Through Scarcity to Prosperity: Toward a Theory of Sustainable Growth

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Highlights

- Paper integrates fertility choice and exhaustible resource dynamics in a tractable model of endogenous technological change.
- The goal is to make progress toward a comprehensive theory of sustainable growth that due to technical difficulties is still lacking.
- The paper proposes a theory of the de-coupling of the growth of living standards from the physical resource base.
- Clear characterization of the conditions for existence of a path leading to steady state with constant exponential growth of consumption per capita despite dependence on an essential natural resource that runs out due to exhaustion.

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Abstract

To make progress toward a comprehensive theory of sustainable growth, this paper integrates fertility choice and exhaustible resource dynamics in a tractable model of endogenous technological change. The model identifies conditions under which the interdependence of population, resources and technology produces a transition that consists of three phases: (1) an initial phase where agents exploit exhaustible natural resources to support population growth; (2) an intermediate phase where agents turn on the Schumpeterian engine of endogenous innovation in response to population-led market expansion; (3) a terminal phase where knowledge accumulation becomes the sole engine of growth. The last phase is crucial: not only economic growth no longer requires growth of physical inputs, but technological change also compensates for the exhaustion of the natural resource.

Keywords Endogenous Growth, Market Structure, Natural Resources.

JEL classification: E10, L16, O31, O40

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1. Introduction

One of the liveliest debates of our times concerns the sustainability of living standards in a world of limited, possibly vanishing, natural resources. To contribute to the debate, this paper integrates fertility choice and exhaustible resource dynamics in a tractable model of endogenous technological change. It then shows that under the right conditions the interdependence of population, resources and technology produces a transition from unsustainable resource-based growth to sustainable knowledge-based growth that consists of three phases:

1. an initial phase where agents build up the economy by exploiting exhaustible natural resources to support population growth;
2. an intermediate phase where agents turn on the Schumpeterian engine of endogenous innovation in response to population-led market expansion;
3. a terminal phase where economic growth becomes fully driven by knowledge accumulation and no longer requires growth of physical inputs.

The last phase is crucial: not only economic growth no longer requires a growing physical resource base, but technological change also compensates for the *exhaustion* of the natural resource stock.

The paper thus proposes a theory of the de-coupling of the growth of living standards from the physical resource base that allows one to investigate analytically issues that to date have been quite challenging. In particular, the theory provides a clear characterization of the conditions under which the economy possesses a steady state with constant exponential growth of consumption per capita despite its dependence on an essential natural resource that runs out due to exhaustion.[1] The characterization provides insights about possible interventions that can ensure sustainability in case the economy fails to meet those conditions.

The paper contributes to a large literature that has accomplished much but that still faces open questions. The analytical framework used to study the relation between re-

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[1] This definition of sustainability focuses the exercise and avoids the vast number of issues that arise when one tries to define the concept of "sustainability" without referring to the behavior of a specific variable in a specific model; see Pezzey and Toman (2002, 2005) for comprehensive discussions.

source scarcity and economic growth emphasizes the role of exhaustible natural resources in generating diminishing returns to other physical inputs that worsen over time as natural resources run out.[2] In the last two decades researchers have extended the scope of the analysis, initially limited to the neoclassical model of capital accumulation, to incorporate insights from the theory of endogenous innovation (see Barbier 1999 for a pioneering contribution and Smulders 2005 for an insightful review of approaches and results). The need to do so emerges clearly from Stiglitz's (1974a,b) classic treatment of the scarcity question, which concluded that technological change is the key force capable of compensating for resource exhaustion. It is thus clear that understanding where it comes from, at what cost, and what possible institutional changes should be implemented to provide the right incentives for it to happen, must be a key component of the analysis.

A similar understanding has gradually emerged concerning demographic forces: it is now widely recognized that population dynamics must be a key *endogenous* component of analyses that project the model forward over long time horizons to explore sustainability (see Bloom and Canning 2001 for a comprehensive discussion). It is thus important to understand the incentives and constraints that drive reproductive decisions. Doing so requires investigating the complex interactions between traditional Malthusian forces—population expansion puts pressure on the natural environment—and modern Schumpeterian forces—population expansion creates the larger market that ignites and sustains endogenous innovation.

Embracing these insights, this paper takes an integrated view that expands the focus from the resource economist's traditional concern with the asymptotic behavior of the economy under increasing scarcity to the system's global dynamics, with special emphasis on the phase transitions that mark shifts to qualitatively different behaviors. This broader focus provides a different vision of the dynamic forces at play. In the first phase of the model's transition, for example, the economy does not invest resources in the generation of technological change and thus it looks like it is just exploiting the natural environment

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[2] The foundations of the framework, often referred to as the DHSS framework, where laid in the 70s by Solow (1974), Stiglitz (1974a,b) and Dasgupta and Heal (1974, 1979). There is now a vast literature elaborating the original insights provided by these contributions. For excellent reviews, see Simpson, Toman and Ayres (2005) and Brock and Taylor (2005). See Barbier and Markandya (1990) for an early attempt at identifying conditions for sustainability in the context of the DHSS framework, in a spirit similar to that of this paper.

to expand the population. Without further consideration, such a situation looks clearly unsustainable. What the model says, however, is that this phase of population expansion is in fact sowing the seeds of future growth because it creates the critical market size needed to support investment in new technology by profit-driven firms. The full fruition of such initial, seemingly unsustainable, development arrives in the third and final phase because—if the conditions are right—the economy reaches a steady state where the rate of *endogenous* technological change is sufficiently fast to compensate for resource exhaustion. Moreover, in this phase the rate of endogenous technological change is divorced from population dynamics so that sustainability is possible even if population ceases growing (or even shrinks).

Given its emphasis on the interaction between population and an exhaustible natural resource, the paper is related to the literature on the rise and fall of civilizations, although most of that literature considers models of renewable resources (see Taylor 2009 for a review). Such models generate rich dynamics, with possible environmental crises that can result in human extinction, and in some examples have been calibrated to replicate the collapse of Easter Island and similar historical episodes (e.g., Brander and Taylor 1998). This literature, however, ignores endogenous technological change and thus provides a very different perspective on sustainable growth from that developed here.

A notable recent contribution is Bretschger (2013) who considers poor substitution (complementarity) between labor and an exhaustible resource in a Romer-style model of endogenous growth that exhibits the strong scale effect. To my knowledge that is the first attempt at integrating in a single model the dynamics of population, exhaustible resources and technology. The analysis developed in this paper builds on the insights developed there and extends the framework to a more comprehensive model of endogenous technological change capable of producing the rich transition described above. Another important difference is that Bretschger (2013) allows for a backstop technology triggered by a sufficiently high resource price. This paper, instead, sets up the harshest possible environment in which economic activity takes place and thereby sets the highest possible bar for technology to clear to deliver sustainability.

