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Invited paper

Efficient predictability of stock return volatility: The role of stock market implied volatility

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ABSTRACT

This study examines the predictability of stock market implied volatility on stock volatility in five developed economies (the US, Japan, Germany, France, and the UK) using monthly volatility data for the period 2000 to 2017. We utilize a simple linear autoregressive model to capture predictive relationships between stock market implied volatility and stock volatility. Our in-sample results show there exists very significant Granger causality from stock market implied volatility to stock volatility. The out-of-sample results also indicate that stock market implied volatility is significantly more powerful for stock volatility than the oil price volatility in five developed economies.

1. Introduction

Stock market volatility is a crucial input for risk management, asset pricing and portfolio management. Therefore, modeling and forecasting stock market volatility remain a hot topic in financial econometrics.

To improve the stock market volatility forecasting, some studies have constructed new and powerful predictors or factors. Schwert (1989) finds limited support for links between volatility and macroeconomic predictors, whereas more recent papers such as Christiansen, Schmeling, and Schrimpf (2012), Paye (2012), Engle, Ghysels, and Sohn (2013), Conrad and Loch (2015), and Nonejad (2017), Mohsen and Sujata (2019) arrive at somewhat more encouraging results by constructing the macroeconomic and financial variables. Very recently, Feng, Wang, and Yin (2017) find that oil volatility risk premium (oil VRP) does exhibit statistically and economically significant in-sample and out-of-sample forecasting power for stock market volatility in G7 countries. Wang, Wei, Wu, and Yin (2018) also show that the crude oil volatility is predictive of stock volatility in the short-term from both in-sample and out-of-sample perspectives. In addition, Bašta and Molnár (2018) study the comovement between volatility of the equity market and the oil market, both for implied and realized volatilities by using the wavelets method.

Some studies have also investigated the relationship between the stock market implied volatility indices and stock volatility and most papers take an in-sample perspective using multivariate GARCH models (see, e.g., Beckers, 1981; Chiras & Manaster, 1978; Christensen & Prabhala, 1998; Engle & Rangel, 2008; Orlowski & Sywak, 2019). In addition, Corrado and Miller (2005) conclude that the volatility forecast based on the VIX and VXN indices, i.e., the IV index based on NASDAQ100 index have the highest information content both for volatility forecasting and for market risk assessment framework. Giot (2005) indicates that combining GARCH with

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implied volatility often improves on the results from either one alone. In a similar vein, Cheng and Fung (2012) show that while IV is more informative than GARCH, and the GARCH forecast improves the predictive ability of IV for the Hong Kong market.

In contrast, studies within a GARCH context also produced mixed results. Ederington and Guan (2002) find that GARCH models and historical volatility models perform well in comparison to VIX. And Canina and Figlewski (1993) show that implied volatilities provide poor forecasts and that the simple historical models perform better. In addition, Becker, Clements, and White (2006) examine whether the S&P 500 implied volatility index (VIX) contains any information relevant to future volatility beyond that available from model based volatility forecasts by using a GARCH model. However, the results of Becker, Clements, and White (2007) show that VIX is not an efficient volatility predictor and does not provide any additional information relevant to future volatility. Hence, there is no clear consensus on the informational efficiency of implied volatility. Such contradiction inspires us to explore the performance of stock market implied volatility by using different data and method.

Although various researchers have investigated the relationship between different implied volatilities and stock market volatility. Most papers employ the multivariate GARCH model from an in-sample perspective. However, good in-sample performance generally does not mean that the prediction model can obtain excellent out-of-sample performance. In addition, implied volatility indices derived from option prices which reflect market's expectation on the future volatility are generally considered to be a better measure of market's uncertainty. Furthermore, most existing research tries to study the relationship between implied volatility index of single stock market and its stock market volatility. Therefore, different from existing research, in this paper, we will investigate that the stock market implied volatility indices whether can be strongly predictive of stock volatility in five developed economies (the US, Japan, Germany, France, and the UK).

We use daily data from January 2001 to December 2017 from the five stock indices. The squared daily returns in each month are summed to construct monthly realized volatility. The corresponding monthly stock implied volatilities spanning January 2001 to December 2017 are used as a predictor. The stock markets under consideration represent five most important stock markets internationally, in terms of capitalization. In addition, these markets are among the most liquid markets of the world.

