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ORIGINAL CONTRIBUTION

Minimization of Real Power Losses of Transmission Lines and Improvement of Voltage Stability in Power System using Recurring MODE Algorithm

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Abstract This paper presents a recently invented recurring multi-objective differential evolution (RMODE) to minimize real power losses of total transmission lines and for voltage stability improvement. This multi-objective reactive dispatch (MORPD) problem has been devised as a constrained nonlinear optimization problem with the objectives of total transmission lines real power losses minimization and voltage stability improvement by minimizing stability index in a power system. In the proposed recurring MODE algorithm, MODE is employed again and again with the efficient solutions in hand and initializing the leftover inhabitants only in each round of RMODE algorithm. Usefulness of the RMODE algorithm is established by solving MORPD problem in the two cases, one standard 30-bus IEEE test system and other Indian practical 75-bus system, and by comparing MORPD results with those obtained using NSGA-II, multi-objective PSO, MODE, hybrid multiple-swarm PSO algorithm and with the reported results. On comparison, RMODE algorithm is found to be a potential approach for handling multi-objective ORPD problem in power systems.

Keywords Pareto optimal solutions · Preferred solution · Total transmission lines real power losses minimization · L-index · Recurring MODE algorithm

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Introduction

Optimal reactive power dispatch problem is a crucial task carried out in modern energy control center. Optimal RPD is carried out for minimization of total transmission real power losses, improvement of voltage profile and enhancement of voltage stability in a power system. In addition to this, power system voltage stability and voltage profile can be improved by controlling reactive power injections. Thus, reactive power dispatch problem can be considered like a nonlinear, combinatorial optimization problem having the objectives such as minimization of transmission lines losses, voltage stability enhancement and improvement of voltage profile in a power network [1–3].

As documented in the literature, a number of conventional optimization techniques such as gradient method, quadratic programming, goal attainment methods, interior point method and linear and nonlinear programming have been implemented for handling ORPD problem [3–6]. However, owing to non-convex, nonlinear, non-differential and multiple modal nature of reactive power dispatch problem, the classical optimization methods occasionally converge to local minimum and also these techniques are sensitive to initial conditions [5, 6]. On the contrary, population-based meta-heuristic random search techniques/evolutionary computing (EC) algorithms have good potential for solving RPD problem [1, 2, 7–38]. Hence, in recent years, several EC techniques such as genetic algorithm (GA) and its modified versions [7–9], PSO [10, 11], evolutionary programming [12], differential evolution (DE) and its modified versions [13–16], bacteria foraging [17], seeker optimization algorithm [18, 19], harmony search algorithm [20], teaching–learning-based optimization [21], artificial bee colony [22], gravitational search

(GS) algorithm [23–25], gray wolf optimization [26, 27], Gaussian bare-bones water cycle algorithm [28], moth-flame optimization [29] and ant lion optimization [30] have been applied for solving RPD problem.

Besides several evolutionary computing techniques, various hybrid algorithms such as hybrid ABC and hybrid PSO have also been employed to handle optimal RPD problem in power system. Hybrids EC techniques generally combine two or more than two evolutionary computing algorithms to improve further the optimization results [31-38] []. These hybrid EC algorithms are usually motivated by biological as well as sociological inspirations and are very much able to solve the multiple-modal, non-sequential, non-comparable and non-convex properties of optimal reactive power dispatch problem. These population-based EC algorithms, though not guaranteeing global optimality, are capable of providing near best optimal solution within adequate computational time.

The optimal RPD problem was considered as one objective problem [7, 8, 10, 12–14, 26, 34] and as a multi-objective problem for minimization of transmission line real power losses and improvement of voltage security/voltage profile/voltage stability [1, 2, 9, 11, 15, 17, 27, 35, 36, 40–43, 45, 46] simultaneously. However, multi-objective RPD (MORPD) problem was converted into a single-objective RPD (SORPD) optimization problem using weighted summation of involved objectives [15, 35, 36]. In such case, a single-objective evolutionary computing (SOEC) technique had to be run again and again after varying the weights of the involved objectives in order to obtain several solutions. Then, from these solutions, the dominated or inferior solutions had to be searched and discarded. The remaining solutions having non-conquered nature were considered to achieve the efficient or optimal Pareto solution set and the best compromised solution. The Pareto optimal solutions (POS) and preferred solution of MORPD problem can also be obtained by running only once a multi-objective evolutionary computing algorithm [1, 2, 27, 40–43, 45, 46]. Owing to this advantage, the multi-objective evolutionary computing (MOEC) algorithms such as SPEA [2], non-dominated sorting genetic algorithm (NSGA-II) [40–42], multi-objective DE (MODE) [43] and multi-objective PSO (MOPSO) [46] are being increasingly applied for solving MORPD problem.

In this paper, a recently developed algorithm, namely recurring MODE (RMODE) algorithm, is proposed for solving constrained, nonlinear MORPD problem for minimization of total transmission lines real power losses and voltage stability improvement simultaneously. The key motivation of the proposed work is to ascertain the potential of the developed RMODE [45] algorithm for solving the nonlinear, constrained, multi-objective reactive power management problem to minimize real power loss and to improve voltage stability simultaneously.

In the proposed and developed recurring MODE algorithm, the MODE algorithm is applied again and again by the utilization of the available POS and re-initialization of residual population only. Afterward, fuzzy membership function-based approach [15, 39] is implemented to obtain preferred and efficient solutions. Superiority of the RMODE algorithm has been established by employing this algorithm for solving MORPD problem in IEEE 30-bus system [3] and in practical Indian 75-bus system [45].

Optimal RPD Problem

In the present paper, the objectives are to minimize the total transmission lines real power losses and to enhance voltage stability simultaneously, satisfying the equality as well as inequality constraints. These two objective functions of the optimal RPD problem with constraints are listed follows [36]:

Objective Functions

(1) ***Minimization of Total Transmission Lines Real Power Losses:***
Total transmission lines real power losses P_{TLOSS} are the total real power losses taking place in different lines of a power network, and this can be computed as

$$F_1 = P_{TLOSS} = \sum_{k=1}^{ntl} g_k \left[V_i^2 + V_j^2 - 2V_iV_j\cos\left(\delta_i - \delta_j\right) \right] \quad (1)$$

where g_k represents conductance of kth line; ntl is the total number of transmission lines in the power system; and $V_i\angle\delta_i$ and $V_j\angle\delta_j$ are the voltages at terminal buses i and j, respectively, of kth line.

(2) ***Improvement of Voltage Stability***

In this paper, for assessment of voltage stability, an indicator, commonly known as *L-index*, has been employed. *L-index* is a scalar quantity that is equal to 0 at no load condition of a power system and 1 at its voltage collapse condition. Voltage stability of a power system can be improved by minimizing the values of this index at each load (PQ) bus or by minimization of global *L-index* [2, 36].

The performance equation in any power network can be expressed as

$$I_{bus} = Y_{bus} \times V_{bus} \quad (2)$$

By separating out the PQ buses and the PV buses (generator buses), (2) can be rewritten as

$$\begin{bmatrix} I_L \\ I_G \end{bmatrix} = \begin{bmatrix} Y_1 Y_2 \\ Y_3 Y_4 \end{bmatrix} \begin{bmatrix} V_L \\ V_G \end{bmatrix} \quad (3)$$

Or

$$\begin{bmatrix} V_L \\ I_G \end{bmatrix} = H \begin{bmatrix} I_L \\ V_G \end{bmatrix} = \begin{bmatrix} H_1 H_2 \\ H_3 H_4 \end{bmatrix} \begin{bmatrix} I_L \\ V_G \end{bmatrix} \quad (4)$$

where V_L represents the voltages at load (PQ) buses, I_L represents the currents at PQ buses, V_G represents the voltages at generator buses and I_G represents the currents at the generator buses. In (4), H is a hybrid matrix having sub-matrices H_1, H_2, H_3 and H_4, which are attained by partial inversion of the Y_{bus} matrix.

