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Xiao-Xue Zheng , Ching-Ter Chang

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Highlights

- ◁ The technique for qualitative and quantitative issues is considered
- ◁ Topology design of remote patient monitoring system is provided.
- ◁ Piecewise linear function is addressed.

Topology design of a remote patient monitoring system concerning qualitative and quantitative issues

Xiao-Xue Zheng

College of Transportation and Civil Engineering, Fujian Agriculture and Forestry University, Fuzhou, Fujian 350002, China.

School of Economics and Management, Fuzhou University, Fuzhou 350002, China

snowwie@126.com

Ching-Ter Chang*

Chang Gung University

Department of Information Management, Kwei-Shan, Tao-Yuan, Taiwan, ROC

Clinical Trial Center, Chang Gung Memorial Hospital, Linkou, Taoyuan, Taiwan, ROC.

Department of Industrial Engineering and Management, Ming Chi University of Technology, Taiwan, ROC.

*corresponding: chingter@mail.cgu.edu.tw

Abstract

Piecewise linear functions have been used to model and solve non-linear problems in science, social science, and technology fields since the 1950s. Multi-choice goal programming (MCGP) has been widely used to solve multiple objective decision-making (MODM) problems in which each goal mapping with multiple aspiration levels is allowed, expanding the original feasible region to obtain better solutions in the MODM problems. This paper integrates the efficient S-shaped penalty method, arbitrary piecewise linear utility functions, trapezoidal utility functions, and MCGP to solve a topology design problem in a remote patient monitoring system (RPMS) by providing universal senior citizen coverage in which quantitative (e.g., patient satisfaction-related cost) and qualitative (e.g., satisfaction) issues are considered simultaneously. In addition, some novel utility functions such as the force utility function, indicator utility function, and arbitrary utility function are proposed to improve the usefulness of MCGP in the field of management science. These proposed utility functions can be easily used to model qualitative issues in real-world problems. A topology design problem for an RPMS is demonstrated to justify the feasibility, usefulness, and compatibility of the proposed

methods. Further, sensitivity analysis and managerial implications are provided using an RPMS in Taiwan.

Keywords: Remote patient monitoring; Piecewise linear function; Utility Function; Multi-choice Goal programming;

1. Introduction

Given the aging society and falling birth rate, Taiwan has been classified as an aged society since 2018 and will turn into a hyper-aged society by 2025. The increasing healthcare costs for the growing population of elderly, chronic diseases, as well as the demand for new technologies and treatments in addition to the decrease in the number of healthcare professionals compared to the elderly increase make the traditional healthcare service model inadequate. Conventional healthcare model inadequacy can be explained by the following two aspects: (1) large hospitals (e.g., accredited hospitals and medical centers) are always overcrowded, and as a result, people often do not have access to outpatient clinics and must wait a long time for inpatient care; and (2) increasing healthcare costs also place a considerable burden on the government's finances. Nowadays, establishing an efficient remote patient monitoring system (**RPMS**) to take care of the greater number of elderly people as well as reduce healthcare costs is essential in Taiwan. Moreover, it is very important to build an RPMS with a more complete topology network under a limited budget to take care of senior citizens. To implement the RPMS for serving senior citizens, several goals and criteria should be considered simultaneously. These goals and criteria contain both quantitative (e.g., the RPMS setup cost and investment cost) and qualitative (e.g., satisfaction of senior citizens) issues, and the balance between them (e.g., balancing the setup cost and the satisfaction of senior citizens), complicates the problem. This is a typical multiple objective decision making (**MODM**) problem with conflicting goals. For example, under the limited budget, the decision maker (**DM**) would like to take care of more people to improve senior citizens' satisfaction level. The biggest challenge in modeling the RPMS in measuring the senior citizens' satisfaction is that the satisfaction level is a qualitative method. To deal with this problem, two nonlinear techniques, piecewise linearization technique and mixed-integer program (**MIP**) are introduced. Then, novel methods for modeling arbitrary piecewise utility functions (**APUF**) and trapezoidal utility functions (**TUF**) are proposed. The proposed methods not only assist in solving the RPMS problem in practice, but also enrich the field of qualitative method in theory. To express the satisfaction, S-shaped utility

