Optik - International Journal for Light and Electron Optics 212 (2020) 164770

Contents lists available at ScienceDirect

Optik

journal homepage: www.elsevier.com/locate/ijleo

Original research article

Efficient Kerr nonlinearity in RTP induced by electro-optically adjustable cascaded second-order process

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A R T I C L E I N F O

Keywords:
Cascaded second-order nonlinear process
Effective nonlinear refractive index
Kerr-lens mode locking
Nonlinear phase shift
Self-focusing & defocusing

A B S T R A C T

This paper reveals an analytical perception of electro-optically tunable cascaded Kerr-lens using $RbTiOPO_4$ (RTP) crystal where the advantageous properties of the crystal have been studied to mitigate the limitations of $KTiOPO_4$ (KTP) and BaB_2O_4 (BBO) in intracavity high power laser generation. An external electric field application to the z-direction of RTP is introducing a phase mismatch condition leading to a nonlinear phase shift within propagating optical waves. Based on simulated endeavors, a maximum effective nonlinear refractive index of $\pm 1.4053 \times 10^{-13}$ cm^2/W is calculated by developing a phase shift of ∓ 4.6 rad. In contrast, KTP and RTP both have almost similar optical properties but due to certain special features like higher damage threshold, less walk-off angle etc., RTP is firmly accepted for high power applications. Additionally, this article is augmented to analyze the self-focusing and defocusing phenomenon owing to the non-uniform distribution of radial intensity of the propagating Gaussian beam. To the best of author's knowledge, the present analysis manifests the superior performance of RTP that may pave the way to devise a configurable lens analogous to Kerr-lens having an electro-optically tunable focal length.

1. Introduction

In the past several decades [1–5], cascaded second-order nonlinearity has played a vital role in the generation of cascaded Kerr lens mode-locking [6] that is nothing but a successive combination of two second harmonic generation (SHG) processes ($\chi^{(2)}:\chi^{(2)}$ development) that yields an intensity-dependent large phase mismatch in propagating fundamental waves. Accordingly, the nonlinear phase shift because of the non phasematched condition is also relative to the intensity of the incident beam that is analogous to Kerr medium ($\chi^{(3)}$ nonlinearity) [6,7]. Moreover, the performance of this feature may enlarge the third-order nonlinearity of a medium having weak third-order coefficient [8] which is grander than the natural third-order nonlinearity [5] associated with the Kerr electro-optic effect as well. As a consequence of that, cascaded second-order nonlinearity equivalent to third-order nonlinearity [2] plays a significant role in several areas like transistor action [9], electro-optical detection [10], electro-optic switching [11], self-focusing and self-defocusing [9] etc. [12–15]. Although different techniques are emphasized to develop cascaded second-order nonlinearity using various types of crystals where problems like undesirable cavity loss, inaccuracy of incident beam angle, etc [5,16]. have occurred that deters a controllable desired output.

In recent years [17,18], an analytical method has been introduced named “electro-optically tunable cascaded nonlinearity” in an exceedingly bulk anisotropic crystals like Potassium Titanyl Phosphate ($KTiOPO_4$, KTP) and Beta Barium Borate (BaB_2O_4, BBO), etc.

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https://doi.org/10.1016/j.ijleo.2020.164770
Received 6 January 2020; Received in revised form 30 March 2020; Accepted 19 April 2020
0030-4026/

where an applied field on the crystal's principal plane has engendered a phase mismatch condition leading to a desirable tunable nonlinear phase shift within the optical beam. This leads to the pivotal requisite of developing an electro-optically tunable configurable lens having a controllable focal length within the crystal which is deliberately a vital aspect of great consequentiality in such self-focusing and defocusing paradigms. The comprehensive analysis of research study inherently converts a linear electro-optically adjustable non-phase matched second-order nonlinearity to a cascaded second-order nonlinearity which is very much relevant in intracavity high power mode-locked laser generation.

