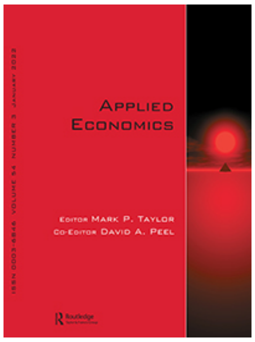

Applied Economics

ISSN: (Print) (Online) Journal homepage: https://www.tandfonline.com/loi/raec20

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APPLIED ECONOMICS
2022, VOL. 54, NO. 3, 234–248
https://doi.org/10.1080/00036846.2021.1876208

Routledge
Taylor & Francis Group

ARTICLE

The Macroeconomic Effects of Positive Trend Inflation in a Small Open Economy

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ABSTRACT
The existing New Keynesian open economy literature tends to make the simplifying assumption that there is no trend inflation. In this paper, we reformulate the standard open economy model to account for positive trend inflation. We then employ the model to understand the effects of macroeconomic shocks in a small open economy when trend inflation is positive. Our main finding is that allowing for trend inflation significantly affects the dynamics of the model through real exchange rate dynamics rather than the slope of the New Keynesian Philips Curve. Specifically, higher trend inflation induces modestly more persistent real exchange rates' responses to the shocks. Further incorporation of trend inflation in an open economy enables us to discuss the Purchasing Power Parity and Delayed Overshooting Puzzles.

KEYWORDS
Trend Inflation; Small Open Economy; New Keynesian models; New Keynesian Phillips Curve; Real Exchange Rate

JEL CLASSIFICATION
E31; E52; F41

I. Introduction

Most of the popular New Keynesian models make a simplifying assumption and presume that there is no inflation in the steady state. However, even during the Great Moderation, average inflation rates in developed economies have been around 2.5%. Also, the central banks target around 2% inflation rate in those economies. It can thus be claimed that zero-trend inflation assumption may make models biased. Starting with King and Wolman (1996), and Ascari (2000), several authors relax the zero-trend inflation assumption, allow for positive trend inflation in their models, and study its effects on macroeconomic dynamics.

Following the Global Financial Crisis of 2008–9, trend inflation again gained importance with the Zero Lower Bound (ZLB) phenomenon. It was exactly at this point that higher inflation targeting became a debate, i.e., whether to use higher inflation targeting as a monetary policy tool to prevent interest rate hitting the ZLB. Broadly speaking, two opposing views can be cited on this debate: Blanchard et al. (2010), Williams (2009) and Ball (2013) suggest that higher inflation targeting leads to a higher inflation environment and thus to higher nominal interest rate. This means that in case of a deflationary shock, the Central Bank has more room to decrease the interest rate. On the other hand, Bernanke (2010) underlines that higher inflation targeting damages inflation expectations and so the pricing behaviour of firms. In turn, higher contemporaneous inflation and higher volatility in inflation may occur.

The effects of positive trend inflation on macroeconomic dynamics in Dynamic Stochastic General Equilibrium (DSGE) models are thoroughly discussed in the literature. Two main paths are identified through which trend inflation can be incorporated into models. The first one is a fixed positive trend inflation rate, which is widely analysed in models by Ascari (2004), Ascari and Ropele (2007, 2009), and Ascari and Sbordone (2014). The second path is a time-varying trend inflation rate whose effects on macroeconomic dynamics are analysed by Cogley and Sbordone (2008), and Lie and Yadav (2017). All papers conclude that the importance of current variables and forward-looking variables both vary as trend inflation increases. Higher trend inflation flattens the slope of the New-Keynesian Phillips Curve (NKPC) but increases the importance of forward-looking variables on the NKPC. Since we use the first-order approximation method, we opt for the former method to analyse the effect of trend inflation.

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When analysing trend inflation so far, the existing literature has focused on the implications of higher trend inflation target using closed-economy models. However, today's global openness could not be ignored for almost all countries. Therefore, using closed-economy models to examine the effects of trend inflation may lead to misleading implications.

In this paper, we take the challenge of extending the small open economy model proposed by Gali and Monacelli (2005) to account for positive trend inflation in Canada. Canada is a typical example of a small open economy that could be significantly affected by large economies. For example, the U.S. Credit Crunch in 2007 adversely influenced the Canadian economy. However, any policy change in the Canadian economy is unlikely to affect the rest of the world. Moreover, the Bank of Canada has implemented an inflation-targeting policy and reduced the nominal interest rate near to zero in order to combat the deflationary effects of the Global Financial Crisis.

When trend inflation is zero, the model in this study collapses to the one described by Gali and Monacelli (2005). To the best of our knowledge, the model developed in this study is one of the first models which incorporates trend inflation in an open economy framework. When the effects of positive trend inflation on the dynamics of the model are analysed, it is found that openness does not affect either the importance of contemporaneous or forward-looking variables at higher trend inflation rates. The implications of higher trend inflation are similar to the closed economy modelled by Ascari (2004), Ascari and Ropele (2007), and Ascari and Sbordone (2014). However, openness provides another channel: real exchange rate dynamics. For a given degree of openness, the impact effect of the monetary policy shock on the real exchange rate increases and the real exchange rate becomes modestly more persistent with increased trend inflation rate. These results are valid for demand and cost-push shocks.

We also study the effects of positive trend inflation in response to monetary policy, demand, and cost-push shocks to understand how the dynamics of the model respond to those shocks at different trend inflation rates. We find that a higher trend inflation decreases the effects of the monetary policy and demand shocks on domestic inflation while it increases the effects of the shocks on output. The primary reason is that the increase in the trend inflation rate flattens the NKPC. Thus, changes in the current demand have smaller impacts on domestic inflation. On the other hand, foreign goods become relatively cheaper (more expensive) as trend inflation rises in the case of the monetary policy (demand) shock. Therefore, the effect of positive trend inflation on CPI inflation is limited for both types of shock. In the case of the cost-push shock, higher trend inflation makes domestic inflation (output) increase (decrease) more. This is mainly because positive trend inflation destabilizes domestic inflation expectations. Thus, domestic prices rise more while output drops more with increasing trend inflation. Since foreign goods become cheaper with higher trend inflation, the effect of trend inflation on CPI inflation is lower relative to the effect on domestic inflation in response to the shock.

The dynamics of real exchange rates are highly debated in the literature. In general, it is claimed that real exchange rates are more persistent, more volatile, and exhibit more hump-shaped dynamics empirically than the models' predictions. Rogoff (1996) refers to the high volatility and high persistence anomalies as the 'Purchasing Power Parity (PPP) Puzzle', while Eichenbaum and Evans (1995) call the hump-shaped dynamics anomaly the 'Delayed Overshooting Puzzle'. Dornbusch (1976) suggests that incorporating sticky prices into models succeeds in predicting the high volatility of the real exchange rate to some extent but also fails to predict the high persistence of the real exchange rate (Chari et al. (2002)).

In the literature, two important questions arise about the PPP Puzzle and the Delayed Overshooting Puzzle. First, what is the source of persistence in generating persistent real exchange rates? Is it persistent shock, policy inertia, or both (Benigno (2004); Steinsson (2008); Carvalho et al. (2019))? Second, is it monetary policy shock or other shocks like demand, technology, and Philips curve shocks that predict the behaviour of real exchange rates (Clarida and Gali (1994); Eichenbaum and Evans (1995); Steinsson (2008))?

In light of these questions, we examine whether the model developed in this paper contributes to the explanation for the PPP Puzzle and the Delayed Overshooting Puzzle in the cases of the monetary, demand, and cost-push shocks under alternative scenarios. We find that higher trend inflation yields modestly more persistent dynamics of the real exchange rate under all alternative scenarios excluding the independent and identically distributed (*i.i.d.*) demand shock with the inertial policy rule and more volatile dynamics of the real exchange rate under all alternative scenarios. Moreover, we find that higher trend inflation induces delayed overshooting of the real exchange rate in response to persistent cost-push shock under the inertial policy rule.

The results obtained for Canada may change for other small open economies since the degree of openness and the degree of price stickiness may differ, though the general trends may be valid.

Section 2 introduces the model. Section 3 presents the Impulse Response Functions (IRFs) in response to shocks and the effects of these shocks at higher levels of trend inflation. Section 4 discusses the PPP Puzzle and the Delayed Overshooting Puzzle. Section 5 concludes the study.

