = 5 dB. It can clearly be seen from the figure that P_{out} drops as the distance the primary from the secondary network increases. When the *PU* is close to the secondary network, like 0.3 Km from it, the outage probability is almost 1. As the distance increases, P_{out} drops significantly thereafter reaching saturation after *PU* reaches 0.9 Km, where it is almost $10^{-4.8}$. It can also be observed that for all the values of I/N_0, the behaviour of the network remains consistent. This is in accordance with the analytical results that predict outage probability to drop as the primary distance from the secondary network increases.

In Fig. 3, the outage probability of the AF, DF and HDAF relays is compared when *PU* moves away from the secondary network along the y-axis. All other secondary nodes continue to be at the initial positions. For this plot, γ_{th} = 5 dB I/N_0= 10 dB and ζ_k =1. It is clearly seen that the outage probability of all the three types of relays drops significantly as the *PU* moves away but for the HDAF relay the drop is significantly more as compared to the other two relays thereby giving the minimum value for P_{out}. Also it is observed that the outage probability at m_k = 1 is higher than at m_k = 2 for all the three types of relays, which is as expected because lower values of m_k indicate higher

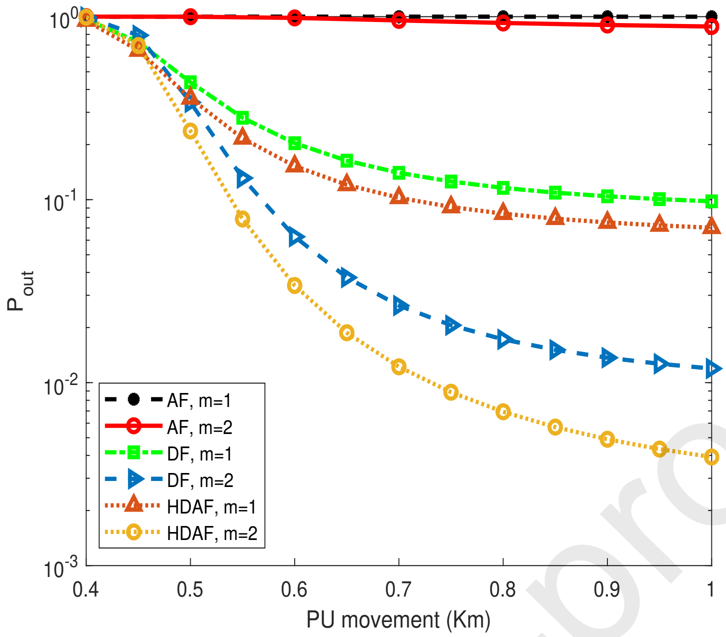

Figure 3: Outage Probability against PU movement for relayed underlay CRN with HDAF, DF and AF relays in the system. The value of γ_{th} for all the three cases is taken to be 5 dB while I/N_0 is taken as 10 dB. The value of ζ_k is taken to be 1 while m_k is considered at the values of 1 and 2 for the comparison in this plot.

fading and shadowing in the radio channel. It is evident from the plot that the HDAF relaying protocol out performs the other protocols as far as outage variation with PU movement is concerned. DF relaying comes next while the AF relaying shows a very high outage probability.

Fig. 4 traces the outage probability against I/N_0 for different values of γ_{th}. All the nodes of the system, i.e. S_s,R_s, D_s and PU continue to be at their initial positions. The severity of fading and shadowing m_k is assigned a value of 2 while the shaping factor ζ_k assumes the values of 1 for this plot. It can be observed from Fig. 4 that the P_{out} drops significantly with the increase in the value of I/N_0. For all the values of γ_{th}, the pattern is consistent. The outage probability is minimum for $\gamma_{th} = 10$ dB and maximum for $\gamma_{th} = 20$ dB. Analytic results also lead to the same conclusions.

