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**Highlights:**

This paper examines sustainability strategies under competition.

We find that sustainability strategies depend on both exposure risk and cost premium.

Firms achieve a win-win equilibrium if the external risk (cost premium) is high (low).

Our results are robust to some extended cases.

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## Analysis of Firm CSR Strategies

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**Abstract**

Corporate social responsibility (CSR) has become increasingly important. From the perspective of operations management, traditional non-CSR-compliant operations are less costly than CSR compliant operations but may be subject to the risk of being exposed to the public by third-party organizations such as Non-Governmental Organizations (NGOs) through external scrutiny. This exposure can  $pgicvkxgn\{ "chhgev" hktou\emptyset" octmgv" ujctg$  when customers are concerned about  $hktou\emptyset"EUT"eqor nkcepg$ . This paper  $uwvfkgu"hktou\emptyset"gpfqigpquw$  CSR compliance strategies, i.e., the incentive to adopt CSR compliant operations.  $Wg"hktuv"eqpukfgt"c"ukping"hktou\emptyset"$  CSR compliance strategies, and then, we extend the analysis to the case of competition. We analyze how exogenous parameters, including the risk of exposure and cost premium, determine equilibrium CSR compliance strategies. We find that CSR compliant operations will be implemented either when the exposure risk is sufficiently high or when the cost premium is sufficiently low. We also discuss how competition affects  $hktou\emptyset"$  CSR compliance strategies and whether firms perform better with CSR compliant operations in equilibrium. **Our results show** that, by adopting CSR compliant operations, firms will engage in a win-win equilibrium if the external risk is high or if the cost premium is low. Besides, we also conduct the analysis under Bertrand (price) competition. Based on these results, we provide managerial insights into when CSR compliant operations should be adopted in practice and how such adoption  $chhgevu" hktou\emptyset"$

performance. Our results also imply that the practice of sustainability requires firms to consider both the external risk of exposure and cost premium.

**Keywords:** corporate social responsibility (CSR); CSR compliant operations; external NGO scrutiny; demand externalities; game theory.

## 1. Introduction

The ongoing social and environmental issues in business practices have made CSR compliant operations ever more important. There is abundant evidence that companies often violate social and environmental standards. In 2013, an eight-story commercial building, Rana Plaza, collapsed in Bangladesh. Rana Plaza produced apparel for multiple well-known brands such as Benetton, Primark, Joe Fresh, and Walmart. This disaster has substantially triggered public concern toward sustainable operations. Based on this concern, some companies adopted measures to enhance their sustainable operations, such as increasing hourly wages and improving future operational standards. Another example of companies violating social and environmental standards is when Cambodian female workers for suppliers of Nike, Asics, Puma, and VF Corporation experienced mass fainting, with many workers collapsing, which was linked to conditions such as body exhaustion, hunger, and excessive room temperatures (Guardian, 2017). Nike launched equitable manufacturing to encourage its suppliers to treat workers better and offered professional training to factory executives in Vietnam, Indonesia, and Thailand. Furthermore, the labor-related deaths and was audited by the Fair Labor Association in 2014 after more than a dozen suicides were reported in its factories (Wall Street Journal, 2016). Following the audit, Foxconn pledged to put more efforts into improving labor conditions, including wage increases by approximately 20% after suicides occurred at its main plant in southern China. Another example is when China Labor investigated four major factories that supply toys to Hasbro and Mattel, where workers experienced inadequate protection and toiled long hours with few, if any, breaks (Fortune, 2015).

In this paper, we study the incentive of firms to adopt CSR compliant operations and how it changes with respect to exogenous parameters, including CSR compliance cost premium and exposure risk. Specifically, our research questions are as follows. Under what circumstances would firms adopt CSR compliant operations? What is the effect of exogenous parameters on firm profits? Do the firms engage in a win-win outcome with CSR compliant operations in equilibrium? To answer these questions, we conduct an analysis using a game-theoretical model. First, we analyze CSR compliance strategies, and next, we generalize the model

to the case of competing firms and analyze equilibrium CSR compliance strategies under competition. In both cases, we find that both the external risk and cost premium are critical in determining hkt ouø CSR compliance strategies in equilibrium.

Based on these results, we also discuss the practical implications of these results on whether and when CSR compliant operations should be adopted. Specifically, our results shed light onto the practice of CSR compliant operations, which should be jointly determined by both the exposure risk with traditional operations and the cost premium of CSR compliance. Overall, we find that CSR compliant operations will normally be adopted either when the external risk is sufficiently high or when the CSR compliance cost premium is sufficiently low. We find that firms are not always better off with CSR compliant operations and characterize the conditions under which firms are worse off when adopting CSR compliant operations. Our results suggest that firms become worse off with CSR compliant operations if the external risk is low and the cost premium is high.

The rest of this paper is organized as follows. Section 2 briefly reviews the related literature. In Section 3, we conduct the benchmarking analysis of the CSR compliance strategies for a single firm. Section 4 analyzes the model by discussing CSR compliance strategies with firms under Cournot (quantity) competition. Section 5 discusses four extensions: 1). continuous CSR compliance strategy; 2). demand uncertainty; 3). updated demand response; and 4). Bertrand (price) competition. The paper concludes in Section 6.

## 2. Related Literature

Our work is related to the literature on corporate social responsibility (CSR), which has been studied since the early 1920s, initially motivated by social rather than economic considerations (Boyd et al., 2007; Bhaduri and Selarka, 2016). CSR initiatives were mainly dedicated to protecting consumer rights and environment, and firms aimed to be philanthropic instead of profit driven with CSR activities. Later, more research emphasized the relationship between CSR activities and firm performance (Vogel, 2005; Porter and Kramer, 2006; Lee, 2008). Departing from existing literature, we consider CSR compliant operations from the perspective of strategic competition instead of philanthropy using a game-theoretical model. Additionally, we provide implications for the common CSR compliance structure and link between CSR cpf" hkt ouø" preferences and performances.

There have also been uvwfkgu" hqewukpi" qp" o gcuwtgu" vq" gpuwtg" uwr rnkgtuø" tgurqpukdkkvkgu" kp" supply chains. Cruz (2008) developed a dynamic model to analyze supply chain networks with CSR via integrated environmental decision-making. In a supply chain, Hsueh (2015) constructed a bilevel programming model to determine the optimal CSR performance and profit levels. These works studied CSR from the perspective of optimization by using mathematical programming

models, while we study sustainable operations by focusing on the effect of strategic competition and the resulting endogenous equilibrium. Ni et al. (2010) examined social responsibility allocation based on both economic and CSR performance criteria under different supply-chain power structures. Xia et al. (2015) proposed a framework that examines the effect of strategic competition on CSR performance under different supply-chain power structures. Bian et al. (2016) integrated corporate social responsibility (CSR) into managerial incentive design to analyze how product relationships affect incentives to adopt CSR activities. Most of the aforementioned studies treated CSR as a quality-type decision, that is, the number of misconducts, but we refer to CSR compliance from the probability perspective. In addition, the above authors studied the effect of CSR on the supply chain relationship, but we have focused on CSR compliance strategies under horizontal competition.

More recently, Plambeck and Taylor (2015) found that the greater effort of a supplier to pass the audit by hiding information, and thus being less worried about preventing harm. Chen and Lee (2016) showed that firms can employ screening mechanisms and incentives to reduce supplier responsibility risks in sourcing contracts and thus make screening more effective and sourcing costs lower. Iyer and Whinston (2013) recommended that a buyer adopt these decisions as a guideline when to promote Material IQ profiles, and recommended that a buyer adopt these decisions as a guideline and inspection to manage the quality of the products provided by suppliers and to identify which is desirable. Unlike these works, we concentrate on endogenous CSR compliance strategies, how they vary with respect to competition, and whether competing firms are better off with CSR compliant practices.

Responsible sourcing has also been studied recently. Chen and Slotnick (2015) discussed that CSR compliance depends on the cost of disclosure and its competitors' strategies from two suppliers and showed that enforcement and penalty, as measures to ensure responsible sourcing, are more desirable than supply-chain transparency. Letizia and Hendrikse (2016) showed that CSR investment incentives can be offered through the supply-chain structure that distributes CSR investment incentives to channel partners. Agrawal and Lee (2018) found that a buyer should adopt a different sourcing policy when partnering with a sole supplier than it would in the case of multiple suppliers. Awasthy and Hazra (2019) identified the optimal policies from the perspectives of the buyer, supplier, and system. Orsdemir et al. (2019) suggested that responsible sourcing can potentially

be achieved through vertical integration and horizontal purchasing. While the aforementioned works studied responsible sourcing, we focus on endogenous choices of whether to adopt sustainable operations. Furthermore, we study how competition can affect such practices and whether firms perform better in equilibrium.

However, another stream of the literature has focused on environmental technology selection and innovation. Debo et al. (2005) examined pricing and technological choices with a remanufacturable product in the presence of heterogeneous consumers. Ohyama and Tsujimura (2008) studied the effect of technological innovation and environmental policy design on emissions reduction under uncertainty. Similarly, Krass et al. (2013) discussed maximizing decisions, such as emissions control technology, in response to environmental policies. Drake et al. (2016) studied the impact of emissions taxation and cap-and-trade policies. Our study showed that an uncertain emissions price under the cap-and-trade policy yields greater expected profits than a constant emissions price under an emissions tax. Unlike the above studies, we concentrate on incentives to switch from traditional to sustainable operations under competition and whether this switch yields a win-win outcome. For a greater understanding of sustainable operations management, we refer to the studies of Drake and Spinler (2013), Brandenburg et al. (2014), Jaehn (2016), and Barbosa-Póvoa et al. (2018).

### 3. Sustainability Strategy of a Monopolist Firm

We first consider the sustainability strategies of a monopolist manufacturer. The decision sequence is as follows. First, the manufacturer decides to choose between traditional operations and CSR compliant operations. Second, the manufacturer sets the production quantity of the products. The decision sequence suggests that CSR compliance choice is more strategic than setting the production quantity. We employ the stylized demand function  $p = \alpha - \beta q$ , where  $\alpha$  is the reservation price,  $p$  is the price, and  $q$  is the product quantity.