We utilize a simple predictive regression for realized volatility of stock index and take past stock market implied volatility indices as a predictor in addition to lagged stock volatility. Our in-sample results show there exists very significant Granger causality from stock market implied volatility to stock volatility in five developed economies. We use both recursive and rolling estimation window to generate one-step-ahead out-of-sample forecasts of stock volatility for January 2006 through December 2017. We compare the out-of-sample forecasting performance of stock market implied volatility model for stock volatility with the benchmark autoregressive model of oil volatility in Wang et al. (2018). Following the literatures (Dai & Zhou, 2020; Paye, 2012; Welch & Goyal, 2008), we also use the out-of-sample $R^2(R^2_{OOS})$ to evaluate out-of-sample performance. This criterion measures the percentage decrease in the mean squared predictive error (MSPE) of the model of interest relative to the MSPE of the benchmark model. The Clark and West (2007) statistic is used to test the equivalence of MSPEs between two nested models. From out-of-sample empirical results, we can see that the stock market implied volatility indices largely improve the predictability of stock volatility out-of-sample perspectives. This predictability is significant during various sample periods in five developed economies.

The remainder of this paper is organized as follows. Section 2 briefly describes the empirical data and descriptive statistics. Section 3 presents the methodology, including the predictive regressions, and the forecast evaluation method. We report the in-sample and out-of-sample results in Sections 4. Section 5 reports robustness tests. Section 6 concludes the paper.

2. Data and descriptive statistics

The goal of this paper is to use stock market implied volatility to predict stock realized volatility. We choose typical stock index from five developed countries to calculate stock return realized volatility including S&P 500 composite index of the US, NIKKEI 225 stock average index of Japan, DAX 30 performance index of Germany, CAC 40 index of France, and FTSE 100 index of the U.K. where the daily data spanning January 2001 to December 2017. Simultaneously, we use end of month closing price spanning January 2000 to December 2017 from the corresponding five implied volatility indices. The implied volatility indices are the following: VIX (S&P500 Volatility Index-US), VXJ (Japanese Volatility Index-Japan), VDAX (DAX 30 Volatility Index-Germany), VCAC (CAC 40 Volatility Index-France), and VFTSE (FTSE 100 Volatility Index-UK). These data are extracted from the Thomson Reuters Database (https://www.thomsonreuters.com/en.html). In addition, Wang et al. (2018) show that crude oil volatility is predictive of stock volatility in the short-term from both in-sample and out-of-sample perspectives. We also use the prices of two oils, West Texas Intermediate (WTI) crude oil and Brent crude oil, which are available at the website of Energy Information Administration (www.eia.gov). We sum the squared daily returns to construct the proxy for the variance of stock and oil returns at the monthly frequency.

Following the literature (e.g., Paye, 2012; Schwert, 1989; Taylor, 1986; Wang et al., 2018), we sum the squared daily returns to construct the proxy for the variance of stock at the monthly frequency. The realized volatility for the specific month t is defined as:

$$\mathrm{V}_t = \sum_{j=1}^{m} r_{t,j}^2,\ j = 1, 2, \ldots, m, \tag{1}$$

where m is the number of business days in each month, and $r_{i,j}$ denotes the j-th daily return of the t-th month. According to the arguments of Andersen and Bollerslev (1997), Andersen, Bollerslev, Diebold, and Ebens (2001), and Andersen, Bollerslev, Diebold, and Labys (2003), this "realized volatility" contains less noise and is a better measure of ex-post variance than the squared monthly returns.

The summary statistics of the five stock realized volatilities and two oil realized volatilities which are obtained by (1) are

Table 1
Summary statistics. This table reports the basis statistics of the realized volatility, which is shown in the first row of the table. S&P, NIKKEI, DAX, CAC and FTSE denote the realized volatility for the corresponding stock index. WTI and BRT denote the realized volatility of West Texas Intermediate (WTI) crude oil and Brent oil, respectively. The mean value, median, standard deviation, range and the number of observations, coefficients of kurtosis and skewness, are shown by row in this table.$\rho(1)$ refers to the first order autocorrelation.

	S&P	NIKKEI	DAX	CAC	FTSE	WTI	BRT
Mean	0.0031	0.0047	0.0048	0.0045	0.0030	0.0127	0.0108
Median	0.0016	0.0031	0.0026	0.0027	0.0017	0.0083	0.0082
S.E.	0.0056	0.0076	0.0063	0.0061	0.0048	0.0143	0.0111
Min.	0.0001	0.0003	0.0003	0.0004	0.0001	0.0010	0.0007
Max.	0.0573	0.1001	0.0542	0.0597	0.0524	0.1209	0.0877
Skew.	6.0275	9.46205	3.8762	4.6493	6.1113	3.6990	3.5480
Kurt.	50.0237	115.1168	23.5024	35.1882	55.5873	21.5372	19.6386
$\rho(1)$	0.7193	0.6283	0.6256	0.6144	0.6017	0.6313	0.5790

presented in Table 1.

