



Contents lists available at ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb



Stochastic inflation with an extremely large number of e -folds

Naoya Kitajima^{a,b}, Yuichiro Tada^c, Fuminobu Takahashi^{b,d}

^a Frontier Research Institute for Interdisciplinary Sciences, Tohoku University, Sendai, 980-8578 Japan

^b Department of Physics, Tohoku University, Sendai, 980-8578 Japan

^c Department of Physics, Nagoya University, Nagoya 464-8602, Japan

^d Kavli IPMU (WPI), UTIAS, The University of Tokyo, Kashiwa, Chiba 277-8583, Japan

ARTICLE INFO

Article history:

Received 4 September 2019

Received in revised form 6 November 2019

Accepted 10 November 2019

Available online xxxx

Editor: J. Hisano

Keywords:

Stochastic inflation

Eternal inflation

Axion

TU-1090

IPMU19-0102

ABSTRACT

We propose a class of single-field, slow-roll inflation models in which a typical number of e -folds can be extremely large. The key point is to introduce a very shallow local minimum near the top of the potential in a hilltop inflation model. In particular, a typical number of e -folds is enhanced if classical behavior dominates around the local minimum such that the inflaton probability distribution is drifted to the local minimum as a whole. After the inflaton escapes from the local minimum due to the stochastic dynamics, the ordinary slow-roll inflation follows and it can generate the primordial density perturbation consistent with observation. Interestingly, our scenario inherits the advantages of the old and new inflation: the typical e -folds can be extremely large as in the old inflation, and slow-roll inflation naturally follows after the stochastic regime as in the new inflation. In our numerical example, the typical number of e -folds can be as large as 10^{10} , which is large enough for various light scalars such the QCD axion to reach the Bunch-Davies distribution.

© 2019 Published by Elsevier B.V. This is an open access article under the CC BY license (<http://creativecommons.org/licenses/by/4.0/>). Funded by SCOAP³.

1. Introduction

How long can inflation last? From the observational point of view, the total number of e -folds, N , must be larger than ~ 50 – 60 , which weakly depends on the inflation scale and thermal history after inflation. On the other hand, an extremely long duration of inflation is often required in certain scenarios. For instance, the relaxation model [1], the stochastic axion scenario [2–5], a quintessence model [6,7] demand the e -folding number of $\log_{10} N \sim \mathcal{O}(10)$, depending on model parameters.¹ The purpose of this Letter is to provide a simple single-field, slow-roll inflation model whose typical number of e -folds is extremely large.

There is a variety of inflation models which last very long. In the string/axion landscape [10–17], there are many local minima where the old inflation takes place and continues until the inflaton tunnels toward one of the adjacent local minima with

a lower energy through bubble formation. In this case, the typical e -folding number can be exponentially large. However, one needs slow-roll inflation after the bubble formation to explain the observed cosmic microwave background (CMB) temperature/polarization anisotropies. For this, one may need another light scalar, in which case the two inflation scales are not related to each other, in general.

It is also well known that a large class of inflation models can be eternal [18–23] (see also [24–26]). The stochastic dynamics of the inflaton plays a crucial role in eternal inflation: quantum fluctuations of the inflaton drive it upward or downward compared to the classical motion. In particular, upward quantum fluctuations keep the inflaton from rolling down the potential, leading to eternal inflation.² In eternal chaotic inflation [21,22], quantum fluctuations overcome the classical motion at sufficiently large fields values, typically much larger than the Planck scale. Eternal inflation also occurs in the new or hilltop inflation [18–20]. This is because the inflaton dynamics becomes stochastic in the vicinity of the potential maximum where the classical motion is suppressed. In both cases, once the universe enters the stochastic regime, the

¹ E-mail addresses: kitajima@tuhep.phys.tohoku.ac.jp (N. Kitajima),

tada.yuichiro@e.mbox.nagoya-u.ac.jp (Y. Tada), fumi@tohoku.ac.jp (F. Takahashi).

² If the e -folding number is extremely large, those scenarios that involve scalars with fine-tuned initial conditions might be significantly modified. For example, such a problem in the curvaton paradigm is reported in Ref. [8]. Also, rare events such as the Higgs tunneling to large-field values could take place somewhere in the entire universe (see e.g. Ref. [9]).

² We note that the downward fluctuations help the inflaton to escape from the eternal inflation regime.

<https://doi.org/10.1016/j.physletb.2019.135097>

0370-2693/© 2019 Published by Elsevier B.V. This is an open access article under the CC BY license (<http://creativecommons.org/licenses/by/4.0/>). Funded by SCOAP³.

inflation continues and never ends in some regions. The volume of the universe is dominated by such regions where the inflation continues. In this sense the inflation is eternal.

It is worth noting that the eternal inflation is based on the idea that the inflating region expands and dominates the volume at later times. In fact, if one randomly picks up a point in space where the inflaton dynamics is in the stochastic regime, the expected e -folds there is not infinite but finite [27] (see also Ref. [28]). In other words, one needs to choose a specific point in space to have a sufficiently long inflation. Such a special choice or fine-tuning can be compensated by large physical volume due to the prolonged inflation. Thus, the eternity of the eternal inflation relies on the volume measure.

