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# A self-adaptive multi-objective dynamic differential evolution algorithm and its application in chemical engineering



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#### ARTICLE INFO

# ABSTRACT

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*Keywords:* Multi-objective optimization Dynamic differential evolution Parameter adaptation Optimal control problem Chemical and biochemical processes This paper proposes a new multi-objective dynamic differential evolution algorithm with parameter self-adaptive strategies, named SA-MODDE. All components of the algorithm are synergically designed to reach its full potential, containing parental selection, mutation strategy, parameter setting, survival selection, constraint handling, and termination criteria. The improvement measures emphasize exploiting Pareto dominance information more efficiently. Particularly, parameter adaptation schemes are introduced based on both prior knowledges of current individual and feedback information on previous promising solutions, and their effectiveness is validated by comparison with three fixed-parameter combinations. Extensive numerical tests are conducted on multiple test suites with five state-of-the-art peer competitors. The statistical results demonstrated that the SA-MODDE exhibits good proximity and diversity in dealing with benchmark functions with various characteristics. Three industrial (bio)chemical processes, including two optimal control and one reformulated constrained tri-objective, are investigated to show the feasibility and robustness of the SA-MODDE.

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#### 1. Introduction

Engineering problems always require the simultaneous optimization of several competing objectives of interests. So far, multi-objective optimization (MOO) has been an active research field in process systems engineering [1,2]. Particularly, various multi-objective evolutionary algorithms (MOEAs), such as NSGA-II, GDE3, and MOPSO, have been widely used to solve both academic and industrial MOO problems [3–5]. Usually, MOEAs have two main advantages: (1) As many diverse non-dominated solutions as possible can be found in a single run; (2) Various types of MOO problems can be handled without assumptions on objective functions and their mathematical characteristics [6].

The algorithm structure and search operator jointly affect the performance of MOEAs. The algorithm structure can be classified into two main categories: Pareto-based [3,7] and decomposition-based [8,9]. The former provides detailed Pareto dominance information of the population to facilitate individual comparison. The latter decomposes MOO problems into a set of scalar aggregation subproblems, each of which is optimized using the current information from neighboring subproblems. The two methods have their own advantages on different types of problems and are considered to be evenly matched [10]. In terms of search operator,

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differential evolution (DE) is simple to implement with only a few control parameters, i.e., scale factor (F) and crossover rate (CR). Except for multi-objective DE (MODE) algorithms, many classic MOEAs also replaced the original evolutionary operators with DE and their performance was significantly enhanced, such as NSGA-II-DE, SPEA2-DE, IBEA-DE, and MOEA/D-DE [11,12]. Through updates by dynamic population rather than generation to generation, Qing [13] presented the dynamic DE (DDE) operator, superior in efficiency, robustness, and storage requirements to the conventional DE. That is, each new individual that performs better than or similar to the corresponding old counterpart will immediately participate in the current population to provide information for subsequent evolution. This makes DDE more responsive to changes in population status. Despite of researches on multi-objective DDE (MODDE) algorithms [14,15], it is still very inadequate compared to MODE. Herein, we propose several improvement measures on the MODDE under Pareto-based structure. For convenience, the background of MOO problems and DDE operators are given in Supplementary Materials.

Maintaining a good balance between the proximity and diversity of obtained non-dominated solutions is not a trivial task for MOEAs [16]. Most existing MODEs conquer this challenge by improving a particular component. Lin et al. [17] embedded three mutation schemes with different features and corresponding adaptive scheme selection strategy in their algorithm, which is better than variants with a single mutation scheme.

# Nomenclature

Abbreviations	
А	A kind of catalyst
CE	Crowding entropy
CSTC	Chi-squared-test based termination crite-
	rion
CVP	Control vector parameterization
DDE	Dynamic differential evolution
DE	differential evolution
DTLZ	Deb-Thiele-Laumanns-Zitzler
GRA	Gray relational analysis
HSS	Hybrid selection strategy
IQR	Interquartile range
MFE	Maximum number of function evaluation
MODDE	Multi-objective DDE
MODE	Multi-objective DE
MOEAs	Multi-objective evolutionary algorithms
MOO	Multi-objective optimization
PM	Performance metric
SA-MODDE	Self-adaptive MODDE
SSDTC	Steady-state detection termination criterion
ZDT	Zitzler–Deb–Thiele
Symbols	
$\chi^2$ -test	Chi-squared-test
$\overline{\mathbf{x}}$	Median
GDm	Modified generational distance
$P^0$	Initialized population
$P_0P_1P_2$	Points
$P^1$	Sorted population
PF*	True Pareto front
X <sub>n</sub>	Parents vector
e <sub>m</sub>	The <i>m</i> th extreme solutions
$\delta_{PM}$	User-defined tolerance value
$\delta_r$	Standard deviation of rankings
$\mu_r$	Average ranking
CR	Crossover rate
D	Decision variables
Ε	Distribution entropy
F	Scale factor
IGD	Inverted Generational Distance
Μ	Number of objective functions
NP	Population size
PF	Obtained Pareto front
PS	Pareto optimal set
R	Rank set
Ratio_CR	Score level of X <sub>i</sub>
S	Substances
SP	Spread
SS	Solution set
U	Trial vector
V	Mutation vector
Χ	Target vector

Euclidian distance

**Objective function** 

d f

n nadir_point $\gamma$ $\lambda + \mu$	Number of solutions or problems Point consisting of maximum Number of generations Original+offspring population
Subscripts	
b	Base vector
best	Best value
i, q	Individual or solution index
init	Initial value
j	Objective function index
т	Modify
р	Parents
r	Ranking
S	Starting vector
t	Terminal vector
Superscripts	
max	maximum value
min	minimum value

Xu et al. [18] partitioned the entire population into several subpopulations and constructed a hybrid selection strategy (HSS). and each subpopulation was assigned a survival selection scenario. Scalability experiment studies indicated that HSS was able to deal with high-dimension MOO problems. Lin et al. [19] adjusted the values of F and CR according to the success rate of offspring in each generation, and confirmed the validity of their work by comparing with two fixed parameter settings. In our work, considering the synergistic effect, several components are carefully devised based on the original MODDE framework. They contain parental selection, mutation strategy, parameter setting, survival selection, constraint handling, and termination criteria. Notably, through simultaneously exploiting prior and posteriori information during the evolutionary process, the new parameter self-adaptive (SA) strategies are devised to customize F and CR for each individual, respectively. Therefore, the proposed algorithm is named as SA-MODDE. Moreover, the external elitist archive typically used for diversity preservation is not required here.

