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Circularity, indispensability, and mathematical explanation in science

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ABSTRACT

In this paper I consider the objection that the Enhanced Indispensability Argument (EIA) is circular and hence fails to support mathematical platonism. The objection is that the explanandum in any mathematical explanation of a physical phenomenon is itself identified using mathematical concepts. Hence the explanandum is only genuine if the truth of some mathematical theory is already presupposed. I argue that this objection deserves to be taken seriously, that it does sometimes undermine support for EIA, but that there is no reason to think that circularity is an unavoidable feature of mathematical explanation in science.

1. Introduction

A distinctive feature of indispensability arguments for mathematical platonism is that they are 'leveraging' arguments. By this I mean that belief in the truth of core mathematical claims is leveraged on belief in the truth of claims from outside of mathematics itself. In the standard Quine-Putnam indispensability argument, these external claims are ordinary scientific claims, and the leveraging takes place by appeal to confirmational holism. Roughly, on the presumption that one takes the totality of our current scientific theories to be our best source for true claims about the world, one ought to also believe in the mathematical theories that form an indispensable component of this totality of scientific theories.

More recently, specifically explanationist versions of the basic indispensability argument have been developed, for example the Enhanced Indispensability Argument (EIA).[1] According to EIA, the leveraging takes place not via confirmational holism, whose blanket plausibility has been increasingly contested, but via inference to the best explanation (IBE): we ought rationally to believe in certain mathematical claims and theories because they play an indispensable explanatory role in science. This has shifted the focus to mathematical explanation in science (MES). Are there good examples of MES from actual scientific practice? Can an adequate philosophical analysis be given of MES? How does such an analysis fit with more general philosophical accounts of scientific explanation and of explanation within pure mathematics?

The leveraging aspect of indispensability arguments in general, and of EIA in particular, is both a strength and a weakness. The main strength is twofold. Firstly, it avoids begging the question against the anti-platonist opponent. No initial assumption concerning the truth of any part of mathematics is required to get the argument off the ground, nor is the existence of any abstract object presumed. This is in sharp contrast to arguments that focus on pure mathematics. As Mary Leng puts it:

> Given the form of Baker's … argument, one might wonder why it is mathematical explanations of physical phenomena that get priority. For if there are … some genuine mathematical explanations [of mathematical facts] then these explanations must also have true explanans. The reason that this argument can't be used is that, in the context of an argument for realism about mathematics, it is question begging. For we also assume here that genuine explanations must have a true explanandum, and when the explanandum is mathematical, its truth will also be in question.[2]

The second, related strength is that by linking mathematical truth to the truth of empirical science, EIA effects the leveraging on a domain about which realism is a considerably more widespread view. It is a matter of sociological fact that the canonical realist position about mathematics, namely platonism, is a minority position among contemporary philosophers of mathematics. By contrast, scientific realism is fairly clearly the majority position among philosophers of science.[3]

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[1] Baker [2005, 2009].

[2] Leng [2005, p. 174].

[3] I won't take the time to defend these claims here. However, a good place to look for supporting data is the PhilPapers survey, which includes questions concerning realism/anti-realism about both science and mathematics.

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The main weakness of the leveraging approach is that the support that EIA provides for mathematics as only as solid as the scientific theories on which it is based. This has the consequence, notoriously, that mathematics seems to come out on a par with science when it comes to features such as revisability, contingency, and non-*a priority*.[4]

Attacks on EIA have tended to focus either on the weaknesses noted above, or on whether inference to the best explanation is always ontologically committing, or on whether particular putative case studies are actually genuine examples of mathematical explanation in science. In this paper, by contrast, I will consider a line of objection that aims to undermine EIA at its point of strength. The criticism is that EIA is *circular*. In other words, contrary to what is advertised, EIA does in fact implicitly presuppose the truth of the very mathematical theories that it is arguing for. I think that this line of criticism is worth taking seriously precisely because the leveraging aspect of EIA is so crucial to its effectiveness. Ultimately, I believe that EIA can survive this attack, but there are valuable lessons to be learned along the way. This criticism also gets to the heart of the nature of mathematical explanation in science. In particular, it raises the question of whether there can be explanations that make essential use of mathematics and yet have explananda that are genuinely non-mathematical.

2. Local circularity and the cicada example

The effectiveness of the Enhanced Indispensability Argument as an argument for platonism depends crucially on whether there are in fact examples of mathematics playing an indispensable explanatory role in science. This is an empirical claim, so supporting its truth involves coming up with good examples of mathematical explanation in science (MES). As a result, much of the discussion of EIA has revolved around consideration of particular case studies. Many of these have become familiar as a result of their repeated appearance in the philosophy of mathematics literature: periodical cicadas, bee honeycombs, the bridges of Konigsberg, the dividing up of strawberries. Conversely, opponents of EIA have two ways to try to undermine the claim that there are genuine examples of MES, one local and one global. Either they can attack specific cases and argue that they do not involve genuine mathematical explanation. Or they can give some general argument for why MES is impossible in principle.