Using (3) and (4),

$$V_L = H_1 I_L + H_2 V_G = Y_1^{-1} I_L - Y_1^{-1} Y_2 V_G v \quad (5)$$

$$H_2 = -Y_1^{-1} \times Y_2. \quad (6)$$

At no load condition of the power system, the currents at PQ buses (I_L) are zero, and in this condition, (5) can be rewritten as:

$$V_{ok} = \sum_{i \in G} H_{2ki.} V_i \quad (7)$$

where V_{ok} represents the voltage at kth bus of the power network at no load condition. This expression has been utilized to define voltage stability indicator L_k (*L-index*) at any PQ bus k, as:

$$L_k = \left| 1 - \frac{V_{ok}}{V_k} \right| \quad (8)$$

where V_k represents the voltage at kth PQ bus. The value of L_k moves toward 1.0, when a PQ bus in a power system approaches a situation of voltage collapse. Hence, in a power system, global or maximum value of L_k should be less than 1.0. The objective function for improvement of voltage stability is written as [2, 36]:

$$F_2 = L - index = \max(L_k) \quad (9)$$

Voltage stability indicator *L-index* should be minimized for improvement of voltage stability. *L-index* can be computed through normal power flow program.

Constraints

i. *Equality constraints*: In ORPD problem, the equality constraints are the static power flow equations written as:

$$P_{gj} - P_{dj} - V_j \sum_{k=1}^{NBUS} V_k [G_{jk} \cos(\delta_j - \delta_k) + B_{jk} \sin(\delta_j - \delta_k)] = 0 \quad (10)$$

$$Q_{gj} - Q_{dj} - V_j \sum_{k=1}^{NBUS} V_k \left[G_{jk} \sin(\delta_j - \delta_k) \right] = 0 \quad (11)$$

for j = 1, 2……*NBUS,* where *NBUS* shows the number of total buses in a power system; P_{gj} represents the real power generated at jth bus, Q_{gj} represents the reactive power generated at j^{th} bus, P_{dj} represents the real power demand at jth bus, Q_{dj} represents the reactive power demand at jth bus, G_{jk} is the transfer conductance of a line connected between buses j and k, while B_{jk} is its susceptance.

ii. Inequality constraints: In optimal RPD problem, operating constraints of a power system are the inequality constraints and can be listed as:

Generator constraints: These constraints include the minimum and maximum limits of generator voltage V_g and its reactive power output Q_g and can be expressed as:

$$V_{gj}^{\min} \le V_{gj} \le V_{gj}^{\max}, j = 1, 2, \ldots, NGR$$

$$Q_{gj}^{\min} \le Q_{gj} \le Q_{gj}^{\max}, j = 1, 2, ..., NGR$$

where *NGR* is the total number of generators in power system.

Transformer constraints: These constraints consist of the maximum and minimum specified limits on the transformer tap settings T_j, and this can be expressed as:where *NTN* represents the total quantity of transformers.

$$T_j^{\min} \le T_j \le T_j^{\max}, j = 1 \ldots\ldots NTN \quad (14)$$

Constraints on Shunt VAR sources: These constraints contain the maximum and minimum limits on shunt reactive power compensators output Q_s as:

$$Q_{sj}^{\min} \le Q_{sj} \le Q_{sj}^{\max}, j = 1 \ldots NQ \quad (15)$$

where *NQ* represents the total count of shunt VAR compensation devices.

Security constraints: The constraints of security include voltage magnitude and line flow limit constraints. Voltage magnitude constraints are the maximum and minimum limits on the voltage magnitudes V_j at the PQ bus j, while line flow security constraint is the maximum limits on the power flows S_{flk} at any transmission line k. Security constraints can be given as follows:

$$V_j^{\min} \le V_j \le V_j^{\max}, j = 1 \ldots LBUS \quad (16)$$

$$S_{flk} \le S_{flk}^{\max}, k = 1 \ldots\ldots ntl \quad (17)$$

where *LBUS* represents the total count of PQ buses.

ORPD as Multi-Objective Optimization Problem

Several optimal solutions are yielded when solving any MOO problem, recognized as efficient or non-dominated solutions. These non-dominated solutions form efficient set (ES) and demonstrate the trade-off that exists among the involved objectives. The solutions in ES are equally important, and to obtain the best compromised solution, further processing is necessary. The MORPD problem having several constraints and variable bounds can be written as [36, 45]:

$$Minimize\ F = [F_1 F_2]. \tag{18}$$

Subject to satisfy the inequality and equality constraints and bounds on variables given as $h\ (x,\ u) \leq 0$ and $g\ (x, u) = 0$, and decision variable u boundaries given as follows:

$$u_k^{(L)} \leq u_k \leq u_k^{(U)}, k = 1,\ 2,\ 3, \ldots, n \tag{19}$$

where x represents the set of dependent variables and $u_k^{(L)}$ and $u_k^{(U)}$ represent the lower and upper bound of kth control variable, respectively.

In MORPD problem $F_1 = P_{TLOSS}(x,u)$, while $F_2 = L\text{-}index(x,u)$ where

$$x^T = \left[P_{g1}, V_1 \ldots V_{LBUS}, Q_{g1} \ldots Q_{gNGR}, S_{fl1} \ldots S_{flntl}\right] \tag{20}$$

where P_{gl} is the slack bus power.

And

$$u^T = \left[V_{g1} \ldots V_{gNGR}, T_1 \ldots T_{NTN}, Q_{s1} \ldots Q_{sNQ}\right]. \tag{21}$$

The various constraints for the MORPD problem are as depicted in (10)–(17). Solutions of MORPD problem obeying the variables' bounds and constraints form feasible control variable space. When solving MOO problem, various non-dominated solutions are filtered out and the dominated or inferior solutions are discarded using the concept of dominance [36, 39, 45]. After attainment of the efficient set, the preferred or best compromised solution can be extracted out of it. Fuzzy relationship function [15, 39] is employed to extract preferred solution in this paper.

Multi-Objective Evolutionary Algorithms

With the development of several MOEC algorithms such as multi-objective GA and multi-objective PSO, there is an increasing trend of implementing these algorithms for handling a variety of numerical and engineering optimization problems.

Non-Sorting GA-II

In non-sorting GA-II, the optimal Pareto set and the Pareto optimal solution are attained using the elitist non-dominated sorting genetic algorithm [39]. In each cycle of non-sorting GA-II, the initial population of parents is generated and this population is sorted based on the rank as well as the crowding distance. Then, to select individuals for mating pool, tournament selection is carried out and to generate the off-spring population, crossover and mutation are used.

The non-dominated sorting is carried out to allocate fitness values to all the individuals of combined population. At last, elitist sorting is done to select the individuals with better fitness, which will turn out to be the parent individuals for next iteration. These steps are to be repeated for pre-specified number of generations. In addition to it, in the last generation, a niche strategy is applied to select the members of the last Pareto front, which are located in the least crowded section in the front [39]. Once non-dominated solutions are achieved, the preferred solution is extracted from the Pareto optimal solutions using fuzzy membership function [15].

MOPSO Algorithm

MOPSO algorithm integrates Pareto optimality concept into PSO to take care of multi-objective functions [46, 47]. In MOPSO algorithm, the local and global best positions are defined in different manners. Every particle of the population is assessed on the basis of its objective functions, and local best is considered as its current position. The non-dominated local set is created by Pareto optimal solutions obtained by a particle up to the present iteration. As this particle travels in the pre-specified search space, new position of this particle is included in this set. This set is modified by implementing the dominance conditions, so that only the efficient Pareto optimal solutions are kept in the set [15, 46]. Afterward, non-inferior global set is formed by storing the non-inferior solution obtained by all particles up to the current iterations.

