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Marked social networks: A new model of social networks based on dynamic behaviors

Ahmet Karadoğan*, Ali Karacı

Department of Computer Engineering, Faculty of Engineering, İnönü University, Malatya, Turkey

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ABSTRACT

Social networks are electronically information sharing systems, due to this case, there are many studies on social networks. When studying social networks, text-based solution methods can be used; this type of study is outside the scope of this paper. Some studies have used mathematical models such as graphs, and graphs are mathematical models to represent many things, and social networks are one of them. However, graphs are static models whose structure cannot match the behaviors of social networks. To get rid of this case, Petri nets have been used in some recent studies, however, they have some deficiencies (obtained models are not complete and sound). Because of this case, we modeled social networks by using Petri nets. The resulting model is called Marked Social Network. The marked social network has two types such as Concurrent Marked Social Network and Parallel Marked Social Network. The obtained models were analyzed in case of behavioral and structural properties, and the major properties of the model were determined. All these properties are described in this study.

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1. Introduction

Social networks were developed after electronics information sharing systems coming out, and social networks are modeled by using graphs. Due to the interests of users and capabilities of social networks, this area is an important emerging area, so, there are many studies of social networks such as community detection, stance detection, privacy-preserving proximity detection, anomaly detection, irony/sarcasm detection, role mining, topic/event detection, and causality detection.

1.1. Community detection

Community detection is the problem to detecting groups in networks whose characteristics are similar and they are tightly-coupled [3]. In other words, a community can be also described as “a group of entities/that are in proximity of each other when compared to other entities of remaining networks” [3]. The community detection can be handled by using clique detection in the graph which is a mathematical model of the related social network, or compact group discovery can be handled by using graphs [8,11].

* Corresponding author.

E-mail address: ahmet.karadogan@inonu.edu.tr (A. Karadoğan).

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1.2. Stance detection

Stance detection is a social network issue that illustrates that an individual who gave an opinion about a certain target is neutral, against, or favor towards the target. In another word, stance detection can be regarded as opinion mining or sentiment analysis [13].

1.3. Privacy-preserving proximity detection

With the increasing use of location-sensitive mobile devices, proximity-based social networks are also becoming widespread. The users disclose some personal during forwarding profiles from one to another and sharing private information increases the private information break-down due to unsecured wireless communication channels and malicious social network service providers [21,23]. Due to all these cases, privacy-preserving (private information protection) is an important problem in social networks.

1.4. Anomaly detection

The anomaly detection is to detect unexpected data, unexpected rules defining computational models, machine learning, and unexpected behaviors of individuals in social networks. Another important point is that incompatible patterns, inequalities, incompatible observations, exceptions, variations, or surprises are regarded as an anomaly. As a result of unexpected changes in

the interaction patterns of the groups, individuals cause anomalies in social networks [10].

1.5. Irony/sarcasm detection

Social life includes an important problem between individuals which causes many social problems such as irony or sarcasm. This is also valid for social networks. The studies on this problem are to detect ironic text (sarcastic) and non-ironic text (non-sarcastic) for social networks, since such rumors propagation in social networks may cause problems in social life such as protests against a governmental policy, management decisions, etc. [5,14].

1.6. Role mining

The aim of role mining is to observe people behaving predictably on the basis of their social positions, and this will illustrate that a person's role let observer guess interactions of a person with the environment or its opposite [2].

1.7. Topic and event detection

An event can be defined as an event is something that happens in a specific location and time with prerequisites and unavoidable results and a topic is an identifier event or activity through related events and activities [22].

1.8. Causality detection

Causality detection is to determine all events that occur within the social networks and their consequences in the context of cause-effect [17].

There are many studies performed on social networks by using text properties such as text clustering, classification, summarization, modeling social networks by using graphs. It is known that graphs are static mathematic models(nodes are static entities which do not have capability to model dynamic events/behaviors), however, social networks are dynamic models according to their nature. So, graphs are inefficient models for modeling social networks. This is our motivation for modeling social networks with a dynamic mathematical model such as Petri nets inspired by marked graphs.