In the no CSR compliance scenario, traditional operations are risky due to the chance of being exposed to the public. To catch this risk, we adopt  $\theta \in [0, 1]$  to indicate the probability that CSR compliance choice of the type of operations will be exposed to the public. Consequently, the demand may be negatively affected if the firm is scrutinized and exposed to the public with traditional non-CSR-compliant operations. CSR compliant operations require a cost premium in production, such as using more expensive, environmentally friendlier materials and wage increases. Each firm can choose between CSR compliant and traditional non-CSR-compliant operations, where the latter makes the firm subject to the market risk of being exposed to the public by third-party NGOs. The exposure of the traditional non-CSR-compliant

operations of a firm can negatively affect its demand because consumers may forgo buying products associated to traditional operations (Guo et al., 2013; Orsdemir et al., 2019). Therefore, in this paper we assume that the demand will decrease by a proportion of  $0, 1$ , depending on the concern for EUT. Hence, a higher exposure probability and demand decrease indicate higher market risks. Specifically, the firm would expect its demand with traditional operations to drop to  $1 - q$ . Given this demand, the profit function is thus given by  $\pi = p(1 - q) - cq$ .

This context is similar to customer segmentation based on CSR, where consumers are exogenously segmented into two types: traditional non-CSR consumers and CSR-conscious consumers: the traditional consumer segment does not care about CSR (i.e., non-CSR consumers), but the CSR-conscious consumers are captive to CSR such that they would only choose to purchase products manufactured with CSR friendly operations whenever available (i.e., CSR consumers). As such, the segment of CSR consumers occupy a proportion of  $0, 1$ , which means consumers make purchasing decisions only when they know whether a firm is of CSR type.  $1 - \alpha$  are non-CSR consumers. Our study is also consistent with the existing literature on CSR, such as Öberseder et al. (2011), Chen and Slotnick (2015), Guo et al. (2013), and Orsdemir et al. (2019). After applying the decision sequence, we can solve the problem by maximizing its profit and obtain the optimal results, as shown in Table 1.

**Table 1.** Optimal values with a single firm.

	Traditional operations	CSR compliant operations
Product quantity	$\frac{c}{2}$	$\frac{c}{2}$
Retail price	$\frac{c}{2}$	$\frac{c}{2}$
Firm profit	$\frac{1 - c^2}{4}$	$\frac{c - c^2}{4}$

Ensure that the firm is active in the market.

A higher production cost would occur when the firm chooses to implement CSR compliant operations, i.e., responsible practice requires a cost premium.



given by  $s = p - c - c q$ , where  $v_j$  is the required cost premium for producing each unit of products with CSR compliant operations. Thus,  $c$  is the required cost premium for producing each unit of products with CSR compliant operations, which enables the firm to avoid the risk of being exposed by external scrutiny. The firm can obtain the corresponding results, which are also summarized in Table 1 in the Appendix. After comparing the results of both traditional and CSR compliant operations, we obtain the following results.

**Proposition 1** CSR compliance strategies are given as follows:

- (a)  $s > 0$  for  $0 < c < 1 - \sqrt{1 - c}$ , but  $s < 0$  for  $1 - \sqrt{1 - c} < c < c$ .
- (b) CSR compliant operations are more likely to occur with larger  $\theta$  or  $\lambda$ .

Proof. See Appendix.

Proposition 1(a) suggests that the firm only chooses to adopt CSR compliant operations if the cost premium is not too high; otherwise, the firm would still prefer traditional operations. In addition, the firm is more likely to adopt CSR compliant operations when the exposure probability ( $\theta$ ) or demand decrease ( $\lambda$ ) is sufficiently high, as shown in Proposition 1(b). This CSR compliance strategy depends on both the external exposure risk and internal cost premium. Later, we shall show that this implication carries over well to the case of competition, while the cost premium and exposure risks are more intertwined in affecting CSR compliance strategies under competition.

#### 4. CSR Compliance Strategies of Competing Firms under Cournot Competition

We proceed to analyze the CSR compliance strategies of two competing firms (firm 1 and firm 2). Suppose that firm 1 and firm 2 produce two substitutable products, product 1 and product 2, respectively. CSR compliant operations require a cost premium in production, such as using more expensive, environmentally friendlier materials and wage increases. Similar to the case of a single firm, each firm can choose between CSR compliant and traditional non-CSR-compliant operations, where the latter may subject the firm to the market risk of being exposed to the public by third-party NGOs. The exposure of the traditional non-CSR-compliant operations of a firm can negatively affect its demand because consumers may forgo buying products associated to the non-CSR-compliant firm if it adopts CSR compliant operations (Guo et al., 2013; Orsdemir et al., 2019). With two competing firms, the decision sequence is as follows. In the first stage, both firms simultaneously choose between CSR compliant and traditional operations. Second, both firms decide their production quantities to pursue profit maximization.

Corresponding to each firm's CSR compliance choice, we employ  $v_j$  to indicate the different CSR compliance structures of two competing firms, where  $X (= T \text{ or } S)$  and  $Y (= T \text{ or } S)$  indicate the CSR compliance choices of firms  $i$  and  $j$ , respectively, with  $\delta T$  indicating traditional operations and  $\delta S$  indicating CSR compliant operations. In what follows, we shall analyze each strategy structure. Next, by comparing the firms' equilibrium profits under different strategy scenarios, we can derive the final equilibrium strategy structures, analyze whether the equilibrium strategy structure consists of CSR compliant operations, and analyze whether adopting CSR compliant operations helps firms better off.

Following Singh and Vives (1984), we employ the stylized quadratic concave utility function expressed by

$$U(q_i, q_j) = \alpha_i q_i + \alpha_j q_j - \frac{1}{2} q_i^2 - 2\beta_{ij} q_i q_j - \frac{1}{2} q_j^2, \quad i, j = 1, 2; i \neq j, \quad (1)$$

with  $\alpha_i \geq 0$ ,  $\alpha_j \geq 0$  and  $\beta_{ij} \in [0, 1]$  measuring reservation utility and the degree of substitutability between product  $i$  and product  $j$ , respectively. Thus, consumer surplus after purchasing products is given by

$$CS(q_i, q_j) = U(q_i, q_j) - p_i q_i - p_j q_j, \quad i, j = 1, 2; i \neq j. \quad (2)$$

Differentiating  $CS(q_i, q_j)$  with respect to  $q_i$  and  $q_j$ , letting  $dCS(q_i, q_j)/dq_i = 0$  and  $dCS(q_i, q_j)/dq_j = 0$ , we have the inverse demand function:

$$p_i(q_i, q_j) = \alpha_i - q_i - \beta_{ij} q_j, \quad i, j = 1, 2; i \neq j. \quad (3)$$

Based on Eq. (3), we can proceed to discuss each CSR compliance strategy structure in detail below.

#### 4.1 Traditional Strategy Structure $TT$

We first consider the traditional strategy structure  $TT$ , where both firm  $i$  and firm  $j$  choose traditional non-CSR-compliant operations.  $\beta_{ij}$  decrease due to the negative concern from consumers (Guo et al., 2013; Orsdemir et al., 2019). Thus, the demand of firm  $i$  would be  $q_i = \alpha_i - p_i$ ,  $i = 1, 2$ . Following the above specified decision sequence, we adopt backward induction to ensure subgame perfection. Under the strategy structure  $TT$ , firm  $i$  solves the following problem:

$$\max_{q_i} \pi_i^{TT}(q_i, q_j) = p_i(q_i, q_j) q_i - c q_i, \quad i, j = 1, 2; i \neq j, \quad (4)$$

that both firms choose the traditional operations, and the firm  $i$ . Substituting Eq. (3) into Eq. (4) and solving the corresponding first-order conditions (FOCs) of Eq. (4) yields the equilibrium quantities

$$q_i^{TT} = \frac{c}{2}, i = 1, 2. \quad (5)$$

Based on Eq. (5),  $q_i^{TT} = \frac{c}{2}$ ,  $i = 1, 2$ . and they are summarized in the first column of Table 2.

**Table 2.** Equilibrium values under Cournot competition.

	Traditional strategy structure $TT$	Mixed strategy structure $TS$		CSR compliance strategy structure $SS$
Product quantity	$\frac{c}{2}$	$\frac{2c}{4}$	$\frac{2c}{2}$	$\frac{c}{2}$
Price	$\frac{2c}{4}$	$\frac{2c}{4}$	$\frac{2c}{2}$	$\frac{c}{2}$
Firm profits	$\frac{1}{2}c^2$	$\frac{2c^2}{4}$	$\frac{2c^2}{2}$	$\frac{1}{2}c^2$

where  $\delta = \frac{1}{2}$  and  $c \triangleq \frac{1}{2}c$  ensure that

$q_i^{TT} = \frac{c}{2}$ ,  $i = 1, 2$ .

#### 4.2 Mixed CSR Compliance Strategy Structures $ST$ or $TS$

In this section, we consider the mixed CSR compliance structure, where one firm chooses CSR compliant operations, while the other adopts traditional operations. Due to symmetry, it suffices to consider CSR compliance structure  $TS$  only, with the other ( $ST$ ) being just the opposite. Specifically, we assume that in CSR compliance structure  $TS$ , firm  $i$  chooses traditional operations, whereas firm  $j$  adopts CSR compliant operations for ease of exposition. With this mixed strategy structure, firm  $i$  may experience a demand drop due to the traditional operations being exposed to the public, which leads to firm  $i$ 's demand being  $(1 - \delta)q_i$ , which means a proportion  $\delta$  of consumers are CSR captive but the rest  $(1 - \delta)$  are non-CSR consumers. This is consistent with previous studies such as Guo et al. (2013) and Orsdemir et al.

(2019). In contrast, firm  $j$  may benefit from demand increase because the customers of firm  $i$  may choose to switch to firm  $j$  as a consequence of such public exposure. Thus, firm  $j$ 's demand is  $q_j = q_i + \dots$ .