Some clustering algorithm is also implemented for reducing the size of non-inferior global set to the pre-specified value. In addition, external set is formed which acts as an archive to accumulate a past record of the Pareto optimal solutions achieved throughout the searching process. After each iteration, this set is modified by implementing the dominance conditions to the combination of the non-inferior global set and this set. Also, on the external set, some clustering algorithm is implemented for reducing its size to pre-specified value. The local best *pBest* and the global best *gBest* are updated. The distances between the members in local best, and the members in global best, are calculated. If the local best *pBest* is a member of non-dominated local set which provides the minimum distance, then it is chosen as the local best of the particles. Similarly, if the global best *gBest* is a member of non-dominated global set, which provides the minimum distance, then it is chosen as the global best of the particles [47].

MODE Algorithm

Multi-objective differential evolution algorithm is extension of DE algorithm, which can be implemented for solving MOO problems. DE algorithm [44] is random search technique based on population, wherein DE variants perturb the members of the population (*NP*) by a scaled difference of the arbitrarily chosen different members of the population. The optimization method utilizes the three main parameters, namely selection, mutation or differential, and crossover operators in this technique [43–45]. Consider the probability of every solution of particle parents search space by selecting randomly solutions of initial candidate given limitations as:

$$y_{jk}(0) = y_k^l + rand(0,1) \times (y_k^u - y_k^l) \quad (22)$$

where y_{jk} is the kth component of jth member of the population, y_k^l and y_k^u are the lower and upper bound of the kth variable, respectively, and (0,1) is the random number that is uniformly distributed between 1 and 0. In each iteration, a donor or mutant vector v_j(mv) is produced to perturb the vector y_j (mv), a population member. The kth component of the donor vector is determined as:

$$v_{j,k}(mv+1) = y_{d1,k}(mv) + F(y_{d2,k}(mv) - y_{d3,k}(mv)) \quad (23)$$

where y_{d1}, y_{d2} and y_{d3} are three vectors chosen arbitrarily out of the existing population except the member y_j and F is the scalar number utilized for controlling perturbation as well as improving convergence. Crossover operator is used to increase the diversity in the population. Mutant vector is used to replace its components with those of member $y_j(t)$. The DE operator crossover produces the trail vectors to be included in the selection procedure. Binomial crossover adopted in this paper can be written as:

$$u_{j,k}(mv) = \begin{cases} v_{j,k}(mv) \\ y_{j,k}(mv) \end{cases} \text{if else rand}(0,1) < Cr \quad (24)$$

Selection process is implemented to select any individual from trail vector and the target vector for next iteration using the principle of survival of the fittest. Thus, the selection process is given as,

$$Y_j(mv+1) = \left\{ \begin{matrix} U_j(mv) \text{ if } f(U_j(mv)) \le f\left(Y_j(mv)\right) \\ Y_j(mv) \text{ if } f(Y_j(mv)) < f\left(U_j(mv)\right) \end{matrix} \right) \quad (25)$$

where $f(Y)$ represents the function, which is to be optimized. If the trail vector $U_j(mv)$ provides a better fitness value, then it is included in the next iteration; otherwise $Y_i(mv)$, the target vector is kept in the population. Thus, in subsequent iterations, either the population gets better or it remains constant.

The MODE algorithm adopted in this paper applies the selection operator to maintain the size of population, *NP* constant but it uses the concept of dominance. In each iteration, a dominated evaluation is made *NP* times. Terminating criterion used in this paper is attainment of a predefined number of iterations *M*. After completion of the last iteration, a non-dominated sorting is carried out to eliminate the inferior solutions [45]. Once the Pareto efficient or Pareto optimal solutions are obtained, the best compromised solution is taken out from the efficient set using a fuzzy membership-based mechanism [15, 45].

Recurring MODE Algorithm

Recurring multi-objective DE algorithm [45] has been applied in this paper, wherein multi-objective DE is employed for some pre-specified iterations M and Pareto optimal solutions say NP* are attained. In the next round of RMODE algorithm, these NP* Pareto optimal solutions have been used along with the re-initialized remaining population, i.e., (NP- NP*), and multi-objective DE algorithm has been employed again for iterations M. This practice has been repeated for a pre-defined number of rounds R. For each next round of the recurrent MODE algorithm, the better quality Pareto optimal solutions have been obtained. As a last step of RMODE algorithm, fuzzy membership-based mechanism [39, 45] has been applied to find the preferred solution out of the available Pareto optimal solutions. As in every next round of the recurring MODE algorithm, better population has been used; the improved quality efficient set and preferred solution have been achieved. Figure 1 depicts the flowchart of the proposed recurring multi-objective DE algorithm.

Solution Steps of RMODE Algorithm

Recurring MODE algorithm is employed to attain optimal setting of decision variables. The algorithm starts using the initial population, which is generated arbitrarily within the given minimum and maximum limits of the variables. For every entity of the population, NRLF program is developed and run to attain the values of the ORPD objective functions, namely P_{TLOSS} and *L-index* as per (1) and (2), respectively. After that, extended objective functions have been computed combining the P_{TLOSS} and *L-index* and the penalty factor (*PF*) as per the violation of constraints. At the end of maximum iteration *M* of multi-objective DE algorithm, a non-dominating sorting (NDS) check is

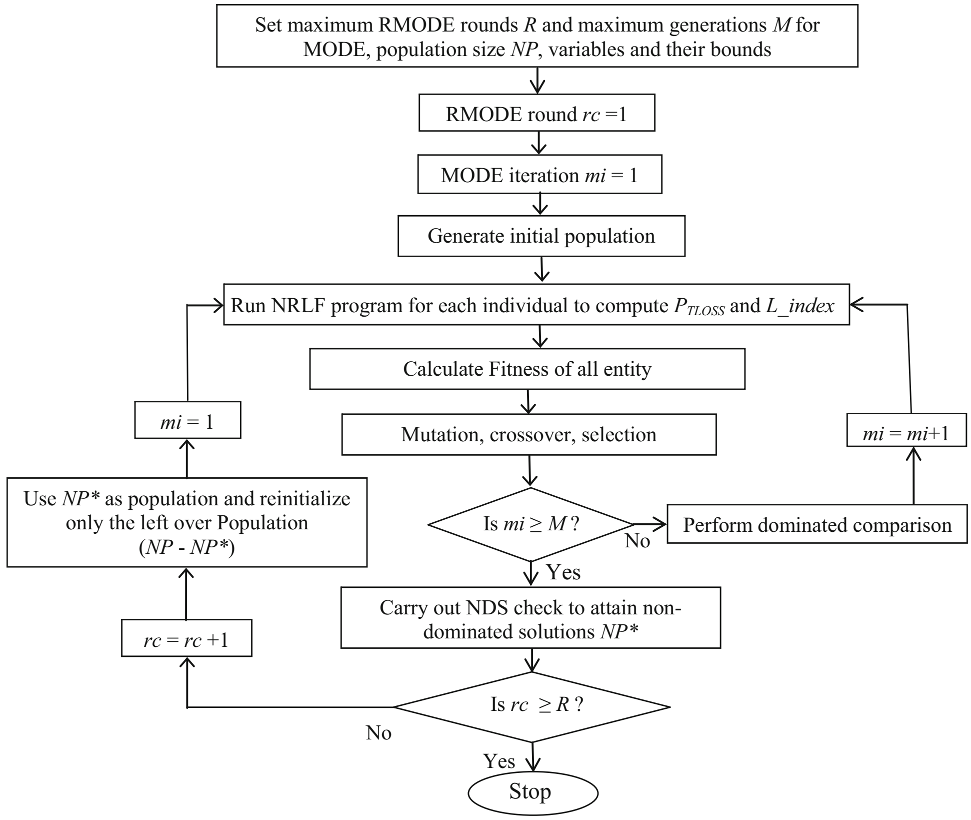

Fig. 1 Flowchart for recurring MODE algorithm

carried out to achieve the non-dominated solutions *NP**. These Pareto optimal solutions along with the re-initialized remaining population (*NP*-*NP**) are considered as the initial population for the subsequent round of the recurring MODE algorithm. This practice is repeated for a pre-defined rounds *R* of RMODE algorithm. The solution steps of the RMODE algorithm are summarized as follows:

i. Initialize the population within the variables' bounds using (22).
ii. Set recurring MODE round $rc = 1$.
iii. Set multi-objective DE iteration $mi = 1$.
iv. For every entity of the population, run NRLF program, to obtain the values of P_{TLOSS} (F_1) and *L-index* ($F2$).
v. Evaluate the extended objective functions by means of the objective functions F_1 and F_2 and the penalty factor as per the violation of the constraints
$FF_1 = F_1 + PF.$
$FF_2 = F_2 + PF.$
vi. Carry out the mutation and crossover as per the literatures (23) and (24).
vii. Apply selection operator to choose individuals for next iteration as per the earlier investigations (25).
viii. If $mi < M$, perform dominated comparison. Put $mi = mi + 1$, then go to step *iv*. Otherwise, continue to step *ix*.
ix. To attain the Pareto optimal or non-dominated solutions *NP**, carry out the non-dominating sorting check.
x. If $rc < R$, put $rc = rc + 1$, $m = 1$ and proceed to *xi*. Otherwise, jump to *xiii*.
xi. Consider the attained non-dominated solutions *NP** as the initial population and reinitialize the residual population, i.e., *NP*—*NP**.
xii. Go to step *iv*.
xiii. Stop. Extract the preferred solution from the non-dominated [45].
xiv. Choose the decision variables setting corresponding to the best compromised solution.

The setting of decision variables as attained for the preferred solution will offer the minimum value of P_{TLOSS} and the minimum value of *L-index* simultaneously for the specified operating condition of a power network. The results achieved by applying the recurring MODE algorithm have been evaluated by comparing the results obtained by applying other MOEC algorithms, namely NSGA-II, MOPSO, MODE and the reported results. Also, to authenticate the Pareto optimal front (POF) achieved from the recurring MODE algorithm, a reference Pareto optimal front has been created. For this purpose, the ORPD problem has been converted into a SOO problem by means of the weighted sum of objective functions P_{TLOSS} and *L-index* given as follows:

$$Minimize w \times P_{TLOSS} + (1 - w) \times L - index. \quad (26)$$

Here, w represents the weighing factor that is a number uniformly spread between 1 and 0. As an example, to attain 25 Pareto optimal solutions, a SOEC algorithm, say HMPSO, has to be implemented 25 times, and in each time, the weighing factor w should be different. Using these Pareto optimal solutions, reference Pareto optimal front can be drawn. Once the reference Pareto optimal front was achieved, the POFs using NSGA-II, MOPSO, MODE and recurring MODE for the same power network have been evaluated with respect to the reference POF.

In multi-objective optimization algorithm, there is trade-off between the two conflicting objective functions and the two objectives are to be considered simultaneously. In this paper, the Pareto optimal solutions achieved using different MOO algorithms are also compared with those of reference Pareto optimal solutions. The MOO algorithm providing the results very close to that of reference POF is considered as the superior one.

Simulation Results

The proposed recurring multi-objective DE algorithm is employed for solving MORPD problem in the 30-bus IEEE test system [3, 45] and 75-bus Indian system [45]. In this paper, for solving MORPD problem, two cases of each power system have been considered. Only the voltage constraints on PQ buses were considered in (i), whereas the constraints of voltage and the constraints of line loading both were implemented in (ii). Performance of the proposed recurring MODE algorithm is evaluated by applying NSGA-II, MOPSO, MODE and HMPSO [35] algorithms for the MORPD problem and comparing the obtained results for both the systems. Simulation results of all the algorithms were carried out using Matab2017Ra software on Intel –i7 3470 Desktop PC computer 2.8 GHz, with 2 GB of RAM.

30-Bus IEEE Test System

The 30-bus IEEE test system consists of 06 number of generators at bus numbers 13, 11, 8, 5, 2 and 1 and total transmission lines 41 with 04 off-nominal transformers tap setting ratio in lines 9–6, 10–6, 12–04 and 28–27. This system has 09 number of shunt VAR sources at bus nos. 29, 24, 23, 21, 20, 17, 15, 12 and 10. Thus, the total number of control variables in 30-bus IEEE test system for MORPD problem is 19 (6 + 4 + 9). At all the load buses, the lower limits on bus voltage magnitude are 0.95pu, while the upper voltage limits are 1.1pu for the PV buses and 1.05pu for PQ buses. Maximum limits are 1.1pu and minimum limits are 0.9 pu for transformer tap settings ratio. The lower limits of the VAR outputs of all the shunt reactive power sources are 0 and upper limits are 0.05pu [3, 14].

With varying parameters' values, several trials were taken when applying various MOEC algorithms for MORPD problem. However, the best results attained and provided here are for recurring multi-objective DE population size NP = 30, number of recurring MODE rounds R equal to 10, number of MODE iterations M = 100, mutation factor F equal to 0.39 and the crossover probability Cr equal to 0.95. For multi-objective DE algorithm, with the similar values of crossover probability Cr and mutation factor F the maximum number of iterations considered = 1000 (10 × 100). In multi-objective PSO algorithm, the key parameters considered were like this, inertia weight w equal to 0.76, cognitive coefficient c_1 and social coefficient c_2 equal to 1.9, the total number of population NP = 30 and number of the particles in repository = 20. Each algorithm method was implemented for 1000 iterations only for Case 1(i) as well as for Case 1(ii).

Case1 (i): Voltage Constraints

With the voltage profile magnitudes constraints at various PQ-type buses of IEEE 30-bus system, multi-objective PSO, non-dominated sorting GA-II, multi-objective DE, recurring MODE and hybrid multi-swarm PSO algorithms were applied for MORPD problem with the objectives of minimization P_{TLOSS} and *L-index*. The Pareto optimal fronts obtained on applying these MOEC algorithms are compared with the reference Pareto optimal solution [35, 36] in Fig. 2. Recurring MODE algorithm obtained Pareto optimal front closest to the reference optimal Pareto front as compared to other multi-objective EC algorithms. Out of the Pareto optimal solutions, the preferred solutions having the maximum value of normalized membership function have been extracted [45] for various algorithms.

The preferred solutions attained for the minimum P_{TLOSS} and minimum *L-index* same time using RMODE algorithm and other algorithms along with the corresponding control variables are shown in Table 1. As shown in Table 1, the recurring MODE algorithm gives the minimum total transmission lines real power losses as 4.87857 MW and minimum *L-index* as 0.12864 which are close to those obtained in reference Pareto optimal solutions using HMOPSO [36]. This clearly establishes the effectiveness of the proposed recurring MODE algorithm over the other MOEC algorithms.

