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# Multi-period pricing in the presence of competition and social influence 

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## Multi-period pricing in the presence of competition and social influence

## Highlights

- We study Single vs. Dynamic Pricing Policies in a duopoly with social influence.
- Assuming two firms adopt the same pricing policy, either policy may be preferable.
- Dynamic Pricing always dominates for a sufficiently large market size in period 2.
- Posted Pricing Policy may dominate when social influence is sufficiently strong.
- If firms freely choose Single or Dynamic Pricing, asymmetric equilibria may exist.


## *Manuscript

# Multi-period pricing in the presence of competition and social influence 


#### Abstract

This paper examines Single and Dynamic Pricing Policies of two competing firms over two periods in the presence of social influence. Assuming two firms adopt the same pricing policy, we find that, under either pricing policy, firm profits always decrease with the degree of social influence. Firms prefer Dynamic Pricing Policy when social influence is either relatively weak or sufficiently strong (for firms under Dynamic Pricing Policy to set zero prices in the first period). Otherwise, Single Pricing Policy is more preferable. The conclusions are similar when the market size varies over periods, except that Dynamic Pricing Policy is always more profitable if the market size in period 2 is sufficiently large. We have further compared the two pricing policies with Posted Pricing Policy. The results show that Dynamic Pricing Policy dominates when social influence is relatively weak, while Posted Pricing Policy dominates when social influence is sufficiently strong because of the synergy between social influence and the reference price effect. Finally, when each firm freely chooses either Single or Dynamic Pricing Policy, we find that, if the degree of social influence is relatively small, two asymmetric equilibria exist where two firms adopt different pricing policies. If the degree of social influence is very large, however, the unique equilibrium is both firms adopting Dynamic Pricing Policy. These findings provide important implications for firms to make more informed pricing decisions in an increasingly competitive environment with strong social influence.


Keywords: Social influence, Multi-period pricing, Price

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competition, Single Pricing Policy, Dynamic Pricing Policy
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## 1. Introduction

Social influence is defined as "any change which a person's relations with other people (individual, group, institution or society) produce on his (sic) intellectual activities, emotions or actions" (Dictionary of Personality and Social Psychology (1986, p.328)). It has been found to significantly affect consumer purchase decision. Substantial research has shown that consumers prefer products that are popular among other consumers in the previous selling periods (Hu et al., 2015). This explains why almost all the online shopping websites (e.g., Amazon and Target) update the sales ranking information from time to time to show which product is more popular. Particularly, Amazon adds a "best seller" tag on the product that is very popular. In some websites, they also show the previous sales quantity of each product. For example, the daily deal site, DailyDeal, and eBay both show the sales quantity of their products. Similarly, T-mall and Taobao, the two most famous shopping websites in China, also reveal the sales quantity in the previous month of each product, which will certainly be an important reference for consumers to make purchase decisions. The underlying motivation of displaying the sales quantity information is similar to having "top seller" lists (Parsons et al., 2014). Empirical studies also show that social influence results in the "the rich get richer" phenomenon (Cai et al., 2009; Carare, 2012). Carare (2012) finds that a consumer would like to pay more for a highly ranked product through investigating the sales ranking in Apple's App Store.

As a consequence, social influence has oftentimes been utilized as a strategic tool for firms to stand out in the increasingly fierce competition. Nowadays various merchandises similar in function and quality are available to consumers from multiple channels-both online and offline. It becomes highly difficult for consumers to evaluate and choose from so many products. Hence, consumers rely more on the previous sales quantity or ranking of the products to make their purchase decisions. This motivates firms to differentiate its product from the competitors by occupying a larger market share quickly in the initial periods
of the selling horizon and benefit from social influence in the following periods. Thus, it is now regular practice for firms to charge a much lower price in the initial periods to quickly accumulate sales, and raise price afterwards to reap the profits.

For example, on Juhuasuan (a famous online daily-deal shopping website in China where retailers have limited time to sell their products), two competing firms ( $A$ and $B$ ) sell raisins with similar quality during the same time period. We observe that the two firms adopt different pricing policies. Firm $A$ cuts its price by 50 percent (the selling price is below its marginal cost) in the first three minutes, while firm $B$ keeps its price constant (at the same level with firm $A$ 's price after the first three minutes). At the end of the three minutes, as expected, $A$ obtains more sales than firm $B$ ( 1459 units vs. 15 units). Interestingly, however, when the selling period is over, $A$ sells a whopping 15080 units, approximately fifty times more than $B$ (295 units). Such an astonishing difference occurs because the larger market share that firm $A$ occupies in the first three minutes has systematically affected the purchase decision of a new cohort of consumers in the following period. This is why firm $A$ 's sales accumulate so fast afterwards, compared to its competitor.

Firms in the above example are not exceptions. Many retailers are trying to "create" hot style (products) by strategically adjusting its prices over different periods. However, it remains unclear whether firms in a competitive setting can actually benefit from social influence under different pricing policies. Oftentimes, a firm competes head-to-head with other firms and may adopt the same pricing policy with its competitors. In the presence of social influence, all the firms may have strong motivations to lower their prices in the initial period (to occupy a larger market share and take advantage of social influence), which may lead to more intense pricing competition and even damage firms' profits. Therefore, in the presence of social influence, competing firms selling horizontally differentiated products may face serious challenges in making optimal pricing decisions. Pricing over multiple periods considering social influence has received significant research attention in recent years; however, there is few
study that examines and compares different multi-period pricing strategies in a competitive setting in the presence of social influence.

In this paper, we examine the effect of social influence in a competitive setting by considering two competing firms' pricing strategies and profits over two consecutive periods. Particularly, we investigate and focus on two pricing schemes: Single Pricing Policy (also referred to as static pricing, uniform pricing, fixed pricing) and Dynamic Pricing Policy (also referred to as contingent pricing). The former means that firms commit to sell at a fixed price at the beginning of the selling horizon. Under Dynamic Pricing, firms decide their prices sequentially at each period during the selling horizon (Cachon and Feldma, 2010; Şen, 2013; Tong et al., 2020). As an extension, in Section 5 we further compare these two pricing policies with another policy, named Posted Pricing Policy (also referred to as preannounced pricing policy, Dasu and Tong, 2010; Papanastasiou and Savva, 2017), where firms announce their prices for the following two periods simultaneously at the beginning of the selling horizon. Firms adopting Posted Pricing Policy intend to leverage a reference price effect to accumulate a even larger market share in the initial period.

Specifically, we intend to address the following research questions. First, what is the impact of social influence on competing firms' equilibrium prices and profits under different pricing schemes? Which pricing scheme is better for the firms, Single Pricing Policy or Dynamic Pricing Policy? Second, how would the conclusions change when the market size varies over two periods? Third, how would Dynamic Pricing Policy and Single Pricing Policy perform when compared to the Posted Pricing Policy with a reference price effect? Finally, what if the two competing firms adopt different pricing policies? Answering these questions may provide important implications for firms to make more informed pricing decisions in an increasingly competitive environment with strong social influence.

The rest of the paper is organized as follows. We review the relevant literature in Section 2. Section 3 introduces and analyzes the main model where both firms adopt the same pricing policy (i.e., either Single Pricing Policy or

Dynamic Pricing Policy). Section 4 examines the model where the market size varies over the two periods (Extension 1). Section 5 further compares the two pricing policies to Posted Pricing Policy with a reference price effect (Extension 2). Section 6 allows the competing firms to adopt different pricing policies and examine the equilibrium pricing strategies (Extension 3). Section 7 summarizes the findings and implications, and provides some directions for future research.

## 2. Literature review

In existing literature, there are a large number of papers studying social learning and strategic consumers (Bhalla, 2012; Papanastasiou and Savva, 2017; Feldman et al., 2019). These studies assume that product quality is unknown, and consumers form their belief in quality by learning from other consumers who purchase in the previous period. Each consumer is assumed to be rational and forward-looking, deciding in which period to make a purchase to maximize the inter-temporal utility. For example, Papanastasiou and Savva (2017) investigate the pricing strategies of a monopolist who sells a new product to forward-looking consumers over two periods. They show that, when there is no social learning, the firm prefers a decreasing price strategy. By contrast, in the presence of social learning, the firm will adopt an increasing price plan under Posted Pricing Policy (preannounced pricing), while prices may either rise or decline under Dynamic Pricing Policy (responsive pricing). Feldman et al. (2019) study the design of a new experience product by a monopolistic firm and argue that when consumers are sufficiently forward-looking, social learning hurts the firm's profit. Peng et al. (2020) investigate price guarantee policies for the advance selling of a seller in the presence of strategic consumers who make purchase decisions through preorder-dependent social learning. They show that, in the advance selling model, price guarantee policies may benefit the seller under certain conditions.

This stream of literature, however, mainly examines experience goods, while our work focuses on search goods for which consumers have complete knowledge regarding quality. Furthermore, the above research primarily models how
consumers learn about product quality from other consumers and strategically decide when to make a purchase. In contrast, we focus on how the sales in the previous period influence the purchase decision of a new cohort of consumers in the following period, and thus firms' multi-period pricing and profits.

Social influence has also been thoroughly investigated in the literature on luxury fashion consumption, where consumers purchase conspicuous goods to "show off" their wealth or social status (Mason, 1984; Bagwell and Bernheim, 1996; Corneo and Jeanne, 1997). In this stream of literature, researchers often distinguish two types of consumers, i.e., fashion leader (snob) and fashion follower (conformist) (Grilo et al. 2001; Amaldoss and Jain, 2005a; Amaldoss and Jain, 2005b; Zheng et al., 2012; Shen et al., 2017). The former desires uniqueness while the later desires conformity, and such a difference may provide important implications for firms. For example, Amaldoss and Jain (2005a) propose a monopoly model and find that the firm's profit increases with the follower effect but decreases with snobbishness. Amaldoss and Jain (2005b) further investigate a duopoly model and show that the desire for uniqueness leads to higher prices and profits, while the desire for conformity leads to lower prices and profits. Moreover, as service design is one of the most important strategies in luxury fashion supply chain management (Brun et al., 2008), Shen et al. (2017) study the price and service strategies of a luxury fashion supply chain with one supplier and one retailer while considering social influence and demand change. They find that the supply chain is more likely to provide better service to fashion leaders than to fashion followers when the impact of social influence becomes stronger. With luxury fashion market as the research context, other researchers also have examined the allocation of advertising budget between the leaders and the followers (Chiu et al., 2018), and the monopoly firm's pricing and production decisions when considering conspicuous consumers' discount sensitivity behavior (Zhou et al., 2018).