2. The model

The economy is closed. All variables are functions of (continuous) time but to simplify the notation the time argument is omitted unless necessary to avoid confusion. The model is of the "endogenous growth and endogenous market structure" class. This particular class of models features two dimensions of technology that play interdependent but distinct roles: the vertical dimension (here, quality) provides the engine of growth, the horizontal dimension provides the endogenous market structure (here, mass of firms). This framework provides a natural way to integrate insights from Industrial Organization in the theory of innovation-driven growth. A large literature uses it to study applied issues ranging from the general role of imperfect competition in the growth process, to taxation (with special focus on corporate taxation), corporate governance, natural resource scarcity, the interaction between demography and technology and so on.[3]

2.1. Final producers

A competitive representative firm produces a final good Y that can be consumed, used to produce intermediate goods, invested in the improvement of the quality of existing intermediate goods, or invested in the creation of new intermediate goods. The final good is the numeraire so its price is $P_Y \equiv 1$. The production technology is

$$Y = \int_0^N X_i^\theta \left(Z_i^\alpha Z^{1-\alpha} \frac{L^\gamma R^{1-\gamma}}{N^{1-\sigma}} \right)^{1-\theta} di, \quad 0 < \theta, \alpha, \gamma, \sigma < 1 \tag{1}$$

where N is the mass of non-durable intermediate goods and L and R are, respectively, services of labor and an exhaustible natural resource.[4] Quality is the good's ability to raise the productivity of the other factors: the contribution of good i depends on its own quality, Z_i, and on average quality $Z = \int_0^N (Z_j/N)\, dj$. The technology features social

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[3] For examples of contributions that laid the foundations of the approach, see among others Peretto (1998 and 1999). This paper builds on the version of the approach developed in Peretto (2015). To avoid repetition, whenever appropriate the interested reader is referred to that paper for details of the model's production structure not discussed here in full. The main innovations here are endogenous fertility choice and endogenous natural resource dynamics. Peretto (2015) works with exogenous constant population growth and a natural resource in exogenous constant inelastic supply (e.g., land).

[4] To keep things simple, there is no physical capital. More precisely, there is no physical capital in the neoclassical sense of a homogenous, durable, intermediate good accumulated through foregone consumption. Instead, there are differentiated, non-durable, intermediate goods produced through foregone consumption. One can think of these goods as capital, albeit with 100% instantaneous depreciation. Introducing neoclassical physical capital complicates the analysis without adding insight.

returns to variety of degree σ and social returns to quality of degree 1.[5]

The first-order conditions for the profit maximization problem of the final producer yield that each intermediate producer faces the demand curve

$$X_i = \left(\frac{\theta}{P_i}\right)^{\frac{1}{1-\theta}} Z_i^{\alpha} Z^{1-\alpha} \frac{L^{\gamma} R^{1-\gamma}}{N^{1-\sigma}}, \tag{2}$$

where P_i is the price of good i. Let w denote the wage and p denote the resource price. The first-order conditions then yield that the final producer pays total compensation

$$\int_0^N P_i X_i di = \theta Y,\ wL = \gamma(1-\theta)Y \text{ and } pR = (1-\gamma)(1-\theta)Y \tag{3}$$

to intermediate goods, labor and resource suppliers, respectively.

Three considerations drive the choice of the Cobb-Douglas structure in equation (1). First, the literature on sustainability has mainly focused on that formulation (see, e.g., Brock and Taylor 2005 and, especially, Stiglitz 1974a) and it is useful to derive the paper's insights in a framework that is directly comparable to that benchmark. Second, the Cobb-Douglas structure is the simplest way to postulate *essentiality* of the inputs, especially the exhaustible natural resource. Third, specifications that allow for the compensation shares in (3) to be endogenous—maintaining essentiality of the natural resource by postulating low elasticity of substitution—typically force researchers to limit the analysis to the asymptotic behavior of the economy or to rely on numerical simulations. Since the goal of this paper is to offer fresh analytical insight on complex dynamics, working with the simpler specification is more fruitful. In addition to these technical considerations, one should also note that, as explained in detail section 3.3 below, in this model final output Y is not GDP and therefore the factor shares traditionally defined differ from the compensation shares in (3) and are endogenous equilibrium objects.[6] Consequently, the Cobb-Douglas structure in (1) is truly just a simplifying assumption.

2.2. *Intermediate producers*

The typical intermediate firm operates a technology that requires one unit of final output per unit of intermediate good and a fixed operating cost $\phi Z_i^{\alpha} Z^{1-\alpha}$, also in units

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[5] See Peretto (2015) for an interpretation of σ in terms of economies of scope and congestion effects in the use of intermediate goods, labor and natural resources.

[6] See Peretto (2015) for a discussion of this property.

of final output. The firm can increase quality according to the technology

$$\dot{Z}_i = I_i, \tag{4}$$

where I_i is R&D in units of final output. Using (2), the firm's gross profit is

$$\Pi_i = \left[(P_i - 1) \left(\frac{\theta}{P_i} \right)^{\frac{1}{1-\theta}} \frac{L^\gamma R^{1-\gamma}}{N^{1-\sigma}} - \phi \right] Z_i^\alpha Z^{1-\alpha}. \tag{5}$$

The firm chooses the time path of its price, $P_i(t)$, and R&D, $I_i(t)$, to maximize

$$V_i(0) = \int_0^\infty e^{-\int_0^t r(s)ds} \left[\Pi_i(t) - I_i(t) \right] dt \tag{6}$$

subject to (4) and (5), where r is the interest rate and 0 is the point in time when the firm makes decisions. The firm takes average quality, Z, in (5) as given. The characterization of the firm's decision yields a unique and symmetric industry equilibrium where

$$r = \alpha \frac{\Pi}{Z} \tag{7}$$

is the return to quality innovation (derivation in the appendix) and α is intuitively interpreted as the elasticity of the firm's gross profit with respect to its own quality.[7]

At time t, an agent who wants to create a new firm must sink $\beta X(t)$ units of final output, where $X = \int_0^N (X_j/N)\, dj$. Because of this sunk cost, the new firm cannot supply an existing good in Bertrand competition with the incumbent monopolist but must introduce a new good. New firms enter at the average quality level, Z, and therefore at average size (this simplifying assumption preserves symmetry of equilibrium at all times), and finance entry by issuing equity. Entry is positive if the value of the firm is equal to its setup cost, i.e., if the free-entry condition $V_i = \beta X$ holds. Taking logs and time derivatives of the free-entry condition and of the value of the firm in (6), and imposing symmetry, yields the return to variety innovation

$$r = \frac{\Pi - I}{\beta X} + \frac{\dot{X}}{X}. \tag{8}$$

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[7] See Peretto (2015) for a review of the conditions that deliver symmetric equilibria in models of this class. In this paper, the conditions essentially reduce to: (a) the firm-specific return to quality innovation is decreasing in Z_i, which follows from the assumption $\alpha < 1$; (b) the economy starts with a symmetric (i.e., degenerate) distribution of initial values $Z_i(0)$ and at any time $t \geq 0$ entrants enter at the average level of quality $Z(t)$ (see below). The first property implies that if one holds constant the mass of firms and starts the model from an asymmetric (i.e., non-degenerate) distribution of firm sizes, then the model converges to a symmetric distribution. The second ensures that entrants do not perturb such initial symmetric distribution. The interested reader can find a thorough discussion of these arguments in the papers that laid the foundations of the endogenous growth and endogenous market structure framework, especially Peretto (1998 and 1999).

2.3. Households

The economy is populated by a continuum of measure one of identical households that supply labor services and purchase financial assets in competitive labor and asset markets. The typical household has preferences:

$$U(0) = \int_0^\infty e^{-\rho t} u(t)\, dt, \quad \rho > 0; \tag{9}$$

$$u(t) = \mu \log\left(C_M(t) M(t)^\eta\right) + (1-\mu)\log\left(C_B(t) B(t)^\eta\right), \quad 0 < \mu, \eta < 1. \tag{10}$$

In equation (9), 0 is the point in time when the household makes decisions and ρ is the discount rate. In equation (10), C_M is consumption per adult, M is the mass of adults, C_B is consumption per child, B is the mass of children. The mass of adults evolves according to

$$\dot{M} = B - \delta M, \quad M_0 > 0,\ \delta > 0, \tag{11}$$

where δ is the exogenous death rate.