However, in case of high power laser generation like second harmonic generation (SHG) and optical parametric oscillator (OPO), a laser-induced damage happens inside KTP known as grey-tracking [19]. In an alternative approach, due to the smaller effective electro-optic coefficient, BBO needs noticeably higher operating voltage [20]. To beat these shortcomings, authors have chosen bulk Rubidium Titanyl Phosphate ($RbTiOPO_4$, RTP) as a current crystal that is an isomorph of KTP. As it is well known, each RTP and KTP crystals have homogeneous nonlinear optical properties and are congruous for varied applications like electro-optic, SHG and OPO [20]. In spite of that, RTP has an optical damage threshold of 1.8 times of KTP [21] and a walk-off angle which is 6 times less than that of KTP [19]. Hence, for intracavity high power optical device applications, RTP is commendably adjudged to exploit enhanced cascaded Kerr electro-optic nonlinearity than KTP [22,11]. At the same time, the efficacious electro-optic coefficient of RTP is better than that of KTP which makes it further subsidiary for electro-optic and low voltage applications [19]. Additionally, it has a wider temperature phase-matching bandwidth than KTP. Therefore, in case of high average power application, RTP is the best opportune for SHG in quasi-continuous operation and high power OPO [20]. Moreover, up to the MHz range, there is no chance of piezoelectric ringing with electrical signals [19] that sanctions this crystal for Q-switching at a hundred kHz range of frequencies in case of maintaining short pulse length [20].

With the approach of such superior benefits, this article convinces to employ a bulk RTP crystal to develop an adjustable electro-optically configured cascaded Kerr lens by including an adjustable electric field to the crystal plane. Though electro-optically induced adjustable nonlinear phase shift within RTP has already been reported earlier [23], except that the higher performance of this anisotropic crystal to create a better configurable cascaded Kerr-lens is yet to be analyzed. Even more, as compared to other crystals, RTP requies a minimum voltage to come up with a required phase mismatch because of its greater electro-optic coefficients. Hence, this methodology engenders the electro-optically adjustable Kerr type nonlinear effect within the bulk RTP that holds validity in high power applications.

2. Theory and proposed scheme

The overall idea of the proposed scheme is illustrated in Fig. 1 describing the fundamental waves (ω) are propagating along the type II phase-matching directions ($\Delta k = 0$) i.e. along the y-principal axis of RTP making a polar angle of θ and azimuthal angle of ϕ with the principal axis. However, when an external field (E_z) is directly applied perpendicular to the propagating direction then a non phasematched condition ($\Delta k \neq 0$) occurs within the propagating optical waves because of the change of refractive indices of the crystal.

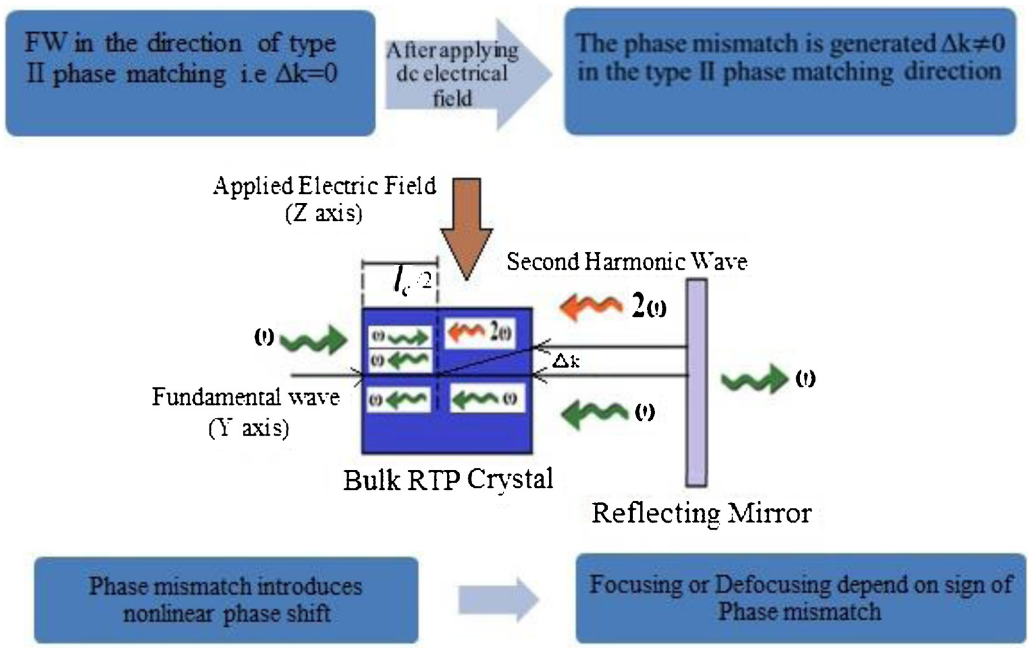

Fig. 1. Schematic block diagram of proposed idea of applying an external electric field to bulk RTP crystal to create the phase mismatch condition within propagating waves.