Fig. 5 determines the optimum position of the relay for minimum outage probability of the hybrid relayed underlay CRN for different values of γ_{th}. All other nodes like PU, S_s and D_s continue to be at their initial positions of (0.5, 0.5), (0, 0) and (1, 0) respectively. Terms m_k and ζ_k are both taken to be 1 for this plot. Hence more severe multipath fading is considered in this case. It is observed from the graph that the optimum relay position, where the outage probability is minimum is around 0.57 Km from the secondary source, i.e. $d_{sr} = 0.\ 57$ Km. As we move the relay from this optimum position, on either side, the outage probability increases. It is true for all the values of γ_{th}, like 0 dB, 3 dB, 5 dB and 8 dB for which the plot has been made. It can thus be concluded that the considered CRN system model will give best results for outage probability if the relay is positioned somewhere near the middle, between the secondary source and destination for all values of γ_{th}.

The advantages of HDAF relay for minimizing the outage probability are demonstrated in Fig. 6, where we compare the outage performance of HDAF relay to that of DF and AF relay in the considered CRN system model. The comparison is made for m_k at the values of 1 (more severe fading) and 2 (less severe fading) for the three relay types. The system nodes PU, S_s, R_s and D_s continue to be at the initial positions of (0.5, 0.5), (0, 0), (0.5, 0) and (1, 0) respectively. The observations are made for γ_{th} at a constant value of 15 dB, for all the relays. It is evident from the figure that the hybrid relay outperforms the AF and the DF relay as the outage probability with HDAF relay in the CRN is less than with AF or DF relay. It is also clear from the figure that there is a marked difference in the values of outage probability of all the three relays for $m_k = 1$ and $m_k = 2$ with the outage probability being more at $m_k = 1$ than

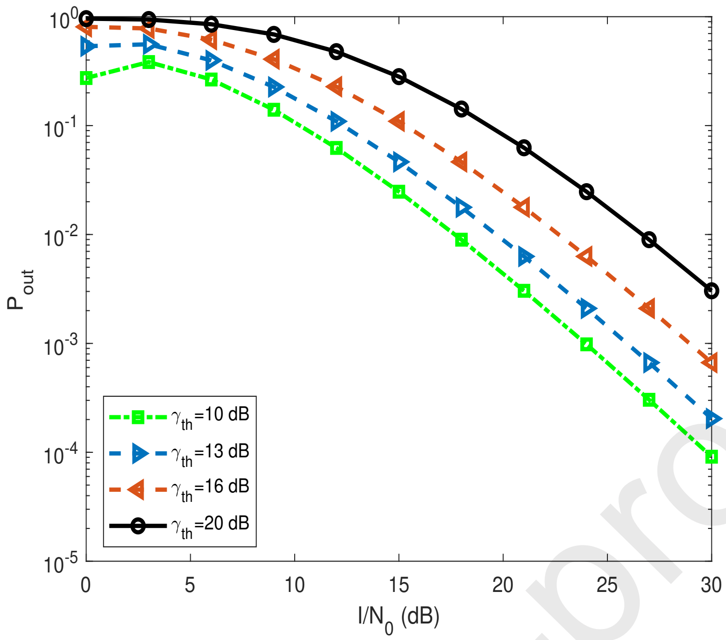

Figure 4: Outage Probability variation against the interference to noise power ratio I/N_0 for HDAF relayed CRN for different values of, keeping the source, the relay and the destination at the initial positions of (0, 0), (0.5, 0) and (1, 0).