The five stock realized volatilities reach from 0.0030 to 0.0048 on average, together with a standard deviation from 0.0048 to 0.0076. In addition, the first-order autocorrelation of five stock realized volatilities exceeds 0.6 which shows the five stock realized volatilities are highly persistent.

From Table 1, we can see that the kurtosis and skewness are large. Following the literature (e.g., Paye, 2012; Wang et al., 2018), we also utilize the natural logarithm of realized volatility, $V_t = \log(RVt)$, to mitigate the impact of leptokurtic for the original realized volatility defined by (1). As we model and forecast volatility using the predictive regressions, the parameters of which are estimated via the ordinary least squares (OLS).

3. Methodology

3.1. Forecasting models

As is pointed out by Christiansen et al. (2012), Paye (2012), Conrad and Loch (2015) and Nonejad (2017), stock realized volatility is highly persistent. From the descriptive statistics in Table 1, we can also see this phenomenon for all five asset markets under study which shows realized volatility can be explained by its past. Therefore, following Christiansen et al. (2012), Paye (2012), Dai, Chen, & Wen (2015) and Wang et al. (2018), autoregressive model will be used to investigate the predictability of stock market implied volatility on realized volatility in five asset markets (the US, Japan, Germany, France, and the UK) from both in-sample and out-of-sample perspectives.

A standard benchmark to forecast stock volatility at the horizon of one month is the following autoregressive model (AR):

$$V_{t+1} = \omega + \sum_{i=0}^{p-1} \alpha_i V_{t-i} + \varepsilon_{i+1}, \tag{2}$$

where $V_t = \log(RV_t)$, the error term ε_{t+1} is assumed to follow an independent and identically normal distribution.

Wang et al. (2018) extended the AR benchmark model (2) by incorporating a log realized volatility of crude oil as an additional predictor:

$$V_{t+1} = \omega + \sum_{i=0}^{p-1} \alpha_i V_{t-i} + \beta\, V_{t,oil} + \varepsilon_{i+1}, \tag{3}$$

where $V_{t,oil}$ is the oil volatility in the t-th month, and the lag order p is set equal to 6 when using monthly data. The use of such long lag length is to sufficiently capture the $V_t = \log(RV_t)$ strong autocorrelation in stock volatility. Wang et al. (2018) showed that crude oil volatility is predictive of stock volatility in the short-term from both in-sample and out-of-sample perspectives.

To investigate the predictive content of the stock market implied volatility indices, we also extend the AR benchmark model (2) by incorporating a log implied volatility of stock as an additional predictor:

$$V_{t+1} = \omega + \sum_{i=0}^{p-1} \alpha_i V_{t-i} + \beta\, IV_t + \varepsilon_{i+1}, \tag{4}$$

where IV_t is the stock market implied volatility in the t-th month. The parameter β captures the effects of stock market implied volatility on future stock volatility. We obtain the parameter estimates in (4) by using the OLS. The null hypothesis of no predictability, $\beta = 0$, can be tested using the standard t-statistic. In order to take into account the possible presence of serial correlation in the data, we employ the Newey-West covariance correction for serial correlation when computing the t-statistics. In this paper, the lag order p is set to be 5, 6, 7 to optimally eliminate the impact caused by the autocorrelation in stock volatility.

The out-of-sample volatility forecasts are generated by predictive regressions based on the rolling and recursive estimation windows. For both techniques of rolling and recursive estimation windows, the total sample of T observations for each oil and stock volatility series are divided into an in sample part containing the first M observations and the out-of-sample part containing the remaining T-M observations. Specifically, the first forecast based on rolling estimation is exactly the same to the first forecast based on recursive window. Differently, after adding a new observation to rolling window method should drop the most distant one to do parameter estimation. In this way, when the estimation window rolls forward, the window M in rolling estimation size is fixed, however the window size M in recursive estimation is increasing.