An exhaustive overview of the application of MOEAs to process systems engineering was given by Rangaiah and Petric. [6], covering the areas of petrochemicals, biofuels, environment, and thermodynamics. In addition, Babu and Gujarathi [20] addressed a three-stage supply chain problem involving a network of suppliers, plants and customer areas, and considered three cases of objective functions using MODE. For the industrial manufacturing process, two styrene reactor configurations [21] considering productivity, selectivity and yield and the oxidation of pure terephthalic acid [22] were optimized using an improved MODE, respectively. The results show that MODE can cover a wider range and a better spread compared to NSGA-II. Additionally, two MODE algorithms were implemented in the problem of maximizing ethylene and propylene yield in naphtha crack unit [23]. In particular, dynamic optimization problems, i.e., problems modeled by a set of non-linear differential and algebraic equations (DAE), are quite challenging. Gujarathi et al. [24] demonstrated the usability of MODE in solid state fermentation processes, with the model formulated as DAE. Also, control vector parameterization (CVP) has been shown to be an effective and efficient method to solve such DAE problems by discretizing the control variables [25]. This work combines CVP to optimize chemical and biochemical processes related to either space or time. Furthermore, a constrained complex bi-objective problem including only

steady-state variables is reformulated as a tri-objective problem to develop more realistic solutions.

Overall, the main contributions of the proposed SA-MODDE can be summarized as follows:

- (1) Each component has been well-designed to reach its full potential. Pareto information is fully utilized in the parental selection, parameter setting and survival selection, even in constraint handling. Also, self-adaptive strategies of *F* and *CR* are presented based on both prior knowledges of current individual and feedback information of previous promising solutions. The effectiveness of the strategy is evaluated by comparing it with three fixed parameter combinations. Thus, only generic control parameters such as the population size (*NP*) and termination criterion are required. Besides, a performance-based termination criterion is incorporated into the SA-MODDE as an alternative to an arbitrary specified maximum number of generation or function evaluation.
- (2) The performance is firstly investigated through the widelyused ZDT, DTLZ test suites and six constrained test instances [26–28]. Credible and thorough comparisons are conducted with 5 state-of-the-art peer competitor algorithms on two performance metrics (PMs). Then, two optimal control problems and a constrained steady-state (bio) chemical problem are employed to test the ability to track complex practical problems. The results show that SA-MODDE performs well on various types of MOO problems in terms of proximity and diversity and provides informative trade-off solutions for decision-makers.

The remainder of the paper is organized as follows. Section 2 expounds on each component of the proposed SA-MODDE in detail. Section 3 carries out extensive numerical experiments using two PMs. In Section 4, the use of the proposed SA-MODDE on three chemical and biochemical processes are discussed. Finally, conclusions are drawn in Section 5.

#### 2. The proposed SA-MODDE algorithm

Algorithm S2 gives the main procedure of the proposed SA-MODDE algorithm. Firstly, the Pareto information of the initial population is provided by fast non-dominated sorting and crowding entropy assignment before the evolutionary iteration precedes. Then, during the mutation operation, the parents, i.e., the vectors participating in the mutation operator, are ranked and selected according to their dominance relationship. Besides, "rand/1" and "rand/2" are selected as mutation strategies with equal probability. Next, the binomial crossover operation is executed and the trial individual is generated. The values of F and CR of each individual are allocated by proposed parameter adaptation schemes. In the selection operation, which is crucial to maintain population diversity, substitution and truncation mechanisms are applied in combination. Finally, a simple dominance filtering operation is performed and the final solution set is obtained. The following subsections elaborate on each component of SA-MODDE.

#### 2.1. Fast non-dominated sorting and crowding entropy assignment

Fast non-dominated sorting approach was proposed by Deb et al. [3], with two entities need to be calculated for each solution: (1) the number of solutions that dominate the *i*th solution, denoted as  $n_i$ ; and (2) the set of solutions that dominated by the *i*th solution, named *SS*<sub>i</sub>. Firstly, the solutions with  $n_i = 0$  forms the first non-dominated front. Then, for the *i*th solution of the first front, each member q in *SS*<sub>i</sub> is visited and the corresponding

 $n_q$  value is reduced by one. Solution q will be collected in the second non-dominated front when  $n_q = 0$ . Next, for each member of the second front, the above procedure is repeated to make up the third front. The process loop continues until all fronts are identified.

The order of solutions in the same non-dominated front is usually determined through crowding degree assignment. The crowding distance is the universal crowding degree estimation measure [3]. It calculates the sum of the average distances between two solutions adjacent to the *i*th solution along with each objective, reflecting the solution density around the specific solution. On its basis, Wang et al. [14] introduced the crowding entropy (CE) concept by considering the distribution of the solution. The solutions in the same non-dominated front are firstly sorted in ascending or descending order by the function values of any objective. Then, the boundary solutions, namely, solutions with the largest or the smallest function values in any dimension, are set to an infinite CE value. Next, the calculation formulas of *i*th intermediate solution are defined as follows.

$$CE_{i} = \sum_{j=1}^{M} (c_{ij} \cdot E_{ij}) / (f_{j}^{max} - f_{j}^{min})$$
(1)

$$E_{ij} = -\left[pl_{ij}\log_2(pl_{ij}) + pu_{ij}\log_2(pu_{ij})\right]$$
(2)

$$pl_{ij} = \frac{dl_{ij}}{c_{ii}} \tag{3}$$

$$pu_{ij} = \frac{du_{ij}}{c_{ii}} \tag{4}$$

$$c_{ij} = dl_{ij} + du_{ij} \tag{5}$$

where  $E_{ij}$  is the distribution entropy of the *i*th solution along with the *j*th objective function, and the parameters  $f_j^{max}$  and  $f_j^{min}$  are the maximum and minimum values of the *j*th objective function; *M* is the number of objective functions;  $dl_{ij}$  and  $du_{ij}$  are the Euclidean distances of the *i*th solution to its lower and upper adjacent solution along the *j*th objective function, respectively. If a point locates in the middle of its two neighbors, then  $pl_{ij} = pu_{ij} = 0.5$  and  $E_{ij} = 1$ , and it is regarded as the best distribution point.

In **Algorithm S2**, the initial population  $P^0$  is first arranged in ascending order of the non-dominated front, and individuals in the same front are then sorted in descending order according to the CE values to obtain the new population  $P^1$ .

