Mathematical Modeling of the Short Circuit Mode of a Voltage Transformer

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Abstract — **When a transformer works under the operating mode an emergency short circuit may occur. This is a dangerous emergency mode, in which the transformer currents are many times higher than their rated values. Such currents may lead to isolation overheating, reliability decreasing or even to transformer failing. In order to prevent damage during the critical mode and increase the transformer reliability by using a protector, it is necessary to determine the short circuit current value. The article considers the problem of optimal accurate design by the method of mathematical transformer modeling by determination of short circuit current value in emergency mode.**

Keywords — mathematical modeling; transformer; short circuit; electromechanical combability.

I. Introduction

Computer technology progress has contributed greatly to the mathematical modeling development in all areas of science. The mathematical modeling allows to research a model of interest with minimizing the utilized resources. Modeling are used when a real experiment performing is impossible, dangerous or difficult and expensive [1]. The main task in modeling is the adoption of an adequate model which properties will be sufficiently close to the real simulated properties.

Now the problem of electromechanical compatibility of electric power system facilities is particularly acute. In [8-11], the authors touched on this important topic for the nominal operation mode of an induction motor. Of particular interest are emergency modes, such as short circuits.

The simplest mathematical model of a single-phase double -winding transformer is considered in the paper. The case of a transformer emergency short-circuiting was investigated and the short-circuit value was calculated by using the model.

A transformer is an inductive coupled circuit (Fig. 1.) which usually used to change voltage and current level [2].

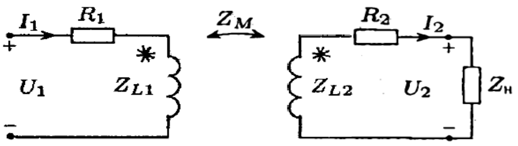

Fig. 1. Transform model shown as two inductively coupled coils circuit.

A transformer is a static electromagnetic converter. When a transformer works under the operating mode an emergency short circuit may occur. This is a dangerous emergency mode, in which the transformer currents are many times higher than their rated values. In such a situation a sharp decrease in reliability of transformer or even transformer failure may occur.

It's important to know a short-circuit current value in case to forecast eventual consequences. Also, it's useful to determine the relation between short-circuit current and input parameters. That way it will be possible to influence on the current value by transformer rating selection during design.

II. Description of model

Transient process ocurred in the circuit (Fig. 2) when the key is closed and the circuit topology changes instantly could be a model of an emergency mode mentioned above. Circuit shown on the figure is a decoupled circuit, where Z_n – a load resistance, r_1, r_2 – internal real coils resistance, M – mutual induction, r_m –real coils resistance of mutual, $L^*_1 = L_1 - M$; $L^*_2 = L_2 - M$ – inductivity of the coils in decoupled circuit, i_1, i_2, i_3 – primary current, secondary current and magnesetion closed loop current, $u_1(t)$ – input voltage, $u_2(t)$ – load voltage.

The discribed mathematical model has following assumptions: such a model can only be applied to low-power transformers as with a large current value nonlinear changes in the magnetic permiability occur and the windings change their magnetic properties.

The second significant assumption is that the transformer switches to short-circuit mode instantly.

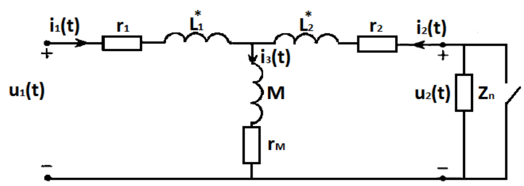

Fig. 2. Transform model shown as decoupled circuit

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The main task of the study is analytical discription of the sudden short-circuit process.

The circuit is in a steady sinusoidal mode before the commutation. Therefore, it’s possible to describe processes occurring in the circuit with simple mathematical equations. At There is a load Z_n at the circuit output in the initial moment. We can find the circuit current to determine the independent initial conditions for the coil current i_1.

The calculation is carrying out using complex amplitude method.

$$\dot{I}_1 = \frac{\dot{U}_1}{R_{eq}}, \tag{1}$$

where $R_{eq} = X_1 + \frac{(Z_n + X_2) \cdot X_m}{Z_n + X_2 + X_m}$

Separately initial conditions for the first coil expres by the formula:

$$\dot{I}_1 = \frac{\dot{U}_1 \cdot (Z_n + X_2 + X_m)}{X_m \cdot X_2 + X_m \cdot Z_n + X_1 \cdot (Z_n + X_2 + X_m)} \tag{2}$$

where $X_1 = Z_{L_1^*} + r_1, X_2 = Z_{L_2^*} + r_2, X_m = Z_m + r_m$

For the second coil separately initial conditions may be found from the following formula:

$$\dot{I}_2 = \dot{I}_1 \cdot \frac{X_m}{X_m + X_2 + Z_n} \tag{3}$$

$$\dot{I}_2 = \frac{\dot{U}_1 \cdot X_m}{Z_n \cdot X_m + X_2 \cdot X_m + X_1 \cdot (X_m + X_2 + Z_n)} \tag{4}$$

In the model, the transition process occurs instantly, after the switch closes. The transient process time is much shorter than the period of the input sinusoidal electrical grid signal. Compose the state equation and find characteristic polynomial poles to calculate the transient process.