II. Model[1]

Following Ascari and Sbordone (2014), this paper incorporates positive trend inflation into the small open economy model by Gali and Monacelli (2005). In this section, we do not repeat the equations and details of the model by Gali and Monacelli (2005), and Ascari and Sbordone (2014). We instead describe the main features of our model. The demand and market equilibrium blocks of the model are identical to the model in Gali and Monacelli (2005). The equations in the supply block are based on Ascari and Sbordone (2014). The monetary policy rule is a standard Taylor rule.

In the model, there is a continuum of countries indexed by $i \in [0, 1]$ and a representative, infinitely lived household residing in each country. The household provides labour force to domestic intermediate firms and purchases a composite consumption basket including both domestically produced and imported goods. The household can purchase internationally traded state-contingent bonds without transaction costs. There is no friction in the labour market. The Law of One Price (LOOP) is assumed to hold for all varieties of goods. The countries initially have identical conditions (i.e., zero-net asset holdings and an ex-ante environment) at time 0. We assume that all countries are symmetric.

In every country, there is a continuum of monopolistically competitive intermediate firms indexed by $j \in [0,1]$ and a final goods-producing firm. The final goods-producing firm buys differentiated intermediate goods $Y_{j,t}$, produced by intermediate goods-producing firms employing the same technology, and produces the final goods Y_t using the Dixit-Stiglitz production function. Moreover, intermediate goods-producing firms use the Calvo (1983) rule on resetting prices. $(1-\theta)$ share of intermediate firms update their nominal prices and θ share of the firms keep their price levels constant in each period.

The log-linearized first-order condition of the profit maximization problem of the intermediate firm j can be expressed as follows[2]:

$$\hat{x}_t = \hat{\psi}_t - \hat{\phi}_t \tag{1}$$

where

$$\hat{\psi}_t = (1-\theta\beta\pi^{\epsilon})\left(\widehat{mc}^r_{H,t}\right) + (\theta\beta\pi^{\epsilon})E_t\left(\epsilon\hat{\pi}_{H,t+1} + \hat{\psi}_{t+1}\right) \tag{2}$$

$$\hat{\phi}_t = \left(\theta\beta\pi^{\epsilon-1}\right)E_t\left((\epsilon-1)\hat{\pi}_{H,t+1} + \hat{\phi}_{t+1}\right) \tag{3}$$

$\hat{x}_t$ is the real reset price, while $\hat{\psi}_t$ and $\hat{\phi}_t$ define the present value of the discounted marginal cost and the present value of the discounted marginal revenue, respectively. E_t is the expectation operator at time t. β is the intertemporal discount factor, ϵ is the elasticity of substitution between the differentiated goods, π is the trend inflation rate, $\hat{\pi}_{H,t}$ is the domestic inflation rate, and $\widehat{mc}^r_{H,t}$ is the real marginal cost defined as:

[1]Variables with a subscript $i \in [0,1]$ indicate that those variables belong to country i. Variables without any index notation indicate that the variables belong to the domestic economy. Variables representing the world economy are indicated by a star superscript.

[2]The superscript hat shows the log-linearization of any variable from its steady state.

$$\widehat{mc}^r_{H,t} = \varphi\hat{z}_t + (\sigma - \sigma_\alpha)\hat{y}^*_t + (\varphi + \sigma_\alpha)\hat{y}_t - (1+\varphi)\hat{a}_t \quad (4)$$

where $\hat{z}_t$ is price dispersion, $\hat{a}_t$ is technology, and $\hat{y}_t$ is output. $\sigma_\alpha = \frac{\sigma}{1-\alpha+\alpha(\sigma\gamma+(1-\alpha)(\sigma\eta-1))}$ where φ is the inverse Frisch elasticity of labour supply, σ is the inverse elasticity of intertemporal substitution of consumption, α is the degree of openness, γ is the elasticity of substitution between goods produced in foreign countries, and η is the elasticity of substitution between domestic and imported goods.

Price dispersion $\hat{z}_t$ is

$$\hat{z}_t = (-\epsilon(1-\theta\pi^\epsilon))(\hat{x}_t) + \theta\pi^\epsilon\left(\epsilon\hat{\pi}_{H,t} + \hat{z}_{t-1}\right) \quad (5)$$

Trend Inflation, Trade Openness, and Macroeconomic Dynamics

Ascari and Sbordone (2014) discuss how the presence of trend inflation affects the dynamics of the NKPC for a closed economy. In the light of their discussions, we evaluate how the presence of positive trend inflation affects the standard small open economy in Gali and Monacelli (2005)'s model. With positive trend inflation, the NKPC can be expressed as in Equation 6.

$$\hat{\pi}_{H,t} = \kappa_1\left(\widehat{mc}^r_{H,t}\right) + \kappa_2 E_t(\hat{\pi}_{H,t+1}) + \kappa_3 E_t(\hat{\psi}_{t+1}) \quad (6)$$

where

$$\kappa_1 = \frac{(1-\beta\theta\pi^\epsilon)(1-\theta\pi^{\epsilon-1})}{\theta\pi^{\epsilon-1}},$$
$$\kappa_2 = \beta\left[\left(1-\theta\pi^{\epsilon-1}\right)(\pi-1)\epsilon+1\right] \text{and}$$
$$\kappa_3 = \beta\left(1-\theta\pi^{\epsilon-1}\right)(\pi-1)$$

Compared to the standard small open economy model in Gali and Monacelli (2005), positive trend inflation changes the dynamics of the model as in Ascari and Sbordone (2014). Three channels can be identified in the background of changes in the dynamics of the model. First, since positive trend inflation makes price-setting firms more forward-looking, higher trend inflation flattens the slope of the NKPC. In other words, while the importance of current variables on the NKPC decreases, the importance of future variables increases. Second, trend inflation causes an extra variable, expected marginal cost, to enter the NKPC. Third, positive trend inflation makes price dispersion significant for the dynamics of the model. In addition, it provides an extra source of persistence to the model due to its backwardness. In turn, it yields more persistent NKPC dynamics. However, for zero-trend inflation, price dispersion loses its importance on the dynamics of the model.

We next discuss whether openness affects the implications of positive trend inflation on the model. There are two possible ways for openness to affect the implications of positive trend inflation on the model through the slope of the NKPC and the real exchange rate dynamics.

First, marginal cost is rewritten as:

$$\widehat{mc}^r_{H,t} = \varphi\hat{z}_t + (\sigma - \sigma_\alpha)\hat{y}^*_t + (\varphi + \sigma_\alpha)\hat{y}_t - (1+\varphi)\hat{a}_t$$

Under the balanced trade condition, σ_α equals σ. Thus, the slope of the NKPC does not depend on the degree of openness. In other words, the degree of openness does not change the importance of either current or forward-looking variables on the NKPC. Considering our model, openness does not affect the slope of the NKPC with increased trend inflation rate.

Second, openness affects the dynamics of variables through the terms of trade variable. More specifically, the terms of trade variable affects the propagation mechanism of each shock so that it affects the variables' dynamics. As indicated in Equation 7, terms of trade is a backward looking variable and so yields more persistence for the dynamics of the model and variables[3]:

$$\hat{s}_t = \hat{s}_{t-1} + \hat{e}_t - \hat{e}_{t-1} + \hat{\pi}^*_t - \hat{\pi}_{H,t} \quad (7)$$

where $\hat{s}_t$ is the terms of trade, $\hat{\pi}^*_t$ is the world inflation rate and $\hat{e}_t$ is the nominal exchange rate.

To show how the interaction between trend inflation and openness affects the dynamics of variables in response to shocks, we use real exchange rate dynamics which is a function of the terms of trade (Gali and Monacelli, 2005, pg. 713). For simplicity, we set calibration values for σ and φ as 1 and 0, respectively. The world interest rate,

[3]It is calculated as the first-order difference of equation 15 in Gali and Monacelli (2005, pg 713).

inflation rate, and consumption are set to 0. Moreover, we simplify the Taylor rule to $\hat{i}_t = \phi_\pi \hat{\pi}_t + v_t$ where $\beta = \frac{1}{\phi_\pi}$. The exogenous shock m_t follows AR(1) with $\rho_m \in [0,1)$ where $m \in (v, d, u)$ implies the persistence of shock. $E_t(m_{t+1})$ equals $\rho_m m_t$. v, d and u imply the monetary policy shock, the demand shock, and the cost-push shock, respectively.