2. Related works

Petri nets are mathematical and graphical modeling tools used to analyze various systems. They are suitable tools for describing and studying systems that are distributed, asynchronous, concurrent, parallel, and/or stochastic [15]. Petri nets have places and transitions and these are interconnected with directional arrows. Petri nets are also called Place-Transition (PT) net [16]. The places represent conditions, and the transitions represent events. The formal definition of Petri nets can be given as follows [15,4]:

Definition 1: [15]. A Petri net is a 5-tuple, $PN = (P, T, F, W, M_0)$ where P is a finite set of places, T is a finite set of transitions, F is a set of arcs, W is a weight function, M_0 is the initial marking.

In the graphical representation of a Petri net model, places are shown as circles, and transitions are shown as bars, and arcs are labeled with their weights. The places marked with tokens and tokens shown as black dots. The tokens indicate the number of data items available in places. If a place has at least one token, then it would be enabled and a transition can be fired. A token is

removed from the input place and added to the output place when a transition is fired. So, the tokens travel through the Petri net depending on transitions firing sequences.

Fig. 1 illustrates an example of a Petri net. For this Petri net

$$P = \{p_1, p_2, p_3, p_4\}, T = \{t_1, t_2, t_3, t_4\}, F = \{(p_1, t_1), (t_1, p_2), (p_2, t_2), (t_2, p_1), (t_4, p_3), (p_3, t_3), (t_3, p_4), (p_4, t_4), (p_1, t_3), (t_3, p_1)\}$$

$$W = \{\text{all weights are equal to } 1\}, M_0 = (1 \ 0 \ 1 \ 0)^T$$

Any Petri net can be represented by the incidence matrix:

Definition 2: [15]. Incidence Matrix: For a Petri net with m places and n transitions, the incidence matrix $A = [a_{ij}]$ is a $n \times m$ matrix and its entry is given by

$$a_{ij} = a_{ij}^+ - a_{ij}^- \quad (1)$$

where $a_{ij}^+ = w(i, j)$ is the weight of the arc that goes from transition i to output place j and $a_{ij}^- = w(j, i)$ is the weight of the arc that to transition i from input place j .

Incidence matrix A is the same as the incidence matrix of a directed graph. To perform the dynamic behaviors of PN according to initial marking M_0 , marking and incidence matrix are used with a firing vector, such as

$$M_i = M_{i-1} + A^T \sum_{k=1}^d u_k \quad (2)$$

where A^T is the transpose of A, and u_k is the firing vector. The incidence matrix for PN in Fig. 1 is as follows:

$$A = \begin{bmatrix} -1 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 \\ \mp 1 & 0 & -1 & 1 \\ 0 & 0 & 1 & -1 \end{bmatrix} \text{ and firing vectors } u_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \text{ and } u_2 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}.$$

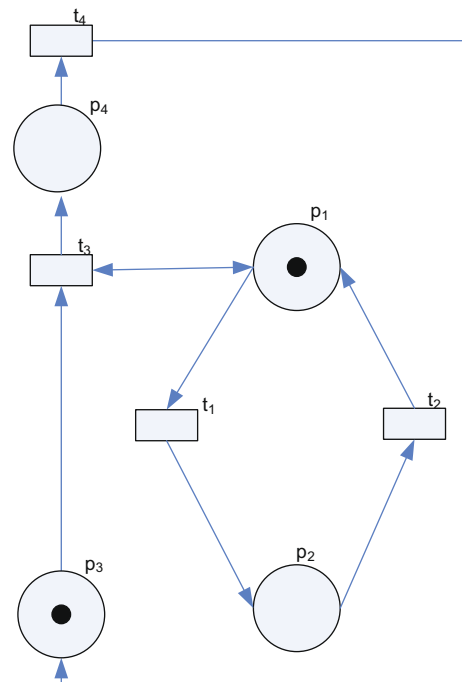


Fig. 1. An example of a Petri net.