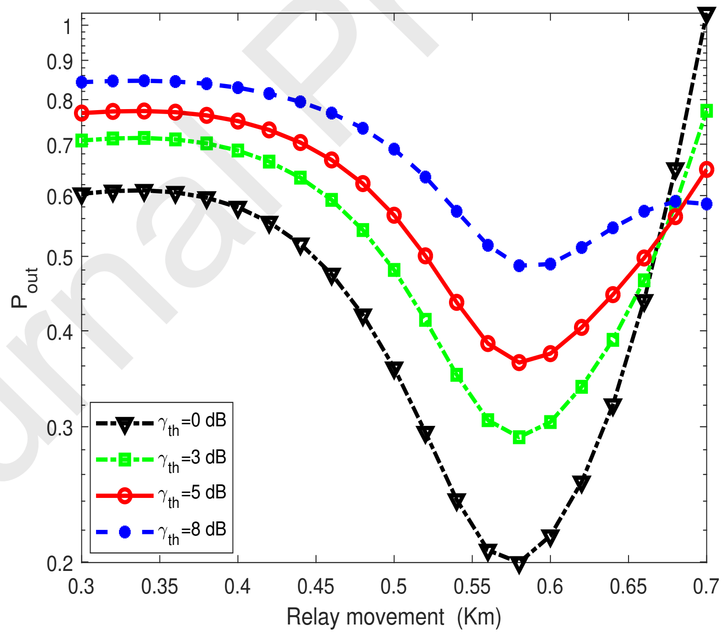

Figure 5: Variation of outage probability against the movement of relay along the x-axis in HDAF relayed CRN system. The positions of PU, S_s and D_s are kept at the initial values while γ_{th} = 5 dB and m_k and ζ_k both are kept at 1

at $m_k = 2$. This resonates with the theory that the lower values of m_k indicate more severe fading and hence higher outage probability. This can be concluded from the analytic results also. As can be observed from the plot, the HDAF relay performs better at both these values of m_k, compared to AF and DF relays, for the considered value of γ_{th}.

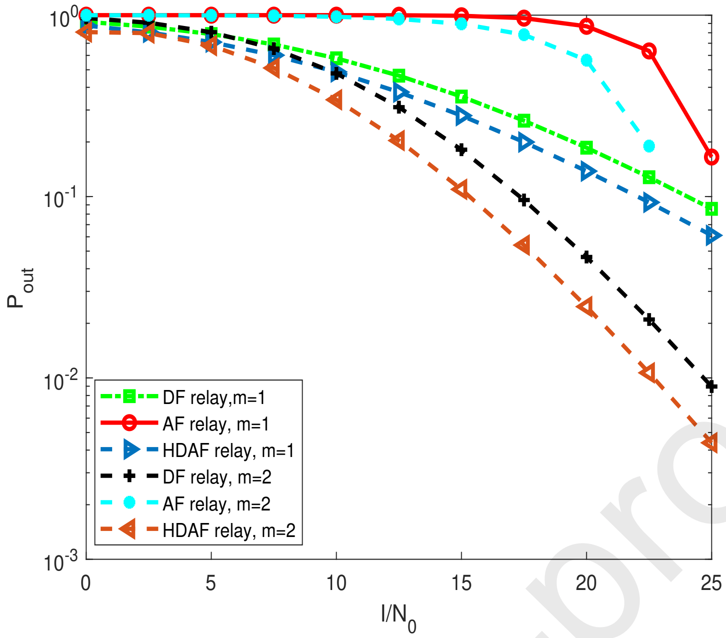

Figure 6: Outage Probability against I/N_0 for relayed underlay CRN with HDAF, DF and AF relays in the system. The value of γ_{th} for all the three cases is taken to be 15 dB. m_k is considered at the values of 1 and 2 for this plot while ζ_k=1.