In this structure, the decision maker cares about utility of adults and utility of children with weights μ and $1-\mu$. Adults and children derive utility from their individual consumption and from the mass of adults and the mass of children. The parameter η regulates the trade-off between consumption per adult (child) and the mass of adults (children). Childhood lasts for one instant and then the child becomes a productive adult. Children consume but do not work.

The household owns an initial stock S_0 of an exhaustible resource and thus faces the constraints

$$S_0 \geq \int_0^\infty R(t)\, dt, \quad R \geq 0, \quad S_0 > 0, \quad \dot{S} = -R, \tag{12}$$

where R is the flow of the resource that the household sells for price p. Each adult is endowed with one unit of labor that he supplies entirely in the labor market. Since children do not work, the household faces the flow budget constraint

$$\dot{A} = rA + wM + pR - C_M M - C_B B, \quad A_0 \gtreqless 0, \tag{13}$$

where A is assets holding, r is the rate of return on assets and w is the wage.

3. The economy's general equilibrium

This section characterizes first the behavior of the household. It then imposes general equilibrium conditions and characterizes how market interactions determine the dynamics of resource supply and use. Finally, it characterizes how these dynamics drive the evolution of the economy.

3.1. Household behavior

The following exposition focuses on intuition, see the appendix for the detailed derivation. Let $C \equiv C_M M + C_B B$ be total household consumption. The first-order conditions for consumption per adult, C_M, and consumption per child, C_B, yield $C = C_M M + C_B B = 1/\lambda_A$, where λ_A is the shadow value of financial wealth. This expression says that at any point in time consumption equals the inverse of the shadow value of financial wealth. Although this is not the traditional condition that the marginal utility of consumption equal the shadow value of wealth, in light of the logarithmic preferences it ends up having the same interpretation (see the appendix for the analytical details), namely that the intertemporal trade-off compares the benefit of consuming today to the benefit of postponing current consumption and investing in financial assets.

Now let the ratio of consumption to final output (henceforth consumption ratio for short) be $c \equiv C/Y$ and births per adult (henceforth birth rate for short) be $b \equiv B/M$. The empirical counterpart of b is the crude birth rate (often called CBR). The empirical counterpart of c is not the traditional one minus the saving rate, because Y is not GDP (see below for the formal mapping between c and GDP), but this variable plays the same role in the characterization of the consumption-saving path. Manipulation of the first-order conditions for consumption and financial wealth, A, yields

$$r = \rho + \frac{\dot{C}}{C} = \rho + \frac{\dot{c}}{c} + \frac{\dot{Y}}{Y}. \tag{14}$$

As anticipated, the household's consumption-saving decision yields the familiar Euler equation from the simpler structure with no fertility decision.

The first-order conditions for fertility, B, financial wealth, A, and adult population,

M, yield the fertility rule

$$\frac{(1-\mu)\,\eta}{B}+\lambda_M=\lambda_A C_B, \tag{15}$$

where λ_M is the shadow of a working adult, and the asset-pricing-like equation

$$\frac{\eta+\lambda_A\,(wM-C)}{\lambda_M M}+\left(\frac{\dot{\lambda}_M}{\lambda_M}+\frac{\dot{M}}{M}\right)=\rho. \tag{16}$$

Equation (15) says that the household equates the marginal benefit of a child to the marginal cost. The former is the child's contribution to current utility, the term $(1-\mu)\,\eta/B$, plus his shadow value as a future working adult, the term λ_M. The marginal cost is the child's consumption, C_B, evaluated at the marginal cost of spending on consumption rather than on wealth accumulation, the term λ_A. Equation (16) says that the household views fertility as investment in an asset, a working adult, that pays a stream of dividends in the future. Along the utility-maximizing path, the household equates the return generated by this asset to the discount rate, ρ. The return has a dividend-price ratio component and a capital gain-loss component. The former, consists of the contribution of adults to current utility, the term η, plus their net contribution to financial wealth accumulation, the term $\lambda_A\,(wM-C)$. The two conditions collapse to

$$\frac{\dot{b}}{b}=\left[\frac{\gamma\,(1-\theta)}{c\,(1-\eta)}-1\right]\frac{b}{1-\mu}-\rho. \tag{17}$$

This simple expression describes the utility-maximizing dynamics of the birth rate.

Finally, the result $C=1/\lambda_A$, the first-order conditions for the extraction flow R and the resource stock S plus the Euler equation (14) yield the traditional Hotelling rule

$$\frac{\dot{p}}{p}=\rho+\frac{\dot{C}}{C}=r, \tag{18}$$

stating that the household wants to follow an extraction path such that the resource price, p, grows at the rate of interest.

3.2. The equilibrium resource extraction path

The natural resource market clears when the flow of the resource supplied by the household equals the final sector demand, i.e., $pR=(1-\gamma)\,(1-\theta)\,Y$. Log-differentiating this expression and using the Hotelling rule (18) yields

$$\frac{\dot{R}}{R}=\frac{\dot{Y}}{Y}-\frac{\dot{p}}{p}=\frac{\dot{Y}}{Y}-r=-\left(\frac{\dot{c}}{c}+\rho\right). \tag{19}$$

Integrating this expression and defining the average growth rate of the extraction flow between time 0 and time t, i.e., $\varepsilon(t) \equiv \frac{1}{t}\int_0^t (\dot{c}(s)/c(s)+\rho)\,ds$, yields $R(t) = R_0 e^{-\varepsilon(t)t}$. Substituting this result into the constraint $S_0 = \int_0^\infty R(t)\,dt$ yields

$$R_0 = \left[\int_0^\infty e^{-\varepsilon(t)t}dt\right]^{-1} \cdot S_0, \tag{20}$$

where the term in brackets is a constant that depends on the fundamentals. Therefore, the resource extraction path is

$$R(t) = \frac{e^{-\varepsilon(t)t}}{\int_0^\infty e^{-\varepsilon(t)t}dt} \cdot S_0 \tag{21}$$

and the resource stock evolves according to

$$S(t) = S_0 - \int_0^t R(s)\,ds = S_0 \cdot \left[1 - \frac{\int_0^t e^{-\varepsilon(s)s}ds}{\int_0^\infty e^{-\varepsilon(t)t}dt}\right], \tag{22}$$

converging to zero as $t \to \infty$.

This path says that the forward-looking representative household chooses the initial extraction flow R_0 as proportional to the endowment S_0 and thereafter follows equation (21), which ties the extraction flow, R, to the growth rate of the consumption ratio, c. The logic is that the household takes into account that in order to sustain faster consumption growth it needs to extract more aggressively and balances the benefit of so extracting against the benefit of leaving the resource in the ground and reaping higher future scarcity rents.

3.3. GDP and market structure dynamics

Labor market clearing yields $L = M$. The equilibrium of the intermediate sector is unique and symmetric because firms make identical decisions and entrants enter at average knowledge. Using the demand schedule (2) to eliminate X, the production function (1) yields

$$Y = \theta^{\frac{2\theta}{1-\theta}} \cdot N^\sigma Z M^\gamma R^{1-\gamma}, \tag{23}$$

where $N^\sigma Z$ is Hicks-neutral TFP in the final output sector.