3.2. Out-of-sample evaluation

Following the convention in stock market return and volatility forecasting (see, e.g., Bildirici & Badur, 2019; Campbell & Thompson, 2008; Dai & Zhu, 2019, 2020; Dai Z.F., & Zhu, 2020; Gong & Lin, 2018, 2019; He, He, & Wen, 2019; Ma, Liu, Wahab, & Zhang, 2018; Neely, Rapach, Tu, & Zhou, 2014; Rapach, Strauss, & Zhou, 2010; Wen, Gong, & Cai, 2016, Wen et al., 2018, 2019; Zhu & Zhu, 2013), we also utilize the out-of-sample $R^2(R^2_{OoS})$ statistic to evaluate the out-of-sample predictive performance of the proposed forecasting model relative to a benchmark model. The R^2_{OoS} statistic is defined as following

$$R^2_{OoS} = 1 - \frac{MSPE_{\bmod el}}{MSPE_{bench}}, \tag{5}$$

where $MSPE_{\bmod el} = \frac{1}{T-M}\sum_{t=M+1}^{T}(V_t - \hat{V}_t)^2$, $MSPE_{bench} = \frac{1}{T-M}\sum_{t=M+1}^{T}(V_t - \bar{V}_t)^2$, and V_t $\bar{V}_t$, $\hat{V}_t$ are the actual stock volatility, the benchmark model forecast of stock volatility, and the stock volatility forecast based on the forecasting model of interest, respectively. $MSPE_{bench}$ and $MSPE_{\bmod el}$ are the mean squared predictive errors (MSPE) of the benchmark model and the tested model, respectively. The R^2_{oos} statistic measures the reduction in MSPE for the volatility forecast relative to the selected benchmark. It is obviously that a positive R^2_{oos} implies that the forecast model of interest has lower MSPE than the benchmark model, implying the greater forecasting accuracy.

We employ the Clark and West (2007) statistic to test the null hypothesis that the MSPE of the benchmark is smaller than or equal to the MSPE of the forecasting model of interest against the alternative hypothesis that the MSPE of the used benchmark is larger than the MSPE of the forecasting model of interest. Mathematically, the Clark and West (2007) statistic is computed by first defining

$$f_t = (V_t - \bar{V}_t)^2 - (V_t - \hat{V}_t)^2 + (\bar{V}_t - \hat{V}_t)^2 \tag{6}$$

By regressing $\{f_s\}_{s=m+1}^{T}$ on a constant, we can conveniently obtain the Clark and West t-statistic and a p-value for the one-sided (upper-tail) test.

4. Empirical results

4.1. In-sample results and analysis

Inoue and Kilian (2004) have emphasized that in-sample predictability is a necessary condition for out-of-sample predictability. Table 2 reports the estimated coefficients of stock market implied volatility expressed by (4), as well as the t-statistics based on the Newey-West covariance correction for serial correlation for the lag order $p = 5, 6, 7$. We also give the increase in R^2 for the regression with stock market implied volatility relative to the benchmark of AR model by (2), expressed as a percentage.

The coefficient estimate of β is significantly positive at 1% level for the VIX, VXJ, VDAX, VCAC, and VFTSE which indicates the strong in-sample predictability from stock market implied volatility to stock volatility for the lag order $p = 5, 6, 7$. For the lag order $p = 5$, the coefficient estimate of β is 1.94, 1.91, 1.92, 1.54 and 1.96 for the VIX, VXJ, VDAX, VCAC, and VFTSE, respectively.

Table 2

In-sample estimation results. This table reports the in-sample estimation results for the predictive regressions for monthly stock volatility with stock market implied volatility. The specification of the predictive model is given by, $V_{t+1} = \omega + \sum_{i=0}^{p-1}\alpha_i V_{t-i} + \beta \cdot IV_t + \varepsilon_{t+1}$, where V_t and IV_t are the natural logarithm of monthly realized volatility of stock and stock market implied volatility, respectively. The lag order is set as $p = 5, 6, 7$. We report the estimate of the slope coefficient, as well as the corresponding heteroskedasticity-adjusted t-statistic based on the Newey-West method. We also show the percent increase in R^2 of the model of interest relative to that of the benchmark of AR(6): $V_{t+1} = \omega + \sum_{i=0}^{p-1}\alpha_i V_{t-i} + \varepsilon_{t+1}$. The asterisks *, **, *** denote rejections of null hypothesis at 10%, 5% and 1% significance levels, respectively.