The literature on luxury goods consumption mainly studies the rationalexpectation equilibrium where consumers' purchase decision is influenced by their expectation regarding other consumers' purchase behaviors. Our research,
however, proposes a two-period model and examines products without attributes that signal social status (e.g., raisins). For such products, in our two-period modelling framework, the sales quantity in the first period will influence consumer purchase decision in the second period, but not the other way around (i.e., consumer purchase decision in the first period are not affected by the sales quantity in the second period). In particular, consumers can derive higher utility from a product with more sales in the previous period (Bensaid and Lesne, 1996; Gabszewicz and Garcia, 2008). Thus, the social influence in our paper is more like the follower effect.

Our work is most closely related to Hu et al. (2015). They consider a monopolist selling two substitutable products to a stream of sequential arrivals whose purchase decisions can be influenced by earlier purchases, and examine how social influence results in demand herding. However, Hu et al. (2015) assume that the prices are exogenously given and examine a monopoly setting. We extend their work to a duopoly setting and examine the impact of social influence on competing firms' multi-period pricing strategies.

Multi-period pricing in the presence of competition and social influence is still under investigation. Prior literature on the comparison of different multiperiod pricing policies has mostly ignored social influence (Koenig and Meissner, 2010; Cachon and Feldma, 2010; Ku and Chang, 2012; Sato and Sawaki, 2013; Şen, 2013). Cachon and Feldma (2010) compare Single Pricing Policy (or static pricing) and Dynamic Pricing Policy in a monopoly setting, and find that when consumers can strategically anticipate firm pricing behavior, a firm may be better off under Single Pricing Policy. Sato and Sawaki (2013) consider a duopoly model where a firm adopts Dynamic Pricing Policy while its competitor uses static pricing, and find that dynamic pricing is not always more profitable. Dasu and Tong (2010) study Dynamic Pricing Policy and Posted Pricing Policy of a firm selling perishable goods to strategic consumers over multiple periods and find that the profit difference between the two is small. Tong et al. (2020) theoretically and empirically prove that dynamic pricing outperforms static pricing for O2O on-demand food service platforms. Different from these studies, we
examine different pricing policies of competing firms in the presence of social influence. To the best of our knowledge, this work is the first attempt to examine and compare different multi-period pricing schemes of competing firms in the presence of social influence.

## 3. The model and analyses

### 3.1. The setup

Consider two firms, labeled $A$ and $B$, respectively, selling two horizontally differentiated products to a market of heterogeneous consumers over a finite selling horizon. The selling horizon consists of two successive periods, labeled 1 and 2. The two firms are located at the two end points of a unit Hotelling line $[0,1]$. In each period, there are one unit of consumers whose preferences are uniformly distributed along the Hotelling line (In Section 4, we relax this assumption by allowing the market size to vary over the two periods). We assume that each consumer only needs one unit of the product and consumers only "live" for one period (i.e., the consumers in the two periods are different) (Gabszewicz and Garcia 2008, Liu et al. 2017). Furthermore, the two firms have the same marginal cost $c$ and their fixed costs are normalized to zero. We assume that consumers are only influenced by the sales information in the previous period.

While examining how social influence affects the pricing and profits of the two competing firms, we only focus on the cases where both firms earn positive profits in the selling horizon (i.e., both firms remain in business). Let $p_{i t}, \Pi_{i t}$, and $D_{i t}(i=A, B ; t=1,2)$ denote firm $i$ 's price, profit, and demand in period $t$, respectively.

A consumer located at $x_{t}\left(x_{t} \in[0,1], t \in\{1,2\}\right)$ in period $t$ on the Hotelling line obtains utility $U_{i t}\left(x_{t}\right)$ from purchasing from firm $i(i \in\{A, B\})$, which can be written as:

$$
\left\{\begin{array}{l}
U_{A t}\left(x_{t}\right)=v_{A}-\theta x_{t}-p_{A t}+\lambda D_{A, t-1} \\
U_{B t}\left(x_{t}\right)=v_{B}-\theta\left(1-x_{t}\right)-p_{B t}+\lambda D_{B, t-1}
\end{array}\right.
$$

Here $v_{i}(i \in\{A, B\})$ denotes the base quality level of firm $i$ 's product, which is assumed to be large enough so that each consumer will buy (i.e., the market is covered). Parameter $\theta$ represents the sensitivity of consumers to product characteristics (Grossman and Shapiro, 1984; Amaldoss and Jain, 2005a). The degree of social influence is captured by $\lambda \geq 0$. The social utility term for firm $i$ in period $t, \lambda D_{i, t-1}$, is increasing with the demand or sales of firm $i$ in the previous period (i.e., $D_{i, t-1}$ ). This model setting is pervasively adopted to capture social influence in prior literature (e.g., Amaldoss and Jain, 2005b; Gabszewicz and Garcia, 2008; Shen et al., 2017). We assume that there is no social influence in period 1 , and thus $D_{i, 0}=0$. For simplicity, we assume $v_{A}=v_{B} \equiv v$.

Let $\bar{x}_{t} \in[0,1]$ be the location of consumers who are indifferent between the two firms in period $t$. That satisfies

$$
\begin{aligned}
U_{A t}\left(\bar{x}_{t}\right) & =v-\theta \bar{x}_{t}-p_{A t}+\lambda D_{A, t-1} \\
& =v-\theta\left(1-\bar{x}_{t}\right)-p_{B t}+\lambda D_{B, t-1}=U_{B t}\left(\bar{x}_{t}\right) .
\end{aligned}
$$

It immediately follows that

$$
\bar{x}_{t}=\frac{1}{2 \theta}\left(\theta+p_{B t}-p_{A t}+\lambda\left(D_{A, t-1}-D_{B, t-1}\right)\right)
$$

Therefore, in period 1, we have

$$
U_{A 1}\left(\bar{x}_{1}\right)=v-\theta \bar{x}_{1}-p_{A 1}=v-\theta\left(1-\bar{x}_{1}\right)-p_{B 1}=U_{B 1}\left(\bar{x}_{1}\right) .
$$

Consequently, the demand and profit of firm $i(i=A, B)$ in period 1 can be computed:

$$
\begin{gather*}
\left\{\begin{array}{c}
D_{A 1}=\bar{x}_{1}=\frac{1}{2 \theta}\left(\theta+p_{B 1}-p_{A 1}\right) \\
D_{B 1}=1-\bar{x}_{1}=\frac{1}{2 \theta}\left(\theta+p_{A 1}-p_{B 1}\right)
\end{array}\right.  \tag{2a}\\
\left\{\begin{array}{l}
\Pi_{A 1}=\left(p_{A 1}-c\right) D_{A 1}=\left(p_{A 1}-c\right) \frac{1}{2 \theta}\left(\theta+p_{B 1}-p_{A 1}\right) \\
\Pi_{B 1}=\left(p_{B 1}-c\right) D_{B 1}=\left(p_{B 1}-c\right) \frac{1}{2 \theta}\left(\theta+p_{A 1}-p_{B 1}\right)
\end{array}\right.
\end{gather*}
$$

Similarly, in period 2, we have:

$$
\begin{align*}
U_{A 2}\left(\bar{x}_{2}\right) & =v-\theta \bar{x}_{2}-p_{A 2}+\lambda D_{A 1} \\
& =v-\theta\left(1-\bar{x}_{2}\right)-p_{B 2}+\lambda D_{B 1}=U_{B 2}\left(\bar{x}_{2}\right) . \tag{4}
\end{align*}
$$

Substituting equations (2a) and (2b) into (4), we can readily obtain

$$
\bar{x}_{2}=\frac{1}{2 \theta^{2}}\left(\theta^{2}+\lambda\left(p_{B 1}-p_{A 1}\right)+p_{B 2}-p_{A 2}\right)
$$

Then, the demand and profit of firm $i(i=A, B)$ in period 2 can be computed:

$$
\begin{gathered}
\left\{\begin{array}{l}
D_{A 2}=\bar{x}_{2}=\frac{1}{2 \theta^{2}}\left(\theta^{2}+\lambda\left(p_{B 1}-p_{A 1}\right)+\theta\left(p_{B 2}-p_{A 2}\right)\right) \\
D_{B 2}=1-\bar{x}_{2}=\frac{1}{2 \theta^{2}}\left(\theta^{2}+\lambda\left(p_{A 1}-p_{B 1}\right)+\theta\left(p_{A 2}-p_{B 2}\right)\right)
\end{array}\right. \\
\left\{\begin{array}{l}
\Pi_{A 2}=\left(p_{A 2}-c\right) D_{A 2}=\left(p_{A 2}-c\right) \frac{1}{2 \theta^{2}}\left(\theta^{2}+\lambda\left(p_{B 1}-p_{A 1}\right)+\theta\left(p_{B 2}-p_{A 2}\right)\right) \\
\Pi_{B 2}=\left(p_{B 2}-c\right) D_{B 2}=\left(p_{B 2}-c\right) \frac{1}{2 \theta^{2}}\left(\theta^{2}+\lambda\left(p_{A 1}-p_{B 1}\right)+\theta\left(p_{A 2}-p_{B 2}\right)\right) .
\end{array}\right.
\end{gathered}
$$

The total profit in the two periods of firm $i(i=A, B)$ is:

$$
\left\{\begin{array}{rl}
\Pi_{A}= & \Pi_{A 1}+\delta \Pi_{A 2}
\end{array}=\left(p_{A 1}-c\right) \frac{1}{2 \theta}\left(\theta+p_{B 1}-p_{A 1}\right) .\right.
$$

Here $\delta(0<\delta \leq 1)$ is the discount factor.
Based on the above model setting, we consider two pricing policies, Single Pricing Policy and Dynamic Pricing Policy. To distinguish the scenario where both firms adopt Single Pricing Policy from the scenario where both firms adopt Dynamic Pricing Policy, their equilibrium results are denoted with superscripts " $S$ " and " $D$ ", respectively.