The dynamics of the real exchange rate $\hat{q}_t$ is summarized below[4]:

$$\hat{q}_t = \frac{1}{\Lambda}[\frac{\alpha}{1-\alpha}\hat{q}_{t-1} + \beta E_t(\hat{q}_{t+1}) - \kappa_2 E_t(\hat{\pi}_{H,t+1}) + \beta E_t(\hat{\pi}_{t+1}) - \kappa_3 E_t(\hat{\psi}_{t+1}) - \beta v_t + \beta d_t - u_t] \tag{8}$$

where $\Lambda = \frac{\kappa_1 + (1-\alpha)\beta + \alpha}{1-\alpha}$. $\kappa_1, \kappa_2,$ and κ_3 are defined previously.

The dynamics of the real exchange rate depends on its backward-looking variable, its expected variable, other expected variables (CPI inflation rate $E_t(\hat{\pi}_{t+1})$ and the domestic inflation rate $E_t(\hat{\pi}_{H,t+1})$, an expected auxiliary variable $E_t(\hat{\psi}_{t+1})$, and shocks. The coefficients of the dependent variables are affected by the rate of trend inflation and the degree of openness. For further analysis of the effects of the trend inflation rate and the degree of openness, *ceteris paribus* is assumed. Thus, $E_t(\hat{\pi}_{H,t+1}), E_t(\hat{\pi}_{t+1})$ and $E_t(\hat{\psi}_{t+1})$ are dropped from Equation 8 and the equation is expressed as follows:

$$\hat{q}_t = \frac{1}{\Lambda}[\frac{\alpha}{1-\alpha}\hat{q}_{t-1} + \beta E_t(\hat{q}_{t+1}) - \beta v_t + \beta d_t - u_t] \tag{9}$$

Since the coefficients for monetary policy shock $-\frac{\beta}{\Lambda}$, demand shock $\frac{\beta}{\Lambda}$, and cost-push shock - $\frac{1}{\Lambda}$ are similar in absolute terms in this equation, we will only discuss real exchange rate dynamics for the monetary policy shock to obtain a general inference about how the interaction between trend inflation and openness affects the dynamics of the real exchange rate. Applying the guess and verify method yields the following real exchange rate dynamics with monetary policy shock:

$$\hat{q}_t = \tau_1 \hat{q}_{t-1} + \tau_2 v_t$$

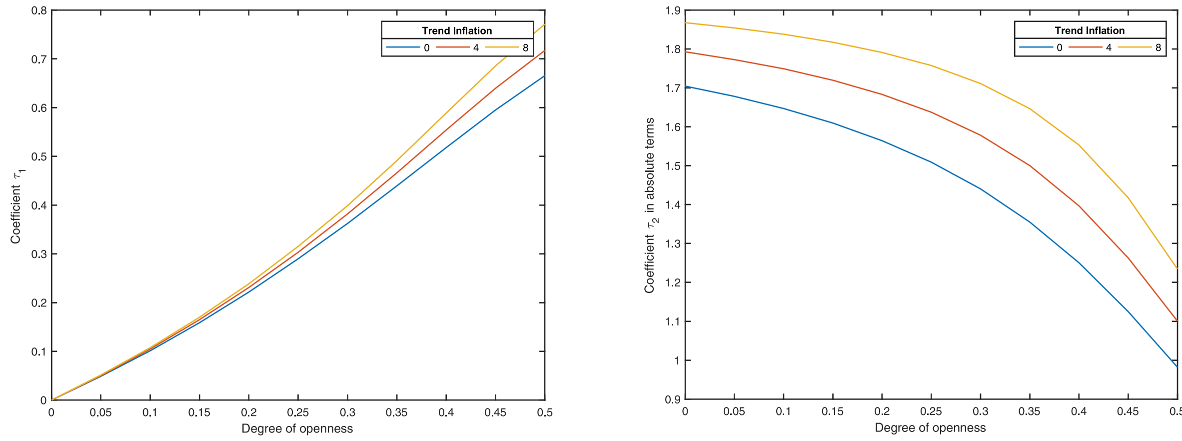
The coefficients τ_1 and τ_2 are as follows:

$$\tau_1 = \frac{1 - \sqrt{1 - \frac{4\beta\alpha}{\Lambda^2(1-\alpha)}}}{\frac{2\beta}{\Lambda}} \quad \text{and} \quad \tau_2 = \frac{-\beta}{\Lambda(1 - \frac{\beta}{\Lambda}\rho_v - \frac{\beta}{\Lambda}\tau_1)} \tag{10}$$

As indicated in Equation 10, both the trend inflation rate and the degree of openness affect coefficients τ_1 and τ_2. To analyse the effect of trend inflation and openness on real exchange rate dynamics, τ_1 and τ_2 are plotted for different values of trend inflation and for degrees of openness in Figure 1. When trend inflation is zero, the coefficient of the inertial variable increases as the economy becomes more open. The impact effect of the shock on the real exchange rate decreases at higher degrees of openness. For a given degree of openness, the coefficients of the inertial variable and the effect of the monetary policy shock on the real exchange rate increase at higher trend inflation rates. With increased trend inflation rate, the real exchange rate becomes more persistent.

The inference behind why openness induces a more persistent real exchange rate is two-fold. First, domestic goods' prices decrease more as trade openness increases in the case of the monetary policy shock. Notably, to compete with foreign goods, domestic firms decrease domestic goods' prices more since households have the opportunity to access relatively cheaper foreign goods. Due to the presence of sticky price dynamics in the domestic price level, increased trade openness intertemporally affects domestic prices. Second, the weight of the price of cheaper foreign goods in the CPI increases with more trade openness. These two dynamics lead to a modestly more persistent real exchange rate as trade openness increases. On the other hand, higher openness provides higher import opportunity for the home economy, and this lessens the impact effect of the shock on the real exchange rate. In other words, openness absorbs the effect of the shock.

[4]See appendix for more detailed derivation.



(a) Backward-looking Variable

(b) Monetary Policy Shock v_t

Figure 1. Coefficients of the Backward-looking Variable and Monetary Policy Shock Variable.

Due to sticky domestic prices, for a given degree of openness, the real exchange rate is modestly more persistent and the impact effect of the monetary policy shock on the real exchange rate increases at higher trend inflation rates.

Since the coefficients for the demand and cost-push shocks are similar to the one for monetary policy shock in absolute terms, the results obtained above for the monetary policy shock can be generalized for the demand and cost-push shocks.

Calibration

We calibrate the model for Canada. Our calibration is fairly standard. We set $\sigma = 1$, $\eta = 1$ and $\gamma = 1$. The discount factor β, taken as standard, is 0.99. The elasticity of substitution, ϵ is set to 6 for Canada by Gali and Monacalli (2005). Labour is indivisible for the country. We calculate the degree of openness, α, which is defined as the ratio of imports to GDP for Canada as 0.33. We set the price stickiness parameter, θ for Canada to 0.75 as suggested by Gali and Monacelli (2005). Persistence for exogenous monetary policy shock is 0.50, 0.80 for exogenous demand shock, and 0.80 for exogenous cost-push shock. These are standard in the Business-Cycle literature.

III. Impulse Response Analyses

Figures 2, 3, and 4 present the impulse response functions (IRFs) of CPI inflation, real interest rate, output, domestic inflation, nominal interest rate, the terms of trade, consumption, and the real exchange rate for Canada in response to positive monetary policy, demand, and cost-push shocks. We discuss how each shock affects the dynamics of the model at different rates of trend inflation. Note that all shocks hit the model economy in Period 1.

Monetary Policy Shock

Figure 2 shows the IRFs of the variables in response to a one per cent positive monetary policy shock at different rates of trend inflation (0%, 4% and 8 %).