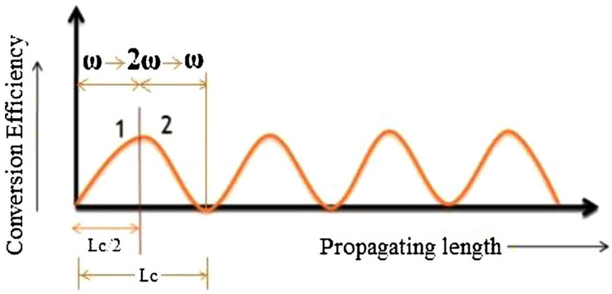

Fig. 2. Up (1) and down (2) conversions of fundamental to second harmonic waves and vice versa.

2.1. Up and down conversion

During this phase mismatch condition, a continuous generation of fundamental (ω) to second harmonic (2ω) wave is occurring up to the half of the coherence length ($l_c/2$) which is known as up conversion. On the other way, in the next half of the coherence length, the generated second harmonic wave is again reconverted into fundamental wave known as down conversion. The frequency conversion graph along with the propagating length (L) is clearly shown in Fig. 2 where up and down conversions are denoted by 1 and 2 numbers respectively due to which a phase difference occurs within the incident and reconverted fundamental waves creating a nonlinear phase shift ($\Delta\phi^{NL}$) [22] that can be obtained by assuming the negligible depletion of fundamental waves.

2.2. Nonlinear phase shift ($\Delta\phi^{NL}$) by creating phase mismatch (Δk) condition

Basically, biaxial crystals like RTP and KTP are belonging to the same crystal family having transparency ranges of 0.35–4.5 µm [20]. These crystals exhibit excellent thermal stability. The refractive indices of both the crystals are very proximate. Consequently, the postulations and formulations of bulk RTP have been considered akin to the bulk KTP as demonstrated in recent work [17]. Therefore, the coupled amplitude equations for the fundamental waves can be directly developed from Maxwell's equations similar to KTP [17]. As external field is applied to the principal plane of RTP due to the change of refractive indices the amplitude equation is also modified that can be solved from the coupled amplitude equations as [17]

$$E_1 = E_0 e^{-i(\Delta k/2)L}[\cos(pL) + i\Delta k sin(pL)/(2p)] \tag{1}$$

Here in Eq. (1), it is clearly understood that the electric field of generated second harmonic wave (E_1) is proportional to three important factors viz. the electric field of incident fundamental wave (E_0), the propagating length (L) and the phase mismatch (Δk). This Δk along the propagation length can be expressed in terms of wave vectors as well as refractive indices as [15]

$$\Delta kL = L(k_1 + k_2 - k_3) = (2\pi L/\lambda)(n_1' + n_2' - n_3') \tag{2}$$

Here, k_1, k_2 and k_3 respectively represent the wave vectors of incident fundamental and generated second harmonic. After applied field to the medium, the modified refractive indices for incident fundamentals are n_1', n_2' and for generated second harmonic is n_3' respectively. However, in Eq. (1), two terms p and q are expressed as

$$p = \sqrt{(\Delta k/2)^2 + \Gamma^2 q} \tag{3}$$

$$q = [4I_1/I_0 - I/I_0] \tag{4}$$

where in Eq. (3), Γ is a nonlinear coupling coefficient [17] that can be defined interms of crystal's parameters. I_0 and I are representing the normalized and total peak intensity that is defined as $I = I_1 + I_2$ and considering $I_1 = I_2$ means peak intensities of propagating fundamental waves for type II phase matching are same. However, $\Delta\phi^{NL}$ during a large non phasematched condition having lower intensity of light ($|\Delta k| >> |\Pi\Omega|$) can be defined at unit intensity as,

$$\Delta\phi^{NL} = (2\pi L n_2^{eff} I)/\lambda \tag{5}$$

Here, $\Delta\phi^{NL}$ is changed linearly with the change of intensity (I) of the incident propagating wave which is comparable to optical Kerr effect [11]. The parameter n_2^{eff} is known as intensity dependent indirect effective nonlinear refractive index comparable to n_2 parameter of optical Kerr effect ($n(I) = n_0 + n_2 I$) and can be defined as [17]

$$n_2^{eff} = -\frac{4\pi}{c\varepsilon_0} \times \frac{L}{\lambda} \times \frac{d_{eff}^2}{n_{\omega1}' n_{\omega2}' n_{\omega3}'} \times \frac{1}{\Delta kL} \tag{6}$$