#### 2.2. Parental selection and mutation strategies

The individuals acting as parents, denoted as  $X_p$ , are usually randomly selected from the whole population. This way is conducive to global search but affects the convergence rate. Abbass et al. [29] selected parents only from among non-dominated solutions based on the recognition that good parents always generate good offspring. However, this approach may in turn undermine global search capability. Chen et al. [30] calculated selection probability for each individual and found that the individual with high selection probability was more likely to be a parent. Although the method accelerated convergence while keeping global exploratory, it required a high computational complexity. Our work puts forward a simpler strategy: Several distinct individuals are randomly picked from the entire population to form parents, and then their own ranking is allocated according to their index in  $P^1$ . And the smaller the index value, the higher the ranking. The parent with the highest ranking is chosen as the base vector  $X_{pb}$ , and the parent with the lowest ranking is chosen as the terminal vector  $X_{pt}$ . Taking a two-dimensional problem as an example, the generation processes of mutation vector  $V_i$  with and



Fig. 1. The generation process of mutation vector  $V_i$ : (a) with parents ranking; (b) without parents ranking.

without parents ranking are illustrated in Fig. 1. As can be seen, under the guidance of the good base vector and the favorable direction of the difference vector ( $X_{ps} - X_{pt}$ ), there are more opportunities to produce fine offspring. Switching  $X_{pb}$  and  $X_{pt}$  while holding individuals positions constant results in  $V_i$  is far from the Pareto optimal set (*PS*). Thus, the proposed parent ranking strategy not only facilitates the propagation of good information to the offspring without extra computational cost but also ensures powerful global exploration capabilities due to that there is no over-mine around specific vectors.

Multi mutation strategies have been recommended to solve MOO problems since no single strategy outperforms all others in the evolutionary process [18]. Several DE mutation strategies exhibit their own unique search characteristics and have different performance on diverse problems. Of these, "rand/1" and "rand/2" are the two most frequently used strategies, as appeared in **Algorithm S2**. Compared to "rand/1", "rand/2" adds a difference vector. This results in a better perturbation and provides more search directions, but may degrade search efficiency. In this work, the two strategies complement each other to exert different advantages and are employed by each individual with equal probability.

#### 2.3. Survival selection

After generating the *i*th trial vector  $U_i$ , survival selection is executed to update the population. The survival schemes can be classified into three categories as summarized by Cheng et al. [31]. The first category conducts one-to-one selection. The replacement is performed when one solution dominates the other. When two solutions are non-dominated, one solution is randomly chosen or the one with less crowding degree is preferred. The second category implements a  $(\lambda + \mu)$  design. In other words, the original population and the offspring population are merged, and then the combined population is truncated into the desired size. The third category considers the above two schemes synthetically, and its performance is better due to the hybridization effect. That is, the one-to-one strategy will be applied when two solutions are comparable, or the  $(\lambda + \mu)$  procedure will be executed on the contrary.

The hybrid selection scheme is adopted in SA-MODDE, as shown in **Algorithm S3**, which includes three steps:

1. If  $U_i$  dominates target vector  $X_i$ ,  $U_i$  will replace  $X_i$ . Then, the new population is reordered after  $U_i$  joins. At this time, it is unnecessary to recalculate the Pareto dominance relationships among all individuals, thanks to the original population that was sorted well in advance. In the worst

case,  $U_i$  will be compared with all other individuals to obtain new non-dominated front classification, and then crowding entropy assignment will be recalculated only for the fronts where individuals' situations have changed.

- 2. If  $X_i$  dominates  $U_i$ ,  $U_i$  is discarded. The original population has no change.
- 3. Otherwise,  $U_i$  is added to the population. The united population has (NP + 1) individuals and is reordered after  $U_i$  joins, which is similar to step 1. Then, it is truncated at the last individual to keep size to be NP.

It is also noted that the  $F_i$  and  $CR_i$  values are collected into the set  $F\_pool$  and  $CR\_pool$  when  $X_i$  cannot dominate  $U_i$ , respectively, as listed in **line 4 and 13 in Algorithm S3**. The medians of the two sets are considered to be the best control parameter of the current evolution process, named  $F_{best}$  and  $CR_{best}$ , which can provide feedback information for parameter adaptation.

#### 2.4. Parameter adaptation

The performances of DE-based algorithms are sensitive to the setting of F and CR. F changes the search step size and CR determines how much information  $U_i$  will inherit from  $X_i$ . Therefore, they affect the convergence speed and population diversity significantly [19]. In SA-MODDE, the parameters of each individual are dynamically adjusted by taking into account both prior and posteriori information during the evolutionary process. The parameter adaptation schemes are given in **Algorithm S4** and include the following three aspects.

- 1. Data normalization. Each objective function value in the obtained solution set is mapped to the range of  $0 \sim 1$ .
- 2. The self-adaptive strategy of F. F controls the scaling degree of the disturbance of the difference vector  $(X_{ps} - X_{pt})$ to the base vector  $X_{pb}$ . When  $X_{ps}$  and  $X_{pt}$  are close in the search space, the generated difference vector value is small, and F should take a larger value at this time, otherwise the disturbance is too small to play the role of mutation, as the generation process of  $V_{i1}$  shown in Fig. 2(a). On the contrary, when  $X_{ps}$  and  $X_{pt}$  are farther apart, the generated difference vector value is large, and F should take a small value to limit the amount of disturbance, otherwise, the mutation vector may cross the boundaries of the feasible region, as the generation process of  $V_{i2}$  reflected in Fig. 2(a). The concept has been successfully applied in a single-objective DE algorithm by evaluating the difference between fitness values of different individuals to roughly reflect their closeness in the search space [32]. Despite



Fig. 2. Self-adaptive mechanism based on prior knowledge: (a) F; (b) CR.

that this mapping method is not always accurate, its computational cost is lower, especially for high-dimensional problems. Therefore, the setting of *F* first gets inspiration from the proximity of  $f(X_{ps})$  and  $f(X_{pt})$  in the solution space. The distance reference point is composed of the normalized maximum value on each objective function, called *nadir\_point*. Then, the initial value of  $F_i$  is obtained through a linear relationship, denoted as  $F_{init}$ . Next,  $F_{init}$ approaches the most suitable control parameter  $F_{best}$  in the current population by a certain step and the final value of  $F_i$  is obtained. Furthermore,  $F_{best}$  is also used directly as  $F_i$ with the same probability. The upper and lower limit of  $F_i$ are 0.05 and 0.55, respectively.

3. The self-adaptive strategy of CR. The better  $X_i$  performs, the more its structure should be inherited, i.e., the smaller  $CR_i$  should be. The performance score of each  $X_i$  is obtained by summing its corresponding normalized objective function values. For minimization optimization problems, the  $X_i$  with the smallest score is considered to perform best. The score level of the *i*th individual in the score set is calculated in line 20, denoted as Ratio\_CR. As shown in Fig. 2(b), CR<sub>init</sub> is assigned within (0, 0.9) according to Ratio\_CR. When Ratio\_CR is below the average, the value of CR<sub>init</sub> is small so that more individual information can be retained. In turn, when Ratio\_CR is above the average, the value of CR<sub>init</sub> increases rapidly, accelerating the elimination of poor individual structure. Then, CR<sub>init</sub> will either learn experience from *CR*<sub>best</sub> in a certain step, or be directly replaced by CR<sub>best</sub>.