Equations (7)-(8) and the definition of gross profit (5) say that the returns to innovation are functions of the *quality-adjusted* size of the firm (henceforth firm size for short)

$x_i \equiv X_i/Z_i$, which in symmetric equilibrium reads $x_i = x = X/Z$. Since the final producer pays total compensation $N \cdot PX = \theta Y$ to intermediate producers and intermediate producers set $P = 1/\theta$, one has $NX = \theta^2 Y$. Substituting these results in the definition of firm size and using the reduced-form production function (23) yields

$$x = \frac{X}{Z} = \frac{NX}{NZ} = \frac{\theta^2 Y}{NZ} = \theta^{\frac{2}{1-\theta}} \cdot \frac{M^{\gamma} R^{1-\gamma}}{N^{1-\sigma}}. \tag{24}$$

Next, let G denote this economy's GDP. Subtracting the cost of intermediate production from the value of final production and using (24) yields GDP per worker (equivalently, adult) as

$$\frac{G}{M} = \underbrace{\theta^{\frac{2\theta}{1-\theta}} \left[1 - \theta^2 \left(1 + \frac{\phi}{x}\right)\right] \cdot N^{\sigma} Z}_{overall\ TFP} \cdot \underbrace{\left(\frac{R}{M}\right)^{1-\gamma}}_{resources\ per\ worker}. \tag{25}$$

This expression says that output per worker rises with efficiency (firms' average scale), technology (product variety and average quality) and with resource abundance per worker. What is different from the typical construct of growth economics is that the flow of the resource $R(t)$ obeys the Hotelling extraction path characterized by equations (19)-(22).

3.4. Key components of the equilibrium dynamical system

The following results describe key properties of the model's general equilibrium.

Lemma 1 *Denote the rates of variety and quality innovation, respectively,* $n \equiv \dot{N}/N$ *and* $z \equiv \dot{Z}/Z$. *Denote the growth rate of adult population (the workforce)* $m \equiv \dot{M}/M$ *and the growth rate of GDP per worker* $g \equiv \dot{G}/G - m$. *Let also*

$$\xi(x) \equiv \frac{\theta^2 \phi/x}{1 - \theta^2 (1 + \phi/x)} \tag{26}$$

be the elasticity of GDP with respect to firm size. At any point in time, the interest rate and the growth rate of GDP per worker are, respectively:

$$r = \sigma n + z + \gamma (m + \dot{c}/c + \rho); \tag{27}$$

$$g = \underbrace{\sigma n + z + \xi(x) \cdot (\dot{x}/x)}_{TFP\ growth} - \underbrace{(1-\gamma)(m + \dot{c}/c + \rho)}_{growth\ drag}. \tag{28}$$

Proof. See the Appendix. ■

In words, the growth rate of GDP per worker is the growth rate of TFP minus the *growth drag* due to the presence of the natural resource. The drag is equal to the share of the natural resource, $1-\gamma$, times the sum of the growth rate of adult population (i.e., the growth rate of the workforce), m, and the rate of exhaustion of the (flow) supply of the resource, $\dot{c}/c+\rho$.

It is worth highlighting the difference between variables expressed as per worker versus per capita. Recall that $b = B/M$ denotes births per adult. The fertility rate defined as births per capita is $B/(B+M)$. Similarly, GDP per capita is $G/(B+M)$. It follows that the growth rate of GDP per capita is

$$g - \frac{b}{b+1}.\frac{\dot{b}}{b}. \tag{29}$$

There is thus an additional drag at play: when births per adult grow, GDP per capita growth falls below GDP per worker growth. Noting that $b/(b+1) = B/(B+M)$ provides the interpretation: this term is the dependency ratio and is itself rising as long as b rises. It follows that a path with rising births per adult exhibits a widening gap between growth of GDP per worker and growth of GDP per capita because the fraction of the population that does not work is rising.

The intermediate goods sector evolves as follows.

Lemma 2 *Using the definition of firm size in equation (24), the returns to innovation in equations (7) and (8) become:*

$$r = \alpha\left[\left(\frac{1}{\theta}-1\right)x-\phi\right]; \tag{30}$$

$$r = \frac{1}{\beta}\left(\frac{1}{\theta}-1-\frac{\phi+z}{x}\right)+\frac{\dot{x}}{x}+z. \tag{31}$$

Firm size obeys the differential equation

$$\frac{\dot{x}}{x} = \underbrace{\gamma m-(1-\gamma)\left(\dot{c}/c+\rho\right)}_{\textit{market growth}} - \underbrace{(1-\sigma)\,n}_{\textit{market fragmentation}}. \tag{32}$$

Proof. See the Appendix. ■

These expressions capture the model's main property: decisions to invest in quality and variety innovation depend on (quality-adjusted) firm size. The evolution of (quality-adjusted) firm size, in turn, is driven by the difference between the term $\gamma m - (1-\gamma)(\dot{c}/c+\rho)$, which captures how adult population growth net of resource exhaustion drives the growth of the market for intermediate goods, and the term $(1-\sigma)\,n$, which captures how product proliferation net of the contribution of product variety to TFP growth fragments the overall market in smaller submarkets and thus reduces the profitability of the individual firm.

According to Lemma 1, whether the firms' investment decisions support positive growth of output per worker depends on whether the resulting rate of growth of TFP is larger than the growth drag; this is the classic condition for sustainability derived by Stiglitz (1974; see also Brock and Taylor 2005), with the difference that in this model TFP growth is endogenous and not necessarily positive. The reason is that from the perspective of the firm, innovation entails a sunk cost that is economically justified only when the anticipated revenue flow is sufficiently large.

Specifically, the non-negativity constraint on variety growth, $n \equiv \dot{N}/N \geq 0$, yields a threshold of firm size below which entry is zero because the return is too low. Similarly, the non-negativity constraint on quality growth, $z \equiv \dot{Z}/Z \geq 0$, yields a threshold of firm size below which incumbents do not do R&D because the return is too low. For simplicity, we focus on the case where the threshold for variety innovation, denoted x_N, is smaller than the threshold for quality innovation, denoted x_Z. The threshold x_N has a special role, stated formally as follows.

Lemma 3 *There are two regimes, one with entry and one with no entry. The expenditure behavior of the household in the two regimes is*

$$c = \begin{cases} \theta^2\left(\frac{1}{\theta}-1-\frac{\phi}{x}\right)+1-\theta & \phi/\left(\frac{1}{\theta}-1\right) < x \leq x_N \\ \rho\beta\theta^2+1-\theta & x > x_N \end{cases}. \tag{33}$$

The associated growth rate of the consumption ratio is

$$\frac{\dot{c}}{c} = \begin{cases} \xi(x)\,\frac{\dot{x}}{x} & \phi/\left(\frac{1}{\theta}-1\right) < x \leq x_N \\ 0 & x > x_N \end{cases}. \tag{34}$$

Proof. See the Appendix. ■

In words, when entry is zero incumbents earn rents that are increasing in firm size, x, and, since they are distributed to the household as dividends, yield that the consumption ratio, $c \equiv C/Y$, is increasing in x. When entry is positive, instead, such rents are arbitraged away and c is constant.

3.5. The equilibrium dynamical system

Despite the seeming complexity of the model, the property in Lemma 3 conveniently compresses the system characterizing the economy's dynamics to just two dimensions. There are two cases.