### 3.2. Single Pricing Policy

In this subsection, we consider the scenario where both firms adopt Single Pricing Policy. In other words, each firm commits to sell at a fixed price in the two periods, i.e., $p_{A 1}=p_{A 2}=p_{A}, p_{B 1}=p_{B 2}=p_{B}$. By analyzing the first- and second-order conditions of the two firms' profit functions, we can easily get the equilibrium prices and profits, which are summarized in Proposition 1.

Proposition 1. Assuming both firms adopt Single Pricing Policy, for $\lambda \geq 0$, there exists a unique equilibrium, $p_{A}^{S *}=p_{B}^{S *}=\frac{(1+\delta) \theta^{2}}{\theta+\delta \theta+\delta \lambda}+c$. The corresponding profits are $\Pi_{A 1}^{S *}=\Pi_{A 2}^{S *}=\Pi_{B 1}^{S *}=\Pi_{B 2}^{S *}=\frac{(1+\delta) \theta^{2}}{2(\theta+\delta \theta+\delta \lambda)}, \Pi_{A}^{S *}=\Pi_{B}^{S *}=\frac{(1+\delta)^{2} \theta^{2}}{2(\theta+\delta \theta+\delta \lambda)}$.

According to Proposition 1, the equilibrium price is a linear function of $c$, while the total profit of each firm is unrelated to $c$. The monotonicity of the equilibrium prices and profits is summarized in Corollary 1.

Corollary 1. When both firms adopt Single Pricing Policy, for $i \in A, B$, we have
(i) $p_{i}^{S *}$ increases with $\theta$ and decreases with $\lambda$ or $\delta ; \Pi_{i}^{S *}$ increases with $\theta$ and decreases with $\lambda$.
(ii) If $\lambda \leq \theta, \Pi_{i}^{S *}$ increases with $\delta$ for $0<\delta \leq 1$; if $\lambda>\theta, \Pi_{i}^{S *}$ increases with $\delta$ for $\frac{\lambda-\theta}{\lambda+\theta}<\delta \leq 1$, and decreases with $\delta$ for $0<\delta \leq \frac{\lambda-\theta}{\lambda+\theta}$.

Corollary 1 indicates that the sensitivity of consumers to product characteristics, $\theta$, has a positive effect on the equilibrium prices and profits. This is expected because the products of the two firms will be more differentiated when $\theta$ becomes larger, and this helps soften the competition between firms.

More importantly, Corollary 1 suggests that social influence will always do harm to the firms under Single Pricing Policy. We observe that when both firms adopt Single Pricing Policy, they obtain less profits in the presence of social influence than the case without social influence. As the degree of social influence increases, the demand in the first period would become more important for firms (to capture a larger market share in the following period). This motivates firms
to compete more fiercely on price, and leads to lower equilibrium prices and profits.

Similarly, a larger discount factor $\delta$ is also associated with lower equilibrium prices. When $\delta$ increases, the profit in period 2 weighs more in the total profit. Due to the existence of social influence, both firms have incentives to decrease their prices to obtain more sales in period 1. In terms of profit, when the degree of social influence is relatively small in comparison to the sensitivity of consumers to product characteristics $(\lambda \leq \theta)$, firms always benefit from a larger discount factor. However, when the degree of social influence is sufficiently large $(\lambda>\theta)$ and the discount factor is relatively small $\left(0<\delta \leq \frac{\lambda-\theta}{\lambda+\theta}\right)$, firms may be worse off as $\delta$ increases. The underlying reason is that, in this parameter space, the increase of the profit due to a relatively larger $\delta$ cannot compensate for the profit loss due to the intensified competition induced by strong social influence.

### 3.3. Dynamic Pricing Policy

In this subsection, we consider the scenario where both firms adopt Dynamic Pricing Policy. In this scenario, firms decide their prices sequentially at each period during the selling horizon.

Solving the game in Subsection 3.1 by backward induction, we can get Proposition 2.

Proposition 2. Assuming both firms adopt Dynamic Pricing Policy, for $0 \leq$ $\lambda<\frac{3 \theta}{\sqrt{\delta}},{ }^{1}$ we have
(i)If $0 \leq \lambda<\frac{3(\theta+c)}{2 \delta}$, there exists a unique equilibrium $p_{A 1}^{D *}=p_{B 1}^{D *}=\theta+c-$ $\frac{2 \delta \lambda}{3}, p_{A 2}^{D *}=p_{B 2}^{D *}=\theta+c$. The corresponding profits are $\Pi_{A 1}^{D *}=\Pi_{B 1}^{D *}=\frac{3 \theta-2 \delta \lambda}{6}$, $\Pi_{A 2}^{D *}=\Pi_{B 2}^{D *}=\frac{\theta}{2}$, and $\Pi_{A}^{D *}=\Pi_{B}^{D *}=\frac{3(1+\delta) \theta-2 \delta \lambda}{6}$.
(ii) If $\lambda \geq \frac{3(\theta+c)}{2 \delta}$, there exists a unique equilibrium $p_{A 1}^{D *}=p_{B 1}^{D *}=0, p_{A 2}^{D *}=$ $p_{B 2}^{D *}=\theta+c$. The corresponding profits are ${ }^{2} \Pi_{A 1}^{D *}=\Pi_{B 1}^{D *}=-\frac{c}{2}, \Pi_{A 2}^{D *}=\Pi_{B 2}^{D *}=\frac{\theta}{2}$,

[^0]and $\Pi_{A}^{D *}=\Pi_{B}^{D *}=\frac{\delta \theta-c}{2}$.
From Proposition 2, we can easily derive Corollary 2.
Corollary 2. Assuming both firms adopt Dynamic Pricing Policy, for $0 \leq \lambda<$ $\frac{3 \theta}{\sqrt{\delta}}$, we have
(i) If $0 \leq \lambda<\frac{3(\theta+c)}{2 \delta}, p_{A 1}^{D *}, p_{B 1}^{D *}, \Pi_{A 1}^{D *}, \Pi_{B 1}^{D *}, \Pi_{A}^{D *}$, and $\Pi_{B}^{D *}$ decrease with $\lambda$ and increase with $\theta ; p_{A 1}^{D *}, p_{B 1}^{D *}, \Pi_{A 1}^{D *}$, and $\Pi_{B 1}^{D *}$ decrease with $\delta ; \Pi_{A}^{D *}$ and $\Pi_{B}^{D *}$ increase with $\delta$ when $\lambda<\frac{3 \theta}{2}$ and decrease with $\delta$ when $\lambda>\frac{3 \theta}{2}$. All profits are unrelated to c. Specially, when $\frac{3 \theta}{2 \delta}<\lambda<\frac{3(\theta+c)}{2 \delta}$, the prices of both firms in period 1 are positive but lower than the marginal cost c, leading to negative profits in the first period.
(ii)If $\lambda \geq \frac{3(\theta+c)}{2 \delta}$, the prices of both firms in period 1 are zero, leading to negative profits in the first period; the total profits of both firms decrease with $c$ but increase with $\delta$ or $\theta$.

Under Dynamic Pricing Policy, the sensitivity of consumers to product characteristics $(\theta)$ has a positive effect on the equilibrium prices and profits (see Corollary 2), which is consistent with that under Single Pricing Policy.

Furthermore, from Corollary 2 (i) we can conclude that when both firms adopt Dynamic Pricing Policy, the total profit of each firm in the presence of social influence is less than the case without social influence, which is similar to that under Single Pricing Policy. Proposition 2 and Corollary 2 also suggest that firms may even sell below their marginal cost in period 1 when the degree of social influence is sufficiently large $\left(\lambda>\frac{3 \theta}{2 \delta}\right)$. This is because strong social influence motivates both firms to occupy a large market share in period 1 by setting lower prices. Thus, social influence makes the competition more intense, which leads to lower profits for both firms. Specially, if the marginal cost is relatively low $(0 \leq c<\theta(2 \sqrt{\delta}-1))$ and the degree of social influence is large enough $\left(\lambda \geq \frac{3(\theta+c)}{2 \delta}\right)$, firms may even set zero prices in period 1 .

[^1]Moreover, when all the equilibrium prices are positive (i.e., interior solution, $\left.0 \leq \lambda<\frac{3(\theta+c)}{2 \delta}\right)$, a larger discount factor will also intensify the competition in period 1, leading to lower prices. When the degree of social influence is relatively small $\left(\lambda<\frac{3 \theta}{2}\right)$, firms always benefit from a larger discount factor. On the contrary, when the degree of social influence is sufficiently large $\left(\lambda>\frac{3 \theta}{2}\right)$, the increase of profit due to a larger $\delta$ cannot compensate for the profit loss due to the intensified competition in period 1. As a result, in this case two firms are worse off as $\delta$ increases.

Comparing Proposition 1 with Proposition 2, we can obtain Theorem 1.
Theorem 1 (Comparison of Single Pricing Policy and Dynamic Pricing Policy). Under the conditions of Propositions 1 and 2, for $i \in\{A, B\}$, we have
(i) For $0 \leq \lambda<\frac{3(\theta+c)}{2 \delta}: \Pi_{i}^{S *}>\Pi_{i}^{D *}$ if $\lambda>\frac{(\delta+1) \theta}{2 \delta}$, and $\Pi_{i}^{S *}<\Pi_{i}^{D *}$ if $\lambda<\frac{(\delta+1) \theta}{2 \delta}$.
(ii) For $\lambda \geq \frac{3(\theta+c)}{2 \delta}: \Pi_{i}^{S *}>\Pi_{i}^{D *}$ if $\lambda<\frac{(\delta+1) \theta(c+\theta)}{\delta(\delta \theta-c)}$, and $\Pi_{i}^{S *}<\Pi_{i}^{D *}$ if $\lambda>\frac{(\delta+1) \theta(c+\theta)}{\delta(\delta \theta-c)}$.

