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New voltage sensitivity analysis for smart distribution grids using analytical derivation: ABCD model

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ABSTRACT

Sensitivity Analysis plays a significant role in voltage prediction and control of power networks. However, the classical sensitivity methods require significant computation time. As active distribution networks require real-time implementation for voltage control, reducing the computation time becomes a necessary task for network operators, especially in the context of optimization techniques. This work develops a new analytical and fast voltage sensitivity analysis method via the derivative of the nodal quantities (power, current and voltage) with respect to power injections. The proposed method mainly depends on the construction of the ABCD matrix. The values of the matrix elements remain the same regardless of the bus on which the power is injected. Thus, it has a high potential to be implemented in online applications. To make a complete separation between the sensitivities to active and the sensitivities to reactive power injections, the analytical formulations are expressed in Cartesian coordinates. A radial distribution network including several DG units is used to verify and assess the proposed sensitivity method under different scenarios.

1. Introduction

Future Power grids will meet new challenges in voltage control due to the high penetration levels of Distributed Generation (DG) units [1]. DG units can be actively involved in power systems for voltage regulation [2]. Voltage control methods mainly depend on the relationship between the system voltages and control variables (i.e. power injections). The sensitivity analysis is usually used to find the voltage sensitivity coefficients with respect to nodal reactive and real power injections. These sensitivities can be actively used to manage control variables to solve voltage problems in an accurate way. Many approaches have been proposed in the literature to compute these sensitivities.

One of the well-known approaches is based on the Jacobian matrix [3,4]. This approach is a classical method and depends on solving a Newton Raphson power flow [5]. The voltage sensitivities are found by taking the Jacobian matrix (J) inverse at one operating condition. However, the sensitivity coefficients have to be updated with any change in system state (e.g. changes of demand, generation, topology, and/or network parameters). This requires performing new Newton Raphson-based power flow calculations and, therefore, more computation time is required. Besides, convergence may not be obtained by this method. Such methods developed for transmission load flow studies are unsuitable for distribution systems due to poor convergence [6]. This is due to the radial structure and the high R/X ratio of distribution networks. Thus, these issues may add a new challenge for real-time voltage control. Although this kind of sensitivity analysis appears in recent studies for real-time applications such as voltage control [7] and voltage stability issues [8], the analysis is done offline and only at normal operating conditions.

Many other sensitivity methods have been discussed in the literature. An approach based on the Gauss-Seidel method of load flow is developed [9]. The approach depends on the impedance matrix of the network and uses an iteration process with a fixed number of iterations. Therefore,

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the accuracy of this method is low. A sensitivity approach based on the network impedance matrix and the constant-current model of loads is developed [10]. However, this approach depends on the approximated representation of the network lines. An approach starting from branch currents is used for sensitivity analysis [11]. This method requires a base load flow solution. Other methods based on the so-called adjoint network are also proposed [12–15]. Another approach based on the Perturb-And-Observe (P &O) Power Flow is proposed [16]. It depends on performing two power flows with a slight change in the power at the interested node. An approach based on historical data is also presented [17]. An approach is developed in [18] based on the system topology and independent of the system operating point. In [19], the voltage sensitivities are approximated based on the historical measurements of the phasor measurement unit at partial nodes of distribution networks. In [20], local estimation of the voltage sensitivities is obtained based on a critical approximation of radial topology networks as the power flow to each node is directly affected by its downstream nodes. In [21], a voltage sensitivity analysis is proposed for low voltage network by means of voltage perturbations, followed by active and reactive power measurements. In [22], a voltage sensitivity analysis is obtained via probabilistic load flow based on polynomial chaos method. In [23], voltage sensitivities models are established based on one initial load flow calculation and valid only for radial networks. In [24], probabilistic voltage sensitivity analysis is developed for radial networks. However, these works suffer from inaccuracy, low-speed calculations, or valid for only a particular type of networks.

As power systems continue in hosting large penetration levels of DG units, the need for online voltage control approaches is advanced. Sensitivity analysis represents the main role in voltage prediction and control. However, most of the common sensitivity techniques may not meet the requirements of future distribution networks to continuously update the sensitivities. The convergence problems and the remarkable calculation time associated with the common sensitivity analysis methods add new challenges for online applications, especially in the context of optimization problems and practical systems. In this regard, this work aims to develop a new and fast approach for voltage sensitivity analysis in power systems. The sensitivities are obtained via the direct derivative of nodal quantities (power, voltages, and currents) with respect to active and reactive power injections. The proposed method, namely ABCD model, mainly depends on the construction of ABCD matrix (refer to Eq. (25)).

The sensitivity method uses the exact power flow equations without any approximation to build up a linear system of equations. The main part of this linear system is the construction of ABCD matrix. The ABCD matrix is derived using a straightforward analytical derivation of nodal quantities, in terms of Cartesian coordinates. ABCD elements represent coefficients for the partial derivatives of node voltages (in Cartesian form). The Cartesian coordinates enable us to separate between the sensitivities to active and the sensitivities to reactive power injections (i.e. the submatrices A and B are used to obtain the sensitivities to active power injections while the submatrices C and D are used to obtain the sensitivities to reactive power injections). The ABCD is a general matrix can be used directly to obtain sensitivities through a linear system of equations, without need for iteration process. The values of the matrix elements remain the same regardless of the bus on which the power is injected. Construction of ABCD matrix is also very important for unbalanced electrical distribution networks. The present work can be used to support the computation of the sensitivities for a generic unbalanced electrical network by using the [ABCD] compound matrix of a generic multi-phase radial unbalanced network. Moreover, ABCD matrix is strongly related to the parts of Y_{bus} and thus has the advantage of being sparse. Another advantage is that with the aim to find the voltage sensitivity of a particular bus, there is no need to find the voltage sensitivities of all network buses.

The method developed in this work is oriented for online applications in smart grids. The problem is formulated such that the sensitivities can be directly obtained using the final expression (refer to Eq. (24)). The final expression illustrated in (24) can be considered to be a general formula for sensitivity analysis in distribution networks. The method is also flexible, such that it is not limited to a particular type of network or governed by a particular power flow method. The method can also be extended to include effect of PV buses and the different types of control variables. The ABCD model can be used as an alternative technique for the classical ones but with an extra advantage of fast computations.

The characteristics associated with ABCD method make the proposed method unique. These significant characteristics are: (a) ABCD matrix is constant regardless of the node on which the power is injected, (b) the expressions are in Cartesian coordinates for fast calculations and (c) the complete separation between the sensitivities to active and the sensitivities to reactive power injections.

The features of the ABCD method can be summarized as:

- It depends on sparse submatrices, which can also speed up the computation.
- It has almost the same level of the accuracy of J^{-1} method. The errors in the sensitivities or in the predicted voltages are very small.
- It does not require to update ABCD matrix with changing the bus on which the power is injected. This will also reduce the computation time.
- It can account for any change in the demand, generation, or network parameters.
- It is strongly related to the parts of Y_{bus}. Thus, it can take into accounts the structural changes in the networks.
- It completely separates between the sensitivities to active and the sensitivities to reactive power injections.
- It can be extended to compute the sensitivities with respect to different types of control variables (i.e. load tap changers).
- It is suitable for any network (transmission or distribution, radial or meshed networks).

The key contributions of this work are:

- Development of a new analytical and fast approach for voltage sensitivity analysis of power systems via the direct derivative of the real and imaginary parts of the nodal quantities with respect to power injections, which to the best to our knowledge, is not done in literature.
- Construction of ABCD matrix, which the proposed method depends on, and derivation of one mathematical expression to find the sensitivities with respect to any power injection. The value of ABCD matrix elements of a particular system remains the same regardless of the bus on which the power is injected.
- The complete separation between the sensitivities to active and the sensitivities to reactive power injections.
- Validation of the fast computation of the proposed method and its applicability in online voltage control.

The remainder of this work is organized as follows. Section 2 derives the mathematical development for nodal power injections in Cartesian coordinates. The proposed sensitivity analysis method is presented in Section 3. Simulation results are presented in Section 4. Section 5 provides the conclusions.

1.1. Mathematical Model in Cartesian Coordinates Formula for Nodal Power Injections

This section aims to find the mathematical relation for the network states and parameters in Cartesian coordinates.

1.2. Nodal current injection in Cartesian coordinates

The nodal bus currents I can be written in terms of nodal voltages V and system admittance Y as:

$$\begin{bmatrix} I_1 \\ . \\ . \\ I_i \\ . \\ . \\ I_N \end{bmatrix} = \begin{bmatrix} Y_{11}..Y_{1i}..Y_{1N} \\ \cdots \\ \cdots \\ Y_{i1}..Y_{ii}..Y_{iN} \\ \cdots \\ \cdots \\ Y_{N1}..Y_{Ni}..Y_{NN} \end{bmatrix} \begin{bmatrix} V_1 \\ . \\ . \\ V_i \\ . \\ . \\ V_N \end{bmatrix} \tag{1}$$

where N denotes the number of system nodes. The element Y_{ij} can be written as $G_{ij} + jB_{ij}$, where G and Bdenote the conductance and susceptance, respectively. Similarly, the voltage V_j can be written as $V_{j,r} + jV_{j,im}$, where $V_{j,r}$ and $V_{j,im}$ denote the real and the imaginary parts, respectively. Accordingly, the current I_i can be expressed as:

$$I_i = \sum_{j\in N} Y_{ij}V_j = \sum_{j\in N}\left(\left(G_{ij}V_{j,r} - B_{ij}V_{j,im}\right) + j\left(G_{ij}V_{j,im} + B_{ij}V_{j,r}\right)\right) \tag{2}$$

Thus, the real and the imaginary parts of the current I_i (i.e. $I_{i,r}$ and $I_{i,im}$, respectively) can be obtained as:

$$I_{i,r} = \sum_{j\in N}\left(G_{ij}V_{j,r} - B_{ij}V_{j,im}\right) \tag{3}$$

$$I_{i,im} = \sum_{j\in N}\left(G_{ij}V_{j,im} + B_{ij}V_{j,r}\right) \tag{4}$$

1.3. Nodal power injections in Cartesian coordinates

The complex power at node i (S_i) can be written in terms of real and reactive power (i.e. P_i and Q_i, respectively) as:

$$\begin{aligned} S_i &= V_i I_i^* \\ &= (V_{i,r} + jV_{i,im})(I_{i,r} - jI_{i,im}) \\ &= (V_{i,r}I_{i,r} + V_{i,im}I_{i,im}) + j(V_{i,im}I_{i,r} - V_{i,r}I_{i,im}) \end{aligned} \tag{5}$$

Thus, we obtain:

$$P_i = (V_{i,r}I_{i,r} + V_{i,im}I_{i,im}) \tag{6}$$

$$Q_i = \left(V_{i,im}I_{i,r} - V_{i,r}I_{i,im}\right) \tag{7}$$

By substituting Eqs. (3) and (4) into Eqs. (6) and (7), we obtain:

$$P_i = V_{i,r}\sum_{j\in N}\left(G_{ij}V_{j,r} - B_{ij}V_{j,im}\right) + V_{i,im}\sum_{j\in N}\left(G_{ij}V_{j,im} + B_{ij}V_{j,r}\right) \tag{8}$$

$$Q_i = V_{i,im}\sum_{j\in N}\left(G_{ij}V_{j,r} - B_{ij}V_{j,im}\right) - V_{i,r}\sum_{j\in N}\left(G_{ij}V_{j,im} + B_{ij}V_{j,r}\right) \tag{9}$$

It is clear from Eqs. (8) and (9) that the real and reactive power are expressed in terms of real and imaginary parts of the network admittance and network voltages.

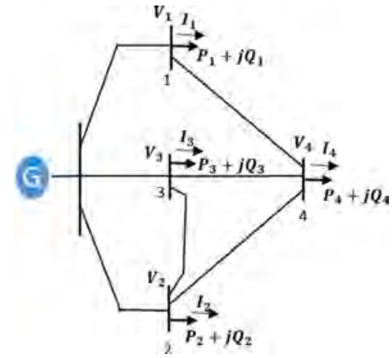

Fig. 1. Simple power system.