	VIX		VXJ		VDAX		VCAC		VFTSE	
	Coefficient	t-stat	Coefficient	t -stat	Coefficient	t -stat	Coefficient	t -stat	Coefficient	t -stat
Parameter estimation results for lag order $p = 5$.										
β	1.94***	5.37	1.91***	5.50	1.92***	5.51	1.54***	4.75	1.96***	5.67
Δ	12.158		20.643		10.890		8.667		14.856	
Parameter estimation results for lag order $p = 6$.										
β	1.95***	5.30	1.93***	5.74	1.93***	5.63	1.52***	4.53	1.95***	5.64
Δ	11.731		20.660		10.725		7.838		14.087	
Parameter estimation results for lag order $p = 7$.										
β	1.98***	5.35	1.93***	5.78	1.90***	5.60	1.50***	4.30	2.01***	5.61
Δ	11.938		20.686		10.055		7.330		14.394	

Table 3
Out-of-sample forecasting results based on recursive window. This table reports the forecasting results for the predictive regressions with stock market implied volatility. We give the results from the regression given by, $V_{t+1} = \omega + \sum_{i=0}^{p-1} \alpha_i V_{t-i} + \beta IV_t + \varepsilon_{t+1}$, where V_t and IV_t are the natural logarithm of monthly realized volatility of stock and stock market implied volatility, respectively. The lag order is set as $p = 5, 6, 7$. The forecasts are generated using a recursive window with the initial sample covers the period of 60 months. The table reports the out-of-sample R^2, defined by the percent reduction of mean squared predictive error (MSPE) of the stock market implied volatility model relative to that of the benchmark of model: $V_{t+1} = \omega + \sum_{i=0}^{p-1} \alpha_i V_{t-i} + \beta V_{t,oil} + \varepsilon_{t+1}$. The p-values of Clark and West (2007) (CW) tests for the equivalence of MSPEs between stock market implied volatility and the benchmark oil volatility model are given in the parentheses. The asterisks *, ** and *** indicate rejections of null hypothesis at 10%, 5% and 1% significance levels, respectively.

	VIX	VXJ	CVCAC	DAX	VFTSE
Out-of-sample forecasting results for lag order $p = 5$.					
R^2_{oos}	15.38***	13.20***	12.87***	8.49***	12.37***
p-value	6.65E-06	0.00030	7.8E-05	0.00031	0.00042
Out-of-sample forecasting results for lag order $p = 6$.					
R^2_{oos}	15.70***	13.78***	12.49***	8.19***	11.99***
p-value	9.87E-06	0.00014	9.13E-05	0.00052	0.00043
Out-of-sample forecasting results for lag order $p = 7$.					
R^2_{oos}	15.42***	14.12***	11.32***	7.41***	11.92***
p-value	1.33E-05	9.28E-05	0.00015	0.00130	0.00058

Therefore, for example, one percent increases in current month's stock market implied volatility of S&P 500 returns will lead to 1.95% increase in next month's volatility of S&P 500 returns. The percent increase in R^2 after adding stock realized volatility to the benchmark regression is 12.158. These values are much greater than the increase in R^2 due to the inclusion of the oil volatility in predictive regressions reported in Table 1 of Wang et al. (2018), implying that stock market implied volatility can provide much greater predictive content than the oil volatility for stock volatility.

For the lag order $p = 6$ or $p = 7$, the coefficient estimate of β is similar as the lag order$p = 5$, which implies the coefficient estimate of β is stable for different countries. The percent increase in R^2 after adding stock market implied volatility to the benchmark regression (2) is also stable for different countries except for VCAC. Comparing the percent increase in R^2 after adding stock realized volatility to the benchmark regression (2) within the tested countries, we note that the VXJ has highest percent increase in R^2, and the VCAC has lowest percent increase in R^2. And the coefficient estimate of β for the VCAC is also smallest for different lag order.

In addition, in place of the slope estimate β, the table also displays the t-statistic testing the null hypothes is of no Granger causality, that is, the null that $\beta = 0$. The null hypothesis is of no Granger causality is strongly rejected for all case, which shows that the stock market implied volatility emerges as a highly significant predictor.

4.2. Out-of-sample results and analysis

In the following out-of-sample analysis, we select the crude oil volatility driven model (3) as benchmark. The main reasons are as follows. Firstly, crude oil is a core input in modern industry. Oil price shocks can certainly lead to changes in stock prices by affecting real economic activities (see, Hamilton, 1983; Kilian, 2009). Secondly, Wang et al. (2018) show that crude oil volatility can be strongly predictive of stock volatility. Finally, choosing a benchmark model with strong predictability can better demonstrate the prediction ability of the proposed forecasting model.