Based on the above analysis, firm  $i$  and firm  $j$  have the following objectives:

$$\begin{aligned} \max_{q_i} \quad & TS_{M_i}(q_i, q_j) - p_i(q_i, q_j) - c_1 q_i \\ \max_{q_j} \quad & TS_{M_j}(q_i, q_j) - p_j(q_i, q_j) - c_2 q_j - c_1 q_i \end{aligned}, \quad i, j = 1, 2; i \neq j. \quad (6)$$

After solving Eq. (6), the quantity responses can be obtained as

$$\begin{aligned} q_i &= \frac{2 - 2c_2 + c_1}{4 - 2c_1} \\ q_j &= \frac{2 - 2c_1 + c_2}{4 - 2c_1} \end{aligned}, \quad i, j = 1, 2; i \neq j. \quad (7)$$

Following Eq. (7), the results, which are summarized in the second column of Table 2 in the Appendix.

**Lemma 1.** Given that firm  $i$  adopts traditional non-CSR-compliant operations, the strategy of the other firm (firm  $j$ ) is given as follows:

(a) If  $c_1 < \dots$ , then  $\dots$  for  $0 < c_1 < \dots$ , where  $\dots$  and

$$c_1 < \frac{2\sqrt{5} - 4}{2}.$$

(b) If  $c_1 > \dots$ , then  $\dots$  for  $0 < c_1 < \dots$ , while  $\dots$  for  $\dots$ , with

the equality holding at  $c_1 = \dots$ , where

$$c_1 = \frac{2 - 2\sqrt{1 - 4c_2^2}}{2}.$$

Proof. See Appendix.

Given that one firm (firm  $i$ ) adopts traditional non-CSR-compliant operations, Lemma 1 discusses the CSR compliance strategies of the other firm (firm  $j$ ). We identify the conditions under which firm  $j$  should practice CSR compliance and what the factors affecting its strategy choices on CSR compliance are. Specifically, Lemma 1(a) suggests that firm  $j$  should always adopt CSR compliant operations, regardless of how expensive they are, when the market risk of exposure is sufficiently high ( $c_1 > \dots$ ). This is because the outside market risk due to exposure is too high for not becoming CSR compliant, and the high risk of market loss outweighs the cost premium in terms of affecting firm  $j$ 's demand.

In contrast, if the exposure risk is low, firm  $j$ 's CSR compliance choices will be contingent on the magnitude of the cost premium. In other words, firm  $j$  should implement CSR compliant operations if doing so is not expensive, i.e., if the cost premium is not too high ( $0 < c < c_1$ ); otherwise, traditional non-CSR-compliant operations are preferred, as shown in Lemma 1(b). This result implies that the practice of CSR compliant operations should be jointly determined by the external risk of exposure and the cost premium of CSR compliance.

### 4.3 CSR Compliance Strategy Structure SS

Next, we examine the strategy structure where both firms adopt CSR compliant operations, i.e., CSR compliance strategy structure SS. In this case, there will be no demand decrease as the price premium is not too high (Guo et al., 2013; Orsdemir et al., 2019), i.e.,  $Q_i^{SS} = q_i, i = 1, 2$ . Mathematically, this means firm  $i$ 's profits are given by

$$\max_{q_i} \pi_i^{SS}(q_i, q_j) = p_i(q_i, q_j) - c_i q_i, i, j = 1, 2; i \neq j. \quad (8)$$

After solving the FOCs from Eq. (8), we have product quantities as

$$q_i^{SS} = \frac{c - c_j}{2}, i, j = 1, 2; i \neq j. \quad (9)$$

Based on Eq. (9), we can obtain all other equilibrium results, which are shown in the last column of Table 2 in the Appendix.

**Lemma 2.** Given that firm  $i$  chooses CSR compliant operations, the strategy of the other firm (firm  $j$ ) is given below:

(a) If  $c_2 > \frac{c_1}{2}$ , then firm  $j$  chooses CSR compliant operations for  $0 < c < \frac{c_1}{2}$ , where

$$c_2 > \frac{1}{2} \sqrt{2c_1^2 - 8c_1c + 8c^2} - \frac{1}{2}c_1 + c.$$

(b) If  $c_2 < \frac{c_1}{2}$ , then firm  $j$  chooses CSR compliant operations for  $0 < c < c_2$  and chooses non-CSR compliant operations for  $c_2 < c < \frac{c_1}{2}$ , with

the equality holding at  $c = c_2$ , where

$$c_2 = \frac{2\sqrt{1 - \frac{2c_1^2 - 4c_1c + 4c^2}{4c_1^2}} + c_1}{2}.$$

Proof. See Appendix.

Lemma 2 discusses the CSR compliance strategies of one firm (firm  $j$ ), provided that the other firm (firm  $i$ ) practices CSR compliant operations. We identify the conditions under which firm  $j$  should employ CSR compliant operations, and the results are structurally similar to those of Lemma 1. Lemma 2(a) shows that firm  $j$  should always adopt CSR compliant operations in the

presence of high exposure risk ( $\lambda_2$ ), regardless of the cost premium of CSR compliance. This is the same reasoning as in Lemma 1. In contrast, firm  $j$  on the magnitude of the cost premium when both exposure risks are low. In other words, firm  $j$  should implement CSR compliant operations if doing so is not too costly; otherwise, traditional non-CSR-compliant operations are preferred, as shown in Lemma 2(b).

Based on the results in Lemma 1 and Lemma 2, we obtain the following results on CSR compliance strategy structures in equilibrium.

**Proposition 2.** The equilibrium structures of CSR compliance strategies are characterized as follows:

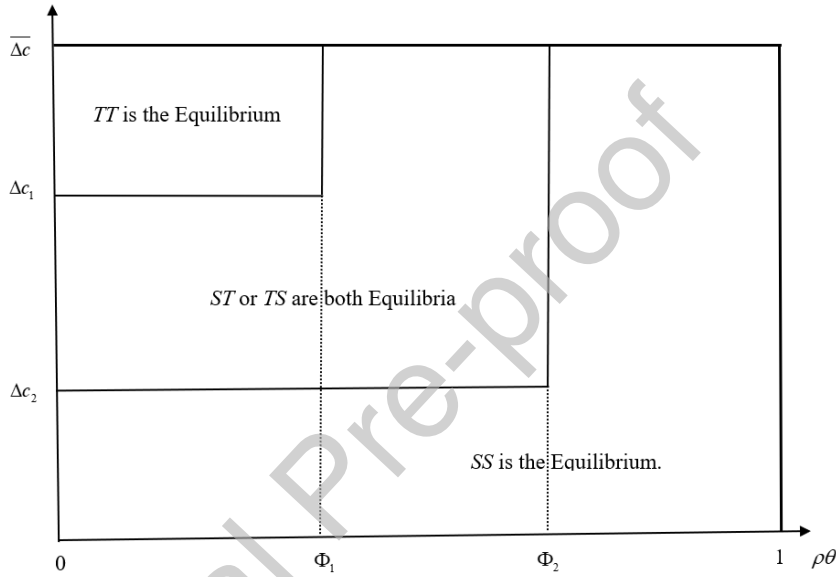
- a). The traditional strategy structure  $TT$  is a coordination equilibrium if  $\lambda_2 < \lambda_1$  and  $c_2 < c_1$ .
- b). The CSR compliance strategy structure  $SS$  is a coordination equilibrium if either of the following two conditions holds:
  - i).  $\lambda_2 < \lambda_1$  and  $c_2 > c_1$ ;
  - ii).  $\lambda_2 > \lambda_1$  and  $c_2 < c_1$ .
- c). Otherwise, the CSR compliance strategy structures  $ST$  or  $TS$  are the equilibria.

Proof. See Appendix.

Proposition 2 identifies the equilibrium CSR compliance strategies for the two competing firms. Specifically, the results are explained as follows. First, when the exposure risk is sufficiently low (Proposition 2 (a)), then CSR compliance strategies critically depend on the magnitude of the cost premium. Both firms will choose to adopt CSR compliant (traditional) operations if the cost premium is low (high), while the mixed CSR compliance strategies will arise if the cost premium is within the intermediate range (i.e.,  $c_2 < c < c_1$ ). The reasoning behind the case of mixed CSR compliance strategies is as follows. On the one hand, when one firm chooses traditional operations, the other firm will adopt CSR compliant operations because being CSR compliant can attract more customers from its competitors and can allow firms to avoid exposure risk. On the other hand, when one firm chooses CSR compliant operations, the other would likely stick with traditional operations to save costs, without being too concerned with the low level of exposure risk. Overall, we show that under low exposure risk, the two competing firms will engage in a coordination equilibrium by choosing the same strategy ( $SS$  or  $TT$ ) when the cost premium either too high or low. In contrast, the firms will choose different strategies to differentiate themselves from each other when the cost premium is intermediate.

Second, when the exposure risk is intermediate (Proposition 2 (b)), then the traditional strategy structure  $TT$  will no longer occur, while the purely CSR compliance strategies  $SS$  (or

mixed strategies  $TS/ST$  would arise in the case of a low (or high) cost premium, i.e., the two coordination equilibria will not arise under the same condition. Finally, when the exposure risk is high, the firms would always choose CSR compliant operations, regardless of the cost to implement CSR compliance, i.e., only the coordination equilibrium  $SS$  will occur. In general, this result indicates that the cost premium of CSR compliant operations plays a less important role when the exposure risk becomes higher. Together with the previous results, Proposition 2 again confirms that the outside exposure risk and cost premium of CSR compliant operations jointly determine whether firms should adopt CSR compliant operations in equilibrium. All the cases in Propositions 2 (a)-(c) are described by different areas in Figure 1 below.



**Figure 1.** Equilibrium Sustainability Strategy Structures

Next, we proceed to discuss whether the firms are better off by switching from traditional to CSR compliant operations, i.e., whether CSR compliance yields a better outcome compared to traditional operations.