The purpose of this Letter is to show that one can make the typical number of e -folds extremely large in a context of a simple single-field, slow-roll inflation. In particular, it does not rely on the volume measure. In Sec. 2 we address our scenario after a brief review on the stochastic formalism. In Sec. 3 we provide an example model which exhibits an extremely long inflation. The last section is devoted for discussion and conclusions.

2. Stochastic inflation with large N

In the simplest setup, the inflationary phase in the early universe can be realized by the potential energy of some scalar field inflaton, ϕ , homogeneously filling the universe. For a sufficiently long inflationary phase, however, the inflaton field must be extremely light, in which case its homogeneity beyond the horizon scale is no longer guaranteed. Specifically, superhorizon inhomogeneities are sourced by subhorizon fluctuations which continuously exit the horizon during inflation. Even though the subhorizon fluctuations originate from the quantum zero-point oscillation, they are well approximated to be classical ones after the horizon exit due to the gravitational decoherence. Accordingly the evolution of the local inflaton field value can be described by a diffusion process. This is known as the stochastic approach [29,30], in which the probability distribution function $P(\phi)$ follows the Fokker-Planck (FP) equation,

$$\frac{\partial P}{\partial N} = \mathcal{L}_{\text{FP}}(\phi) \cdot P, \quad \mathcal{L}_{\text{FP}} = -\frac{\partial}{\partial \phi} h(\phi) + \frac{1}{2} \frac{\partial^2}{\partial \phi^2} D(\phi). \quad (1)$$

Here N is the e -folding number used as a time variable, and it is related to the Hubble parameter H and the cosmic time t by $dN = H dt$. For single-field and slow-roll inflation, the drift and diffusion coefficients are given by $h = -M_{\text{pl}}^2 V'/V$ and $D = V/12\pi^2 M_{\text{pl}}^2$ in terms of the inflaton potential $V(\phi)$, respectively. Due to such a random walk behavior, the e -folds $\mathcal{N}(\phi)$ elapsed for the first passage from an initial field value ϕ to ϕ_f at the end of inflation also becomes a random parameter. It's generating function $\chi_{\mathcal{N}}(\phi; J) = \langle e^{iJ\mathcal{N}(\phi)} \rangle$ is known to follow the adjoint FP equation [31,32],

$$\mathcal{L}_{\text{FP}}^\dagger \cdot \chi_{\mathcal{N}} = -iJ\chi_{\mathcal{N}}, \quad \mathcal{L}_{\text{FP}}^\dagger = h(\phi) \frac{\partial}{\partial \phi} + \frac{1}{2} D(\phi) \frac{\partial^2}{\partial \phi^2}, \quad (2)$$

with the boundary condition $\chi_{\mathcal{N}} = 1$ at the end of inflation. The bracket represents an average over realizations. An arbitrary moment of \mathcal{N} is obtained from this generating function by $\langle \mathcal{N}^n \rangle = \left(\frac{1}{i} \frac{\partial}{\partial J} \right)^n \chi_{\mathcal{N}} \Big|_{J=0}$. Thus, one can estimate the typical e -folding number of inflation by calculating the average of the elapsed e -folds, $\langle \mathcal{N} \rangle$.

For the single-field slow-roll inflation, these FP equations are reduced to the ordinary differential equations, for which the mo-

ments of \mathcal{N} have recursive analytic solutions. For example, the expectation value of \mathcal{N} is given by

$$\langle \mathcal{N} \rangle(\phi) = \int_{\phi_f}^{\phi} \frac{dx}{M_{\text{pl}}} \int_x^{\bar{\phi}} \frac{dy}{M_{\text{pl}}} \frac{1}{v(y)} \exp \left[\frac{1}{v(y)} - \frac{1}{v(x)} \right], \quad (3)$$

where $v = \frac{V}{24\pi^2 M_{\text{pl}}^4}$ is the normalized potential in the Planck unit, and $\bar{\phi}$ represents a constant of integration at which the first derivative vanishes $\partial_\phi \langle \mathcal{N} \rangle |_{\bar{\phi}} = 0$. For a Z_2 -symmetric hilltop model we consider later, $\bar{\phi} = 0$ is obviously suitable for this boundary condition. As shown in Ref. [31], when the following conditions are satisfied,

$$v \ll 1 \quad \text{and} \quad \eta_{\text{cl}} = \left| \frac{v'' v^2}{v'^2} \right| \ll 1, \quad (4)$$

one can use the saddle-point approximation to simplify this analytic solution (3) to the well-known formula for the classical drift-dominated case,

$$\langle \mathcal{N} \rangle(\phi) \simeq N_{\text{cl}}(\phi) = \int_{\phi_f}^{\phi} \frac{dx}{M_{\text{pl}}} \frac{v(x)}{v'(x)}. \quad (5)$$

Note that this formula is valid as long as the conditions (4) are satisfied everywhere between ϕ and ϕ_f . Since the first condition in Eq. (4) is trivially satisfied in our context, it is the classicality parameter η_{cl} that controls the stochastic effect on the averaged e -folding number.