When $0 \leq \lambda<\frac{3(\theta+c)}{2 \delta}$, interior equilibrium exists. In this case, if the degree of social influence is relatively small $\left(\lambda<\frac{(\delta+1) \theta}{2 \delta}\right)$, firms will be better off when they both adopt Dynamic Pricing Policy than when they adopt Single Pricing Policy. Compared to Single Pricing Policy, firms have more flexibility to adjust their prices over two periods under Dynamic Pricing Policy. That is, firms can strategically lower their prices in period 1 (to obtain more sales), and raise prices in the following period $\left(p_{i 1}^{D *}<p_{i}^{S *}<p_{i 2}^{D *}, i \in\{A, B\}\right)$ and take advantage of social influence. If the degree of social influence is relatively large $\left(\frac{(\delta+1) \theta}{2 \delta}<\right.$ $\left.\lambda<\frac{3(\theta+c)}{2 \delta}\right)$, however, Single Pricing Policy can outperform Dynamic Pricing Policy. This is because a relatively large degree of social influence induces firms under Dynamic Pricing Policy to compete too fiercely on pricing in period 1 (and even set prices below the marginal cost), and hence the profit loss induced by intensified competition is too large. As a consequence, Dynamic Pricing Policy can be less profitable than Single Pricing Policy which will soften the price competition in the first period.

When $\lambda$ is very large $\left(\lambda \geq \frac{3(\theta+c)}{2 \delta}\right)$, however, the competition is so intense that there exists a boundary solution where both firms under Dynamic Pricing Policy set zero prices in period 1. Under this circumstance, firms under Dynamic Pricing Policy can benefit from social influence in period 2 without further intensifying the price competition in period 1, since they are unable to further decrease their prices. Firm profits under Single Pricing Policy, however, always decrease with the degree of social influence (see Corollary 1). Thus, when $\lambda$ is large enough (i.e., $\lambda>\frac{(\delta+1) \theta(c+\theta)}{\delta(\delta \theta-c)}$ ), Dynamic Pricing Policy dominates Single Pricing Policy.

In the above model, we assume that the market size remains constant across the two periods and both firms adopt the same pricing policy. In the following sections, we will relax these assumptions with some extensions. Particularly, in Section 4, we examine the model where market size varies over the two periods (Extension 1). In Section 5, we further compare Dynamic Pricing Policy and Single Pricing policy to the Posted Pricing Policy with a reference price effect (Extension 2). In Section 6, we allow the competing firms to adopt different pricing policies, i.e., one firm adopts Single Pricing Policy and the other adopts Dynamic Pricing Policy (Extension 3).

## 4. Extension 1: Model with market size varying over periods

In this section, we assume that the market size of the two periods is 1 and $\beta(\beta>0)$, respectively. In addition, this information is known to both firms. To focus on social influence $(\lambda)$ and market size change $(\beta)$, without loss of generality, we normalize $\theta=1, \delta=1$, and $c=0$.

According to Subsection 3.1, the profit of firm $i(i=A, B)$ in period $t$ $(t=1,2)$ is:

$$
\left\{\begin{array}{l}
\Pi_{A 1}=p_{A 1} D_{A 1}=\frac{1}{2} p_{A 1}\left(1+p_{B 1}-p_{A 1}\right) \\
\Pi_{B 1}=p_{B 1} D_{B 1}=\frac{1}{2} p_{B 1}\left(1+p_{A 1}-p_{B 1}\right)
\end{array}\right.
$$

$$
\left\{\begin{array}{l}
\Pi_{A 2}=p_{A 2} D_{A 2}=\frac{1}{2} p_{A 2} \beta\left(1+\lambda p_{B 1}-\lambda p_{A 1}+p_{B 2}-p_{A 2}\right) \\
\Pi_{B 2}=p_{B 2} D_{B 2}=\frac{1}{2} p_{B 2} \beta\left(1+\lambda p_{A 1}-\lambda p_{B 1}+p_{A 2}-p_{B 2}\right)
\end{array}\right.
$$

Then, the total profit in the two periods of firm $i(i=A, B)$ is:

$$
\left\{\begin{aligned}
\Pi_{A}=\Pi_{A 1}+\Pi_{A 2} & =\frac{1}{2} p_{A 1}\left(1+p_{B 1}-p_{A 1}\right) \\
& +\frac{1}{2} p_{A 2} \beta\left(1+\lambda p_{B 1}-\lambda p_{A 1}+p_{B 2}-p_{A 2}\right) \\
\Pi_{B}=\Pi_{B 1}+\Pi_{B 2} & =\frac{1}{2} p_{B 1}\left(1+p_{A 1}-p_{B 1}\right) \\
& +\frac{1}{2} p_{B 2} \beta\left(1+\lambda p_{A 1}-\lambda p_{B 1}+p_{A 2}-p_{B 2}\right)
\end{aligned}\right.
$$

### 4.1. Single Pricing Policy

In this scenario, $p_{A 1}=p_{A 2}=p_{A}, p_{B 1}=p_{B 2}=p_{B}$. We can readily obtain the equilibrium prices and profits, which are summarized in Proposition 3.

Proposition 3. Assuming both firms adopt Single Pricing Policy, when $\lambda \geq 0$ and $\beta>0$, there exists a unique equilibrium

$$
p_{A}^{S *}=p_{B}^{S *}=\frac{1+\beta}{1+\beta+\beta \lambda}
$$

The corresponding profits are
$\Pi_{A 1}^{S *}=\Pi_{B 1}^{S *}=\frac{1+\beta}{2(1+\beta+\beta \lambda)}, \Pi_{A 2}^{S *}=\Pi_{B 2}^{S *}=\frac{\beta(1+\beta)}{2(1+\beta+\beta \lambda)}, \Pi_{A}^{S *}=\Pi_{B}^{S *}=\frac{(1+\beta)^{2}}{2(1+\beta+\beta \lambda)}$.
From Proposition 3, we can easily derive Corollary 3.
Corollary 3. Under the conditions of Proposition 3, $p_{i}^{S *}(i=A, B)$ decreases with both $\lambda$ and $\beta$, and $\Pi_{i}^{S *}$ decreases with $\lambda$. If $0 \leq \lambda \leq 1, \Pi_{i}^{S *}$ increases with $\beta$; If $\lambda>1, \Pi_{i}^{S *}$ increases with $\beta$ when $\beta>\frac{\lambda-1}{\lambda+1}$, and it decreases with $\beta$ when $0<\beta<\frac{\lambda-1}{\lambda+1}<1$.

Similar to the results of our main model, the presence of social influence (i.e., $\lambda>0$ ) intensifies competition and leads firms to charge lower prices and end up with lower profits. When the market size in period $2(\beta)$ increases, both firms have the potential to obtain higher profits. In this circumstance, however, the profit in period 2 weighs more in the total profit and hence firms have incentives
to obtain a larger market share in period 2 , which further motivates them to decrease their prices in the presence of social influence. Thus, the equilibrium prices under Single Pricing Policy decrease with $\beta$ (see Corollary 3).

Whether firms can be better off with a larger $\beta$ depends on the profit gain owing to the increase of market size and the profit loss due to the intensified price competition in the presence of social influence. Under Single Pricing Policy, firm profits increase with $\beta$ when the degree of social influence is relatively small $(0 \leq \lambda \leq 1)$, or when the degree of social influence is large $(\lambda>1)$ but the market size in period 2 is relatively large $\left(\beta>\frac{\lambda-1}{\lambda+1}\right)$. However, when the degree of social influence is large and the market size in period 2 is relatively small $\left(\lambda>1\right.$ and $\left.0<\beta<\frac{\lambda-1}{\lambda+1}<1\right)$, the profit gain brought by a larger $\beta$ cannot make up for the profit loss due to the intensified competition caused by social influence. As a result, in this case firms will be worse off when $\beta$ becomes larger.

Moreover, we can easily derive from Corollary 3 that when the market size in the second period is no smaller than that in the first period (i.e., $\beta \geq 1$ ), firm profits always increase with $\beta$. The underlying reason is that when the market size of period 2 is large enough, the profit increase due to a larger $\beta$ is more than the profit loss due to the intensified competition induced by social influence.

### 4.2. Dynamic Pricing Policy

When both firms adopt Dynamic Pricing Policy, they decide their prices in the two periods sequentially at the beginning of each period. Solving the game by backward induction, we can obtain Proposition 4.

Proposition 4. Considering the two-period pricing game, if both firms adopt Dynamic Pricing Policy, when $0 \leq \lambda<\frac{3}{\sqrt{\beta}}^{3}$ and $\beta>0$, we have
(i) If $0 \leq \lambda<\frac{3}{2 \beta}$, there exists a unique equilibrium $p_{A 1}^{D *}=p_{B 1}^{D *}=1-\frac{2 \beta \lambda}{3}$, $p_{A 2}^{D *}=p_{B 2}^{D *}=1$. The corresponding profits are $\Pi_{A 1}^{D *}=\Pi_{B 1}^{D *}=\frac{3-2 \beta \lambda}{6}, \Pi_{A 2}^{D *}=$ $\Pi_{B 2}^{D *}=\frac{\beta}{2}, \Pi_{A}^{D *}=\Pi_{B}^{D *}=\frac{3+\beta(3-2 \lambda)}{6}$.

[^2](ii) If $\frac{3}{2 \beta} \leq \lambda<\frac{3}{\sqrt{\beta}}\left(\Rightarrow \beta>\frac{1}{4}\right)$, there exists a unique equilibrium $p_{A 1}^{D *}=$ $p_{B 1}^{D *}=0, p_{A 2}^{D *}=p_{B 2}^{D *}=1$. The corresponding profits are $\Pi_{A 1}^{P *}=\Pi_{B 1}^{P *}=0$, $\Pi_{A 2}^{D *}=\Pi_{B 2}^{D *}=\frac{\beta}{2}$, and $\Pi_{A}^{D *}=\Pi_{B}^{D *}=\frac{\beta}{2}$.

Under Dynamic Pricing Policy, when the market size of period 2 is not too small (i.e., $\beta>\frac{1}{4}$ ), a strong enough social influence $\left(\lambda \geq \frac{3}{2 \beta}\right.$ ) will induce firms to set zero prices in the first period. This finding is similar to that of our main model where market size remains the same across different periods.