The circularity objection, at its core, is that the explanandum in a genuine MES cannot be identified independently of the mathematical theory involved in the explanation. Hence, the explanandum is only legitimate if the truth of the mathematical theory is already assumed. The MES therefore fails as a leveraging argument against the nominalist, because the nominalist will not recognize the putative explanandum. For the circularity objection, a similar distinction between strategies can be seen to that pointed out above. One way to press the objection is *local*, by showing how specific case studies have circular explananda. A second strategy is *global*, by showing that circularity is unavoidable in principle in any genuine MES.

I shall begin, in this section, by looking at the local version of the circularity objection, in particular as directed against the well-known example of the mathematical explanation of the life-cycle lengths of the North American periodical cicada. I shall be discussing the cicada example quite extensively in what follows, so it is worth allaying a couple of potential worries at the outset. Although the cicada example is the most discussed case study in the philosophical literature, I don't want to be taken as pinning the success of the Enhanced Indispensability Argument on the fate of this one example. Relevant here is a recent paper by Wakil and Justus, in which they argue that the cicada explanation is not a good explanation by scientific standards.[5] I do not have the space to respond to their arguments here, however there is circumstantial evidence that supports the scientific legitimacy of the cicada explanation. Firstly, the basic number-theoretic explanation of cicada periods (which will be summarized shortly) comes from the scientific literature and was not concocted by philosophers interested in mathematical explanation. Secondly, although Wakil and Justus do manage to unearth a couple of scientific papers that offer very different explanations of cicada life-cycles, explanations that bypass any appeal to number-theory, these papers have been largely ignored (and have rarely been cited) in the ensuing scientific debate.[6]

Returning to the 'local' circularity objection, in a 2008 paper Sorin Bangu attacks the cicada case study as support for platonism on the grounds that what is being explained by appeal to number theory is not a *purely physical* phenomenon:

> Baker begs the question against the nominalist. … If ["Cicadas have prime periods"] is taken to be true, this can't hold unless there is a mathematical object (specifically: a number) to which the property 'is prime' applies. Therefore, by taking the explanandum as being true (to comply with the requirements of the IBE strategy), Baker assumes realism before he argues for it.[7]

Bangu's concern is well-placed if the explanandum in the Cicada case is "Cicadas have prime periods." Insofar as primality is an ineliminably mathematical concept, then "Cicadas have prime periods" is true only if mathematical objects exist. Thus the nominalist can simply reject the truth of the explanandum and thus brush off the request to explain it.

However, while the Cicada case is occasionally presented as providing an explanation for why cicada periods are prime, this is not the 'official' formulation of the argument that is endorsed by proponents of the Enhanced Indispensability Argument such as Baker and Colyvan. Below is the formulation that was given in Baker [2005], which was the paper that introduced the Cicada case into the philosophical literature[8]

(1) Having a life-cycle period which minimizes intersection with other (nearby/lower) periods is evolutionarily advantageous. [**biological law**]
(2) Prime periods minimize intersection (compared to non-prime periods). [**number theoretic theorem**]
(3) Hence organisms with periodic life-cycles are likely to evolve periods that are prime. [**'mixed' biological/mathematical law**]
(4) Cicadas in ecosystem-type, E, are limited by biological constraints to periods from 14 to 18 years. [**ecological constraint**]
(5) Hence cicadas in ecosystem-type, E, are likely to evolve 17-year periods.

The key point to notice here is that the explanandum does not invoke the concept of primality. Rather it is the claim, "Cicadas in ecosystem-type, E, have 17-year periods." At first blush, this does not seem to blunt the force of Bangu's argument. After all, having a period (in years) that is 17 is just as much a mathematical property as is having a period (in years) that is prime. However, there is an important difference between

[4] On the other hand, there is nothing to stop the proponent of EIA bringing in other arguments to bolster the strength of the conclusion. She could use EIA to get the platonist's 'foot in the door,' establishing the existence of abstract mathematical objects, and then use other arguments to support the thesis that mathematics is necessary, *a priori*, etc.

[5] Wakil & Justus [2017].

[6] There is more that can (and perhaps should) be said here about Wakil and Justus's arguments. In particular, I am presuming a defeasible link between an explanation being offered – and frequently cited – by scientists and that explanation being 'good' by scientific standards. There may perhaps be more direct ways of assessing the scientific acceptability of a given explanation, while still maintaining a broadly naturalist background. I am grateful to an anonymous referee for pressing me on this point.

[7] Bangu [2008, p. 18].

[8] Baker [2005, pp. 232–3].

the two explananda and this concerns the *eliminability* of the mathematical concept in question.

In the case of "The number of years in a cicada period is 17," there is a straightforward paraphrase available into first-order logic. The paraphrase for 17 is fairly lengthy, but the core strategy – familiar from introductory logic courses – can be illustrated with the example.

(6) The number of F's is 2

Can be paraphrased as.

(6*) $\exists x \exists y\ (Fx \wedge Fy \wedge x \neq y \wedge \forall z\ (Fz \supset (z = x \vee z = y)))$,

The crucial feature of (6*) is that, although it involves quantifiers, there is no quantification over numbers (or over any other mathematical objects). Hence, there is an important sense in which (6) is 'mathematically innocuous.' What (6) says about the world, namely that the world contains two F's, can be expressed without ontological commitment to numbers.