For a better understanding of the second condition in Eq. (4), let us consider the change of the inflaton potential value due to the inflaton dynamics in one Hubble time: $\delta v \simeq v' \delta \phi + \frac{1}{2} v'' \delta \phi^2$, where $\delta \phi$ is the sum of the classical motion $\delta \phi_{\text{cl}} \sim h = -M_{\text{pl}}^2 v'/v$ and the quantum jump $\delta \phi_{\text{sto}} \sim \sqrt{D} = \sqrt{2} v M_{\text{pl}}$. Note that the ensemble average and the variance of $\delta \phi$ are respectively given by $\langle \delta \phi \rangle = \delta \phi_{\text{cl}}$ and $\langle \delta \phi^2 \rangle = \delta \phi_{\text{cl}}^2 + \delta \phi_{\text{sto}}^2$, and hence one obtains $\langle \delta v \rangle \simeq v' \delta \phi_{\text{cl}} + \frac{1}{2} v'' (\delta \phi_{\text{cl}}^2 + \delta \phi_{\text{sto}}^2)$. Thus, the second condition in Eq. (4) requires that the averaged-change of the inflaton potential value is dominated by the linear term of the classical motion, i.e. $|v' \delta \phi_{\text{cl}}| \gg |v'' \delta \phi_{\text{sto}}^2|$. Note that $v' \delta \phi_{\text{cl}} \gg v'' \delta \phi_{\text{cl}}^2$ is satisfied under the second slow-roll condition: $\eta_v \ll 1$ with the second slow-roll parameter $\eta_v = M_{\text{pl}}^2 v''/v$.

Let us emphasize here that the above classical formula (5) can be valid even if there is a highly stochastic region between ϕ and ϕ_f , as long as the curvature of the potential is sufficiently small. To see this, let us first define the stochasticity of the inflaton dynamics by $\xi_{\text{sto}} \equiv \delta \phi_{\text{sto}}^2 / \delta \phi_{\text{cl}}^2 (\equiv h^2/D) = 2v^3/v'^2 M_{\text{pl}}^2$, which is nothing but the amplitude of the curvature perturbations in the classical limit. For $\xi_{\text{sto}} \gtrsim 1$, the size of the quantum jump is greater than the classical field excursion over one Hubble time, i.e. the inflaton dynamics is in the stochastic regime (and this is nothing but the necessary condition for eternal inflation). The stochasticity parameter is related to η_{cl} by $\eta_{\text{cl}} = \frac{1}{2} |\eta_v| \xi_{\text{sto}}$. Therefore, the conditions (4) can be satisfied in a highly stochastic region $\xi_{\text{sto}} \gtrsim 1$ if $\eta_v \ll 1$, in which case the expected e -folds can be well approximated by the classical formula (5).

As an extreme example, let us consider a slightly tilted linear potential for which $\xi_{\text{sto}} \gtrsim 1$ and η_{cl} vanishes by definition. Suppose that the inflaton is initially located at some point. Then, the inflaton distribution spreads out due to the quantum diffusion because of $\xi_{\text{sto}} \gtrsim 1$. However, the quantum diffusion does not change the inflaton potential value on average because up and downhill occur with the same probability. It is the classical motion that changes

the potential value on average, as it uniformly drifts the whole inflaton distribution.

If $\eta_{cl} \gtrsim 1$, the diffusion dynamics overcomes the uniform drift due to the classical motion. In other words, the evolution of the universe relies upon the stochastic process, and the classical dynamics is irrelevant. In the usual hilltop model (without a local minimum), η_{cl} is large around the potential maximum, and the stochastic diffusion sweeps out the inflaton from the hilltop. This is the reason why the averaged e -folds are saturated and does not grow as the initial value ϕ approaches the hilltop (see Fig. 2).

In fact, one can slightly modify the hilltop potential to realize an extremely large e -folds. To this end, we introduce a shallow local minimum surrounded by a region with $\eta_{cl} \ll 1$ around the hilltop. There, the inflaton distribution tends to be pushed back to the local minimum as a whole even in the stochastic regime, and then the expected e -folds can be significantly enhanced. In such a case, the escape rate can be analytically estimated by the expansion around the local maximum and minimum for x and y integrals respectively in Eq. (3), as given by [33]

$$\langle \mathcal{N} \rangle_{\text{esc}} \simeq \frac{\pi v(\phi_+)}{2M_{\text{Pl}}^2 \sqrt{|v''(0)| |v''(\phi_+)|}} e^{\frac{1}{v(0)} - \frac{1}{v(\phi_+)}} \quad (6)$$

for a symmetric potential. Here ϕ_+ denotes the local maximum and we take the local minimum at the origin $\phi = 0$. As suggested by the Hawking-Moss factor $e^{1/v(0) - 1/v(\phi_+)}$ [34], this escape time can be extremely large for a sufficient low scale potential $v \ll 1$. In the next section, we concretely provide such a model which generates the primordial density perturbations consistent with the CMB observations after exiting the local minimum.