Firing t_1 concludes in marking M_1 and firing t_3 concludes in marking M_2 . $M_1 = M_0 + A^T u_1 = [0 \ 1 \ 1 \ 0]^T$ and $M_2 = M_0 + A^T u_2 = [1 \ 0 \ 0 \ 1]^T$.

PNs have behavioral and structural properties. The behavioral properties are; *Reachability, Boundedness, Liveness, Reversibility and Home State, Coverability, Persistence, and Fairness*. The structural properties are; *Controllability, Boundedness, Liveness, Conservativeness, Repetitiveness, Consistency, Structural B-Fairness, and S- and T-Invariants*. The definitions of these properties can be found in [15].

PNs are dynamic systems since they have behavioral properties, however, graphs are not dynamic models; they are static models. Due to this case, social networks cannot be model with graphs so good. PNs are better than graphs to model social networks. However, there are many studies have used graphs as modeling tools for social networks [20]. The behaviors of individuals in social networks can be modeled by using graphs or information used in networks [7], and this information is text-based. The behaviors of individuals were modeled by using Petri nets [9]. Some studies modeled and simulated rumor propagation in social networks by using Petri nets [19], however, this study just includes rumor propagation by modeling token behaviors in Petri nets. There is no individual modeling and simulation in this study. The friendships in social networks also modeled and simulated by using Petri nets [18], and this study does not include individual's behavior models. The voice traffic between users in social networks was modeled by using Petri nets [12].

The aim of this study is to model and simulate the behaviors of individuals in social networks using the Petri nets. The model obtained was analyzed mathematically and the properties of the model were determined.

3. Material and methods

Social networks can be defined as a collection of social or interpersonal relationships within a social group. To better understand the social concepts class and community were analysed [1]. After this stage, the number and variety of theoretical and experimental studies also increased. Another aspect is that social networks are used to explain the characteristics and behavior of individuals and communities; they are used to analyze the social processes of large and small groups.

The basic data of social networks can be defined as a set of social units, for example, individuals can be defined as pairs connected by a certain social connection [6]. Examples include a group of friends, a group of people in a tribe, a group of employees at a workplace, etc. examples. Graphs do not have the dynamic capability to model social networks, and there are draft studies to model social networks by using Petri nets, but they have some deficiencies. Due to this case, we modeled social networks by using Petri nets and then depicted the properties of the obtained model.

An individual in social networks can be modeled by using the Petri net as seen in Fig. 2. The definitions for an individual and social network in Definition 5 and Definition 6 belong to this study. These definitions cannot be found in the literature.

Definition 5. An individual in social networks can be modeled by a Petri net such as $I=(P,T,F,W,M_0)$ where

$P=\{\text{Offline, Online, Critical Section Entry, Critical Section Exit}\}$
 $T=\{\text{Login, Logout, Sharing Message, Preparing for Message}\}$
 $F=\{(\text{Offline, Login}), (\text{Login, Online}), (\text{Online, Logout}), (\text{Logout, Offline}), (\text{Critical Section Entry, Sharing Message}), (\text{Message Sharing, Critical Section Exit}), (\text{Online, Message Sharing}), (\text{Message Sharing,}$

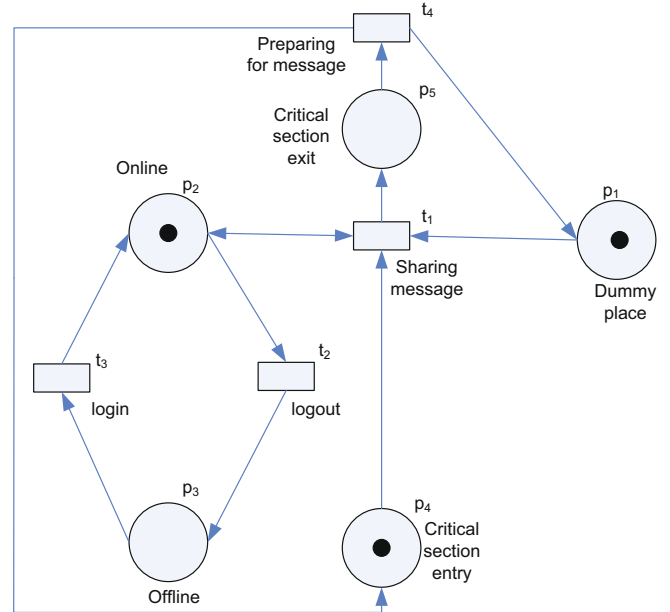


Fig. 2. Petri net model of an individual in social networks.