**Proposition 3.** Compared to the benchmark traditional structure  $TT$ , we have

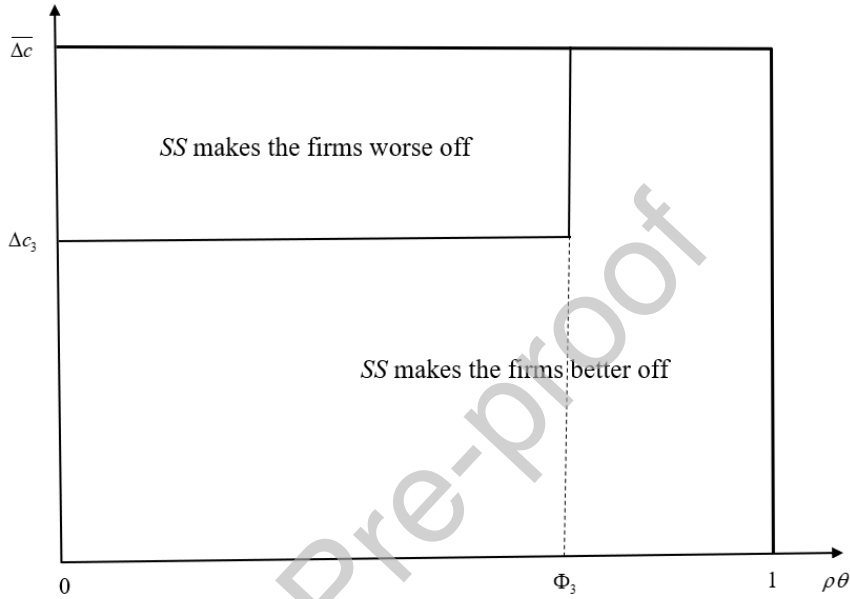
(a) If  $0 < \rho\theta < \Phi_1$ , then CSR compliance structure  $SS$  makes the firms better off when  $0 < c_3 < c_2$ , while the firms become worse off when  $c_3 > c_2$ , where  $c_2 = 2\sqrt{2} - 2$  and  $c_3 = 1 - \sqrt{1 - c}$ .

(b) If  $\rho\theta > \Phi_2$ , then CSR compliance structure  $SS$  always leads to higher profits for both firms.

Proof. See Appendix.

Proposition 3 identifies the conditions under which the firms are better off under equilibrium CSR compliance structure  $SS$  compared to the traditional strategy structure  $TT$ . The results suggest that the firms do not necessarily perform better with CSR compliant operations.

Specifically, the firms are better off with CSR compliant operations under two conditions: a) when the exposure risk is high or b) when the exposure risk is low and CSR compliance is not very costly. In contrast, when the external risk is low and the CSR compliance cost premium is sufficiently high, the firms are worse off by adopting CSR compliant operations. This result is also illustrated in Figure 2 and provides a practical guideline in the sense that, by estimating the external risk and CSR compliance cost premium, firms can judge whether their performance can become better/worse in equilibrium.



**Figure 2.** Compliance Strategy Structure  $SS$

Comparing between the cases of monopoly and duopoly, we have the following two observations:

- 1). In the area of  $TT$  ( $SS$ ) equilibrium, the monopolistic firm would choose  $T$  ( $S$ ) as the optimal strategy.
- 2). The area of mixed strategy structure  $ST$  (or  $TS$ ) is split into two parts with low and high cost premiums respectively. In the low (high) cost premium part, the monopolistic firm would choose  $S$  ( $T$ ) as the optimal strategy.

## 5. Discussion of Extensions

In this section, we extend the base model to include four extensions: 1). continuous CSR compliance strategy; 2). demand uncertainty; 3). updated demand response; and 4). Bertrand competition. In what follows, we shall discuss each scenario separately.

### 5.1. Analysis of Continuous CSR Compliance Strategy

Following the binary CSR compliance strategy above, we proceed to discuss the case with continuous strategy spectrum. We employ  $i = 0, 1, 2$  to capture the level of CSR



compliance, with  $\delta_i = 0$  and  $\delta_i = 1$  representing the cases of no and full CSR compliance choices, respectively. The probability of detection is given by  $p_i = 1 - \delta_i$ , which is a decreasing function of  $\delta_i$ .  $\delta_i = 0$  and  $\delta_i = 1$  correspond to the cases when firm  $i$  has no and full CSR compliance strategies, respectively, where the former shows a generic probability of detection while the latter no probability of detection. In this setting, firm  $i$ 's expected profit function can be expressed as

$$M_i = \frac{p_i c + c - \delta_i q_i + (1 - \delta_j) q_i - \delta_j q_j}{p_i c + 1 - \delta_i + \delta_i q_i}, \quad i, j = 1, 2; i \neq j. \quad (10)$$

Although it is hard to derive analytical results for the general case of continuous CSR compliance and decreasing probability of detection, we can link the case of continuous CSR compliance strategies with the base model scenarios based on Eq. (10), which are also shown as follows in Table 3.

**Table 3.** Profile of CSR Compliance Strategies

		Firm $j$ 's EUT Eq. (10)	
		$\delta_j = 0$	$\delta_j = 1$
Firm $i$ 's EUT Compliance Strategy	$\delta_i = 0$	CSR Compliance Strategy Structure $TT$	CSR Compliance Strategy Structure $TS$
	$\delta_i = 1$	CSR Compliance Strategy Structure $ST$	CSR Compliance Strategy Structure $SS$

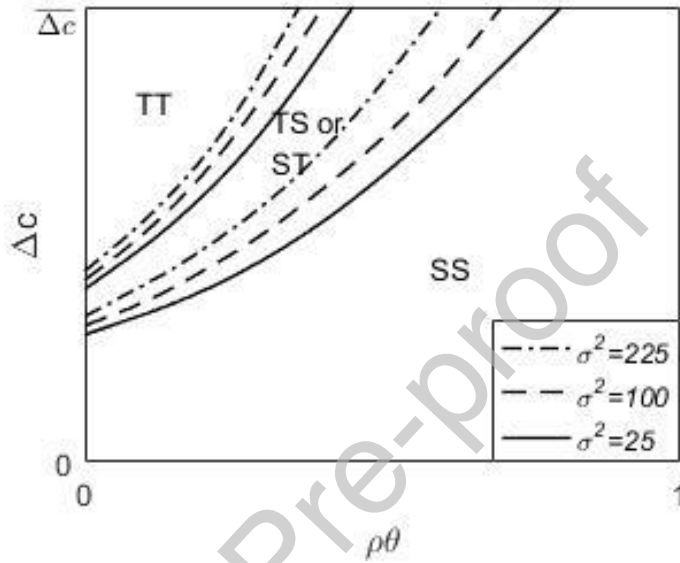
Based on Eq. (10) and Table 3, it can be easily seen that our previous models and results carry over to the general case qualitatively.

### 5.2. Analysis of CSR Compliance Strategies Under Uncertainty

Following the base model, we proceed to extend the model to include demand uncertainty. To this end, we assume the demand function is given by  $\tilde{p}_i = q_i + \tilde{\epsilon}_i$ ,  $\tilde{q}_j = q_j + \tilde{\epsilon}_j$ ,  $i, j = 1, 2; i \neq j$ , where  $\tilde{\epsilon}_i \sim N(\mu, \sigma^2)$ , with  $\mu$  constant and normally distributed with mean 0 and variance  $\sigma^2$ . Based on this, each firm will make decisions considering the expected profit given the CSR compliance structure.

We conduct an extensive numerical study to investigate the impact of cost premium and exposure risk on the equilibrium strategies by using different parameters. For example, we choose  $\mu = 50$ ,  $c = 10$ ,  $c \sim U(0, 8)$  with 20 discrete values,  $\sigma \sim U(0.05, 0.95)$  with 20 discrete values. The stochastic factor follows truncated normal distribution  $N(a, b, \sigma^2)$  in  $a, b$ ,

where  $\rho = 0$  to ensure that the expected value of the potential market equals  $\bar{c}$ . We choose three different intervals such as  $a = 15, b = 15$ ,  $a = 15, b = 30$ , and  $a = 30, b = 15$  with three different variations such as  $\sigma^2 = 25, 100$ , and  $225$ . The numerical results show that using different parameters do not qualitatively change the main conclusions derived in the deterministic demand case. Thus, we choose  $a = 15, b = 30$  as the representative example to present the results, as shown in Figure 3 and Figure 4 below.



**Figure 3.** Equilibrium Strategies Under Demand Uncertainty

### 5.3. CSR Compliance Strategies with Updated Demand Response

Following the base model, we proceed to extend the model to the case where prices have been updated with demand changes. To this end, we assume the demand function is given by  $p_i = Q_i, Q_j = Q_i - Q_j, i, j = 1, 2; i \neq j$ , where the actual demand levels  $Q_i$  and  $Q_j$  are different under different strategy structures, which are discussed in detail below.

We first consider strategy structure  $TT$ , where both firm 1 and firm 2 choose traditional non-CSR-compliant operations, so the demand of firm  $i$  would be  $Q_i^{TT} = 1 - q_i, i = 1, 2$ , that both firms choose traditional operations. Under the strategy structure  $TT$ , firm  $i$  solves the following problem:

$$\text{Max}_{M_i} \pi_i^{TT}(Q_i^{TT}, Q_j^{TT}) = p_i(Q_i^{TT}, Q_j^{TT}) - c_i Q_i^{TT}, i, j = 1, 2; i \neq j. \quad (11)$$

Next, under the mixed CSR compliance strategy structure, one firm chooses CSR compliant operations, while the other adopts the traditional non-CSR-compliant operations. Due to symmetry, it suffices to consider the mixed CSR compliance strategy structure  $TS$  only, with the

other (*ST*) being symmetric. Specifically, we assume that in strategy structure *TS*, firm *i* chooses traditional non-CSR-compliant operations, whereas firm *j* adopts CSR compliant operations for ease of exposition. With strategy structure *TS*, firm *i* may experience a demand drop due to the non-CSR-compliant operations being exposed to the public, which leads to firm *i*'s demand being  $Q_i^{TS} = 1 - \frac{1}{4} q_i$ . In contrast, firm *j* may benefit from demand increase because the customers of firm *i* may choose to switch to firm *j* because of such public exposure. Thus, firm *j*'s demand is  $Q_j^{TS} = \frac{1}{4} q_i + q_j$  (Guo et al., 2013; Orsdemir et al., 2019).