Corollary 4. Under the conditions of Proposition 4, for $i \in\{A, B\}$, we have
(i) If $0 \leq \lambda<\frac{3}{2 \beta}, p_{i 1}^{D *}$ decreases with both $\lambda$ and $\beta ; \Pi_{i}^{D *}$ decreases with $\lambda$; $\Pi_{i}^{D *}$ increases with $\beta$ when $0 \leq \lambda<\frac{3}{2}$, and it decreases with $\beta$ when $\lambda>\frac{3}{2}$.
(ii) If $\frac{3}{2 \beta} \leq \lambda<\frac{3}{\sqrt{\beta}}, \Pi_{i}^{D *}$ increases with $\beta$.

According to Corollary 4 (i), the equilibrium total profit of each firm decreases with $\lambda$, indicating that social influence will always hurt the firms under Dynamic Pricing Policy. Moreover, when $\beta \geq 1$, from Corollary 4 (i), we have $0 \leq \lambda<\frac{3}{2 \beta} \leq \frac{3}{2}$. Therefore, combining with Corollary 4 (ii), we can conclude that when the market size in the second period is no smaller than that in the first period (i.e., $\beta \geq 1$ ), the total profit of each firm always increases with $\beta$. This result is similar to that under Single Pricing Policy.

Corollary 4 (i) also indicates that when the market size in period 2 increases, firms have incentives to decrease their prices to capture a larger market share in period 1 (so that they can take advantage of the social influence in period $2)$. When the degree of social influence is relatively small $\left(\lambda<\frac{3}{2}\right)$, firms always benefit from a larger market size in period 2 . However, when the degree of social influence is relatively large and the market size in period 2 is relatively small $\left(\lambda>\frac{3}{2}\right.$ and $\left.\beta<1\right)$, firms are worse off as $\beta$ becomes larger. The underlying reason is that in this case the profit increase due to a larger $\beta$ cannot compensate for the profit loss due to the intensified competition in period 1 induced by social influence.

Comparing Proposition 3 with Proposition 4, we can obtain Theorem 2.

Theorem 2 (Comparison of Single Pricing Policy and Dynamic Pricing Policy). Under the conditions of Propositions 3 and 4, for $i \in\{A, B\}$, we have
(i) For $\beta<2: \Pi_{i}^{S *}<\Pi_{i}^{D *}$ if $0<\lambda<\frac{1+\beta}{2 \beta}$ or $\lambda>\frac{1+\beta}{\beta^{2}}, \Pi_{i}^{S *}<\Pi_{i}^{D *}$ if $\frac{1+\beta}{2 \beta}<\lambda<\frac{1+\beta}{\beta^{2}}$.
(ii) For $\beta \geq 2: \Pi_{i}^{S *}<\Pi_{i}^{D *}$.

When the market size in period 2 is not very large (i.e., $\beta<2$ ), the key conclusions are similar to those of our main model. Specifically, if the degree of social influence is relatively small (i.e., $0<\lambda<\frac{1+\beta}{2 \beta}$ ), Dynamic Pricing Policy outperforms Single Pricing Policy. If the degree of social influence is relatively large (i.e., $\frac{1+\beta}{2 \beta}<\lambda<\frac{1+\beta}{\beta^{2}}$ ), relatively strong social influence would make the competition too intense under Dynamic Pricing Policy so that Single Pricing Policy dominates. Moreover, if the degree of social influence becomes very large (i.e., $\lambda>\frac{1+\beta}{\beta^{2}}$ ), firms under Dynamic Pricing Policy set zero prices in period 1 and Dynamic Pricing Policy may dominate Single Pricing Policy again.

When the market size in period 2 is sufficiently large (i.e., $\beta \geq 2$ ), Dynamic Pricing Policy always outperforms Single Pricing Policy. Different from the case when $\beta$ is relatively small, a sufficiently large market size in period 2 would induce firms to decrease their prices sharply in period 1 to occupy large market shares, so that they can benefit from social influence and obtain more profits in period 2. In this case, under Dynamic Pricing Policy, even though relatively strong social influence may intensify the competition and hurt firm profits, the profit gain from a sufficiently large market size can more than compensate for the profit loss, and firms can always benefit from flexibly adjusting prices over two periods. Under Single Pricing Policy, however, firms have limited room to adjust the prices over two periods, and thus they obtain less profits.

## 5. Extension 2: Model with Posted Pricing Policy and reference price effect

In this section, we further compare Single and Dynamic Pricing with another commonly adopted policy in practice named "Posted Pricing Policy" (or "Pre-
announced Pricing Policy"). Under Posted Pricing Policy, the firm announces all the prices at the beginning of the selling horizon (Dasu and Tong, 2010; Papanastasiou and Savva, 2017), i.e., consumers in period 1 are informed of the prices in both periods. Consequently, the price information in the following period might serve as a reference and affect consumer purchase decision in the current period ${ }^{4}$, known as the reference price effect in the marketing and economics literature (Kopalle et al., 1996; Fibich et al., 2003; Hu et al., 2016; Zhang and Chiang, 2018). Reference price is the price expectation that consumers form, which is used to judge the current selling price. When the current selling price is lower than the reference price, consumers are likely to perceive a gain. Contrarily, when the current selling price is higher than the reference price, consumers are likely to perceive a loss. Prior research has provided plenty of empirical support for the existence of reference price effect (Krishnamurthi et al., 1992; Raman and Bass, 2002; Kopalle et al., 2012).

Note that in online shopping websites, the prices across periods under Posted Pricing Policy are only posted in the initial period. When the second period starts, the price of the previous period is no longer available to consumers. This is because it is not wise for firms to inform the consumers of the price increase in comparison to the previous period. Therefore, we only consider the reference price effect for consumers in the first period.

We consider the scenario when the market size remains the same in different periods and assume $\delta=1, \theta=1, c=0$ without loss of generality. Note that there is no reference price effect under Single Pricing Policy or Dynamic Pricing Policy, and the results are the same as those in Propositions 1-2. Next, we consider the game when both firms adopt Posted Pricing Policy. The equilibrium results in this scenario are denoted with superscript " $P$ ".

[^3]In period 1, the location of indifferent consumers satisfies

$$
\begin{aligned}
U_{A 1}\left(\bar{x}_{1}\right) & =v-\bar{x}_{1}-p_{A 1}+\alpha\left(p_{\mathrm{A} 2}-p_{\mathrm{A} 1}\right) \\
& =v-\left(1-\bar{x}_{1}\right)-p_{B 1}+\alpha\left(p_{\mathrm{B} 2}-p_{\mathrm{B} 1}\right)=U_{B 1}\left(\bar{x}_{1}\right)
\end{aligned}
$$

In the above equation, $\alpha$ represents the reference price effect. It immediately follows that

$$
\bar{x}_{1}=\frac{1}{2}\left(1-(1+\alpha)\left(p_{\mathrm{A} 1}-p_{\mathrm{B} 1}\right)+\alpha\left(p_{\mathrm{A} 2}-p_{\mathrm{B} 2}\right)\right)
$$

Then, the demand and profit of firm $i(i=A, B)$ in period 1 can be readily obtained:

$$
\left\{\begin{array}{l}
D_{A 1}=\bar{x}_{1}=\frac{1}{2}\left(1-(1+\alpha)\left(p_{\mathrm{A} 1}-p_{\mathrm{B} 1}\right)+\alpha\left(p_{\mathrm{A} 2}-p_{\mathrm{B} 2}\right)\right)  \tag{11a}\\
D_{B 1}=1-\bar{x}_{1}=\frac{1}{2}\left(1+(1+\alpha)\left(p_{\mathrm{A} 1}-p_{\mathrm{B} 1}\right)-\alpha\left(p_{\mathrm{A} 2}-p_{\mathrm{B} 2}\right)\right)
\end{array}\right.
$$

$$
\left\{\begin{array}{l}
\Pi_{A 1}=p_{A 1} D_{A 1}=p_{A 1} \frac{1}{2 \theta}\left(\theta+p_{B 1}-p_{A 1}\right) \\
\Pi_{B 1}=p_{B 1} D_{B 1}=p_{B 1} \frac{1}{2 \theta}\left(\theta+p_{A 1}-p_{B 1}\right)
\end{array}\right.
$$

Similarly, the location of indifferent consumers in period 2 satisfies

$$
\begin{align*}
U_{A 2}\left(\bar{x}_{2}\right) & =v-\bar{x}_{2}-p_{A 2}+\lambda D_{A 1} \\
& =v-\left(1-\bar{x}_{2}\right)-p_{B 2}+\lambda D_{B 1}=U_{B 2}\left(\bar{x}_{2}\right) \tag{13}
\end{align*}
$$

Substituting equations (11a) and (11b) into (13), we immediately obtain

$$
\bar{x}_{2}=\frac{1}{2}\left(1+\lambda(1+\alpha)\left(p_{B 1}-p_{A 1}\right)+(1-\alpha \lambda)\left(p_{B 2}-p_{A 2}\right)\right)
$$

Thus, the demand and profit of firm $i(i \in A, B)$ in period 2 can be computed:

$$
\begin{gathered}
\left\{\begin{array}{c}
D_{A 2}=\bar{x}_{2}=\frac{1}{2}\left(1+\lambda(1+\alpha)\left(p_{B 1}-p_{A 1}\right)+(1-\alpha \lambda)\left(p_{B 2}-p_{A 2}\right)\right) \\
D_{B 2}=1-\bar{x}_{2}=\frac{1}{2}\left(1-\lambda(1+\alpha)\left(p_{B 1}-p_{A 1}\right)-(1-\alpha \lambda)\left(p_{B 2}-p_{A 2}\right)\right)
\end{array}\right. \\
\left\{\begin{array}{l}
\Pi_{A 2}=p_{A 2} D_{A 2}=p_{A 2} \frac{1}{2}\left(1+\lambda(1+\alpha)\left(p_{B 1}-p_{A 1}\right)+(1-\alpha \lambda)\left(p_{B 2}-p_{A 2}\right)\right) \\
\Pi_{B 2}=p_{B 2} D_{B 2}=p_{B 2} \frac{1}{2}\left(1-\lambda(1+\alpha)\left(p_{B 1}-p_{A 1}\right)-(1-\alpha \lambda)\left(p_{B 2}-p_{A 2}\right)\right)
\end{array}\right.
\end{gathered}
$$

Then, we can get the total profit of firm $i(i=A, B)$ as follows:

$$
\left\{\begin{array}{l}
\Pi_{A}=\Pi_{A 1}+\Pi_{A 2} \\
\Pi_{B}=\Pi_{B 1}+\Pi_{B 2}
\end{array}\right.
$$

By analyzing the first- and second-order derivatives of $\Pi_{i}$ with respect to $p_{i 1}$ and $p_{i 2}(i \in A, B)$, we can obtain Proposition 5.