We generate volatility forecasts for January 2005 through December 2017 and evaluate the forecasting performance. Table 3 displays the evaluation results of the predictive regression based on the recursive estimation window. In the crude oil volatility driven model (3), we select the WTI oil volatility as an additional predictor for S&P 500 composite index of the US, NIKKEI 225 stock average index of Japan, and select the Brent oil volatility as an additional predictor for DAX 30 performance index of Germany, CAC 40 index of France, and FTSE 100 index of the UK Because Brent oil price is determined by oil market situation in the Europe and Africa, and WTI price is relevant to the US and Japan. We give the values of out-of-sample R^2_{oos}, as well as the p-values of CW test for the equivalence of MSPE of two tested models.

Firstly, we look at the forecasting performance of the implied volatility-VIX for S&P500 Volatility Index-US. The values of R^2_{oos} suggest that the inclusion of the implied volatility-VIX in the right-hand side of predictive regression can lead to a reduction of MSPE of 15.38% in the full out-of-sample period for the lag order $p = 5$. The P-value of CW test is 6.65E-06 which suggests that the improvement of forecasting accuracy is very significant comparing with the benchmark of the crude oil volatility driven model (3). For the lag order $p = 6$ or $p = 7$, the values of R^2_{oos} and the p-value of CW test are similar as the lag order $p = 5$, which shows that the predictive regressions with stock market implied volatility can significantly beat the oil volatility benchmark model (3) for different lag order.

Secondly, we look at the forecasting performance of the other implied volatilities. The values of R^2_{oos} for VXJ, VDAX, VCAC, and VFTSE suggest that the inclusion of the implied volatility in the right-hand side of predictive regression can lead to a reduction of MSPE from 7.41% to 14.12% in the full out-of-sample period for different lag order. The largest P-value of CW test is 0.0013, which suggests that the improvement of forecasting accuracy is very significant comparing with the benchmark of the crude oil volatility driven model (3).

Comparing the values of R^2_{oos} after adding stock market implied volatility to the bench-mark of the crude oil volatility driven

Table 4
Out-of-sample forecasting results based on rolling window. This table reports the forecasting results for the predictive regressions with stock market implied volatility. We present the results from the regression given by, $V_{t+1} = \omega + \sum_{i=0}^{p-1} \alpha_i V_{t-i} + \beta IV_t + \varepsilon_{t+1}$, where V_t and IV_i are the natural logarithm of monthly realized volatility of stock and stock market implied volatility, respectively. The lag order is set as $p = 5, 6, 7$. The forecasts are generated using a rolling window with each window covers a period of 60 months. The table reports the out-of-sample R^2, defined by the percent reduction of mean squared predictive error (MSPE) of the stock market implied volatility model (4) relative to that of the benchmark of model $V_{t+1} = \omega + \sum_{i=0}^{p-1} \alpha_i V_{t-i} + \beta V_{t,oil} + \varepsilon_{t+1}$. The p-values of Clark and West (2007) (CW) tests for the equivalence of MSPEs between stock market implied volatility and the benchmark oil volatility model are given in the parentheses. The asterisks *, ** and *** indicate rejections of null hypothesis at 10%, 5% and 1% significance levels, respectively.

	VIX	VXJ	CVCAC	DAX	VFTSE
Out-of-sample forecasting results for lag order $p = 5$.					
R^2_{oos}	17.71***	23.25***	25.06***	17.22***	10.75***
p-value	5.27E-07	1.8E-07	4.56E-08	1.32E-06	0.00046
Out-of-sample forecasting results for lag order $p = 6$.					
R^2_{oos}	16.56***	23.66***	24.09***	16.67***	10.31***
p-value	2.37E-06	3.09E-07	3.04E-08	8.84E-07	0.00054
Out-of-sample forecasting results for lag order $p = 7$.					
R^2_{oos}	17.02***	22.35***	24.92***	17.67***	11.92***
p-value	2.4E-06	7.29E-07	4.88E-08	6.94E-07	0.00034

model (3) for VIX VXJ, VDAX, VCAC, and VFTSE, we note that the VIX has largest values of R^2_{oos}, and the VDAX has smallest values of R^2_{oos}. In addition, all of the P-values of CW test are less 0.01. Hence, we can find significant predictability from stock realized volatility to stock volatility for different countries and different lag order.