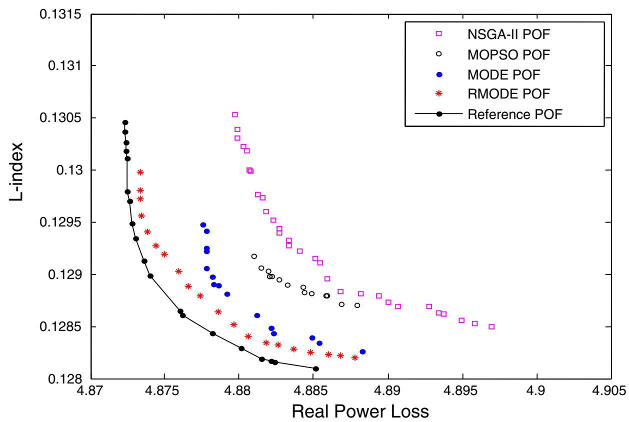

Fig. 2 POFs for minimization of P_{TLOSS} and *L-index—Case 1(i)*

Case 1(ii): Voltage and Line Loading Constraints

With the constraints on voltage magnitude at PQ buses and line loading limits at various lines, the NSGA-II, MOPSO, MODE, RMODE and HMPSO algorithms were implemented to solve the MORPD problem in the IEEE 30-bus test system. The POFs obtained have been compared with the reference Pareto front in Fig. 3. As shown in Fig. 3, the POF obtained using RMODE algorithm is very close to reference POF as compared to those obtained using other MOEC algorithms.

From the Pareto optimal solutions as obtained above, the preferred solutions have been extracted using fuzzy membership-based mechanism [45]. The preferred solutions thus achieved for the minimum P_{TLOSS} and minimum *L-index* at the same time using NSGA-II, MOPSO, MODE, RMODE and HMPSO algorithms and as reported in [46] along with decision variables are depicted in Table 2. The same as given in Table 2, the recurring MODE algorithm gives the minimum real power loss of total transmission lines as 4.87758 MW and minimum *L*-index as 0.12897. This reveals the usefulness of the recurring MODE algorithm over other MOEC algorithms.

The better performance of recurring MODE algorithm over the multi-objective DE algorithm can be clearly observed from the POFs attained by using them for Case

Table 1 Decision variables setting for preferred solution—Case 1(i)

S. no.	Decision variable	Initial setting	Algorithms				
			NSGA-II	MOPSO	MODE	RMODE	HMPSO [36]
Generator voltage settings							
1	V_1	1.0500	1.0720	1.07208	1.07037	1.07154	1.07179
2	V_2	1.0400	1.06334	1.06269	1.06168	1.06232	1.06269
3	V_5	1.0100	1.03955	1.03795	1.03843	1.04049	1.04036
4	V_8	1.0100	1.04023	1.04061	1.03869	1.04023	1.04079
5	V_{11}	1.0500	1.06994	1.07499	1.09185	1.08044	1.08444
6	V_{13}	1.0500	1.05576	1.05084	1.05782	1.05358	1.0492
Transformer tapping setting							
7	T_{11}	1.078	0.99654	1.01871	1.07551	1.0238	1.05655
8	T_{12}	1.069	0.99257	0.95274	0.91382	0.96341	0.92951
9	T_{15}	1.032	0.99536	0.99300	0.99578	0.99221	0.98776
10	T_{36}	1.068	0.97476	0.97642	0.97669	0.9766	0.97364
Shunt VAR source settings							
11	Q_{s10}	0.0	0.02981	0.05000	0.04871	0.04942	0.05000
12	Q_{s12}	0.0	0.04648	0.04983	0.02483	0.04240	0.04965
13	Q_{s15}	0.0	0.04370	0.050000	0.03791	0.04227	0.04955
14	Q_{s17}	0.0	0.05000	0.04963	0.04807	0.04969	0.04979
15	Q_{s20}	0.0	0.04946	0.05000	0.0493	0.04417	0.04456
16	Q_{s21}	0.0	0.04989	0.05000	0.04903	0.04998	0.04997
17	Q_{s23}	0.0	0.04536	0.04152	0.03389	0.04025	0.03418
18	Q_{s24}	0.0	0.04886	0.05000	0.04922	0.04954	0.04997
19	Q_{s29}	0.0	0.04307	0.04127	0.0429	0.04307	0.03358
P_{TLOSS}		5.8423	4.88684	4.88233	4.88323	4.87857	4.87396
L-index		0.1772	0.12883	0.12843	0.1289	0.12864	0.12899

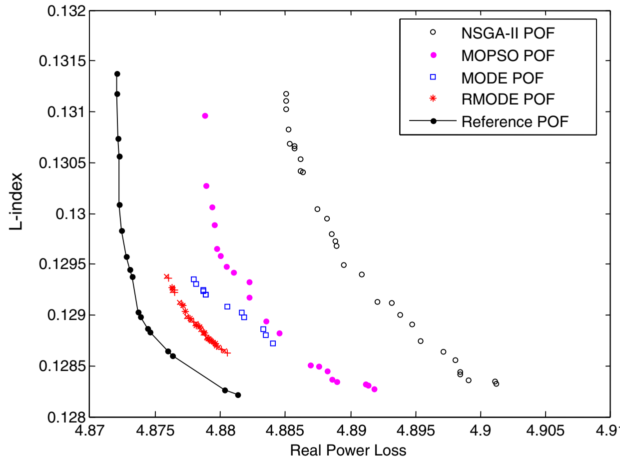

Fig. 3 POFs for minimization of P_{TLOSS} and *L-index—Case 1(ii)*

1(i) and Case 1(ii) and is shown, respectively, in Fig. 2 and Fig. 3. As shown in Figs. 2–3, the recurring MODE algorithm offers improved quality and more number of non-dominated solutions in the POF in comparison with those obtained using multi-objective DE algorithm.

Comparison of P_{TLOSS} and *L-index* objectives as attained using the recurring MODE algorithm with NSGA-II, MOPSO, MODE and HMPSO algorithms for Cases 1(i) and 1(ii) is summarized in Table 3. As is clear from Table 3, the recurring MODE algorithm gives improved results for P_{TLOSS} and *L-index* minimization when optimized simultaneously. Also, this assessment confirms the better performance of the recurring MODE algorithm over NSGA-II, MOPSO and MODE algorithms for solving multi-objective ORPD problem.

75-Bus Practical Power System

The 75-bus system is a practical power system representing 400 kV and 220 kV buses of Uttar Pradesh State Electricity Board [35, 45]. This highly stressed practical power system contains 15 generators at bus numbers 1 to15, 60 load buses from bus numbers 16 to 75 and 17 total number off-nominal tap settings of transformers in line numbers 16,17, 18, 19, …, 32 and 114 transmission lines. This system also contains 12 shunt reactors Q_s as given in Table 4. In this power system, total control variables for optimal reactive power dispatch are 44 (15 + 17 + 12). The upper and lower boundaries on voltage magnitudes at the generator and load buses and on the transformer tap settings considered are same as in IEEE 30-bus system.

Minimum limit of shunt reactors' outputs has been considered as zero, while maximum limit of shunt reactors' outputs is the same as shown in column 3 of Table 4. This column shows the initial settings of 44 control variables and the corresponding objective functions, P_{TLOSS} and *L-index* values. In 75-bus system also, ORPD problem was handled for the two cases. In the first instance, only the voltage constraints were imposed. The results obtained thus are included in Case 2(i), as well as Case 2(ii).

Here also, various trials were taken considering different values of the key parameters for various MOEC algorithms. However, the best results obtained here are for the population size NP = 30, recurring MODE rounds R = 8, maximum number of MODE iterations M equal to 150, mutation factor F equal to 0.44 and Cr equal to 0.97. For implementation of multi-objective DE algorithm and comparison of results, the equivalent number of iterations = 1200 (8 × 150), while the values of F and Cr are the same as in case of RMODE algorithm. For implementation of MOPSO algorithm with number of particles NP = 30, best results obtained were with inertia weight w = 0.73, $c_1 = c_2 = 1.5$ and number of particles in the repository equal to 30. For the purpose of performance comparison, every algorithm was implemented for 1200 iterations for both the cases of 75-bus Indian system.