Since higher trend inflation makes the NKPC flatter, the relation between the contemporaneous variable (e.g. demand for home goods) and domestic inflation weakens. Thus, the initial decrease in demand for home goods by the shock causes domestic inflation to decrease less as trend inflation rises. On the other hand, the real interest rate is higher at higher trend inflation rates. Since the uncovered interest rate parity holds, the nominal domestic currency initially appreciates more. Sluggish dynamics of domestic prices combined with fast response of the domestic nominal currency lowers the terms of trade and the real exchange rate more. In other words, imported goods become relatively cheaper at higher trend inflation rates. The opposite dynamics of domestic inflation and terms of trade in response to changes

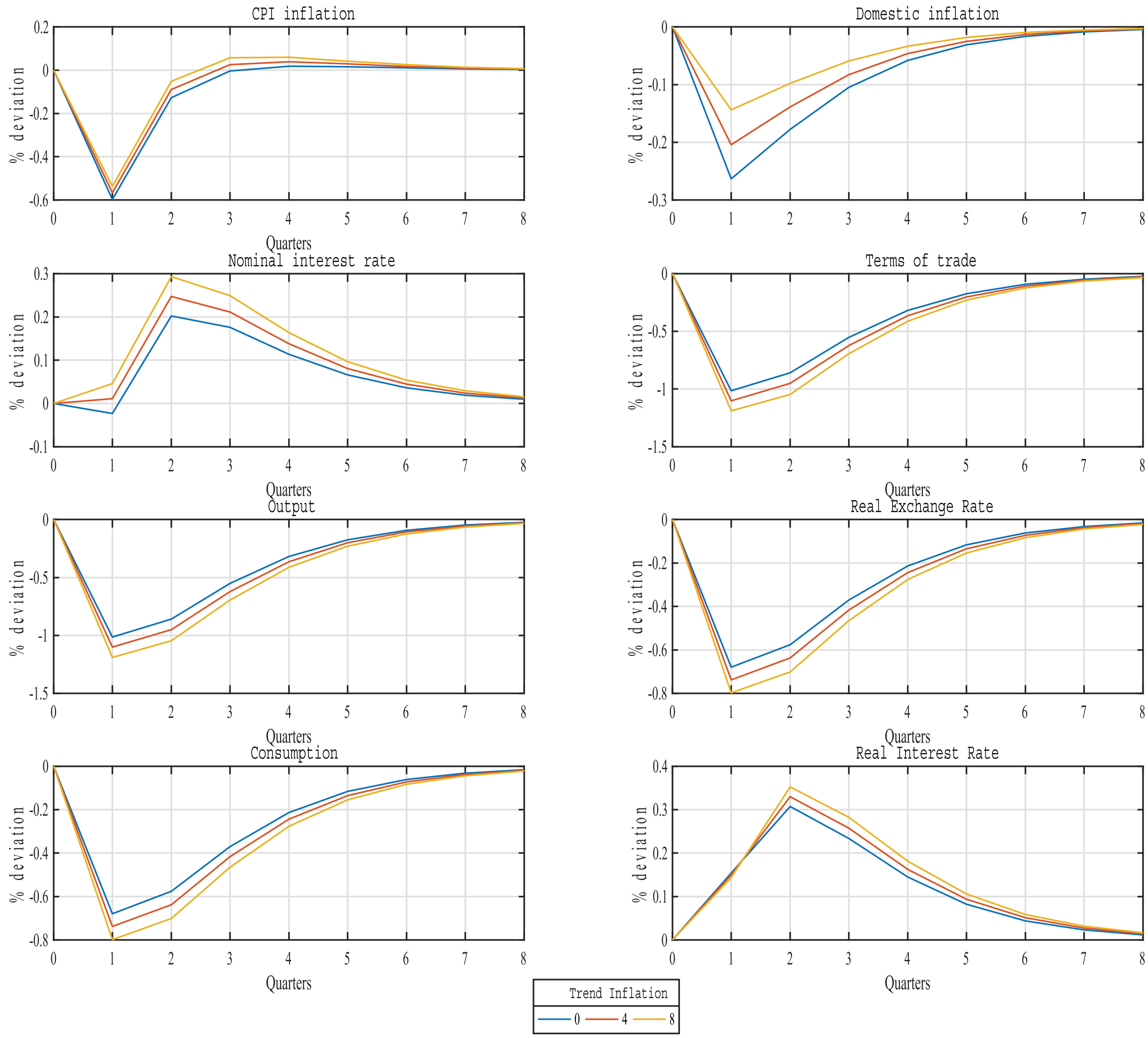


Figure 2. IRFs of One Percent Positive Monetary Policy Shock. Note that all IRFs are percentage deviations from steady states in this section.

in trend inflation cause the effect of trend inflation on CPI inflation to be limited. Moreover, aggregate domestic output decreases more as trend inflation rises. The nominal interest rate increases more at higher trend inflation rates.

On the other hand, the effects of monetary policy shock on the variables are short-lived. All variables, except for the real interest rate and the nominal interest rate, reach their bottoms at Quarter 1, but the real interest and nominal interest rates reach their peaks at Quarter 2. All the variables converge to their steady states at Quarter 8. Prior to Quarter 6, the effects of trend inflation on the variables exist but weaken thereafter.

Demand Shock

Figure 3 presents the IRFs of the variables in response to a one per cent positive demand shock at different rates of trend inflation (0%, 4% and 8 %).

Due to the flatter slope of the NKPC, an increase in demand for home goods causes domestic prices and domestic inflation to increase less as trend inflation increases. The real exchange rate and terms of trade increase more, and imported goods become relatively more expensive as trend inflation increases. The opposite dynamics of domestic inflation and the real exchange rate in response to demand shock are what limit the effect of trend

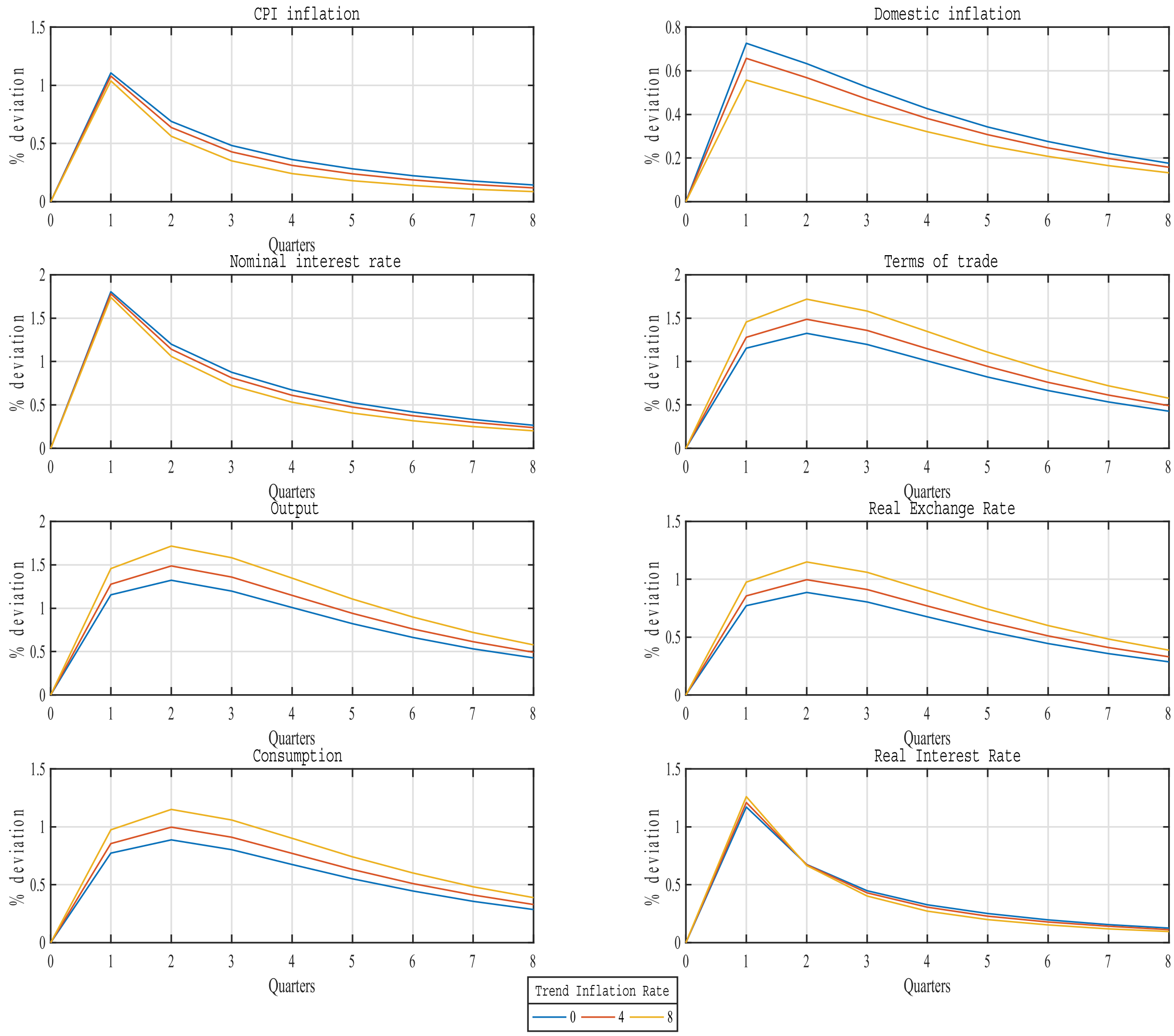


Figure 3. IRFs of One Percent Positive Demand Shock.

inflation on CPI inflation. On the other hand, high domestic and foreign demand for home goods induce increase in aggregate domestic output more as trend inflation increases. As the trend inflation rate rises, the nominal interest rate increases less.