A precisely corresponding point applies to the Cicada explanandum.

(7) The number of years in a cicada period (in ecosystem-type E) is 17.

The above translation strategy makes available a paraphrase, (7*), that avoids commitment to numbers but which expresses the *nominalistic content* of (7). This means that the explanandum in the Cicada case does not beg the question against the opponent of platonism.

The situation would be very different if the explanandum in the Cicada case invoked the property of primality. For example.

(8) The number of years in a cicada period is prime.

This is because "the number of F's is prime" cannot be paraphrased into a first-order logic so as to eliminate any mention of primality. The reason, in a nutshell, is that there is an infinite number of ways for a number to be prime; hence, any paraphrase of (8) would have to involve an infinite disjunction of the form, 'X has period length 2 or period length 3 or period length 5 or period length 7 or … '.

Before proceeding, it is worth mentioning a couple of potential pitfalls for this paraphrase strategy, one general and one specific. The general pitfall is highlighted in recent work by Alisa Bokulich. In a 2018 paper presenting what she terms an "eikonic conception" of scientific explanation, Bokulich argues that scientists do not explain facts *simpliciter* but rather facts *as represented* in a particular way:

> [S]cientists do not study phenomena in their full complexity, rather they study a simplified representation of the phenomenon contextualized within a particular field, research program, or explanatory project.[9]

This in turn mains that there may be more than one distinct explanandum associated with a particular fact:

> Even within a single subfield, one can often find a plurality of representations of an explanandum phenomenon …. This is because some representations may be more appropriate for answering certain kinds of questions, while other representations are needed for answering other sorts of questions. When scientists explain a phenomenon … they do so within the context of one of these (several possible) representations.[10]

If Bokulich is correct then this raises the following worry for the paraphrase strategy. The number-involving claim, (7), and its nominalistic paraphrase, (7*), are two different representations of the same fact about cicada period lengths. So just because the Cicada MES is a good explanation of (7) does not mean that it is a good explanation of the alternative representation, (7*).

On further reflection, however, this worry is fairly straightforward to dissolve. The cases Bokulich has in mind involve alternative representations that attribute starkly different – even incompatible – features to the physical phenomenon in question. For example, she discusses three classes of representations of water: continuum representations, classical atomistic representations, and quantum representations. It is hardly surprising that an explanation given within one of these models does not easily transfer to another. By contrast, (7) and its paraphrase (7*) are designed to have precisely the *same* implications for the physical phenomenon of cicada periods. Furthermore, (7*) is a direct consequence of (7). Hence it seems plausible that any good explanation of (7) will also be a good explanation of (7*). In summary, we can take Bokulich's general point on board while still allowing that – sometimes – alternative representations can receive one and the same explanation.

The paraphrase strategy canvased above has also been criticized in a more specific way in a recent paper by Heylen and Tump.[11] These authors point to a further distinction between the number claim, (7), and its logical paraphrase, (7*):

> Note that, on the one hand, (7*) treats of a *plurality*, that is, the collection of objects expressed by the sentence in first-order logic with identity. But (7), on the other hand, treats of an *entity*, which has the property of being prime.[12]

Heylen and Tump conclude that there is a logical gap between (7) and (7*), and that some kind of bridge principle is required to connect the claim about pluralities to the claim about entities. Their suggestion is to appeal to Hume's Principle, that for any concepts F, G, the number of F is identical to the number of G if and only if F and G are equinumerous.[13] Heylen and Tump go on to argue that invoking Hume's Principle (HP) makes the success of the Enhanced Indispensability Argument – at least in its Cicada instantiation – dependent on the outcome of the ongoing debate about the status of HP. As they point out, there is disagreement about whether HP is itself analytic. If it is not, then it cannot be appealed to non-question-beggingly in EIA-based arguments against nominalism. To put the point another way, if HP is best seen as a principle whose truth *presupposes* the existence of numbers, then it cannot be used as part of an argument to justify the existence of abstract mathematical objects.

To summarize, the Heylen-Tump objection – although interesting – is hardly a fatal blow against the proposed defense of the Cicada example against Bangu's local circularity worry. If Hume's Principle is analytic, then the objection is dissolved. And even if HP is non-analytic, there is still the option for the proponent of EIA to fold in HP into the overall explanatory inference, using inference to the best explanation to justify belief in the truth both of HP and of the specific claims about primality that are germane to the Cicada example. I conclude that the Cicada example can and does successfully evade Bangu's objection that is problematically circular.

3. Global circularity

Before turning to the global circularity argument, it will be helpful to have an explicit formulation of the Enhanced Indispensability Argument in front of us:

3.1. EIA

(1) We ought rationally to believe in the existence of any entities that play an indispensable explanatory role in science.

[9] Bokulich [2018, p. 801].
[10] *ibid.*

[11] Heylen & Tump (2019).
[12] *op. cit.*, p. 6. Italics in the original. The numbering of the claims has been altered to match the current discussion.
[13] See Shapiro [2000, p. 110].