Table 4 reports the forecasting results by rolling window. The forecasts are generated using a rolling with the initial sample covers the period of 60 months. Firstly, we look at the forecasting performance of the implied volatility for stock volatility Index. The values of R^2_{oos} suggest that the inclusion of the implied volatility in the right-hand side of predictive regression can lead to a reduction of MSPE from 10.31% to 25.06% in the full out-of-sample period for the lag order p = 5, 6, 7. The P-values of CW test are less than 0.001 for VIX, VXJ, VDAX, VCAC, and VFTSE. We can find that the predictive regressions with the stock market implied volatility model (4) can significantly beat the oil volatility benchmark model (3), regardless of whether WTI or Brent oil is included. This evidence indicates the strong predictive content of stock market implied volatility for stock volatility.

Comparing the values of R^2_{oos} after adding stock realized volatility to the benchmark of the crude oil volatility driven model (3) for VIX VXJ, VDAX, VCAC, and VFTSE, we note that the VCAC has largest values of R^2_{oos}, and the VFTSE has smallest values of R^2_{oos}. The values of R^2_{oos} significantly increase comparing rolling window method to recursive window method, especially for the VCAC. The reason is that the revealed predictability is stronger over more recent periods.

Overall, we find the existence of significant predictability from stock market implied volatility to stock volatility and the predictability of stock market implied volatility is much stronger than the predictability of oil volatility.

5. Robustness tests

Rossi and Inoue (2012) have emphasized that the choice of the estimation window size has always been a concern for practitioners, since the use of different window sizes may lead to different empirical results in practice. That is, the choice of forecasting window sizes plays an important role in out-of-sample evaluation. If a predictor has significant predictability for stock volatility, it ought to be insensitive to the choice of time period. Therefore, it is necessary and reasonable to test the predictive content of the stock market implied volatility for stock volatility whether is robust to the choice of the estimation and evaluation window size. Thus, we additionally consider another three forecasting windows, where the length of the initial estimation windows is 8 years, 11 years and 14 years. As a result, all three forecasting windows considered in this paper have a desirable trade-off between an initial estimation period that has enough in-sample observations to precisely estimate parameters and an out-of sample period that is relatively long for forecast evaluation. In the robustness test, the lag order is set as $p = 6$, and the out-of-sample volatility forecasts are also generated by predictive regressions based on the recursive and rolling estimation windows.

Tables 5 and 6 report the out-of-sample forecasting performance for alternative forecasting windows based on the recursive estimation method and the rolling estimation method, respectively. From Table 5, we can see that the values of R^2_{oos} suggest that the inclusion of the implied volatility in the right-hand side of predictive regression can lead to a reduction of MSPE from 6.81% to 21.09% relative to that of the benchmark of model (3) in different out-of-sample periods for the recursive estimation method. But the values of R^2_{oos} decrease for VIX VXJ, VDAX, VCAC, and VFTSE during 2014.01–2017.12, the reason is that the predictive ability of the oil volatility driven benchmark model (3) becomes stronger during more recent subperiods which has been shown in Wang et al. (2018).

From Table 6, we can find that the values of R^2_{oos} suggest that the inclusion of the implied volatility in the right-hand side of predictive regression can lead to a reduction of MSPE from 10.75% to 32.24% relative to that of the benchmark of model (3) in different out-of-sample periods for the rolling estimation method. In addition, all of the P-values of CW test are less 0.01, except for the VFTSE during 2014.01–2017.12.

We can observe a robust result that the stock market implied volatility regression model can significantly beat the oil volatility

Table 5
Out-of-sample forecasting results based on recursive window for different sample period. This table reports the forecasting results for the predictive regressions with stock market implied volatility. We present the results from the regression given by, $V_{t+1} = \omega + \sum_{i=0}^{p-1} \alpha_i V_{t-i} + \beta IV_t + \varepsilon_{t+1}$, where V_t and IV_i are the natural logarithm of monthly realized volatility of stock and stock market implied volatility, respectively. The lag order is set as $p = 6$. The forecasts are generated using a recursive window with the initial sample covers the period of 96 months. The table reports the out-of-sample R^2, defined by the percent reduction of mean squared predictive error (MSPE) of the stock market implied volatility model (4) relative to that of the benchmark of model $V_{t+1} = \omega + \sum_{i=0}^{p-1} \alpha_i V_{t-i} + \beta V_{t,oil} + \varepsilon_{t+1}$. The p-values of Clark and West (2007) (CW) tests for the equivalence of MSPEs between stock market implied volatility and the benchmark oil volatility model are given in the parentheses. The asterisks *, ** and *** indicate rejections of null hypothesis at 10%, 5% and 1% significance levels, respectively.