- *Equilibrium with no entry ($n \equiv \dot{N}/N = 0$):*

$$\frac{\dot{b}}{b} = \left[\frac{\gamma(1-\theta)}{(1-\eta)c} - 1\right]\frac{b}{1-\mu} - \rho, \quad c = \theta^2\left(\frac{1}{\theta} - 1 - \frac{\phi}{x}\right) + 1 - \theta; \tag{35}$$

$$\frac{\dot{x}}{x} = \frac{\gamma(b-\delta) - (1-\gamma)\rho}{1 + (1-\gamma)\xi(x)}. \tag{36}$$

- *Equilibrium with entry ($n \equiv \dot{N}/N > 0$):*

$$\frac{\dot{b}}{b} = \left[\frac{\gamma(1-\theta)}{(1-\eta)\left(\rho\beta\theta^2 + 1 - \theta\right)} - 1\right]\frac{b}{1-\mu} - \rho; \tag{37}$$

$$\frac{\dot{x}}{x} = \gamma(b-\delta) - (1-\gamma)\rho - (1-\sigma)n. \tag{38}$$

In the first case there is analytical solution for the relation between the jumping variable c and the state variable x; in the second the unstable differential equation for c says that the consumption ratio jumps to its steady state value and remains constant throughout the transition. The combination of the two properties is that overall the model features a global closed-form solution for the relation between the consumption ratio, c, and the state variable firm size, x. This allows solving out for c and reducing the dynamics to two piece-wise differential equations in the birth rate, b, and firm size, x, plus the associated boundary conditions.

Inspecting the system, moreover, reveals that in the regime with entry: (i) the fertility rate, b, jumps to its steady-state value, denoted b^*, and remains constant throughout the transition driven by the evolution of firm size, x; (ii) the resource input, R, follows an exponential process with constant rate of exhaustion ρ, i.e., $R(t) = \rho S_0 e^{-\rho t}$. In other words, the regime with entry exhibits constant, but *endogenous*, consumption ratio, birth rate (births per adult and births per capita are proportional to each other) and extraction rate. The questions then are whether the economy converges to such a regime and whether such a regime constitutes a sustainable growth path. For the second question, the key issue is whether endogenous innovation can overcome the fact that the resource stock vanishes at a constant exponential rate. The following characterization of innovation behavior, that exploits the features of fertility and extraction behavior just established, aids in answering these questions.

Lemma 4 *Assume*

$$\phi\alpha \frac{\rho\beta}{\frac{1}{\theta} - 1 - \rho\beta} < \gamma\left(m^* + \rho\right), \tag{39}$$

where $m^* = b^* - \delta$ *is the growth rate of population in the regime with entry. Then, the activation thresholds for variety and quality innovation are*

$$x_N = \frac{\phi}{\frac{1}{\theta} - 1 - \rho\beta} \tag{40}$$

and

$$x_Z = \arg\text{solve}\left\{\left[\left(\frac{1}{\theta} - 1\right)x - \phi\right]\left(\alpha - \frac{\sigma}{\beta x}\right) = \gamma\left(m^* + \rho\right) - \sigma\rho\right\}, \tag{41}$$

with $x_N < x_Z$.[8] *Assume also* $\beta x > \sigma \ \forall x > \phi$, *i.e.,* $\beta\phi > \sigma$. *Then, for* $x > x_N$ *the equilibrium rates of variety and quality innovation are:*

$$n = \begin{cases} \frac{1}{\beta}\left(\frac{1}{\theta} - 1 - \frac{\phi}{x}\right) - \rho & x_N < x \leq x_Z \\ \frac{(1-\alpha)\left[\left(\frac{1}{\theta}-1\right)x - \phi\right] - \rho\beta x + \gamma(m^*+\rho)}{\beta x - \sigma} & x > x_Z \end{cases}; \tag{42}$$

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[8] The equation in the argsolve function in (41) is quadratic in x and thus yields a closed-form expression for x_Z. The expression, however, is cumbersome (see the appendix) and not particularly informative. Using the argsolve format keeps the exposition cleaner. Also, as shown in Peretto (2015), models of this class allow for the reversed ordering of the activation thresholds, i.e., $x_Z < x_N$. The key qualitative features of the transition path change little. Since the goal of the paper is to identify novel mechanisms rather than proving general theorems, the exposition focuses on the ordering $x_N < x_Z$ which delivers the desired insight with minimal mathematical complexity. The parametric restriction that delivers this ordering of the thresholds is inequality (39).

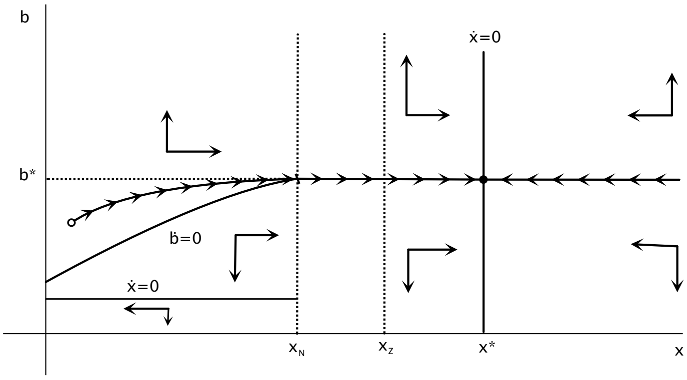

Figure 1: success story

$$z = \begin{cases} 0 & x_N < x \leq x_Z \\ \frac{\left[\left(\frac{1}{\theta}-1\right)x-\phi\right]\left(\alpha-\frac{\sigma}{\beta x}\right)-\gamma(\rho+m^*)+\sigma\rho}{1-\frac{\sigma}{\beta x}} & x > x_Z \end{cases}. \tag{43}$$

Proof. See the Appendix. ■

We now have all the ingredients needed to study the process of convergence, or failure thereof, to a sustainable growth path.

4. The transition

The model produces three scenarios: the success story, where the economy makes the full transition to sustainable growth; failure to launch, where the economy remains trapped in a downward spiral of no innovation, resource exhaustion and falling population; premature market saturation, where the economy turns on the engine of innovation only partially and converges to a steady state in which income per capita growth requires population growth. The model is remarkably tractable and delivers rich analytical results. Nevertheless, the qualitative analysis is sufficient to develop the main insights. Therefore, the following exposition focuses on the phase diagram and the narrative it produces. The reader interested in the model's analytics can consult the appendix.

4.1. The success story

Figure 1 illustrates a path consisting of the three phases discussed in the Introduction.

The hollow circle denotes the initial choice of consumption, fertility and extraction; the star denotes the sustainable steady state. It is worth stressing again that the initial choice x_0 is not determined solely by the initial stocks but depends on the associated path of consumption. The following proposition states the result formally.