*Online), (Critical Section Exit, Preparing for Message), (Preparing for Message, Critical Section Entry)}
 $W=\{\text{all weights are equal to 1}\}$
 $M_0=(1 \ 0 \ 1 \ 0)$ for initial marking.*

Individuals can be in online or offline states. In the case of offline, an individual cannot share messages and he/she is inactive in social networks since transition t_1 is not firable. In the case of an online state, an individual can share messages in social networks concurrently by using a semaphore principle. In a concurrent system, only one process can take a clock cycle, due to this case, there should be a critical section for taking the clock cycle. So that online individuals can share messages at a time by using critical section entry and after shared message, individuals should use critical section exit. To share the next message, the individual should use critical section entry again, and so on. The initial marking should be (1 0 1 0) for an active individual in the system. "Dummy place" represents the existence of the social network system (Fig.1). An individual is represented with four places and four transitions.

Definition 6 ((Concurrent Marked Social Network)). A social network is a collection of individuals where each individual is represented by using Petri net in Definition 5 and a "Dummy place". The obtained social network called as **Marked Social Network (MSN)** where $MSN=\{\text{Dummy Place, } I_1, I_2, \dots, I_r\}$, I_i are individuals. If the dummy place includes a single token whatsoever the number of individuals, this marked social network is called **Concurrent Marked Social Network (C-MSN)**.

A Marked Social Network is seen in Fig. 3. Fig. 3 illustrates a MSN for three individuals and the first individual: $P = \{p_2, p_3, p_4, p_5\}$ and $T = \{t_1, t_2, t_3, t_4\}$. The second individual: $P = \{p_6, p_7, p_8, p_9\}$ and $T = \{t_5, t_6, t_7, t_8\}$. The third individual: $P = \{p_{10}, p_{11}, p_{12}, p_{13}\}$ and $T = \{t_9, t_{10}, t_{11}, t_{12}\}$. The incidence matrix for this MSN is as below:

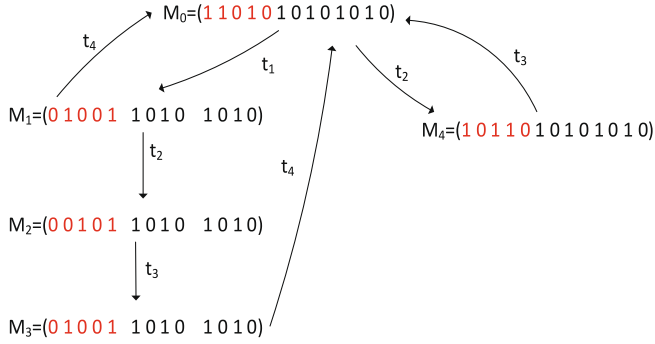
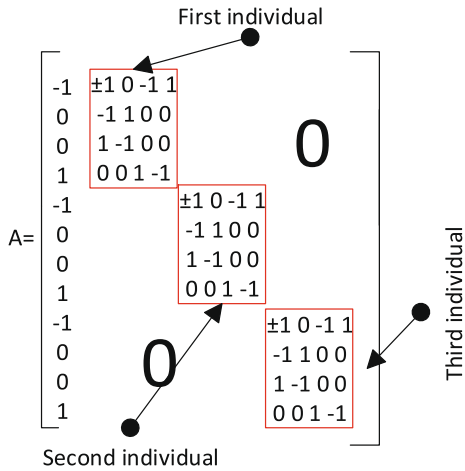
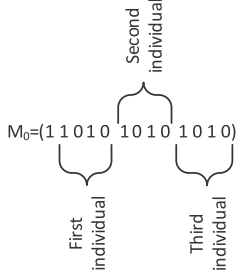


Fig. 3. The first individual performs all possible alternative actions and obtained markings.