	VIX	VXJ	CVCAC	DAX	VFTSE
Out-of-sample forecasting results for 2008.01–2017.12.					
R^2_{oos}	14.62***	9.05***	12.21***	6.96 ***	12.63***
p-value	0.00018	0.00697	0.00157	0.00680	0.00269
Out-of-sample forecasting results for 2011.01–2017.12.					
R^2_{oos}	21.09***	7.18***	14.96***	8.66**	14.85***
p-value	0.00015	0.01730	0.00208	0.01483	0.00867
Out-of-sample forecasting results for 2014.01–2017.12.					
R^2_{oos}	18.61***	6.81**	13.19**	8.11**	13.24**
p-value	0.00792	0.01948	0.01796	0.03677	0.01275

Table 6
. Out-of-sample forecasting results based on rolling window for different sample period. This table reports the forecasting results for the predictive regressions with stock market implied volatility. We present the results from the regression given by, $V_{t+1} = \omega + \sum_{i=0}^{p-1} \alpha_i V_{t-i} + \beta IV_t + \varepsilon_{t+1}$, where V_t and IV_i are the natural logarithm of monthly realized volatility of stock and stock market implied volatility, respectively. The lag order is set as $p = 6$. The forecasts are generated using a rolling window with each window covers a period of 60 months, with different initial sample period. The table reports the out-of-sample R^2, defined by the percent reduction of mean squared predictive error (MSPE) of the stock market implied volatility model relative to that of the benchmark of model $V_{t+1} = \omega + \sum_{i=0}^{p-1} \alpha_i V_{t-i} + \beta V_{t,oil} + \varepsilon_{t+1}$. The p-values of Clark and West (2007) (CW) tests for the equivalence of MSPEs between stock market implied volatility and the benchmark oil volatility model are given in the parentheses. The asterisks *, ** and *** indicate rejections of null hypothesis at 10%, 5% and 1% significance levels, respectively.

	VIX	VXJ	CVCAC	DAX	VFTSE
Out-of-sample forecasting results for 2008.01–2017.12.					
R^2_{oos}	12.58***	21.58***	23.00***	17.94***	11.42***
p-value	0.00031	3.62E-05	9.33E-07	5.94E-06	0.00144
Out-of-sample forecasting results for 2011.01–2017.12.					
R^2_{oos}	22.98***	32.24***	26.40***	21.70***	11.51***
p-value	0.00031	1.41E-05	2.01E-06	1.1E-05	0.00505
Out-of-sample forecasting results for 2014.01–2017.12.					
R^2_{oos}	20.32***	19.57***	29.65***	23.55***	10.75*
p-value	0.00453	0.00587	0.00015	0.00056	0.05372

driven benchmark model (3) in different out-of-sample periods for the recursive and rolling estimation method. In other words, the novelty of the methodologies we propose is that they are robust to the choice of the estimation and evaluation window size which shows the predictability of stock market implied volatility is much stronger than the predictability of oil volatility.

Overall, when we use stock market implied volatility to predict stock realized volatility, the rolling estimation method performs better than the recursive method, the prediction accuracy is high and robust to the choice of the estimation and evaluation window size.

6. Conclusions and implication

This paper investigates the role of stock market implied volatility in predicting stock return volatility among five developed economies (the US, Japan, Germany, France, and the UK). The in-sample result shows the slope coefficients in predictive regressions of stock volatilities on the stock market implied volatilities are significantly positive at 1% level for the VIX, VXJ, VDAX, VCAC, and VFTSE. Interestingly, not only the values of out-of-sample R^2_{oos}, but also the *p*-values of CW test, show that stock market implied volatility can strongly predict stock return volatility among five developed economies. Our results suggest that the predictability is not affected by the change of the estimation and evaluation window size to all extent. In short, results from in-sample and out-of-sample indicate the stock market implied volatility has significant predictability of stock return volatility.

Our findings have some implications for market participants. Firstly, stock realized volatility is highly persistent and the auto-regressive model is efficient for stock volatility prediction. Secondly, the predictive power of stock market implied volatility is robust to controlling for lagged volatility. Finally, the prediction ability of stock market implied volatility is much better than that of oil volatility.

Author contributions

Dr. Zhifeng Dai has made substantial contributions to the conception or design of the work, and drafted the work or revised it.
Miss Huiting Zhou has made substantial contributions to the acquisition, analysis, or interpretation of data for the work.
Professor Fenghua Wen has made substantial contributions to the conception or design of the work, and drafted the work or revised it.
Professor Shaoyi He has made contributions to the acquisition, analysis, drafted the work.
All persons who have made substantial contributions to the work are reported in the manuscript.
All persons have approved the final version to be published.

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