While consumption, output, terms of trade, and the real exchange rate reach their peaks at Quarter 2 the rest of the variables reach their peaks at Quarter 1. All variables converge to their steady states after Quarter 8. The effects of the trend inflation rate on output, consumption, CPI inflation rate, domestic inflation rate, terms of trade, nominal exchange rate, and the real exchange rate persist for more than 2 years, but the effects of trend inflation on the real interest rate weaken earlier.

Cost-Push Shock

Figure 4 presents the IRFs of the variables in response to a one per cent positive cost-push shock at different rates of trend inflation (0%, 4% and 8%).

The implications of positive trend inflation are in effect through two channels. The first is the importance of contemporaneous and future variables on the NKPC as discussed above. The second works through the expectations channel. This second channel is so strong that it dominates the effects of the first channel on domestic inflation. Thus, higher trend inflation increases domestic prices and domestic inflation more. The terms of trade and the real exchange rate decrease more as trend inflation increases. This implies that imported goods become relatively cheaper and so

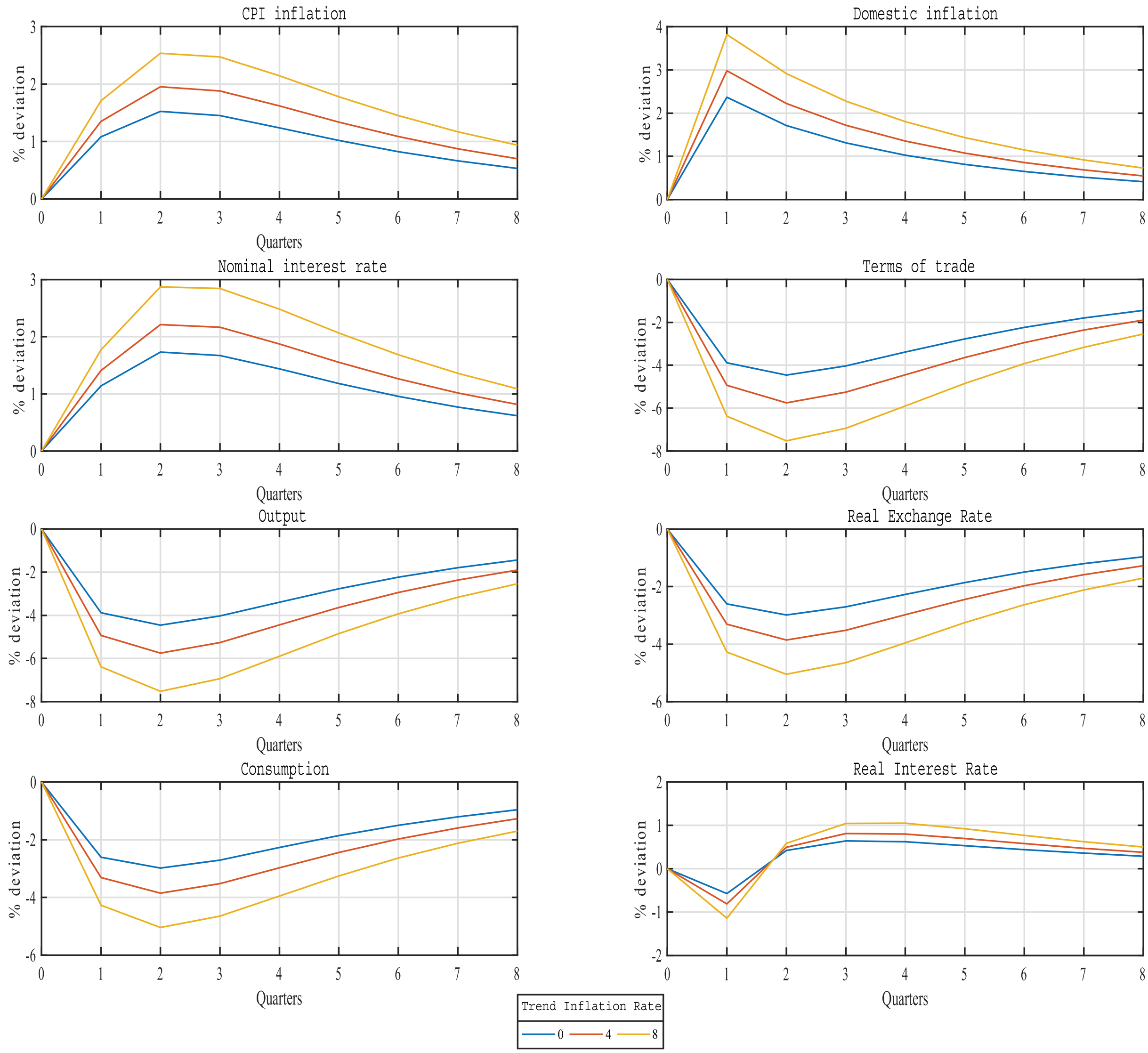


Figure 4. IRFs of One Percent Positive Cost-Push Shock.

the increase in CPI inflation is less than the increase in domestic inflation. Furthermore, the decline in aggregate domestic output is more significant at higher trend inflation rates. The nominal interest rate increases more, as trend inflation increases.

The effects of the cost-push shock on the variables are long-lived and last more than 2 years. While CPI inflation, nominal interest rate, output, consumption, terms of trade, and the real exchange rate reach their peaks (or bottoms) at Quarter 2, domestic inflation reaches its peak at Quarter 1 and the real interest rate at Quarter 3. The effects of the trend inflation rate on these variables persist for more than 2 years.

Our model can also be utilized to consider the effects of world output shock. In response to a positive world output shock as trend inflation increases, CPI inflation decreases more while output increases more.

IV. Real Exchange Rate Puzzles

In this section, we evaluate whether the model developed in this study contributes to resolving real exchange rate puzzles. We extend the sources of persistence with positive trend inflation. Tables 1, 2, and 3 present persistence specifications and standard deviations, and Figures 5, 6, and 7 present the IRFs of the real exchange rate to the shocks at different levels of trend inflation under three alternative scenarios. We

Table 1. Real Exchange Rate Properties under different Monetary Policy Shock Specifications and Policy Rules.

	$\rho_i = 0$ and $\rho_v = 0.50$					$\rho_i = 0.80$ and $\rho_v = 0.50$					$\rho_i = 0.80$ and $\rho_v = 0$				
π	HL	QL	UL	ρ	St. Dev.	HL	QL	UL	ρ	St. Dev.	HL	QL	UL	ρ	St. Dev.
0	2.13	4.26	1	0.72	1.00	2.30	4.61	1	0.74	4.46	1.61	3.22	1	0.65	2.27
4	2.21	4.42	1	0.73	1.10	2.59	5.17	1	0.76	4.87	1.79	3.58	1	0.68	2.44
8	2.29	4.58	1	0.74	1.21	2.94	5.87	1	0.79	5.34	2.01	4.02	1	0.71	2.64

consider three cases: 1) the shocks are persistent; 2) the shocks are persistent, and the policy rule has inertia; and 3) the shocks are *i.i.d.*, and the policy rule has inertia to analyse whether any source of persistence helps to obtain persistent dynamics of real exchange rates.

Purchasing Power Parity (PPP) Puzzle

In this section, we discuss whether the level of trend inflation plays any role in explaining the PPP Puzzle under alternative scenarios. Tables 1, 2, and 3 summarize persistence and volatility results, which are the half-lives (HLs), the quarter-lives (QLs), the up-lives (ULs)[5] , the first-order autocorrelation coefficients ρ, and the standard deviations under alternative scenarios in response to the shocks.

Tables 1 and 2 present persistence specifications and standard deviations of the real exchange rate under different sources of persistence at different trend inflation rates in the cases of a one per cent positive monetary policy shock and a one per cent positive demand shock, respectively. Higher trend inflation rates tend to increase the persistence of the real exchange rate under all alternative scenarios (except for *i.i.d.* demand shock under the inertial policy), but the increase is limited. It can be argued that in the light of the effect of higher trend inflation on the real exchange rate, the main source of the real exchange rate's persistence is the persistence of the shock rather than the inertial variable of the Taylor rule and positive trend inflation rate in both types of shock. Furthermore, the inertial variable of the Taylor rule strengthens the effect of persistent monetary policy shock on the real exchange rate. On the other hand, volatility increases with increased trend inflation rates under all alternative scenarios in both types of shock.