The demand functions will be  $\tilde{p}_i = Q_i^{TS}, Q_j^{TS}$  if *i* chooses traditional operations and firm *j* adopts CSR compliant operations. Under the mixed strategy structure *TS*, firm *i* and firm *j* solve the following problem:

$$\begin{aligned} \text{Max}_{M_i} \quad & Q_i^{TS}, Q_j^{TS} \quad p_i(Q_i^{TS}, Q_j^{TS}) - c Q_i^{TS}, i, j = 1, 2; i \neq j. \\ \text{Max}_{M_j} \quad & Q_j^{TS}, Q_i^{TS} \quad p_j(Q_j^{TS}, Q_i^{TS}) - c Q_j^{TS} \end{aligned} \quad (12)$$

Finally, we examine the strategy structure where both firms adopt CSR compliant operations, i.e., CSR compliance strategy structure *SS*. In this case, there will be no demand changes as the market is symmetric. Mathematically,  $Q_i^{SS} = q_i, i = 1, 2$ .

$$\text{Max}_{M_i} \quad Q_i^{SS}, Q_j^{SS} \quad p_i(Q_i^{SS}, Q_j^{SS}) - c Q_i^{SS}, i, j = 1, 2; i = j. \quad (13)$$

The results under different strategy structures are summarized in Table 4 below.

**Table 4.** Equilibrium values with updated demand information

	Traditional strategy structure <i>TT</i>	Mixed strategy structure <i>TS</i>				CSR compliance strategy structure <i>SS</i>
Product quantity	$\frac{2c}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{c}{3}$	$\frac{c}{3}$	$\frac{c}{3}$
Price	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{c}{3}$	$\frac{c}{3}$	$\frac{c}{3}$

Firm	$\frac{c^2}{2}$	1	1	2	2	$c$	$c^2$
profits		$\frac{1}{4}$		$\frac{2}{2^2}$			
		2	2	$\frac{2}{4}$	$\frac{c}{2^2}$	2	$\frac{2}{c}$
		$\frac{4}{4}$		$\frac{2}{2^2}$			

Proof. See Appendix.  $c \leq \frac{1}{2}$  ensure that

hkt o u r t q f w e k q p " s w c p v k v k g u " c t g " r q u k v k x g " k p " v j g " o c t m g v 0

**Lemma 3.** Given that firm  $i$  adopts traditional operations, the strategy of the other firm (firm  $j$ ) is given as follows:

(a) If  $u_1$ , then for , where and

$$u_1 = \frac{2}{\sqrt{2}}$$

(b) If  $u_1$ , then for  $0 < c < c_1^U$ , while for  $c_1^U < c < \bar{c}$ ,

with the equality holding at  $c = c_1^U$ , where

$$c_1^U = \frac{\sqrt{4^2 + 2^2 + 16^2 + 3^3 + 8^2 + 8^2 + 1^2 + 2^3 + 5^2 + 8^2 + 4^2 + 2^2}}{2^2 + 2^2 + 2^2}$$

Proof. See Appendix.

With updated information, Lemma 3 is the result corresponding to Lemma 1 under Cournot competition. Thus, the results in Lemma 3 and Lemma 1 are structurally the same, and the rationale behind the results is also the same, which suggests that the result of Lemma 1 carries over to the case with updated information.

**Lemma 4.** Given that firm  $i$  chooses CSR compliant operations, the strategy of the other firm (firm  $j$ ) is given below:

(a) If  $u_2$ , then for , where

$$u_2 = \frac{6^2 + 4^2 + \sqrt{2^2 + 3^3 + 3^2 + 5^2}}{2^3 + 3^2 + 5}$$

(b) If  $u_2$ , then for  $0 < c < c_2^U$  and for  $c_2^U < c < \bar{c}$ , with

the equality holding at  $c = c_2^U$ , where

$$c_2^U = \frac{\sqrt{1 - \frac{2^2 \cdot 2^2 \cdot 4^2 \cdot 2^2 \cdot 2^2 \cdot 2^2}{4 \cdot 1 \cdot 2^2 \cdot 2^2 \cdot 4 \cdot 2^2 \cdot 2^2 \cdot 3^3}}}{4 \cdot 1 \cdot 2^2 \cdot 2^2 \cdot 4 \cdot 2^2 \cdot 2^2 \cdot 3^3} \cdot c$$

Proof. See Appendix.

Similarly, the result in Lemma 4 is structurally identical to that of Lemma 2 under Cournot competition. Based on the results in Lemma 3 and Lemma 4, we obtain the following results on

**Proposition 4.** The equilibrium structures of CSR compliance strategies are characterized as follows:

a). The traditional strategy structure  $TT$  is a coordination equilibrium if  $\frac{c_2}{c} > \frac{1}{2}$  and  $\frac{c_1}{c} > \frac{1}{2}$ .

b). The CSR compliance strategy structure  $SS$  is a coordination equilibrium if either of the following two conditions holds:

i).  $\frac{c_1}{c} > \frac{1}{2}$  and  $\frac{c_2}{c} > \frac{1}{2}$ ; or

ii).  $\frac{c_1}{c} > \frac{1}{2}$  and  $\frac{c_2}{c} > \frac{1}{2}$ .

c). Otherwise, the CSR compliance strategy structures  $ST$  or  $TS$  are the equilibria.

Proof. See Appendix.

The equilibrium structures characterized by Proposition 4 are also structurally the same as those in Proposition 2.

#### 5.4. CSR Compliance Strategies of Competing Firms under Bertrand Competition

In this section, we shall check whether the results under Cournot (quantity) competition carry over to the Bertrand competition (price) mode. To this end, we need to invert Eq. (3) to express the demand functions with prices as decision variables:

$$q_i = p_i - \frac{1}{2} \frac{p_i - p_j}{c_i - c_j}, \quad i, j = 1, 2; \quad i \neq j. \tag{14}$$

CSR compliance structures are expressed as follows. Traditional operations strategy structure  $TT$  are given by

$$\text{Max}_{M_i^B} \pi_i = (p_i - c_i) q_i, \quad i, j = 1, 2; \quad i \neq j, \tag{15}$$

where  $\pi_i$  is the profit of firm  $i$ .

CSR compliance structures  $SS$  are given by

$$\begin{aligned} \text{Max}_{M_i}^{TS-B} p_i, p_j &= p_i - c - 1 - q_i p_i, p_j, \quad , i, j = 1, 2; i \neq j. \quad (16) \\ \text{Max}_{M_j}^{TS-B} p_i, p_j &= p_j - c - c - q_j p_i, p_j - q_i p_i, p_j \end{aligned}$$

Vjg"hkt o uø" qdlgevkvxgu"wpfjgt"vjg"CSR compliance strategy structure SS are given by

$$\text{Max}_{M_i}^{SS-B} p_i, p_j = p_i - c - c - q_i p_i, p_j, \quad , i, j = 1, 2; i \neq j. \quad (17)$$

Following the solution procedure as stated before, we can solve the above problems correspondingly. The results under Bertrand competition are summarized in Table 5.

**Table 5.** Equilibrium values under Bertrand competition.

	Traditional strategy structure <i>TT</i>	Mixed strategy structure <i>TS</i>	CSR compliance strategy structure <i>SS</i>
Product quantity	$\frac{1}{2} \frac{c}{2}$	$\frac{1}{2} \frac{2}{4} \frac{c}{3} \frac{1}{2} \frac{c}{2} \frac{1}{2} \frac{c}{2}$	$\frac{1}{2} \frac{c}{2}$
Price	$\frac{1}{2} \frac{c}{2}$	$\frac{1}{2} \frac{2}{4} \frac{2}{3} \frac{1}{2} \frac{c}{2} \frac{1}{2} \frac{c}{2}$	$\frac{1}{2} \frac{c}{2}$
Firm profits	$\frac{1}{2} \frac{1}{2} \frac{c^2}{2}$	$\frac{1}{2} \frac{1}{2} \frac{2}{4} \frac{c}{3} \frac{1}{2} \frac{c^2}{2}$	$\frac{1}{2} \frac{c}{2} \frac{c^2}{2}$

$$P_{qv} v_j c v^e q p f k v k p^o \delta = c \frac{1}{2} \frac{2}{2} \frac{3}{1} \frac{c}{2} \triangleq \frac{c^B}{c} \delta^u j q w n f^d g^$$

satisfied to ensure that vjg"hkt o uø" r t q f w e v k q p^s w c p v k v k g u^c t g^ r q u k v k x g^k p^v j g^o c t m g v^0

Based on these solutions, we can obtain the following results.

**Lemma 5.** Given that firm *i* adopts traditional operations, the strategy of the other firm (firm *j*) under Bertrand competition is given as follows:

(a) If  $\frac{B}{1}$ , then  $\frac{TS}{M_j} = \frac{TT}{M_j}$  for  $0 < c < \frac{c^B}{c}$ , where

$$\frac{B}{1} = \frac{2}{2} \frac{2}{4} \frac{\sqrt{20}}{3} \frac{8}{2} \frac{12}{2} \frac{2}{3} \frac{4}{2} \frac{2}{4} \frac{5}{5} \frac{6}{1} \frac{1}{2} \frac{2}{2}$$

(b) If  $c_1^B < c$ , then  $\frac{TS}{M_j} > \frac{TT}{M_j}$  for  $0 < c < c_1^B$ , while  $\frac{TS}{M_j} < \frac{TT}{M_j}$  for  $c_1^B < c < \frac{c}{2}$ ,

where

$$c_1^B = \frac{1 + \sqrt{1 - \frac{1}{4} \left( \frac{4}{3} - \frac{2}{c} \right)^2}}{2}.$$

Proof. See Appendix.

Under Bertrand competition, Lemma 5 is the result corresponding to Lemma 1 under Cournot competition. Thus, the results in Lemma 5 and Lemma 1 are structurally the same, and the rationale behind the results is also the same, which suggests that the result of Lemma 1 carries over to the case of Bertrand competition.