3. Example model

Let us explicitly show a concrete model of the extreme long inflation. We consider a Z_2 -symmetric hilltop model given by

$$V(\phi) = \frac{1}{2}m^2\phi^2 - \frac{1}{4}\lambda\phi^4 + \left(\Lambda^2 - g \frac{\phi^6}{M_{\text{Pl}}^4} \right)^2 \quad (7)$$

where Λ determines the inflation scale, λ and g are positive coupling constants, and m is the mass at the origin. For a positive mass-squared in the range of $0 < m^2 < H^2$, this potential has a shallow local minimum around the origin $\phi = 0$, where the inflaton spends a large number of e -folds. After diffusing out from the shallow local minimum, the inflaton dynamics is driven by the tilt of the quartic and hexic terms as an ordinary hilltop inflation, toward the true minimum at $\phi_{\text{min}} \simeq (\Lambda^2 M_{\text{Pl}}^4 / g)^{1/6}$. Some of the parameters are fixed by the CMB data. For example, the following parameters

$$\left(\frac{\Lambda}{M_{\text{Pl}}}, \lambda, \left(\frac{\Lambda^2 M_{\text{Pl}}^4}{g} \right)^{1/6} \right) = (0.998 \times 10^{-4}, 3.39 \times 10^{-14}, 0.1 M_{\text{Pl}}) \quad (8)$$

result in the curvature perturbations with the amplitude $A_s = 2.1 \times 10^{-9}$ and spectral index $n_s = 0.958$ consistent with the recent Planck observation [35].³ We illustrate the potential form in Fig. 1 with these parameters and the typical mass value $m^2/H^2 = 10^{-7}$. Here $H = \Lambda^2 / \sqrt{3} M_{\text{Pl}}$ is the Hubble parameter around the

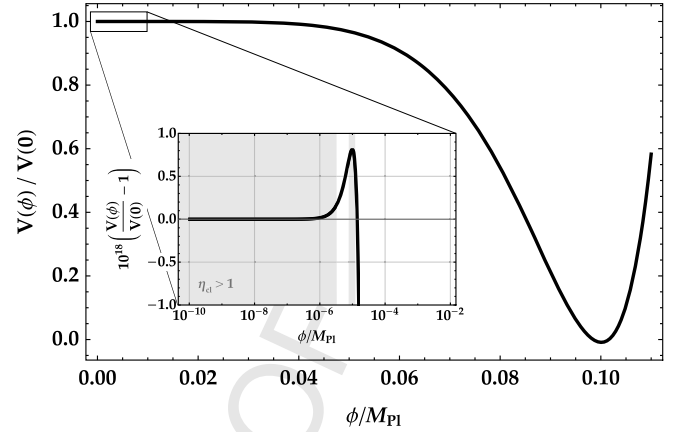


Fig. 1. The potential (7) with the parameter values (8) and the typical mass $m^2/H^2 = 10^{-7}$. The potential has a shallow local minimum surrounded by the local maximum as shown in the magnification. The shaded region corresponds with $\eta_{cl} > 1$. The classical region with $\eta_{cl} < 1$ appears between the local minimum and maximum that will increase the expected e -folds for $m^2/H^2 \gtrsim 10^{-7}$, because the inflaton is pushed back to the local minimum by the classical dynamics.

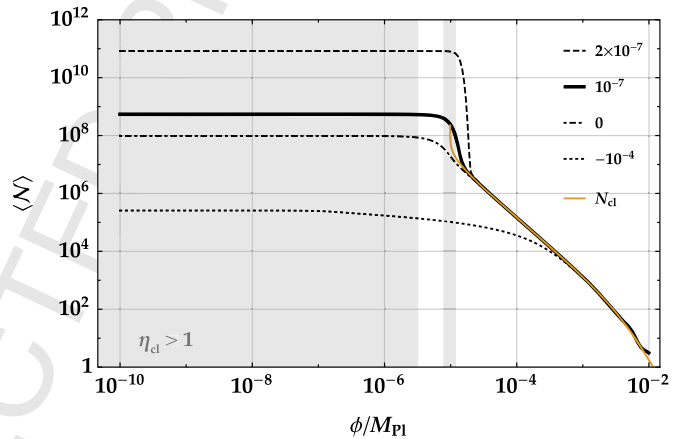


Fig. 2. The averaged e -folds $\langle \mathcal{N} \rangle(\phi)$ given by Eq. (3) for different masses, $m^2/H^2 = (2 \times 10^{-7}, 10^{-7}, 0, -10^{-4})$, where $H = \Lambda^2 / \sqrt{3} M_{\text{Pl}}$ is the Hubble parameter around the origin. The other parameters are fixed by the CMB data as Eq. (8). The shaded region and the orange line represent $\eta_{cl} > 1$ and $N_{cl}(\phi)$ given by Eq. (5), respectively, for $m^2/H^2 = 10^{-7}$. For $\phi \gtrsim 10^{-5} M_{\text{Pl}}$, the inflaton is to the right of the local maximum, and $\langle \mathcal{N} \rangle$ is reduced to the classical formula N_{cl} without the diffusion. Then, a sufficiently long standard slow-roll inflation follows. For $m^2/H^2 \gtrsim 10^{-7}$ the expected e -folds are enhanced due to the classical region with $\eta_{cl} < 1$ between the local minimum and maximum.

origin. For this or larger mass, the local minimum becomes deep enough so that the classical region with $\eta_{cl} < 1$ appears between the local minimum and maximum. The classical region efficiently pushes the inflaton back to the local minimum and the expected e -folds will be enhanced significantly as shown below. The stochasticity itself is high enough as $\xi_{\text{sto}} > 1$ for $\phi \lesssim 10^{-4} M_{\text{Pl}}$ enclosing the whole local minimum and maximum.