In addition, considering that  $F_{best}$  and  $CR_{best}$  have different ranges along with search stages,  $F_{pool}$  and  $CR_{pool}$  are emptied after each iteration, as shown in **line 5** of **Algorithm S2**. Thus, more appropriate and accurate values of the best control parameters can be obtained.

#### 2.5. Constraint handling

Many real-world application problems are constrained besides bounds on decision variables. Three main categories of constraint handling approaches have been summarized [33]: (a) Penalty function method. This approach can be conveniently used in the formulations of various problems by adding penalty terms into the objective functions [34]. Thus, it is the most popular technique for constraint handling. However, its difficulty lies in choosing the suitable penalty factor. (b) Separation of objectives and constraints. A representative method is the feasibility approach proposed by Deb et al. [3]. It selects a feasible solution a prior over an infeasible solution based on the extent of constraint violation. (c) Repair algorithms. This approach converts the infeasible solution into a feasible or less feasible solution [35]. Recently, Chih [36,37] proposed two self-adaptive check and repair operators motivated by the fact that alternative pseudo-utility ratios must vary the approach directions in repairing infeasible solutions. They had been demonstrated to perform well on the multidimensional knapsack problem.

The feasibility approach is adopted in this work, as it can be seamlessly integrated with the proposed algorithm. The entire population is divided into a feasible group and an infeasible group based on whether or not individuals violate the constraints. The former is sorted according to the principle of non-dominance and crowding entropy, while the latter is first sorted according to the fewer number of constraint violations and then fewer total constraints violation. Four possible scenarios with corresponding handling methods are as follow.

- 1. If  $U_i$  and  $X_i$  are both feasible, the survival selection scheme introduced in Section 2.3 are applicable. The difference is that when they are non-dominated,  $U_i$  is accepted in the feasible group and the last individual of the infeasible group is discarded.
- 2. If *U<sub>i</sub>* is feasible while *X<sub>i</sub>* is infeasible, then *U<sub>i</sub>* is added to the suitable position in the feasible group and *X<sub>i</sub>* is abandoned from the infeasible group.
- 3. If  $U_i$  is infeasible while  $X_i$  is feasible, then  $U_i$  is refused.
- 4. If *U<sub>i</sub>* and *X<sub>i</sub>* are both infeasible, then *U<sub>i</sub>* enters the infeasible group and the last individual of the reordered infeasible group is discarded.

#### 2.6. Computational complexity

The proposed SA-MODDE algorithm is developed by integrating the above ingredients. The complexity of fast non-dominated sorting is  $O(M \cdot NP^2)$ , while the crowding entropy assignment needs  $O(M \cdot NP \cdot \log_2 NP)$ . In the survival selection step, the complexity is  $O(M \cdot NP)$  when conducting dominant relationship comparison between  $U_i$  and  $X_i$ , and under the worst scene, reordering population requires  $O(M \cdot NP^2)$ . In the parameter adaptation step, the main computational complexity lies in data normalization and distance calculation of two solutions, which are  $O(NP^2)$  and  $O(M^2)$ , respectively. Besides, parental selection and constraint handling do not introduce additional complexity. Therefore, the overall complexity of SA-MODDE is  $O(M \cdot NP^2)$  at each iteration, which is the same as NSGA-II.

## 3. Numerical experiments

In the section, the performance of SA-MODDE is assessed through numerical experimental studies. Firstly, test problems and performance metrics are stated. Secondly, the proposed algorithm is compared with 5 state-of-the-art MOEAs. Thirdly, the effectiveness of parameter adaptation strategies is verified by comparing it with three fixed parameter settings. Finally, a termination criterion based on evolution performance is applied. See **Supplementary Materials** for relevant tables and figures.

## 3.1. Benchmark problems and performance metrics

12 unconstrained test instances, including 5 bi-objective MOPs from the ZDT test suite and 7 tri-objective MOPs from the DTLZ test suite, and 6 constrained benchmark functions are covered in experimental studies. They have a variety of characteristics, such as multi-local optimal fronts, non-uniform, and discontinuity, making the numerical testing comprehensive and reliable. The parentheses in the first column of **Table S1** list the number of the objective function (M) and decision variables (D) for each unconstrained test instance, and the parentheses in the first column of **Table S3** list M, D, and the number of constraints for each constrained problem.

Two quantitative assessment metrics, Inverted Generational Distance (*IGD*) and Spread (*SP*), are employed to reflect the performance of different algorithms. Assume that *PF* is a set of achieved non-dominated solutions and *PF*<sup>\*</sup> is a set of uniformly distributed solutions along the true Pareto front. *IGD* metric mirrors both the convergence and diversity of *PF* and a lower value mean a better approximation [12]. *SP* metric measures the solution distribution of *PF* and a lower value means a more even distribution [3]. Each objective in *PF* and *PF*<sup>\*</sup> is firstly normalized and the calculation formulas are defined as follows.

$$IGD = \frac{\sqrt{\sum_{i \in PF^*} d^2(i, PF)}}{|PF^*|}$$
(6)

where d(i, PF) is the minimum Euclidian distance between *i*th solution in *PF*<sup>\*</sup> and solutions in *PF* and  $|PF^*|$  is the number of solutions in *PF*<sup>\*</sup>.

$$SP = \frac{\sum_{m=1}^{M} d(e_m, PF) + \sum_{i=1}^{|PF|} |d_i - \overline{d}|}{\sum_{m=1}^{M} d(e_m, PF) + |PF| \cdot \overline{d}}$$
(7)

where  $(e_1, e_2, \ldots, e_M)$  are *m* extreme solutions of  $PF^*$ ;  $d(e_m, PF)$  is the Euclidian distance between the extreme solution of *m*th objective in *PF*<sup>\*</sup> and its nearest solution in *PF*;  $d_i$  is the Euclidian distance between *i*th solution in *PF* and its nearest solution in the same set;  $\overline{d}$  is the average of  $d_i$ ; and |PF| is the number of solutions in *PF*.

## 3.2. Peer algorithms and experimental settings

Four of the recent MOEAs with different structural features, including two DE variants, are chosen as peer competitors on unconstrained test problems. They outperformed several classical algorithms, such as GDE3, MOEA/D-DE, and MOPSO. A representative algorithm, NSGA-II, is chosen as a rival on constrained problems. A brief introduction of these five algorithms is given below.