**Lemma 6.** Given that firm  $i$  chooses CSR compliant operations, the strategy of the other firm (firm  $j$ ) under Bertrand competition is given below:

(a) If  $c_2^B < c$ , then  $\frac{SS}{M_j} > \frac{ST}{M_j}$  for  $0 < c < c_2^B$ , where  $c_2^B$  is the solution of the following equation:

$$2^2 X^3 - 14X^2 + 16X - 8 = 0.$$

(b) If  $c_2^B > c$ , then  $\frac{SS}{M_j} < \frac{ST}{M_j}$  for  $0 < c < c_2^B$  and  $\frac{SS}{M_j} > \frac{ST}{M_j}$  for  $c_2^B < c < \frac{c}{2}$ ,

where  $c_2^B = \frac{1 + H_1 + \sqrt{H_2}}{H_3} c$ , with

$$H_1 = \frac{3^2 - 2^2 + 3^2 - 7 + 2^3}{2^2 - 16 + 8^2 - 3^2},$$

$$H_2 = \frac{9^4 - 2^2 + 5^3 - 3^3 - 2^2 + 20 - 3^8 - 2^4 - 4^4}{2^2 - 74 + 30^2 - 58^2 - 12^3 - 11^4 - 3^3},$$

$$H_3 = \frac{2^2 - 80 + 74^2 - 92^2 - 58^3 - 34^4 - 11^5 - 4^6 - 2^2}{2^2 - 4^2 + 8^2 - 20^2 - 6^2 - 8^3 - 4^4 - 2^2 - 2^2 - 4^2 - 2^2},$$

$$H_3 = \frac{4^2 - 2^2 + 3^3 - 2^9 - 26^2 - 13^2 - 2^3 - 4^2 - 2^2}{2^2 - 12 + 20^3 - 3^2 - 8^3 - 2^2 - 2^2 - 2^2 - 2^2}.$$

Proof. See Appendix.

Similarly, the result in Lemma 6 is structurally identical to that of Lemma 2 under Cournot competition.

Based on Lemma 5 and Lemma 6, we have the following result.

**Proposition 5.** The equilibrium sustainability strategy structures under Bertrand competition are characterized as follows.

a). The traditional strategy structure  $TT$  is a coordination equilibrium if  $\beta > \beta_c$  and  $\alpha > \alpha_c$ .

b). The CSR compliance strategy structure  $SS$  is a coordination equilibrium if either of the following two conditions holds:

i).  $\beta > \beta_c$  and  $\alpha > \alpha_c$ ;

ii).  $\beta > \beta_c$  and  $\alpha > \alpha_c$ .

c). Otherwise, the CSR compliance strategy structures  $ST$  or  $TS$  are the equilibria.

Proof. See Appendix.

Again, the equilibrium structures characterized by Proposition 5 are also structurally the same as those in Proposition 2.

## 6. Discussion and Conclusion

In this study, we have discussed the incentive to adopt CSR compliant operations and how equilibrium CSR compliance strategies vary with respect to exogenous parameters, including the cost premium of CSR compliance and risk of external exposure. We first consider Cournot and Bertrand competition and analyze the CSR compliance strategies in equilibrium. Specifically, we first analyze the CSR compliance incentive under different scenarios and identify the equilibrium CSR compliance strategies. Following this approach, we examine how the risk of exposure and cost premium interacts with each other in determining the appropriate equilibrium CSR compliance strategies. In summary, we find that CSR compliant operations tend to be implemented either when the external exposure risk is sufficiently high or when the cost premium is sufficiently low. This implies that it is important for the policy makers to understand the risk and cost of non-CSR-compliant and CSR compliant operations before making EUT compliance strategies. We also discuss whether firms engage in a win-win outcome with CSR compliant operations in equilibrium. We demonstrate that firms are not necessarily better off with CSR compliant operations. To be precise, by adopting CSR compliant operations, firms will become better off if the external risk is high and the cost premium is low; otherwise, the firms are worse off. This indicates that firms also need to be vigilant after implementing CSR compliant operations to ensure that their performances are better. Policy makers may also provide some incentives to induce firms to adopt CSR compliant operations.



Apart from Cournot (quantity) competition, we also extend the analysis by discussing four additional scenarios: 1). continuous CSR compliance strategy; 2). demand uncertainty; 3). updated demand response; and 4). Bertrand (price) competition and find that our key results carry over to these scenarios qualitatively. Based on the above analysis, our results shed light on when CSR compliant operations should be adopted in practice and how they affect firm performance. Our results also imply that the practice of CSR compliance requires that the relevant firms consider both the external risk of exposure and cost premium of CSR compliance.

In terms of future research, there are several possible directions to consider. First, technology uncertainty can also be included to reveal how these factors impact CSR compliant operations because green technological investment is also important to ensure that the operations are CSR compliant. Furthermore, sequential decisions and asymmetric market sizes can be studied to further check how the results will be affected. Second, it is also worthwhile to extend the analysis to a multiperiod scenario. Finally, it would be worthwhile to incorporate the policy- $\theta$  such as penalty for non-CSR-compliant practices, e.g., through environmental policies such as taxation.

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## References

- Citcy.cn." X0." (" Ngg." F0" \*423;+0" Vjg" ghghev" qh" uqwtelkpi" rqnkekgu" qp" c" uwrrnkgtøu" uwuvckpcdng" practices. *Production and Operations Management*, <https://doi.org/10.1111/poms.12943>.
- Awasthy, P., & Hazra, J. (2019). Responsible sourcing by improving workplace safety through buyer-supplier collaboration. *European Journal of Operational Research*, 274(1), 155-164.
- Barbosa-Póvoa, A. P., da Silva, C., & Carvalho, A. (2018). Opportunities and challenges in sustainable supply chain: An operations research perspective. *European Journal of Operational Research*, 268(2), 399-431.
- Bhaduri, S. N., & Selarka, E. (2016). Corporate Social Responsibility Around the World-An Overview of Theoretical Framework, and Evolution. In *Corporate governance and corporate social responsibility of Indian companies* (pp. 11-32). Springer, Singapore.
- Bian, J., Li, K. W., & Guo, X. (2016). A strategic analysis of incorporating CSR into managerial incentive design. *Transportation Research Part E: Logistics and Transportation Review*, 86, 83-93.
- Boyd, D. E., Spekman, R. E., Kamauff, J.W., & Werhane, P. (2007). Corporate social responsibility in global supply chains: a procedural justice perspective. *Long Range Planning*, 40(3), 341-356.
- Brandenburg, M., Govindan, K., Sarkis, J., & Seuring, S. (2014). Quantitative models for sustainable supply chain management: Developments and directions. *European Journal of Operational Research*, 233(2), 299-312.
- Brandenburg, M., Govindan, K., Sarkis, J., & Seuring, S. (2014). Quantitative models for sustainable supply chain management: Developments and directions. *European Journal of Operational Research*, 233(2), 299-312.
- Chain Reaction Research (2018). <https://chainreactionresearch.com/the-chain-unilever-and-nestle-publish-detailed-supplier-lists/>.
- Chen, J. Y., & Slotnick, S. A. (2015). Supply chain disclosure and ethical sourcing. *International Journal of Production Economics*, 161, 17-30.
- Chen, L., & Lee, H. L. (2016). Sourcing under supplier responsibility risk: The effects of certification, audit, and contingency payment. *Management Science*, 63(9), 2795-2812.
- Cruz, J. M., 2008. Dynamics of supply chain networks with corporate social responsibility through integrated environmental decision-making. *European Journal of Operational Research*, 184(3), 1005-1031.
- Debo, L. G., Toktay, L. B., & Van Wassenhove, L. N. (2005). Market segmentation and product technology selection for remanufacturable products. *Management Science*, 51(8), 1193-1205.

- Drake, D. F., & Spinler, S. (2013). OM Forum-Sustainable Operations Management: An Enduring Stream or a Passing Fancy? *Manufacturing & Service Operations Management*, 15(4), 689-700.
- Drake, D. F., Kleindorfer, P. R., & Van Wassenhove, L. N. (2016). Technology choice and capacity portfolios under emissions regulation. *Production and Operations Management*, 25(6), 1006-1025.
- Forbes, 2017. New Global Smartphone Boom Gives Tech-Reliant Taiwan a Second Chance. <https://www.forbes.com/sites/ralphjennings/2017/11/14/new-boom-in-smartphones-gives-tech-reliant-taiwan-a-second-chance/#7cde55557e74>.
- Fortune, 2015. Hasbro, Mattel Toy Suppliers Slammed in Labor Report. <http://fortune.com/2015/11/22/hasbro-mattel-china-labor/>.
- Guardian, 2017. Cambodian female workers in Nike, Asics and Puma factories suffer mass faintings. <https://www.theguardian.com/business/2017/jun/25/female-cambodian-garment-workers-mass-fainting>.
- Guo, R., Lee, H. L., & Swinney, R. (2015). Responsible sourcing in supply chains. *Management Science*, 62(9), 2722-2744.
- Guo, R., Lee, H., & Swinney, R. (2013). The impact of supply chain structure on responsible sourcing. In Proceedings of M&SOM Sustainable Operations SIG Conference (Vol. 28), INSEAD, Fontainebleau, France.
- Hsueh, C. F. (2015). A bilevel programming model for corporate social responsibility collaboration in sustainable supply chain management. *Transportation Research Part E: Logistics and Transportation Review*, 73, 84-95.
- Jaehn, F. (2016). Sustainable operations. *European Journal of Operational Research*, 253(2), 243-264.
- Karaer, Ö., Kraft, T., & Khawam, J. (2017). Buyer and nonprofit levers to improve supplier environmental performance. *Production and Operations Management*, 26(6), 1163-1190.
- Kim, S., Wagner, S. M., & Colicchia, C. (2018). The impact of supplier sustainability risk on shareholder value. *Journal of Supply Chain Management*, 55(1), 71-87.
- Kleindorfer, P. R., Singhal, K., & Van Wassenhove, L. N. (2005). Sustainable operations management. *Production and Operations Management*, 14(4), 482-492.
- Krass, D., Nedorezov, T., & Ovchinnikov, A. (2013). Environmental taxes and the choice of green technology. *Production and Operations Management*, 22(5), 1035-1055.
- Lee, H. H., & Li, C. (2018). Supplier quality management: Investment, inspection, and incentives. *Production and Operations Management*, 27(2), 304-322.

Letizia, P., & Hendrikse, G. (2016). Supply chain structure incentives for corporate social responsibility: An incomplete contracting analysis. *Production and Operations Management*, 25(11), 1919-1941.

