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Topology design of remote patient monitoring system concerning qualitative and quantitative issues

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Highlights

- The technique for qualitative and quantitative issues is considered
- Topology design of remote patient monitoring system is provided.
- Piecewise linear function is addressed.

Topology design of a remote patient monitoring system concerning qualitative and quantitative issues

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Abstract

Piecewise linear functions have been used to model and solve non-linear problems in science, social science, and technology fields since the 1950's. Multi-choice goal programming (**MCGP**) has been widely used to solve multiple objective decision-making (**MODM**) problems in which each goal mapping with multiple aspiration levels is allowed, expanding the original feasible region to obtain better solutions in the MODM problems. This paper integrates the efficient S-shaped penalty method, arbitrary piecewise linear utility functions, trapezoidal utility functions, and MCGP to solve a topology design problem in a remote patient monitoring system (**RPMS**) by providing universal senior citizen coverage in which quantitative (e.g., patient satisfaction-related cost) and qualitative (e.g., satisfaction) issues are considered simultaneously. In addition, some novel utility functions such as the force utility function, indicator utility function, and arbitrary utility function are proposed to improve the usefulness of MCGP in the field of management science. These proposed utility functions can be easily used to model qualitative issues in real-world problems. A topology design problem for an RPMS is demonstrated to justify the feasibility, usefulness, and compatibility of the proposed

methods. Further, sensitivity analysis and managerial implications are provided using an RPMS in Taiwan.

Keywords: Remote patient monitoring; Piecewise linear function; Utility Function; Multi-choice Goal programming;

1. Introduction

Given the aging society and falling birth rate, Taiwan has been classified as an "aged" society since 2018 and will turn into a "hyper-aged" society by 2025. The increasing healthcare costs for the growing population of elderly, chronic diseases, as well as the demand for new technologies and treatments in addition to the decrease in the number of healthcare professionals compared to the elderly increase make the traditional healthcare service model inadequate. Conventional healthcare model inadequacy can be explained by the following two aspects: (1) large hospitals (e.g., accredited hospitals and medical centers) are always overcrowded, and as a result, people often don't have access to outpatient clinics and must wait a long time for inpatient care; and (2) increasing healthcare costs also place a considerable burden on the government's finances. Nowadays, establishing an efficient remote patient monitoring system (**RPMS**) to take care of the greater number of elderly people as well as reduce healthcare costs is essential in Taiwan. Moreover, it is very important to build an RPMS with a more complete topology network under a limited budget to take care of senior citizens. To implement the RPMS for serving senior citizens, several goals and criteria should be considered simultaneously. These goals and criteria contain both quantitative (e.g., the RPMS setup cost and investment cost) and qualitative (e.g., satisfaction of senior citizens) issues, and the balance between them (e.g., balancing the setup cost and the satisfaction of senior citizens), complicates the problem. This is a typical multiple objective decision making (**MODM**) problem with conflicting goals. For example, under the limited budget, the decision maker (**DM**) would like to take care of more people to improve senior citizens' satisfaction level. The biggest challenge in modeling the RPMS in measuring the senior citizens' satisfaction level is it is a nonlinear problem. To deal with this problem, two nonlinear techniques, piecewise linearization technique and mixed-integer program (**MIP**) are introduced. Then, novel methods for modeling arbitrary piecewise utility functions (**APUF**) and trapezoidal utility functions (**TUF**) are proposed. The proposed methods not only **assist** in measuring senior citizens' satisfaction **in practice**, but also **enrich** the field of qualitative method **in theory**. To express senior citizens' satisfaction, S-shaped utility functions (**UF**) and linear UF are introduced. In addition, force UF (**FUF**) and indicate UF (**IUF**) are also derived to enrich the qualitative research. To force the solution of the investment cost of RPMS further away (approach) from (to) the target value to attract investors into the development of RPMS, penalty functions (**PF**) are introduced. Multi-choice goal programming (**MCGP**) has widely been used to solve multiple criteria decision making (**MCDM**) and MODM problems. Thus, the MCGP is used to model and solve the topology design problem of an RPMS with quantitative issues**.** All the above mentioned points improve the linkage between the related techniques and the development of RPMS. In the field of management science, few works exist that address qualitative and quantitative issues at the same time in one model. Therefore, this paper proposes an integrated method to resolve the RPMS problem considering quantitative, qualitative, and balance issues simultaneously. This reduces the gap between the research and health sector to improve the usefulness of proposed methods in solving the MCDM problems.

In the 1950's, it was shown a nonlinear problem can be modeled using piecewise linear functions (**PLF**) and binary variables within a MIP (Markowitz and Manne 1957, Sherali 2001, Croxton et al. 2003, Li et al. 2009, Toriello and Vielma 2012) and that such problems can be solved by specialized heuristic algorithms (Keha et al. 2006). This type of linearization was instrumental in formulating optimization problems in multiple domains including the fields of science, economics, and social science, and specifically in Very-Large-Scale Integrated circuit design, big data analytics, network analysis, portfolio optimization, and supply chain optimization. Comprehensive monographs on related subjects have been offered (Bazaraa et al. 1993, Floudas 1999). However, such techniques (i.e., linearization by adding binary variables) may lead to significantly increased computational burden when the problem size becomes too large. Hence, a special ordered set of the type 2 (SOS 2) method was proposed by Beale and Tomlin (1970) to reduce the complexity of these MIP problems; the SOS 2 is defined as: a set of variables $\{\lambda_1,...,\lambda_n\}$ having at most two adjacent variables that are nonzero to indicate which line segment is activated. Thereafter, many extensions of these models have been studied (Vielma and Nemhauser 2011, Li et al. (2009, 2012), Vielma and Nemhauser (2010, 2011)). Recently, Chang (2018) proposed an efficient S-shaped penalty method (**ESPM**) to model an S-shaped PF enabling the solution of a realistic large-scale problem within a reasonable amount of time. This paper uses the principle of goal programming (**GP**), plus newly created techniques to deal with PLF, so we first introduce the related technology of GP.

The initial goal programming (**GP**) model was proposed by Charnes et al. (1955) with further development by Tamiz et al. (1998), Ignizio (1985), and Lee (1972). Recently, more advanced variants were proposed for more complex decision problems with multiple conflicting goals such as fuzzy GP (Narasimhan 1980), meta-GP (Rodríquez et al. 2002), extended GP (Romero 2001), and MCGP (Chang 2007). Fuzzy GP is used to resolve MODM problems with imprecisely defined model goals and/or constraints in a decision-making environment. The concept of a meta-goal is proposed by the meta-GP method as a high-level goal going beyond a single goal and giving an overall measure of satisfaction for the DM. An extended GP comprises two meta-goals: normalizing unwanted deviations and minimizing the maximal goal. MCGP is used to solve MODM problems where each goal can be achieved from different aspiration levels (Chang 2007, 2008). Subsequently, MCGP has been widely applied to resolve various practical problems, including coffee shop location selection (Ho et. al., 2013), house selection (Ho et. al., 2015), stochastic transportation problem (Mahapatra et al., 2013), e-learning system evaluation (Lin et. al., 2014), a supply chain order allocation problem (Chang et al., 2014), course planning (Kırış, 2014), supplier selection problems (Jadidi, et al. (2015), renewable energy facilities location selection problem (Chang, 2015), reverse logistics providers selection problem (Govindan et al. 2017), consumer choice behavior (Swait et al., 2018) and others (Shalabh and Sonia, 2017). Multi-objective transportation problems are solved using various methods such as grey parameters, the conic scalarization approach, UF, dual-hesitant fuzzy number, fuzzy MCGP, and multi-choice interval GP (Roy et al. 2017a, 2017b, Roy and Maity 2017, Maity et al. 2016, 2019, Maity and Roy 2016).

The purpose of the study is to integrate ESPM, APUF, TUF, and MCGP to solve a topology design problem of an RPMS considering quantitative, qualitative, and balance issues. The contributions of the paper are (1) the qualitative and quantitative issues can be simultaneously considered in solving the topology design problem for an RPMS, (2) new utility functions such as FUF, IUF, APUF, and TUF are proposed to enhance the usefulness of the UF, and (3) the optimal topology of the RPMS in Taiwan is obtained to provide universal senior citizen coverage by considering multiple criteria, subject to resource limitations. This will help Taiwan’s long-term care planning for the “hyper-aged” society in the future.

The following section briefly reviews previous methods such as piecewise linear methods, MCGP, and MCGP with utility functions. New utility functions (UIF, APUF and TUF) are proposed in Section 3. Section 4 demonstrates a topology design problem of RPMS. Discussion, sensitivity analysis, and the management implications are also provided in Section 4. Conclusions are presented in Section 5.

2. Previous related methods for qualitative and quantitative issues

In the past, several methods have been proposed for MCDM problems involving qualitative or quantitative objectives. These methods should be addressed as follows.

2.1 Piecewise penalty functions for qualitative issues

Charnes and Collomb (1997) proposed an interval GP method to deal with the importance of marginal changes in goal achievement based on distance to the goal target. These penalty functions serve to increasingly penalize the objective function of the problem as the solution becomes further away from the goal. Later, various methods were proposed to model the increasing PF, reverse PF, and nonlinear PF. These methods were used to solve many practical problems such as constrained optimal control problems, tax payment problems, and search engine advertising problems. The S-shaped PF is a more general model appearing in models of technological innovation, labor, and economics (Chang and Lin, 2009). Therefore, a simple S-shaped PF is discussed here; the penalized behavior is depicted in Figure 1, and the scale data is listed in Table 1. Yang et al. (1991) derived a novel method to solve an S-shaped PF problem. To formulate the problem, one additional binary variable is needed to interpret the 'either-or' relationship for the union of the ramp-type linear functions. Chang and Lin (2009) derived an efficient mixed-integer model to solve the S-shaped PF problem. Later, an efficient model was proposed by Lu and Chen (2013) for the S-shaped PF in which the number of extra binary variables can be reduced from n down to $\lceil \log_2(n-1) \rceil$. However, all the above methods formulate the S-shaped PF as an MIP problem by adding extra binary variables.

Table 1. Scale data of the S-shaped PF

Goals	Unit (%)	Marginal penalty	The value of $f_i(\mathbf{x})$	Marginal penalized	Directions
g_1	below 80	0.8	75	34	Left
	80-90	2	85	20	four-sided
	90-100	1	95	5	penalty
	over 100	0	105	0	

below 110	0	105	0	Right four-sided penalty
110-120	1	115	5	
120-130	2	125	20	
over 130	0.8	135	34	

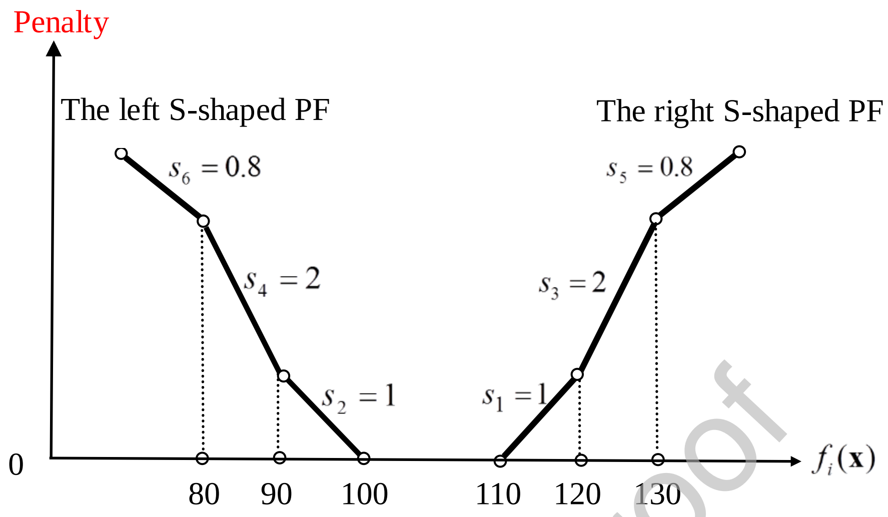

Figure 1. An example of a two-side S-shaped PF

Adding binary variables to an MIP problem does not improve the linear programming relaxation, as indicated by Keha et al. (2004, 2006). Therefore, it is essential to create a new efficient model to deal with a large size MIP problem. A typical right-side S-shaped PF with marginal penalties equal to 1, 2, and 0.75 is depicted in Figure 2.