1. MODE-RMO [30]: Multi-objective differential algorithm evolution with ranking-based mutation operator. The base and terminal vector in the mutation operator had a large probability to be selected from better vectors instead of random assignment.

- 2. MODE-PMSMO [38]: Multi-objective differential evolution with performance-metric-based self-adaptive mutation operator. The highlight of the algorithm was the automatic selection of a suitable mutation operator in different evolution stages.
- 3. NS-GWO [39]: Non-Dominated Sorting Gray Wolf Optimizer algorithm. The algorithm was proposed based on the natural hunting process of gray wolves, including searching, encircling, and attacking, and a leader selection mechanism was established.
- 4. INM-TLBO [10]: Multi-objective Individualized-Instruction Teaching Learning Based Optimization Algorithm. The algorithm assigned a specific teacher or interactive object for each individual and emphasized the guiding role of the non-dominated solution.
- 5. NSGA-II [3]: Non-dominated Sorting Genetic Algorithm II. The algorithm proposed a series of innovative concepts along with low computational complexity and has been successfully applied in many real-world constrained optimization problems [40].

Each algorithm performs 25,000 function evaluations and 30 independent runs for each test problem. *NP* is set to 100 for algorithms except for INM-TLBO, which is set to 20 due to its optimization mechanism. Other control parameters adopt recommended values in their original publications and all algorithms are implemented in the MATLAB platform. Additionally, the Wilcoxon rank sum test at a 0.05 significance level is applied to determine whether the experimental results of different algorithms are statistically significantly different. In **Table S1-6**, the symbols "+ ", "--", and " $\approx$ " denote that other algorithms perform better than, worse than, and similar to SA-MODDE, respectively, and the best results are bolded. At last, the average ranking method is used to give a comprehensive ranking of each algorithm, and the calculation formulas are as follows [41].

$$\mu_r = \frac{\sum_{i=1}^n R_i}{n} \tag{8}$$

$$\delta_r = \sqrt{\frac{\sum_{i=1}^n (R_i - \mu_r)^2}{n}}$$
(9)

where  $R = \{R_1, R_2, ..., R_n\}$  is a rank set of one algorithm and n is the number of test problems.  $\mu_r$  is the average ranking and  $\delta_r$  is the standard deviation of the rankings.

#### 3.3. Experimental results and discussions

The Median  $(\bar{x})$  and interguartile range (IQR) of *IGD* metric on ZDT and DTLZ test suites of all algorithms are recorded in Table S1. SA-MODDE wins the first place in most problems, except for losing to INM-TLBO on ZDT6 and DTLZ7. The results of ZDT6, DTLZ1, and DTLZ7 are no statistically different from those of NS-GWO, MODE-PMSMO and MODE-RMO, and MODE-PMSMO, respectively. Moreover, the results of DTLZ3, DTLZ5, and DTLZ6 are several orders of magnitude lower than those of other competitors. In particular, regarding DTLZ3, which has many local optima, only the results of SA-MODDE are acceptable, while the data median and dispersion of other algorithms are not satisfactory, especially NS-GWO. To visually display the evolutionary behaviors of different algorithms, the curses of the median IGD value of 30 runs versus the number of function evaluations on each test instance are illustrated in Fig. S1 (a-l). It can be observed that (1) INM-TLBO exhibits convergence speed advantage on the ZDT test suite, while SA-MODDE has fast declines on the DTLZ test suite except for DTLZ6 and DTLZ7. (2) SA-MODDE can quickly reach a stable lowest platform area on most problems within 25,000 function evaluations. (3) The DE-based algorithms offer advantages in descending speed and solution quality on DTLZ1, DTLZ4, and DTLZ5, while exposing the weaknesses on these two aspects on ZDT6 and DTLZ7.

The statistical numerical results of SP metric on ZDT and DTLZ test suites are presented in Table S2. All competitors are inferior to SA-MODDE in the final average ranking. INM-TLBO provides the best results on ZDT6, DTLZ2, and DTLZ7, and obtains the runner-up again, while other competitors only have better or similar performance on DTLZ7. Fig. S2 (a-l) depicts the evolutionary curses of the median SP value of 30 runs versus the number of function evaluations. Compared with IGD curves, the SP curves oscillate significantly, especially the tri-objective problems. This is due to the characteristic that SP value is affected by the distribution uniformity of the currently obtained solutions. Besides, the SP curves of SA-MODDE drop rapidly within a certain abscissa interval on most problems, and the decline is larger than other algorithms, such as the interval [3,000-7,000] of ZDT1. In order to intuitively reflect the approximation and distribution of the solution set, the typical fronts obtained by each algorithm on ZDT6 and DTLZ3 after 25,000 function evaluations are exemplified in Fig. S3 (a-b).

Table S3 shows the comparison results of NSGA-II and SA-MODDE in six constrained test problems. The first five are biobjective problems with 2 or 6 constraints, and the last one is a five-objective problem with 7 constraints. In terms of statistical numerical results, SA-MODDE outperforms NSGA-II except for the IGD performance of OSY. Fig. S1 (m-r) and Fig. S2 (m-r) show the evolutionary behaviors corresponding to IGD and SP, respectively. SA-MODDE performs well on the first three problems, while the evolving speed on the fourth problem is slow along with a slight advantage in solution quality. For the fifth problem, which has five separated regions, SA-MODDE is prone to stagnation. For the sixth problem with 5 objectives, IGD curve of NSGA-II increases first and then stays steady. To explain the phenomena, Fig. S3 (c-d) plot the projections of the typical fronts realized by NSGA-II after 100 function evaluations, NSGA-II after 25,000 function evaluations, and SA-MODDE after 25,000 function evaluations on the space of objective 1  $(f_1)$  and objective 2  $(f_2)$ , as well as objective 3  $(f_3)$  and objective 4  $(f_4)$ , respectively. In the early stage of evolution, the solutions of NSGA-II in the feasible region (coved by blue dots) are relatively uniform distributed, while in the later stage of evolution, all solutions are concentrated on a certain boundary. This leads to an increase in the IGD curve instead of a decrease, indicating that NSGA-II deteriorates population diversity during the evolution process. On the contrary, SA-MODDE evenly covers the entire projection region including boundary points, which proves its ability to simultaneously maintain good proximity and diversity when dealing with constrained complex problems.

## 3.4. Effectiveness testing of parameter adaptation

To verify the effectiveness of proposed parameter adaptation strategies in SA-MODDE, three fixed control parameters are adopted for performance comparison, i.e., (1) Case1: F = 0.3, CR = 0.1; (2) Case2: F = 0.5, CR = 0.5; (3) Case3: F = 0.7, CR = 0.9, and other settings remain the same.