Proposition 5. If both firms adopt Posted Pricing Policy, when $0 \leq \lambda<2$ and $0 \leq \alpha<\frac{2-\lambda-\lambda^{2}+2 \sqrt{\lambda+2}}{(\lambda+1)^{2}},{ }^{5}$ we have
(i) If $\lambda<\frac{1}{1+2 \alpha}$, there exists a unique equilibrium $p_{A 1}^{P *}=p_{B 1}^{P *}=\frac{1-\lambda(1+2 \alpha)}{1+\alpha}$, $p_{A 2}^{P *}=p_{B 2}^{P *}=\frac{1+2 \alpha}{1+\alpha}$. The corresponding profits are $\Pi_{A 1}^{P *}=\Pi_{B 1}^{P *}=\frac{1-\lambda(1+2 \alpha)}{2(1+\alpha)}$, $\Pi_{A 2}^{P *}=\Pi_{B 2}^{P *}=\frac{1+2 \alpha}{2(1+\alpha)}, \Pi_{A}^{P *}=\Pi_{B}^{P *}=\frac{2(1+\alpha)-(1+2 \alpha) \lambda}{2(1+\alpha)}$.
(ii) If $\lambda \geq \frac{1}{1+2 \alpha}$, there exists a unique equilibrium $p_{A 1}^{P *}=p_{B 1}^{P *}=0, p_{A 2}^{P *}=$ $p_{B 2}^{P *}=\frac{1}{1-\alpha \lambda}$. The corresponding profits are $\Pi_{A 1}^{P *}=\Pi_{B 1}^{P *}=0, \Pi_{A 2}^{P *}=\Pi_{B 2}^{P *}=$ $\frac{1}{2(1-\alpha \lambda)}$, and $\Pi_{A}^{P *}=\Pi_{B}^{P *}=\frac{1}{2(1-\alpha \lambda)}$.

Furthermore, we can derive the monotonicity of firms' equilibrium prices and profits with regard to $\lambda$ and $\alpha$, respectively.

Corollary 5. Under the conditions of Proposition 5, for $i=\{A, B\}$,
(i) If $\lambda<\frac{1}{1+2 \alpha}, p_{i 1}^{*}, \Pi_{i 1}^{P *}$, and $\Pi_{i}^{P *}$ decrease with both $\lambda$ and $\alpha$; $p_{i 2}^{*}$ and $\Pi_{i 2}^{P *}$ increase with $\alpha$.
(ii) If $\lambda \geq \frac{1}{1+2 \alpha}, p_{i 2}^{P *}, \Pi_{i 2}^{P *}$, and $\Pi_{i}^{P *}$ increase with both $\lambda$ and $\alpha$.

According to Proposition 5 and Corollary 5, if the degree of social influence is relatively small (i.e., $\lambda<\frac{1}{1+2 \alpha}$ ), the price competition between firms is not too intense and all the equilibrium prices are positive. The changes of firms' equilibrium prices and profits with regard to $\lambda$ are similar to the results when both firms adopt Dynamic Pricing Policy (see Corollary 2). That is, as the degree of social influence increases, the market share in the first period would

[^4]become more important for firms (to capture a larger market share in the following period). This motivates firms to compete more fiercely on price in the first period, leading to lower prices in period 1 and lower total profits. In addition, as the reference price effect $(\alpha)$ becomes stronger, firms have more incentives to take advantage of it (by enlarging the price difference between two periods) to increase their market share in period 1. This motivation becomes especially strong in the presence of social influence, because a larger market share in period 1 will further help firms take advantage of social influence in period 2. Thus, in this case firms lower their prices in period 1 and increase their prices in period 2. As a result, the profit in period 1 of each firm decreases and the profit in period 2 increases with $\alpha$. However, the profit gain in period 2 cannot make up for the loss in period 1 , so the total profit decreases when $\alpha$ becomes larger.

On the other hand, when the degree of social influence is sufficiently large (i.e., $\lambda \geq \frac{1}{1+2 \alpha}$ ), price competition between two firms is rather intense, and hence the equilibrium prices in period 1 are zero. Interestingly, under this circumstance firms may even benefit from a larger degree of social influence. The underlying reason is that, when the degree of social influence becomes larger, firms have incentives to capture a larger market share in period 1 but there is no room for them to decrease their prices in the first period. At the same time, the presence of reference price effect can accelerates the acquisition of market share in period 1 when firms charge higher prices in period 2. Thus, as the degree of social influence becomes larger, its synergy with the reference price effect induces firms to charge higher prices in period 2, which leads firm profits to increase with $\lambda$. This finding is different from models without the reference price effect, in which firms are always worse off with a larger degree of social influence.

Comparing the results under Posted Pricing Policy with those under Single Pricing Policy and Dynamic Pricing Policy (recalling Theorem 1 and assuming $\delta=1, \theta=1, c=0$ ), we can obtain Theorem 3.

Theorem 3 (Comparison of Three Pricing Policies). When $0 \leq \lambda<2$ and $0 \leq \alpha<\frac{2-\lambda-\lambda^{2}+2 \sqrt{\lambda+2}}{(\lambda+1)^{2}}$, ${ }^{6}$ we have
(i) For $0 \leq \alpha<\frac{1}{4}: \Pi_{i}^{D *}>\max \left\{\Pi_{i}^{S *}, \Pi_{i}^{P *}\right\}$ if $0<\lambda<1$; $\Pi_{i}^{S *}>$ $\max \left\{\Pi_{i}^{D *}, \Pi_{i}^{P *}\right\}$ if $1<\lambda<\frac{2}{1+4 \alpha} ; \Pi_{i}^{P *}>\max \left\{\Pi_{i}^{D *}, \Pi_{i}^{S *}\right\}$ if $\frac{2}{1+4 \alpha}<\lambda<$ $\frac{2 \sqrt{1+\alpha}-\alpha}{1+\alpha}$.
(ii) For $\alpha \geq \frac{1}{4}: \Pi_{i}^{D *}>\max \left\{\Pi_{i}^{S *}, \Pi_{i}^{P *}\right\}$ if $0<\lambda<\frac{1+3 \alpha-\sqrt{1+9 \alpha^{2}}}{2 \alpha}$; $\Pi_{i}^{P *}>$ $\max \left\{\Pi_{i}^{D *}, \Pi_{i}^{S *}\right\}$ if $\frac{1+3 \alpha-\sqrt{1+9 \alpha^{2}}}{2 \alpha}<\lambda<\frac{2 \sqrt{1+\alpha}-\alpha}{1+\alpha}$.

As shown in Theorem 3, without reference price effect (i.e., $\alpha=0$ ), Posted Pricing Policy is always the worst choice for firms (Note that the condition $\frac{2}{1+4 \alpha}<\lambda<\frac{2 \sqrt{1+\alpha}-\alpha}{1+\alpha}$ cannot be satisfied for $\alpha=0$ in Theorem 3(i)). When the reference price effect is relatively small $\left(0<\alpha<\frac{1}{4}\right)$, any of the three pricing policies can dominate. In particular, when the degree of social influence is relatively small $(0<\lambda<1)$, Dynamic Pricing Policy is the most profitable pricing strategy for firms since it allows firms to adjust their prices strategically to take advantage of social influence. When the degree of social influence is relatively large $\left(1<\lambda<\frac{2}{1+4 \alpha}\right)$, Single Pricing Policy dominates. This is because, compared with the other two pricing policies, it limits the room for firms to adjust prices and hence the price competition is relatively less intense under the strong social influence. When the degree of social influence further increases $\left(\frac{2}{1+4 \alpha}<\lambda<\frac{2 \sqrt{1+\alpha}-\alpha}{1+\alpha}\right)$, however, Posted Pricing Policy will dominate because of the synergy between social influence and the reference price effect as explained below Corollary 5.

When the reference price effect is relatively large ( $\alpha \geq \frac{1}{4}$ ), however, Single Pricing Policy can never be an optimal pricing strategy. In this case, again, Dynamic Pricing Policy can make firms better off if the degree of social influence is relatively small $\left(0<\lambda<\frac{1+3 \alpha-\sqrt{1+9 \alpha^{2}}}{2 \alpha}\right.$ ), while Posted Pricing Policy is optimal when the degree of social influence is large $\left(\frac{1+3 \alpha-\sqrt{1+9 \alpha^{2}}}{2 \alpha}<\lambda<\frac{2 \sqrt{1+\alpha}-\alpha}{1+\alpha}\right)$. The underlying reason is that, when $\lambda$ is relatively small, the prices under Sin-

[^5]gle Pricing Policy are less adjustable than those under Dynamic Pricing Policy, while when $\lambda$ is relatively large, firms under Single Pricing Policy can not take advantage of the synergy between social influence and the reference price effect as those under Posted Pricing Policy.

## 6. Extension 3: Model with firms adopting different pricing policies

In previous sections, we examine the cases where both firms adopt the same pricing policy. In this section, we consider the scenario where two firms adopt different pricing policies. Without loss of generality, we assume that firm $A$ adopts Single Pricing Policy (decides $p_{A}$ ), while firm $B$ adopts Dynamic Pricing Policy (decides $p_{B 1}$ in period 1 and $p_{B 2}$ in period 2 ). We will further show the results when two firms can freely choose either one of the two pricing policies. To focus on social influence and make our analysis more concise, without loss of generality, we assume that $\delta=1, \theta=1$, and $c=0$. Such a normalization will not change our key conclusions qualitatively. The equilibrium results in this scenario are denoted with superscript "SD".