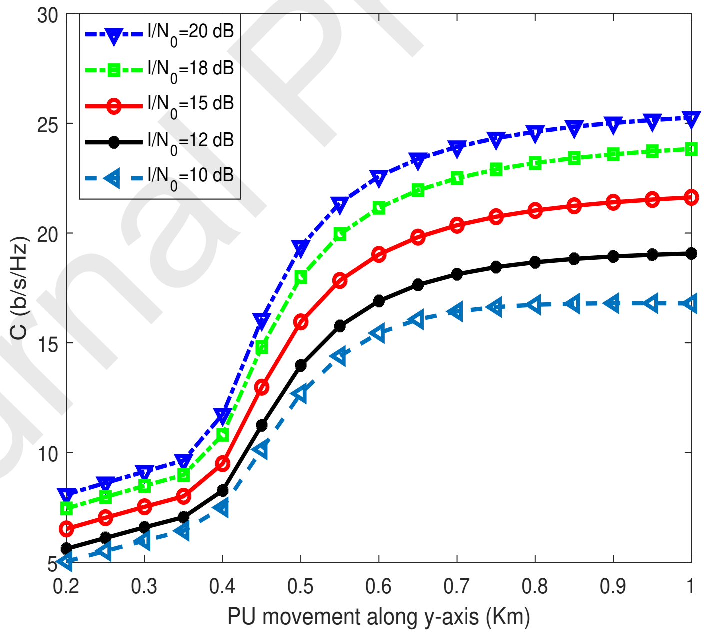

Figure 7: Impact of *PU* movement along the y-axis on the capacity of the HDAF relayed underlay CRN with all other nodes at their initial positions of (0, 0), (0.5, 0) and (1, 0), γ_{th} = 5 dB , m_k =2 and ζ_k=1

In Fig. 7, the impact of *PU* movement on the capacity of the considered CRN with a hybrid relay is studied. All the secondary nodes are at the initial positions of (0, 0), (0.5, 0) and (1, 0). Shaping factor and severity parameters, ζ_k

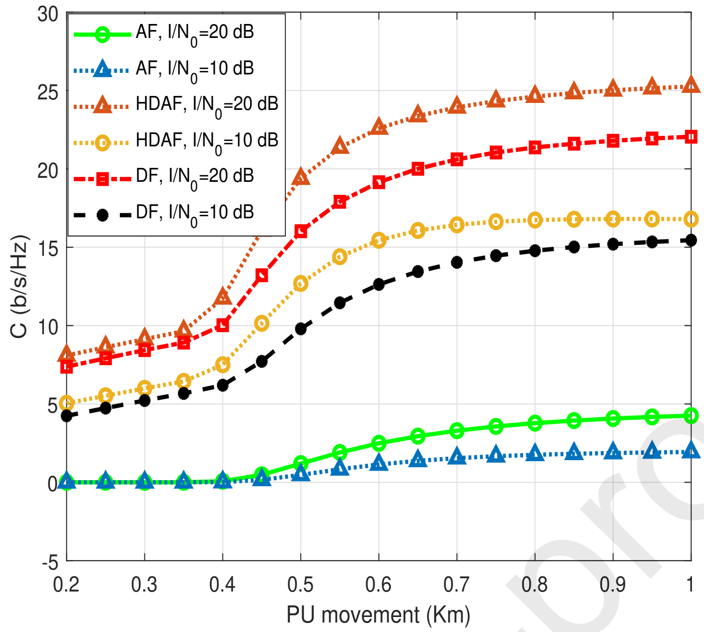

Figure 8: Capacity against the movement of the PU for relayed underlay CRN with HDAF, DF and AF relays for I/N_0 at the values of 10 and 20 dB. The value of γ_{th} for all the three cases is taken to be 5 dB. m_k is taken to be 2 for this plot while ζ_k=1.