**Table S4** gives the compared results in terms of *IGD*. Case 1 has 6 problems worse than, 12 problems similar to, and no problems better than SA-MODDE, respectively. Case 2 has a slight advantage over ZDT3 and CONSTR, is on the downside on 3 problems, and obtains comparable results on 13 problems. Case 3 scores higher on SRN and CONSTR, ties on 3 problems and loses on 13 problems. Finally, SA-MODDE prevails in the final average ranking. Notably, SA-MODDE obtains a *IGD* result with

an accuracy level of  $10^{-4}/(10^{-6})$  and  $10^{-4}/(10^{-5})$  for ZDT4 and DTLZ3 respectively, while the best accuracy level of the results for three cases are  $10^{-3}/(10^{-2})$  and  $10^{-3}$  /  $(10^{-4})$ . These two problems include many local Pareto fronts, and are suitable for testing the ability of algorithms to handle multimodality problems. Conversely, on the test problems that SA-MODDE lost, its results are in the same order of magnitude as those obtained by the first-place. These demonstrate the critical role of parameter adaptation. In addition, the performance of three cases and SA-MODDE on DTLZ6 and BNH are statistically equivalent, indicating that these two problems are not sensitive to control parameters.

**Table S5** gives the compared results in terms of *SP*. Among the 18 test problems, the performances of case 1 are weaker than SA-MODDE in 7 problems, and are equivalent in the others. Case 2 outperforms SA-MODDE on ZDT series (except ZDT4) and CONSTR, and achieves the same ranking on 9 problems and poor ranking on 4 problems. Case 3 yields better results on ZDT1 and 4 constrained test problems, comparable results on 7 problems, and worse results on 6 problems. Finally, despite the  $\mu_r$  value of case 2 is the same as SA-MODDE, the latter is ranked first by virtue of the lower  $\delta_r$  value.

There is no doubt that a certain of test problems perform well under some specific parameter combinations. However, SA-MODDE enables the selection of control parameters less dependent on the type of optimization problems, and consistently yields reliable and high-quality results in terms of convergence and diversity.

#### 3.5. Performance-based termination criterion

As shown in Fig. S1 and S2, before the evolution processes reach the maximum number of function evaluation (MFE), IGD values of SA-MODDE no longer decrease significantly on most problems, and SP values fluctuate in a narrow range. This suggests that the current stopping condition overuse computational resources. Therefore, it is necessary to adopt a more reliable and efficient termination criterion. Recently, performance-based termination criteria have been growing studied, which can stop search progress in time by monitoring the improvement of PMs. Sharma and Rangaiah [42] screened out two suitable PMs, namely, modified generational distance  $(GD_m)$  and modified spread  $(SP_m)$ , and proposed the chi-squared-test based termination criterion (CSTC). Wong et al. [43] successfully applied a steady-state detection termination criterion (SSDTC) in heat exchangers design problems. Rangaiah et al. [44] evaluated the performance of these two criteria through several chemical processes and found that CSTC was more reliable and time-saving over SSDTC. Concerning the outstanding performance of CSTC, it is adopted in the work, with the difference that modified *IGD* and *SP*, i.e.,  $IGD_m$  and  $SP_m$ , are chosen as PMs. Firstly, PMs are calculated using the nondominated solution set in the previous and current generations. Then, a statistical verification of PM variations over the latest  $\gamma$ generations is required. The related calculation formulas are as follows.

$$IGD_m = \frac{\sqrt{\sum_{i \in N_{curr}} d_{i,IGD_m}^2}}{N_{curr}}$$
(10)

where  $d_{i,IGD_m}$  is the minimum Euclidian distance between *i*th solution in the current generation and solutions in the previous generation, and  $N_{curr}$  is the number of non-dominated solutions in the current generation.

$$SP_m = \frac{\sum_{i=1}^{n_{curr}} |d_{i,SP_m} - d_m|}{N_{curr} \cdot \overline{d}_m}$$
(11)

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$$Max f_1 = 1 - x_1(z_f) - x_2(z_f)$$
$$Min f_2 = \int_0^{z_f} u(z) dz$$

Subject to:

inlet  

$$S_1$$
 $S_1$ 
 $S_1$ 
 $S_1$ ,  $S_2$ ,  $S_3$ 
 $S_1$ ,  $S_2$ ,  $S_3$ 
 $(x_1, x_2, x_3)$ 
 $Z_f$ 
 $Z_f$ 
 $S_1$ 
 $S_2$ ,  $S_3$ 
 $(x_1, x_2, x_3)$ 
 $Z_f$ 
 $dx_1$ 
 $dx_2$ 
 $dx$ 

 $0 \le u(z) \le 1$   $x(0) = [1 \ 0]^T, \ k = [1 \ 10 \ 1]^T$  $0 \le z \le z_f, z_f = 1$ 

**Fig. 3.** The schematic diagram and model equation of catalysts mixing process. Where  $x_1$  and  $x_2$  (state variables) are the concentrations of  $S_1$  and  $S_2$ , respectively. u (control variable) is the fraction of catalyst A, and z denotes the spatial coordinate.  $k_1$ ,  $k_2$ ,  $k_3$  are the kinetic factors of reactions 1–3. The rates of reaction 1 and 2 are affected by catalyst A, while reaction 3 does not occur unless catalyst B is present.

where  $d_{i,SP_m}$  is the Euclidian distance between *i*th solution in the current generation and its nearest solution in the same set;  $\overline{d}_m$  is the average of  $d_{i,SP_m}$ .

$$Chi(PM) = \frac{Variance\left[PM_1, PM_2, \dots, PM_{\lambda}\right](\gamma - 1)}{\delta_{PM}^2}$$
(12)

$$P(PM) = \chi^2 [Chi(PM), (\lambda - 1)]$$
(13)

where  $\delta_{PM}$  is the user-defined tolerance value for standard deviation of *PM*, and *P*(*PM*) is the probability that  $\chi^2$ -test supports the hypothesis that the variance of *PM* is lower than  $\delta_{PM}^2$ . A value of 10 is used for  $\gamma$ . The iterative search will stop if each *P*(*PM*) exceeds 99% simultaneously at a certain generation. Moreover, the stopping condition based on MFE is also used, which acts as a guarantee to avoid indefinite looping.