Case 2(i): Voltage Constraints

With the voltage constraints at various PQ buses, the P_{TLOSS} and *L-index* objective functions were minimized simultaneously using NSGA-II, MOPSO, MODE, RMODE and HMPSO algorithms. The POFs achieved using multi-objective evolutionary techniques are compared in Fig. 4. Figure 4 clearly shows that the POF provided by recurring MODE algorithm is in closed vicinity to reference POF in comparison with the POFs provided by other MOEC algorithms.

The preferred solutions extracted out of non-dominated solutions for the simultaneous minimization of P_{TLOSS} and *L-index* and the corresponding control variables are depicted in Table 4. It is clear from Table 4 that recurring MODE algorithm offers minimum total transmission lines real power losses of 1.85742 MW and minimum voltage stability index of 0.42442, which are better than those of other MOEC algorithms and comparable with those of Reference Pareto set. This expresses the efficiency of the recurring MODE algorithm.

Case 2(ii): Voltage and Line Flows Constraints

NSGA-II, MOPSO, MODE, RMODE and HMPSO algorithms were applied for minimization of P_{TLOSS} and *L-index* with voltage security constraints on PQ buses and line loading constraints on various lines of the power network. The Pareto optimal fronts thus attained are compared in Fig. 5. Figure 5 clearly shows that in comparison with the POFs provided by the other MOEC algorithms, the

Table 2 Decision variables setting for preferred solution—*Case 1(ii)*

S. no.	Decision variable	Methods of algorithm							
		NSGA-II	MOPSO	MODE	RMODE	HMPSO	MOCIPSO [46]	MOIPSO [46]	MOPSO [46]
Generator voltage settings									
1	V_1	1.07296	1.07069	1.07348	1.07174	1.0718	1.10000	0.90000	0.90000
2	V_2	1.06485	1.06207	1.06435	1.06288	1.06271	1.10000	0.90000	1.10000
3	V_5	1.04123	1.03815	1.04119	1.04005	1.04024	1.10000	0.90817	1.10000
4	V_8	1.04258	1.04025	1.0405	1.04054	1.04079	1.10000	1.10000	0.90000
5	V_{11}	1.06443	1.08145	1.08284	1.08381	1.08444	1.10000	1.10000	0.90000
6	V_{13}	1.04601	1.05591	1.04909	1.05415	1.04984	1.10000	1.10000	1.10000
Transformer tapping setting									
7	T_{11}	0.99965	1.01564	1.0434	1.02168	1.03948	0.94000	0.95000	1.01000
8	T_{12}	0.99316	0.9706	0.93533	0.96714	0.94601	1.10000	1.10000	0.91000
9	T_{15}	0.98572	0.99598	0.98261	0.98940	0.9887	1.10000	1.10000	1.10000
10	T_{36}	0.98028	0.98112	0.97433	0.97786	0.97338	0.94000	0.95000	0.94000
Shunt VAR source settings									
11	Q_{s10}	0.05000	0.04965	0.04939	0.04942	0.04992	0.22000	0.28000	0.00000
12	Q_{s12}	0.05000	0.05000	0.0274	0.0205	0.04999	0.30000	0.30000	0.30000
13	Q_{s15}	0.05000	0.02056	0.04832	0.04266	0.04712	0.12000	0.08000	0.00000
14	Q_{s17}	0.05000	0.05000	0.04982	0.04895	0.04998	0.09000	0.00000	0.00000
15	Q_{s20}	0.05000	0.05000	0.02949	0.04908	0.04415	0.00000	0.05000	0.00000
16	Q_{s21}	0.04989	0.05000	0.04972	0.04988	0.04999	0.11000	0.17000	0.23000
17	Q_{s23}	0.03930	0.04879	0.0449	0.04199	0.03525	0.01000	0.06000	0.09000
18	Q_{s24}	0.05000	0.05000	0.04998	0.04991	0.04998	0.07000	0.00000	0.00000
19	Q_{s29}	0.03991	0.03954	0.03505	0.04107	0.0333	0.03000	0.04000	0.05000
P_{TLOSS}		4.88947	4.88043	4.87890	4.87758	4.87386	5.2320	5.2790	5.3080
L-index		0.12949	0.12948	0.12920	0.12897	0.12901	0.11821	0.11904	0.12191

Table 3 Preferred solution for minimization of P_{TLOSS} and *L-index* in IEEE 30-bus system

Case	Method	Preferred solution	
		P_{TLOSS} *(MW)*	*L-index*
1(i)	NSGA-II algorithm	4.88684	0.12883
	MOPSO algorithm	4.88233	0.12843
	MODE algorithm	4.88323	0.12890
	RMODE algorithm	**4.87857**	**0.12864**
	HMOPSO algorithm [36]	4.87396	0.12899
1(ii)	NSGA-II algorithm	4.88947	0.12949
	MOPSO algorithm	4.88043	0.12948
	MODE algorithm	4.87890	0.12920
	RMODE algorithm	**4.87758**	**0.12897**
	HMOPSO algorithm	4.87386	0.12901
	MOCIPSO algorithm [46]	5.2320	0.11821
	MOIPSO algorithm [46]	5.2790	0.11904
	MOPSO algorithm [46]	5.3080	0.12191

Table 4 Decision variables setting for preferred solution—*Case 2(i)*

S. no.	Decision variable	Initial setting	Method/algorithm				
			NSGA-II	MOPSO	MODE	RMODE	HMPSO
Generator voltage settings							
1	V_1	1.030	1.06378	1.06338	1.06193	1.06377	1.06036
2	V_2	1.050	1.09155	1.03696	1.04848	0.99148	1.0746
3	V_3	1.030	1.08172	1.06429	1.05208	1.03209	1.07554
4	V_4	1.050	1.03555	1.0179	1.0827	1.04722	1.06474
5	V_5	1.050	0.95	1.03093	1.02667	1.04235	0.99973
6	V_6	1.050	1.06676	1.00562	1.01639	1.04604	0.96395
7	V_7	1.050	0.99141	1.02341	1.05555	1.03043	1.07137
8	V_8	1.050	0.98524	0.98852	0.99824	1.0218	1.00281
9	V_9	1.050	1.02816	0.96154	1.07671	1.03792	1.00812
10	V_{10}	1.020	1.01253	1.05256	1.02946	1.04961	1.09002
11	V_{11}	1.020	1.08069	1.06526	0.99383	1.03178	1.07837
12	V_{12}	1.050	1.05041	1.03656	1.07798	1.09583	1.04255
13	V_{13}	1.050	1.04073	1.0721	1.04633	1.0962	1.07957
14	V_{14}	1.030	0.96969	0.98311	1.09243	1.06829	1.04157
15	V_{15}	1.010	1.02876	0.99295	0.98236	0.98488	0.99195
Transformer tapping setting							
16	T_{16}	1.0	0.95928	1.01328	0.98664	0.99537	0.99783
17	T_{17}	1.0	1.02299	0.96161	0.95511	0.95022	0.98323
18	T_{18}	1.0	1.01107	0.98146	0.98067	0.98268	0.98163
19	T_{19}	1.0	1.00241	1.03865	1.0235	1.02463	0.99035
20	T_{20}	1.0	0.9691	1.0053	0.95007	0.9549	0.98884
21	T_{21}	1.0	0.95777	0.98148	0.9502	0.95331	0.96383
22	T_{22}	1.0	0.96191	0.9504	0.97198	0.96943	0.95665
23	T_{23}	1.0	0.97802	1.00009	0.99454	0.98098	0.97551
24	T_{24}	1.0	0.97063	0.96384	0.95057	0.96705	0.97902
25	T_{25}	1.0	1.1	0.9549	0.95017	0.95159	0.95001
26	T_{26}	1.0	0.95503	1.03285	0.95051	1.06421	0.97276
27	T_{27}	1.0	0.9501	1.02003	1.04369	0.9507	1.05082
28	T_{28}	1.0	1.00668	0.98765	0.97445	0.98136	1.00867
29	T_{29}	1.0	0.95913	1.0042	1.01855	1.00584	1.00066
30	T_{30}	1.0	1.02188	0.98143	0.99163	0.98795	0.97989
31	T_{31}	1.0	1.01128	0.95075	1.01909	1.01635	0.98695
32	T_{32}	1.0	0.95319	1.01241	0.95002	0.95005	0.97569
Shunt reactors settings							
33	Q_{s17}	1.00	0.24482	0.00895	0.1767	0.00386	0.00926
34	Q_{s19}	0.50	0.24219	0.14997	0.00127	0.0005	0.00022
35	Q_{s22}	0.50	0.27856	0.48956	0.21042	0.00271	0.4671
36	Q_{s23}	1.00	0.94969	0.3301	0.92521	0.94147	0.7183
37	Q_{s26}	1.63	0.00874	0.00273	0	0.00116	0.00433
38	Q_{s29}	1.00	0.70346	0.52397	0.43845	0.6547	0.48107
39	Q_{s35}	0.50	0.11575	0.14741	0.0204	0.01492	0.05602
40	Q_{s36}	0.50	0.04468	0.30179	0.00001	0.0005	0.01086
41	Q_{s41}	2.23	2.1593	1.19611	1.19871	1.49052	1.4605
42	Q_{s42}	0.63	0.36542	0.31331	0.03409	0.05239	0.13921
43	Q_{s73}	0.50	0.49909	0.5	0.49957	0.4995	0.49943
44	Q_{s74}	2.83	1.71797	2.80696	2.49491	2.36265	2.59054
P_{TLOSS}		2.05591	1.86935	1.8575	1.85809	1.85742	1.85668
L-index		0.5025	0.42600	0.42604	0.42448	0.42442	0.42385