Liu, X., Mishra, A., Goldstein, S., & Sinha, K. K. (2018). Toward Improving Factory Working Conditions in Developing Countries: An Empirical Analysis of Bangladesh Ready-Made Garment Factories. *Manufacturing & Service Operations Management*. <https://doi.org/10.1287/msom.2017.0679>.

Ni, D., Li, K. W., Tang, X. (2010). Social responsibility allocation in two-echelon supply chains: insights from wholesale price contracts. *European Journal of Operational Research*, 207(3), 1269-1279.

Ohyama, A., & Tsujimura, M. (2008). Induced effects and technological innovation with strategic environmental policy. *European Journal of Operational Research*, 190(3), 834-854.

Orsdemir, A., Hu, B., & Deshpande, V. (2019). Ensuring corporate social and environmental responsibility through vertical integration and horizontal sourcing. *Manufacturing & Service Operations Management*. 21(2) 251-477.

Panda, S., Modak, N. M., Basu, M., Goyal, S. K. (2015). Channel coordination and profit distribution in a social responsible three-layer supply chain. *International Journal of Production Economics*, 168, 224-233.

Rnc o dgem."G0" N0." ( "Vc {nqt." V0" C0" \*4237+0" Uw r rnkgt" gxcukqp" qh" c" dw {gtøu" cwfkv<" Ko rnkecvkqpu" hqt" motivating supplier social and environmental responsibility. *Manufacturing & Service Operations Management*, 18(2), 184-197.

Porter, M. E., Kramer, M. R. (2006). The link between competitive advantage and corporate social responsibility. *Harvard Business Review*, 84(12), 78-92.

Singh, N., & Vives, X. (1984). Price and quantity competition in a differentiated duopoly. *The RAND Journal of Economics*, 15(4), 546-554.

Tang, C.S., & Zhou, S. (2012). Research advances in environmentally and socially sustainable operations. *European Journal of Operational Research*, 223(3), 585-594.

Tong, X., Lai, K. H., Zhu, Q., Zhao, S., Chen, J., & Cheng, T. C. E. (2018). Multinational gpgtr tkug" dw {gtuø" e j kkegu" hqt" gzvgpfkpi" eqtrqtcvg" uqekcn" tgu r qpukdknk v { " r tcevkegu" vq" uw r rnkgtu" kp" emerging countries: A multi-method study. *Journal of Operations Management*, 63, 25-43.

Vogel, D. J. (2005). Is there a market for virtue? The business case for corporate social responsibility. *California Management Review*, 47(4), 19-45.

Wall Street Journal, 2016. Deaths of Foxconn Employees Highlight Preuwrtgu"Hceg f" d {"E j kpcøu" Factory Workers. <https://www.wsj.com/articles/deaths-of-foxconn-employees-highlight-p pressures-faced-by-chinas-factory-workers-1471796417>.

Xia, Y., Zu, X., Shi, C. (2015). A profit-ftkxgp" crrtqcej" vq" dwknfkpi" c" ++rgqrng-tgurqpukngö" supply chain. *European Journal of Operational Research*, 241(2), 348-360.

## Appendix

### Proof of Proposition 1.

Eqo rctkpi "hkt o" rrtqhkvu" wpfgt" vjg" öTraditional Orgtcvkqpuö" cpf" öCSR Compliant Orgtcvkqpuö" scenarios in Table 1, we have  $s^T = \frac{1}{4}c^2 - \frac{1}{2}c + \frac{1}{4}c^2$ , which is a quadratic function of  $c$ . It can be easily checked that  $s^T \geq 0$  for  $0 < c < 1 + \sqrt{1 - c}$ , but  $s^T < 0$  for  $1 + \sqrt{1 - c} < c < c$ , with the equality holding at  $c = 1 + \sqrt{1 - c}$ . With larger  $c$ ,  $\alpha$ , or  $f$ , the parameter range supporting CSR Compliant operations becomes larger; thus, it is more likely that the firm would adopt such operations.

### Proof of Lemma 1.

Comparing firm jöu" rrtqhkvs wpfgt" vjg" ötraditional strategy structure TTö" cpf" ömixed strategy structure TSö" uegpctkqu in Table 2, we have

$$\frac{\pi_j^S}{\pi_j^T} = \frac{F_1 c^2 + F_2 c + F_3}{2c^2 + 4c + 2}, \text{ where}$$

$$F_1 = 2c^2 + 2c^2;$$

$$F_2 = 2c^2 + 2c^2 - 2c^2 = 2c^2; \quad c;$$

$$F_3 = 2c^2 + 2c^2 - 4c^2 + 4c^3 + 4c^2 + 12c^2 + 4c^2 + 3c^4 + 20c^2 + 32c^2 = c^2.$$

The sign of  $\frac{\pi_j^S}{\pi_j^T}$  is the same as that of  $F_1 c^2 + F_2 c + F_3$ , which is a quadratic function of  $c$ . It can be verified that  $F_1 c^2 + F_2 c + F_3 > 0$  always holds when  $c < \frac{2 + \sqrt{5 - 4c^2} + 1}{2}$ , where

$$c_1 = \frac{2 + \sqrt{5 - 4c^2} + 1}{2}. \text{ However, when } c > c_1, \text{ then } F_1 c^2 + F_2 c + F_3 < 0 \text{ for } c > c_1,$$

while  $F_1 c^2 + F_2 c + F_3 > 0$  for  $c_1 < c < \frac{2 + \sqrt{1 - 4c^2} + 2}{2}$ , with the equality holding at  $c = \frac{2 + \sqrt{1 - 4c^2} + 2}{2}$ , where

$$c_1 = \frac{2 + \sqrt{1 - 4c^2} + 2}{2}.$$

**Proof of Lemma 2.**

Comparing firm  $j$ 's  $SS$  and  $TS$   $\bar{c}SR$  compliance strategy structure  $SS$  and  $TS$  in Table 2, we have  $\frac{SS_j}{TS_j} = \frac{F_4 c^2 + F_5 c + F_6}{2}$ ,

where

$$\begin{aligned} F_4 &= 2^2 + 2^2 + 2^2 + 4^2 + 6^2 + 4^2 + 8^2 + 4^2 + 12^2 + 16^2; \\ F_5 &= 2^2 + 2^2 + 2^2 + 4^2 + 4^2 + 4^2 + 16^2 + 2^2 + 4^2 + 8^2 + 16^2 + c; \\ F_6 &= 2^2 + 2^2 + 2^2 + 4^2 + 4^2 + 4^2 + 12^2 + 4^2 + 3^2 + 20^2 + 32^2 + c^2. \end{aligned}$$

The sign of  $\frac{SS_j}{TS_j}$  is the same as that of  $F_4 c^2 + F_5 c + F_6$ , which is a quadratic function of  $c$ . It can also be verified that  $F_4 c^2 + F_5 c + F_6 > 0$  always holds when  $c > \frac{1}{2}$ , where  $c > \frac{1}{2} \sqrt{2^2 + 8^2 + 8^2} = \frac{1}{2} \sqrt{2^2 + 3^2 + 2^2}$ . However, when  $c < \frac{1}{2}$ , then  $F_4 c^2 + F_5 c + F_6 < 0$  for  $c < \frac{1}{2}$ , while  $F_4 c^2 + F_5 c + F_6 > 0$  for  $c > \frac{1}{2}$ , with the equality holding at  $c = \frac{1}{2}$ , where

$$c_2 = \frac{2^2 + 2^2 + 2^2 + 4^2 + 4^2 + 2^2 + 2^2 + 2^2 + 2\sqrt{1 + 2^2 + 4^2 + 2^2}}{2^2 + 2^2 + 2^2 + 2^2 + 4^2 + 4^2 + 2^2 + 4^2 + 2^2} c.$$

**Proof of Proposition 2.**

The result of Proposition 2 is directly based on Lemma 1 and Lemma 2, which can be discussed as follows.

- (a) If  $c > \frac{1}{2}$  and  $\frac{SS_j}{TS_j} > 1$ , the result can be derived by Lemma 1(a) and Lemma 1(b).
- (b) If  $c < \frac{1}{2}$  and  $\frac{SS_j}{TS_j} < 1$ , or  $c > \frac{1}{2}$  and  $\frac{SS_j}{TS_j} < 1$ , the result can be derived by Lemma 2(a) and Lemma 2(b).
- (d) The result can be derived by Lemma 1 and Lemma 2.

**Proof of Proposition 3.**

Comparing firm  $j$ 's  $SS$  and  $TT$   $\bar{c}SR$  compliance strategy structure  $SS$  and  $TT$  in Table 2, we have  $\frac{SS_j}{TT_j} = \frac{c^2 + 2c + 2}{2}$ , with the numerator being a quadratic function of  $c$ .

$$\frac{SS_j}{TT_j} = \frac{c^2 + 2c + 2}{2}, \text{ with the numerator being a quadratic function of } c.$$

Following the same analysis as before, it can be easily checked that  $\frac{SS_j}{TT_j} > 0$  always holds

when  $c_3 \geq 1$ . If  $c_3 < 1$ , then  $c_3 = 1 - \sqrt{1 - c}$  and  $c_4 = 1 + \sqrt{1 - c}$ . Together with the condition  $c \leq \bar{c}$ , we do not consider the root  $c_4$ , which completes the proof of Proposition 3.