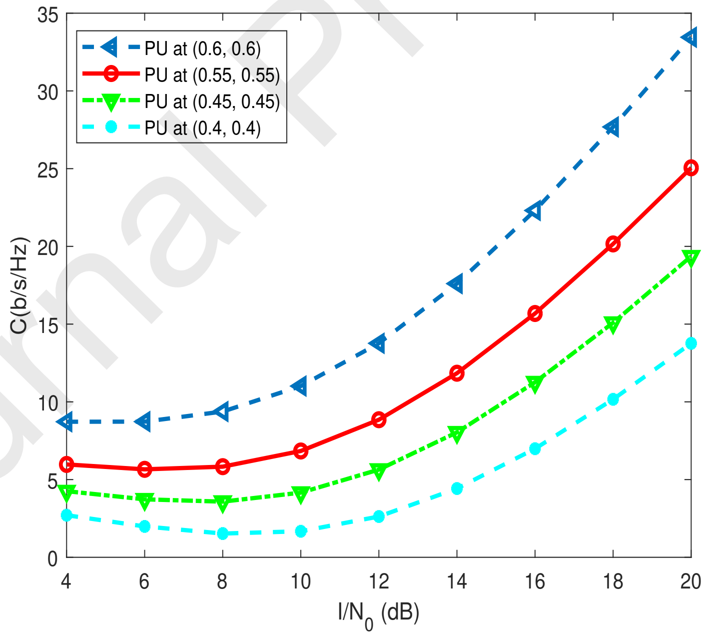

Figure 9: Change in the Capacity against the variation in the I/N_0 for different PU positions for the considered HDAF relayed CRN. The value of γ_{th} = 5 dB while m_k and ζ_k are kept at a value of 1. Apart from PU, all other nodes are at their initial positions.

and m_k are held at the values of 1 and 2 respectively and γ_{th} is kept at 5 dB. The capacity C in (b/s/Hz) is observed for different values of I/N_0. It is observed that when the PU is close to the secondary network, e.g. at 0.2 Km from it, the capacity is almost 0. But as the PU moves away from the secondary network, the capacity increases sharply and thereafter saturates after reaching the value of 0.7 Km from the secondary network. This again reiterates the analytic concept that less the proximity of the primary user to the secondary network, the higher the capacity of the system. The behaviour of the system remains the same for all the values of the I/N_0 that have been considered. Another observation that can be made is that higher the value of I/N_0 (like 20 dB), the higher the capacity of the considered system (like 25 b/s/Hz). On the other hand, I/N_0 of 10 dB corresponds to a capacity of 15 b/s/Hz.

Better performance of the HDAF relay compared to DF and AF relays, as far as capacity variation with PU movement goes, is fully demonstrated in Fig. 8. For this plot, all other secondary nodes are at the initial positions while γ_{th} is at 5 dB, $m_k = 2$ and $\zeta_k = 1$. In general, the capacity(b/s/Hz) for all the relay types shows an increase as the PU distance from the secondary network increases. To go further, the capacity of the relaying protocols is then compared with I/N_0 at the values 10 dB first where it is observed that HDAF relaying shows higher capacity when compared to AF and DF relaying protocols. Secondly, when compared with I/N_0 at 20 dB, similar results are observed thereby highlighting better performance of hybrid relaying protocol for capacity when PU is moving. AF relaying shows the worst performance while DF relaying is somewhere in the middle.

The capacity variation with I/N_0 at different PU positions is illustrated in Fig. 9. The secondary nodes S_s, R_s and D_s are at the original positions of (0, 0), (0.5, 0) and (1, 0) respectively. Terms m_k and ζ_k are taken to be 2 and 1 respectively and γ_{th} is kept at 5 dB. Increase in the capacity with the increase in the value of I/N_0 is observed in this plot in general. It is further observed that when PU is closer to the secondary network like at (0.4, 0.4), the capacity is less compared to when PU is away from the secondary network like at (0.6, 0.6). This outcome is along the expected lines as it is well known from analysis that the more the separation between the primary and the secondary networks, the better the effect on the capacity of the underlay CRN system where both the networks communicate simultaneously.

6. Conclusion

In the presented work, the end-to-end performance of a relayed underlay CRN system using HDAF relaying, is investigated. A long link length or excessive fading and shadowing attribute to the non-existence of a direct link between the source and the destination nodes in the secondary network. As a result, the communication between the two is possible only through a relay (an HDAF relay in this case). The system is constrained by interference temperature due the proximity of the primary user destination to secondary source and relay nodes. Due to this, there is a constraint on the transmit powers of these nodes to avoid interfering with the primary communication. The RF links in the system have an EGK distribution as it is a generalized distribution on which several key distributions can be modeled under special and limiting conditions. Also it can model the RF link according to the prevalent channel conditions. The closed form expressions for the outage probability and channel capacity of the underlay CRN system are derived. It can easily be concluded that the proposed HDAF-relayed underlay CRN system has an improved system performance, both for the outage probability and the system capacity, as compared to the more commonly used AF and DF relaying systems. It can further be concluded that optimal relay positioning can improve the performance of the system even more as will the increased distance between the primary network from the secondary network.

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