2. Proposed sensitivity analysis approach

2.1. Change in power injections in Cartesian coordinates

The simple system presented in Fig. 1 explains the concept behind the proposed sensitivity method. In this figure, V and I denote for load voltage and current, respectively. $P + jQ$ denotes load power. It is worth noting that the developed expressions in this section have an assumption of a constant power model for load and DG units.

Any increment in the power injection (ΔP or ΔQ) at any node "x" will increase the voltage at node "i" by $\Delta V_i = \Delta V_{r,i} + j\Delta V_{im,i}$. The new voltage of node $i\in M$ can be then expressed as $V_i + \Delta V_i = (V_{r,i} + \Delta V_{r,i}) + j(V_{im,i} + \Delta V_{im,i})$. Similarly, the load current will also be varied by $\Delta I_i = \Delta I_{r,i} + j\Delta I_{im,i}$, resulting in an expression for the new current as $I_i + \Delta I_i = (I_{r,i} + \Delta I_{r,i}) + j(I_{im,i} + \Delta I_{im,i})$. To find an expression for the voltage sensitivity to real power injection, let a load of node "i" at the 1st instant be $S_{i,1} = P_{i,1} + jQ_i$ and the load at the 2nd instant be $S_{i,2} = P_{i,2} + jQ_i$. According to Eq. (6), the real power at both instants can be written as:

$$P_{i,1} = V_{r,i}I_{r,i} + V_{im,i}I_{im,i} \tag{10}$$

$$P_{i,2} = (V_{r,i} + \Delta V_{r,i})(I_{r,i} + \Delta I_{r,i}) + (V_{im,i} + \Delta V_{im,i})(I_{im,i} + \Delta I_{im,i}) \tag{11}$$

The change in the real power $\Delta P_i = P_{i,2} - P_{i,1}$ is written as:

$$\Delta P_i = I_{r,i}\Delta V_{r,i} + V_{r,i}\Delta I_{r,i} + \Delta V_{r,i}\Delta I_{r,i} + I_{im,i}\Delta V_{im,i} + V_{im,i}\Delta I_{im,i} + \Delta V_{im,i}\Delta I_{im,i} \tag{12}$$

It is worth mentioning that the smaller the value of ΔP_i (implying terms of ΔV_i & ΔI_i are also small), the closer the two operating points are, the better the sensitivity coefficients' estimation. The terms $\Delta V_{r,i}\Delta I_{r,i}$ and $\Delta V_{im,i}\Delta I_{im,i}$ of Eq. (12) represents very small values and therefore can be ignored. Accordingly, Eq. (12) can be written as:

$$\Delta P_i = I_{r,i}\Delta V_{r,i} + V_{r,i}\Delta I_{r,i} + I_{im,i}\Delta V_{im,i} + V_{im,i}\Delta I_{im,i} \tag{13}$$

To find an expression for the voltage sensitivity to reactive power injection, let the load at node "i" at the 1st instant be $S_{i,1} = P + jQ_{i,1}$ and the load at the 2nd instant be $S_{i,2} = P + jQ_{i,2}$. According to Eq. (7), the reactive power at both instants can be expressed as:

$$Q_{i,1} = V_{im,i}I_{r,i} - V_{r,i}I_{im,i} \tag{14}$$

$$Q_{i,2} = (V_{im,i} + \Delta V_{im,i})(I_{r,i} + \Delta I_{r,i}) - (V_{r,i} + \Delta V_{r,i})(I_{im,i} + \Delta I_{im,i}) \tag{15}$$

The reactive power change $\Delta Q_i = Q_{i,2} - Q_{i,1}$ can be written as:

$$\Delta Q_i = V_{im,i}\Delta I_{r,i} + \Delta V_{im,i}I_{r,i} + \Delta V_{im,i}\Delta I_{r,i} - V_{r,i}\Delta I_{im,i} - \Delta V_{r,i}I_{im,i} - \Delta V_{r,i}\Delta I_{im,i} \tag{16}$$

By ignoring the terms $\Delta V_{im,i}\Delta I_{r,i}$ and $\Delta V_{r,i}\Delta I_{im,i}$, (16) becomes:

$$\Delta Q_i = -I_{im,i}\Delta V_{r,i} + V_{im,i}\Delta I_{r,i} + I_{r,i}\Delta V_{im,i} - V_{r,i}\Delta I_{im,i} \tag{17}$$

It is clear from Eqs. (13) and (17) that the real and reactive power changes are expressed in terms of real and imaginary parts of the network voltages and currents.

2.2. Analytical derivation of power injection

In this section, mathematical expressions that link network voltages to node power injections are derived.

The expressions presented in Eqs. (13) and (17) represent the change in active or reactive power injection in Cartesian coordinates. Dividing Eq. (13) by amount of real power injection at bus "x" (ΔP_x) yields:

$$\frac{\Delta P_i}{\Delta P_x} = I_{r,i}\frac{\Delta V_{r,i}}{\Delta P_x} + V_{r,i}\frac{\Delta I_{r,i}}{\Delta P_x} + I_{im,i}\frac{\Delta V_{im,i}}{\Delta P_x} + V_{im,i}\frac{\Delta I_{im,i}}{\Delta P_x} \tag{18}$$

By taking the limit of the expression in Eq. (18) as $\Delta P_x \to 0$, Eq. (18) becomes:

$$\frac{\partial P_i}{\partial P_x} = I_{i,r}\frac{\partial V_{i,r}}{\partial P_x} + V_{i,r}\frac{\partial I_{i,r}}{\partial P_x} + I_{i,im}\frac{\partial V_{i,im}}{\partial P_x} + V_{i,im}\frac{\partial I_{i,im}}{\partial P_x} \tag{19}$$

Similarly, dividing Eq. (17) by an amount of ΔQ_x and taking the limit of the expression as $\Delta Q_x \to 0$, we obtain:

$$\frac{\partial Q_i}{\partial Q_x} = -I_{i,im}\frac{\partial V_{i,r}}{\partial Q_x} + V_{i,im}\frac{\partial I_{i,r}}{\partial Q_x} + I_{i,r}\frac{\partial V_{i,im}}{\partial Q_x} - V_{i,r}\frac{\partial I_{i,im}}{\partial Q_x} \tag{20}$$

where $\frac{\partial P_i}{\partial P_x}$ and $\frac{\partial Q_i}{\partial Q_x}$ represent the partial derivatives of the real and reactive power of node "i" with respect to active and reactive power injected into node x, respectively. $\frac{\partial V_{i,r}}{\partial P_x}$, $\frac{\partial V_{i,im}}{\partial P_x}$, $\frac{\partial I_{i,r}}{\partial P_x}$, $\frac{\partial I_{i,im}}{\partial P_x}$and $\frac{\partial V_{i,r}}{\partial Q_x}$, $\frac{\partial V_{i,im}}{\partial Q_x}$, $\frac{\partial I_{i,r}}{\partial Q_x}$, $\frac{\partial I_{i,im}}{\partial Q_x}$represent the partial derivatives of the voltage and current (real and imaginary parts) of bus "i" with respect to active and reactive power injected into node "x", respectively. The partial derivatives $\frac{\partial I_{i,r}}{\partial P_x}$, $\frac{\partial I_{i,im}}{\partial P_x}$, $\frac{\partial I_{i,r}}{\partial Q_x}$ and $\frac{\partial I_{i,im}}{\partial Q_x}$ can be obtained by taking the derivation of Eqs. (3) and (4) with respect to active or reactive power injection as follows:

$$\frac{\partial I_{i,r}}{\partial P_x} = \sum_{j\in N}\left(G_{ij}\frac{\partial V_{j,r}}{\partial P_x} - B_{ij}\frac{\partial V_{j,im}}{\partial P_x}\right) \quad i\in M \tag{21a}$$

$$\frac{\partial I_{i,r}}{\partial Q_x} = \sum_{j\in N}\left(G_{ij}\frac{\partial V_{j,r}}{\partial Q_x} - B_{ij}\frac{\partial V_{j,im}}{\partial Q_x}\right) \quad i\in M \tag{21b}$$

$$\frac{\partial I_{i,im}}{\partial P_x} = \sum_{j\in N}\left(G_{ij}\frac{\partial V_{j,im}}{\partial P_x} + B_{ij}\frac{\partial V_{j,r}}{\partial P_x}\right) \quad i\in M \tag{21c}$$

$$\frac{\partial I_{i,im}}{\partial Q_x} = \sum_{j\in N}\left(G_{ij}\frac{\partial V_{j,im}}{\partial Q_x} + B_{ij}\frac{\partial V_{j,r}}{\partial Q_x}\right) \quad i\in M \tag{21d}$$

where $\frac{\partial V_{j,r}}{\partial P_x}$, $\frac{\partial V_{j,r}}{\partial Q_x}$, $\frac{\partial V_{j,im}}{\partial P_x}$ and $\frac{\partial V_{j,im}}{\partial Q_x}$ are partial derivations referred to the node j. M represents the number of PQ buses. By substituting Eq. (3), Eq. (4) and Eqs. (21a)–(21d) into Eqs. (19) and (20), we obtain:

$$\begin{aligned}\frac{\partial P_i}{\partial P_x} = &\sum_{j\in M}\left(G_{ij}V_{i,r}\frac{\partial V_{j,r}}{\partial P_x} - B_{ij}V_{i,r}\frac{\partial V_{j,im}}{\partial P_x}\right) + \frac{\partial V_{i,r}}{\partial P_x}\sum_{j\in N}\left(G_{ij}V_{j,r} - B_{ij}V_{j,im}\right)\\ &+\sum_{j\in M}\left(G_{ij}V_{i,im}\frac{\partial V_{j,im}}{\partial P_x} + B_{ij}V_{i,im}\frac{\partial V_{j,r}}{\partial P_x}\right)\\ &\frac{\partial V_{i,im}}{\partial P_x}\sum_{j\in N}\left(G_{ij}V_{j,im} + B_{ij}V_{j,r}\right) \quad i\in M\end{aligned} \tag{22}$$

$$\begin{aligned}\frac{\partial Q_i}{\partial Q_x} = &\sum_{j\in M}\left(G_{ij}V_{i,im}\frac{\partial V_{j,r}}{\partial Q_x} - B_{ij}V_{i,im}\frac{\partial V_{j,im}}{\partial Q_x}\right) + \frac{\partial V_{i,im}}{\partial Q_x}\sum_{j\in N}\left(G_{ij}V_{j,r} - B_{ij}V_{j,im}\right)\\ &-\sum_{j\in M}\left(G_{ij}V_{i,r}\frac{\partial V_{j,im}}{\partial Q_x} + B_{ij}V_{i,r}\frac{\partial V_{j,r}}{\partial Q_x}\right)\\ &\frac{\partial V_{i,r}}{\partial Q_x}\sum_{j\in N}\left(G_{ij}V_{j,im} + B_{ij}V_{j,r}\right) \quad i\in M\end{aligned} \tag{23}$$