The first column of the incidence matrix corresponds to the dummy place (system) and the surrounded region represents an individual. The incidence matrix A is a matrix of the marked social network of three individuals. The initial marking for all online individuals is as below:



The initial marking of any MSN with many individuals is $M_0 = (1 (1 0 1 0) +)$ where $(1 0 1 0) +$ means that the "1 0 1 0" may occur at least once or many times. The properties of C-MSN can be analyzed as follow.

4. Behavioral properties of C-MSN

Behavioral properties of C-MSN can be listed as below:

4.1. Reachability

If the reachability of an individual in the net with the dummy place is illustrated, then this case can be enlarged to the whole net. Fig. 4 depicts the possible actions of the first individual. It can be seen that the initial marking can be recovered and the obtained markings are also reachable from the other markings. It

is easy to see that this case is also valid for the remaining individuals. Due to this case, C-MSN is reachable.

Theorem 1. Every possible marking in a C-MSN is reachable.

Proof. Assume that the initial marking $M_0 = (1 1 0 1 0 1 0 1 0 \dots 1 0 1 0)^T$ and any marking $M_i = (b t_{11} t_{12} t_{13} t_{14} t_{21} t_{22} t_{23} t_{24} \dots t_{j1} t_{j2} t_{j3} t_{j4} \dots t_{k1} t_{k2} t_{k3} t_{k4})^T$ where $t_{j1..4}$ is the tokens in places corresponding to individual j and b corresponds to the dummy place. Without losing generality, firing the transitions for individual j is as follows (there are three paths):

$$M_i = (1 T_{11} T_{12} T_{13} T_{14} T_{21} T_{22} T_{23} T_{24} \dots 1 0 1 0 \dots T_{k1} T_{k2} T_{k3} T_{k4})^T \rightarrow (0 T_{11} T_{12} T_{13} T_{14} T_{21} T_{22} T_{23} T_{24} \dots 1 0 0 1 \dots T_{k1} T_{k2} T_{k3} T_{k4})^T \rightarrow (1 T_{11} T_{12} T_{13} T_{14} T_{21} T_{22} T_{23} T_{24} \dots 1 0 1 0 \dots T_{k1} T_{k2} T_{k3} T_{k4})^T$$

and another path is

$$M_i = (1 T_{11} T_{12} T_{13} T_{14} T_{21} T_{22} T_{23} T_{24} \dots 1 0 1 0 \dots T_{k1} T_{k2} T_{k3} T_{k4})^T \rightarrow (1 T_{11} T_{12} T_{13} T_{14} T_{21} T_{22} T_{23} T_{24} \dots 0 1 1 0 \dots T_{k1} T_{k2} T_{k3} T_{k4})^T \rightarrow (1 T_{11} T_{12} T_{13} T_{14} T_{21} T_{22} T_{23} T_{24} \dots 1 0 1 0 \dots T_{k1} T_{k2} T_{k3} T_{k4})^T$$

and the last path is

$$M_i = (1 T_{11} T_{12} T_{13} T_{14} T_{21} T_{22} T_{23} T_{24} \dots 1 0 1 0 \dots T_{k1} T_{k2} T_{k3} T_{k4})^T \rightarrow (0 T_{11} T_{12} T_{13} T_{14} T_{21} T_{22} T_{23} T_{24} \dots 1 0 0 1 \dots T_{k1} T_{k2} T_{k3} T_{k4})^T \rightarrow (0 T_{11} T_{12} T_{13} T_{14} T_{21} T_{22} T_{23} T_{24} \dots 0 1 0 1 \dots T_{k1} T_{k2} T_{k3} T_{k4})^T \rightarrow (0 T_{11} T_{12} T_{13} T_{14} T_{21} T_{22} T_{23} T_{24} \dots 1 0 0 1 \dots T_{k1} T_{k2} T_{k3} T_{k4})^T \rightarrow (1 T_{11} T_{12} T_{13} T_{14} T_{21} T_{22} T_{23} T_{24} \dots 1 0 1 0 \dots T_{k1} T_{k2} T_{k3} T_{k4})^T$$