functions (**UF**) and linear UF are introduced. In addition, force UF (**FUF**) and indicate UF (**IUF**) are also derived to enrich the qualitative research. To force the solution of the investment cost of RPMS further away (approach) from (to) the target value to attract investors into the development of RPMS, penalty functions (**PF**) are introduced. Multi-choice goal programming (**MCGP**) has widely been used to solve multiple criteria decision making (**MCDM**) and MODM problems. Thus, the MCGP is used to model and solve the topology design problem of an RPMS with quantitative issues. All the above mentioned points improve the linkage between the related techniques and the development of RPMS. In the field of management science, few works exist that address qualitative and quantitative issues at the same time in one model. Therefore, this paper proposes an integrated method to resolve the RPMS problem considering quantitative, qualitative, and balance issues simultaneously. This reduces the gap between the research and health sector to improve the usefulness of proposed methods in solving the MCDM problems.

In the 1950s, it was shown a nonlinear problem can be modeled using piecewise linear functions (**PLF**) and binary variables within a MIP (Markowitz and Manne 1957, Sherali 2001, Croxton et al. 2003, Li et al. 2009, Toriello and Vielma 2012) and that such problems can be solved by specialized heuristic algorithms (Keha et al. 2006). This type of linearization was instrumental in formulating optimization problems in multiple domains including the fields of science, economics, and social science, and specifically in Very-Large-Scale Integrated circuit design, big data analytics, network analysis, portfolio optimization, and supply chain optimization. Comprehensive monographs on related subjects have been offered (Bazaraa et al. 1993, Floudas 1999). However, such techniques (i.e., linearization by adding binary variables) may lead to significantly increased computational burden when the problem size becomes too large. Hence, a special ordered set of the type 2 (SOS 2) method was proposed by Beale and Tomlin (1970) to reduce the complexity of these MIP problems; the SOS 2 is defined as: a set of variables $\{x_1, \dots, x_n\}$ having at most two adjacent variables that are nonzero to indicate which line segment is activated. Thereafter, many extensions of these models have been studied (Vielma and Nemhauser 2011, Li et al. (2009, 2012), Vielma and Nemhauser (2010, 2011)). Recently, Chang (2018) proposed an efficient S-shaped penalty method (**ESPM**) to model an S-shaped PF enabling the solution of a realistic large-scale problem within a reasonable

amount of time. This paper uses the principle of goal programming (**GP**), plus newly created techniques to deal with PLF, so we first introduce the related technology of GP.

The initial goal programming (**GP**) model was proposed by Charnes et al. (1955) with further development by Tamiz et al. (1998), Ignizio (1985), and Lee (1972). Recently, more advanced variants were proposed for more complex decision problems with multiple conflicting goals such as fuzzy GP (Narasimhan 1980), meta-GP (Rodríguez et al. 2002), extended GP (Romero 2001), and MCGP (Chang 2007). Fuzzy GP is used to resolve MODM problems with imprecisely defined model goals and/or constraints in a decision-making environment. The concept of a meta-goal is proposed by the meta-GP method as a high-level goal going beyond a single goal and giving an overall measure of satisfaction for the DM. An extended GP comprises two meta-goals: normalizing unwanted deviations and minimizing the maximal goal. MCGP is used to solve MODM problems where each goal can be achieved from different aspiration levels (Chang 2007, 2008). Subsequently, MCGP has been widely applied to resolve various practical problems, including coffee shop location selection (Ho et. al., 2013), house selection (Ho et. al., 2015), stochastic transportation problem (Mahapatra et al., 2013), e-learning system evaluation (Lin et. al., 2014), a supply chain order allocation problem (Chang et al., 2014), course planning (Kiri, 2014), supplier selection problems (Jadidi, et al. (2015), renewable energy facilities location selection problem (Chang, 2015), reverse logistics providers selection problem (Govindan et al. 2017), consumer choice behavior (Swait et al., 2018) and others (Shalabh and Sonia, 2017). Multi-objective transportation problems are solved using various methods such as grey parameters, the conic scalarization approach, UF, dual-hesitant fuzzy number, fuzzy MCGP, and multi-choice interval GP (Roy et al. 2017a, 2017b, Roy and Maity 2017, Maity et al. 2016, 2019, Maity and Roy 2016).