Compose the equation according the Kirchhoff’s laws:

$$\begin{cases} i_1 + i_2 = i_3 \\ i_1 \cdot r_1 + L' \cdot \frac{di_1}{dt} + M \cdot \frac{di_1}{dt} + M \cdot \frac{di_2}{dt} + i_1 \cdot r_m + i_2 \cdot r_m = u_1 \\ i_2 \cdot r_2 + L'' \frac{di_2}{dt} + M \cdot \frac{di_1}{dt} + M \cdot \frac{di_2}{dt} + i_1 \cdot r_m + i_2 \cdot r_m = 0 \end{cases} \tag{5}$$

For Laplace transform the equations are following:

$$\begin{cases} I_1 + I_2 = I_3 \\ I_1 \cdot r_1 + L_1^* \cdot (p \cdot I_1 - I_1(0^-)) + M \cdot \left(p \cdot (I_1 + I_2) - I_1(0^-) - I_2(0^-) \right) + (I_1 + I_2) \cdot r_m = U_1 \frac{p}{p^2 + \omega^2} \\ I_2 \cdot r_2 + L_2^* \cdot (p \cdot I_2 - I_2(0^-)) + M \cdot \left(p \cdot (I_1 + I_2) - I_1(0^-) - I_2(0^-) \right) + (I_1 + I_2) \cdot r_m = 0 \end{cases} \tag{6}$$

The equation system can be reduced to the following form:

$$\begin{pmatrix} u_1 \\ 0 \end{pmatrix} = \begin{pmatrix} r_1 + L_1^* \cdot \frac{d}{dt} + M \cdot \frac{d}{dt} + r_m & M \cdot \frac{d}{dt} + r_m \\ M \cdot \frac{d}{dt} + r_m & r_2 + L_2^* \cdot \frac{d}{dt} + M \cdot \frac{d}{dt} + r_m \end{pmatrix} \cdot \begin{pmatrix} i_1 \\ i_2 \end{pmatrix} \tag{7}$$

State equations can be find by expressing the system (7) for $\frac{di_1}{dt}$, $\frac{di_2}{dt}$. The characteristic matrix shown below:

$$\begin{pmatrix} \frac{-r_1 \cdot \left(L_2^* + M \right)}{L_1^* \cdot L_2^* + L_1^* \cdot M + L_2^* \cdot M} - p & \frac{M \cdot r_2}{L_1^* \cdot L_2^* + L_1^* \cdot M + L_2^* \cdot M} \\ \frac{M \cdot r_1}{L_1^* \cdot L_2^* + L_1^* \cdot M + L_2^* \cdot M} & \frac{-r_2 \cdot \left(L_1^* + M \right)}{L_1^* \cdot L_2^* + L_1^* \cdot M + L_2^* \cdot M} - p \end{pmatrix}$$

The characteristic polynomial has following form.

$$\left(L_1^* \cdot L_2^* + L_1^* \cdot M + L_2^* \cdot M \right) p^2 +$$

$$\left(r_1 \cdot \left(L_2^* + M \right) + r_2 \left(L_1^* + M \right) \right) p + r_1 \cdot r_2 = 0$$

The characteristic polynomial poles are determined of transform input parameters only. That means that we can influence a mode observed in the circuit during the transient process by changing the input perameters.

Substitute $Z_n \to 0$ in the previously derived formula (4). In this case the relation between short-circuit current and input transform parameters be as follows.

$$i_2(t) = \frac{u_1(t) \cdot X_m}{X_2 \cdot X_m + X_1 \cdot (X_m + X_2)} \tag{8}$$

where $X_1 = Z_{L_1^*} + r_1, X_2 = Z_{L_2^*} + r_2, X_m = Z_m + r_m$

Using the derived mathematical formulas, we will calculate the short circuit current. Linear transform parameters using in the calculation are taken from the source [3].