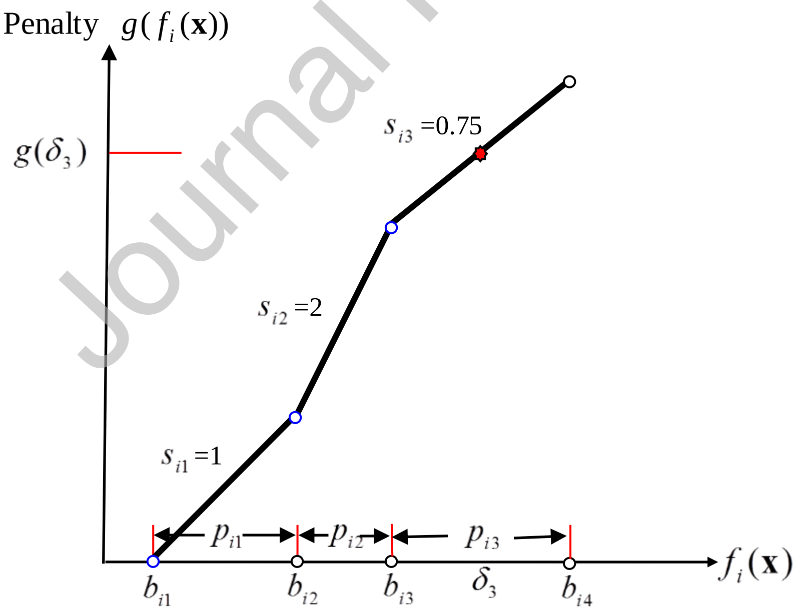

Figure 2. An example of the right-side S-shaped PF

Based on a right-side S-shaped PF, as shown in Figure 2, Chang (2018) proposed an ESPM to approximate the S-shaped function. To save space, we only express the right-side S-shaped PF as follows.

(P1)

Min $\sum_{j=1}^{3} w_{ij} p_{ij} + g(f_i(\mathbf{x}))$

s.t.

$g(f_i(\mathbf{x})) = g(b_{i1}) + s_{i1} p_{i1} + s_{i2} p_{i2} + s_{i3} p_{i3}$, for right-side S-shaped function (1)

$f_i(\mathbf{x}) - p_{i1} - p_{i2} - p_{i3} = b_{i1}$, (2)

$0 \le p_{ij} \le b_{ij+1} - b_{ij}$, $j = 1,2,3$, (3)

$\mathbf{x} \in \mathbf{F}$, ($\mathbf{F}$ is a feasible set), (4)

where $g(f_i(\mathbf{x}))$ denotes the right-side S-shaped PF; the slope of the line segment [b_{ij}, b_{ij+1}], $j = 1,2,3$ is denoted as $s_{ij} = \dfrac{g(b_{ij+1}) - g(b_{ij})}{b_{ij+1} - b_{ij}}$, $j = 1,2,3$; the deviational variable is denoted as p_{ij}; PF is represented by $g(f_i(\mathbf{x}))$; the weight w_{ij} must be appropriately assigned to p_{ij} in the objective function to ensure the active priority as $p_{i1} \succ p_{i2} \succ p_{i3}$ in (2) automatically.

In general, organizations often have a predefined budget for their system constructions, and the budget is expected to be balanced, not either too high or too low. Hence, the S-shaped PF is a useful technique to guide the construction cost of the RPMS to approach the predefined target cost.

2.2 MCGP with utility functions on qualitative issues

A popular linear UF, $\mu_i(y_i)$, can be found in Lai and Hwang (1994). It is widely used to represent individual or organizational preferences in various decision and management models. Thus, the UF is used to formulate the satisfaction of senior citizens in the RPMS. The senior citizens' satisfaction is measured using the system coverage rate. The function is characterized as follows.

$$\mu_i(y_i) = \begin{cases} 1, & if \quad y_i \le g_{i,\min} \\ \dfrac{g_{i,\max} - y_i}{g_{i,\max} - g_{i,\min}}, & if \quad g_{i,\min} \le y_i \le g_{i,\max} \\ 0, & if \quad y_i \ge g_{i,\max} \end{cases}$$ Case I - left linear UF (LLUF)

$$\mu_i(y_i) = \begin{cases} 1, & if \quad y_i \geq g_{i,\max} \\ \dfrac{y_i - g_{i,\min}}{g_{i,\max} - g_{i,\min}}, & if \quad g_{i,\min} \leq y_i \leq g_{i,\max} \\ 0, & if \quad y_i \leq g_{i,\min} \end{cases}$$ Case II - right linear UF (RLUF)

where $g_{i,\max}$ ($g_{i,\min}$) is the upper (lower) bound for the i th goal. According to the principle of maximizing expected utility, the DM would like to increase the utility value $\lambda_i = \mu_i(y_i)$ as much as possible. Therefore, the left linear UF and the right linear UF should be addressed (see Figure 3).

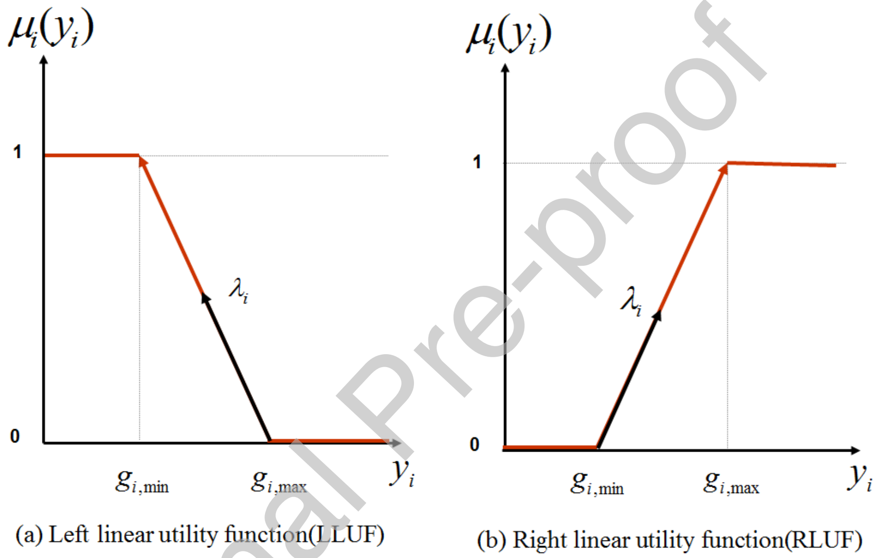

Figure 3. The left and right linear utility function

In some situations, the DM prefers to increase the value of UF, $\mu_i(y_i)$, as much as possible in the above cases (Case I and Case II) for their decision models (e.g., DM would like to increase the senior citizens' satisfaction level of using the RPMS), called FUF. That is, the achievement model will add some deviational variables to force the value of y_i to approach $g_{i,\min}$ or $g_{i,\max}$ (i.e., $\mu_i(y_i) \to 1$). This simple case has been addressed by Chang (2011); please refer to Appendix 1. The MCGP with FUF has been applied to solve some practical problems such as renewable energy portfolios under uncertainty (Hocine et al., 2018), the supplier selection problem (Alizadeh and Yousefi, 2018), and consensus models (Gong, 2015).

2.3 MCGP methods on quantitative issues

To solve an MODM problem with multiple goals and each goal being achieved from some specific aspiration levels, MCGP was proposed by Chang (2007, 2008). In the MCGP method, the DM is allowed to set multiple goals for their MODM problems, and each goal maps to multiple aspiration levels (i.e., discrete aspiration levels). MCGP can extend the feasible solution region to improve solution quality. MCGP can be expressed as in Appendix 2. Later, Chang (2008) proposed a revised MCGP for MODM problems with multiple goals with each goal being able to be achieved from some vector aspiration levels (i.e., continuous aspiration levels), as shown in Appendix 3. The history of MCGP is listed in Figure 4. In fact, MCGP related methods have been constructed as a preliminary processing system for qualitative and quantitative issues. It is easy to see there are fewer methods to handle qualitative issues than quantitative issues. Therefore, if more qualitative methods (e.g., IUF, APUF, and TUF) can be added by this paper, the MCGP system will be more colorful and powerful in its contribution to solving MODM and MCDM problems in management science. The RPMS topology design problem is a typical MODM problem with qualitative and quantitative issues. Therefore, we integrate MCGP and other methods (e.g., FUF, IUF, APUF, and TUF) to be one model for the problem. A comprehensive analysis of MCGP theories and applications was addressed by Shalabhs and Sonia (2017). Other improved MCGP methods were proposed: Behzad et al. (2012) derived a fuzzy MCGP approach to solving the fuzzy MODM problem; the conic scalarizing function utilized in MCGP was proposed by Ustun (2012); the computation of an MCGP problem was addressed by Patro et al. (2015).

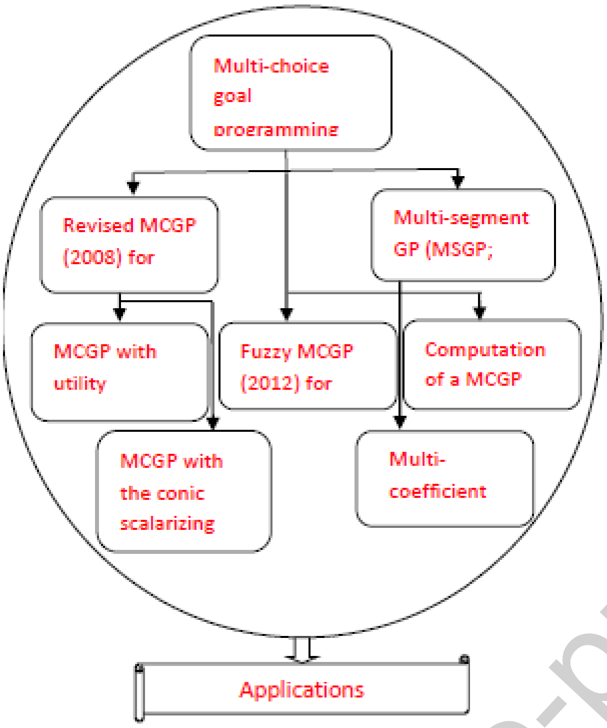

Figure 4. The history of MCGP

3. Proposed new utility functions for MCGP

3.1 Indicator utility function for MCGP

In some situations, the multiple-step decision process is required in the decision model. In this case, the DM prefers to know the current value of UF, $\mu_i(y_i)$, for the next step of the decision-making process. Thus, the DM does not need to force λ_i to approach $g_{i,\min}$ or $g_{i,max}$. In other words, the action of λ_i is as an indicator of the value of UF, $\mu_i(y_i)$, namely IUF. Although this method is not used in modeling the RPMS, it is worth introducing to enrich qualitative research. The idea of IUF is first proposed in this paper to improve the usefulness of UF in multiple-step decision-making methods. Generally, using mathematical programming to solve decision problems involves formulating the decision problem into a decision model and then solving it in one step. In contrast, many decision problems exist in real situations that may require using many solution steps (Swait et al., 2018). This issue is critical, however, it is rarely discussed in the literature. Therefore, we propose a new IUF to express this case as follows.

(P2) expresses the left linear UF shown in Figure 3(a)

Min $\sum_{i=1}^{n} w_i (d_i^+ + d_i^-)$

s.t.

$$\lambda_i = \frac{g_{i,\max} - f_i(\mathbf{x})}{g_{i,\max} - g_{i,\min}}, \quad i = 1,2,...,n, \tag{5}$$

$$f_i(\mathbf{x}) - d_i^+ + d_i^- = g_i, i = 1,2,...,n \tag{6}$$

$$d_i^+, \quad d_i^- \geq 0, \quad i = 1,2,...,n, \tag{7}$$

$$\mathbf{x} \in \mathbf{F}, \quad (\mathbf{F} \text{ is a feasible set}), \tag{8}$$

where w_i is the weight attached to deviational variables d_i^+ and d_i^-; indicator variable λ_i represents the value of the utility function of $\frac{g_{i,max} - f_i(\mathbf{x})}{g_{i,max} - g_{i,min}}$; the lower (upper) bound is denoted as $g_{i,\min}$ ($g_{i,\max}$) for the i th goal.