Table 3 presents persistence specifications and standard deviations of the real exchange rate under different sources of persistence at different trend inflation rates in the case of a one per cent positive cost-push shock. Trend inflation magnifies the persistence of the real exchange rate under all alternative scenarios. Comparing the effect of trend inflation on the real exchange rate with other alternatives, it can be argued that persistence of cost-push shock is the main source of the real exchange

Table 2. Real Exchange Rate Properties under different Demand Policy Shock Specifications and Policy Rules.

	$\rho_i = 0$ and $\rho_d = 0.80$					$\rho_i = 0.80$ and $\rho_d = 0.80$					$\rho_i = 0.80$ and $\rho_d = 0$				
π	HL	QL	UL	ρ	St. Dev.	HL	QL	UL	ρ	St. Dev.	HL	QL	UL	ρ	St. Dev.
0	6.44	12.89	2	0.90	1.83	1.61	3.22	1	0.65	2.27	0.50	0.25	1	−0.04	0.56
4	6.68	13.37	2	0.90	2.07	1.79	3.58	1	0.68	2.44	0.50	0.25	1	−0.03	0.57
8	6.92	13.84	2	0.90	2.40	2.01	4.02	1	0.71	2.64	0.50	0.25	1	−0.02	0.58

Table 3. Real Exchange Rate Properties under different Cost-Push Shock Specifications and Policy Rules.

	$\rho_i = 0$ and $\rho_u = 0.80$					$\rho_i = 0.80$ and $\rho_u = 0.80$					$\rho_i = 0.80$ and $\rho_u = 0$				
π	HL	QL	UL	ρ	St. Dev.	HL	QL	UL	ρ	St. Dev.	HL	QL	UL	ρ	St. Dev.
0	6.44	12.89	2	0.90	6.15	16.04	32.07	3	0.95	5.54	1.61	3.22	1	0.65	0.68
4	6.68	13.37	2	0.90	7.99	17.84	35.69	3	0.96	7.02	1.79	3.59	1	0.68	0.73
8	6.92	13.84	2	0.90	10.53	20.04	40.08	4	0.97	9.12	2.01	4.02	1	0.71	0.79

[5]HL is the number of periods at which the effect of shock on the real exchange rate reduces by half; and QL is the number of periods at which the effect of shock on the real exchange rate reduces by quarter; and UL is the number of period at which the real exchange rate reaches its peak (or bottom).

rate's persistence. The inertial variable of the Taylor rule strengthens the effect of persistent cost-push shock on the real exchange rate. Volatility increases with increased trend inflation rates under all alternative scenarios.

In short, the real exchange rate's persistence rises in response to the shocks under all alternative scenarios with the exception of *i.i.d.* demand shock under the inertial policy rule at higher trend inflation rates. The real exchange rate becomes more volatile as the level of trend inflation rises in response to all types of shock under all alternative scenarios.

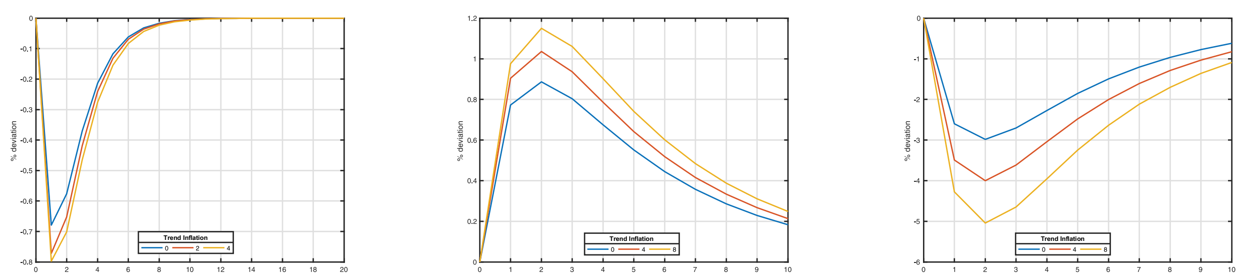
Delayed Overshooting Puzzle

In this section, we discuss whether the level of trend inflation plays any role in justifying the Delayed Overshooting Puzzle under alternative scenarios or not.

Figure 5 shows the IRFs of the real exchange rate in response to a one per cent positive persistent monetary policy shock, a one per cent positive persistent demand shock, and a one per cent positive persistent cost-push shock with the standard Taylor rule at different rates of trend inflation.

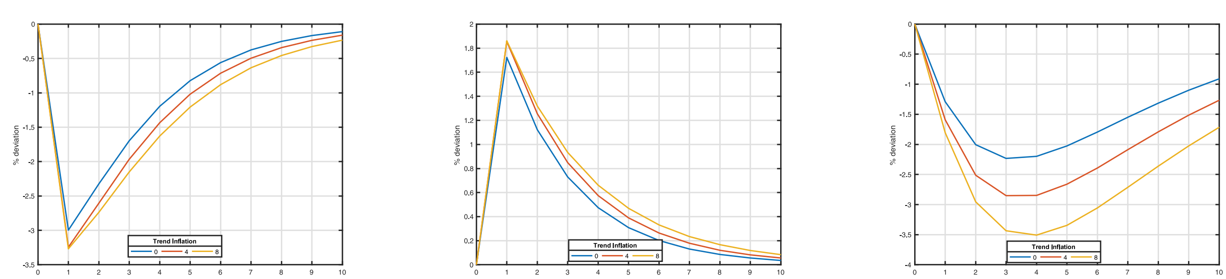
While the real exchange rate reaches its bottom at Period 1 in the case of monetary policy shock, it reaches its peak (bottom) at Period 2 in the case of demand (cost-push) shock at different rates of trend inflation. A higher trend inflation rate increases the response of the real exchange rate to all types of shocks, but it does not cause delayed overshooting of the real exchange rate for all types of shocks. It can be argued that the source of the delayed overshooting is the persistence of the shock for the demand and cost-push shocks.

Figure 6 shows the IRFs of the real exchange rate in response to a one per cent persistent monetary policy shock, a one per cent persistent demand



(a) Monetary Policy Shock (b) Demand Shock (c) Cost-Push Shock

Figure 5. Standard Monetary Policy Rule with One Percent Persistent Shocks.



(a) Monetary Policy Shock (b) Demand Shock (c) Cost-Push Shock

Figure 6. Inertial Monetary Policy Rule with One Percent Persistent Shocks.

shock, and a one per cent persistent cost-push shock with the inertial Taylor rule at different rates of trend inflation.

Higher trend inflation rates increase the response of the real exchange rate to all types of shocks. The real exchange rate reaches its bottom (peak) in Period 1 in the case of monetary policy (demand) shock at different rates of trend inflation. Although it reaches its bottom in Period 3 at 0% and 4% trend inflation rates, it reaches its bottom in Period 4 at 8% trend inflation rate in the case of cost-push shock. It can be claimed that higher trend inflation causes delayed overshooting of the real exchange rate in the case of cost-push shock. However, it does not do this in the cases of monetary policy shock and demand shock per se.

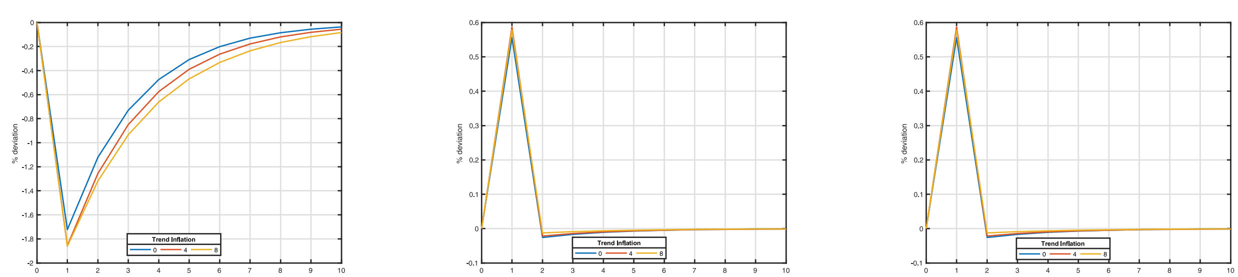
The source of persistence from the inertial component of the Taylor rule increases the effect of shock persistence on the real exchange rate for monetary policy and demand shocks. However, for monetary policy and demand shocks, both sources of persistence do not lead to delayed overshooting of the real exchange rate. On the other hand, higher trend inflation does not suffice to lead to delayed overshooting of the real exchange rate in both types of shock. For the cost-push shock, the inertial component helps to magnify the effect of the cost-push shock's persistence on the real exchange rate. The reason is that the inertial component of the Taylor rule prevents the nominal interest rate from increasing sufficiently, and the effect of the shock is not absorbed. Thus, the shock induces delayed overshooting of the real exchange rate irrespective of the rate of trend inflation. However, since inflation expectations are significantly deteriorated at higher trend inflation rates, the interactions between trend inflation and the other two sources of persistence lead to a more delayed peak time of the real exchange rate at 8% trend inflation rate.