TABLE I. PARAMETERS OF THE INVESTIGATED TRANSFORMER

Designations	Value
Apparent power, S	16 [kW]
The primary winding voltage, V_1	660 [Vrms]
The secondary winding voltage, V_2	400 [Vrms]
The mutual induction real resistance, r_m	128 [pu]
Mutual induction, M	17,4 [pu]
The primary winding induction, L_1^*	0,0223 [pu]
The primary winding real resistance, r_1	0,0125 [pu]
The secondary winding induction, L_2^*	0,0223 [pu]
The secondary real resistance, r_2	0,0125 [pu]

The transform no-load operation and the transform operating mode are considered in the paper.

Use the rate valued load for the transform operating mode calculation. The value obtain by sabstiturion the parameters to the following formula:

$$Z_n = \frac{S}{V_2{}^2} \tag{9}$$

$$Z_n = \frac{16 \cdot 10^3 W}{40^2 V} = 10\Omega$$

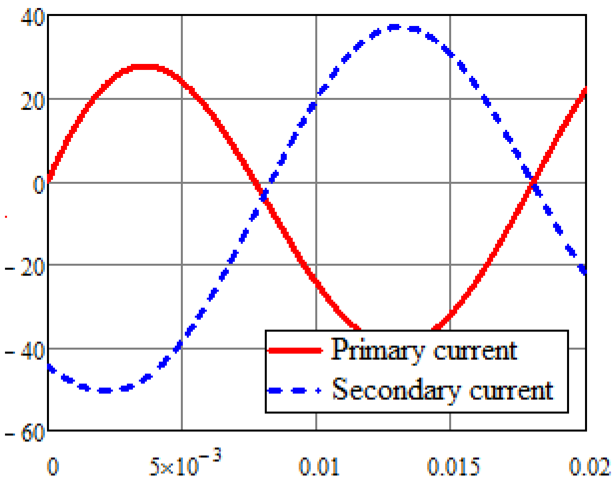

Fig. 3. Primary amd secondary winding current for the transformer under the operating mode.

When short circuit occurs the current has the following view.

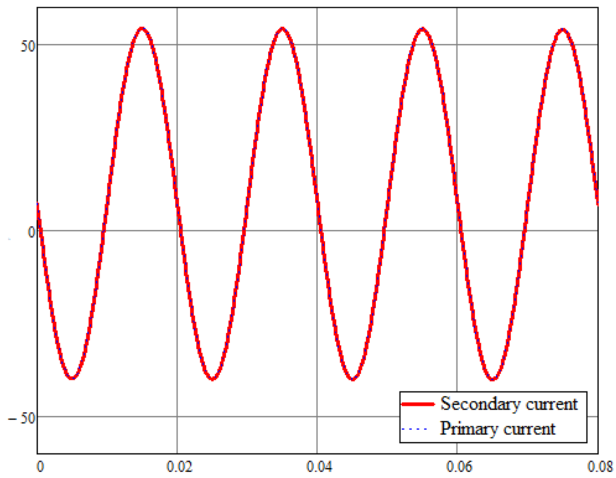

Fig. 4. Primary amd secondary winding current for the short-circuit transformer conditions

The short-circuit current for windings increase three times.

Richter R. gave an estimation to the transform transient process for no-load to short circuit mode commutation in his book [4]. Richter supposed that standart transform short-circuit current increased 30-40 times in relation to the rate current.

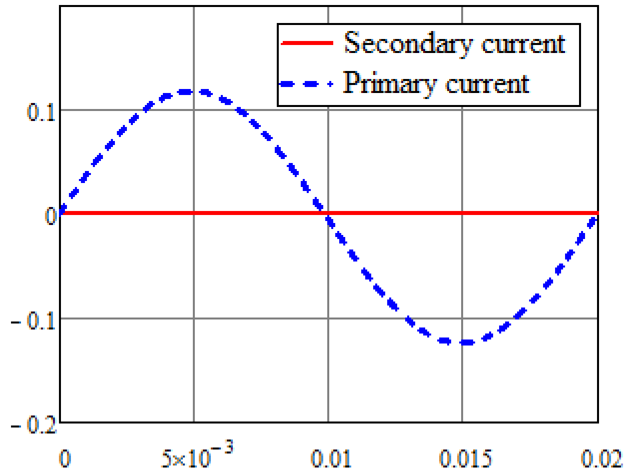

Fig. 5. Primary amd secondary winding current for the transformer under the no-load conditions