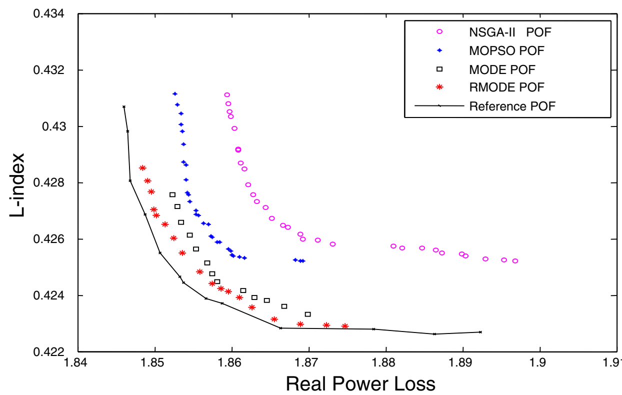

Fig. 4 POFs for minimization of P_{TLOSS} and *L-index—Case 2(i)*

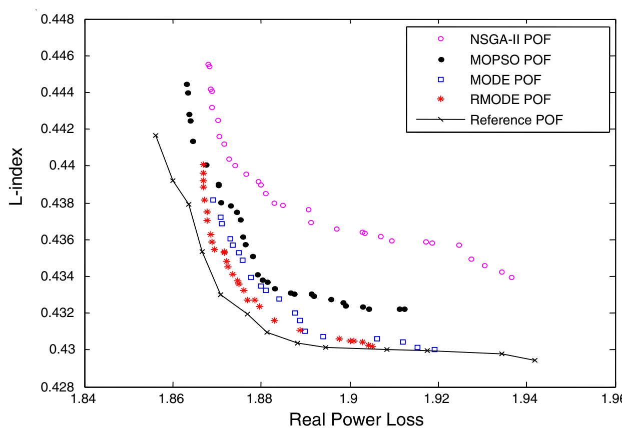

Fig. 5 POFs for minimization of P_{TLOSS} and *L-index—Case2 (ii)*

Pareto optimal front provided by the recurring MODE algorithm is very close to reference POF. From the Pareto optimal solutions obtained using various algorithms, the preferred solutions have been extracted and are shown along with their corresponding decision variables in Table 5. As per the observation of Table 5, the recurring MODE algorithm gives the minimum total transmission lines real power losses of 1.87698 MW with minimum *L-index* of 0.43265, which are better than those of other MOEC algorithms and comparable with those of reference Pareto set. This clearly reveals the efficacy of the proposed recurring MODE algorithm.

The superior performance of the recurring MODE over the multi-objective DE algorithm can be noticed from the POFs attained by using them for both cases and is shown, respectively, in Fig. 4 and Fig. 5. This is noticed from Figs. 4 and 5 that the recurring MODE algorithm offers improved quality and larger set of non-dominated solutions in the POF in comparison with those attained using MODE algorithm.

Comparison of P_{TLOSS} and *L-index* objectives as attained using the recurring MODE algorithm with NSGA-II, MOPSO, MODE and HMPSO algorithms for both cases is summarized in Table 6. As clearly shown in Table 6, the recurring MODE algorithm gives improved results for P_{TLOSS} and *L-index* minimization when optimized concurrently. Also, this assessment confirms the superiority of recurring MODE algorithm over NSGA-II, MOPSO and MODE algorithms for solving MORPD problem.

Performance Comparison of Various MOEC Algorithms

Performance of various single-objective evolutionary computing or multi-objective evolutionary computing techniques cannot be compared only on the basis of the preferred or best compromised solution provided by them. Some standard tests like analysis of variance (ANOVA) test are to be carried out in evaluating the performance of single-objective evolutionary computing algorithms [48]. However, ANOVA test has also been carried out in multi-objective evolutionary computing (MOEC) algorithms, but for determining the importance of various parameters on the given objective function and selecting the more influential parameters [51–53].

To measure the performance of various MOEC algorithms, many performance metrics such as generational distance (GD), inverted generational distance (IGD), maximum Pareto front error (MPFE) and spacing are calculated to declare the best one [54, 55]. The performance metrics GD, IGD, MPFE and spacing are calculated for all the four MOEC algorithms applied for solving MORPD problem and listed in Table 7 and Table 8 for 30-bus IEEE test system and for 75-bus practical power system, respectively. On the basis of various metrics, it can be observed that RMODE algorithms are the better algorithms out of the four MOEC algorithms.

Performance Metrics

Nowadays, there are many metrics to measure performance of MOEAs. Among them, the following five metrics are widely employed. They can reveal the convergence and diversity of MOEAs very well. However, many researches just employ a few of them to evaluate algorithms and argue that their proposed algorithms are the best. In fact, it is unfair to give the conclusion without comprehensive metrics and evaluations. Therefore, the five metrics are selected to make the comprehensive comparisons. Generational distance

$$GD = \frac{\sqrt{\sum_{i=1}^{n} d_i^2}}{n}$$

where $d_i = \min j \| f(xi) — PF_{true}(xj) \|$ is the distance between non-dominated solution $f(xi)$ and the nearest Pareto front