Figure 7 shows the IRFs of the real exchange rate in response to a one per cent *i.i.d.* monetary policy shock, a one per cent *i.i.d.* demand shock, and a per cent *i.i.d.* cost-push shock with the inertial Taylor rule at different rates of trend inflation.

The real exchange rate reaches its peak (or bottom) at Quarter 1 for all types of shock at different rates of trend inflation. A higher trend inflation rate increases the response of the real exchange rate to all types of shocks but does not cause delayed overshooting of the real exchange rate for all types of shocks. Therefore, it can be inferred that both the trend inflation rate and the inertial component of the Taylor rule do not lead to delayed overshooting of the real exchange rate per se.

V. Conclusion

We develop an alternative version of a small open economy model based on Gali and Monacelli (2005) with positive trend inflation. Gali and Monacelli (2005)'s model assumes that trend inflation is zero. However, this is a counterfactual assumption mainly for two reasons. First, the average inflation rates for developed countries in recent decades are well above zero. Second, the central banks do not target zero inflation rate. As a policy, higher inflation targeting has been a priority on the agenda of central banks,



(a) Monetary Policy Shock (b) Demand Shock (c) Cost-Push Shock

Figure 7. Inertial Monetary Policy Rule with One Percent *i.i.d.* Shocks.

economists and policy-makers since the ZLB incident observed in the post-Global Financial Crisis in 2008–9. There have been debates about the effectiveness of higher inflation-targeting policy in economic environments of low interest rates, deflation, and stagnation. Various studies analyse different aspects of this policy in terms of welfare and indeterminacy. Predictably, results change according to the assumptions, specifications, and estimation methods of the models used.

Positive trend inflation has three-fold effects on the model. First, increased trend inflation flattens the NKPC. Second, it induces the expected marginal cost to enter the NKPC. Third, due to positive trend inflation, price dispersion has effects on the model and thus the NKPC. These implications resemble the closed economy modelled by Ascari (2004), Ascari and Ropele (2007), and Ascari and Sbordone (2014). On the other hand, while openness affects the model through the real exchange rate, it does not affect the slope of the NKPC. In the case of monetary policy shock, higher degree of openness leads to more persistent real exchange rate. On the other hand, foreign trade dynamics absorb the impact effect of the shock at a higher degree of openness. However, for a given degree of openness, trend inflation aggravates the persistence of the real exchange rate and the impact effect of the shock on the real exchange rate. The demand and cost-push shocks lead to similar results with the monetary policy shock.

We next plot the IRFs to analyse the role of trend inflation on the dynamics of variables and find that the trend inflation rate plays a key role in the macroeconomic dynamics of variables in response to different types of shock. For monetary policy shock, increased trend inflation decreases the effect of the shock on CPI inflation and domestic inflation but increases the effects of the shock on the other variables. At higher trend inflation rates, the Central Bank increases the nominal interest rate more to stabilize the changes in CPI inflation and output. For demand shock, increased trend inflation decreases the effects of the shock on CPI inflation, domestic inflation, and the nominal interest rate while increasing the effects of the shock on the other variables except for the real interest rate. The Central Bank increases the nominal interest rate less to stabilize the changes in CPI inflation and output in response to the demand shock. In the case of cost-push shock at higher trend inflation rates, the Central Bank increases the nominal interest rate more to stabilize the changes in CPI inflation and output. Also, households and firms face higher domestic inflation rate and CPI inflation rate. The effects of the shock on all variables increases at higher trend inflation rates.

We analyse the effects of positive trend inflation on both persistence and volatility of the real exchange rate in the context of the PPP puzzle. It is found that trend inflation increases the persistence of the real exchange rate under all scenarios with the exception of *i.i.d.* demand shock under the inertial policy rule. Trend inflation increases the volatility of the real exchange rate under all alternative scenarios. However, these effects are modest compared to those of zero-inflation rate. Moreover, it can be claimed that the main source of the real exchange rate's persistence shocks is the persistence of shock in all types of shocks. Finally, we discuss whether trend inflation and sources of persistence lead to delayed overshooting of the real exchange rate under alternative scenarios. Higher trend inflation rates increase the effects of each shock on the real exchange rate, but this only matters for delayed overshooting of the real exchange rate in response to persistent cost-push shock under the inertial policy rule. On the other hand, the sources of persistence lead to delayed overshooting of the real exchange rate in response to the persistent demand and cost-push shocks under the standard Taylor rule and persistent cost-push shock under the inertial Taylor rule regardless of the trend inflation rate.

The analyses conducted in this study may be useful in formulating policies for central banks in a trend inflation environment for open economies. Higher inflation-targeting policies followed by central banks have benefits and costs for policy-makers in both the short and the long run. These benefits and costs depend on the type of shock. In the cases of monetary policy and cost-push shocks, higher trend inflation de-anchors inflationary expectations. Central banks could substantially increase their nominal interest rates to combat fluctuations of inflation rate and output. For this reason, central banks should adopt the long-run trend inflation rate as their target level. However, in the case of demand shock, central banks do not need to respond to higher trend inflation by increasing the nominal interest rate. This is because the effect of higher trend inflation rates on the nominal interest rate is limited. Thus, central banks could target higher inflation rates in the case of demand shock. From our analyses, clues about the duration of the effects of shocks on the economy's dynamics can also be inferred. In formulating policies, the interwoven behavioural relations among agents, their expectations, and developments in the rest of the world should be considered. Accounting for this necessitates balancing the costs of these policies with their benefits. However, recent global experiences, like the Covid-19 pandemic, question the capabilities of the central banks to increase trend inflation in the current situation.

Acknowledgments

We thank Engin Kara for helpful comments.

Disclosure statement

There is no financial interest or benefit from the application of our research.

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References

Ascari, G. 2000. "Optimising Agents, Staggered Wages and the Persistence of the Real Effects of Money Shocks." *Economic Journal* 110 (465): 664–686. doi:10.1111/1468-0297.00560.

Ascari, G. 2004. "Staggered Prices and Trend Inflation: Some Nuisances." *Review of Economic Dynamics* 7 (3): 642–667. doi:10.1016/j.red.2004.01.002.

Ascari, G., and A. M. Sbordone. 2014. "The Macroeconomics of Trend Inflation." *Journal of Economic Literature. 52(3)* 679–739. doi:10.1257/jel.52.3.679.

Ascari, G., and T. Ropele. 2007. "Optimal Monetary Policy under Low Trend Inflation." *Journal of Monetary Economics* 54 (8): 2568–2583. doi:10.1016/j.jmoneco.2007.06.024.

Ascari, G., and T. Ropele. 2009. "Trend Inflation, Taylor Principle, and Indeterminacy." *Journal of Money, Credit and Banking, 41(8)*, no. 8: 1557–1584. doi:10.1111/j.1538-4616.2009.00272.x.

Ball, L. 2013. "The Case for Four Percent Inflation." *Central Bank Review* 13 (2): 17–31.

Benigno, G. 2004. "Real Exchange Rate Persistence and Monetary Policy Rules." *Journal of Monetary Economics* 51 (3): 473–502. doi:10.1016/j.jmoneco.2003.06.001.

Bernanke, B. S. 2010. "Opening Remarks: The Economic Outlook and Monetary Policy." *In Proceedings-Economic Policy Symposium-Jackson Hole*: 1–16. Federal Reserve Bank of Kansas City.

Blanchard, O., G. Dell'Ariccia, and P. Mauro. 2010. *Rethinking Macroeconomic Policy. IMF Staff Position Note. IMF. SPN/10/03*. Retrieved from https://www.imf.org/external/pubs/ft/spn/2010/spn1003.pdf

Calvo, G. 1983. "Staggered Prices in a Utility Maximepsilong Framework." *Journal of Monetary Economics* 12 (3): 383–398. doi:10.1016/0304-3932(83)90060-0.