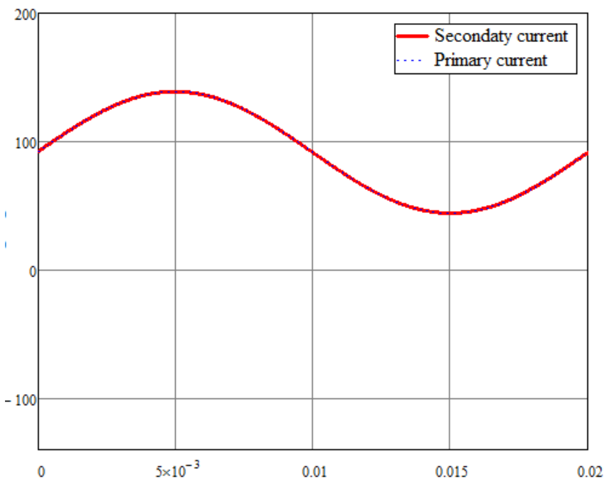

Fig. 6. Primary winding current for the transformer under the no-load conditions for a short time period

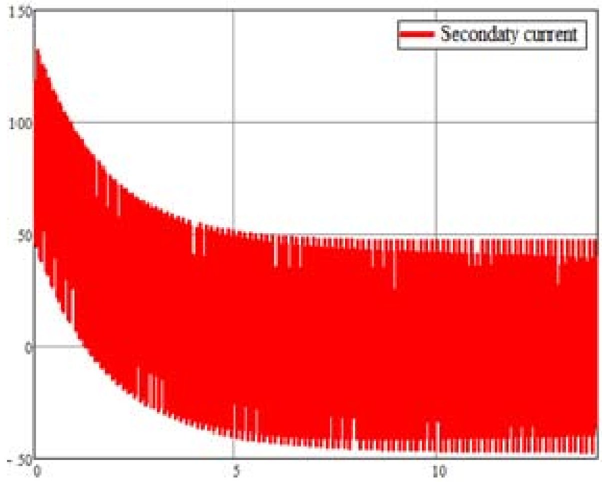

Fig. 7. Secondary winding current for the transformer under the no-load conditions for a long time period

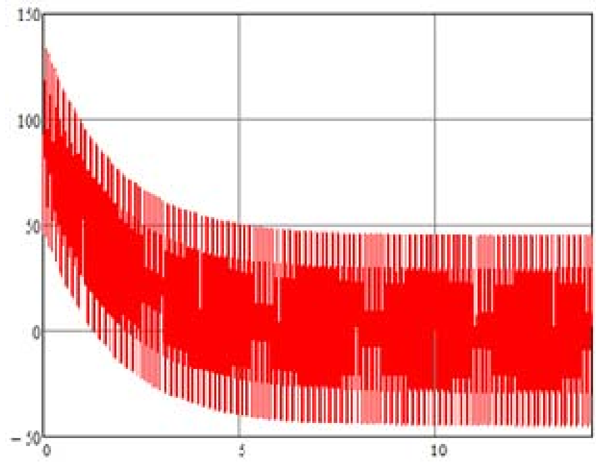

Fig. 8. Primary winding current for the transformer under the no-load conditions for a long time period

The current values in the primary and secondary windings after swithing have values above 100 A.

$$I_n = \frac{S}{V_2} \tag{10}$$

$$I_n = \frac{16 \cdot 10^3 W}{40V} = 40A$$

where, I_n – nominal current

Short-circuit current is about three times more than the nominal current.

III. Conclusions

1. The simple mathematical transform model is considered in the paper. Mathematical modeling of the transition to the short circuit mode from the operating mode. On its basis, mathematical modeling of the transition to the short circuit mode from the nominal operating mode, as well as from the idle mode, was performed.

2. For the transformer presented in the work, the short circuit current is calculated, when switching the circuit from the initial states:

- The operating mode of the circuit with a load of Zn=10

- The no-load mode

As a result of calculations according to the model, it was found that for the first and second cases, the short-circuit current increased 3 times.

3. This model does not take into account the nonlinear change in the magnetic permeability and saturation of the core steel, as well as the increase in the scattering fluxes of the transformer windings, therefore it is applicable only for low-power transformers and for small currents, in further works it is planned to refine the mathematical model to achieve greater accuracy and adequacy of the model. At the same time, in this model, the process of circuit closure occurs instantly, which is also a mathematical assumption and may affect the accuracy of the description of the transformer by this model.

4. In the course of the study, analytical methods and formulas for calculating the short circuit currents of the transformer were obtained. Indeed, these currents exceeded the rated ones by several times; therefore, long-term operation of the transformer in this mode is undesirable, as it causes additional overheating of the winding insulation and a decrease in the reliability and service life of the transformer. Carrying out similar studies for other transformers, it is possible to analyze their short circuit currents, based only on the passport data, which can significantly reduce the accident rate and the cost of conducting full-scale tests.

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