4.2. Liveness

All transitions are fireable infinitely and this case can be seen in Fig. 4 for the first individual. This case is valid for all remaining individuals, so, C-MSN is at least L_3 -live. To describe the liveness in C-MSN, Eq. (2) can be redefined as in Eq.(3).

Theorem 2.

$$M_i = M_{i-1} + \left[\bigwedge_{k=1}^n m_{i-1}(p_k) \right]^T A u_j \quad (3)$$

where $m_{i-1}(p_k)$ denotes the existence of tokens in place p_k of marking M_{i-1} , p_k is input place of transition t_j and $[f(x)]$ is a true predicate (numerical result is 1) when $f(x) > 0$, false (numerical result is 0) otherwise. If $\left[\bigwedge_{k=1}^n m_{i-1}(p_k) \right]^T A u_j \neq 0$, then this C-MSN is liveness net.

Proof. To fire any transition, $M_i = M_{i-1} + \left[\bigwedge_{k=1}^n m_{i-1}(p_k) \right]^T A u_j \neq M_{i-1}$. If $\left[\bigwedge_{k=1}^n m_{i-1}(p_k) \right]^T A u_j \neq 0$, this means that $\left[\bigwedge_{k=1}^n m_{i-1}(p_k) \right] = 1$. This is a requirement for firing t_j transition, so, transition t_j is fireable

The example in Fig. 3 can be used to depict this case. Its initial marking is $M_0 = (1 1 0 1 0 1 0 1 0 1 0)^T$ and incidence matrix A can be used. The firing vectors for the first individual are $u_1 = (1 0 0 0 0 0 0 0 0 0 0)^T$, $u_2 = (0 1 0 0 0 0 0 0 0 0 0)^T$, $u_3 = (0 0 1 0 0 0 0 0 0 0 0)^T$ and $u_4 = (0 0 0 1 0 0 0 0 0 0 0)^T$.

$M_1 = M_0 + A^T u_1 = (0 1 0 0 1 1 0 1 0 1 0 1 0)^T$, $M_2 = M_0 + A^T u_2 = (1 0 1 1 0 1 0 1 0 1 0 1 0)^T$, $M_3 = M_0 + A^T u_3 = M_0$ and t_3 is not fireable, and $M_4 = M_0 + A^T u_4 = M_0$ and t_4 is also not fireable. After firing t_2 , t_3 is fireable and after firing t_1 makes t_4 be fireable.

4.2.1. Boundedness

Fig. 4 depicts that C-MSNs are bounded Marked Social Network since all place include maximum one token.

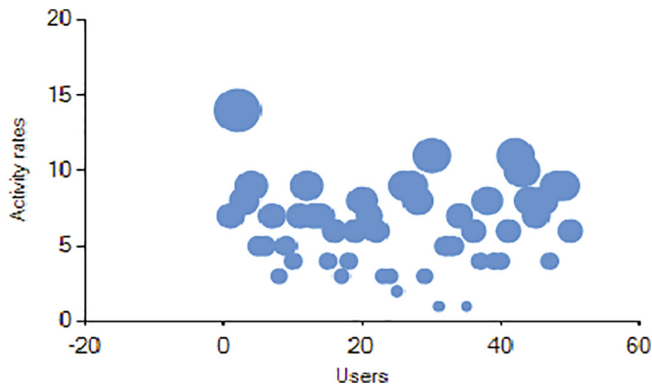


Fig. 5. Activity rates of users.