Fig. 2 shows the results of the numerical integration of $\langle \mathcal{N} \rangle(\phi)$ in Eq. (3) for different masses. One can see that, at sufficiently small ϕ , the expected e -folds is saturated due to the stochastic effects, which sweep out the inflaton from the hilltop, $\phi \lesssim 10^{-5} M_{\text{Pl}}$. For $\phi \gtrsim 10^{-5} M_{\text{Pl}}$, the inflaton already passes to the right of the local maximum (in the case of $m^2 > 0$), and $\langle \mathcal{N} \rangle$ is reduced to the classical formula N_{cl} in Eq. (5) even though the stochasticity ξ_{sto} itself is large until around $\phi \lesssim 10^{-4} M_{\text{Pl}}$. The usual standard slow-roll inflation will take place afterwards, generating the primordial density perturbations consistent with the CMB observation. There is a slight wiggling around $\phi \sim 10^{-2} M_{\text{Pl}}$, which, however, is caused

³ By adding a linear term [36] or the Coleman-Weinberg correction [37], one can increase n_s to give a better fit to observation.

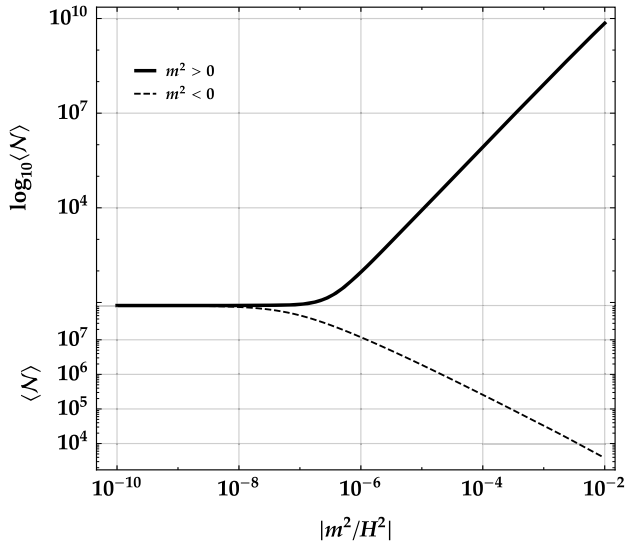


Fig. 3. The dependence of $\langle \mathcal{N} \rangle$ ($\phi = 0$) on m^2 . Again the other parameters are given by Eq. (8). For sufficiently large and positive mass squared (solid line) (but still smaller than H^2), extremely long inflation is realized. Typical e -folds can be as large as $\langle \mathcal{N} \rangle \sim 10^{10}$. The numerical result is consistent with the analytic estimate (6). The case of the negative mass squared (dashed line) is also shown for comparison.

by the numerical error, and not due to any physical effect. The expected e -folds become enhanced for $m^2/H^2 \gtrsim 10^{-7}$ as mentioned.

We show in Fig. 3 the m^2 -dependence of the e -folds from the origin $\phi = 0$. We have confirmed that the analytic estimate of the escape time (6) agrees well with our numerical result. One can see that an extremely large number of e -folds as $\langle \mathcal{N} \rangle \sim 10^{10}$ can be realized with a deep enough local minimum. Even in such a case, the inflaton mass-squared itself is still smaller than the Hubble scale, so that the slow-roll stochastic approach remains valid. The typical e -folds in this model is large enough for various light scalars such the QCD axion to reach the Bunch-Davies distribution [38]. For instance, the required e -folds for the axion to reach the so-called Bunch-Davies distribution is $N \gtrsim 10^{26} (H_{\text{int}}/100 \text{ MeV})^2 / (m_a/10^{-5} \text{ eV})^2$ [2,3].⁴ For comparison, we also show the result for the usual case with the negative mass-squared, where the averaged e -folding number decreases as expected.

4. Discussion and conclusions

So far we have studied a simple hilltop inflation with a shallow local minimum at the origin. One can realize a hilltop inflation in terms of an axion field which enjoys discrete symmetry [39,40]. In this case a small modulation(s) can be introduced, which may induce multiple local minima around the hilltop. See Refs. [41–43] for cosmological implications of such small modulations on the inflaton potential. More comprehensive study for wider classes of the inflaton potential is left for future work.