The purpose of the study is to integrate ESPM, APUF, TUF, and MCGP to solve a topology design problem of an RPMS considering quantitative, qualitative, and balance issues. The contributions of the paper are (1) the qualitative and quantitative issues can be simultaneously considered in solving the topology design problem for an RPMS, (2) new utility functions such as FUF, IUF, APUF, and TUF are proposed to enhance the usefulness of the UF, and (3) the optimal topology of the RPMS in Taiwan is obtained to provide universal senior citizen coverage by considering multiple criteria, subject to resource limitations. This will help Taiwan's long-term care planning for the hyper-aged society in the future.

The following section briefly reviews previous methods such as piecewise linear methods, MCGP, and MCGP with utility functions. New utility functions (UIF, APUF and TUF) are proposed in Section 3. Section 4 demonstrates a topology design problem of RPMS. Discussion, sensitivity analysis, and the management implications are also provided in Section 4. Conclusions are presented in Section 5.

2. Previous related methods for qualitative and quantitative issues

In the past, several methods have been proposed for MCDM problems involving qualitative or quantitative objectives. These methods should be addressed as follows.

2.1 Piecewise penalty functions for qualitative issues

Charnes and Collomb (1997) proposed an interval GP method to deal with the importance of marginal changes in goal achievement based on distance to the goal target. These penalty functions serve to increasingly penalize the objective function of the problem as the solution becomes further away from the goal. Later, various methods were proposed to model the increasing PF, reverse PF, and nonlinear PF. These methods were used to solve many practical problems such as constrained optimal control problems, tax payment problems, and search engine advertising problems. The S-shaped PF is a more general model appearing in models of technological innovation, labor, and economics (Chang and Lin, 2009). Therefore, a simple S-shaped PF is discussed here; the penalized behavior is depicted in Figure 1, and the scale data is listed in Table 1. Yang et al. (1991) derived a novel method to solve an S-shaped PF problem. To formulate the problem, one additional binary variable is needed to interpret the union of the ramp-type linear functions. Chang and Lin (2009) derived an efficient mixed-integer model to solve the S-shaped PF problem. Later, an efficient model was proposed by Lu and Chen (2013) for the S-shaped PF in which the number of extra binary variables can be reduced from n down to $\lceil \log_2(n+1) \rceil$. However, all the above methods formulate the S-shaped PF as an MIP problem by adding extra binary variables.

Table 1. Scale data of the S-shaped PF

Goals	Unit (%)	Marginal penalty	The value of $f_i(\mathbf{x})$	Marginal penalized	Directions
g_1	below 80	0.8	75	34	Left
	80-90	2	85	20	four-sided penalty
	90-100	1	95	5	
	over 100	0	105	0	

below 110	0	105	0	Right
110-120	1	115	5	four-sided
120-130	2	125	20	penalty
over 130	0.8	135	34	

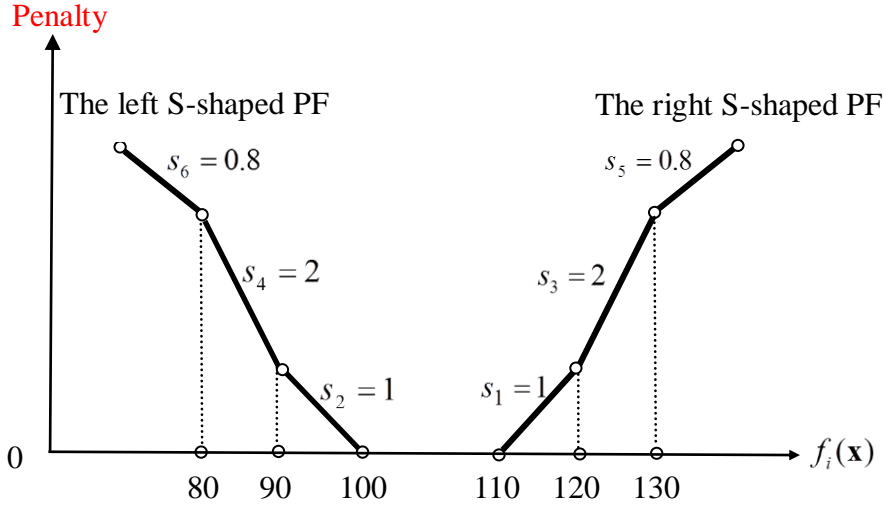


Figure 1. An example of a two-side S-shaped PF

Adding binary variables to an MIP problem does not improve the linear programming relaxation, as indicated by Keha et al. (2004, 2006). Therefore, it is essential to create a new efficient model to deal with a large size MIP problem. A typical right-side S-shaped PF with marginal penalties equal to 1, 2, and 0.75 is depicted in Figure 2.