From the above Eq. (6), it is observed that n_2^{eff} is proportional to ΔkL, $[d_{eff}^2/(n_{\omega1}' n_{\omega2}' n_{\omega3}')]$ from where we can say that the non phasematched condition within the propagating optical waves are strongly dependent on the crystal parameter, d^{eff} which can be compared to noncentrosymmetric materials having $\chi^{(2)}$ nonlinearity as

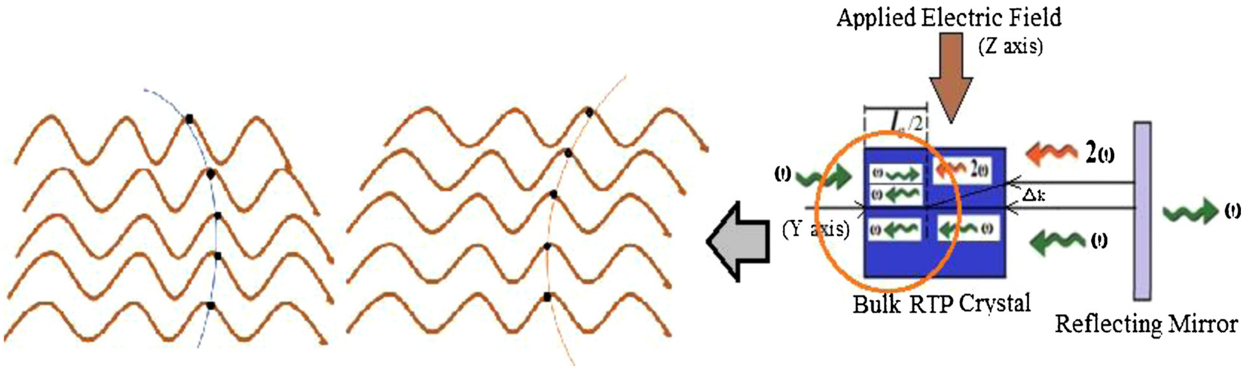

Fig. 3. Self-focusing and self-defocusing of fundamentals inside RTP depending upon the beam intensity distribution.

$$n_2^{eff} \cong d_{eff}^2/n^3 \tag{7}$$

In this Eq. (7), n^3 is represnting $n'_{\omega 1}n'_{\omega 2}n'_{\omega 3}$ combinely.

2.3. Kerr-lens generation in bulk RTP crystal

Fig. 3 depicts the incident and reconverted fundamental waves during propagation within the crystal where the black dots are displaying the nonlinear phase shifts ($\Delta\phi^{NL}$) making a curvature patterns within propagating wavefronts. Due to this curvature, a self-focusing and self-defocusing phenomenon happens within the crystal that makes the crystal to behave as a configurable lens depending upon distribution of the intensity of incident Gaussian beam. Significantly, we can say that analogous to optical Kerr media, cascaded second-order nonlinear method is additonally acting as a self-focusing and self-defocusing configurable lens within $\chi^{(2)}$ medium having a focal length of F that can be calculated in terms of n_2^{eff} and $\Delta\phi^{NL}$ as

$$F = r^2/(4In_2^{eff}L)$$
$$or,\ F = (r^2\pi)/(2\Delta\phi^{NL}\lambda) \tag{8}$$

Here, r and F are representing the radius of the incident beam [17] and focal length of the cascaded configurable lens. In comparison, we can say that in optical Kerr medium, the variation of nonlinear phase and self-focusing or self-defocusing of propagating beam is dependent on only the sign of n_2^{eff}, whereas, in cascaded nonlinearity, the non phasematched condition between propagating optical waves is also responsible. Similar to Kerr medium, fundamental waves focus or defocus while travelling through the medium having cascaded nonlinear property. The self focusing or self-defocusing of the fundamental waves relies upon the sign of Δk. That said while propagating through the crystal, the fundamental waves focus for positive ($\Delta k > 0$) values of Δk and vice-versa [23][24].