If m^2 is taken to be larger than H^2 in the inflation model (7), the local minimum becomes deeper, and the stochastic formalism breaks down. The inflaton trapped in the local minimum may tunnel to the right of the potential barrier by the bubble formation, followed by slow-roll inflation as in the case of the original new inflation [44,45]. For even larger m^2 , the situation will be similar to the old inflation [46–48]. In this sense, our scenario corresponds to a limit of the shallow local minimum for which the stochastic formalism can be applied. Interestingly, our scenario inherits the

advantages of the old and new inflation: the typical e -folds can be extremely large as in the old inflation, and slow-roll inflation naturally follows afterward as in the new inflation.

In this Letter, we have studied a possibility to realize extremely long inflation in the simple single-field slow-roll framework. In particular we work on a small-field inflation, which can be well described by an effective field theory. In the usual eternal inflation picture, the quantum diffusion dominates over the classical dynamics, and it keeps the inflaton from rolling down the potential in some spatial regions. The inflation continues in such regions, which subsequently dominate the volume of the universe even if the probability to continue inflation is small. On the other hand, we have explored a possibility to realize an extremely large number of typical e -folds $\langle \mathcal{N} \rangle$, i.e. the expected e -folding number without resort to the volume measure. We have shown that this is indeed possible by introducing a shallow local minimum around the hilltop, which was numerically confirmed using a simple toy model: see Fig. 3 showing the dependence of $\langle \mathcal{N} \rangle$ on the curvature of the local minimum, which can be understood as the Hawking-Moss decay rate (6) [33,34]. It is the classical region with $\eta_{\text{cl}} < 1$ to the left of the local maximum that pushes back the inflaton to the local minimum, and forces the inflaton to stay there for an extremely long time. Although extremely unlikely, it is possible for the inflaton to move to the right of the local maximum due to the accumulated quantum diffusion. Then, after moving to the right of the local maximum, standard slow-roll inflation follows as shown in Fig. 2 and generates the primordial density perturbation consistent with observation.

We emphasize that our scenario does not rely on the volume measure and thus does not need any fine-tuning of the spatial position to realize extremely long inflation. Moreover, the eternal inflation and the slow-roll inflation phases are smoothly connected in a single-field regime, without any processes of tunneling and bubble nucleation. Therefore, it provides an alternative possibility to naturally realize extremely large number of e -folds, which is often necessary in some cosmological scenarios.

Acknowledgements

F.T. thanks Wen Yin for discussion on the eternal inflation. This work is supported by JSPS KAKENHI Grant Numbers JP15H05889 (F.T.), JP15K21733 (F.T.), JP17H02875 (F.T.), JP17H02878 (F.T.), JP18J01992 (Y.T.), JP19K14707 (Y.T.), JP18H01243 (N.K.), JP19K14708 (N.K.), JP19H01894 (N.K.), and by World Premier International Research Center Initiative (WPI Initiative), MEXT, Japan.

References

- [1] P.W. Graham, D.E. Kaplan, S. Rajendran, Cosmological relaxation of the electroweak scale, *Phys. Rev. Lett.* 115 (22) (2015) 221801, <https://doi.org/10.1103/PhysRevLett.115.221801>, arXiv:1504.07551.
- [2] P.W. Graham, A. Scherlis, Stochastic axion scenario, *Phys. Rev. D* 98 (3) (2018) 035017, <https://doi.org/10.1103/PhysRevD.98.035017>, arXiv:1805.07362.
- [3] F. Takahashi, W. Yin, A.H. Guth, QCD axion window and low-scale inflation, *Phys. Rev. D* 98 (1) (2018) 015042, <https://doi.org/10.1103/PhysRevD.98.015042>, arXiv:1805.08763.
- [4] S.-Y. Ho, F. Takahashi, W. Yin, Relaxing the cosmological moduli problem by low-scale inflation, *J. High Energy Phys.* 04 (2019) 149, [https://doi.org/10.1007/JHEP04\(2019\)149](https://doi.org/10.1007/JHEP04(2019)149), arXiv:1901.01240.
- [5] F. Takahashi, W. Yin, QCD axion on hilltop by phase shift of π , arXiv:1908.06071.
- [6] C. Ringeval, T. Suyama, T. Takahashi, M. Yamaguchi, S. Yokoyama, Dark energy from primordial inflationary quantum fluctuations, *Phys. Rev. Lett.* 105 (2010) 121301, <https://doi.org/10.1103/PhysRevLett.105.121301>, arXiv:1006.0368.
- [7] C. Ringeval, T. Suyama, M. Yamaguchi, Large mass hierarchy from a small non-minimal coupling, *Phys. Rev. D* 99 (12) (2019) 123524, <https://doi.org/10.1103/PhysRevD.99.123524>, arXiv:1903.03544.
- [8] J. Torrado, C.T. Byrnes, R.J. Hardwick, V. Vennin, D. Wands, Measuring the duration of inflation with the curvature, *Phys. Rev. D* 98 (6) (2018) 063525, <https://doi.org/10.1103/PhysRevD.98.063525>, arXiv:1712.05364.

⁴ The typical e -folds can be even larger if one considers a lower inflation scale.