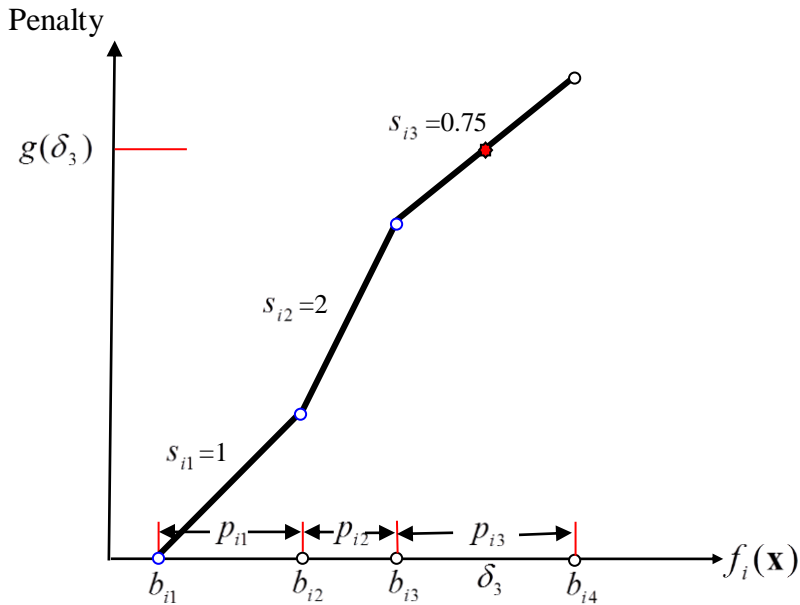


Figure 2. An example of the right-side S-shaped PF

Based on a right-side S-shaped PF, as shown in Figure 2, Chang (2018) proposed an ESPM to approximate the S-shaped function. To save space, we only express the right-side S-shaped PF as follows.

		been covered
		z Satisfaction rate being 50%
4	Balance the setup cost and the number of senior citizens covered with the trapezoidal utility function	z 11 cities have been selected z Setup cost equals 14.1 million dollars z 1254 thousand senior citizens have been covered z Satisfaction rate being 100% in TUT

4.1 Discussion

As seen in Table 3, DMs can add different utility functions, constraints, and models to the proposed method, demonstrating its feasibility and compatibility for MCDM/MODM problems involving quantitative and qualitative issues. Given the evident falling birth rate and aging society in Japan, Italy, Germany, and Taiwan, more and more countries are having a similar problem; country productivity is declining, and more elderly people need to be taken care of, but medical expansion simply cannot catch up. The RPMS can take care of more elderly people than the traditional method of care in the hospital and saves on medical resources to reduce the burden on the government. The topology design problem of the RPMS is a major element of success in implementing RPMS for aging societies. This paper provides a novel integrated method to contribute to the development of the RPMS considering qualitative, quantitative, and balance issues, reducing the gap between management science and the healthcare sector to implement the RPMS. In addition, the authors hope the proposed method can increase the adoption of MCDM methods that are used by companies to solve practical problems.

4.2 Sensitivity analysis

In establishing an RPMS, the most important issue for DMs is to understand the relationship between setup cost and senior citizens covered following sensitivity analysis, and the results of the analysis can provide a reference for DMs. To understand how a change in the weight values of the setup cost from 1000% to 1600% in the objective function of P13 will affect the achieved target value (i.e., the rate of satisfaction, setup cost, number of senior citizens covered), sensitivity analysis is performed. LINGO (Schrage, 2004) is used to solve P13 again to obtain the optimal solution shown in Table 4. The value of M1 and M2 is 1, regardless of the weight increase. The setup cost and the number of senior citizens covered change slightly when the weight increases from 1300% to 1500%.

Table 4. Results of sensitivity analysis

weight	Set up cost	Number of senior citizens coverage	M1	M2
1000%	16.7	1259	1	1
1100%	16.7	1259	1	1
1200%	16.7	1259	1	1

1300%	16.7	1259	1	1
1400%	13.2	1232	1	1
1500%	15.8	1277	1	1
1600%	15.8	1277	1	1

Consequently, the management implications of the proposed methods are: (1) the proposed integrated method can easily be used to help the DM find an appropriate topology CCS solution with various weights, (2) the non-linear UF can easily be approximated by the proposed piecewise method where binary variables are no longer needed. This result makes it possible to solve a realistic large-scale problem within a reasonable amount of time, and (3) the APUF and TUF can easily be used to solve MODM problems with quantitative and qualitative issues.