(2) Mathematics plays an indispensable explanatory role in science.
(3) Hence, we ought rationally to believe in the existence of abstract mathematical objects.

In an early section of his Ph. D thesis,[14] David Michael Price argues that there are "good *scientific* reasons" to be suspicious of premise 1 of EIA, based on concerns connected to circularity. Price considers an episode from the history of physics, after quarks had been postulated but before their existence had been conclusively established. As part of the theory of quantum chromodynamics, particles known as "gluons" were postulated to mediate the strong force in interactions between quarks. Price imagines a hypothetical physicist arguing as follows: "Quarks must exist! They are part of our best possible explanation of the behaviour and properties of gluons!"[15] As Price points out, this is hardly a convincing line of argument, since gluons are a posit of the very same theory – quantum chromodynamics – that posits the existence of quarks. More carefully, we have (or had at that point in time) no reason to believe in gluons unless we are *already* ontologically committed to quarks.

Price extracts from this example what he takes to be a more general principle that ought rationally to constrain our ontological commitments:

> An explanation is only ontologically committing when [it is] possible to identify the phenomena in the explanandum independently of the theory from which the explanans is taken.[16]

Price refers to this as the *No Circularity Condition* (NCC).

In the case of the Cicada example, Price's No Circularity Condition (NCC) initially plays out in a way very similar to that discussed earlier by Bangu. The explanandum is the prime-numbered life cycles of periodical cicadas, and the mathematical theory doing the explanatory work is number theory. However, primality is itself a number-theoretic property. Hence number theory is "explaining a fact that could only have been identified and characterized within number theory itself," thus violating NCC.[17]

Where Price moves beyond Bangu's analysis is in his explicit claim that the circularity problem is likely to generalize, and thus pose a problem for *any* putative example of mathematical explanation in science. Thus Price writes.

> It seems likely that all mathematical explanations will suffer from this same defect: they will attempt to derive the truth of a mathematical theory from the fact that it explains phenomena that could only have been identified by the self-same theory.

Below I give a formulation of Price's global circularity argument against the Enhanced Indispensability Argument:

Price's Circularity Argument.

(1) An explanation is only ontologically committing when it is possible to identify the phenomena in the explanandum independently of the theory from which the explanans is taken. [NCC]
(2) All mathematical explanations in science have explananda that can only be identified using the mathematical theory that is doing the explaining.
(3) Hence, no mathematical explanation in science is ontologically committing.

There are two main problems with the Circularity Argument. The first problem is that Price is not clear about just what he means by "identifying" a phenomenon (or explanandum), and this makes it difficult to assess the plausibility of both premise (1) and premise (2) of the Argument. The second problem is that Price fails to give any compelling reasons for believing the universal claim in premise (2). In other words, even if circularity is problematic, why think that it is *unavoidable*?

The clearest case of circularity is when the explanandum is itself mathematical, and I readily concede that the explanandum in any genuine MES must not feature any distinctively mathematical properties or relations. Thus, in the cicada case, the explanandum cannot be that periodical cicadas have prime periods. As I argued in my reply to Bangu, the correct explanandum here is a claim such as, "Periodical cicadas in ecosystem E have a period of 17 years." The property of having a 17-year period is not distinctively mathematical because it can be paraphrased, in familiar fashion, using the apparatus of first-order logic plus identity.

Generalizing from this particular case study, let E_M stand for a mathematical explanandum (such as "Periodical cicadas in ecosystem E have prime periods") and let E_P denote any non-mathematical (i.e. purely physical) explanandum (such as "Sample S burned with a blue flame in the presence of oxygen"). If E_M is the explanandum of a mathematical explanation, then we have circularity. But what about E_P? Can an explanandum be 'purely' physical and yet still fall foul of Price's NCC condition? If the answer is no then Price's argument is in trouble. For we have already seen just from an examination of the Cicada example that there are cases of genuine MES where the explanandum is non-mathematical. If circularity is only a problem for E_M-style explananda, then the circularity problem is not universal.

Fortunately for Price, however, the answer to the above question is yes. In what follows, I shall focus on two particular ways in which circularity can occur even when the explanandum is purely physical. Ultimately I don't think that this saves Price's argument, because not all E_P explananda of MES's fall foul of circularity. But the extension of the problem beyond mathematical explananda does show that the worry is more pressing than might initially be expected.

4. Type 1 circularity: justification

The first kind of circularity occurs when our belief in the *truth* of the physical explanandum, E_P, rests solely on the mathematical theory, M, that is doing the explaining. The one case that Price mentions (other than the Cicada case), that falls in this category is an example due to Mark Colyvan that concerns antipodal points and the Borsuk-Ulam theorem.