- [9] K. Enqvist, T. Meriniemi, S. Nurmi, Higgs dynamics during inflation, *J. Cosmol. Astropart. Phys.* 1407 (2014) 025, <https://doi.org/10.1088/1475-7516/2014/07/025>, arXiv:1404.3699.
- [10] R. Bousof, J. Polchinski, Quantization of four form fluxes and dynamical neutralization of the cosmological constant, *J. High Energy Phys.* 06 (2000) 006, <https://doi.org/10.1088/1126-6708/2000/06/006>, arXiv:hep-th/0004134.
- [11] L. Susskind, The anthropic landscape of string theory, in: Bernard Carr (Ed.), *Universe or Multiverse?*, 2003, pp. 247–266, arXiv:hep-th/0302219.
- [12] B. Freivogel, M. Kleban, M. Rodriguez Martinez, L. Susskind, Observational consequences of a landscape, *J. High Energy Phys.* 03 (2006) 039, <https://doi.org/10.1088/1126-6708/2006/03/039>, arXiv:hep-th/0505232.
- [13] T. Higaki, F. Takahashi, Natural and multi-natural inflation in axion landscape, *J. High Energy Phys.* 07 (2014) 074, [https://doi.org/10.1007/JHEP07\(2014\)074](https://doi.org/10.1007/JHEP07(2014)074), arXiv:1404.6923.
- [14] T. Higaki, F. Takahashi, Axion landscape and natural inflation, *Phys. Lett. B* 744 (2015) 153–159, <https://doi.org/10.1016/j.physletb.2015.03.052>, arXiv:1409.8409.
- [15] G. Wang, T. Battefeld, Vacuum selection on axionic landscapes, *J. Cosmol. Astropart. Phys.* 1604 (04) (2016) 025, <https://doi.org/10.1088/1475-7516/2016/04/025>, arXiv:1512.04224.
- [16] A. Masoumi, A. Vilenkin, Vacuum statistics and stability in axionic landscapes, *J. Cosmol. Astropart. Phys.* 1603 (03) (2016) 054, <https://doi.org/10.1088/1475-7516/2016/03/054>, arXiv:1601.01662.
- [17] T.C. Bachlechner, K. Eckerle, O. Janssen, M. Kleban, The axidental universe, arXiv:1902.05952.
- [18] A.D. Linde, *Nonsingular Regenerating Inflationary Universe*, 1982.
- [19] P.J. Steinhardt, Natural inflation, in: *Nuffield Workshop on the Very Early Universe*, Cambridge, England, June 21–July 9, 1982, pp. 251–266.
- [20] A. Vilenkin, The birth of inflationary universes, *Phys. Rev. D* 27 (1983) 2848, <https://doi.org/10.1103/PhysRevD.27.2848>.
- [21] A.D. Linde, Eternal chaotic inflation, *Mod. Phys. Lett. A* 1 (1986) 81, <https://doi.org/10.1142/S0217732386000129>.
- [22] A.D. Linde, Eternally existing selfreproducing chaotic inflationary universe, *Phys. Lett. B* 175 (1986) 395–400, [https://doi.org/10.1016/0370-2693\(86\)90611-8](https://doi.org/10.1016/0370-2693(86)90611-8).
- [23] A.S. Goncharov, A.D. Linde, V.F. Mukhanov, The global structure of the inflationary universe, *Int. J. Mod. Phys. A* 2 (1987) 561–591, <https://doi.org/10.1142/S0217751X87000211>.
- [24] A.H. Guth, Inflation and eternal inflation, *Phys. Rep.* 333 (2000) 555–574, [https://doi.org/10.1016/S0370-1573\(00\)00037-5](https://doi.org/10.1016/S0370-1573(00)00037-5), arXiv:astro-ph/0002156.
- [25] A.H. Guth, Eternal inflation and its implications, *J. Phys. A* 40 (2007) 6811–6826, <https://doi.org/10.1088/1751-8113/40/25/S25>, arXiv:hep-th/0702178.
- [26] A. Linde, A brief history of the multiverse, *Rep. Prog. Phys.* 80 (2) (2017) 022001, <https://doi.org/10.1088/1361-6633/aa50e4>, arXiv:1512.01203.
- [27] G. Barenboim, W.-I. Park, W.H. Kinney, Eternal hilltop inflation, *J. Cosmol. Astropart. Phys.* 1605 (05) (2016) 030, <https://doi.org/10.1088/1475-7516/2016/05/030>, arXiv:1601.08140.
- [28] H. Assadullahi, H. Firouzjahi, M. Noorbala, V. Vennin, D. Wands, Multiple fields in stochastic inflation, *J. Cosmol. Astropart. Phys.* 1606 (06) (2016) 043, <https://doi.org/10.1088/1475-7516/2016/06/043>, arXiv:1604.04502.
- [29] A.A. Starobinsky, Stochastic de Sitter (inflationary) stage in the early universe, *Lect. Notes Phys.* 246 (1986) 107–126, https://doi.org/10.1007/3-540-16452-9_6.
- [30] A.A. Starobinsky, J. Yokoyama, Equilibrium state of a selfinteracting scalar field in the de Sitter background, *Phys. Rev. D* 50 (1994) 6357–6368, <https://doi.org/10.1103/PhysRevD.50.6357>, arXiv:astro-ph/9407016.