(P3) expresses the right linear UF shown in Figure 3(b)

Min $\sum_{i=1}^{n} w_i (d_i^+ + d_i^-)$

s.t.

$$\lambda_i = \frac{f_i(\mathbf{x}) - g_{i,\min}}{g_{i,\max} - g_{i,\min}}, \quad i = 1,2,...,n, \tag{9}$$

Eqs.(6)-(8), (10)

where all variables are defined as in P2.

As seen in P2 and P3, the value of λ_i provides useful information for the DM to resolve a multiple-step decision-making problem. This new idea can enrich the formulation of qualitative methods. To briefly describe the idea of IUF, a two-step decision-making process is shown in Figure 5. As seen in Figure 5, the IUF can be classified into the following two types of roles.

(i) IUF as an IF condition.

When the role of IUF is an IF condition control, it guides the direction of the decision-making process to either feedback to step one of the decision-making process or progress towards step two of the decision-making process.

(ii) IUF as a CASE condition.

When the role of IUF is a CASE condition control, it guides the direction of the decision-making process to different paths of the step two decision making process.

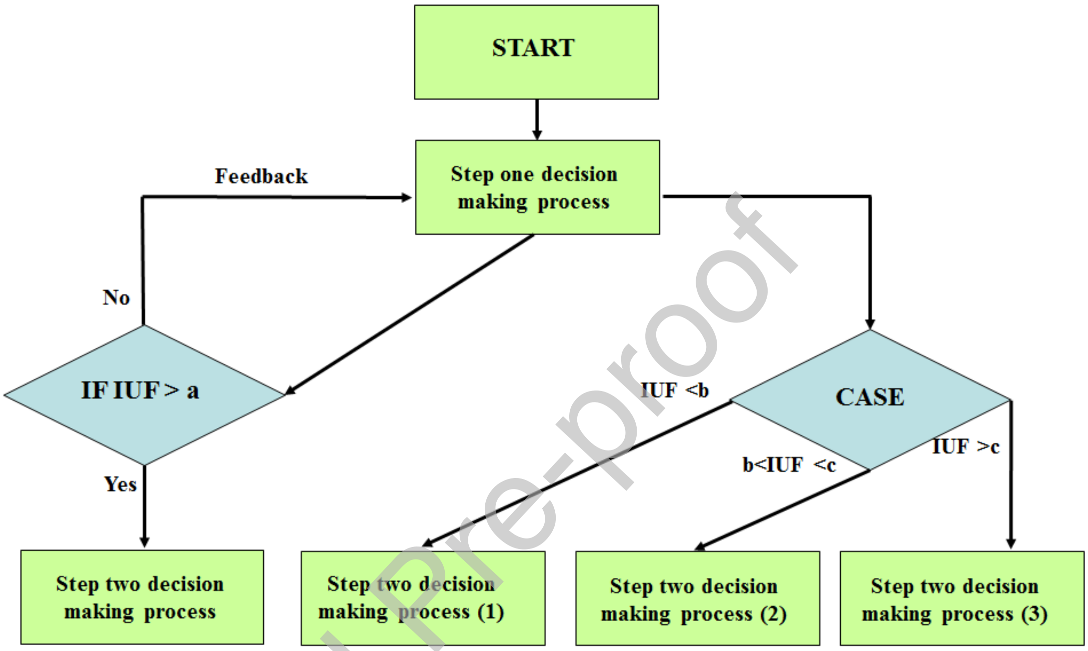

Figure 5. An example of two steps decision making process

3.2 Piecewise utility function for MCGP

To enrich the usefulness of UFs in the field of mathematical programming, an APUF should be provided because of its generality. For simplicity, an APUF example with a negative slope is depicted in Figure 6.

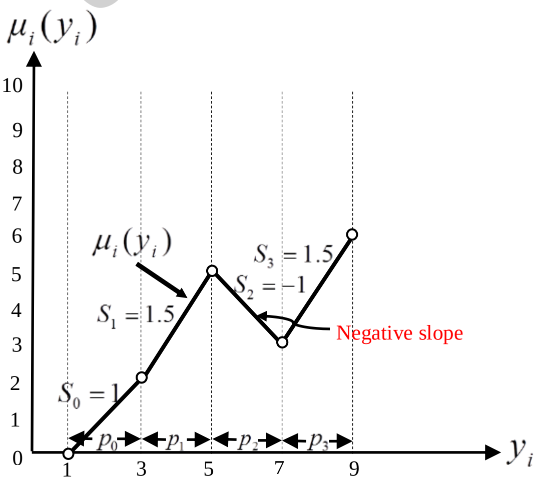

Figure 6. A simple example of APUF

Figure 6 can be intuitively formulated as follows.

(P4)

Min $\mu_i(y_i) + \sum_{i=0}^{3} w_i p_i$

s.t.

$$\mu_i(y_i) = \mu_i(a_0) + \sum_{i=0}^{2} s_i p_i + s_3 p_3 \ , \tag{11}$$

$$y_i - \sum_{i=0}^{3} p_i = a_0 \ , \tag{12}$$

$$a_i \le p_i \le a_{i+1}, i = 0,1,2,3 \ , \tag{13}$$

where w_i ($w_0 < w_1 < w_2 < w_3$) is the weight attached to deviational variable p_i in the objective function and p_i is bounded by Eq.(13); Eq.(11) is the summation of the deviations used to represent the APUF, $\mu_i(y_i)$; Eq.(12) is the control constraint utilized to indicate the exact value of the APUF in y_i axis; the slope of the line segment $[a_i, a_{i+1}]$ is denoted as $s_i = \frac{\mu_i(a_{i+1}) - \mu_i(a_i)}{a_{i+1} - a_i}$, $i = 0,1,2,3$; An appropriate weight, w_i, is used to ensure the activation sequence of p_i in Eq.(12) as $p_0 \succ p_1 \succ p_2 \succ p_3$, automatically.

Proposition 1. $\mu_i(y_i)$ in P4 and $\mu_i(y_i)$ in Figure 6 are equivalent in the sense that they have the same optimal solutions.

Proof.

Clearly, $y_i = a_0 + p_0 + p_1 + p_2 + p_3 \ge a_0 + p_0 + p_1 + p_2$, the constraint, $y_i - p_0 - p_1 - p_2 - p_3 \ge a_0$ is therefore covered by the constraint, $y_i - p_0 - p_1 - p_2 \ge a_0$ in Eq.(12). In addition, $w_i < w_{i+1}$, are the weights, in the objective function, attached to p_i, $\forall i$. This essentially forces the activation priority of p_i to be higher than p_{i+1}, $\forall i$ in the minimization problem (from Eq.(12)). This automatically ensures the activation sequence of p_i in Eq.(12) is as $p_0 \succ p_1 \succ p_2 \succ p_3$. Furthermore, the active priority must be guaranteed by an appropriate weight, w_i, which is assigned to $p_i, \forall i$. $\mu_i(y_i)$ in the objective function of P4 can be replaced by Eq.(11). $\mu_i(a_0)$ in Eq.(11) is a constant that can be ignored. Thus, P4' objective function can be expressed as follows.

$$s_0 p_0 + s_1 p_1 + s_2 p_2 + s_3 p_3 + w_0 p_0 + w_1 p_1 + w_2 p_2 + w_3 p_3 \tag{14}$$

$$=(w_0+s_0)p_0+\ldots+(w_3+s_3)p_3 \tag{15}$$

To guarantee the active priority: $p_0 \succ p_1 \succ p_2 \succ p_3$ in Eq.(12) , the following weights sequence is given.

$$(w_0+s_0)<(w_1+s_1)<(w_2+s_2)<(w_3+s_3) \tag{16}$$

This essentially forces $\mu_i(y_i)=a_0+s_0p_0+s_1p_1+s_2p_2+s_3p_3$ in Eq.(11). $\mu_i(y_i)$ in P4 is then equivalent to $\mu_i(y_i)$ in Figure 6. The proof of Proposition 1 is completed.

To demonstrate P4 in finding the appropriate weights, Figure 6 is used as an example below. The objective function of P4 can be replaced by Eq.(11); we yield the following equation:

$$p_0+1.5p_1-p_2+1.5p_3+w_0p_0+w_1p_1+w_2p_2+w_3p_3$$

$$=(1+w_0)p_0+(1.5+w_1)p_1+(w_2-1)p_2+(1.5+w_3)p_3 \tag{17}$$

Based on Eq.(17), we obtain the following weights sequence for p_0, p_1, p_2 and p_3, respectively.

$$(1+w_0)<(1.5+w_1)<(w_2-1)<(1.5+w_3) \tag{18}$$

It is observed the appropriate weight sequence of Eq.(18) can be applied to any APUF problem.

Based on Proposition 1, the weights sequence of Eq.(18) is strictly increasing to ensure the value of $\mu_i(y_i)$ can be computed exactly corresponding to the value of y_i in Figure 6. However, if the weights sequence of Eq.(18) is violated, then the value of y_i will be invalidated in Eq.(12). This means the value of $\mu_i(y_i)$ cannot be obtained exactly in Eq.(11).

Eqs.(11)-(12)$\in R^n$ with a nonempty interior is robust because P4 is a convex set $\in R^n$. Proposition 1 is very easy to deduce for all cases where $\delta_\theta \in [a_0, a_m] \in R^n$ and $\mu_i(\delta_\theta) \in R^n$ are valid.

Remark 1. Expressing $\mu_i(y)$ with $m+1$ breakpoints using P4 requires zero additional binary variables and two auxiliary constraints (i.e., Eqs.(11)-(12)), regardless of the number of breakpoints.

Based on P4, the following multiple APUF is established in far greater generality than S-shaped UF.

(P5)

Min $\sum_{j=1}^{n} \mu_{ij}(y_{ij}) + \sum_{j=1}^{n}\sum_{i=0}^{3} w_{ij} p_{ij}$

s.t.

$$\mu_{ij}(y_{ij}) = \mu_{ij}(a_{0j}) + \sum_{i=0}^{2} s_{ij} p_{ij} + s_{3j} p_{3j}, \quad j = 1,2,\ldots,n, \tag{19}$$

$$y_{ij} - \sum_{i=0}^{3} p_{ij} = a_{0j}, \quad j = 1,2,\ldots,n, \tag{20}$$

$$a_{ij} \le p_{ij} \le a_{i+1,j}, \quad i = 0,1,2,3, \quad j = 1,2,\ldots,n, \tag{21}$$

where all variables are similarly defined as in P4.

3.3 Trapezoidal utility function for MCGP

Triangular UF and TUF are often used to measure the satisfaction levels of individuals or organizations (Paksoy and Pehlivan 2012). In addition, triangular UF is a subset of TUF. Thus, TUF is discussed here. For simplicity, a TUF in the left graph of Figure 7 can be expressed as Eq.(22).

$$M_i(y) = \begin{cases} 0, & if \quad y \le g_{i1}^{\min} \quad or \quad y \ge g_{i1}^{\max} \\ \dfrac{y - g_{i1}^{\min}}{\tilde{g}_{i1b} - g_{i1}^{\min}}, & if \quad g_{i1}^{\min} \le y \le \tilde{g}_{i1b} \\ 1, & if \quad \tilde{g}_{i1b} \le y \le \tilde{g}_{i1c} \\ \dfrac{y - \tilde{g}_{i1c}}{g_{i1}^{\max} - \tilde{g}_{i1c}}, & if \quad \tilde{g}_{i1c} \le y \le g_{i1}^{\max} \end{cases} \tag{22}$$

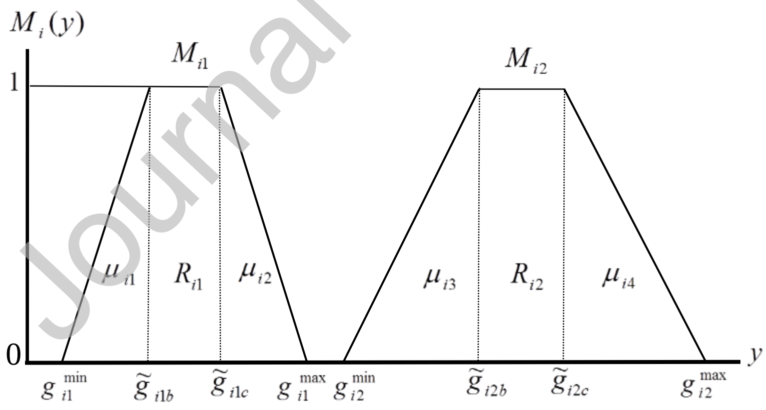

Figure 7. Two trapezoidal utility functions

An example of two TUFs in Figure 7 can be represented by $M_{i1} = \mu_{i1} \cup R_{i1} \cup \mu_{i2}$ and $M_{i2} = \mu_{i3} \cup R_{i2} \cup \mu_{i4}$. Therefore, Figure 7 can be formulated as follows.