It is clear from Eqs. (22) and (23) that the right-hand side is written in terms of the partial derivation of node voltages (real and imaginary parts) with respect to active or reactive power injected at node "x". Eqs. (22) and (23) can be organized as:

$$\begin{aligned}\frac{\partial P_i}{\partial P_x} = &\sum_{\substack{j\in M\\ i\neq j}}\left(G_{ij}V_{i,r} + B_{ij}V_{i,im}\right)\frac{\partial V_{j,r}}{\partial P_x}\\ &+\left(G_{ii}V_{i,r} + B_{ii}V_{i,im} + \sum_{j\in N}\left(G_{ij}V_{j,r} - B_{ij}V_{j,im}\right)\right)\frac{\partial V_{i,r}}{\partial P_x}\\ &+\sum_{\substack{j\in M\\ i\neq j}}\left(G_{ij}V_{i,im} - B_{ij}V_{i,r}\right)\frac{\partial V_{j,im}}{\partial P_x}\\ &+\left(G_{ii}V_{i,im} - B_{ii}V_{i,r} + \sum_{j\in N}\left(G_{ij}V_{j,im} + B_{ij}V_{j,r}\right)\right)\frac{\partial V_{i,im}}{\partial P_x} \quad i\in M\end{aligned} \tag{24}$$

$$\begin{aligned}\frac{\partial Q_i}{\partial Q_x} = &\sum_{\substack{j\in M\\ i\neq j}}\left(G_{ij}V_{i,im} - B_{ij}V_{i,r}\right)\frac{\partial V_{j,r}}{\partial Q_x}\\ &+\left(G_{ii}V_{i,im} - B_{ii}V_{i,r} - \sum_{j\in N}\left(G_{ij}V_{j,im} + B_{ij}V_{j,r}\right)\right)\frac{\partial V_{i,r}}{\partial Q_x}\\ &-\sum_{\substack{j\in M\\ i\neq j}}\left(G_{ij}V_{i,r} + B_{ij}V_{j,im}\right)\frac{\partial V_{j,im}}{\partial Q_x}\\ &+\left(-G_{ii}V_{i,r} - B_{ii}V_{i,im} + \sum_{j\in N}\left(G_{ij}V_{j,r} - B_{ij}V_{j,im}\right)\right)\frac{\partial V_{i,im}}{\partial Q_x} \quad i\in M\end{aligned} \tag{25}$$

2.3. Build up the proposed model: ABCD matrix

To find the partial derivatives of node voltages (real and imaginary parts) with respect to active and reactive power injection at node "x", Eqs. (24) and (25) are performed for each bus i ∊M. In a matrix form, the system of equations can be organized as:

$$\underbrace{\begin{bmatrix} \overbrace{\begin{matrix} a_{11} \ .. \ a_{1i} \ .. a_{1M} \\ . \quad . \quad . \\ . \quad . \quad . \\ a_{i1} \ .. \ a_{ii} \ .. a_{iM} \\ . \quad . \quad . \\ . \quad . \quad . \\ a_{M1} \ .. \ a_{Mi} \ .. a_{MM} \end{matrix}}^{A} & \overbrace{\begin{matrix} b_{11} \ .. \ b_{1i} \ .. b_{1M} \\ . \quad . \quad . \\ . \quad . \quad . \\ b_{i1} \ .. \ b_{ii} \ .. b_{iM} \\ . \quad . \quad . \\ . \quad . \quad . \\ b_{M1} \ .. \ b_{Mi} \ .. b_{MM} \end{matrix}}^{B} \end{bmatrix}}_{R} \begin{bmatrix} \frac{\partial V_r}{\partial P_x} \\ \frac{\partial V_{im}}{\partial P_x} \end{bmatrix} = \begin{bmatrix} \frac{\partial P_1}{\partial P_x} \\ . \\ . \\ \frac{\partial P_i}{\partial P_x} \\ . \\ . \\ \frac{\partial P_M}{\partial P_x} \end{bmatrix} \tag{26}$$

$$\begin{bmatrix} \overbrace{c_{11}\ ..\ c_{1i}\ ..c_{1M}}^{C} & \overbrace{d_{11}\ ..\ d_{1i}\ ..d_{1M}}^{D} \\ . & . \\ . & . \\ c_{i1}\ ..\ c_{ii}\ ..c_{iM} & d_{i1}\ ..\ d_{ii}\ ..d_{iM} \\ . & . \\ . & . \\ c_{M1}\ ..\ c_{Mi}\ ..c_{MM} & d_{M1}\ ..\ d_{Mi}\ ..d_{MM} \end{bmatrix} \begin{bmatrix} \frac{\partial V_r}{\partial Q_x} \\ \frac{\partial V_{im}}{\partial Q_x} \end{bmatrix} = \begin{bmatrix} \frac{\partial Q_1}{\partial Q_x} \\ . \\ . \\ \frac{\partial Q_i}{\partial Q_x} \\ . \\ . \\ \frac{\partial Q_M}{\partial Q_x} \end{bmatrix} \tag{27}$$

where $\frac{\partial V_r}{\partial P_x} = \left[\frac{\partial V_{1,r}}{\partial P_x} .. \frac{\partial V_{i,r}}{\partial P_x} .. \frac{\partial V_{M,r}}{\partial P_x}\right]^T$, $\frac{\partial V_{im}}{\partial P_x} = \left[\frac{\partial V_{1,im}}{\partial P_x} .. \frac{\partial V_{i,im}}{\partial P_x} .. \frac{\partial V_{M,im}}{\partial P_x}\right]^T$, $\frac{\partial V_r}{\partial Q_x} = \left[\frac{\partial V_{1,r}}{\partial Q_x} .. \frac{\partial V_{i,r}}{\partial Q_x} .. \frac{\partial V_{M,r}}{\partial Q_x}\right]^T$ and $\frac{\partial V_{im}}{\partial Q_x} = \left[\frac{\partial V_{1,im}}{\partial Q_x} .. \frac{\partial V_{i,im}}{\partial Q_x} .. \frac{\partial V_{M,im}}{\partial Q_x}\right]^T$ are sensitivity vectors of real and imaginary parts of PQ voltages with respect to active and reactive power injection at node "x", respectively. "T" denotes for transpose. From Eq. (26), we can see that the matrix R consists of two submatrices A and B. The elements of A represent the coefficients associated with the sensitivities $\frac{\partial V_r}{\partial P_x}$ while the elements of B represent the coefficients associated with the sensitivities $\frac{\partial V_{im}}{\partial P_x}$. Similarly, we can see from Eq. (27) that the matrix T consists of two submatrices C and D. The elements of C represent the coefficients associated with the sensitivities $\frac{\partial V_r}{\partial Q_x}$ while the elements of D represent the coefficients associated with the sensitivities $\frac{\partial V_{im}}{\partial Q_x}$. In a general form, Eq. (26) and Eq. (27) can be expressed as:

$$[A\ \ B]\begin{bmatrix} \frac{\partial V_r}{\partial P_x} \\ \frac{\partial V_{im}}{\partial P_x} \end{bmatrix} = \left[\frac{\partial P}{\partial P_x}\right], \qquad [C\ \ D]\begin{bmatrix} \frac{\partial V_r}{\partial Q_x} \\ \frac{\partial V_{im}}{\partial Q_x} \end{bmatrix} = \left[\frac{\partial Q}{\partial Q_x}\right] \tag{28}$$

where $\frac{\partial P}{\partial P_x}$ and $\frac{\partial Q}{\partial Q_x}$ are sensitivity vectors of the active and reactive power of PQ nodes with respect to active and reactive power injection at node "x", respectively. The ijth element of the matrix can be found as (where both i and j ∊M):

$$a_{ij} = \begin{cases} G_{ii}V_{i,r} + B_{ii}V_{i,im} + \sum_{j\in N}(G_{ij}V_{j,r} - B_{ij}V_{j,im}) & j = i \\ G_{ij}V_{i,r} + B_{ij}V_{i,im} & otherwise \end{cases} \tag{29a}$$

$$b_{ij} = \begin{cases} G_{ii}V_{i,im} - B_{ii}V_{i,r} + \sum_{j\in N}\left(G_{ij}V_{j,im} + B_{ij}V_{j,r}\right) & j = i \\ G_{ij}V_{i,im} - B_{ij}V_{i,r} & otherwise \end{cases} \tag{29b}$$

$$c_{ij} = \begin{cases} G_{ii}V_{i,im} - B_{ii}V_{i,r} - \sum_{j\in N}\left(G_{ij}V_{j,im} + B_{ij}V_{j,r}\right) & j = i \\ G_{ij}V_{i,im} - B_{ij}V_{i,r} & otherwise \end{cases} \tag{29c}$$

$$d_{ij} = \begin{cases} -G_{ii}V_{i,r} - B_{ii}V_{i,im} + \sum_{j\in N}(G_{ij}V_{j,r} - B_{ij}V_{j,im}) & j = i \\ -G_{ij}V_{i,r} - B_{ij}V_{i,im} & otherwise \end{cases} \tag{29d}$$

To obtain the voltages V_{re} and V_{im}, one may use the information gathered from SCADA and nodal measurements. Alternatively, the linear power flow method developed in Cartesian coordinates in [25] can be used as:

$$\begin{bmatrix} G & -B \\ B & G \end{bmatrix}\begin{bmatrix} V_{re} \\ V_{im} \end{bmatrix} = \begin{bmatrix} I_p \\ I_q \end{bmatrix} \tag{30}$$

where I_p and I_q are vectors of load current parts. G and B are submatrices of a modified admittance matrix. These quantities and parameters are explained in detail in [25].

The power injection at a particular bus is independent of power injections of other buses. Therefore, the partial derivation $\frac{\partial P_i}{\partial P_x}$ and $\frac{\partial Q_i}{\partial Q_x}$ can be found as:

$$\frac{\partial P_i}{\partial P_x} = \frac{\partial Q_i}{\partial Q_x} = \begin{cases} 1 & i = x \\ 0 & otherwise \end{cases} \tag{31}$$

A similar system of equations is presented in Eq. (28) which can be used to find the partial derivatives with respect to other power injections (i.e. for power injections at node x∊N). Based on the expressions illustrated in Eq. (28–31), we can conclude the following points:

- A and B are developed to find the voltage sensitivities to active power injections while B and C are developed to find the voltage sensitivities to reactive power injections.
- Regardless of the node at which active or reactive power is supplied, the submatrices A, B, C, and D are identical. But, the only change has happened to the value of $\frac{\partial P_i}{\partial P_x}$ or $\frac{\partial Q_i}{\partial Q_x}$.
- It is clear that D = −A, and B = C for non-diagonal elements.
- The size of A, B, C, and D is M × M while the size of $\frac{\partial V_r}{\partial P_x}, \frac{\partial V_{im}}{\partial P_x}, \frac{\partial V_r}{\partial Q_x}, \frac{\partial V_{im}}{\partial Q_x}$, $\frac{\partial P}{\partial P_x}$ and $\frac{\partial Q}{\partial Q_x}$ is M × 1.

Once the partial derivations $\frac{\partial V_{i,r}}{\partial P_x}, \frac{\partial V_{i,im}}{\partial P_x}$ or $\frac{\partial V_{i,r}}{\partial Q_x}, \frac{\partial V_{i,im}}{\partial Q_x}$ are obtained, the voltage sensitivity coefficients can be easily computed. Since the voltage magnitude of the bus "i" $|V_i|$can be represented by $|V_i| = ({V_{i,r}}^2 + {V_{i,im}}^2)^{1/2}$, the voltage sensitivities of the bus "i" to power injections at node "x" can be found as:

$$\frac{\partial |V_i|}{\partial P_x} = \frac{1}{|V_i|}\left(V_{i,r}\frac{\partial V_{i,r}}{\partial P_x} + V_{i,im}\frac{\partial V_{i,im}}{dP_x}\right) \tag{32a}$$

$$\frac{\partial |V_i|}{\partial Q_x} = \frac{1}{|V_i|}\left(V_{i,r}\frac{\partial V_{i,r}}{\partial Q_x} + V_{i,im}\frac{\partial V_{i,im}}{dQ_x}\right) \tag{32b}$$

2.4. Effect of PV buses

The PV buses can also be considered in the sensitivity analysis. When real or reactive power is injected at any network node, some system voltages may change. Consequently, reactive power injections will be needed at PV nodes (i.e. ΔQ_g) to keep constant voltages at those buses. Any increment in real or reactive power injection at node "x" (i.e. Δu_x) will cause a change in the voltages of PV buses (i.e. ΔV_g). This can be represented as:

$$\frac{\partial |V_g|}{\partial u_x}\Delta u_x + \frac{\partial |V_g|}{\partial Q_g}\Delta Q_g = 0 \tag{33}$$

where $\frac{\partial |V_g|}{\partial u_x}$ is the vector of PV bus sensitivities with respect to real or reactive power injection at bus "x". $\frac{\partial |V_g|}{\partial Q_g}$is a matrix of PV bus sensitivities with respect to reactive power injection at these buses, and ΔQ_g denotes the vector of change for reactive power injections at PV buses. For any increment in real or reactive power injection at any node, the change in the node voltage sensitivities (including the impact of PV buses) can be expressed as:

$$\Delta |V| = \frac{\partial |V|}{\partial u_x}\Delta u_x + \frac{\partial |V|}{\partial Q_g}\Delta Q_g \tag{34}$$

where |V| is a vector of bus voltage magnitudes. $\frac{\partial |V|}{\partial u_x}$ is the vector of node voltage sensitivities with respect to active or reactive power injection at "x" node. $\frac{\partial |V|}{\partial Q_g}$is a matrix of the voltage sensitivities with respect to reactive power injection at PV buses. Dividing Eq. (34) by Δu_x and substituting $\frac{\Delta Q_g}{\Delta u_x}$ (Eq. (33)) into Eq. (34) we obtain:

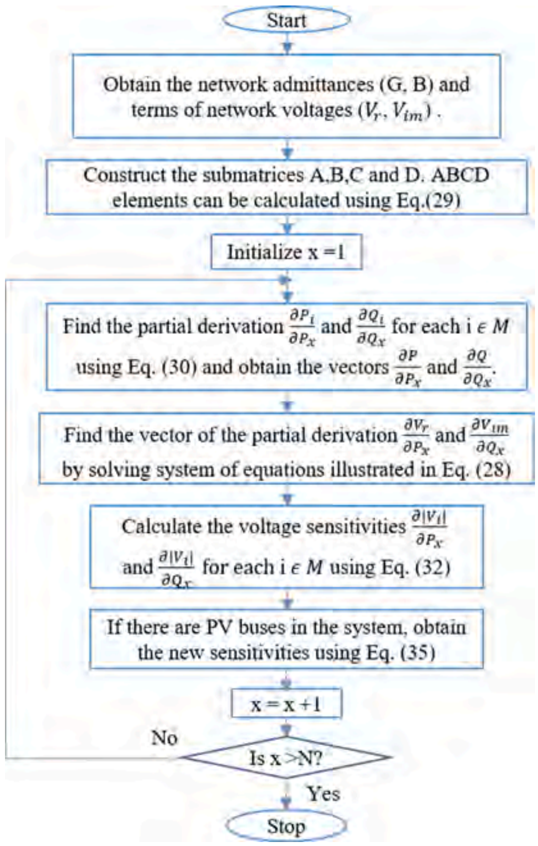

Fig. 2. A flowchart of the proposed voltage sensitivity method.

$$\frac{\Delta|V|}{\Delta u_x} = \frac{\partial|V|}{\partial u_x} - \frac{\partial|V|}{\partial Q_g}\left[\frac{\partial|V_g|}{\partial Q_g}\right]^{-1}\frac{\partial|V_g|}{\partial u_x} \tag{35}$$

From Eq. (35), it is clear that $\frac{\Delta|V|}{\Delta u_x}$ depends on the voltage sensitivity calculation illustrated in Eq. (32).

A flowchart of the proposed voltage sensitivity analysis is shown in Fig. 2.

Remark. It is generally known that the network parameters and measurements may have some errors. Additional analysis is required to consider the measurement-parameter error processing problem while calculating ABCD matrices. This is out of the scope of this work and can be regarded as in our future work.

2.5. Implementation of the proposed sensitivity method

It is proposed that the proposed voltage sensitivity approach is embedded in the distribution management system (DMS), along with other function softwares (like voltage control software) for online applications [26,27]. It is assumed that the DMS is a centralized substation system. The input of the DMS are SCADA measurements (or the pseudo measurements of load demands and DG production), and information of system admittance matrix (i.e. structural changes). Since system reconfiguration is done online, any structural change can easily be obtained. If there is no change in the system configuration, the previous system admittance matrix is used for voltage sensitivity analysis. Power networks are normally equipped with the SCADA system to transmit such information onward to the control center.

The DMS can involve function modules to solve any problem (like voltage control) to achieve different tasks such as sensitivity analysis. According to the proposed voltage sensitivity method, three function modules are needed: admittance module to obtain the admittance system matrix, ABCD module to calculate the values of the ABCD elements, and sensitivity module to calculate the voltage sensitivities.

Since the proposed voltage sensitivity analysis is mainly based on SCADA measurements (or the pseudo measurements of load demands and DG production), it is suggested that the uncertainties of DG outputs can be considered in sensitivity calculation. Based on such measures, the voltage sensitivities can be updated.

3. Simulation results

To validate the accuracy of the proposed sensitivity analysis method, 75-bus, 11 kV distribution system (Fig. 3) is considered in this work. It is assumed that the network hosts 22 DG units (each with a rating of 3MVA). No output powers are generated by DG units unless otherwise is mentioned. The system data and parameters can be found in [28]. The study network and the proposed algorithm are implemented in MATLAB environment. To improve the figures' readability, the first two digits of each bus number are omitted (i.e. 1175 will be 75).

3.1. Verification at base load condition

In this section, the voltage sensitivity coefficients ($d|V|/dP$and $d|V|/dQ$) are computed at base load conditions using the ABCD method. The coefficients are demonstrated in the matrices shown in Fig. 4. From both matrices, we can see that the sensitivity coefficients are positive. This demonstrates the fact that injecting active or reactive power into power network will definitely increase the voltage magnitudes. The higher values of the self-sensitivity coefficients compared with the cross-sensitivity coefficients also demonstrate the accuracy of the proposed method. Indeed, power injection at a specific bus can increase the voltage magnitude of that bus more than the voltage magnitudes of other buses. This is because system impedances mainly affect the power flow. Furthermore, it is obvious that the sensitivity coefficients of a particular node as a result of power injections into the same feeder are significantly greater than the coefficients as a result of power injections into other feeders. Fig. 5 shows sensitivity matrices obtained using the inverse of J (J^{-1}). By comparing the matrices obtained using the ABCD method with the ones obtained using J^{-1}, it can be noticed that the matrices are very close to each other, rather they appear to be the same matrices.

It is worth noting that accurate sensitivity analysis (without approximation) should involve the variation of load powers with system voltages, the actual network impedances and the actual network operation point. However, this information is not well-known in practice and some approximations are required [29]. The inverse of the J method can be considered to overcome this issue if the sensitivity coefficients are updated with any change in the system state. Thus, in this study, the proposed sensitivity method's results are compared to the inverse of J's results.

Power flow calculations point out that V_{75} is the lowest voltage among network buses. The $d|V|/dP$ and $d|V|/dQ$ coefficients of bus 75 obtained via the ABCD method are illustrated in Fig. 6. Since the sensitivities to power injections into other feeders are very small, they aren't included in this figure. From Fig. 6, it is clear that the sensitivity coefficients of bus 75 due to power injections Q_{75} or P_{75} are higher than the sensitivity coefficients to power injections into other buses. Moreover, the sensitivity coefficients of bus 75 due to power injections Q_{66} or P_{66} are the second largest among all the sensitivity coefficients. This is demonstrated by the formulas for the submatrices A, B, C, and D in Eq. (29) which show that their elements are directly dependent on the coupling admittances between system nodes. Since bus 66 is the nearest node to bus 75, the sensitivity coefficients of bus 75 to power injections into node 66 is the largest cross-sensitivity. Verification of the self-sensitivity coefficients and the largest cross-sensitivity coefficients of bus 75 validates the proposed method for sensitivity analysis.

The verification of voltage self-sensitivity coefficients and the largest cross-sensitivity coefficients of node 75 (which has the lowest voltage) at

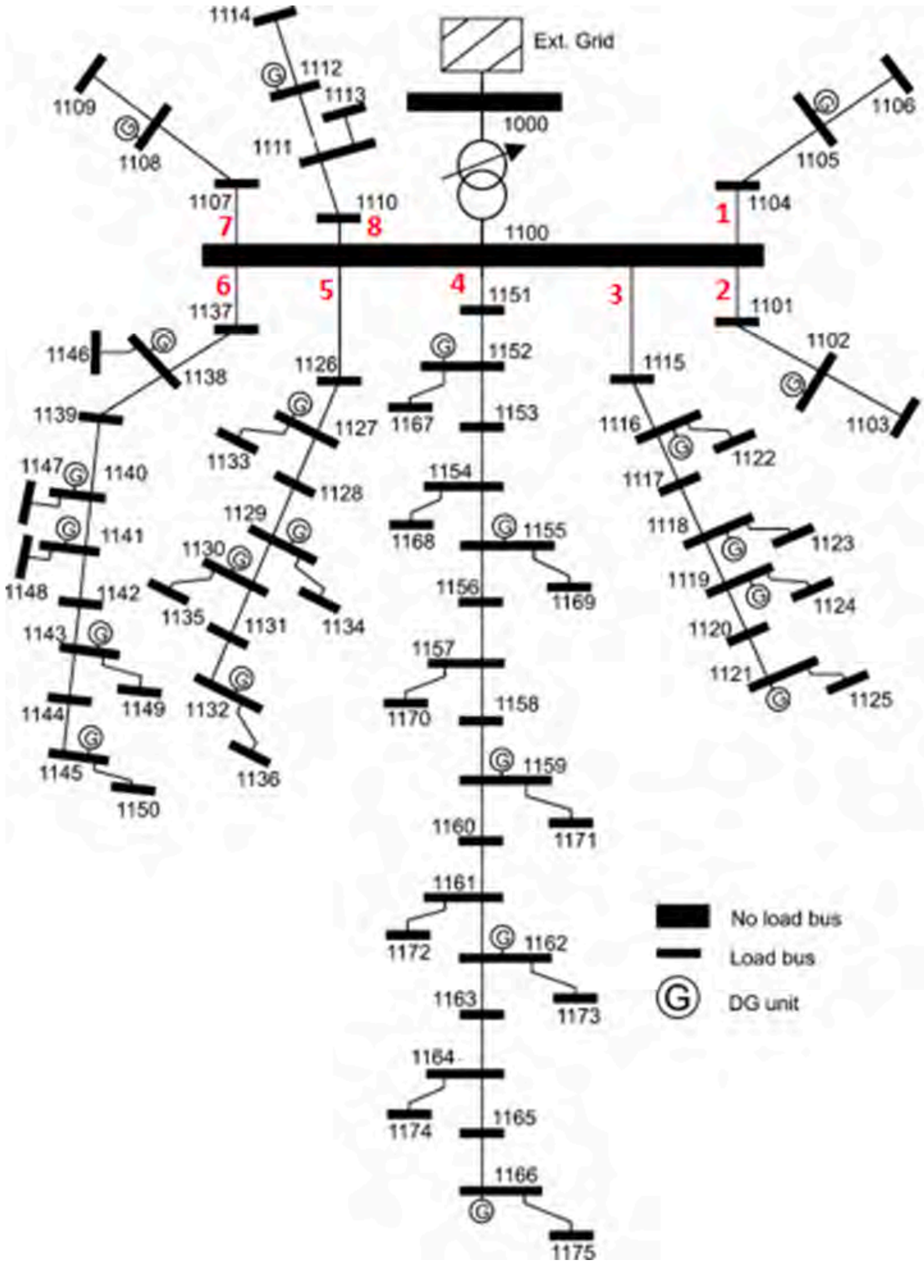

Fig. 3. Topology of the test system [7].