- [31] V. Vennin, A.A. Starobinsky, Correlation functions in stochastic inflation, *Eur. Phys. J. C* 75 (2015) 413, <https://doi.org/10.1140/epjc/s10052-015-3643-y>, arXiv:1506.04732.
- [32] C. Pattison, V. Vennin, H. Assadullahi, D. Wands, Quantum diffusion during inflation and primordial black holes, *J. Cosmol. Astropart. Phys.* 1710 (10) (2017) 046, <https://doi.org/10.1088/1475-7516/2017/10/046>, arXiv:1707.00537.
- [33] M. Noorbala, V. Vennin, H. Assadullahi, H. Firouzjahi, D. Wands, Tunneling in stochastic inflation, *J. Cosmol. Astropart. Phys.* 1809 (09) (2018) 032, <https://doi.org/10.1088/1475-7516/2018/09/032>, arXiv:1806.09634.
- [34] S.W. Hawking, I.G. Moss, Supercooled phase transitions in the very early universe, *Phys. Lett. B* 110 (1982) 35–38, [https://doi.org/10.1016/0370-2693\(82\)90946-7](https://doi.org/10.1016/0370-2693(82)90946-7), *Adv. Ser. Astrophys. Cosmol.* 3 (1987) 154.
- [35] N. Aghanim, et al., Planck 2018 results. VI. Cosmological parameters, arXiv:1807.06209.
- [36] F. Takahashi, New inflation in supergravity after Planck and LHC, *Phys. Lett. B* 727 (2013) 21–26, <https://doi.org/10.1016/j.physletb.2013.10.026>, arXiv:1308.4212.
- [37] K. Nakayama, F. Takahashi, PeV-scale supersymmetry from new inflation, *J. Cosmol. Astropart. Phys.* 1205 (2012) 035, <https://doi.org/10.1088/1475-7516/2012/05/035>, arXiv:1203.0323.
- [38] T.S. Bunch, P.C.W. Davies, Quantum field theory in de Sitter space: renormalization by point splitting, *Proc. R. Soc. Lond. A* 360 (1978) 117–134, <https://doi.org/10.1098/rspa.1978.0060>.
- [39] M. Czerny, F. Takahashi, Multi-natural inflation, *Phys. Lett. B* 733 (2014) 241–246, <https://doi.org/10.1016/j.physletb.2014.04.039>, arXiv:1401.5212.
- [40] M. Czerny, T. Higaki, F. Takahashi, Multi-natural inflation in supergravity, *J. High Energy Phys.* 05 (2014) 144, [https://doi.org/10.1007/JHEP05\(2014\)144](https://doi.org/10.1007/JHEP05(2014)144), arXiv:1403.0410.
- [41] R. Flauger, L. McAllister, E. Pajer, A. Westphal, G. Xu, Oscillations in the CMB from axion monodromy inflation, *J. Cosmol. Astropart. Phys.* 1006 (2010) 009, <https://doi.org/10.1088/1475-7516/2010/06/009>, arXiv:0907.2916.
- [42] T. Kobayashi, F. Takahashi, Running spectral index from inflation with modulations, *J. Cosmol. Astropart. Phys.* 1101 (2011) 026, <https://doi.org/10.1088/1475-7516/2011/01/026>, arXiv:1011.3988.
- [43] F. Takahashi, The spectral index and its running in axionic curvaton, *J. Cosmol. Astropart. Phys.* 1306 (2013) 013, <https://doi.org/10.1088/1475-7516/2013/06/013>, arXiv:1301.2834.
- [44] A.D. Linde, A new inflationary universe scenario: a possible solution of the horizon, flatness, homogeneity, isotropy and primordial monopole problems, *Phys. Lett. B* 108 (1982) 389–393, [https://doi.org/10.1016/0370-2693\(82\)91219-9](https://doi.org/10.1016/0370-2693(82)91219-9), *Adv. Ser. Astrophys. Cosmol.* 3 (1987) 149.
- [45] A. Albrecht, P.J. Steinhardt, Cosmology for grand unified theories with radiatively induced symmetry breaking, *Phys. Rev. Lett.* 48 (1982) 1220–1223, <https://doi.org/10.1103/PhysRevLett.48.1220>, *Adv. Ser. Astrophys. Cosmol.* 3 (1987) 158.
- [46] A.H. Guth, The inflationary universe: a possible solution to the horizon and flatness problems, *Phys. Rev. D* 23 (1981) 347–356, <https://doi.org/10.1103/PhysRevD.23.347>, *Adv. Ser. Astrophys. Cosmol.* 3 (1987) 139.
- [47] A.A. Starobinsky, A new type of isotropic cosmological models without singularity, *Phys. Lett. B* 91 (1980) 99–102, 771, [https://doi.org/10.1016/0370-2693\(80\)90670-X](https://doi.org/10.1016/0370-2693(80)90670-X).
- [48] K. Sato, First order phase transition of a vacuum and expansion of the universe, *Mon. Not. R. Astron. Soc.* 195 (1981) 467–479.