(P6)

Min $\sum_{i=1}^{n} (d_i^+ + d_i^- + \sum_{j=1}^{6} w_{ij} e_{ij}^-)$

s.t.

$$f_i(y) - d_i^+ + d_i^- = y_{i1}z_{i1}z_{i2}z_{i3} + y_{i2}z_{i1}z_{i2}(1 - z_{i3}) + y_{i3}z_{i1}(1 - z_{i2})z_{i3} + y_{i4}z_{i1}(1 - z_{i2})(1 - z_{i3}) + y_{i5}(1 - z_{i1})z_{i2}z_{i3} + y_{i6}(1 - z_{i1})z_{i2}(1 - z_{i3}), \quad i = 1,...,n \tag{23}$$

$$\mu_{i1} = 1 - \frac{\tilde{g}_{i1b} - y_{i1}}{\tilde{g}_{i1b} - g_{i1}^{\min}}, \quad \mu_{i2} = 1 - \frac{y_{i2} - \tilde{g}_{i1c}}{g_{i1}^{\max} - \tilde{g}_{i1c}}, \quad i = 1,...,n \tag{24}$$

$$\mu_{i1} + e_{i1}^- = 1, \quad \mu_{i2} + e_{i2}^- = 1, \quad i = 1,...,n \tag{25}$$

$$R_{i1} = z_{i1}(1 - z_{i2})z_{i3}, \quad R_{i1} + e_{i3}^- = 1, \quad i = 1,...,n \tag{26}$$

$$\mu_{i3} = 1 - \frac{\tilde{g}_{i2b} - y_{i4}}{\tilde{g}_{i2b} - g_{i2}^{\min}}, \quad \mu_{i4} = 1 - \frac{y_{i5} - \tilde{g}_{i2c}}{g_{i2}^{\max} - \tilde{g}_{i2c}}, \quad i = 1,...,n \tag{27}$$

$$\mu_{i3} + e_{i4}^- = 1, \quad \mu_{i4} + e_{i5}^- = 1, \quad i = 1,...,n \tag{28}$$

$$R_{i2} = (1 - z_{i1})z_{i2}(1 - z_{i3}), \quad R_{i2} + e_{i6}^- = 1, \quad i = 1,...,n \tag{29}$$

$$g_{i1}^{\min} \le f_i(y) \le g_{i2}^{\max}, \quad i = 1,...,n \tag{30}$$

where d_i^+ (d_i^-) is the positive (negative) deviational variable for $|f_i(y) - \tilde{g}_i|$; $\tilde{g}_i = y_{i1}z_{i1}z_{i2}z_{i3} + ... + y_{i6}(1 - z_{i1})z_{i2}(1 - z_{i3})$; the weight w_{ij} is the weight attached to e_{ij}^-; the ith objective function is denoted as $f_i(y)$; the binary variable is denoted as z_{iu}; the μ_{ix} is the subset of the TUF; the negative deviational variable e_i^- is used to force μ_{ix} and R_{iv} approaching 1 (i.e., max value of TUF); the $g_{iv}^{\max}(g_{iv}^{\min})$ is an upper (lower) bound on y_{ix}; $y_{i1} \in [g_{i1}^{min}, \tilde{g}_{i1b}]$, $R_{i1} \in [\tilde{g}_{i1b}, \tilde{g}_{i1c}]$, $y_{i2} \in [\tilde{g}_{i1c}, g_{i1}^{max}]$, $y_{i3} \in [g_{i2}^{\min}, \tilde{g}_{i2b}]$, $R_{i2} \in [\tilde{g}_{i2b}, \tilde{g}_{i2c}]$ and $y_{i4} \in [\tilde{g}_{i2c}, g_{i2}^{max}]$ are additional variables; indices are $i = 1,...,n$, $u = 1,2,3$, $x = 1,..,6$, and $v = 1,2$.

Proposition 2. P6 and Figure 7 are equivalent in the sense that they have the same optimal solutions.

Proof. To prove P6 and Figure 7 are equivalent, all values of y fall in all intervals $[g_{i1}^{min}, \tilde{g}_{i1b}]$, $[\tilde{g}_{i1b}, \tilde{g}_{i1c}]$, $[\tilde{g}_{i1c}, g_{i1}^{max}]$, $[g_{i2}^{\min}, \tilde{g}_{i2b}]$, $[\tilde{g}_{i2b}, \tilde{g}_{i2c}]$ and $[\tilde{g}_{i2c}, g_{i2}^{max}]$ in Figure 7 should be checked below.

Case (i). $y \in [g_{i1}^{min}, \tilde{g}_{i1b}]$: $\mu_{i1} = 1 - \dfrac{\tilde{g}_{i1b} - y_{i1}}{\tilde{g}_{i1b} - g_{i1}^{\min}}$ is activated (from Eq.(24)). This forces $z_{i1} = z_{i2} = z_{i3} = 1$ (from Eq.(23)) and μ_{i1} approaching 1 (from Eq.(25)).

Case (ii). $y \in [\tilde{g}_{i1b}, \tilde{g}_{i1c}]$: $R_{i1} = 1$ is activated (from Eq.(26)). This forces $z_{i1} = 1, \quad z_{i2} = 0, \quad z_{i3} = 1$ (from Eq.(23)).

Case (iii). $y \in [\tilde{g}_{i1c}, g_{i1}^{\max}]$: $\mu_{i2} = 1 - \frac{y_{i2} - \tilde{g}_{i1c}}{g_{i1}^{\max} - \tilde{g}_{i1c}}$ is activated (from Eq.(24)). This forces $z_{i1} = z_{i2} = 1$, $z_{i3} = 0$ (from Eq.(23)) and μ_{i2} approaching 1 (from Eq.(25)).

Case (iv). $y \in [g_{i2}^{min}, \tilde{g}_{i1b}]$: $\mu_{i3} = 1 - \frac{\tilde{g}_{i2b} - y_{i4}}{\tilde{g}_{i2b} - g_{i2}^{\min}}$ is activated (from Eq.(27)). This forces $z_{i1} = 1$, $z_{i2} = 0$, $z_{i3} = 1$ (from Eq.(23)) and μ_{i3} approaching 1 (from Eq.(28)).

Case (v). $y \in [\tilde{g}_{i2b}, \tilde{g}_{i2c}]$: $R_{i2} = 1$ is activated (from Eq.(29)). This forces $z_{i1} = 0$, $z_{i2} = 1$, $z_{i3} = 0$ (from Eq.(23)).

Case (vi). $y \in [\tilde{g}_{i2c}, g_{i2}^{\max}]$: $\mu_{i4} = 1 - \frac{y_{i5} - \tilde{g}_{i2c}}{g_{i2}^{\max} - \tilde{g}_{i2c}}$ is activated (from Eq.(27)). This forces $z_{i1} = 0$, $z_{i2} = 1$, $z_{i3} = 1$ (from Eq.(23)) and μ_{i4} approaching 1 (from Eq.(28)). Based on the above checked, $\mu_{i1} \cup R_{i1} \cup \mu_{i2}$ and $\mu_{i3} \cup R_{i2} \cup \mu_{i4}$ in P6 are then equivalent to $M_i(y)$ in Figure 7. The proof of Proposition 2 is completed.

4. A topology design problem of a remote patient monitoring system

A feasible way of solving the RPMS problem concerning qualitative and quantitative issues is demonstrated in the proposed method. A presentation structure is shown in Figure 8. As seen in Figure 8, two goals are designed for the presentation: (1) to demonstrate the proposed method is a good way of solving the topology design problem for an RPMS with feasible, compact and integrated characteristics, and (2) to provide useful information to the government for future RPMS planning references.

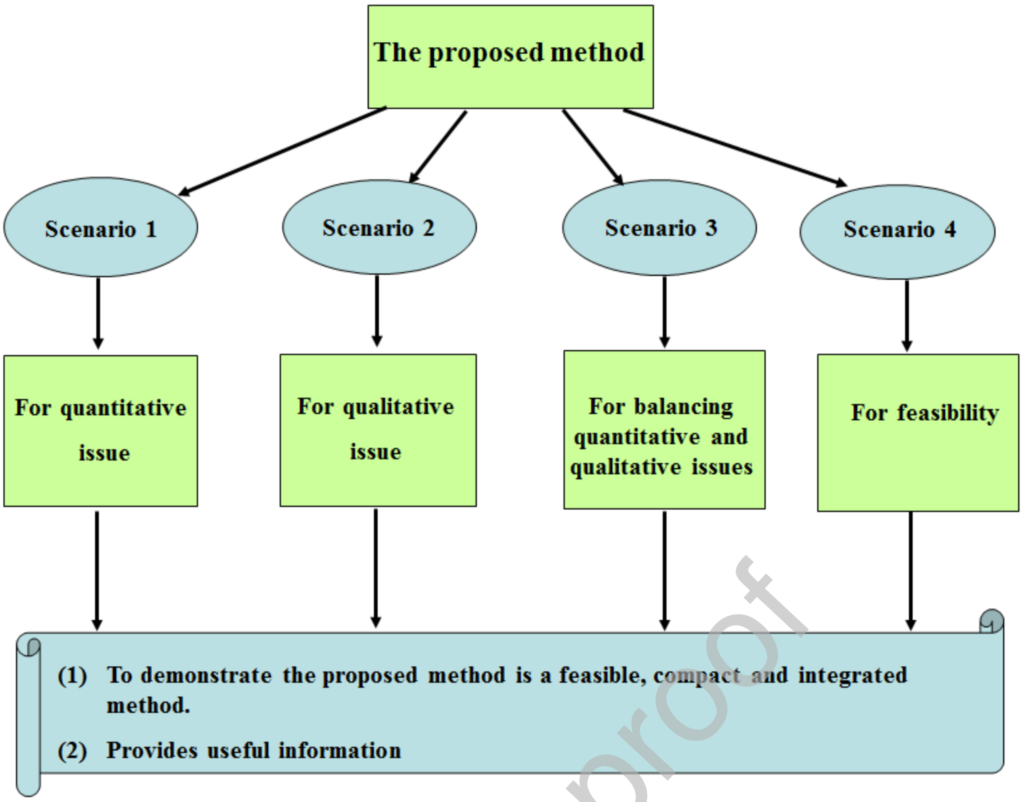

Figure 8. The presentation structure of the topology design problem of RPMS

A company would like to promote its RPMS to senior citizens in Taiwan. The RPMS platform is depicted in Figure 9. An RPMS can help patients confined to their home, who are perhaps aged or disabled, to obtain timely healthcare anytime they need. Accompanied by a fast-growing aging population, the booming demand for RPM makes it an essential part of the future healthcare system. There are seven components, including foreign care, sensor, patient, platform administrator, physician, and agent server, as seen in Figure 9. The function of these components is as follows: (1) Nursing staff: Nursing staff frequently contact physicians to discuss regular follow-up, medical referral, intervention, and to understand disease changes. (2) Agent Server: The agent server is used to ensure the RPMS operates automatically, such as monitoring elderly people with a diaper sensor. (3) Physician: Physicians and nurses often consult with one another in relevant medical treatments. (4) Platform Administrator: A Platform Administrator controls the platform, including network monitoring, software deployment, abnormal message monitoring, data transmission, and hardware sensor management. (5) Patient: According to patients' physiological data received using sensing devices, appropriate medical treatment can be given. (6) Sensor: Different kinds of instruments collecting the patients' physiological data. (7) Foreign care: Responsible for training patients to wear the sensing devices in the correct way to ensure the system can receive the emergency alert signal.