5. Experimental results

In this section, a simulation was performed on developed C-MSN and P-MSN models and experimental results were presented. Social network groups consisting of different numbers of people were simulated and each person in the group was shown as an agent. The activities of the individuals in the group were analyzed by following the flow of the tokens in places in the Petri net model. After any action of individuals in the network, the transitions in the Petri net model are fired and the number of tokens in the affected places increases or decreases. As a result, the values of the M initial matrix also changed. The M matrix is used to track the activities of individuals in the network. In the M matrix, each individual is normally represented by four places. A fifth place was added to the output of the “sharing message” transition to obtain the number of messages from individuals. Petri net models proposed in this study can be revised according to different social networks and different analyzes can be obtained.

Within the scope of simulation, experimental tests were carried out on groups of 10, 50, 100, 250, 500, and 1000 individuals using C-MSN and P-MSN models. For each group, active individuals in the group were determined according to the number of activities of the users and the time they spent. Fig. 5 shows the activity rates of individuals in a social network group of 50 people. The average time spent by these individuals is shown in Fig. 6.

Since there are as many tokens as the number of users in the dummy place in P-MSN, users can write messages in parallel. But since there is only one token in the dummy place in C-MSN, other users wait while a user write a message. This situation causes a delay in the social network and as the number of users on the network increases, the waiting times increase. Therefore, the P-MSN

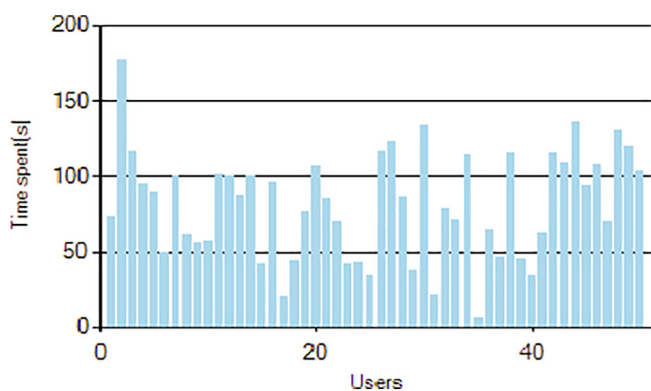


Fig. 6. Average time spent by the users.

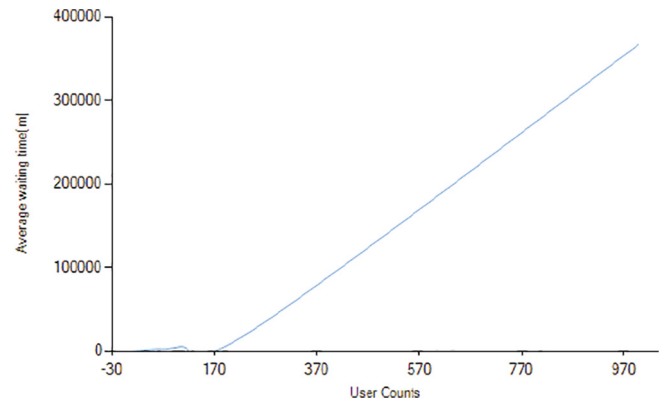


Fig. 7. Average waiting time by the number of users in the group for C-MSN.

model is more suitable for real-life applications. Fig. 7 shows the average waiting times in social network groups with different numbers of users for C-MSN model.

6. Conclusions

The social networks are modeled by using graphs; however, graphs are static models and they cannot model the dynamic properties of social networks. Due to this case, we modeled social networks by using Petri nets. The obtained Petri net model was named Marked Social Networks. The Marked Social Networks have two types such as Concurrent Marked Social Network and Parallel Marked Social Network. The major properties of these networks were analyzed in this paper. An important point is that both models (C-MSN and P-MSN) are deadlock-free, so they can be used to model real-life applications. However, the P-MSN model is more suitable because of the waiting times in the C-MSN model.

In this study, a general mathematical model for social networks is presented. This model can be customized for different social networks. In future studies, this model will be expanded and applied for different analyzes on various social network groups.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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