> We discover that at some time t_0 there are two antipodal points p_1 and p_2 on the earth's surface with exactly the same temperature and barometric pressure. What is the explanation for this coincidence? … [The Borsuk-Ulam Theorem] guarantees that there will be two such antipodal points at any time, and, furthermore, the explanation makes explicit appeal to non-causal entities such as continuous functions and spheres.[18]

Here the physical phenomenon, E_P, that it is always possible to find points opposite one another on the surface of the Earth that have the same temperature and pressure, can be described straightforwardly enough in non-mathematical terms. However, there is no direct observational evidence that E_P holds ever, let alone always. After all, it is not as if meteorologists have gone around collecting examples of antipodal points with this property!

Elsewhere, this phenomenon has been referred to as *mathematics-driven explanation in science* (MDES).[19] Such cases typically take as their starting point some interesting, unexpected, or otherwise surprising result in pure mathematics. The striking nature of the mathematical result prompts mathematicians, and perhaps also philosophers of mathematics, to seek out 'applications' in the real world. As another example of this process in action, consider the Stone-Tukey Theorem. This states that given *n* measurable 'objects' (i.e. sets of finite measure) in *n*-

[14] Price [2017].
[15] *op. cit.*, p. 40.
[16] *op. cit.*, p. 42.
[17] *op. cit.*, p. 41. Price does have a response to my suggested shift of focus from claims about primality to specific claims about period length as the proper explananda in the Cicada case, and we shall consider this response in Section 5 below.
[18] Colyvan [2001, p. 49].
[19] See Baker [2012].

dimensional space, it is possible to divide all of them in half according to volume with a single ($n-1$)-dimensional hyperplane. So far so abstract. But if we consider the case where $n = 3$, and take the three objects to be a piece of ham and two slices of bread, then a consequence of the Stone-Tukey Theorem is that this ‘sandwich’ can be bisected with a single cut of the knife so that equal volumes of ham and bread are in each portion. Indeed it is for this reason that the above theorem is popularly referred to as the “Ham Sandwich Theorem.”

Moving from prediction to explanation, it seems we can equally well cite the Stone-Tukey Theorem in an explanation of our repeated success in fairly bisecting ham sandwiches of a variety of shapes and sizes, and this counts as a genuine MES. However, as with the antipodal point example discussed earlier, the impetus for constructing this MES comes from the discovery of the associated mathematical theorem rather than from some particular physical phenomenon that we find puzzling. It is for this reason that I classify both of these examples as “mathematics-driven” MES, and see them as falling withing the category of circularity of justification.

I agree with Price that this kind of circularity is problematic when it occurs, and it ought to undermine our confidence in such examples as legitimate instances of the Enhanced Indispensability Argument. However, there are several reasons for thinking that core exemplars of mathematical explanation in science will not be undermined in this fashion.

Firstly, it is clear that the particular case of the Cicada MES does not fall foul of this variety of circularity. Recall that the core non-mathematical explanandum is that particular cicada species have periods (in years) that are either 13 or 17. (More carefully, the explanandum is the logical paraphrase of one or other of these specific number claims.) Our belief in the truth of the claim, say, that cicada species in ecosystem-type E has a life-cycle period of 13 years, in no way depends on any mathematical theory. Rather it is based on empirical observations in the field of mass cicada emergences in particular locales. Hence there is no circularity involved in our belief in the truth of the E_P explanandum in the Cicada case.

Secondly, the subcategory of MES that is affected by this form of circularity, mathematics-driven explanations in science, is of marginal relevance both in terms of their explanatory credentials and in terms of their role in scientific practice. MDES's are explanatorily marginal because it is a mathematical result that is the driving force behind the development of a given MDES, hence the physical phenomenon it gets linked to is often a prediction rather than an explanandum. Thus in the Borsuk-Ulam example discussed earlier, the existence of antipodal points with the same temperature and pressure was not a physical fact whose truth was known prior to the prediction made using this theorem.[20] Although MDES's do forge links between mathematics and physical phenomena, the phenomena that are linked to by MDES's are typically of little scientific significance in their own right. Knowing about the existence of antipodal points of the sort predicted by the Borsuk-Ulam theorem, for example, does not help climate scientists develop better models, or explain existing climate trends.

To summarize, I would agree that on this first, strong way of interpreting “identifying” the physical explanandum, E_P, in which there is no reason independent of the explaining theory to think that E_P is true, the path to ontological commitment is blocked. However, there is no reason to think that all mathematical explanations in science are ‘mathematics-driven’ in this way. Indeed, mathematics-driven explanations seem to be of marginal relevance to actual scientific practice and may not even count as genuine scientific explanations at all.[21] For a platonist aiming to bolster her position using the Enhanced Indispensability Argument, there are good reasons – independently of circularity concerns – for seeking out examples of mathematical explanation in science that are more methodologically central, and these will tend to be examples that do not fall foul of justification-based circularity.

[20] For more criticism along these lines, see Baker [2005, pp. 226–7].

[21] It is worth noting that Marc Lange is happy to include this kind of MDES within the category of what he terms “distinctively mathematical explanations.” His own interest in this topic is not motivated by indispensability considerations or the platonist/nominalist debate, hence the issue of circularity does not worry him. Indeed, Lange readily concedes that the facts such as “that there are always antipodal equatorial points at the same temperature would presumably have gone entirely unnoticed had they not been recognized as having distinctively mathematical explanations.” (Lange [2017, p. 405]).