3. Result and discussion

The typical speciman dimension of bulk RTP is considered as $10 \times 5 \times 5$ mm^3 which is analytically examined for linear electro-optically induced phase mismatched second harmonic generation process to create the cascaded Kerr nonlinearity within the medium. The propagating fundamental wave is travelling in the direction of type II phase-matching parallel to the y -axis of the medium making an angle of $\theta = 90°$and $\phi = 58.147°$. The refractive indices of RTP along different principle axes at different frequency having a wavelength of 1.0642 μm and respective second harmonic wave along the principal axes of the crystal are tabulated [21] in Table 1.

The linear electro-optic coefficients of RTP are assumed to be r_{13} =10.6 pm/V, r_{23} =12.5 pm/V and r_{33} =35 pm/V [25] and the fundamental beam waist is considered 80 μm having an intensity of 0.4 GW/cm^2. Besides that the diffraction limited length of the propagating beam should be higher than the propagation length. A calculated field of ∓2 KV/mm is given to $x - y$ plane of the medium having 5 mm of thickness for creating the phase mismatch factor (ΔkL) of ∓9.2 radians.

Fig. 4 is comprenhending that the magnitude and sign of ΔkL can simply be adjusted by adjusting the magnitude and polarity of the given voltage to the medium. ΔkL is the most essential parameter in cascaded second-order nonlinearity for creating the sufficient

Table 1
Refractive indices of RTP crystal.

Refractive indices along principle axes	Fundamental frequency (ω)	Second harmonic frequency (2ω)
n_x	1.7740	1.8179
n_y	1.7874	1.8355
n_z	1.8675	1.9384

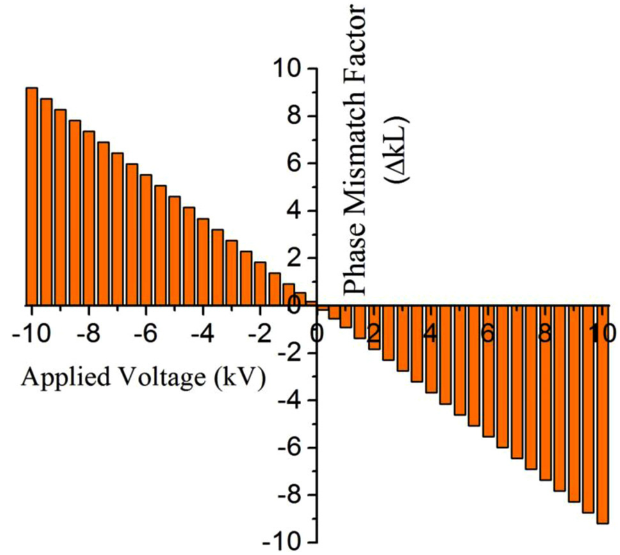

Fig. 4. Tuning of ΔkL with respect to voltage applied (KV) to the crystal.

$\Delta\phi^{NL}$ within the regenerated and incident initial fundamental waves. This $\Delta\phi^{NL}$ can also be varied at different intensities of propagating fundamental waves along the crystal's length.

Fig. 5(a) and (b) are depicting a linear variation of nonlinear phase shifts of the propagating fundamental waves along the various propagating length of the crystal at various intensities of incident beam viz. 0.2 GW/cm^2 and 0.5 GW/cm^2 respectively according to the given voltage for different value of ΔKL. As a result the salient characteristic that is a stepwise change in along with the increasing propagation distance. In fact for large values of ΔKL the increase occurs via steps, the larger the phase-mismatch the smaller (and more frequent) the steps. Finally the net $\Delta\phi^{NL}$ depends upon the ΔKL which is commonly known as SHG detuning [26].

However, in these two figures it is seen that, once the applied voltage is zero, $\Delta\phi^{NL}$ of the fundamental waves is zero whereas