The setting of  $\delta_{IGD_m}$  and  $\delta_{SP_m}$  needs to consider a balance between solution quality and computational expense. The small values will produce better results but will consume a lot of time, while large values will reduce computational costs but may cause premature stops. Here,  $\delta_{IGD_m} = 0.0002$  and  $\delta_{SP_m} = 0.05$  are recommended for bi-objective problems and these values are 0.0008 and 0.02 for problems with higher objective numbers. Table S6 summarizes the comparison results applying CSTC and MFE on 18 test instances. As expected, the quality of the solutions obtained by the former is slightly inferior to the latter on most problems. Remarkably, there is no compromise for CSTC on the IGD values of 2 problems and the SP values of 6 problems. Besides, the median number of function evaluations required by CSTC are all located at the beginning of platform areas in Fig. S1. This indicates that adopted  $\delta_{PM}$  values are suitable to stop the search timely without premature or overripe. In short, CSTC is well incorporated into the SA-MODDE framework, avoiding the waste of computing resources caused by arbitrarily specifying the maximum number of generation or function evaluation.

#### 4. Chemical engineering processes optimization

Three process applications taken from literature, namely, catalysts mixing policy, Lee–Ramirez bioreactor, and alkylation process, are optimized by SA-MODDE to verify its performance in practical engineering problems. The first two problems are unconstrained optimal control problems, and the third is a constrained steady-state optimization problem. The number of independent runs is setting to 30 for all problems. Process model equations and optimization results are listed in **Supplementary Materials**.

#### 4.1. Catalysts mixing policy

The problem studies optimal mixing policies of two catalysts packed in a plug-flow reactor. These catalysts promote a series of reactions including one reversible and one irreversible, that is,  $S_1 \iff S_2 \rightarrow S_3$ . The optimization objectives are maximizing the conversion of  $S_3$  and minimizing the consumption of expensive catalyst A. The schematic diagram and model equation [45,46] are depicted in Fig. 3.

In order to solve and optimize the problem, the spatial interval is divided into 10 stages of equal length, and NP and MFE are set to 100 and 10,000. The typical PFs obtained by SA-MODDE under CSTC and MFE are plotted in Fig. S4 (a). It can be observed that the non-dominated solutions under CSTC are close enough to that under MFE, except for an extreme solution  $P_0$ , while the median of the number of function evaluations under CSTC is only 4850. In addition, the conflicting relationship between the two objectives is clearly visible. In terms of maximum value of  $f_1$ , i.e., the abscissa value of  $P_0$  under MFE or  $P_1$  under CSTC, the median value of 30 runs is 0.047928 and 0.047837, respectively. Moreover, gray relational analysis (GRA), as a decision-making approach without objectives weight or other user inputs [47], is employed to select the appropriate solution. Fig. S6 (c-d) illustrates the control trajectories of boundary points ( $P_1$  and  $P_3$ ) and the recommended point  $P_2$  under CSTC. The common thread is that catalyst A should be loaded more at the reactor inlet.

#### 4.2. Lee-Ramirez bioreactor

Lee and Ramirez [48] presented a strong nonlinear optimal control problem for a fed-batch reactor using recombinant bacteria to induce foreign protein production, containing 7 state variables and 2 control variables. The objectives are maximizing



$$Min f_{2} = \int_{0}^{t_{f}} u_{2}(t) dt$$
  
Subject to:  

$$\frac{dx_{1}}{dt} = u_{1} + u_{2}$$
  

$$\frac{dx_{2}}{dt} = \mu x_{2} - \frac{u_{1} + u_{2}}{x_{1}} x_{2}$$
  

$$\frac{dx_{3}}{dt} = C_{nf} \frac{u_{1}}{x_{1}} - \frac{u_{1} + u_{2}}{x_{1}} x_{3} - Y^{-1} \mu x_{2}$$
  

$$\frac{dx_{4}}{dt} = R_{fp} x_{2} - \frac{u_{1} + u_{2}}{x_{1}} x_{4}$$
  

$$\frac{dx_{5}}{dt} = C_{if} \frac{u_{2}}{x_{1}} - \frac{u_{1} + u_{2}}{x_{1}} x_{5}$$
  

$$\frac{dx_{6}}{dt} = -k_{1} x_{6}$$
  

$$\frac{dx_{7}}{dt} = k_{2}(1 - x_{7})$$
  

$$\mu = \frac{x_{3}}{(14.35 + x_{3} + x_{3}^{2}/111.5)} \left(x_{6} + \frac{0.22x_{7}}{0.22 + x_{5}}\right)$$
  

$$R_{fp} = \frac{0.233x_{3}(0.0005 + x_{3})}{(14.35 + x_{3} + x_{3}^{2}/111.5)(0.22 + x_{5})}$$
  

$$k_{1} = k_{2} = \frac{0.09x_{5}}{0.034 + x_{5}}$$
  

$$x(0) = [1 \ 0.1 \ 40 \ 0 \ 0 \ 1 \ 0]^{T}$$
  

$$0 \le u_{1}, u_{2} \le 1, \ t_{0} = 0, \ t_{f} = 10$$
  

$$C_{nf} = 100.0, \ C_{if} = 4.0, \ Y = 0.51$$

 $Max f_1 = x_1(t_f)x_4(t_f)$ 

**Fig. 4.** The schematic diagram and model equation of Lee–Ramirez bioreactor. Where  $x_i(t)$  (i = 1, 2, ..., 7) are state variables, indicating the reactor volume, cell density, nutrient concentration, foreign protein concentration, inducer concentration, the inducer shock and inducer recovery factors on cell growth rate, respectively.  $u_1$  and  $u_2$  are control variables, representing nutrient feeding rate and inducer feeding rate.  $C_{nf}$  and  $C_{if}$  are concentration of nutrient and inducer.  $\mu$  is the specific growth rate,  $R_{fp}$  is the foreign protein production rate, and Y is the growth yield coefficient.  $k_1, k_2, k_3$  are the shock and recovery parameters, respectively.

the yield of foreign protein and minimizing the consumption of inducer. The schematic diagram and model equation are shown in Fig. 4.

The time interval is partitioned into 10 stages of equal length, and *NP* of 100 and MFE of 30,000 are set. A visual comparison result for non-dominated solutions obtained by SA-MODDE under CSCT and MFE are depicted in **Fig. S5 (a)**. The results obtained by CSTC in a lower computation cost are in good agreement with those of MFE, except that some extreme solutions are not covered. When only  $f_1$  is considered, the medians of maximum values achieved by MFE and CSTC are  $6.1191(P_0)$  and  $6.0859(P_1)$ , respectively. Also, there is a trade-off between two objectives and  $P_2$  is suggested to decision-makers by GRA. The control trajectories of  $P_1$ ,  $P_2$ , and  $P_3$  are plotted in **Fig. S5 (b–d)**, indicating that the nutrient and inducer feeding rates at the onset of reactions should remain low.