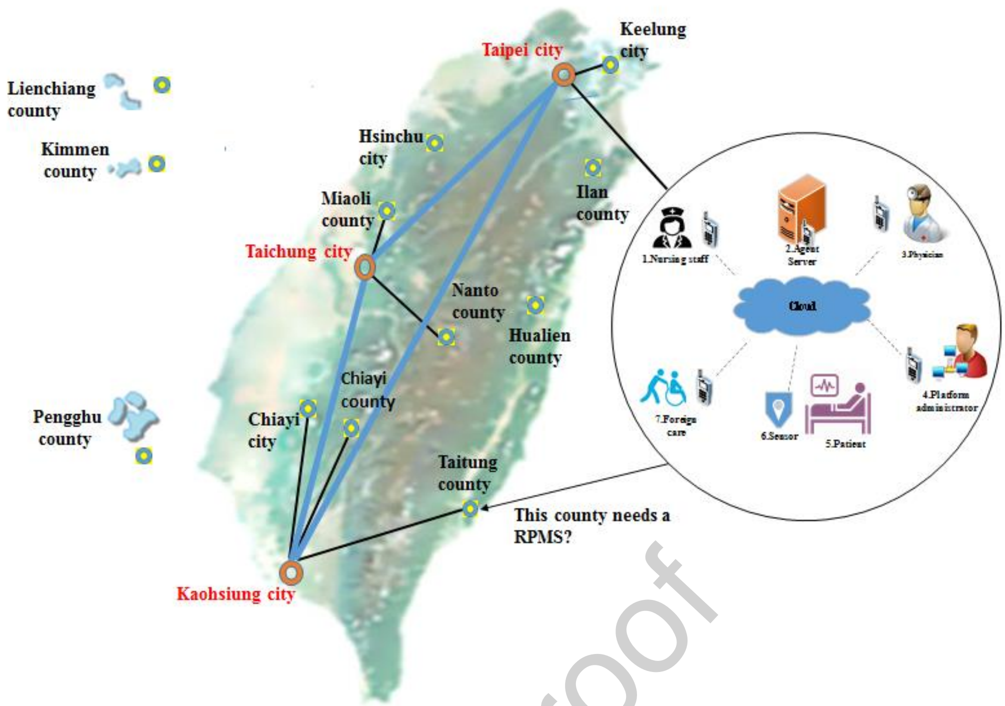

Figure 9. The remote patient monitoring system

The RPMS contains cloud computing home monitoring, servers (**CCS**), healthcare devices, distributed databases, audio/video tools, and a graphic user interface. As seen in Figure 9, Taipei is a political and economic center in Taiwan, so the establishment of an RPMS is required, but does Taitung county or other countries need the establishment an RPMS? Under budget limitations, which cities are the priorities? This is a typical 0-1 MIP problem. To implement the RPMS for serving senior citizens, several goals and criteria should be considered simultaneously. This is a typical MODM problem. Therefore, RPMS is a mixed-integer MODM problem. A major element of RPME is CCS. To consider the RPMS risk, an S-shaped PF is given in Figure 10, which guides the CCS setup cost. The DM assigns a penalty of 0.75 to a deviation surpassing 16 million dollars because when the total setup cost is over 16 million dollars, the investor may prefer to join the RPMS, a penalty of 2 to a deviation falling between 10 and 16 million dollars, a penalty of 1 to a deviation falling between 1 and 10 million dollars, and zero penalty to a deviation falling under 1 million dollars. The risk of the RPMS will be decreased when the new investor joins, so the DM assigns a penalty of 0.75 to a deviation surpassing 16 million dollars. For achieving reliability in the RPMS, the three major cities of Taiwan, i.e., Kaohsiung, Taichung, Taipei, must install CCS connecting to one another as a basic three-hub-node ring trunking network. The achievement model can be expressed as follows.

Notations:

y is the additional variable.

p_i, e_1^+, e_1^-, e_2^+, e_2^- are deviational variables.

Decision variables:

$x_i \in \{0,1\}$ is ith city/county in Taiwan

λ is the value of the penalty function

λ_1 is the value of the utility function

To demonstrate the usefulness of the proposed methods, scenario 1 is given.

Scenario 1.

G1: the RPMS can cover senior citizens in Taiwan, and the more citizens covered, the better. (major goal)

The DM would like to know senior citizens' satisfaction with the use of RPMS for improving its functions in the future. The satisfaction function, $\mu_i(y_i)$, is represented by the RPMS coverage rate, which can be depicted as shown in Figure 11. That is, the role of $\mu_i(y_i)$ is IUF. The parameters of the CCS, including all cities in Taiwan, capital requirements in each city, and the number of senior citizens in each city are given in Table 2. Subject to the resource limitations of the company, three constraints should be considered as follows.

(i) The total investment budget must not exceed 30 million dollars.

(ii) The RPMS must serve at least 500 thousand senior citizens.

(iii) At least 8 CCSs should be implemented.

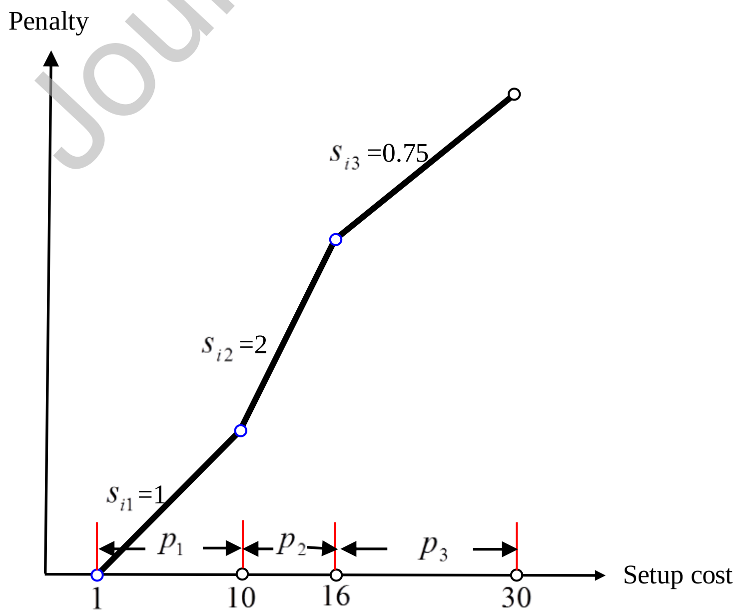

Figure 10. S-shaped penalty function for building cost

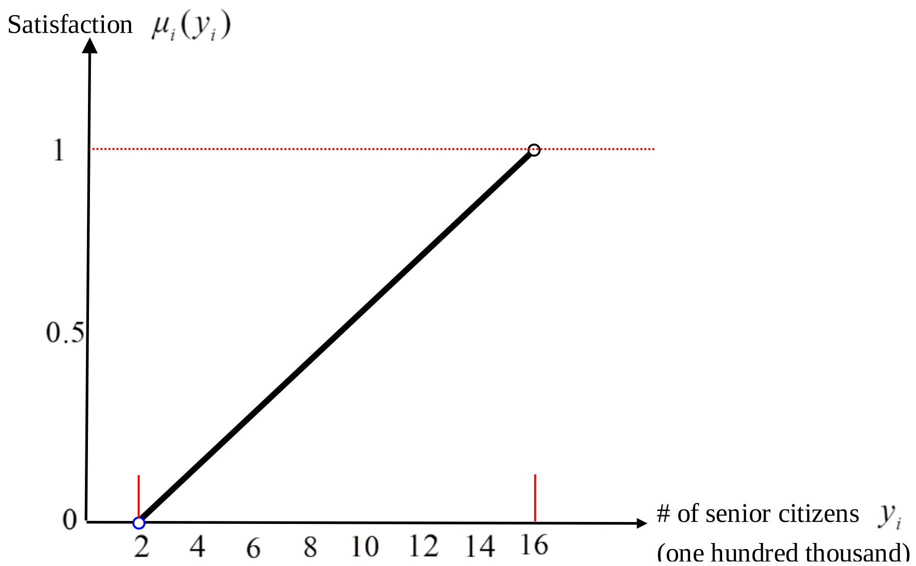

Figure 11. Satisfaction for number of senior citizens coverage

Table 2. CCS parameters

Decision	Decision variable	Capital requirements	The number of senior citizens
CCS in New Taipei city	x_1	2 million	399 thousand
CCS in Taichung city	x_2	2 million	279 thousand
CCS in Kaohsiung city	x_3	2 million	277 thousand
CCS in Taipei city	x_4	2 million	266 thousand
CCS in Taoyuan city	x_5	2 million	221 thousand
CCS in Tainan city	x_6	1.5 million	188 thousand
CCS in Changwa county	x_7	1.5 million	127 thousand
CCS in Pingtung county	x_8	1.3 million	82 thousand
CCS in Yunlin county	x_9	1.2 million	68 thousand
CCS in Hsinchu county	x_{10}	1.2 million	55 thousand
CCS in Miaoli county	x_{11}	1.2 million	54 thousand
CCS in Chiayi county	x_{12}	1.1 million	50 thousand
CCS in Nanto county	x_{13}	1 million	49 thousand
CCS in Ilan county	x_{14}	1 million	45 thousand
CCS in Hsinchu city	x_{15}	1 million	44 thousand
CCS in Keelung city	x_{16}	0.9 million	37 thousand
CCS in Hualien county	x_{17}	0.8 million	32 thousand
CCS in Chiayi city	x_{18}	0.7 million	26 thousand
CCS in Taitung county	x_{19}	0.7 million	21 thousand
CCS in Kinmen county	x_{20}	0.5 million	13 thousand

CCS in Penghu county	x_{21}	0.4 million	10 thousand
CCS in Lienchiang county	x_{22}	0.2 million	1 thousand

Based on MCGP, FUF, and APUF, the topology design problem for the RPMS can be formulated as follows.

(P7)

Min $\lambda + \sum_{i=1}^{3} w_i p_i + \sum_{i=1}^{2} \alpha_i (e_i^+ + e_i^-)$

s.t.

$$399x_1 + 279x_2 + ... + x_{22} - e_1^+ + e_1^- = y \tag{31}$$

$$y - e_2^+ + e_2^- = upperbound \tag{32}$$

$$\left.\begin{array}{l}\lambda = p_1 + 2p_2 + 0.75p_3, \\ 2x_1 + 2x_2 + ... + 0.2x_{22} - p_1 - p_2 - p_3 = 1,\end{array}\right\} \quad \text{(S-shaped penalized)} \tag{33}$$

$$2x_1 + 2x_2 + ... + 0.2x_{22} \le 30, \text{ (Budget)} \tag{34}$$

$$x_1 + x_3 + x_4 = 3, \text{ (three-hub-node ring trunking network)} \tag{35}$$

$$x_1 + x_2 + ... + x_{22} \ge 8, \text{ (At least 8 CCS)} \tag{36}$$

$$399x_1 + 279x_2 + ... + x_{22} \ge 500, \text{ (At least 500 thousand senior citizens coverage)} \tag{37}$$

$$0 \le p_1 \le 10,\ 0 \le p_2 \le 6,\ 0 \le p_3 \le 14,\ 0 \le y \le upperbound \tag{38}$$

$$\lambda_1 = (399x_1 + 279x_2 + ... + x_{22} - 200)/1400, \text{ (Satisfaction)} \tag{39}$$

where x_i ($i = 1,2,...,22$) is the binary variable indicating whether the ith city is selected or not; "upperbound" is a large value; the α_i is the weight attached to deviations e_i^+ and e_i^-; Eqs.(31)-(32) are used to force universal senior citizen coverage, the more, the better; Eq.(33) represents the S-shaped penalty; Eqs.(34)-(37) are constraints for budget, ring trunking, number of CCS, and coverage; Eq.(39) is the senior citizens' satisfaction with the use of the RPMS.