5. Conclusions

FUF, IUF, APUF, and TUF often appear in business and industrial decision-making models. However, any given efficient mathematical method is yet to be able to define these types of problem very well. A novel integrated method is proposed by this paper to solve the S-shaped PF where extra binary variables are no longer needed. This reduces the complexity of the S-shaped PF formulation. It can improve the efficiency in processing the solution for the S-shaped PF and APUF in MCGP. In addition, ESPM, APUF, TUF, and MCGP approaches also proved conducive to addressing a topology design problem for an RPMS with qualitative and quantitative issues. By demonstrating the RPMS in Section 4, several merits were shown: (1) RPMS implementation benefited from the satisfaction of senior citizens, (2) the proposed method provided good guidance for RPMS planning, and (3) constraints and scenarios could easily be added to the proposed model to show the feasibility of RPMS. These improve the usefulness of the MCGP method in dealing with qualitative and quantitative issues in MODM problems. The framework and contributions of the paper are shown in Figure 18. As seen in Figure 18, the TUF, FUF, IUF, and UFU methods are created by this paper to contribute to the field of qualitative methods. The proposed model is an integrated MCGP method containing qualitative and quantitative functions to resolve MCDM, MODM, and the topology design of RPMS problems.

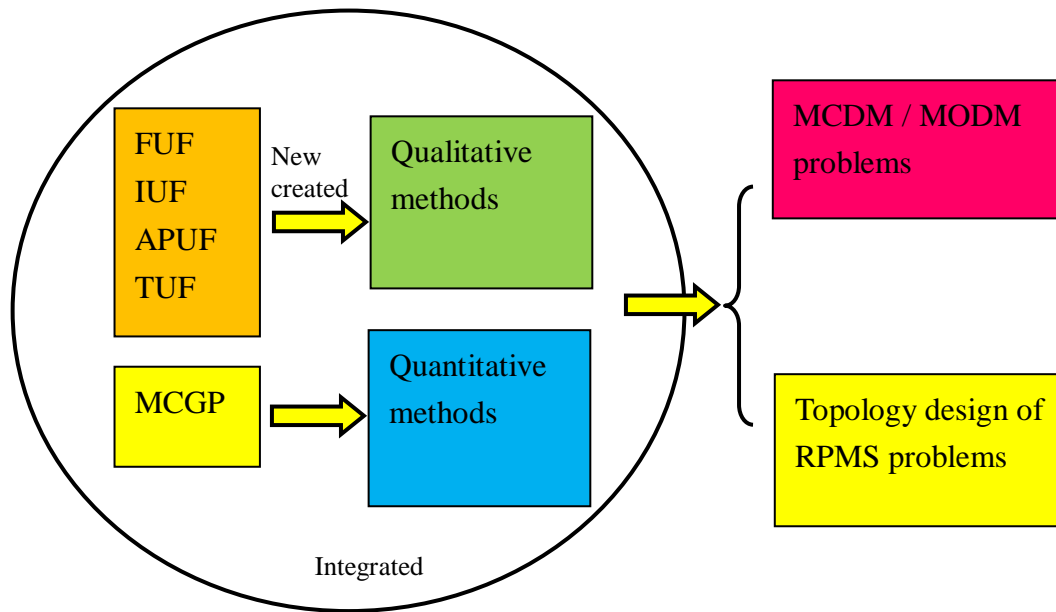


Figure 18. Framework and contributions of the paper

It is clear that the proposed methods are potentially serviceable, the promising results also shed light on future directions, such as all-unit discount cost structure in supply chain management (Chan et al. 2002), inventory models with controllable lead time (Chang et al, 2006), and more real-world oriented adaptation.