Proposition 5 ***(Success Story)*** *Assume:*

$$\frac{\rho\left(1-\mu\right)}{\frac{\gamma}{1-\eta}-1} \geq \delta + \frac{1-\gamma}{\gamma}\rho; \tag{44}$$

$$\frac{\left[\left(\frac{1}{\theta}-1\right)\bar{x}^{*}-\phi\right]\left(\alpha-\frac{\sigma}{\beta\bar{x}^{*}}\right)-\gamma\left(b^{*}-\delta+\rho\right)+\sigma\rho}{1-\frac{\sigma}{\beta\bar{x}^{*}}} > 0; \tag{45}$$

$$\frac{\left(1-\sigma\right)\left(1-\alpha\right)}{\gamma\left(b^{*}-\delta+\rho\right)-\sigma\rho} > \frac{\beta}{\frac{1}{\theta}-1} > \frac{1}{\phi}; \tag{46}$$

$$\alpha\frac{\phi\beta-\frac{1}{\theta}+1}{\frac{(1-\sigma)(1-\alpha)}{\gamma(b^{*}-\delta+\rho)-\sigma\rho}\left(\frac{1}{\theta}-1\right)-\beta} > b^{*}-\delta+\rho. \tag{47}$$

Then, there is a unique equilibrium path: the economy chooses the pair (x_0, b_0)*, where*

$$x_0 = \theta^{\frac{2}{1-\theta}}\frac{M_0^{\gamma}\left(\left[\int_0^{\infty}e^{-\varepsilon(t)t}dt\right]^{-1}S_0\right)^{1-\gamma}}{N_0^{1-\sigma}} < x_N, \tag{48}$$

and rides the saddle path that converges to (x^{*}, b^{*})*, where:*

$$x^{*} = \frac{\frac{(1-\sigma)(1-\alpha)}{\gamma(b^{*}-\delta+\rho)-\sigma\rho}\phi-1}{\frac{(1-\sigma)(1-\alpha)}{\gamma(b^{*}-\delta+\rho)-\sigma\rho}\left(\frac{1}{\theta}-1\right)-\beta} > x_Z; \tag{49}$$

$$b^{*} = \frac{\rho\left(1-\mu\right)}{\frac{\gamma(1-\theta)}{(1-\eta)\left(\rho\beta\theta^{2}+1-\theta\right)}-1}. \tag{50}$$

Proof. See the Appendix. ■

The four conditions in Proposition 5 deliver the *success story*, which is the paper's best-case scenario. Collectively they say that the economy starts with positive growth of final output, Y, and thus of firm size, x. As firm size grows, it eventually crosses the threshold that activates horizontal innovation (entry) but the process of product proliferation does not weakens firm profitability so much that firm size stops growing before crossing the threshold that activates vertical in-house innovation. Consequently, the economy makes the complete transition from the first phase of growth based on natural resource exploitation to the last phase of growth based on knowledge accumulation divorced from the exhaustion dynamics of the natural resource. The specifics and the associated economic insights are as follows.

Condition (44) guarantees (sufficient condition) that the first phase of the transition has the property that agents make consumption, fertility and extraction decisions that ensure positive growth of market size and thus of firm size. More precisely, the condition guarantees that the first phase exhibits $\dot{x}/x = \dot{Y}/Y > 0$. Using equations (28), (34) and (36) we obtain that the rate of growth of GDP per worker is

$$g\left(x\right) = \left(1-\gamma\right) \frac{\left[\frac{2\gamma-1}{1-\gamma}\xi\left(x\right)-1\right]\left[b\left(x\right)-\delta\right]-\left[1+\xi\left(x\right)\right]\rho}{1+\left(1-\gamma\right)\xi\left(x\right)}. \tag{51}$$

This expression says that $g > 0$ for

$$\frac{\frac{2\gamma-1}{1-\gamma}\xi\left(x\right)-1}{1+\xi\left(x\right)}\left[b\left(x\right)-\delta\right] > \rho. \tag{52}$$

This is possible only if (necessary condition) the coefficient of population growth on the left-hand side is positive. This in turn requires $\gamma > 1/2$ and $\xi\left(x\right) > \left(1-\gamma\right)/\left(2\gamma-1\right)$ for $x \in \left[\phi/\left(\frac{1}{\theta}-1\right), x_N\right]$. The interpretation is that the exhaustible resource cannot be too important in production and economies of scale must be sufficiently strong. Given this necessary condition, growth is positive if (sufficient condition)

$$b-\delta > \rho\frac{1+\xi\left(x\right)}{\frac{2\gamma-1}{1-\gamma}\xi\left(x\right)-1}. \tag{53}$$

Given the elasticity $\xi\left(x\right)$ defined in Lemma 1, this inequality defines the boundary of two regions in (x, b) space, one where $g \leq 0$ because population growth is too slow and one where $g > 0$ because population growth is sufficiently fast.

Now observe that the elasticity $\xi\left(x\right)$ has the property $d\xi\left(x\right)/dx < 0$. Hence, the slope of the locus above is positive and, moreover, we have:

$$\frac{db\left(x\right)}{dx} > 0; \tag{54}$$

$$\frac{d\left(\dot{x}/x\right)}{dx} = \frac{d}{dx}\left(\frac{\gamma\left(b\left(x\right)-\delta\right)-\left(1-\gamma\right)\rho}{1+\left(1-\gamma\right)\xi\left(x\right)}\right) > 0. \tag{55}$$

In words, the first phase of the equilibrium path exhibits rising fertility and accelerating firm size growth. The rising growth rate of firm size, $\dot{x}/x$, is critical for the sign of the growth rate of GDP per worker. First, as argued, growth is positive only if the elasticity

$\xi(x)$ is above a critical threshold. Second, the elasticity $\xi(x)$ is decreasing in x because static economies of scale are bounded above. Therefore, the growth rate of GDP per worker can be positive throughout the first phase, and can even be increasing for a while, if and only if the net effect of economies of scale exhaustion and rising firm size growth dominates the rising birth rate. Technically, to ensure that this is the case, it is enough to impose $\xi(x_N) > (1-\gamma)/(2\gamma-1)$, which guarantees that the upward sloping locus (52) intersects the $x = x_N$ boundary below the value b^*.

This calculation complements the phase diagram's visual message and says that the initial phase does not necessarily exhibit falling GDP per worker but that the relentless downward pressure due to the growth drag eventually must result in falling GDP per worker if the economy takes too long to activate innovation. A similar calculation makes another point not apparent from the phase diagram: the rate of exhaustion is falling over time as the rate of growth of the consumption ratio, c, falls toward zero. The downward pressure from exhaustion is nevertheless relentless because the exhaustion rate has a strictly positive floor given by the discount rate, ρ.

Now refer back to condition (44), which ensures that aggregate output grows, $\dot{Y}/Y > 0$, and thus that the economy crosses the threshold x_N at a finite time T_N (see the appendix for the analytics of this results). The condition actually says that the economy follows a version of the Hartwick rule (Hartwick 1977; see also Solow 1974): agents transform natural resources into productive adults and the net effect is aggregate economic growth. Although the Hartwick rule has been derived in models of physical capital accumulation, the mechanism at its heart operates in this model. Stripping away the normative interpretation of the rule, since we are characterizing a market equilibrium, what we have here is that (i) households invest the revenues from extraction of the exhaustible resource in the accumulation of a productive assets and (ii) the net effect of such extraction-reinvestment process is overall growth of output. Although for simplicity the model abstracts from education, it treats the reproduction decision as a costly investment in future wage earners (adults) and thus it is appropriate to say that the key component of the first phase is the transformation of natural capital into human capital.