Appropriate weights are given as (w_1, w_2, w_3)= (1,1,3) based on Proposition 1. LINGO (Schrage 2004) is used to solve P7 to obtain the solution as (x_1, x_2,…, x_{22})=(0,1,1,1,0,0,0,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1), with the penalized value of $\lambda = 23.4$ where α_i=5 is given. The CCS topology is shown in Figure 12. The CCS are established in 18 cities such as Taichung city, Kaohsiung city,…, and Lienchiang county. The total setup cost is 19.2 million dollars. The RPMS serves 1409 thousand senior citizens. The senior citizen satisfaction is 86.35%. The investor will choose to join the new RPMS when the satisfaction rating is high, and the total setup cost is over 16 million dollars.

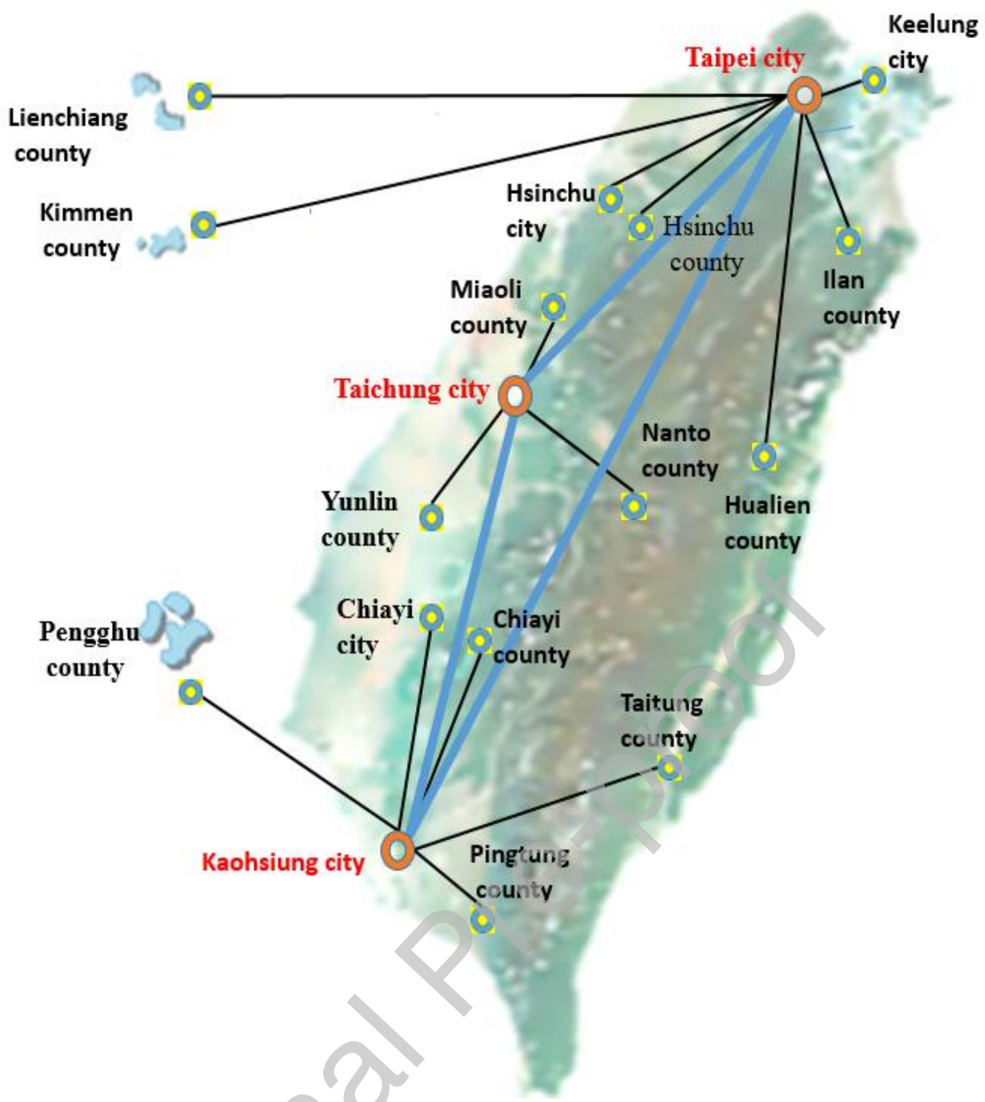

Figure 12. The topology of the CCS

To demonstrate accurate satisfaction using APUF, scenario 2 is given.

Scenario 2.

In general, senior citizen satisfaction will be diminished because Scenario 1 assumes the senior citizen satisfaction is a linear function. Therefore, a more accurate satisfaction function is used to represent it as shown in Figure 13.

Based on P4, an accurate satisfaction function can be formulated as follows.

(P8)

Min $\mu_i(y_i) + \sum_{i=0}^{2} w_i v_i$

s.t.

$$\mu_i(y_i) = \mu_i(a_0) + \sum_{i=0}^{2} s_i v_i \ , \tag{40}$$

$$y_i - \sum_{i=0}^{2} v_i = a_0, \tag{41}$$

$$a_i \le v_i \le a_{i+1}, i = 0,1,2, \tag{42}$$

Based on Proposition 1, the following the weight sequence is given for v_0, v_1, and v_2 as

$$(1+w_0) < (0.7+w_1) < (0.3+w_2) \tag{43}$$

Based on Eq.(43), we can choose the weight values as (w_0, w_1, w_2)=(1, 1.5, 2.5). P8 can be transformed into the following program.

(P9)

Min $\mu_i(y_i) + v_0 + 1.5v_1 + 2.5v_2$

s.t.

$$\mu_i(y_i) = (v_0 + 0.7v_1 + 0.3v_2)/1400, \tag{44}$$

$$y_i - v_0 - v_1 - v_2 = 200, \tag{45}$$

$$0 \le v_0 \le 400,\ 0 \le v_1 \le 600,\ 0 \le v_2 \le 400, \tag{46}$$

Combine P7 and P9 into an achievement program as follows.

(P10)

Min $\lambda + \sum_{i=1}^{3} w_i p_i + \sum_{i=1}^{2} \alpha_i (e_i^+ + e_i^-) + \mu_i(y_i) + v_0 + 1.5v_1 + 2.5v_2$

s.t. Eqs.(31)-(38) and Eqs.(40)-(42),

LINGO (Schrage 2004) is used to solve P10 to obtain the same solution as P7 where α_i =300 is given, except for the solution of senior citizens' satisfaction being 63.05%. In addition, the value of α_i is increased to 500. P10 is again solved to obtain the solutions as ($x_1, x_2, \ldots, x_{22}$)= (0,1,1,1,0,0,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1), with the penalized value of $\lambda = 24.5$. The new system serves 1536 thousand senior citizens. The total setup cost is 20.7 million dollars. The senior citizens satisfaction is 65.77%.

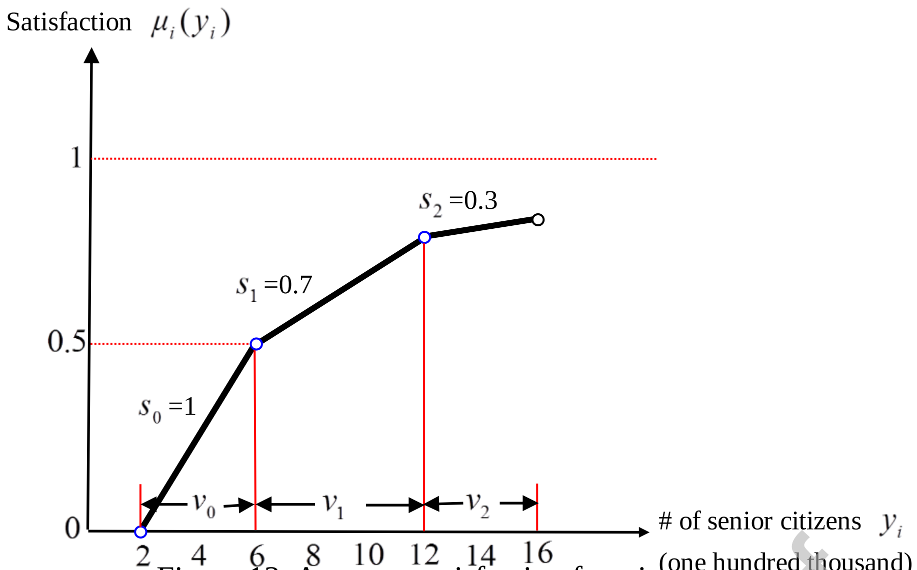

Figure 13. Accurate satisfaction function

To demonstrate the feasibility of the proposed methods, scenario 3 is given.

Scenario 3.

One more goal is added to Scenario 1 for balancing citizens' satisfaction (i.e., qualitative issue) and investment cost (i.e., quantitative issue) as follows.

G2: under the condition the senior citizens' satisfaction is not less than 50%, the lower the investment cost, the better.

Based on MCGP, the following model is formulated for G2.

(P11)

$$\text{Min} \quad \sum_{i=1}^{2} \beta_i (z_i^+ + z_i^-)$$

s.t.

$$2x_1 + 2x_2 + ... + 0.2x_{22} - z_1^+ + z_1^- = y_1, \tag{47}$$

$$y_1 - z_2^+ + z_2^- = lowerbound, \tag{48}$$

Combine P10, P11, and P7 into an achievement program as follows.

(P12)

$$\text{Min} \quad \lambda + \sum_{i=1}^{3} w_i p_i + \sum_{i=1}^{2} \alpha_i (e_i^+ + e_i^-) + \mu_i(y_i) + v_0 + 1.5v_1 + 2.5v_2 + \sum_{i=1}^{2} \beta_i (z_i^+ + z_i^-)$$

s.t.

$$\mu_i(y_i) \ge 0.5, \tag{49}$$

Eqs.(31)-(38), Eqs.(44)-(46), and Eqs.(47)-(48),

LINGO (Schrage 2004) is used to solve P12 again to obtain the solutions as ($x_1, x_2, \ldots,$

x_{22})= (0,1,1,1,0,0,0,0,0,0,0,1,1,0,0,1,0,1,1,1,1,1). The topology of the CCS is shown in Figure 14. The total setup cost is 11.5 million dollars. The RPMS serves 1029 thousand senior citizens. The senior citizens satisfaction is 50%.

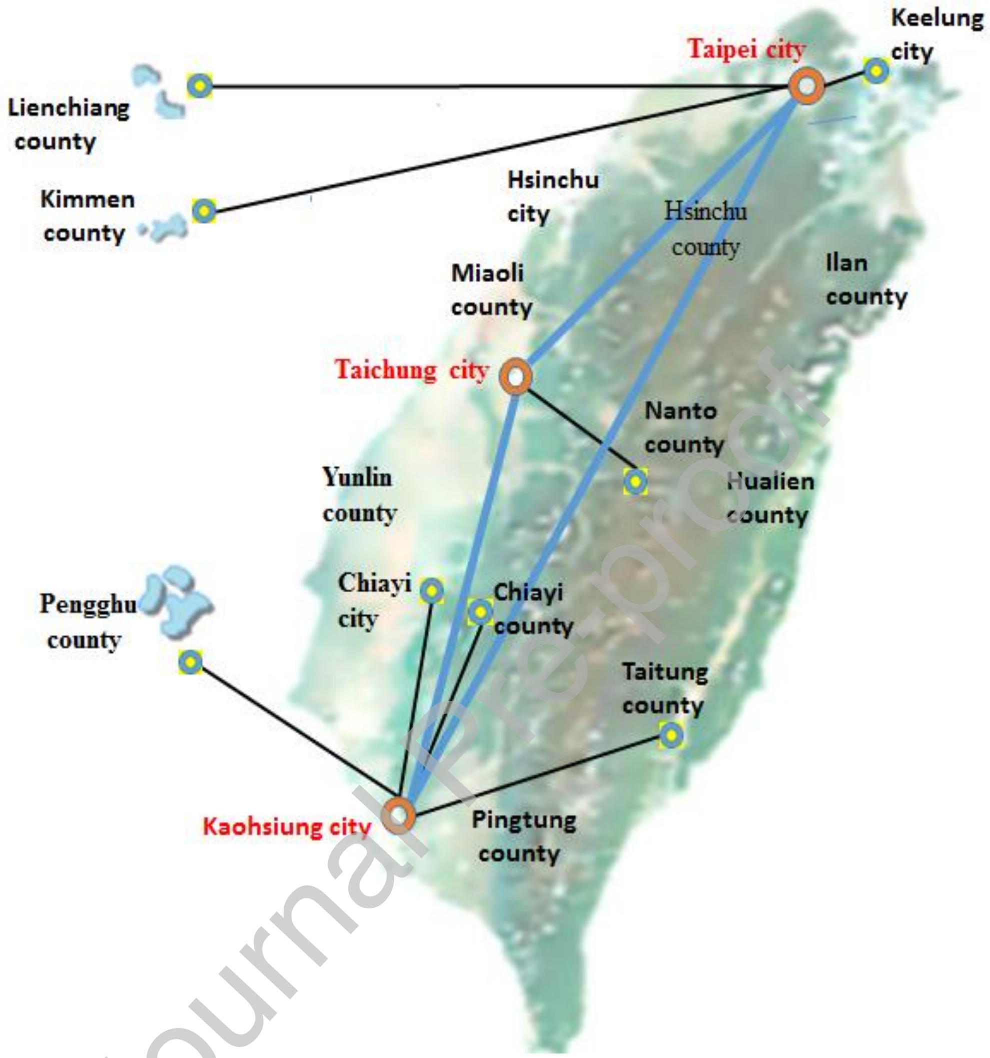

Figure 14. The topology of the CCS

To demonstrate the feasibility and reality of the proposed methods using TUF, scenario 4 is given.