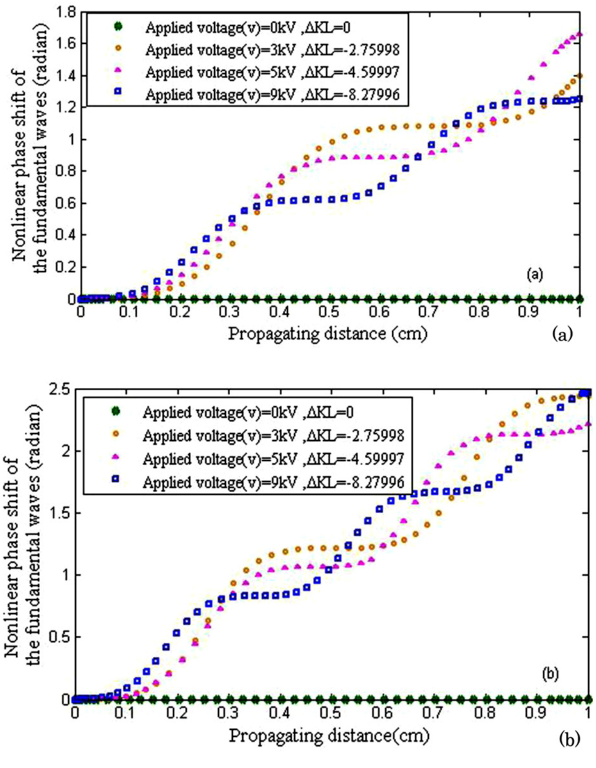

Fig. 5. Nonlinear phase shift of the fundamental waves along the propagation length at (a) 0.2 GW/cm^2 (b) 0.5 GW/cm^2.

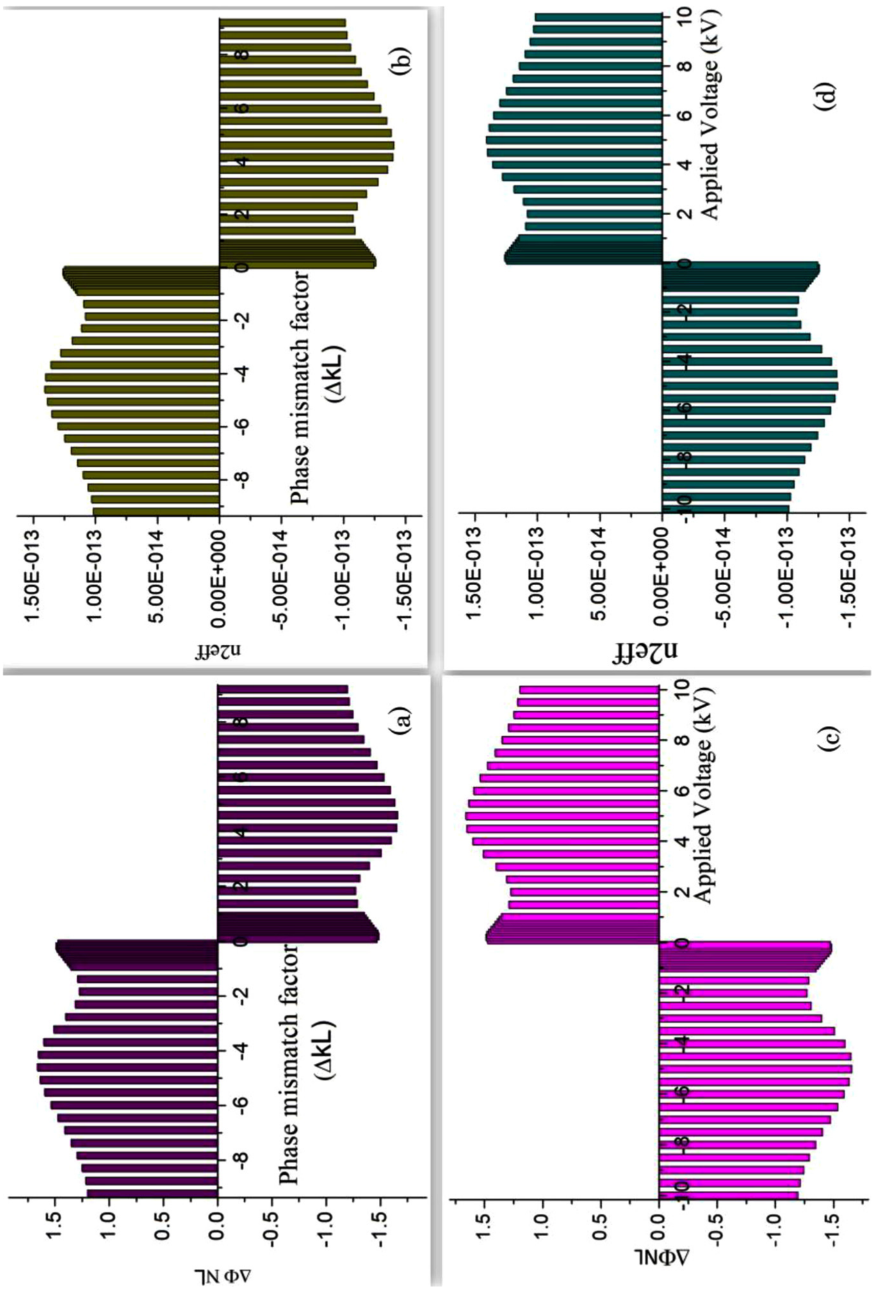

Fig. 6. Tuning of (a) $\Delta\phi^{NL}$ & (b) n_2^{eff} with respect to ΔkL and Tuning of (c) $\Delta\phi^{NL}$ & Tuning of (d) n_2^{eff} with respect to applied voltage (KV).