Carvalho, C., F. Nechio, and F. Yao. 2019. *Monetary Policy and Real Exchange Rate Dynamics in Sticky-Price Models*. Federal Reserve Bank of San Francisco Working Paper 2014-17 Retrieved from http://www.frbsf.org/economic-research/publications/working-papers/wp2014-17.pdf

Chari, V. V., P. Kehoe, and E. Mcgratten. 2002. "Can Sticky Price Models Generate Volatile and Persistent Real Exchange Rates?" *Review of Economic Studies* 69 (3): 533–563. doi:10.1111/1467-937X.00216.

Clarida, R., and J. Gali. 1994. "Sources of Real Exchange-rate Fluctuations: How Important are Nominal Shocks?" *Carnegie-Rochester Conference Series on Public Policy* 41 (1): 1–56. doi:10.1016/0167-2231(94)00012-3.

Cogley, T., and A. M. Sbordone. 2008. "Trend Inflation, Indexation, and Inflation Persistence in the New Keynesian Phillips Curve." *The American Economic Review* 98 (5): 383–398. doi:10.1257/aer.98.5.2101.

Dornbusch, R. 1976. "Expectations and Exchange Rate Dynamics." *Journal of Political Economy* 84 (6): 1161–1176. doi:10.1086/260506.

Eichenbaum, M., and C. Evans. 1995. "Some Empirical Evidence on the Effects of Shocks to Monetary Policy on Exchange Rates." *The Quarterly Journal of Economics* 110 (4): 975–1009. doi:10.2307/2946646.

Gali, J., and T. Monacelli. 2005. "Monetary Policy and Exchange Rate Volatility in a Small Open Economy." *Review of Economic Studies* 72 (3): 707–734. doi:10.1111/j.1467-937X.2005.00349.x.

King, R. G., and A. L. Wolman. 2013. "Inflation Targeting in a St." *Louis Model of the 21st century.Federal Reserve Bank of St. Louis Review* 95 (6): 543.

Lie, D., and A. S. Yadav. 2017. "Time-Varying Trend Inflation and the New Keynesian Phillips Curve in Australia." *The Economic Record* 93 (300): 42–66.

Rogoff, K. 1996. "The Purchasing Power Parity Puzzle." *Journal of Eco- Nomic Literature* 34: 647–668.

Steinsson, J. 2008. "The Dynamic Behaviour of the Real Exchange Rate in Sticky Price Models." *American Economic Review* 98 (1): 519–533. doi:10.1257/aer.98.1.519.

Williams, J. C. 2009. "Heeding Daedalus: Optimal Inflation and the Zero-lower Bound." *Brookings Paper on Economic Activity* 2: 1–37.

Appendix

In this appendix, the real exchange dynamics are derived.

The NKPC is as follows:

$$\hat{\pi}_{H,t} = \kappa_1\left(\widehat{mc}^r_{H,t}\right) + \kappa_2 E_t\left(\hat{\pi}_{H,t+1}\right) + \kappa_3 E_t\left(\hat{\psi}_{t+1}\right) + u_t$$

where $\kappa_1 = \frac{(1-\beta\theta\pi^{\epsilon})(1-\theta\pi^{\epsilon-1})}{\theta\pi^{\epsilon-1}}$, $\kappa_2 = \beta[(1-\theta\pi^{\epsilon-1})(\pi-1)\epsilon+1]$, $\kappa_3 = \beta(1-\theta\pi^{\epsilon-1})(\pi-1)$ and $\hat{\psi}_t = (1-\beta\theta\pi^{\epsilon})\widehat{mc}^r_{H,t} + \beta\theta\pi^{\epsilon}E_t\left(\epsilon\hat{\pi}_{H,t+1} + \hat{\psi}_{t+1}\right)$

Plugging $\widehat{mc}^r_t$ into NKPC yields:

$$\hat{\pi}_{H,t} = \kappa_1(\varphi\hat{z}_t + (\sigma-\sigma_\alpha)\hat{y}^*_t + (\sigma_\alpha+\varphi)\hat{y}_t - (1+\varphi)\hat{a}_t) + \kappa_2 E_t\left(\hat{\pi}_{H,t+1}\right) + \kappa_3 E_t\left(\hat{\psi}_{t+1}\right) + u_t \quad \text{I.1}$$

For simplicity, $\varphi = 0$ and $\sigma = 1$ are set. Since no technology shock occurs in the domestic economy, $\hat{a}_t = 0$. Equation I.1 becomes:

$$\hat{\pi}_{H,t} = \kappa_1\left(\hat{y}_t\right) + \kappa_2 E_t\left(\hat{\pi}_{H,t+1}\right) + \kappa_3 E_t\left(\hat{\psi}_{t+1}\right) + u_t \quad \text{I.2}$$

Since no world shock occurs, all variables related to the world economy are assumed to be 0. Thus, the international risk sharing condition yields the following relation:

$$\hat{c}_t = \hat{q}_t \quad \text{I.3}$$

Simplifying the relation between domestic output, domestic consumption, and terms of trade yields $\hat{y}_t = \hat{c}_t + \frac{\alpha\omega}{\sigma}\hat{s}_t$ where $\omega = \sigma\gamma + (1-\alpha)(\sigma\eta - 1)$. By using the above parameters, σ and φ yields the following equation:

$$\hat{y}_t = \hat{c}_t + \alpha\hat{s}_t \quad \text{I.4}$$

Substituting Equations I.3 and $\hat{q}_t = (1-\alpha)\hat{s}_t$ into Equation I.4 yields:

$$\hat{y}_t = \frac{1}{1-\alpha}\hat{q}_t \quad \text{I.5}$$

Then, using this equation, NKPC is written as follows:

$$\hat{\pi}_{H,t} = \frac{\kappa_1}{1-\alpha}\hat{q}_t + \kappa_2 E_t\left(\hat{\pi}_{H,t+1}\right) + \kappa_3 E_t\left(\hat{\psi}_{t+1}\right) + u_t \quad \text{I.6}$$

Substituting equation $\hat{q}_t = (1-\alpha)\hat{s}_t$ into equation $\hat{\pi}_t = \hat{\pi}_{H,t} + \alpha(\hat{s}_t - \hat{s}_{t-1})$ yields the following relation:

$$\hat{\pi}_t = \hat{\pi}_{H,t} + \frac{\alpha}{1-\alpha}\left(\hat{q}_t - \hat{q}_{t-1}\right) \quad \text{I.7}$$

Substituting Equation I.3 into the standard Euler equation:

$$\hat{i}_t = E_t\left(\hat{q}_{t+1}\right) - \hat{q}_t + E_t\left(\hat{\pi}_{t+1}\right) + d_t \quad \text{I.8}$$

Next, we assume that $\phi_y = 0$, $\phi_\pi\beta = 1$, and the Taylor rule is expressed as follows: $\hat{i}_t = \phi_\pi\hat{\pi}_t + v_t$. When the new Taylor rule is multiplied with β:

$$\beta\hat{i}_t = \hat{\pi}_t + \beta v_t \quad \text{I.9}$$

Substituting $\hat{\pi}_t = \hat{\pi}_{H,t} + \frac{\alpha}{1-\alpha}\left(\hat{q}_t - \hat{q}_{t-1}\right)$ into Equation I.9:

$$\beta\hat{i}_t = \hat{\pi}_{H,t} + \frac{\alpha}{1-\alpha}\left(\hat{q}_t - \hat{q}_{t-1}\right) + \beta v_t \quad \text{I.10}$$

Multiplying Equation I.8 by β
:

$$\beta\hat{i}_t = \beta E_t\left(\hat{q}_{t+1}\right) - \beta\hat{q}_t + \beta E_t\left(\hat{\pi}_{t+1}\right) + \beta d_t \quad \text{I.11}$$

Equating Equations I.10 and I.11 and substituting Equation I.6 into them generates the following dynamic equation of the real exchange rate:

$$\hat{q}_t = \frac{1}{\Lambda}\left[\frac{\alpha}{1-\alpha}\hat{q}_{t-1} + \beta E_t\left(\hat{q}_{t+1}\right) - \kappa_2 E_t\left(\hat{\pi}_{H,t+1}\right) + \beta E_t\left(\hat{\pi}_{t+1}\right) - \kappa_3 E_t\left(\hat{\psi}_{t+1}\right) - \beta v_t + \beta d_t - u_t\right]$$

$$\text{where } \Lambda = \frac{\kappa_1 + (1-\alpha)\beta + \alpha}{1-\alpha} \quad \text{I.12}$$