Table 5 Decision variables setting for preferred solution—*Case 2(ii)*

S. no.	Decision variable	Methods/algorithm				
		NSGA-II	MOPSO	MODE	RMODE	HMPSO
Generator voltage setting						
1	V_1	1.07844	1.03977	1.04358	1.03811	1.06656
2	V_2	1.07628	1.06513	1.09586	1.06513	1.07995
3	V_3	1.06258	1.09988	1.01322	1.09988	1.07554
4	V_4	0.96453	1.04984	1.02059	1.04984	1.00451
5	V_5	0.99154	0.96216	0.95346	0.96216	0.99478
6	V_6	1.00927	0.97634	1.06123	0.97634	0.98919
7	V_7	0.96339	0.99952	0.95618	0.99952	1.00178
8	V_8	1.07241	1.01544	0.99792	1.01544	1.02198
9	V_9	0.98472	1.07856	1.04893	1.07856	1.0547
10	V_{10}	1.06295	1.08922	1.07405	1.08922	1.05737
11	V_{11}	1.05288	1.07986	1.06639	1.07986	1.07521
12	V_{12}	1.07109	1.0023	1.06455	1.0023	1.05313
13	V_{13}	1.04388	1.0676	1.03389	1.0676	1.05493
14	V_{14}	0.95115	1.0212	1.01539	1.0212	1.0031
15	V_{15}	0.98481	0.99926	0.96922	0.99926	1.03926
Transformer tapping setting						
16	T_{16}	1.06408	0.95	0.98722	0.95	0.99289
17	T_{17}	0.93845	1.02522	1.02147	1.02522	0.97923
18	T_{18}	0.98167	1.00345	1.01096	1.00345	0.99867
19	T_{19}	1.03072	0.95526	1.0536	0.95526	0.97982
20	T_{20}	0.98347	1.05075	0.95964	1.05075	1.02422
21	T_{21}	0.92521	0.97286	0.95826	0.97286	0.95343
22	T_{22}	0.96952	0.99362	0.95163	0.99362	0.98804
23	T_{23}	0.97625	1.04446	0.95307	1.04446	1.02741
24	T_{24}	0.96596	0.9773	0.98756	0.9773	0.96901
25	T_{25}	0.96752	1.00056	1.0334	1.00056	0.9892
26	T_{26}	0.98921	1.00581	0.9931	1.00581	1.06335
27	T_{27}	1.06939	0.97676	0.98759	0.97676	0.98059
28	T_{28}	1.03455	1.03282	1.00197	1.03282	0.98776
29	T_{29}	0.93871	0.95	1.01865	0.95	0.99536
30	T_{30}	1.02028	1.00937	1.02157	1.00937	1.0343
31	T_{31}	1.07432	0.95015	0.98186	0.95015	0.98995
32	T_{32}	0.90000	1.01538	0.98844	1.01538	0.98997
Shunt reactors settings						
33	Q_{s17}	0.26679	0.48228	0.57711	0.48228	0.00002
34	Q_{s19}	0.29141	0.23175	0.0	0.23175	0.00004
35	Q_{s22}	0.40243	0.49908	0.05012	0.49908	0.5
36	Q_{s23}	0.96378	0.87482	0.60796	0.87482	0.41272
37	Q_{s26}	1.44517	0.09452	0.36718	0.09452	0.00335
38	Q_{s29}	0.63971	0.7051	0.82353	0.7051	0.99965
39	Q_{s35}	0.06779	0.08292	0	0.08292	0.39505
40	Q_{s36}	0.29510	0.02816	0.04764	0.02816	0.02804
41	Q_{s41}	0.80627	0.31598	0.39383	0.31598	2.09884
42	Q_{s42}	0.15941	0.22083	0.43324	0.22083	0.39485
43	Q_{s73}	0.10109	0.47887	0.39569	0.47887	0.49943
44	Q_{s74}	1.87438	2.59483	2.76236	2.59483	2.21748
P_{TLOSS}		1.88995	1.88031	1.88979	1.87698	1.88244
L-index		0.43799	0.43384	0.43101	0.43265	0.43144

Table 6 Preferred solution for minimization of P_{TLOSS} and *L-index* in 75-bus power system

Case	Method	Preferred solution	
		P_{TLOSS} *(MW)*	*L-index*
2(i)	NSGA-II algorithm	1.86935	0.42600
	MOPSO algorithm	1.85750	0.42604
	MODE algorithm	1.85809	0.42448
	RMODE algorithm	**1.85742**	**0.42442**
	HMPSO algorithm	1.85668	0.42385
2(ii)	NSGA-II algorithm	1.88995	0.43799
	MOPSO algorithm	1.88031	0.43384
	MODE algorithm	1.88979	0.43101
	RMODE algorithm	**1.87698**	**0.43265**
	HMPSO algorithm	1.88244	0.43144

Table 7 Performance comparison for 30-bus IEEE test system

Performance metrics	*Case 1(i)*				*Case 1(ii)*			
	NSGA-II	MOPSO	MODE	RMODE	NSGA-II	MOPSO	MODE	RMODE
GD	0.00088	0.00032	0.00028	0.00021	0.00243	0.00228	0.00052	0.00026
IGD	0.00501	0.00573	0.00311	0.00062	0.01415	0.011	0.00418	0.00254
MPFE	0.01186	0.00289	0.00315	0.00268	0.0155	0.01693	0.00262	0.00168
Spacing	0.00322	0.00054	0.00068	0.00054	0.00244	0.00383	0.00055	0.00047

Table 8 Performance comparison for 75-bus practical power system

Performance metrics	*Case 2(i)*				*Case 2(ii)*			
	NSGA-II	MOPSO	MODE	RMODE	NSGA-II	MOPSO	MODE	RMODE
GD	0.00093	0.00066	0.00063	0.00054	0.00132	0.00071	0.00056	0.00051
IGD	0.00799	0.00711	0.00612	0.00363	0.00737	0.00719	0.00613	0.00615
MPFE	0.00746	0.00583	0.00356	0.00355	0.00975	0.00687	0.00461	0.00461
Space	0.00131	0.00115	0.00085	0.00084	0.00124	0.00111	0.00094	0.00093

solution in objective space. It is to measure the closeness of the solutions to the real Pareto front. If GD is equal to zero, this reveals that all the non-dominated solutions generated are located in the real Pareto front. Therefore, the lower value of GD indicates that the algorithm has better performance [54, 55].

- *IGD*: PF_{true} is a set of uniformly distributed points in the objective space. *PA* is the no dominated solution set obtained by an algorithm and the distance from PF_{true} to P_A is defined as

$$IGD(P_A, PF_{true}) = \frac{\sum_{v \in PFtrue} d(v, PA)}{|PF_{true}|}$$

where (υ, *PA*) is the minimum Euclidean distance between v and the points in P_A. Algorithms with smaller IGD values are desirable [54, 55].

- *Hyper volume:* This hyper volume metric calculates the volume (in the objective space) covered by members of non-dominated solutions sets obtained by MOEAs where all objectives are to be minimized [54]. A hypervolume can be calculated as follows:

$$HV = volume \bigcup_{i=1}^{P_A} \upsilon_i.$$

The larger the HV value is, the better the algorithm is.

Spacing: The metric spacing is to measure how uniformly the non-dominated set is distributed. It can be formulated as follows:

$$s = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(d_i - d)^2}$$

where *di* is the same as the *di* in GD metric, *d* is the average value of *di* and *n* is the number of individuals in non-dominated set. The smaller the spacing is, the better the algorithm performs [55].

- *Maximum Pareto front error***:** It is to measure the worst case and can be formulated as follows:

$MPFE = max(d_i)$

where *di* is the same as *di* employed in GD. MPFE is the largest distance among these *di*. The lower the value of MPFE is, the better the algorithm is [55].

Conclusions

In this paper, a new approach RMODE algorithm has been implemented for solving multi-objective reactive power dispatch problem having two competing objectives, namely total transmission lines real power losses minimization and enhancement of voltage stability. Superiority of the proposed RMODE algorithm has been established by implementing RMODE algorithm to solve the MORPD problem in the standard 30-bus IEEE test system and 75-bus practical power system and by comparing the results with those provided by NSGA-II, MOPSO, MODE and HMPSO algorithms and the reported results. Comparison of the results and the performance metrics clearly demonstrates the efficiency of the recurring MODE algorithm in terms of improved solution quality and confirms its potential to solve the multi-objective optimization problem in practical power systems as well.

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Declarations

Conflict of interest The authors declare that they have no conflict of interest.

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