In this scenario, the location of indifferent consumers in period 1 satisfies

$$
U_{A 1}\left(\bar{x}_{1}\right)=v-\bar{x}_{1}-p_{A}=v-\left(1-\bar{x}_{1}\right)-p_{B 1}=U_{B 1}\left(\bar{x}_{1}\right)
$$

It immediately follows that

$$
\bar{x}_{1}=\frac{1}{2}\left(1-p_{\mathrm{A}}+p_{\mathrm{B} 1}\right) .
$$

Consequently, the demand and profit of firm $i(i=A, B)$ in period 1 can be obtained:

$$
\begin{gather*}
\left\{\begin{array}{c}
D_{A 1}=\bar{x}_{1}=\frac{1}{2}\left(1-p_{\mathrm{A}}+p_{\mathrm{B} 1}\right) \\
D_{B 1}=1-\bar{x}_{1}=\frac{1}{2}\left(1+p_{\mathrm{A}}-p_{\mathrm{B} 1}\right)
\end{array}\right.  \tag{17a}\\
\left\{\begin{array}{l}
\Pi_{A 1}=p_{A} D_{A 1}=p_{A} \frac{1}{2}\left(1-p_{\mathrm{A}}+p_{\mathrm{B} 1}\right) \\
\Pi_{B 1}=p_{B 1} D_{B 1}=p_{B 1} \frac{1}{2}\left(1+p_{\mathrm{A}}-p_{\mathrm{B} 1}\right)
\end{array}\right.
\end{gather*}
$$

In a similar vein, the location of indifferent consumers in period 2 satisfies

$$
\begin{align*}
U_{A 2}\left(\bar{x}_{2}\right) & =v-\bar{x}_{2}-p_{A}+\lambda D_{A 1} \\
& =v-\left(1-\bar{x}_{2}\right)-p_{B 2}+\lambda D_{B 1}=U_{B 2}\left(\bar{x}_{2}\right) \tag{19}
\end{align*}
$$

Substituting equations (17a) and (17b) into (19), we immediately obtain

$$
\bar{x}_{2}=\frac{1}{2}\left(1+\lambda\left(p_{B 1}-p_{A}\right)+p_{B 2}-p_{A}\right) .
$$

Thus, the demand and profit of firm $i(i=A, B)$ in period 2 can be computed:

$$
\begin{gather*}
\left\{\begin{array}{c}
D_{A 2}=\bar{x}_{2}=\frac{1}{2}\left(1+\lambda\left(p_{B 1}-p_{A}\right)+p_{B 2}-p_{A}\right) \\
D_{B 2}=1-\bar{x}_{2}=\frac{1}{2}\left(1-\lambda\left(p_{B 1}-p_{A}\right)-p_{B 2}+p_{A}\right)
\end{array}\right. \\
\left\{\begin{array}{l}
\Pi_{A 2}=p_{A} D_{A 2}=p_{A} \frac{1}{2}\left(1+\lambda\left(p_{B 1}-p_{A}\right)+p_{B 2}-p_{A}\right) \\
\Pi_{B 2}=p_{B 2} D_{B 2}=p_{B 2} \frac{1}{2}\left(1-\lambda\left(p_{B 1}-p_{A}\right)-p_{B 2}+p_{A}\right)
\end{array}\right. \tag{21a}
\end{gather*}
$$

Then we can readily obtain the total profit of each firm $\Pi_{i}(i=A, B)$ as follows:

$$
\left\{\begin{array}{l}
\Pi_{A}=\Pi_{A 1}+\Pi_{A 2}  \tag{22a}\\
\Pi_{B}=\Pi_{B 1}+\Pi_{B 2}
\end{array}\right.
$$

The sequence of the game is as follows. In stage 1 , firm $A$ and $B$ decide $p_{A}$ and $p_{B 1}$, respectively. In stage 2 , given $p_{A}$ and $p_{B 1}$, firm $B$ decides $p_{B 2}$. Solving the game by backward induction, we can obtain Proposition 6.

Proposition 6. Considering the two-period pricing game, if firm A adopts Single Pricing Policy and firm B adopts Dynamic Pricing Policy, when $0 \leq$ $\lambda<2$, we have
(i) If $0 \leq \lambda<\frac{\sqrt{665}-7}{14} \approx 1.3420$, there exists a unique equilibrium $p_{A}^{S D *}=$ $\frac{6(2-\lambda)}{10-\lambda-\lambda^{2}}, p_{B 1}^{S D *}=\frac{22-7 \lambda-7 \lambda^{2}}{(2+\lambda)\left(10-\lambda-\lambda^{2}\right)}, p_{B 2}^{S D *}=\frac{22+5 \lambda-\lambda^{2}}{(2+\lambda)\left(10-\lambda-\lambda^{2}\right)} ; x_{1}^{*}=\frac{18+\lambda-4 \lambda^{2}-\lambda^{3}}{2(2+\lambda)\left(10-\lambda-\lambda^{2}\right)}$, $x_{2}^{*}=\frac{18+11 \lambda-5 \lambda^{2}-2 \lambda^{3}}{2(2+\lambda)\left(10-\lambda-\lambda^{2}\right)}$.

The corresponding profits are
$\Pi_{A 1}^{S D *}=\frac{3(2-\lambda)\left(18+\lambda-4 \lambda^{2}-\lambda^{3}\right)}{(2+\lambda)\left(10-\lambda-\lambda^{2}\right)^{2}}, \Pi_{B 1}^{S D *}=\frac{\left(22-7 \lambda-7 \lambda^{2}\right)\left(22+15 \lambda-2 \lambda^{2}-\lambda^{3}\right)}{2(2+\lambda)^{2}\left(10-\lambda-\lambda^{2}\right)^{2}}$,
$\Pi_{A 2}^{S D *}=\frac{3(2-\lambda)\left(18+11 \lambda-5 \lambda^{2}-2 \lambda^{3}\right)}{(2+\lambda)\left(10-\lambda-\lambda^{2}\right)^{2}}, \Pi_{B 2}^{S D *}=\frac{\left(22+5 \lambda-\lambda^{2}\right)^{2}}{2(2+\lambda)^{2}\left(10-\lambda-\lambda^{2}\right)^{2}} ;$
$\Pi_{A}^{S D *}=\frac{9(2-\lambda)^{2}(3+\lambda)}{\left(10-\lambda-\lambda^{2}\right)^{2}}, \Pi_{B}^{S D *}=\frac{484-44 \lambda-139 \lambda^{2}+8 \lambda^{3}+7 \lambda^{4}}{2(2+\lambda)\left(10-\lambda-\lambda^{2}\right)^{2}}$.
(ii) If $\frac{\sqrt{665}-7}{14} \leq \lambda<2$, there exists a unique equilibrium $p_{A}^{S D *}=\frac{5}{2(3+\lambda)}, p_{B 1}^{S D *}=0, p_{B 2}^{S D *}=\frac{11+7 \lambda}{12+4 \lambda} ; x_{1}^{*}=\frac{1+2 \lambda}{12+4 \lambda}, x_{2}^{*}=\frac{13+\lambda}{24+8 \lambda}$.

The corresponding profits are
$\Pi_{A 1}^{S D *}=\frac{5(1+2 \lambda)}{8(3+\lambda)^{2}}, \Pi_{B 1}^{S D *}=0, \Pi_{A 2}^{S D *}=\frac{5(13+\lambda)}{16(3+\lambda)^{2}}, \Pi_{B 2}^{D *}=\frac{(11+7 \lambda)^{2}}{32(3+\lambda)^{2}} ;$ $\Pi_{A}^{S D *}=\frac{25}{16(3+\lambda)}, \Pi_{B}^{S D *}=\frac{(11+7 \lambda)^{2}}{32(3+\lambda)^{2}}$.

From Proposition 6, we can observe that when the degree of social influence is sufficiently large (i.e., $\lambda \geq \frac{\sqrt{665}-7}{14}$ ), the firm adopting Dynamic Pricing Policy will set its price at zero in period $1\left(p_{B 1}^{S D *}=0\right)$. Furthermore, we can easily derive Corollary 6.

Corollary 6. Assuming $\lambda_{0} \approx 0.5596$ is the first root of $\lambda^{4}-10 \lambda^{3}-73 \lambda^{2}-$ $92 \lambda+76=0$, under the conditions of Proposition 6, we have
(i) $\Pi_{A}^{S D *}<\Pi_{B}^{S D *} ; \triangle \Pi=\Pi_{B}^{S D *}-\Pi_{A}^{S D *}$ increases with $\lambda$.
(ii) $p_{A}^{S D *}, \Pi_{A 1}^{S D *}, \Pi_{A 2}^{S D *}$, and $\Pi_{A}^{S D *}$ decrease with $\lambda$; $p_{B 1}^{S D *}$ and $\Pi_{B 1}^{S D *}$ decrease with $\lambda$ when $0 \leq \lambda<\frac{\sqrt{665}-7}{14} ; p_{B 2}^{S D *}$ and $\Pi_{B 2}^{S D *}$ decrease with $\lambda$ when $0 \leq \lambda \leq \lambda_{0}$ and increase with $\lambda$ when $\lambda_{0}<\lambda<2$; $\Pi_{B}^{S D *}$ decreases with $\lambda$ when $0 \leq \lambda \leq$ $\frac{\sqrt{665}-7}{14}$ and increases with $\lambda$ when $\frac{\sqrt{665}-7}{14}<\lambda<2$.
(iii) $x_{1}^{*}$ decreases with $\lambda$ when $0 \leq \lambda \leq \frac{\sqrt{665}-7}{14}$ and increases with $\lambda$ when $\frac{\sqrt{665}-7}{14}<\lambda<2$; $x_{2}^{*}$ increases with $\lambda$ when $0 \leq \lambda \leq \lambda_{0}$ and decreases with $\lambda$ when $\lambda_{0}<\lambda<2$.