Scenario 4.

The DM would like to balance the setup cost and the number of senior citizens covered. Therefore, two TUFs are given in Figures 15 and 16. Four constraints are also given as follows.

1. The RPMS must serve at least 1000 thousand senior citizens.
2. The total investment budget must not exceed 20 million dollars.
3. At least eight CCSs should be established.

4. Three major cities in Taiwan, i.e., Kaohsiung, Taichung, and Taipei, must install CCSs.

Based on MCGP, FUP, and APUF, the topology design problem of the RPMS can be formulated as follows.

(P13)

Min $\sum_{i=1}^{2}(d_i^+ + d_i^-) + \sum_{i=1}^{6} w_i e_i^-$

s.t.

$$2x_1 + 2x_2 + ... + 0.2x_{22} \le 20, \text{ (Set up cost } \le \text{ Budget)} \quad (50)$$

$$x_1 + x_3 + x_4 = 3, \text{ (three-hub-node ring trunking network)} \quad (51)$$

$$x_1 + x_2 + ... + x_{22} \ge 8, \text{ (At least 8 CCS)} \quad (52)$$

$$399x_1 + ... + x_{22} \ge 1000, \text{ (At least 1000 thousand senior citizens coverage)} \quad (53)$$

$$f_1(\mathbf{x}) + d_1^+ - d_1^- = y_1 z_1 z_2 + y_2 z_1(1 - z_2) + y_3(1 - z_1) z_2, \text{ (for } M_1\text{)} \quad (54)$$

$$\mu_1 = 1 - \frac{\tilde{g}_{1b} - y_1}{\tilde{g}_{1b} - g_1^{min}}, \quad \mu_2 = 1 - \frac{y_2 - \tilde{g}_{1c}}{g_1^{max} - \tilde{g}_{1c}}, \quad (55)$$

$$\mu_1 + e_1^- = 1, \quad \mu_2 + e_2^- = 1, \quad (56)$$

$$R_1 = (1 - z_1) z_2, \quad R_1 + e_3^- = 1, \quad (57)$$

$$f_2(\mathbf{x}) + d_2^+ - d_2^- = y_4 z_3 z_4 + y_5 z_3(1 - z_4) + y_6(1 - z_3) z_4, \text{ (for } M_2\text{)} \quad (58)$$

$$\mu_3 = 1 - \frac{\tilde{g}_{2b} - y_4}{\tilde{g}_{2b} - g_2^{min}}, \quad \mu_4 = 1 - \frac{y_5 - \tilde{g}_{2c}}{g_2^{max} - \tilde{g}_{2c}}, \quad (59)$$

$$\mu_3 + e_4^- = 1, \quad \mu_4 + e_5^- = 1, \quad (60)$$

$$R_2 = (1 - z_3) z_4, R_2 + e_6^- = 1, \quad (61)$$

$$g_1^{\min} \le y_1 \le \tilde{g}_{1b}, \quad \tilde{g}_{1c} \le y_2 \le g_1^{\max}, \quad g_2^{min} \le y_4 \le \tilde{g}_{2b}, \quad \tilde{g}_{2c} \le y_5 \le g_2^{max}, \quad (62)$$

$$g_1^{\min} \le f_1(\mathbf{x}) \le g_1^{\max}, \quad g_2^{min} \le f_2(\mathbf{x}) \le g_2^{max}, \quad (63)$$

where x_i ($i = 1,2,...,22$) is defined as in P7; Eqs.(50)-(53) are the constraints for the budget, ring trunking, number of CCS, and coverage; the $f_1(\mathbf{x})$ is the setup cost; the $f_2(\mathbf{x})$ is the number of senior citizens covered; the deviational variable e_i^- is used to force TUF to approach the highest value of "1"; Eqs.(54)-(61) represent two of the TUFs as shown in Figures 15 and 16; Eq.(62) represents the upper (lower) bound of μ_1, μ_2, μ_3, and μ_4. Eq.(63) represents the upper (lower) bound of the setup cost and the number of senior citizens covered.

For simplicity, we assume $w_i = 1, \ \forall i$. LINGO (Schrage 2004) is used to solve P13 to obtain solutions as ($x_1, x_2, \ldots, x_{22}$)= (0,1,1,1,0,0,1,1,1,1,0,1,0,0,0,0,0,1,0,1,1,1). The CCS topology is shown in Figure 17. The total setup cost is 14.1 million dollars. The RPMS serves 1254 thousand senior citizens. Two TUF satisfactions are fully achieved (e.g., M_1 =1 and M_2 =1).

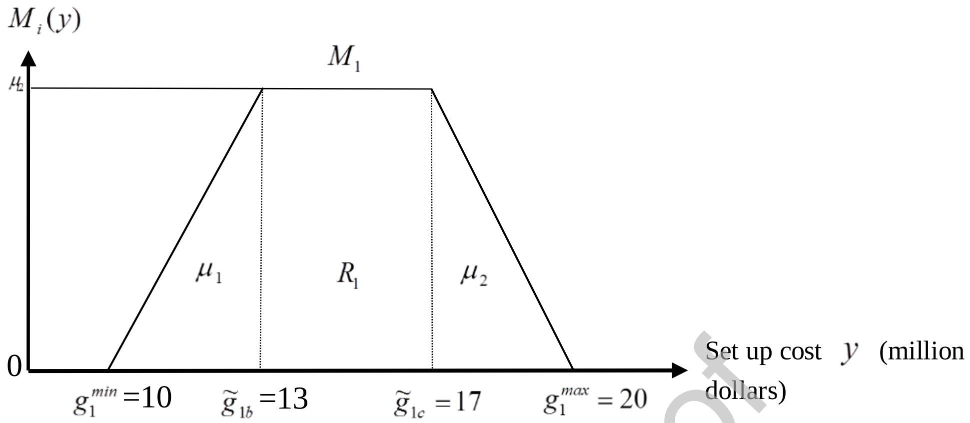

Figure 15. Trapezoidal utility function for setup cost

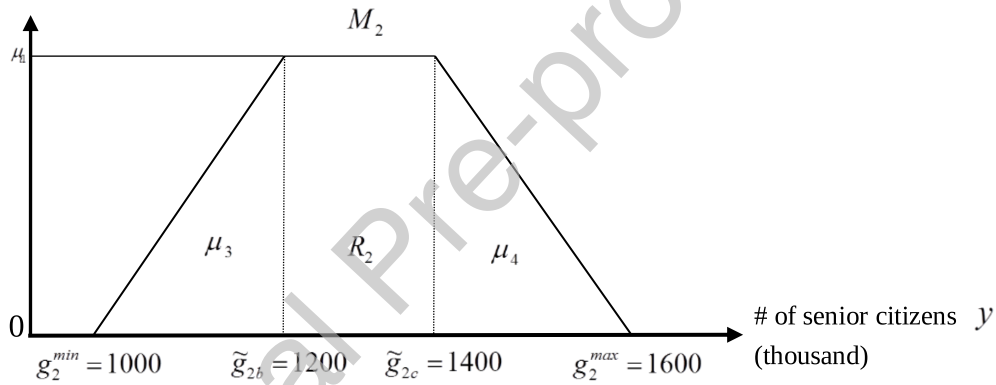

Figure 16. Trapezoidal utility function for senior citizens coverage

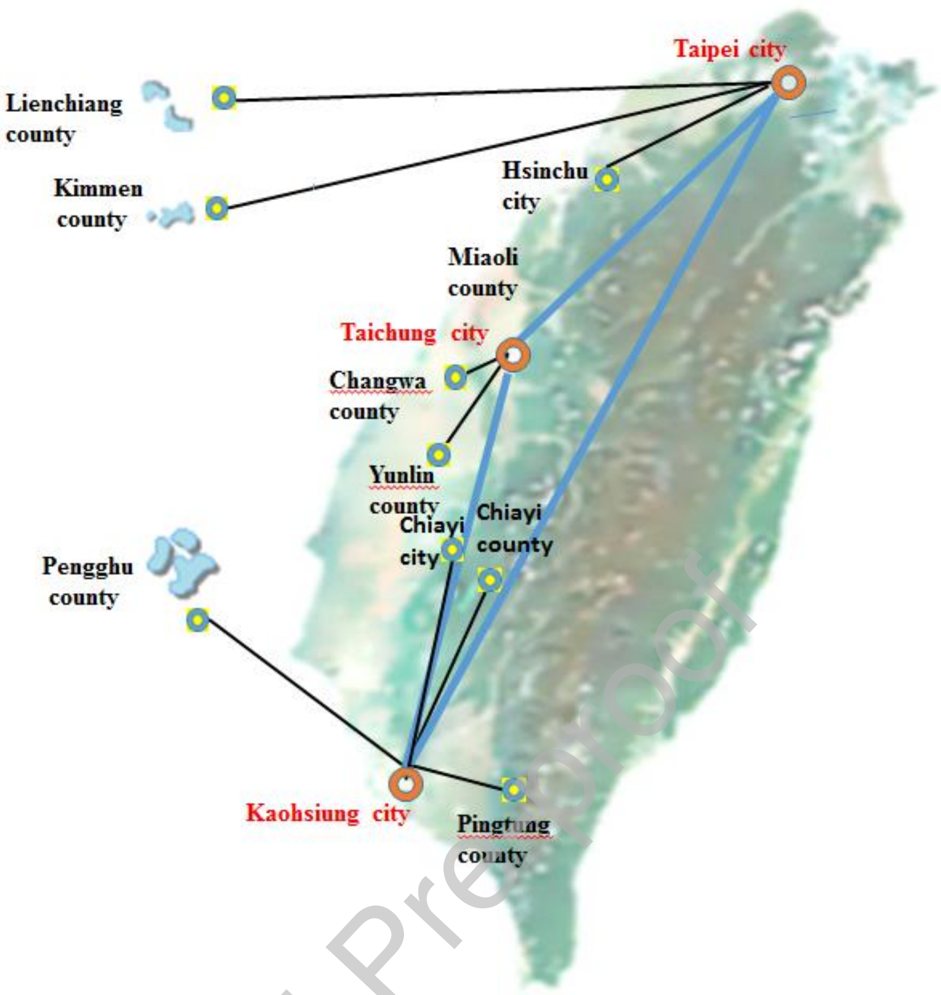

Figure 17. The topology of the CCS

All the goals and results of scenarios are listed in Table 3. This information is of great value to the development of RPMS.