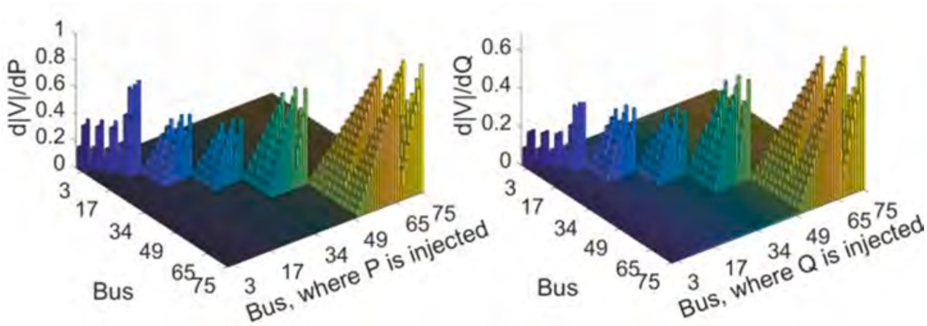

Fig. 4. $d|V|/dP$and $d|V|/dQ$ sensitivity coefficients at base load condition using ABCD method.

base load condition is also investigated, in detail. This verification is investigated by assessing the performance of the sensitivity coefficients in voltage prediction. The predicted voltages are obtained by multiplying the sensitivity with the amount of the active or reactive power

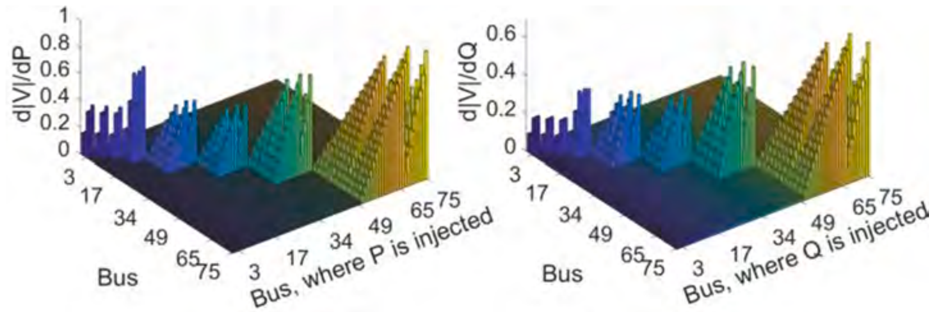

Fig. 5. $d|V|/dP$and $d|V|/dQ$ sensitivity coefficients at base load condition using inverse of J.

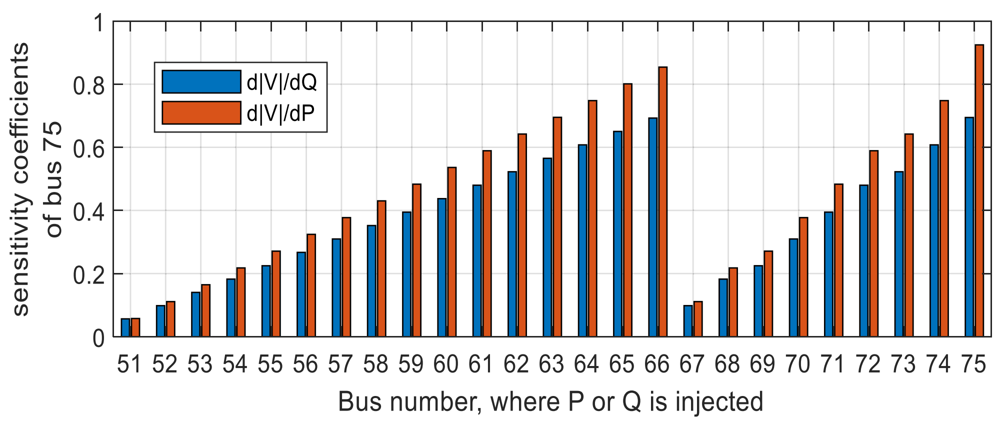

Fig. 6. Voltage Sensitivity coefficients of bus 75 for active and reactive power injections at base load condition.

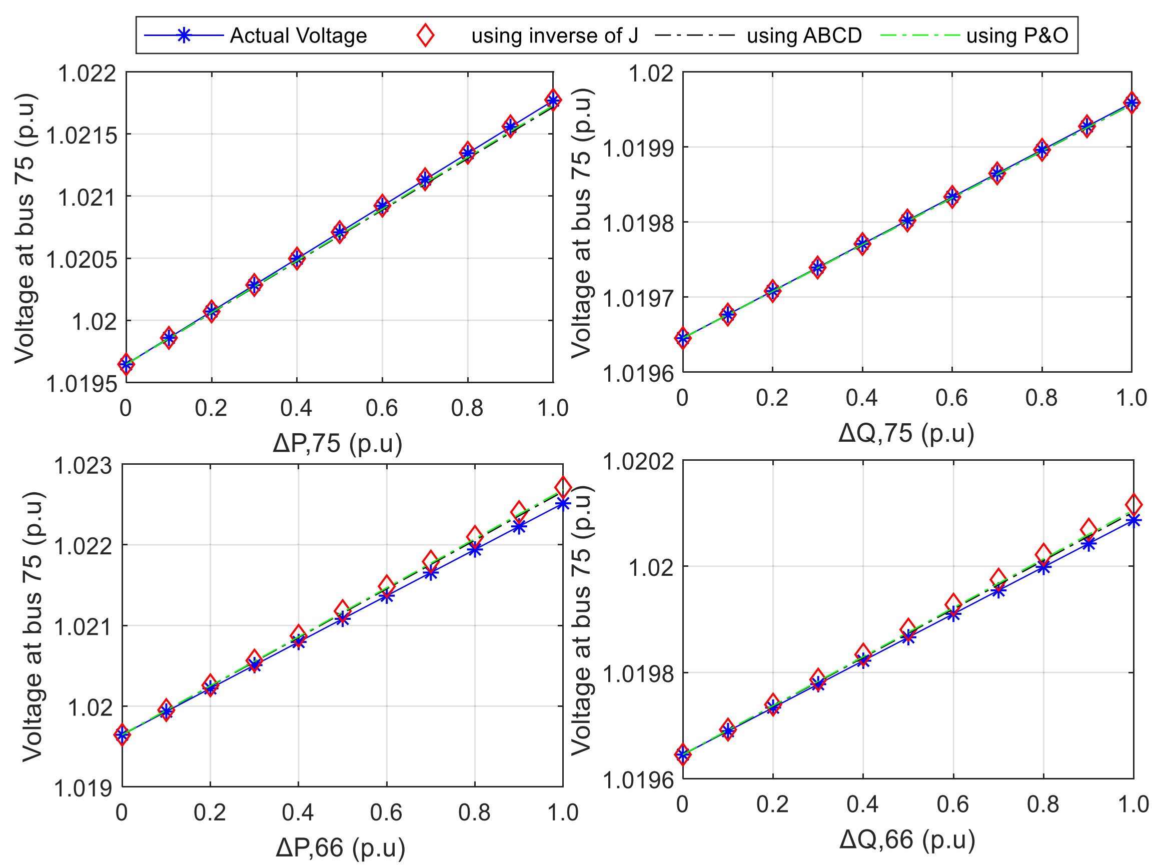

Fig. 7. A comparison between the actual and the predicted voltages of bus 75 due to active and reactive power reductions at buses (75 and 66).

injection (or reduction). A comparison of the predicted voltages (obtained using the proposed method, inverse of J method, and perturb-and-observe (P&O) method presented in [16]) with the actual voltages are then done. The actual voltages are obtained using power flow calculations. Power flow was performed after each step of power reduction or power injection, at a time.

To validate the self-sensitivity coefficients of bus 75 (obtained at base case conditions), 1.0p.u of its own active (or reactive) load power is deducted at a step of 0.1p.u. However, to verify the largest cross-sensitivity coefficients of bus 75, 1.0p.u of bus 66 active (or reactive) load power with step 0.1p.u are deducted. Fig. 7 shows a comparison between the predicted and the actual voltages of bus 75 due to active (or reactive) power reductions at bus 75 (or bus 66). It shows that the errors between the predicted voltages using the ABCD method and those obtained using other methods are very small. It is also clear that as the load power reduction increases, the errors increase. Moreover, the errors due to the self-sensitivities are smaller than the ones due to the cross-sensitivities. Besides, the errors due to $d|V|/dQ$sensitivities are smaller than the ones due to $d|V|/dP$ sensitivities. These characteristics coincide with the results obtained using the inverse of J.

The errors between the actual voltages and the predicted ones of node 75 are summarized in Table 1. The errors in Table I are very small (in the order of 10^{-4}–10^{-9}). These errors are common in the sensitivity coefficients used for prediction and control in nonlinear power networks. This provides a rigorous justification for the accuracy of the ABCD method.

3.2. Verification at different loading conditions

The self-sensitivity and the largest cross-sensitivity of bus 75 are also chosen in order to verify the voltage sensitivity coefficients under varied operating conditions. As section 4.1 of simulation results, 1.0p.u of the active (or reactive) load powers with step 0.1p.u are deducted. The only difference is that as the operating condition changes, the sensitivity coefficients will also be updated. Accordingly, the predicted voltages are obtained by taking the summation of the individual multiplication (multiplication of the sensitivity with the amount of power change for each step). Table 2 shows the errors between the sensitivities obtained using ABCD method and those obtained via inverse of the J method.

A comparison between the actual and the predicted voltages of bus 75 due to active (or reactive) power reductions at bus 75 (or bus 66) is shown in Fig. 8. The errors in the predicted voltage using the proposed method are summarized in Table 3. It can be concluded from these results that the errors in the sensitivities and the predicted voltages are also very small.

By comparing the predicted voltages using the ABCD method shown in Table I with the ones shown in Table 3, it can be seen that the errors are smaller in case of updating the sensitivities in each step (i.e. with the changes in system operating conditions). This demonstrates the necessity for updating the sensitivities in voltage control.

Table 2
The errors in the updated sensitivities.

ΔQ Or ΔP (p.u)	Self-sensitivity to:		Cross-sensitivity to:	
	$d\|V_{75}\|/dP_{75}$	$d\|V_{75}\|/dQ_{75}$	$d\|V_{75}\|/dP_{66}$	$d\|V_{75}\|/dQ_{66}$
0	2.597×10^{-3}	5.444×10^{-4}	1.473×10^{-3}	1.983×10^{-3}
0.1	2.565×10^{-3}	5.383×10^{-4}	1.445×10^{-3}	1.979×10^{-3}
0.2	2.533×10^{-3}	5.323×10^{-4}	1.416×10^{-3}	1.976×10^{-3}
0.3	2.502×10^{-3}	5.263×10^{-4}	1.387×10^{-3}	1.973×10^{-3}
0.4	2.470×10^{-3}	5.203×10^{-4}	1.358×10^{-3}	1.969×10^{-3}
0.5	2.438×10^{-3}	5.143×10^{-4}	1.330×10^{-3}	1.966×10^{-3}
0.6	2.407×10^{-3}	5.083×10^{-4}	1.302×10^{-3}	1.962×10^{-3}
0.7	2.375×10^{-3}	5.023×10^{-4}	1.273×10^{-3}	1.959×10^{-3}
0.8	2.477×10^{-3}	4.963×10^{-4}	1.245×10^{-3}	1.955×10^{-3}
0.9	2.312×10^{-3}	4.903×10^{-4}	1.217×10^{-3}	1.952×10^{-3}
1.0	2.281×10^{-3}	4.843×10^{-4}	1.189×10^{-3}	1.949×10^{-3}

The results show that the ABCD model can overcome the inverse of the J technique in the case of cross-sensitivities. This is because the line impedances can affect the results of power flow (i.e. Jacobian matrix). It is worth noting that it is hard to update the sensitivities via the inverse of the J method in a practical power system. If the sensitivity coefficients obtained via the inverse of J are not updated with the operating condition changes, it cannot be considered as an accurate method. However, to show the accuracy of the proposed method, the updated sensitivities via the ABCD model are compared with the updated sensitivities via the inverse of the J method.

3.3. Verification during DG power influences

To verify voltage sensitivities under the effect of DG power, it is assumed that DG units can deliver reactive power of up to 3 MVAR and active power curtailments of up to 3 MW. This amount of power supply or curtailment represents a high value and can cause distinct variations in network voltage as well as voltage sensitivities. Thus, the verification during DG power influences can provide a good demonstration of the accuracy of the proposed sensitivity method. Moreover, different voltages far away from the DG unit are considered for verification. 3.0 MW curtailment (or MVAR injection) by DG_{59} with step 0.3 MW (or 0.3MVAR) are done. The sensitivity coefficients will be updated during each step. Accordingly, the predicted voltages are obtained by taking the summation of the

individual multiplication (multiplication of the sensitivity with the amount of power change for each step). A comparison between the actual and the predicted voltages due to reactive power injections (or active power curtailments by DG_{59}) can be found in Fig. 9. The predicted voltages are done using the ABCD method and the inverse of the J method. Tables 4 and 5 summarise the errors in both the sensitivities and the predicted voltages, respectively. Although large power

Table 1
The Errors In The Voltage V_{75} At Load Base Condition.