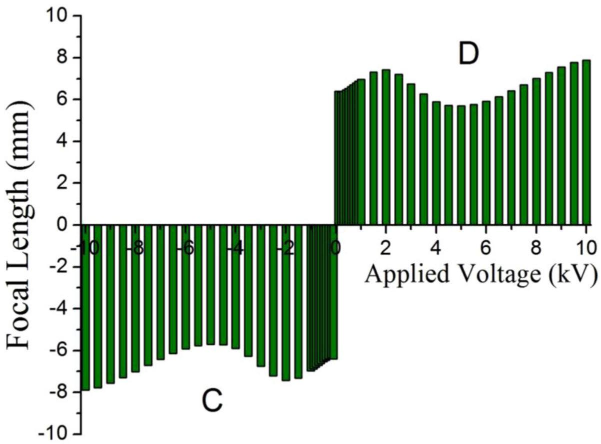

Fig. 7. Changing of focal length (mm) of configurable lens with respect to given voltage (KV).

propagating in the phase-matching direction, as indicated by green solid lines in these graphical potrayal appeared. But at the variation of given voltages viz. 3 KV, 5 KV and 9 KV at different ΔKL, $\Delta\phi^{NL}$ at phase matching direction are modified with the change of beam intensity as specified by orange, pink and blue solid lines severally in both Fig. 5(a) and (b). In the wake of investigating the above Fig. 5, it is clearly seen that $\Delta\phi^{NL}$ fluctuates more in higher intensity (0.5 GW/cm$^{2)}$ than the lower (0.2 GW/cm^2). On that account, it can be said that as the intensity increases, $\Delta\phi^{NL}$ within the propagating fundamental waves will be more at a distinctive rising lengths of the crystal (Eq. (5)). Although, this $\Delta\phi^{NL}$ is also very much dependent on n_2^{eff} of the crystal as explained in Eq. (5) whereas n_2^{eff} is linearly dependent on the crystal length L as shown in Eq. (6). Moreover, the large value of d^{eff} can enhances n_2^{eff} as it is proportional to the square of d^{eff} parameter of the crystal.

As one can see from Fig. 6(a), (b), (c) and (d), $\Delta\phi^{NL}$ and n_2^{eff} can also be tuned by varying phase mismatch factor (ΔkL) and applied voltage (kV) to the crystal. Based on simulation outcomes, the maximum calculated $\Delta\phi^{NL}$ and n_2^{eff} have been achieved ± 1.6589 and ± 1.4053 × 10^{-13} cm^2/W respectively at a phase mismatch factor (ΔkL) of ∓4.6 rad when the applied voltage is 5 kV. As the radial intensity distribution of the incident Gaussian beam is non uniform, so this will establish curvature to propagating fundamental wavefront. As a consequence, the cascaded $\chi^{(2)}:\chi^{(2)}$ will work like self-focusing or self-defocusing lens with which is having intensity-dependent focal length. In this regard the crystal itself performs similar to a configurable lens which is comparable to the cascaded Kerr-lens. The focal length of this cascaded Kerr-lens can also be modulated by adjusting the magnitude and polarity of the given voltage to the medium as shown in Fig. 7.

In the Fig. 7, the area around C (0 to −10 kV) and D (0–10 kV) are depicting the respective positive and negative focal length of cascaded configurable Kerr-lens which is based on the self-focusing and defocusing of the optical beam. More significantly, it is seen that when no voltage is applied to the medium then the focal length of the configurable lens becomes zero. The minimum as well as maximum positive and negative focal length is calculated ± 5.6927 mm and ± 7.8927 mm for applied voltages of ± 5 KV and ± 10 KV respectively. Eventually, it may be stated that by changing the applied voltage to the medium, it is possible to get the desired cascaded configurable Kerr-lens in the crystal having tunable focal length from minimum to maximum values.