5. Type 2 circularity: explanatory urgency

A second way in which the identification of a physical explanandum, E_P, can fail to be independent of the mathematical theory, M, doing the explaining is when the *surprise* of E_P obtaining depends on M. Price argues that this is the case for the Cicada MES. In particular, he argues that this violation of NCC will occur if we replace the original (and unacceptably mathematical) explanandum, that cicadas have prime period lengths, with the more specific explanandum that cicadas have periods of length 13 or 17. Price comments:

> Were the phenomenon in question just the period length (either 13 or 17), it is difficult to see why anyone would have bothered looking for an explanation at all.[22]

Price's objection is that the unlikeliness of this coincidence is only a datum if we assume the mathematical theory (in this case, number theory) from which any ‘explanation’ is likely to come from. A nominalist can simply deny that there is any striking fact here to be explained. The feeling of surprise here is both generated by, and – if the investigation is successful – dissipated by a mathematical theory. So it seems as if someone who denies the truth of the mathematical theory can simply opt out of this explanatory detour.

To avoid the subjective and psychological connotations of surprise, I shall refer to this notion as *explanatory urgency*. Relatively little work has been done on articulating this notion, despite its potential importance to explanationist projects in philosophy of science and in metaphysics. In a 2005 paper, Roger White makes some helpful initial moves. He suggests, for example, that “a fact calls for an explanation when it challenges our assumptions about the circumstances that brought it about.“[23] This may be because the fact in question has low probability, given our antecedent beliefs, but having low probability is neither necessary nor sufficient in itself to make a fact explanatorily urgent. White compares the example of a monkey typing “I want a banana” with a monkey typing “l; kawje oiflkm wesdf.” Both have the same antecedent probability, on the assumption that the monkey is typing randomly. But only the first seems to call for an explanation. Perhaps a further element here concerns the explanatory urgency of *patterns*. If some shared property is repeatedly instantiated, without any prior reason to favor it over other alternative properties, then – other things being equal – the resulting pattern calls for an explanation. Thus in the typing monkey example above, part of what makes “I want a banana” call for an explanation is the repeated instantiation of the property of being an English word.

I suspect that Price is right to insist both that some kind of explanatory urgency is required to motivate a good explanation, and that this urgency cannot be dependent on the mathematical theory being argued for, on pain of circularity. This being said, there does seem to be a way out of the explanatory urgency objection in the specific Cicada case that Price discusses. For it turns out that there is a surprising aspect to cicada period lengths that can be identified independently of the notion of primality (and of any other essentially mathematical notions).

Firstly, note that the possibility of dividing a given specific collection of objects into smaller collections of equal size is something that can be recognized even in the absence of more general mathematical concepts such as primality. For example, a group of 17 objects cannot be divided into two groups of equal size, or three groups, or any number of groups

[22] Price, *op. cit.*, p. 38.

[23] White [2005, p. 3].

between two and sixteen. To put it slightly more formally, the nominalist can recognize the truth of the following fact:

(9) If there are exactly 17 F's, then the F's cannot be divided into collections of 2 such that every F is in exactly one collection, or into collections of 3 such that …, …, or into collections of 16 such that every F is in exactly one collection.

The references in (9) to specific numbers (as sizes of collections) can then be eliminated into first-order logic in the familiar fashion.

It might be thought that the nominalistic acceptability of facts of the above sort puts pressure on the claim that the mathematical notion of primality is ineliminable, a claim that is crucial to the role that the Cicada case (and others) play in supporting the Enhanced Indispensability Argument. However, the availability of non-mathematical paraphrases of *particular* instances of primality, such as when there are exactly 17 F's, does not mean that primality is eliminable in the *general* case. For example, the above procedure will not help us eliminate reference to primality in either of the following two sentences:

(10) There is a prime number of F's.
(11) If there is a prime number of F's and there are more than two F's then there is an odd number of F's.

So we have a nominalistically recognizable fact that is associated with each specific case where we have a prime number of F's (and in the Cicada example, the F's are years). The second part of the counterargument to Price is to show that the concatenation of these nominalistic facts is surprising for reasons that do not depend on number theory (or any other mathematical theory). If we examine periods in the 12 to 18 length range, we find that only two period lengths are such that it is impossible to divide them up in the way outlined in (9). Hence the probability of a single period length with this property occurring at random is 1/3. An important fact about periodical cicadas is that there are seven distinct sub-species, with different geographical ranges and years of mass emergence. Two of these sub-species have 13-year periods and five of them have 17-year periods. We can model the situation as picking randomly seven times (with replacement) from among the numbers 12 to 18. The resulting probability is $(1/3)^7 \approx 0.00046$.[24] This probability is clearly low enough to qualify the fact, E_P, that all periodical cicada subspecies have 13- or 17-year periods, as explanatorily urgent. I concluded that even the nominalist can recognize the explanatory urgency of the explanandum in the Cicada example, and hence there is no circularity here of this second sort.[25]

What, then, would constitute an example of circularity of explanatory urgency? In principle, there seems to be two main categories of case. The first category would involve cases where there is no nominalistic paraphrase to be found, even for individual instances of the mathematical property. The second category would involve cases where there is a nominalistic paraphrase of individual instances, but there is no non-mathematical way to show that the concatenation of instances is surprising (and thus explanatorily urgent).