**Proof of Lemma 3.**

Comparing firm  $j$ 's  $TS$  structure  $J_1 c^2 - J_2 c - J_3$  with  $TT$  structure  $J_1 c^2 - J_2 c - J_3$ , we have  $\frac{TS}{j} - \frac{TT}{j} = \frac{J_1 c^2 - J_2 c - J_3}{2}$ , where

$$\begin{aligned} J_1 &= 2^2 - 2^2 - 2^2 - 2^2; \\ J_2 &= 2^2 - 2^2 - 2^2 - 3 - 1 - 2^2 - 2^2 - 3 - 8 - 4 - c; \\ J_3 &= 5^4 - 4^5 - 5^3 - 2^2 - 8 - 4 - 5^4 - 8^3 - 4^2 - 16 - c^2. \end{aligned}$$

The sign of  $J_1 c^2 - J_2 c - J_3$  is the same as that of  $J_1 c^2 - J_2 c - J_3$ , which is a quadratic function of  $c$ . It can be verified that  $J_1 c^2 - J_2 c - J_3 \geq 0$  always holds for  $0 < c \leq \bar{c}$  when  $c_1 \geq \frac{2}{\sqrt{2}}$ , where  $c_1 = \frac{2}{\sqrt{2}}$ . However, when  $c_1 < \frac{2}{\sqrt{2}}$ , then  $J_1 c^2 - J_2 c - J_3 < 0$  for  $c < c_1$ , while  $J_1 c^2 - J_2 c - J_3 \geq 0$  for  $c_1 < c \leq \bar{c}$ , with the equality holding at  $c = c_1^U$ , where

$$c_1^U = \frac{\sqrt{4^2 - 2^2 - 16 - 3^3 - 8^2 - 8 - 1 - 2^3 - 5^2 - 8 - 4^2 - 2^2}}{2^2 - 2^2 - 2^2 - 2^2} c.$$

**Proof of Lemma 4.**

Comparing firm  $j$ 's  $SS$  structure  $J_4 c^2 - J_5 c - J_6$  with  $TS$  structure  $J_4 c^2 - J_5 c - J_6$ , we have  $\frac{SS}{j} - \frac{TS}{j} = \frac{J_4 c^2 - J_5 c - J_6}{2}$ , where

$$\begin{aligned} J_4 &= 2^5 - 3^3 - 3^2 - 2^2 - 4^4 - 4^3 - 6^2 - 8 - 16 - 12^2 - 4^3; \\ J_5 &= 2^4 - 4^3 - 6^2 - 5 - 8 - 2^2 - 4^4 - 2^3 - 2^2 - 8 - 8 - 2^3 - 4^2 - 8 - 16 - c; \\ J_6 &= 5^4 - 4^8 - 8^3 - 7^2 - 16 - 16 - 5^5 - 6^3 - 8 - c^2. \end{aligned}$$

The sign of  $J_4 c^2 - J_5 c - J_6$  is the same as that of  $J_4 c^2 - J_5 c - J_6$ , which is a quadratic function of  $c$ . It can also be verified that  $J_4 c^2 - J_5 c - J_6 = 0$  always holds when  $c = \frac{U}{2}$ ,

where  $\frac{U}{2} = \frac{6^2 - 4^2 + \sqrt{2^2 - 2^3 - 3^2 - 5}}{2^3 - 3^2 - 5}$ . However, when  $c = \frac{U}{2}$ , then

$J_4 c^2 - J_5 c - J_6 = 0$  for  $c = \frac{U}{2}$ , while  $J_4 c^2 - J_5 c - J_6 = 0$  for  $c = \frac{U}{2}$ , with the equality holding at  $c = \frac{U}{2}$ , where

$$c_2 = \frac{\sqrt{1 - \frac{2^2 - 2^3 - 3^2 - 5}{4 \cdot 1 - 2^2 - 2^2 - 4 \cdot 2 - 2^2 - 3^3 - 3^3 - 3^2 - 5^2 - 2^2 - 2^2}}}{4 \cdot 1 - 2^2 - 2^2 - 4 \cdot 2 - 2^2 - 3^3 - 3^3 - 3^2 - 5^2 - 2^2 - 2^2} c$$

**Proof of Proposition 4.**

The result of Proposition 4 is directly based on Lemma 3 and Lemma 4, which can be discussed as follows.

- (a) If  $c < \frac{U}{2}$  and  $c > \frac{U}{2}$  the result can be derived by Lemma 3(a) and Lemma 3(b).
- (b) If  $c < \frac{U}{2}$  and  $c > \frac{U}{2}$ , or  $c = \frac{U}{2}$ , the result can be derived by Lemma 4(a) and Lemma 4(b).
- (c) The result can be derived by Lemma 3 and Lemma 4.

**Proof of Lemma 5.**

Comparing firm  $j$  structure  $TS_j$  and  $TT_j$  we have

$$\frac{TS_j - TT_j}{j} = \frac{G_1 c^2 - G_2 c - G_3}{1 - 2^2 - 2^2 - 4^2 - 3^2}, \text{ where}$$

$$G_1 = 1 - 2^2 - 2^2 - 2^2;$$

$$G_2 = 2 - 1 - 1 - 2^2 - 2^2 - 2^2 - 2^2 - c;$$

$$G_3 = 1 - 2^3 - 13^2 - 4^2 - 2^2 - 2^4 - 10^3 - 44 - 4 - c^2.$$

The sign of  $\frac{TS_j - TT_j}{j}$  is the same as that of  $G_1 c^2 - G_2 c - G_3$ , which is a quadratic function of  $c$ . It can be verified that  $G_1 c^2 - G_2 c - G_3 = 0$  always holds when  $c = \frac{B}{1}$ ,



where  $c_1^B = \frac{\sqrt{20 \cdot 8 \cdot 12^2 \cdot 4 \cdot 2^5 \cdot 6} \cdot 1 \cdot 2^2}{2 \cdot 4 \cdot 3^2 \cdot 2^3 \cdot 4^5}$ . However, when

$c_1^B$ , then  $G_1 c^2 - G_2 c - G_3 = 0$  for  $0 < c < c_1^B$ , while  $G_1 c^2 - G_2 c - G_3 = 0$  for  $c_1^B < c < \frac{1}{c_1^B}$ , with the equality holding at  $c = c_1^B$ , where

$$c_1^B = \frac{1 + \sqrt{1 + \frac{1}{4} \left( \frac{3}{2} \right)^2}}{2} = \frac{1 + \sqrt{1 + \frac{9}{16}}}{2} = \frac{1 + \sqrt{\frac{25}{16}}}{2} = \frac{1 + \frac{5}{4}}{2} = \frac{9}{8}.$$

**Proof of Lemma 6.**

Comparing firm  $j$ 's CSR compliance strategy structure  $SS_j$  with strategy structure  $TS_j$ , we have

$$\frac{SS_j - TS_j}{j} = \frac{G_4 c^2 - G_5 c - G_6}{1 - 2c^2 + 4c^2 - 3c^2}, \text{ where}$$

$$G_4 = \frac{6^6 \cdot 4^5 \cdot 4^4 \cdot 3^3 \cdot 6^2 \cdot 5 \cdot 13^4 \cdot 26^3 \cdot 9^2 \cdot 2^2}{8^5 \cdot 19^4 \cdot 14^3 \cdot 52^2 \cdot 24^2 \cdot 6^2 \cdot 5 \cdot 8^4 \cdot 20^3 \cdot 4^2 \cdot 32 \cdot 16};$$

$$G_5 = \frac{2 \cdot 1 \cdot 5^4 \cdot 4^4 \cdot 4^3 \cdot 3^3 \cdot 2^5 \cdot 5^4 \cdot 13^3 \cdot 21^2 \cdot 2^2}{5^5 \cdot 6^4 \cdot 16^3 \cdot 16^2 \cdot 32^2 \cdot 5^2 \cdot 2^4 \cdot 6^3 \cdot 12^2 \cdot 8 \cdot 16} c;$$

$$G_6 = \frac{1 \cdot 2^4 \cdot 4^3 \cdot 4^2 \cdot 2^2 \cdot 3^4 \cdot 8^3 \cdot 13^2 \cdot 16}{3^4 \cdot 2^3 \cdot 16^2 \cdot 8 \cdot 16} c^2.$$

The sign of  $\frac{SS_j - TS_j}{j}$  is the same as that of  $G_4 c^2 - G_5 c - G_6$ , which is a quadratic function of  $c$ . It can also be verified that  $G_4 c^2 - G_5 c - G_6 = 0$  always holds when  $c = c_2^B$ , where  $c_2^B$  is the real root of the following equation:

$$2^2 \cdot 1 - 2^2 \cdot 2^2 \cdot X^3 + 1 \cdot 8 \cdot 16^2 \cdot 2^3 \cdot 11^4 \cdot 6^6 \cdot X^2 - 4 \cdot 14 \cdot 5^2 \cdot 12^3 \cdot 3^4 \cdot 2^5 \cdot X - 2^2 \cdot 2^2 \cdot 2^2 = 0.$$

However, when  $c = c_2^B$ , then  $G_4 c^2 - G_5 c - G_6 = 0$  for  $0 < c < c_2^B$ , while  $G_4 c^2 - G_5 c - G_6 = 0$  for  $c_2^B < c < \frac{1}{c_2^B}$ , with the equality holding at  $c = c_2^B$ , where

$$c_2^B = \frac{1 + \frac{H_1 X + \sqrt{H_2 X}}{H_3 X}}{c}, \text{ with}$$

$$\begin{aligned}
 H_1 X &= \frac{3^2 X^3 - 2^7 X^2 + 2^3 X^2}{2^{16} 8^2 X^2 - 2^3 X^2 + 2^2 X^2} , \\
 H_2 X &= \frac{9^4 X^5 - 3^3 X^2 + 2^2 X^4}{2^2 74 X^3 - 30 X^3 + 58 X^3 - 12 X^3 + 11 X^3} , \\
 &= \frac{2^2 80 X^4 - 74 X^4 + 92 X^4 - 58 X^4 + 34 X^4 - 11 X^5 + 4 X^6}{2^2 4 X^2 + 8 X^2 + 20 X^2 + 6 X^2 + 8 X^3 + 4 X^2 + 2^2 X^2 + 2^2 X^2 + 4 X^2} , \\
 H_3 X &= \frac{4^2 X^3 - 2^9 X^2 + 2^6 X^2}{2^{12} X^3 - 20 X^3 + 3^2 X^3 - 8 X^3 + 2^2 X^2 + 2^2 X^2 + 2^2 X^2} .
 \end{aligned}$$

**Proof of Proposition 5.**

The result of Proposition 4 is directly based on Lemma 5 and Lemma 6, which can be discussed as follows.

- (a) If  $\dots$  and  $\dots$ , the result can be derived by Lemma 5(a) and Lemma 5(b).
- (b) If  $\dots$  and  $\dots$ , or  $\dots$ , the result can be derived by Lemma 6(a) and Lemma 6(b).
- (c) The result can be derived by Lemma 5 and Lemma 6.