To complete the characterization of this scenario, note that in the second and third phases the growth rate is given by equation (28) in Lemma 1 while the rates of innovation are given by equations (42)-(43) in Lemma 4. Since the transition features rising firm size, x, it features a rising rate of variety innovation (entry), $n(x)$. Under conditions (45)-(46), the economy crosses the threshold x_Z at a finite time T_Z (see the appendix for the analytics), displays rising rates of variety innovation, $n(x)$, and quality innovation, $z(x)$, and converges from below to the growth rate

$$g^* = \alpha\left[\left(\frac{1}{\theta}-1\right)x^* - \phi\right] - m^* - \rho, \quad m^* = b^* - \delta. \tag{56}$$

Note that because the birth rate, b, is constant, this is the rate of growth of both GDP per worker and GDP per capita. Similarly, because both the ratios of consumption to final output, c, and of final output to GDP, Y/G, are constant, this is the growth rate of consumption per capita. The associated sustainability condition is condition (47), which says that

$$g^* > 0 \quad iff \quad \alpha\left[\left(\frac{1}{\theta}-1\right)x^* - \phi\right] = \alpha\frac{\phi\beta - \frac{1}{\theta} + 1}{\frac{(1-\sigma)(1-\alpha)}{\gamma(m^*+\rho)-\sigma\rho}\left(\frac{1}{\theta}-1\right) - \beta} > m^* + \rho. \tag{57}$$

This inequality holds for small values of m^*, that is, given ρ it holds for sufficiently *slow* population growth. In fact, this growth rate is compatible with zero, or even negative, population growth. Formally, it holds for $m^* \in (m^*_{\min}, m^*_{\max})$ with $m^*_{\min} < 0$ and $m^*_{\max} > 0$. This interval includes 0 and allows for negative population growth.

4.2. Failure to launch

There are two potential pitfalls on the path of this economy. The first is that when condition (44) in Proposition 5 fails, either the $\dot{x} = 0$ locus intersects the $\dot{b} = 0$ locus from above for some value $\tilde{x} \in \left[\phi/\left(\frac{1}{\theta}-1\right), x_N\right]$, or it is above the $\dot{b} = 0$ locus for all $x \in \left[\phi/\left(\frac{1}{\theta}-1\right), x_N\right]$. The latter is just a special case of the former and thus the following discussion focuses only on the case where the intersection $\tilde{x}$ exists.

Proposition 6 ***(Failure to Launch)*** *Assume*

$$\frac{\rho(1-\mu)}{\frac{\gamma}{1-\eta} - 1} < \delta + \frac{1-\gamma}{\gamma}\rho. \tag{58}$$

and consider the case where the $\dot{x} = 0$ locus intersects the $\dot{b} = 0$ locus from above at the value $\tilde{x} \in \left[\phi/\left(\frac{1}{\theta}-1\right), x_N\right]$. Then, two outcomes are possible. If the economy has a

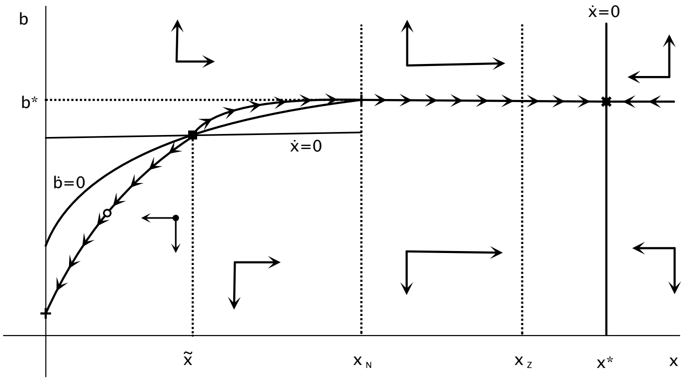

Figure 2: failure to launch

sufficiently large endowment S_0*, it chooses a pair* (x_0, b_0) *with* $x_0 \in (\tilde{x}, x_N)$ *and places itself on the saddle path that converges to* x^**. If, instead, the economy has an insufficient endowment* S_0*, it must choose a pair* (x_0, b_0) *with* $x_0 \in \left(\phi/\left(\frac{1}{\theta}-1\right), \tilde{x}\right)$ *and is thus doomed to collapse.*

Proof. See the Appendix. ■

Figure 2 illustrates this case, in which society fails to build up the economy.

The square denotes the unstable steady state in the no innovation region. The cross denotes the economic collapse point where firms become non-viable. The hollow circle denotes the initial choice of fertility, consumption and extraction when the path that leads to the sustainable steady state is not accessible.

Many factors enter the conditions for this worst-case scenario to occur. Most prominent is the size of the initial endowment. When S_0 is too small, holding constant all the other determinants of fertility and consumption behavior, the economy might be constrained to an initial choice of R_0 resulting in $x_0 < \tilde{x}$. A second prominent factor not immediately apparent from the phase diagram is the regime of property rights over the natural resource. Recall that the model posits a continuum of mass one of household each one with endowment S_0. What this means is that the model posits decentralized resource management by atomistic agents with full property rights. It follows that the initial value R_0 does not allow for (i) coordination among agents and (ii) over-exploitation in the sense of the Tragedy of the Commons (Hardin 1968).

Coordination is potentially crucial because the scenario discussed here hinges on a clear externality: in their extraction decisions agents do not account for the dynamics of aggregate market size and thus extract less than what would allow the economy to cross the threshold $\tilde{x}$. A potentially paradoxical implication is that weaker property rights that result in some form of the tragedy of the commons—in the sense of more aggressive extraction motivated by the expectation that failure to extract today leaves nothing to extract tomorrow—might allow the economy to cross the threshold $\tilde{x}$ for unchanged parameters. In this light, the model poses interesting questions and sheds a different light on issues that traditionally have had straightforward interpretations.

One way to think about these dynamics is that the existence of the threshold $\tilde{x}$ opens the door to temporary changes in extraction behavior that have permanent effects on the growth path of the economy. A simple example could be a temporary suppression of property rights. Obviously, it cannot be desirable to engineer a full blown tragedy of the commons whereby $R_0 = S_0$. So, a temporary intervention has to achieve higher extraction but not complete exhaustion. Thinking about schemes that might accomplish it, two come to mind. The first is temporary nationalization of the resource. The second is temporary subsidies. Both schemes achieve coordination on a more aggressive extraction path but they have different features that yield different potential costs. Nationalization, interpreted as total suppression of property rights, might turn out to be irreversible and might result in less efficient resource management, both for political-economy reasons. Subsidization also can turn out to be irreversible and produce inefficiencies of its own for political-economy reasons. A third scheme with similar trade-offs is regulation, e.g., extraction mandates. The debate on such issues is very old and very lively. However, it has mostly taken place in a context where the market failure is typically taken to be over-exploitation. The scenario discussed here, in contrast, is one of under-exploitation with potentially fatal long-term consequences.