Table 3 Remote patient monitoring system scenarios

Scenarios	Goals	Results
1	Cover senior citizens, the more, the better	● 18 cities have been selected ● Setup cost equals 19.2 million dollars ● 1409 thousand senior citizens have been covered ● Satisfaction rate being 86.35%
2	Measures the satisfaction of senior citizens with accurate satisfaction function	● 19 cities have been selected ● Setup cost equals 20.7 million dollars. ● 1536 thousand senior citizens have been covered ● Satisfaction rate being 65.77%
3	Minimize the investment cost under senior citizens satisfaction > 50%	● 11 cities have been selected ● Setup cost equals 11.5 million dollars ● 1029 thousand senior citizens have

		been covered ● Satisfaction rate being 50%
4	Balance the setup cost and the number of senior citizens covered with the trapezoidal utility function	● 11 cities have been selected ● Setup cost equals 14.1 million dollars ● 1254 thousand senior citizens have been covered ● Satisfaction rate being 100% in TUT

4.1 Discussion

As seen in Table 3, DMs can add different utility functions, constraints, and models to the proposed method, demonstrating its feasibility and compatibility for MCDM/MODM problems involving quantitative and qualitative issues. Given the evident falling birth rate and aging society in Japan, Italy, Germany, and Taiwan, more and more counties are having a similar problem; countries' productivity is declining, and more elderly people need to be taken care of, but medical expansion simply cannot catch up. The RPMS can take care of more elderly people than the traditional method of care in the hospital and saves on medical resources to reduce the burden on the government's finances. The topology design problem of the RPMS is a major element of success in implementing RPMS for aging societies. This paper provides a novel integrated method to contribute to the development of the RPMS considering qualitative, quantitative, and balance issues, reducing the gap between management science and the healthcare sector to implement the RPMS. In addition, the authors hope the proposed method can increase the adoption of MCDM methods that are used by companies to solve practical problems.

4.2 Sensitivity analysis

In establishing an RPMS, the most important issue for DMs is to understand the relationship between setup cost and senior citizens' satisfaction, so we conducted the following sensitivity analysis, and the results of the analysis can provide a reference for DMs. To understand how a change in the weight values of the setup cost from 1000% to 1600% in the objective function of P13 will affect the achieved target value (i.e., the rate of satisfaction, setup cost, number of senior citizens covered), sensitivity analysis is performed. LINGO (Schrage, 2004) is used to solve P13 again to obtain the optimal solution shown in Table 4. The value of M1 and M2 is 1, regardless of the weight increase. The setup cost and the number of senior citizens covered change slightly when the weight increases from 1300% to 1500%.

Table 4. Results of sensitivity analysis

weight	Set up cost	Number of senior citizens coverage	M1	M2
1000%	16.7	1259	1	1
1100%	16.7	1259	1	1
1200%	16.7	1259	1	1

1300%	16.7	1259	1	1
1400%	13.2	1232	1	1
1500%	15.8	1277	1	1
1600%	15.8	1277	1	1

Consequently, the management implications of the proposed methods are: (1) the proposed integrated method can easily be used to help the DM find an appropriate topology CCS solution with various weights, α_i (2) the non-linear UF can easily be approximated by the proposed piecewise method where binary variables are no longer needed. This result makes it possible to solve a realistic large-scale problem within a reasonable amount of time, and (3) the APUF and TUF can easily be used to solve MODM problems with quantitative and qualitative issues.

5. Conclusions

FUF, IUF, APUF, and TUF often appear in business and industrial decision-making models. However, any given efficient mathematical method is yet to be able to define these types of problem very well. A novel integrated method is proposed by this paper to solve the S-shaped PF where extra binary variables are no longer needed. This reduces the complexity of the S-shaped PF formulation. It can improve the efficiency in processing the solution for the S-shaped PF and APUF in MCGP. In addition, ESPM, APUF, TUF, and MCGP approaches also proved conducive to addressing a topology design problem for an RPMS with qualitative and quantitative issues. By demonstrating the RPMS in Section 4, several merits were shown: (1) RPMS implementation benefited from the satisfaction of senior citizens, (2) the proposed method provided good guidance for RPMS planning, and (3) constraints and scenarios could easily be added to the proposed model to show the feasibility of RPMS. These improve the usefulness of the MCGP method in dealing with qualitative and quantitative issues in MODM problems. The framework and contributions of the paper are shown in Figure 18. As seen in Figure 18, the TUF, FUF, IUF, and UFU methods are created by this paper to contribute to the field of qualitative methods. The proposed model is an integrated MCGP method containing qualitative and quantitative functions to resolve MCDM, MODM, and the topology design of RPMS problems.

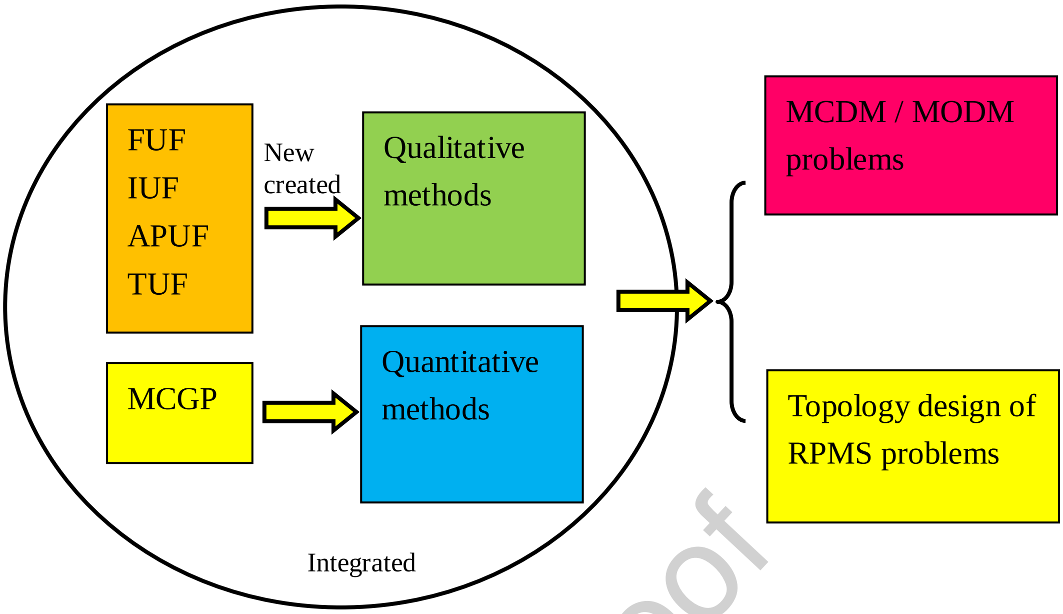

Figure 18. Framework and contributions of the paper

It is clear that the proposed methods are potentially serviceable, the promising results also shed light on future directions, such as all-unit discount cost structure in supply chain management (Chan et al. 2002), inventory models with controllable lead time (Chang et al, 2006), and more real-world oriented adaptation.

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Appendix 1

(AP1) for LLUF

$$\text{Min} \quad \sum_{i=1}^{n} [w_i(d_i^+ + d_i^-) + \beta_i f_i^-]$$

s.t.

$$\lambda_{\text{i}} \leq \frac{g_{i,\max} - y_i}{g_{i,\max} - g_{i,\min}}, \quad i = 1,2,...,n, \tag{A1}$$

$$f_i(\mathbf{x}) - d_i^+ + d_i^- = y_{\text{i}}, i = 1,2,...,n \tag{A2}$$

$$\lambda_{\text{i}} + f_i^- = 1, \quad i = 1,2,...,n, \tag{A3}$$

$$g_{\text{i},\min} \leq y_{\text{i}} \leq g_{i,\max}, \quad i = 1,2,...,n, \tag{A4}$$

$$d_i^+, \quad d_i^-, \quad f_i^-, \quad \lambda_{\text{i}} \geq 0, \quad i = 1,2,...,n, \tag{A5}$$

$$\mathbf{x} \in \mathbf{F}, \quad (\mathbf{F} \text{ is a feasible set}), \tag{A6}$$

where w_{i} and β_{i} are weights attached to deviations d_i^+, d_i^- and f_i^-; the role of weight β_{i} can be seen as a preferential component for the utility value $\mu_i(y_i)$; the utility value of $\mu_i(y_i)$ is denoted as λ_{i}; Eq.(A1) is the LLUF; the highest possible value of the LLUF is 1 , as described in Eq.(A5).

(AP2) for RLUF

$$\text{Min} \quad \sum_{i=1}^{n}[w_i(d_i^+ + d_i^-) + \beta_i f_i^-]$$

s.t.

$$\lambda_{\mathrm{i}} \le \frac{y_i - g_{i,\min}}{g_{i,\max} - g_{i,\min}}, \quad i = 1,2,...,n, \tag{A7}$$

Eqs.(A2)-(A6) (A8)

where all variables are defined as in AP1.

Appendix 2

(AP3)

$$\text{Min} \quad \sum_{i=1}^{n} w_i(d_i^+ + d_i^-)$$

s.t.

$$f_i(\mathbf{x}) - d_i^+ + d_i^- = \sum_{j=1}^{m} g_{ij} S_{ij}(B), \quad i = 1,2,...,n, \tag{A9}$$

$$d_i^+, \ d_i^- \ge 0, \quad i = 1,2,...,n, \tag{A10}$$

$$S_{ij}(B) \in R_i(x), \quad i = 1,2,...,n, \tag{A11}$$

$$\mathbf{x} \in \mathbf{F}, \quad (\mathbf{F} \text{ is a feasible set}), \tag{A12}$$

where $S_{ij}(B)$ represents a function of the binary serial number; the function of resources limitations is denoted as $R_i(x)$; other variables are defined as in GP.

Appendix 3

(AP4) for the case of the more the better of MCDM

$$\text{Min} \quad \sum_{i=1}^{n}[w_i d_i^+ + \alpha_i(e_i^+ + e_i^-)]$$

s.t.

$$f_i(\mathbf{x}) - d_i^+ \ge y_{\mathrm{i}}, \quad i = 1,2,...,n, \tag{A13}$$

$$y_i - e_i^+ + e_i^- = g_{i,max}, \quad i = 1,2,...,n, \tag{A14}$$

$$g_{\mathrm{i,min}} \leq y_i \leq g_{\mathrm{i,max}}, \tag{A15}$$

$$d_i^+,\ e_i^+,\ e_i^- \geq 0 \quad i = 1,2,\ldots,n, \tag{A16}$$

$$\mathbf{x} \in \mathbf{F},\ \ (\mathbf{F}\ \text{ is a feasible set}), \tag{A17}$$

where d_i^+ is the positive deviation attached to the i th goal $|f_i(\mathbf{x}) - y_i|$ in Eq.(A13); the positive and negative deviations e_i^+ and e_i^- are attached to $|y_i - g_{i,max}|$ in Eq.(A14); the weight α_{i} is attached to $|y_i - g_{i,max}|$; other variables are defined as in AP3.

(AP5) for the case of the less the better of MCDM

$$\text{Min}\ \sum_{i=1}^{n}[w_i d_i^+ + \alpha_{\mathrm{i}}(f_i^+ + f_i^-)]$$

s.t.

$$f_i(\mathbf{x}) - d_i^+ \geq y_i,\ \ i = 1,2,\ldots,n, \tag{A18}$$

$$y_i - f_i^+ + f_i^- = g_{i,min},\ \ i = 1,2,\ldots,n, \tag{A19}$$

$$g_{\mathrm{i,min}} \leq \mathrm{y_i} \leq g_{\mathrm{i,max}}, \tag{A20}$$

$$d_i^+,\ \mathrm{f}_i^+,\ \mathrm{f}_i^- \geq 0,\ \ i = 1,2,\ldots,n, \tag{A21}$$

$$\mathbf{x} \in \mathbf{F},\ \ (\mathbf{F}\ \text{ is a feasible set}), \tag{A22}$$

where d_i^+ is the positive deviation attached to the i th goal $|f_i(\mathbf{x}) - y_i|$ in Eq.(A18); the positive (negative) deviational variable f_i^+ (f_i^-) is attached to $|y_i - g_{i,min}|$ in Eq.(A19); the weight α_{i} is attached to $|y_i - g_{i,min}|$; other variables are defined as in AP3.