Corollary 6 (i) indicates that when two firms adopt different pricing policies, the firm adopting Dynamic Pricing Policy (i.e., firm $B$ ) will always be better off than the one adopting Single Pricing Policy (i.e., firm $A$ ). This finding is intuitive, since Dynamic Pricing Policy allows firm $B$ to more flexibly adjust its prices and take advantage of social influence. Furthermore, the profit advantage of the firm adopting Dynamic Pricing Policy relative to its competitor that adopts Single Pricing Policy will be more remarkable when the degree of social influence becomes larger. Corollary 6 (i) and (ii) also indicate that when the
degree of social influence is large enough $\left(\lambda>\lambda_{0} \approx 0.5596\right)$, the equilibrium price, demand $\left(D_{2}^{*}=1-x_{2}^{*}\right)$, and profit in period 2 of the firm adopting Dynamic Pricing Policy will increase with $\lambda$. Particularly, when the degree of social influence is sufficiently large $\left(\lambda>\frac{\sqrt{665}-7}{14} \approx 1.3420\right)$, the total profit of the firm adopting Dynamic Pricing Policy will also increase with $\lambda$.

According to Theorem 1, when the degree of social influence is relatively large, firms will be better off if both of them adopt Single Pricing Policy than if they both adopt Dynamic Pricing Policy. The underlying reason is that Single Pricing Policy helps alleviate the price competition and thus is better than Dynamic Pricing Policy. As a result, a natural question is whether firm $B$ would be better off by switching from Dynamic Pricing Policy to Single Pricing Policy, given that firm $A$ adopts Single Pricing Policy. Comparing Proposition 6 and Proposition 1, we obtain Theorem 4.

Theorem 4. Assume that firm A adopts Single Pricing Policy. Comparing the profit of firm B under Dynamic Pricing Policy $\left(\Pi_{B}^{S D *}\right)$ with that under Single Pricing Policy $\left(\Pi_{B}^{S *}\right)$, for $0<\lambda<2$, we have $\Pi_{B}^{S D *}>\Pi_{B}^{S *}$.

Theorem 4 indicates that it is always more profitable for firm $B$ to stay with Dynamic Pricing Policy. The underlying reason is that, given that firm $A$ adopts Single Pricing Policy, adopting Dynamic Pricing Policy gives firm $B$ more room than its competitor to adjust its prices over the two periods. In addition, assuming that firms can freely choose either one of the two pricing policies, this theorem also indicates that the case where both firms adopt Single Pricing Policy cannot be an equilibrium since either firm could be better off by changing its pricing policy to Dynamic Pricing Policy. Furthermore, combining Proposition 1-2 with Proposition 6, we can obtain Theorem 5 as follows.

Theorem 5 (Comparison of different pricing schemes). Define $\lambda_{1} \approx 0.4716$ as the second root of $24-56 \lambda+10 \lambda^{2}+2 \lambda^{3}-\lambda^{4}+\lambda^{5}=0$. Assuming firms can freely choose either Single Pricing Policy or Dynamic Pricing Policy, when
$0<\lambda<2,{ }^{7}$ we have
(i) If $0<\lambda<\lambda_{1}$, there exists two asymmetric equilibria where one firm adopts Single Pricing Policy and the other adopts Dynamic Pricing Policy.
(ii) If $\lambda>\lambda_{1}$, there exists a unique equilibrium where both firms adopt Dynamic Pricing Policy.

Interestingly, when the degree of social influence is relatively small $(0<\lambda<$ $\lambda_{1}$ ), the cases where one firm adopts Single Pricing Policy and the other adopts Dynamic Pricing Policy can be the equilibria (see Theorem 5 (i)). From the perspective of the firm adopting Single Pricing Policy, though it has a disadvantage compared to its competitor that adopts Dynamic Pricing Policy, the disadvantage is relatively small because of the relatively weak social influence (Corollary 6). If it chose to adopt Dynamic Pricing Policy (rather than Single Pricing Policy), however, the price competition would become more fierce, which leads to lower profits for both firms. As a result, it would be better for the firm to stay with Single Pricing Policy.

On the other hand, when the degree of social influence is very large $(\lambda>$ $\lambda_{1}$ ), both firms adopting Dynamic Pricing Policy is the unique equilibrium (see Theorem 5 (ii)). In this case, social influence is so strong that both firms have incentives to dynamically adjust their prices and take advantage of social influence (Note that when faced with strong social influence, if one firm chose to adopt Single Pricing Policy, it would have a huge disadvantage compared to its competitor that adopts Dynamic Pricing Policy.). As a consequence, firms prefer to compete head-to-head and both adopt Dynamic Pricing Policy. Remember that when social influence is relatively strong, both firms adopting Single Pricing Policy can lead to higher profits than adopting Dynamic Pricing Policy (see Theorem 1). Thus, when firms freely choose pricing policy, they may be caught in a dilemma that makes them both worse off.

[^6]
## 7. Conclusion

In this paper we construct a two-period pricing model in the presence of social influence, and consider different pricing policies for two competing firms. We get some interesting findings which have not been stressed in existing literature. Main results are summarized as follows.

First, assuming two firms adopt the same pricing policy (either Single Pricing Policy or Dynamic Pricing Policy), firm prices and profits always decrease with the degree of social influence. When the degree of social influence is large enough, the competition under Dynamic Pricing Policy would be so intense that firms will set the prices lower than the marginal cost or even at zero in the first period. In addition, firms prefer Dynamic Pricing Policy when the degree of social influence is relatively small or sufficiently large (boundary solutions). Otherwise, Single Pricing Policy is more preferable.

Second, when considering the cases where the market size varies across two periods, Dynamic Pricing Policy always outperforms Single Pricing Policy if the market size in period 2 is sufficiently large. Otherwise, either pricing policy may dominate, which is similar to the results of our main model. When the market size in the second period is smaller than that in the first period and social influence is strong enough, under either pricing policy, a larger market size is period 2 may intensify price competition and make both firms worse off.

Furthermore, Dynamic Pricing Policy dominates when social influence is relatively weak, while Posted Pricing Policy with a reference price effect outperforms both Single and Dynamic Pricing Policy when social influence is sufficiently strong. When both firms adopt Posted Pricing Policy, under certain conditions (such that the equilibrium prices in period 1 are zero due to fierce competition), firms can benefit from a larger degree of social influence or a larger reference price effect because of the synergy between these two.

Finally, when two firms adopt different pricing policies, the firm adopting Dynamic Pricing Policy is always more profitable than the one adopting Single Pricing Policy, and the profit difference will be more remarkable when the degree
of social influence becomes larger. Moreover, when firms can strategically choose either one of the two pricing policies, as long as the degree of social influence is relatively small, asymmetric equilibria exist where one firm adopts Single Pricing Policy and the other adopts Dynamic Pricing Policy. On the other hand, when the degree of social influence is very large, both firms adopting Dynamic Pricing Policy is the unique equilibrium.

This study provides several implications for competing firms' multi-period pricing strategies in the presence of social influence. First, while many firms try to take advantage of social influence, they may have overlooked its dark side. When a firm adopts the same pricing policy with its competitor, social influence can intensify the price competition, and thus make both firms worse off. Second, in markets where firms are likely to make a similar move (i.e., adopting the same pricing policy), it would be wise for firms to adopt Dynamic Pricing Policy (rather than Single Pricing Policy) when facing either relatively weak or sufficiently strong social influence. Otherwise, Single Pricing Policy would be a better choice. Third, in markets where consumers are highly susceptible to both social influence and reference price, Posted Pricing Policy would be more profitable than either Single or Dynamic Pricing Policy. Only with its synergy with a reference price effect can social influence benefit firms in a competitive setting. Finally, in markets where firms change their pricing policies frequently and may not make similar moves (i.e., freely adopting either pricing policy), it is more profitable for the firm to take the lead in adopting Dynamic Pricing Policy if the social influence is relatively weak. In this case, adopting Dynamic Pricing Policy would help the firm obtain a competitive advantage, leaving the other firm the only option to adopt Single Pricing Policy. Moreover, when facing a market with relatively stronger social influence, both firms adopting Dynamic Pricing Policy is the unique outcome, though both firms adopting Single Pricing Policy might lead to higher profits. Thus, when firms freely choose pricing policy, they may be caught in a dilemma that makes them both worse off.

There are various related directions to be explored in future research. First of all, we can investigate two asymmetric firms which produce two vertically
differentiated products or with different marginal costs. In addition, we can also consider a variable marginal cost that is related to quality and study the quality decision of two competing firms. Finally, we can also examine the competing duopoly with social influence from the perspective of supply chain management.

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Multi-period pricing in the presence of competition and social influence

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[^0]:    ${ }^{1}$ The condition $0 \leq \lambda<\frac{3 \theta}{\sqrt{\delta}}$ ensures that all the profit functions are concave and that the equilibrium is unique.
    ${ }^{2}$ In (ii), the condition for this equilibrium is $0 \leq \lambda<\frac{3 \theta}{\sqrt{\delta}}$ and $\lambda \geq \frac{3(\theta+c)}{2 \delta}$. Thus, we

[^1]:    have $\frac{3(\theta+c)}{2 \delta}<\frac{3 \theta}{\sqrt{\delta}} \Longleftrightarrow c<\theta(2 \sqrt{\delta}-1)$, and $\Pi_{A}^{D *}=\Pi_{B}^{D *}=\frac{\delta \theta-c}{2}>\frac{1}{2}(\delta \theta-\theta(2 \sqrt{\delta}-1))=$ $\frac{\theta}{2}(1-\sqrt{\delta})^{2} \geq 0$.

[^2]:    ${ }^{3}$ This condition ensures that all profit functions are concave and the equilibrium is unique.

[^3]:    ${ }^{4}$ We thank the anonymous reviewers for their insightful comments, which inspire us to further examine Posted Pricing Policy with the reference price effect, and compare it with Single and Dynamic Pricing Policies.

[^4]:    ${ }^{5}$ This condition ensures that all the profit functions are concave and that the equilibrium is unique, and is equivalent to $\frac{1}{4}(\alpha(\lambda-1)+\lambda)^{2}+(1+\alpha)(\alpha \lambda-1)<0$. In addition, from this condition we can obtain $\alpha \lambda<1$.

[^5]:    ${ }^{6}$ This condition is equivalent to $0 \leq \alpha<2(1+\sqrt{2})$ and $0 \leq \lambda<\frac{2 \sqrt{1+\alpha}-\alpha}{1+\alpha}$.

[^6]:    ${ }^{7}$ This condition ensures that the two firms have a unique equilibrium under different pricing policies.