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Appendix 1
(AP1) for LLUF

$$\text{Min } \sum_{i=1}^n [w_i (d_i^{\bar{}} \bar{z} d_i^{\underline{}}) \bar{z} V_i f_i^{\underline{}}]$$

s.t.

$$y_i \in \frac{g_{i,\max} \bar{z} y_i}{g_{i,\max} \bar{z} g_{i,\min}}, \quad (A1)$$

$$f_i(\mathbf{x}) \bar{z} d_i^{\bar{}} \bar{z} d_i^{\underline{}} \bar{z} 1 y_i, \quad (A2)$$

$$y_i \bar{z} f_i^{\underline{}} \bar{z} 1, \quad (A3)$$

$$g_{i,\min} \bar{z} y_i \bar{z} g_{i,\max}, \quad (A4)$$

$$d_i^{\underline{}}, f_i^{\underline{}}, y_i \bar{z} 0, \quad (A5)$$

$$(\text{ is a feasible set}), \quad (A6)$$

where w_i and v_i are weights attached to deviations d_i^z , d_i^l and f_i^l ; the role of weight v_i can be seen as a preferential component for the utility value; the utility value of is denoted as u_i ; Eq.(A1) is the LLUF; the highest possible value of the LLUF is 1, as described in Eq.(A5).

(AP2) for RLUF

$$\text{Min } \sum_{i=1}^n [w_i(d_i^z - d_i^l) + v_i f_i^l]$$

s.t.

$$u_i \in \left[\frac{y_i^l - g_{i,\min}}{g_{i,\max} - g_{i,\min}}, \dots \right], \quad (A7)$$

$$\text{Eqs.(A2)-(A6)} \quad (A8)$$

where all variables are defined as in AP1.

Appendix 2

(AP3)

$$\text{Min } \sum_{i=1}^n w_i(d_i^z - d_i^l)$$

s.t.

$$f_i(\mathbf{x}) - d_i^z - d_i^l + \sum_{j=1}^m g_{ij} S_{ij}(B), \quad (A9)$$

$$d_i^l \geq 0, \quad (A10)$$

$$S_{ij}(B) \in R_i(x), \quad (A11)$$

$$(\text{ is a feasible set}), \quad (A12)$$

where $S_{ij}(B)$ represents a function of the binary serial number; the function of resources limitations is denoted as $R_i(x)$; other variables are defined as in GP.

Appendix 3

(AP4) for the case of the more the better of MCDM

$$\text{Min } \sum_{i=1}^n [w_i d_i^z + v_i (e_i^z - e_i^l)]$$

s.t.

$$f_i(\mathbf{x}) - d_i^z \geq y_i, \quad (A13)$$

$$y_i - e_i^z - e_i^l + g_{i,\max}, \quad (A14)$$

$$g_{i,\min} \cup y_i \cup g_{i,\max}, \quad (\text{A15})$$

$$, e_i^{\bar{z}}, e_i^{\underline{z}} \geq 0, \quad (\text{A16})$$

$$\mathbf{x} \in \mathbf{F}, \quad (\text{ is a feasible set}), \quad (\text{A17})$$

where $d_i^{\bar{z}}$ is the positive deviation attached to the i th goal $|f_i(\mathbf{x}) - y_i|$ in Eq.(A13); the positive and negative deviations $e_i^{\bar{z}}$ and $e_i^{\underline{z}}$ are attached to $|y_i - g_{i,\max}|$ in Eq.(A14); the weight U_i is attached to $|y_i - g_{i,\max}|$; other variables are defined as in AP3.

(AP5) for the case of the less the better of MCDM

$$\text{Min } \sum_{i=1}^n [w_i d_i^{\bar{z}} + U_i (f_i^{\bar{z}} - f_i^{\underline{z}})]$$

s.t.

$$f_i(\mathbf{x}) - d_i^{\bar{z}} \leq y_i, \quad (\text{A18})$$

$$y_i - f_i^{\bar{z}} \leq f_i^{\underline{z}} - 1 g_{i,\min}, \quad (\text{A19})$$

$$g_{i,\min} \cup y_i \cup g_{i,\max}, \quad (\text{A20})$$

$$d_i^{\bar{z}}, f_i^{\bar{z}}, f_i^{\underline{z}} \geq 0, \quad (\text{A21})$$

$$(\text{ is a feasible set}), \quad (\text{A22})$$

where $d_i^{\bar{z}}$ is the positive deviation attached to the i th goal $|f_i(\mathbf{x}) - y_i|$ in Eq.(A18); the positive (negative) deviational variable $f_i^{\bar{z}}$ ($f_i^{\underline{z}}$) is attached to $|y_i - g_{i,\min}|$ in Eq.(A19); the weight U_i is attached to $|y_i - g_{i,\min}|$; other variables are defined as in AP3.