On the other hand, the simulated results of KTP [17], BBO [18] and RTP are showing almost close to comparable results. But the major thing is observed that BBO [18] is more sustainable than KTP [17] and RTP is even more sustainable than BBO which is the key thing in this work. The higher quality of properties of RTP crystal can really cover all the limitations of KTP and BBO in high power laser generation. Other than all simulation is detailed to find the parameters which is similar to KTP with little bit variation. The only

Table 2
Table 2 : Comparison of simulated results of KTP [17], BBO [18] and RTP crystals.

Parameters	KTP CRYSTAL	BBO CRYSTAL	RTP CRYSTAL
Crystal Size(W × H × L)	5 × 4×10 mm^3	5 × 4×10 mm^3	5 × 5×10 mm^3
Applied voltage	∓2 KV/mm	∓ 6 KV/mm	∓2 KV/mm
Phase Mismatch	∓7.86 rad for an applied voltage of ± 8 kV	± 4.7699 rad for an applied voltage of ∓ 34 KV	∓9.2 rad for an applied voltage of ± 10 kV
$\Delta\phi^{NL}$	1.44	± 1.5702 at ± 34 KV	± 1.6589 at 5 kV
n_{2eff}	± 6.9872 × 10^{-13} cm^2/W at 1.5 KV	± 6.6508 × 10^{-14} cm^2/W at ± 34 KV	± 1.4053 × 10^{-13} cm^2/W at 5 kV
Focal length	Min ± 3.68 mm at 0.3 KV and max ± 4.57 nm at 2.5 KV	Min ± 2.947 mm at ± 34 KV and max ± 3.7603 mm at ± 14 KV	Min ± 5.6927 mm at ± 5 KV and max ± 7.8927 mm at ± 10 KV
Phase Matching Angle	θ = 24.534° and ϕ = 90° (x–y plane) (y-propagation direction)	θ = 22.39 and ϕ = 90° (Y–Z plane) (Z-propagation direction)	θ = 90°and ϕ = 58.147° (x–y plane) (y-propagation direction)

important differences between these crystals are the sustainability and applied voltages limit to the crystal which is completely depends on the damage threshold of the individual crystals and their particular some properties which are tabulated below (Table 2).

From the tabular analysis, it is observed that all these crystals are considered with almost similar dimensions. But due to their different damage threshold property KTP and RTP are having similar applied voltage range while BBO is having more value than them. The other parameters like Phase Matching Angle, phase mismatch, $\Delta\phi^{NL}$, n_{2eff} and focal length are different for different crystals which can be controlled and tuned by tuning the applied voltage to the crystal. However, the analytical simulations and comparison of properties of these crystals reveals that RTP is only best suitable for this phenomenon that really diminishes all the limitations of KTP and BBO in high power laser generation.

4. Conclusion

In this paper, the authors successfully explicated that a bulk RTP crystal is very efficient to form an electro-optically adjustable cascaded Kerr-lens in intracavity high power laser generation. In point of fact, the proposed idea is analytically studied on the avenue of KTP [15] where the simulation endeavours entail that RTP and KTP both crystals act efficiently as an adaptable configurable lens. But higher effective electro-optic coefficients and damage threshold are making RTP more convenient for electro-optic applications in high power applications. The analytical hegemony of bulk RTP reveals that by applying a voltage of ± 5 kV on x–y plane of 5 mm thick crystal, maximum ΔkL and is attained $\pm\ 1.4053\ \times\ 10^{-13}$ cm^2/W and ∓ 4.6 respectively. Moreover, the focal length of the configurable lens is tuned in between ± 5.6927 mm to ± 7.8927 mm by adjusting the given voltage to the medium from ± 5 kV to ± 10 KV that highly efficient and commendable for Kerr-lens mode-locking to generate high power mode-locked laser.

The novelty of this work is the preface of the bulk RTP along with its favourable advantages over KTP and BBO that make it a lot more acceptable for an amended cascaded second harmonic generation. Moreover, RTP is congruous for the creation of the mode-locked pulse train of high peak power by minimizing the thermal lensing effect as an adaptive optical component within the laser cavity. A free space optical component tunable focusing of high power laser may additionally be achieved with RTP crystal once the dimension is customized and applied voltage is varied as per demand.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Acknowledgments

The authors would like to thank the Department of Electrical & Electronics Engineering, CMR Institute of Technology, Bengaluru, India and National Institute of Technology Agartala, Tripura, India for the valuable motivation and support.

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