I do not have a fully worked-out example of the first category, however it seems plausible that there are examples from particle physics that illustrate this kind of circularity. Consider, for example, the discovery of patterns of symmetry that are shared by different families of fundamental particles, and the subsequent realization that these map onto particular mathematical groups. The proponent of EIA will advocate that an investigation be carried out to look for a mathematical explanation of this phenomenon. And – crucially – such an investigation ought to be carried out even if we have no idea, initially, what the relevance of group theory here could possibly be.[26]

Price's objection is that, in the absence of the mathematical theory (in this case, group theory), there is no explanatory urgency. And in this case he would be correct. There is no effective way for the nominalist to recognize the property of instantiating a given group structure, nor is there likely to be any clear nominalistic property that can be associated with this mathematical property. Hence there is nothing surprising, for the nominalist, about this pattern of symmetries of the various families of fundamental particles. From the nominalist's perspective, therefore, there is nothing that needs to be explained.

The second category of potential circularity of explanatory urgency is where a nominalistic paraphrase of individual instances is available, but there is no non-mathematical route to recognizing the surprising nature of the concatenation of these instances. Here I do have an actual example.

Consider electromagnetism as a relativistic theory (Maxwell's theory is inherently compatible with special relativity). We can unite the electric field and magnetic field, which are each represented as spatial vectors (with 3 components), into a space-time quantity, a 4-by-4 matrix known as the *Faraday tensor*, denoted by Fab. It is an antisymmetric matrix Fab = −Fba, and transforms covariantly under coordinate transformations of Minkowski space. The following mathematical theorem divides electromagnetic fields into three broad categories:

Theorem(Classification of Electromagnetic Fields): Given an arbitrary electromagnetic field, there is an associated linear transformation, $\tilde{F}$, such that one of the following three must hold:

1. $\tilde{F}$ is the zero linear transformation: it sends all vectors to zero.
2. $\tilde{F}$ has only one real eigenvector. It is a repeated eigenvector of multiplicity two, and its eigenvalue is 0.
3. $\tilde{F}$ has exactly two real eigenvectors. They each have multiplicity one, and their eigenvalues agree in magnitude but differ in sign.

Each of the three cases can be given a physical interpretation (although for Case 1 the eigenvalue is zero so there is not much to interpret!). Mathematically an eigenvector of a linear transformation is just a vector whose direction is fixed under the transformation, while its eigenvalue is the amount its magnitude changes by. In the case where F is non-zero, the eigenvectors are called the principal null directions of the electromagnetic field. In Case 2, where F has only one real eigenvector, the unique principal null direction represents the direction of travel for the electromagnetic wave (recall that light always travels at the speed of light and on a light-like curve). Case 3 is more complicated, but the key point is that the resulting electromagnetic field is described not by the propagation of just one wave alone, but by the superposition of multiple waves.

The important point for current purposes is not the mathematical details of the above theorem and its proof, but the fact that the three cases

[24] I am using 12 and 18 as the bounds on cicada periods because this is the surmised ecologically imposed constraint that features in the standard version of the Cicada MES. However, the probability-based argument given here does not hinge on this particular choice of bounds. For a larger bound, we can ask the same question about the probability of independently picking a prime number seven times (with replacement). In every range that includes 12 and 18, the proportion of prime numbers is less than half. Hence the overall probability is less than $(1/2)^7$, i.e. less than 0.008.

[25] One might have misgivings about the role of probability theory in the above argument, since probability theory is a mathematical theory. I don't think that the worry here is too pressing. Firstly, it is a very different mathematical theory than the one being argued for in the Cicada example, so there is no strict circularity. Secondly, probability theory is just being used to make the degree of explanatory urgency quantitatively precise. We could run essentially the same argument in a more qualitative way without bringing in the formal apparatus of the probability calculus.

[26] A real-life example with some similarities to this case concerns the 'Monstrous Moonshine' conjecture that began with a striking numerical coincidence and ended connecting together two apparently very distant areas of pure mathematics. For a book-length treatment of this episode, see Ronan [2006].

have a common mathematical form but no shared physical description. What this example shares with the Cicada example is that each instance of the mathematical property (in this case, an electromagnetic field with a certain configuration of eigenvectors) can be given a physical description.[27] Where this example diverges from the Cicada example is that the physical description is sharply different in different cases. There is something mathematically striking about electromagnetic fields, that the number of eigenvectors of the associated linear transformation is always either 0, 1, or 2, but there is no corresponding general physical property. In the absence of the mathematical theory, therefore, there is no striking pattern of instances to be explained because there is no non-mathematical way to see unity across the three kinds of case. Hence there is no explanatory urgency.