ΔQ Or ΔP (p.u)	Self-sensitivity to:				Cross-sensitivity to:			
	P_{75}		Q_{75}		P_{66}		Q_{66}	
	Using J^{-1}	Using ABCD	Using J^{-1}	Using ABCD	Using J^{-1}	Using ABCD	Using J^{-1}	Using ABCD
0	0	0	0	0	0	0	0	0
0.1	5.613×10^{-8}	5.648×10^{-6}	1.789×10^{-9}	2.373×10^{-7}	1.837×10^{-5}	1.340×10^{-5}	2.806×10^{-6}	1.468×10^{-6}
0.2	2.243×10^{-7}	1.118×10^{-5}	7.156×10^{-9}	4.712×10^{-7}	3.694×10^{-5}	2.700×10^{-5}	5.620×10^{-6}	2.944×10^{-6}
0.3	5.045×10^{-7}	1.660×10^{-5}	1.609×10^{-8}	7.014×10^{-7}	5.571×10^{-5}	4.081×10^{-5}	8.440×10^{-6}	4.426×10^{-6}
0.4	8.963×10^{-7}	2.190×10^{-5}	2.861×10^{-8}	9.280×10^{-7}	7.468×10^{-5}	5.481×10^{-5}	1.126×10^{-5}	5.914×10^{-6}
0.5	1.399×10^{-6}	2.710×10^{-5}	4.471×10^{-8}	1.151×10^{-6}	9.385×10^{-5}	6.902×10^{-5}	1.409×10^{-5}	7.410×10^{-6}
0.6	2.014×10^{-6}	3.217×10^{-5}	6.437×10^{-8}	1.370×10^{-6}	1.132×10^{-4}	8.342×10^{-5}	1.693×10^{-5}	8.912×10^{-6}
0.7	2.739×10^{-6}	3.714×10^{-5}	8.761×10^{-8}	1.586×10^{-6}	1.327×10^{-4}	9.803×10^{-5}	1.978×10^{-5}	1.042×10^{-5}
0.8	3.575×10^{-6}	4.199×10^{-5}	1.144×10^{-7}	1.798×10^{-6}	1.525×10^{-4}	1.128×10^{-4}	2.263×10^{-5}	1.193×10^{-5}
0.9	4.522×10^{-6}	4.673×10^{-5}	1.448×10^{-7}	2.007×10^{-6}	1.724×10^{-4}	1.278×10^{-4}	2.549×10^{-5}	1.345×10^{-5}
1.0	5.579×10^{-6}	5.136×10^{-5}	1.787×10^{-7}	2.212×10^{-6}	1.926×10^{-4}	1.430×10^{-4}	2.836×10^{-5}	1.498×10^{-5}

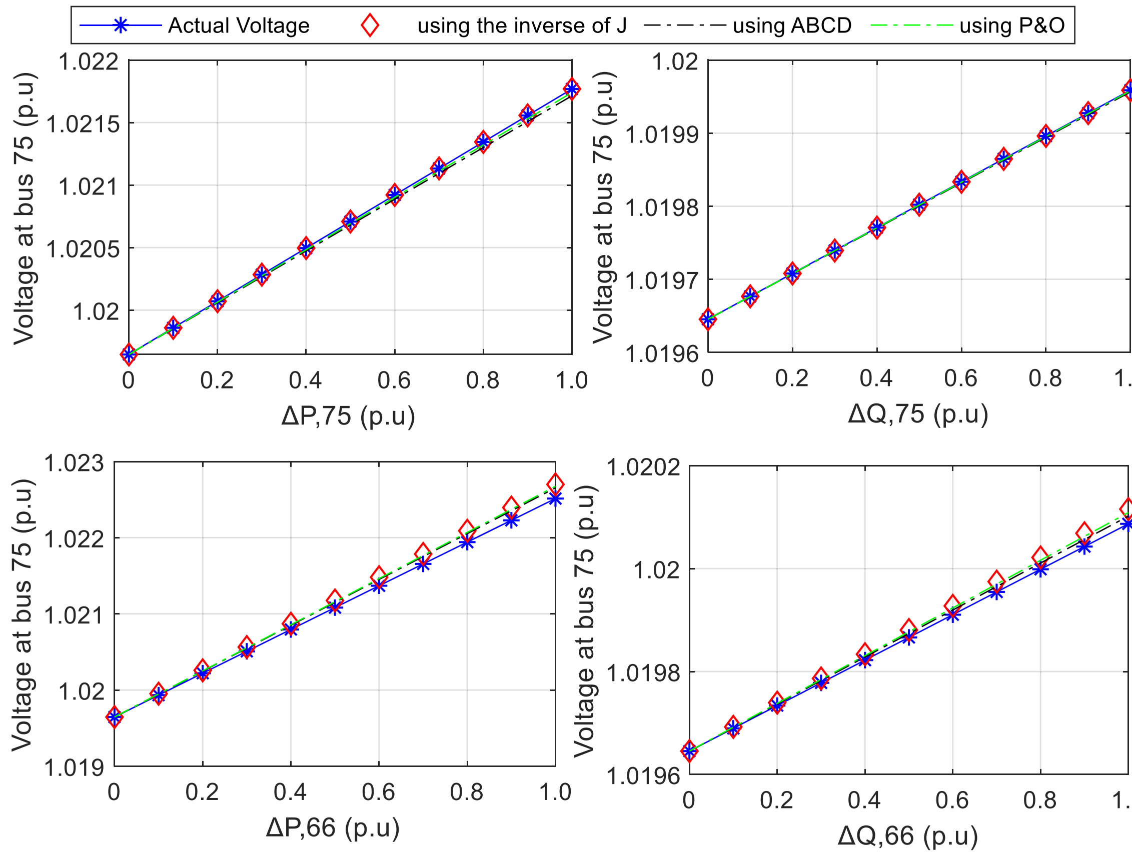

Fig. 8. A comparison between the actual and the predicted voltages of bus 75 due to active and reactive power reductions at buses (75 and 66).

Table 3
The errors in V_{75} due to the updated sensitivities.

ΔQ Or ΔP (p.u)	Self-sensitivity due to:		Cross-sensitivity due to:	
	P_{75}	Q_{75}	P_{66}	Q_{66}
0	0	0	0	0
0.1	5.648×10^{-7}	2.373×10^{-8}	1.340×10^{-6}	1.468×10^{-7}
0.2	1.122×10^{-6}	4.721×10^{-8}	2.686×10^{-6}	2.938×10^{-7}
0.3	1.672×10^{-6}	7.042×10^{-8}	4.039×10^{-6}	4.409×10^{-7}
0.4	2.216×10^{-6}	9.336×10^{-8}	5.398×10^{-6}	5.882×10^{-7}
0.5	2.752×10^{-6}	1.160×10^{-7}	6.763×10^{-6}	7.355×10^{-7}
0.6	3.281×10^{-6}	1.384×10^{-7}	8.134×10^{-6}	8.830×10^{-7}
0.7	3.803×10^{-6}	1.606×10^{-7}	9.511×10^{-6}	1.030×10^{-6}
0.8	4.317×10^{-6}	1.824×10^{-7}	1.089×10^{-5}	1.178×10^{-6}
0.9	4.825×10^{-6}	2.041×10^{-7}	1.228×10^{-5}	1.326×10^{-6}
1.0	5.325×10^{-6}	2.254×10^{-7}	1.367×10^{-5}	1.474×10^{-6}

injections are considered in this case and considering different voltages far away from the DG unit, it is clear that the errors are still very small. The errors in both the sensitivities and the predicted voltages are in order of 10^{-4} (or 10^{-3}) and 10^{-6} (or 10^{-5}), respectively. This also provides a rigorous justification for the accuracy of ABCD method. It is also clear that the errors in the sensitivities of nodes in the same feeder (the feeder owning the DG) are higher than the errors in the sensitivities of nodes of other feeders.

3.4. The performance assessment in online voltage control

In this section, a multi-step optimization problem stated in [29] is employed for voltage control to evaluate the performance of the suggested sensitivity approach through transient simulation simulations. This problem is formulated as:

$$min\sum_{k=0}^{n-1}\|\Delta Q(t+k)\|_F^2+\|\varepsilon\|_G^2 \tag{36}$$

Subjected to:

$$\begin{aligned}
&-\varepsilon_1 A+V_i^{min}\leq V_i(t+k)\leq V_i^{max}+\varepsilon_2 A\\
&V_i(t+k)=V_i(t+k-1)+\frac{\partial|V|}{\partial Q}\Delta Q_i(t+k)\\
&\Delta Q^{min}\leq\Delta Q(t+k)\leq\Delta Q^{max}\\
&Q^{min}\leq Q(t+k)\leq Q^{max}
\end{aligned}\tag{37}$$

where ΔQ represents the vector of changes in reactive power injection by DG units. $\varepsilon=[\varepsilon_1,\varepsilon_2]^{\mathrm{T}}$ is the vector of slack variables used to relax the voltage constraints. n is the number of prediction steps. ‘T’ represents array transposition. ‘A’ denotes a unitary vector. F and G are weight matrices used to penalize the reactive power injections and the slack variables, respectively. $V_i(t+k)$ is the predicted voltage magnitude of bus i. $V_i(t+k-1)$ is the previous voltage magnitude. $\frac{\partial|V|}{\partial Q}$ is the sensitivity matrix of bus voltage magnitudes with respect to reactive power injection by DG units. Q denotes a vector of the reactive power injected by DG units. It is clear from Eqs. (36) and (37) that the voltages are evaluated inside the optimization problem using the sensitivities $\frac{\partial|V|}{\partial Q}$. These sensitivities are obtained using Eqs. (27) and (31), and updated after each step. It is worth noting that the sensitivities can be updated due to changing the submatrices C and D.

In this scenario, all DG units are installed in the grid. No more than 0.3 MVAR of reactive power is allowed to be injected by each DG unit for each control action. The cost of using the slack values is higher than using the reactive power by 800 times. The acceptable limits for voltages are assumed to be [0.98, 1.04] p.u. This work randomly assumes that the control actions take place every 10 s, and this period can be replaced by

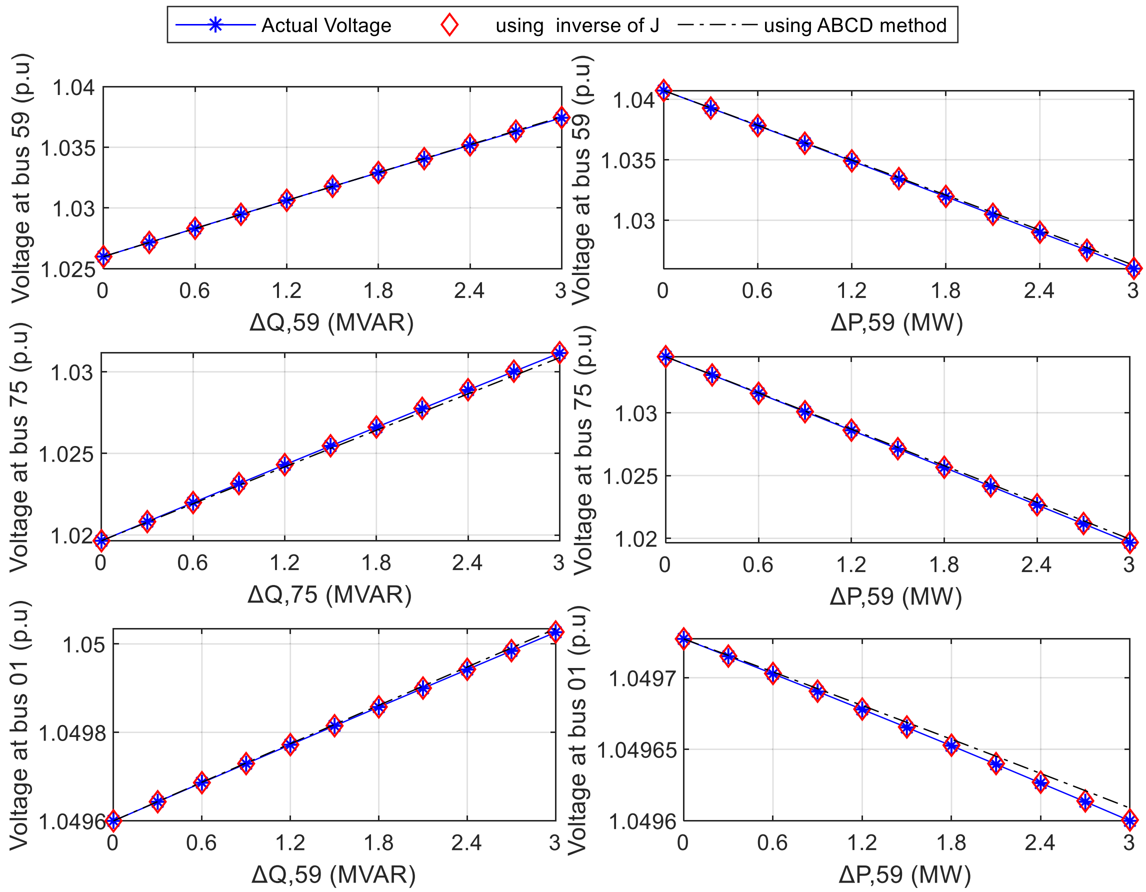

Fig. 9. A comparison between the actual and the predicted voltages of some buses due to reactive power injection (active power curtailment) by.DG_{59}

Table 4
The errors in the sensitivities during DG influence.