Another surprising implication of the dynamics driving this scenario is the following. Consider an economy that at time zero can select $x_0 > \tilde{x}$ and starts on the path that leads to success. Now imagine that such economy at some future date is hit by a shock, say an epidemic, that kills a large fraction of the population. Because of its past extraction, the economy at the time of the shock has a smaller endowment and therefore is *vulnerable* in the following sense. The fall in the size of the workforce resets the state variable x at a

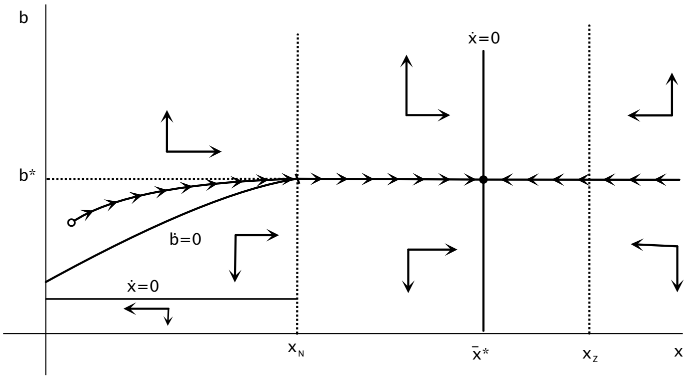

Figure 3: premature market saturation

smaller value. Say that such value is below $\tilde{x}$. The economy now needs to make a new set of initial decisions but, because the endowment is smaller, might well be unable to set the new initial R at a value that yields $x > \tilde{x}$ and therefore be doomed to collapse. One could think of this scenario as far-fetched. In fact, it is consistent with the most recent re-interpretation of the history of Easter Island proposed in archeology (Hunt and Lipo 2011). Easter Island is typically proposed as the archetypical example of a society that collapsed due to over-exploitation of its resource base (a colorful expression often used is eco-suicide). The recent evidence suggests instead that it collapsed because (i) upon first contact with Europeans the native population crashed from diseases against which it had no defense and (ii) the local environment had suffered greatly from the spread of the rats that came with the Europeans.

These reflections cannot be pushed too far, since the analysis is mostly qualitative and much more work is called for to fully flesh out the implications and the empirical validity of the mechanism at the heart of this model. They clearly suggest, however, that the model offers a new perspective on important issues.[9]

4.3. Premature market saturation

Figure 3 illustrates the second potential pitfall: premature market saturation.

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[9] Readers familiar with Unified Growth Theory (Galor 2011) might note that allowing for a Malthusian feedback such that population size responds to a physical resource constraint would only make things worse. If fertility falls as the natural resource runs out, eventually it must fall below the mortality rate yielding shrinking population. The paper abstracts from these forces but reflecting on them adds perspective to the main point, namely, that a period of rising population that seemingly exacerbates natural resource scarcity is the key to success. If achieving such rising population requires overcoming Malthusian constraints, success is harder to achieve but not necessarily impossible.

The dark circle denotes the steady state with no quality innovation. This scenario occurs when condition (45) does not hold and thus the economy fails to cross the threshold for vertical innovation, x_Z, and converges instead to the steady state $(\bar{x}^*, b^*)$. This steady state exhibits the semi-endogenous growth rate

$$\bar{g}^* = \sigma \cdot \underbrace{\frac{\gamma\left(m^* + \rho\right) - \rho}{1-\sigma}}_{n^*} - (1-\gamma)\left(m^* + \rho\right), \quad m^* = b^* - \delta. \tag{59}$$

The associated sustainability condition is

$$\bar{g}^* > 0 \quad iff \quad \sigma > (1-\gamma)\frac{m^* + \rho}{m^*}. \tag{60}$$

Note, first, that the condition is possible in the first place only if $\sigma > 1-\gamma$. Moreover, since the right-hand side is decreasing in m^*, given σ, γ, ρ, the condition holds for

$$m^* > \frac{\rho}{\frac{\sigma}{1-\gamma} - 1}, \tag{61}$$

which says that, because of the non-zero exhaustion rate, sustainable growth requires sufficiently *fast* population growth.

This is an important point in light of the evidence and arguments discussed in, among others, Strulik et al. (2013). Sustainability predicated on population growth runs counter to first-principles and to facts because (i) an infinite population is not possible on a finite planet and (ii) population growth is not only slowing down everywhere, but in many countries it is negative. At most, one should expect it to settle at zero in the long run. The difference between the two scenarios, therefore, is that the semi-endogenous growth outcome ensures sustainability only under implausible conditions. The fully endogenous growth outcome, in contrast, does not need population growth and therefore does not tie sustainability to implausible assumptions about the interdependence of population and resources.

5. Conclusion

This paper has proposed a Schumpeterian approach to the study of the interactions among population, technology and exhaustible resources. Relative to the traditional approach of resource economics based on the DHSS (Dasgupta, Heal, Solow, Stiglitz) foundation, the focus on firms' incentives and the endogeneity of the structure of the market in which they operate provides a novel view of the interplay of population and resources and stresses the key role of market size. The framework is remarkably tractable and allows one to obtain a transparent characterization of dynamics that are typically very complex. The analysis of sustainability, defined as the ability of the economy to achieve positive, constant, exponential growth of consumption per capita in the long run stresses the following insights.

First, it is not just the *rate* of technological change that matters for sustainability, but also the *type*. The concept of de-coupling is broader than simply overcoming resource exhaustion: it refers to a qualitative change in economic activity, from economic growth based on larger use of natural inputs to economic growth divorced, as much as the laws of nature allow, from such inputs.

Second, the same first-principles that drive the concerns about increasing scarcity of physical inputs drive the concerns about the planet's ability to withstand a perpetually growing population—which, after all, *is* a physical input subject to physical constraints. Therefore, de-coupling requires divorcing economic growth from demographic growth as well. In this perspective, productivity growth as the amplification of the growth of the number of people operates in the opposite direction of what the notion of scarcity at the heart of the sustainability debate entails. In the language of the model, quality innovation can deliver sustainable growth while variety innovation cannot. The reason is that the former stands in for the accumulation of intangibles and the increase in the flow of services that we obtain from goods for *unchanged use of physical resources*. The latter, instead, stands in for innovation whose implementation requires the accumulation of tangible productive assets (firms, plants), which requires larger use of physical resources.

The literature has debated these ideas for some time but formal modeling has lagged behind. This paper's goal is to partially fill the gap and hopefully make the debate more concrete and precise. To further develop the approach in the future, the following aspects require careful reflection.

First, the adopted definition of sustainability might strike some as too narrow. Similarly, the harshness of the environment postulated in the paper might strike some as extreme. It is not hard to extend the framework to: (i) regeneration in the resource dynamics (renewable), which would allow for a steady state with constant, positive stock of the resource; (ii) a backstop technology triggered by sufficiently high resource price, which would make the analysis much more difficult and yield conclusions in line with what we already know, namely, that at some point the economy switches to the alternative source. The meaningful counter-argument to such observations, however, is not that such modifications are feasible but that they assume scarcity away with respect to the paper's baseline case, which instead sets the highest possible bar for technology to clear to deliver sustainability. This is not a crucial reason not to consider such extensions, but it suggests that the paper strikes at the core of the sustainability question precisely because it strips away all forces that weaken the scarcity problem.

Second, the paper lacks an internal mechanism that forces population growth to zero. It is possible to introduce feedbacks that stabilize the population but doing so attenuates the scarcity problem, because population pressure on the natural resource eventually ceases growing, and complicates the analysis while the paper privileges transparency. Moreover, an important caveat applies: the stabilizing mechanism cannot be Malthusian, in the sense that population becomes proportional to the resource base, since the latter is always shrinking. In other words, in studying population-resources interdependence one must be very careful: potential exhaustion changes drastically the nature of the problem.

Third, the paper uses the simplest model of exhaustible resource dynamics. Because such model equates the Hotelling rents to the spot market price of the extracted resource, it produces counterfactual behavior: it says that the price of the resource grows all the time at the rate of interest. It is possible to use more sophisticated versions (especially versions that allow technological change in extraction) and obtain conclusions quite similar to those described above. Because the complexity of such analysis is substantial and the goal of this paper is to illuminate mechanisms rather than fit the data, the elaboration of such extensions is left to future work.

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Credit Author Statement

I did all the work for this solo-authored paper.