6. The roles of mathematics in science

One fruitful result of considering circularity objections is that it forces the proponent of the Enhanced Indispensability Argument to get clearer about distinguishing the different roles that mathematics can play in scientific explanation. Some work in this direction has already been done by Yablo (who is himself unsympathetic to EIA).[28]

Yablo distinguishes three grades of mathematical involvement in scientific explanations. He considers a basic template in which outcome *E* is explained as arising out of circumstances *C* by way of a generalization *G* that links them.

> Math helps *descriptively* to the extent that we need it to specify *C* or *E*, or to formulate the generalization. … Math helps *structurally* if it lets us run the explanation at the right level of generality …. Math helps *substantively* if it provides the covering generalization *G*.[29]

The definitional distinctions that Yablo makes here between these three levels are far from clear. (For example, formulation of the generalization *G* seems to potentially apply to all three levels.) However, he goes on to give a (deliberately simplified) example to help illustrate what he has in mind:

> Certain numbers of tiles are never seen on the floors of rectangular rooms. One finds rooms with 18 tiles but not 19, 36 but not 37, 70 but not 71. … This is first grade involvement; we are using numbers to pick out the phenomenon that puzzles us.
>
> The second grade is reached when we realize that the unsuitable bunches of tiles have something in common; they are prime in number. … That is the explanatorily relevant feature and it is mathematical in nature, which is what second-grade involvement requires.[30]

The parallels between this tile example and the Cicada example are clear. However, I am not convinced that Yablo's 3-fold categorization is particularly helpful. To pick on just one aspect of the tile example, according to his own definition, mathematics only has first grade involvement if it is *necessary* for picking out the phenomena of interest. But we already know that logical paraphrases can be given of each particular claim about the number of tiles found on a given floor. Hence it would seem that there is not even first grade involvement in this case.

A more useful classification scheme for present purposes is the following. We start with some instances with a range of properties E_P, E'_P, E''_P, …as our potential explanandum. Either each individual instance can be fully described non-mathematically or it cannot. If not, then we have an intrinsically mathematical phenomenon, so any mathematical explanation if it will fall foul of the circularity objection. If each instance can be non-mathematically described, then there are two further possibilities. If the instances have no mathematical property in common then there is no mathematical explanation to be given0.[31] However, if there is some mathematical property, P_M, that all these instances have in common (according to some associated mathematical theory, M), then there are three basic scenarios:

(i) There is no general explanation for why the property P_M holds. Call this a *mathematical coincidence.*
(ii) The mathematical theory, M, can be used to help explain why P_M holds, but P_M does not explain why the E_P, E'_P, E''_P, …pattern holds. Call this a *mathematical spandrel0.*[32]
(iii) The mathematical theory, M, can be used to help explain why P_M holds, and P_M helps to explain why the E_P, E'_P, E''_P, …pattern holds. Call this a *salient mathematical property.*

Only in case (iii) do we have the potential for a genuine mathematical explanation of the E_P, E'_P, E''_P, …pattern. However, in this case the worry about circularity is avoided because neither the truth nor the explanatory urgency of the pattern of instances presupposes the mathematical theory appealed to in the explanation.

7. Conclusions

In this paper I have argued that self-undermining circularity is sometimes a problem for specific cases of mathematical explanation in science that are offered in support of the Enhanced Indispensability Argument. Supporters of EIA do need to be careful of circularity, since this would undermine a key dialectical advantage of this line of argument for mathematical platonism. And there is a genuine question about whether the indispensability of mathematics for explanation – which is required for EIA to go through – entails the indispensability of mathematics for identifying the explanandum – which would make EIA question-begging against the nominalist.

However, there is no reason to think that circularity will always be a problem. Even in the canonical Cicada example, I have shown that the circularity charge can be avoided. Shifting to a more general viewpoint, although cases can be found where either the truth or the explanatory urgency of a physical explanandum presupposes a mathematical theory, nothing in the structure of the Enhanced Indispensability Argument forces one or other of these kinds of circularity to occur. As long as she is careful, the proponent of EIA can be confident in finding cases of mathematical explanation in science that are not circular and which can lend support to the platonist conclusion of this important line of argument.

Author statement

Alan Baker wrote the manuscript.

[27] At least of the eigenvector component. I am presupposing that the background theory of electromagnetic fields can somehow be nominalized, perhaps along the lines of Hartry Field's nominalization of Newtonian gravitational theory, but any such scheme would have to be worked out in more detail.
[28] Yablo [2012].
[29] *op. cit.*, pp. 1020–1.
[30] *op. cit.*, p. 1021.

[31] Presuming that any such mathematical explanation would take the form of showing how the various instances fall under some shared mathematical descriptor. For example, if periodical cicada species were to have exhibited period lengths of 13, 16, and 28 years, it may be that these three numbers have no non-trivial mathematical property in common. In this case, the fact that each period-length has a distinctively mathematical property – such as being prime, being square, and being perfect (respectively) – does nothing to boost the prospects of there being a mathematical explanation of this physical phenomenon.
[32] For more on this notion, see Baker [2017].

Appendix A. Supplementary data

Supplementary data to this article can be found online at https://doi.org/10.1016/j.shpsa.2021.05.017.

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