ΔQ (MVAR) Or ΔP (MW)	$\frac{d\|V_{59}\|}{dQ_{59}}$ ($\times 10^{-3}$)	$\frac{d\|V_{75}\|}{dQ_{59}}$ ($\times 10^{-3}$)	$\frac{d\|V_{01}\|}{dQ_{59}}$ ($\times 10^{-4}$)	$\frac{d\|V_{59}\|}{dP_{59}}$ ($\times 10^{-3}$)	$\frac{d\|V_{75}\|}{dP_{59}}$ ($\times 10^{-3}$)	$\frac{d\|V_{01}\|}{dP_{59}}$ ($\times 10^{-4}$)
0	3.191	12.209	1.407	3.253	5.252	1.767
0.3	2.051	11.721	1.652	4.357	5.960	1.986
0.6	0.920	11.240	1.867	5.473	6.679	2.208
0.9	0.199	10.768	2.080	6.601	7.409	2.432
1.2	1.309	10.304	2.291	7.743	8.151	2.659
1.5	2.409	9.847	2.500	8.898	8.904	2.889
1.8	3.500	9.398	2.707	10.066	9.669	3.121
2.1	4.581	8.957	2.912	11.247	10.446	3.355
2.4	5.652	8.524	3.116	12.443	11.235	3.593
2.7	6.715	8.097	3.317	13.652	12.036	3.833
3.0	7.768	7.678	3.517	14.876	12.850	4.076

Table 5
The errors in different voltages during DG influence.

ΔQ (MVAR) Or ΔP (MW)	power injection Q_{59}			power injection P_{59}		
	V_{59} ($\times 10^{-6}$)	V_{75} ($\times 10^{-5}$)	V_{01} ($\times 10^{-6}$)	V_{59} ($\times 10^{-5}$)	V_{75} ($\times 10^{-5}$)	V_{01} ($\times 10^{-6}$)
0	0	0	0	0	0	0
0.3	7.014	3.353	0.4729	1.194	1.784	0.5656
0.6	10.70	6.558	1.007	2.713	3.783	1.194
0.9	11.11	9.616	1.602	3.787	5.999	1.887
1.2	8.276	12.53	2.258	6.746	8.436	2.644
1.5	2.237	15.30	2.974	9.270	11.10	3.467
1.8	6.965	17.94	3.748	12.13	13.99	4.356
2.1	19.29	20.44	4.582	15.36	17.12	5.312
2.4	34.71	22.81	5.473	18.93	20.49	6.336
2.7	53.18	25.05	6.423	20.46	24.11	7.428
3.0	74.66	27.16	7.429	27.18	27.97	8.589

any other period (i.e. 1 s, 5 s, … etc.). It is worth mentioning that this duration is chosen to consider the calculation time, measurement collection time, the time required to transmit the new control set-points of DG units, and the dead time required to avoid making decisions based on measurements taken during transients. The measurements in this work is the reactive power output by DG units.

It is also assumed that the loads are operated at their maximum to create undervoltage problem. Thus, reactive power outputs by DG units are the controls for voltage regulations. The optimization software LINGO and MATLAB are both used to investigate the results.

Some of the network voltages and reactive power outputs by DG units are shown in Figs. 10 and 11, respectively. It is clear that the controller, with the aid of the proposed sensitivity analysis, was able to regulate the voltages gradually. Fig. 11 shows that some units (i.e. DG at bus 05) are operated at lower power, and this is to prevent any violation of the upper voltage near bus 05.

Fig. 11 also shows that the DG installed at bus 66 has to participate in power injection more than other units. This is because V66 is the most problematic voltage. It can also be seen that the nearest DG units to the region of the violated voltages have to participate more than the other units. The total compensated amount of reactive powers and the voltage profiles of two cases, namely, Vuncontrolled and Vcontrolled, are illustrated in Figs. 12 and 13, respectively. Based on the simulation results, it is concluded that the proposed sensitivity analysis is suitable for online applications and has a logical performance for managing reactive powers among DG units.

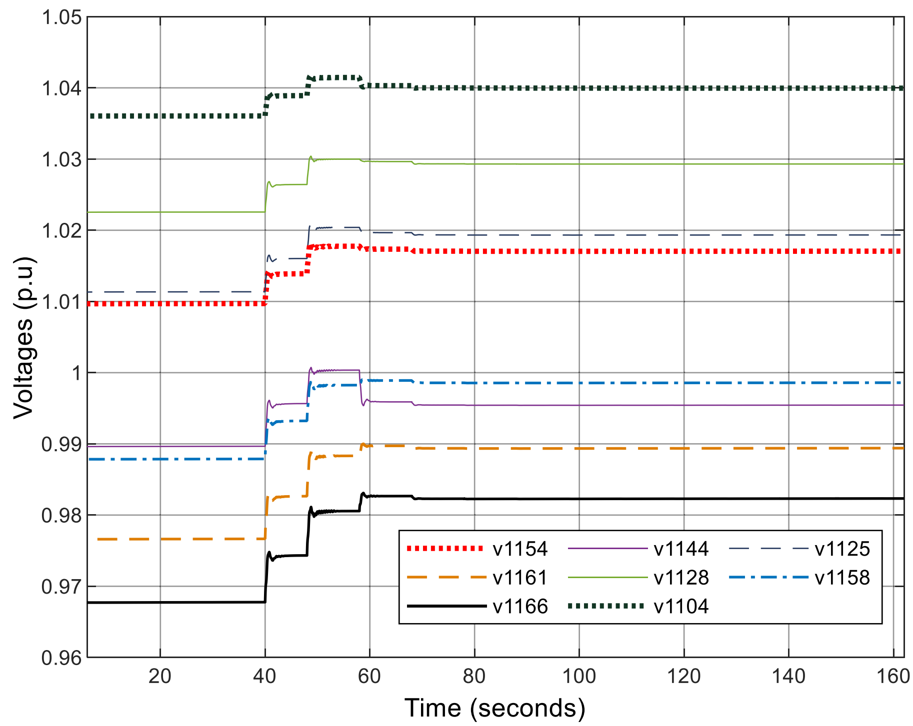

Fig. 10. Some bus voltages during transient analysis.

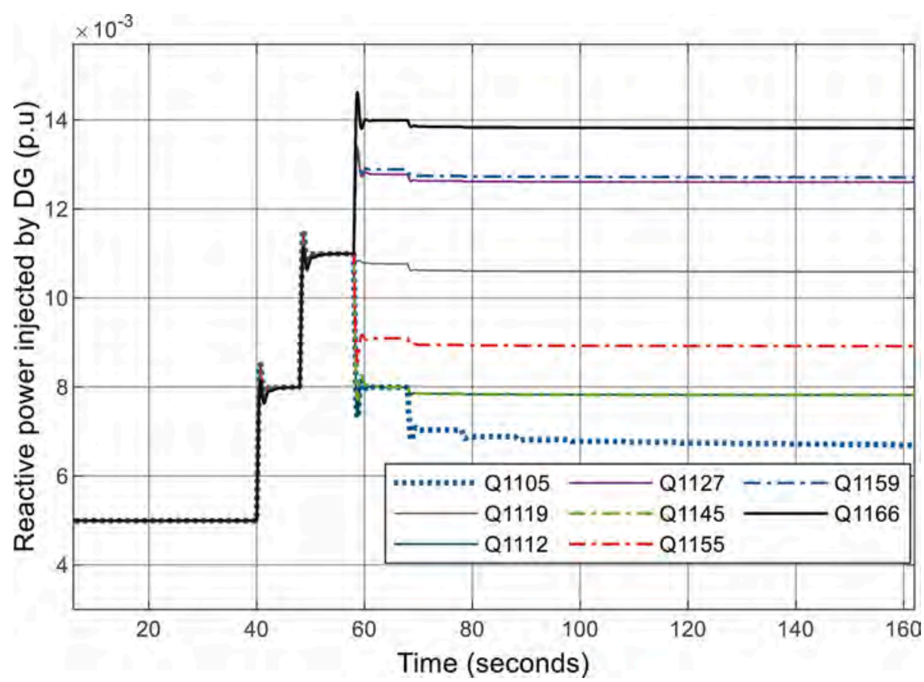

Fig. 11. Some reactive power outputs by DG units.

3.5. *Dynamic simulation studies*

The actual performance of the proposed sensitivity analysis through dynamic simulation is assessed in this section. The system is operated at a condition where there is no violation in the voltage. It is assumed that DG_{75} has the power profile (for 15 s) shown in Fig. 14. It is also assumed that the lower voltage limit is 0.98p.u. Control action takes place when the voltage exceeds the lower limit. New settings are then held for 4 s. The obtained voltage at bus 66 (where the DG is connected) and the set points of the DG unit are presented in Fig. 14. The results show that both sensitivity approaches provide almost the same amount of reactive power and, therefore, nearly the same voltage profile.

3.6. *Calculation speed*

To show the calculation speed of the proposed approach, the scenario presented in section 4.1 is selected for this purpose. The results show that the execution time of the proposed method is 0.41 s. In

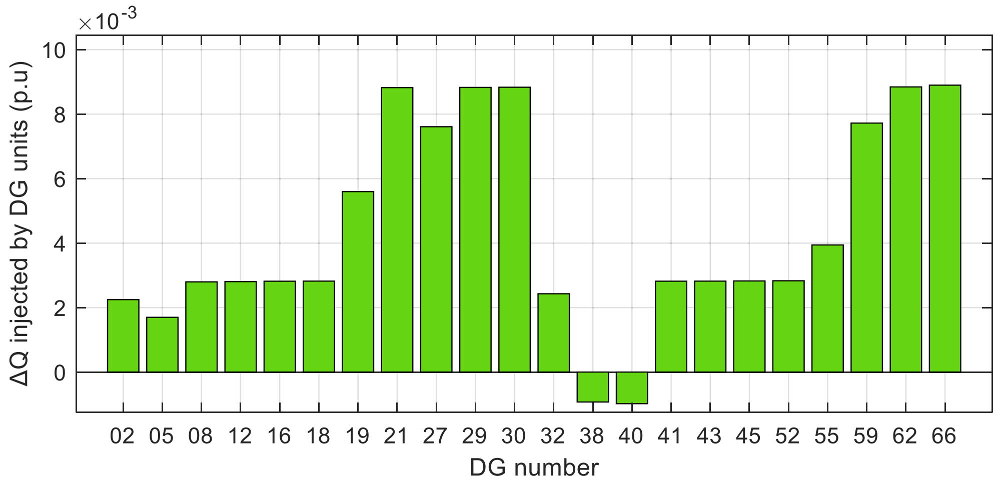

Fig. 12. The total change in reactive power injections by DG units.