In order to verify the feasibility and robustness of SA-MODDE, **Table S7** and **S8** present a comparison of the maximum of  $f_1$  with

earlier studies and four peer algorithms with respect to catalysts mixing and Lee-Ramirez bioreactor problems, respectively. In fact, since there are differences in terms of optimization type. the maximum number of function evaluation, even discretization level, earlier studies involving single-objective optimization are regarded as references. Four multi-objective peer algorithms, as well as the current work, are compared fairly under the same number of function evaluations and 30 independent runs. For catalyst mixing problem, the best conversion value of S<sub>3</sub> obtained by SA-MODDE is sufficiently close to the theoretical solution [49], and the Wilcoxon rank sum test shows that NS-GWO has a slight advantage in solution quality. For Lee-Ramirez bioreactor problem, INM-TLBO wins the first place and SA-MODDE second in terms of the yield of foreign protein. Remarkably, NS-GWO and INM-TLBO were the worst performers on the Lee-Ramirez bioreactor and catalyst mixing problems, respectively. Furthermore, as expected, the best values of multi-objective optimization

$$Max f_{1} = 0.063x_{4}x_{7} - 5.04x_{1} - 0.035x_{2}$$
$$-10x_{3} - 3.36x_{5} [\$/day]$$
$$Max f_{2} = x_{7}$$
$$Min f_{3} = x_{2} [barrels/day]$$
Subject to:  
$$0 \le [x_{2} \equiv x_{1}x_{8} - x_{5}] \le 16000$$
$$0 \le [x_{3} \equiv 0.001x_{4}x_{6}x_{9} / (98 - x_{5})] \le 120$$
$$0 \le [x_{4} \equiv x_{1}(1.12 + 0.13167x_{8} - 0.0066667x_{8}^{2})] \le 5000$$
$$0 \le [x_{5} \equiv 1.22x_{4} - x_{1}] \le 2000$$
$$85 \le [x_{6} \equiv 89 + (x_{7} - 86.35 - 1.098x_{8} + 0.038x_{8}^{2}) / 0.325] \le 93$$
$$1.2 \le [x_{9} \equiv 35.82 - 0.222x_{10}] \le 4$$
$$145 \le [x_{10} \equiv -133 + 3x_{5}] \le 162$$
$$0 \le x_{1} \le 2000$$
$$90 \le x_{7} \le 95$$
$$3 \le x_{8} \le 12$$



**Fig. 5.** The schematic diagram and model equation of the alkylation process. Where  $x_i(t)$  (i = 1, 2, ..., 10) are the olefin feed rate (barrels/day), isobutane recycle rate (barrels/day), acid addition rate ( $10^3 \times$  pounds/day), alkylate production rate (barrels/day), isobutane feed rate (barrels/day), spent acid strength (wt%), octane number, isobutane to olefins ratio, acid dilution factor, and F-4 performance number, respectively.

are inferior to that of single-objective optimization, due to the fact that the former requires more effort to explore the entire Pareto fronts, whereas the latter is exploited around the optimal solution.

#### 4.3. Alkylation process

Light olefins react with isobutane under the catalysis of acid to produce alkylate products, before mixing with refinery products to increase the octane number. The process consists of the reactor and fractionator modules as well as recycle streams. Sharma and Rangaiah [42] discussed two different bi-objective problems for the process. Here, in order to test SA-MODDE's ability to deal with complex engineering problems, a tri-objective problem is considered for the first time, i.e., maximum profit, maximum octane number, and minimum isobutane recycling. Moreover, the results obtained by SA-MODDE are compared with that of NSGA-II. The schematic diagram and model equation are shown in Fig. 5, including 10 variables and 7 inequality constraints [42,50].

**Fig. S6 (a–b)** shows the front and side views of typical *PFs* achieved by SA-MODDE and NSGA-II, with *NP* of 100 and MFE of 50,000. Apparently, the solutions of NSGA-II account for only

a fraction of that of SA-MODDE. This once again proves SA-MODDE's strengths in upholding proximity and diversity. From the projections on  $f_1$ - $f_2$  and  $f_1$ - $f_3$  planes (**Fig. S6 (c-d)**), the correlation between profit and isobutane recycling can be approximated by a linear relationship, while the correlation between profit and octane number is irregular. However, profit and octane number existed a strong linear correlation when isobutane recycling was not considered [42]. This shows that more accurate and realistic solutions can be obtained considering three objectives at the same time. Fig. S7 illustrates the results of SA-MODDE under different termination criteria. The results of CSTC are close enough to that of MFE and consume less computational resources. In addition, when the non-dominated solutions obtained by SA-MODDE under MFE are regarded as the known Pareto-optimal fronts, median IGD and SP values of 30 runs for NSGA-II are 0.0250 and 0.6400, and these values are 0.0069 and 0.4019 for SA-MODDE under CSTC.

#### 5. Conclusion

In this work, each component of SA-MODDE is designed in the following the ways: (1) Crowding entropy rather than crowding

distance is used to improve the uniformity and dispersion of nondominated solutions. (2) Parents selection scheme based on the Pareto dominance relationship provides good guidance information for offspring generation. (3) Multi mutation strategies cooper with each other to balance search capability and diversity maintenance. (4) Parameter self-adaptive strategies of F and CR enable the algorithm less sensitive to the types of optimization problems and their effectiveness is experimentally quantified. (5) A hybrid survival selection is implemented to update the population, avoiding the need for an external elitist archive. (6) The feasibility approach for constraint handling is fine-tuned to accelerate the phase-out of infeasible individuals. (7) A performance-based termination criterion greatly improves computing efficiency with slightly compromise solution quality. Furthermore, low computational complexity enhances the availability and efficiency of the SA-MODDE algorithm.

18 benchmark functions including various types of MOPs, make the numerical experimental results more comprehensive and convincing. Judging from the overall statistical performance of 30 runs and the average evolutionary behavior of a single run, SA-MODDE exhibits stronger exploration and exploitation capabilities with respect to five powerful competitors. Additionally, the test results also revealed that no algorithms can completely conquer others due to the unique advantages of different frameworks on specific problems. Two optimal control problems and a constrained steady-state problem are studied to further examine the applicability of SA-MODDE in tracking chemical engineering problems. The results show that SA-MODDE is efficient to locate and obtain a set of diversified trade-off solutions for decision-makers to refer to.

#### **CRediT authorship contribution statement**

**Xiaodong Zhang:** Conceptualization, Methodology, Software, Validation, Visualization, Writing – original draft. **Lu Jin:** Validation, Data curation. **Chengtian Cui:** Writing – review & editing. **Jinsheng Sun:** Supervision, Reviewing and editing.

#### **Declaration of competing interest**

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

#### Appendix A. Supplementary data

Supplementary material related to this article can be found online at https://doi.org/10.1016/j.asoc.2021.107317.

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