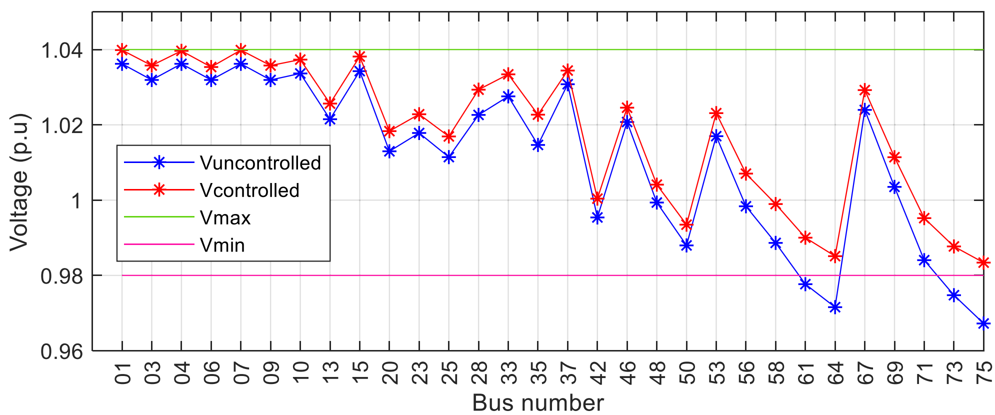

Fig. 13. Voltage profile during the two cases: controlled and uncontrolled.

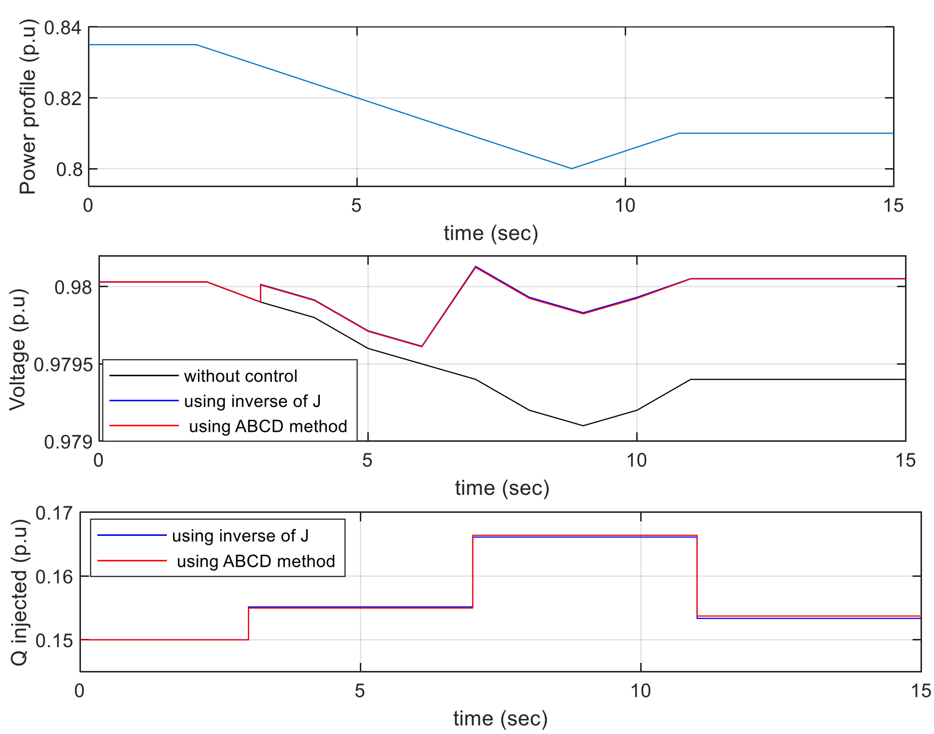

Fig. 14. Dynamic simulation studies.

contrast, 0.64 s is required to execute the Jacobian-based sensitivity analysis method. It is clear that the proposed method was able to reduce the computation time by 38.91%. This reduction in the calculation time will be much higher in cases of practical networks or in the context of optimization techniques. It is well-known commercially that modern monitoring units have a fast sampling rate and efficient communication links [29]. It is assumed that the measurements can be collected every 0.2 s. Assuming that an extra 1 s is also required to filter out the noises, the proposed method will still be computationally efficient. In the worse cases, the ABCD model is still more computationally efficient than the inverse of the J method due to the convergence problems. The results are obtained using an i7–8850 CPU@ 2.60 GHz laptop.

3.7. The performance under changes in network impedances

The network parameters are likely to differ due to the variations in ambient conditions, loading conditions, or even inaccurate manufacturing data. This can continuously cause variations in voltage sensitivity coefficients. An advantage of the proposed sensitivity method is that it can use SCADA measurements to continuously estimate network parameters and, hence, mitigate the inaccuracy in the sensitivity analysis. For example, the line impedance can be estimated using the SCADA measurements as:

$$R = \frac{\left(P_s + P_r - \left(V_s^2 + V_r^2\right)\frac{G}{2}\right)}{I^2} \tag{38}$$

$$X = \frac{\left(Q_s + Q_r + \left(V_s^2 + V_r^2\right)\frac{B}{2}\right)}{I^2} \tag{39}$$

where G and B are shunt conductance and susceptance, respectively. R and X are line resistance and reactance, respectively. P_s, Q_s and P_r, Q_r are the real and reactive power at sending and receiving end, respectively. V_s and V_r are the voltage at sending and receiving end, respectively. I is the line current. To reduce the noise impact from SCADA measurement, multiple points of time can be used to average the results.

To demonstrate the robustness of the proposed method in the presence of such parameter errors, a numerical test for obtaining voltage sensitivities is performed under changes in network impedances. These changes consider line impedances to increase (or decrease) by 15% and 30% of their nominal data. The ABCD model uses the model presented in Eq. (38) and Eq. (39) to estimate the line impedances while the classical inverse of J method is not able to estimate the impedances continuously. Indeed for the inverse of the J method, it is not possible to continuously estimate the network parameters without using sensing or measuring devices [30]. It is worth noting that there are many advanced estimation techniques can be used for network parameters [30,31]. However, such these techniques require an extra calculation effort for either the proposed method or the inverse of J method.

A comparison between the actual and the predicted voltages of bus 75 due to active and reactive power injections at buses (75 and 66) is shown in Fig. 15. The predicted voltage is calculated by multiplying the sensitivity with the amount of power injection at the corresponding bus. This test assumes that the amount of power injection at the corresponding bus is 0.5 MW (or 0.5MVAR). The actual voltages are obtained by performing power flow calculations after each step of impedance change, at a time. It is clear from Fig. 15 that the predicted voltages obtained using the inverse of J are maintained constant. In contrast, the predicted voltages obtained via the ABCD model can continuously vary with impedance change and relatively match the actual voltages (the relative error percent around 0.098% – 0.22%). This means that the ABCD model performs better than the inverse of J under impedance changes and achieve accurate results, compared with the inverse of J. This is because the ABCD model, with the aid of SCADA measurements, can accurately update the line impedances while the classical Jacobian method cannot capture the operating condition changes. It is also clear that the predicted voltages obtained using the ABCD model are higher

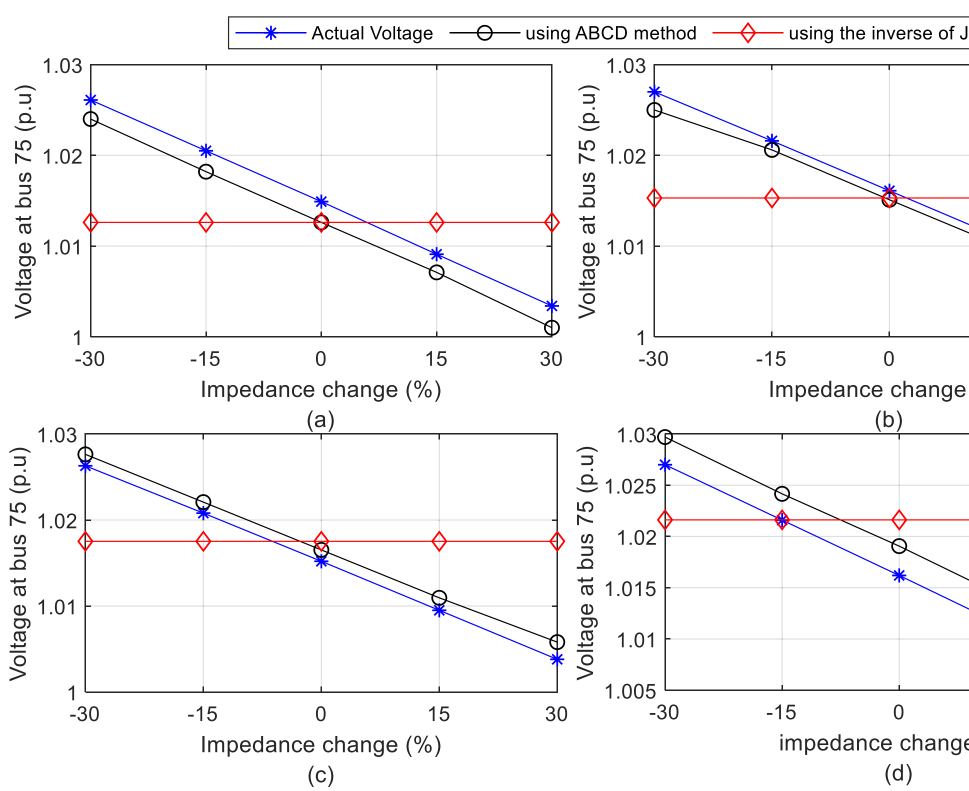

Fig. 15. A comparison between the actual and the predicted voltages of bus 75 due to (a) active and (b) reactive power injection at bus 75, and (c) active and (d) reactive power injection at bus 66.

(or smaller) than the actual voltages in case of power injection at bus 75 (or 66). This is due to the fact that the proposed method overestimates the self-sensitivity coefficients and underestimates the cross-sensitivity coefficients.

As a result, the impedance estimation using SCADA measurements can verify the parameter errors and enhance the accuracy of the voltage sensitivity analysis. This means that the proposed sensitivity method can be considered robust enough when applied to real systems.

4. Conclusions

This study developed a new and fast sensitivity approach (i.e. ABCD) to find the voltage sensitivity coefficients in smart grids. The proposed approach has been validated on a radial distribution network including several DG units under different scenarios. The numerical values of the sensitivity coefficients and the comparison with the results of other techniques validate the accuracy of the proposed method. The comparison is done in terms of the accuracy and the computation time. The results show that the errors in the values of sensitivity coefficients or in voltage prediction are very small, demonstrating that the ABCD model has almost the same level of accuracy as the inverse of the J method. The computational benefit of the proposed approach was investigated by showing the relevant improvements compared with the inverse of J. It is clear that the proposed method was able to reduce the computation time by 38.91%. The results also showed that the proposed method could successfully account for any change in the operating conditions.

Moreover, the proposed method is validated using dynamic and transient analysis. This also demonstrates the potential application of the proposed sensitivity method in online applications. The results also showed that the proposed method is able to manage the voltages by accurate dispatching of control variables efficiently.

As part of our future-work, we plan to extend the ABCD model to consider the measurement-parameter error processing problem and include the probabilistic analysis in the modeling.

CRediT authorship contribution statement

Khaled Alzaareer: Conceptualization, Methodology, Software, Writing – original draft, Writing – review & editing. **Maarouf Saad:** Conceptualization, Validation, Supervision, Visualization, Funding acquisition, Writing – review & editing. **Hasan Mehrjerdi:** Conceptualization, Validation, Visualization, Supervision, Funding acquisition. **Hussein M.K. Al-Masri:** Conceptualization, Writing – review & editing. **Ali Q. Al-Shetwi:** Conceptualization, Writing – original draft. **Dalal Asber:** Conceptualization, Writing – review & editing. **Serge Lefebvre:** Conceptualization, Writing – review & editing.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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