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ORIGINAL CONTRIBUTION

Hierarchical Fuzzy Logic Systems

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Abstract The concept of fuzzy logic has created an immense interest for various research workers in the different fields. Various offshoots of fuzzy logic appeared in the literature during the last four decades or so. As the data involved for several applications has grown considerably, the number of rules of fuzzy systems for real-life applications has increased exponentially and is unmanageable. To reduce the complexity of a fuzzy system, hierarchical fuzzy logic emerged as one of the most viable options. This paper gives an introductory approach to design a system that includes various small dimension fuzzy subsystems, where all subsystems are arranged in a hierarchical structure. This approach handles large numbers of rules and paves a way to design advanced big data applications such as IoT, Intelligent systems, cyber security and WSNs.

Keywords Fuzzy logic · Hierarchical systems · Neuro-fuzzy systems · Fuzzy hierarchical model · Hierarchical logic

Introduction

In 1965, the term 'fuzzy logic' was introduced by Lotfi Zadeh in one of his papers [1] on fuzzy sets. Fuzzy logic provides an adequate route for conflict resolution and making real assessments. Fuzzy logic has the capability to deal with imprecise, uncertain and vague information. Fuzzy logic comprises linguistic variables that support the design of mathematical and realistic models. The fuzzy logic facilitates a huge number of real-life applications. In this paper, a concise review of the hierarchical fuzzy logic has been presented. This will pave a way for researchers to gain the necessary foundations keeping in view the explosion of large datasets in various fields.

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A. Fuzzy systems.

The fuzzy logic [1, 2] has been derived from the conventional logic, i.e., the fuzzy set theory. The fuzzy logic consolidates the smooth transformation between false and true. Instead of presenting the output as extreme '0' or '1,' the output results in the form of degree of truth that includes [0, 1]. The fuzzy logic follows the notion of partial truth. In fuzzy logic, the value fluctuates between '0' and '1.' Fuzzy logic [2, 3] constitutes the set theory in which set A in universe U is categorized by various elements defined by 'x' that appear as real values for each element in universe U. Equation (1a) represents the membership relation for set 'A,' where $\mu(x)$ is the membership function for elements 'x' in set 'A.'

$$A = \{x, \mu(x) | x \in U\} \tag{1a}$$

For more than one fuzzy set, the binary mapping is used in general to aggregate the membership function of all the sets. Consider two fuzzy sets as 'A' and 'B,' then the binary mapping can be shown in Eq. (1b) as:

$$\mu(A, B) = T\{\mu_A(x), \mu_B(x)\} \tag{1b}$$

where $\mu_A(x)$ and $\mu_B(x)$ are the membership functions for elements 'x' in the set 'A' and the set 'B,' respectively.

Interpretability is the main constituent of a fuzzy system that characterizes linguistic approach and human interpretation. Not all the conventional systems derived from

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sample data can comprehend human language. Mamdani and Takagi–Sugeno–Kang proposed a model [4] to boost the objectives of interpretability and accuracy. The Mamdani model defines a linguistic fuzzy modeling concept to enhance the interpretability, whereas the Takagi–Sugeno–Kang model [4] focuses on precision modeling to increase the accuracy. The fuzzy logic consists of a nonlinear behavior and presents a white-box system that facilitates transparency between analysis and its logical interpretation.

B. Real-life examples of fuzzy systems.

With the advent of various complex systems for real-life applications [3] in recent times, the fuzzy logic has benefited many researchers, scientists, mathematicians, analysts, etc. Fuzzy logic has been used in endless real-life applications in the areas such as automotive, defense and security [5, 6], biomedical, Internet of things, electronics and forecasting of weather.

One of the realistic applications [3] of the fuzzy logic is the Sendai subway in Japan. Hitachi designed Nanboku line to manage the railway transport systems using the fuzzy logic control system. Another example of the fuzzy logic is its appearance for the consumer applications, i.e., air conditioner, HVAC systems, ventilator, etc. to manage the thermostat for cooling, heating and making energy efficient systems.

Nowadays, in most of the 3D animated movies, fuzzy logic-based animation systems have been used for generating crowds. The extensive use of these systems can be visible in one of the world's best movies like The Lion, Avatar, the Lord of the Rings, the wardrobe films, etc.

C. Neuro-fuzzy systems.

The term neuro-fuzzy [6, 7] is extracted from the combination of fuzzy logic and neural networks. This hybridization gives an intelligent system, where the fuzzy logic adds linguistic reasoning style, and the neural networks connect the cognitive science concepts. The neuro-fuzzy system incorporates the set theory alongside fuzzy systems that consists of linguistic behavior with fuzzy rule base using IF–THEN statements. The neuro-fuzzy approach adds the interpretability from the fuzzy behavior and accuracy from the neural networks. The fuzzy system restricts the usage of the system within specific data size, whereas the neuro-fuzzy can manage data with large dimensions and can handle higher computation than a conventional fuzzy system.

D. Problem statement.

Despite several advancements in the literature, both the fuzzy and neuro-fuzzy systems have limitations to the dimension, i.e., maximum number of inputs, maximum number of rules, etc. These limitations restrict the use of fuzzy and neuro-systems to address complex real-life applications with large dimension datasets. This paper presents an introductory approach to design hierarchical fuzzy systems [6–8] to leverage a large rule base without reducing the performance of the overall system.

Hierarchical Fuzzy Systems

The term 'hierarchical fuzzy systems' is an arrangement of several fuzzy logic units connected in the form of hierarchy.

Due to transparency, the fuzzy logic system has been preferred for designing complex systems with large dataset [7], but at the same time, the fuzzy logic possesses several functional limitations [6, 7, 9] such as dimensions of rules, dimension of parameters and dimensions of data. These limitations have become a bottleneck in various complex real-life applications. To overcome these limitations, the hierarchical fuzzy system has emerged as one of the most efficient options.

The system loses both generalization and accuracy due to the over-fitting of large data. In the 1990s, Raju, Zhou and Kisner [9] presented the concept of hierarchical fuzzy systems. Several substructures with low data dimensions, commonly known as fuzzy units, are linked together in the form of the hierarchical structure. For various real-life applications, several researchers leveraged hierarchical fuzzy systems with the conventional fuzzy to filter and refine results.

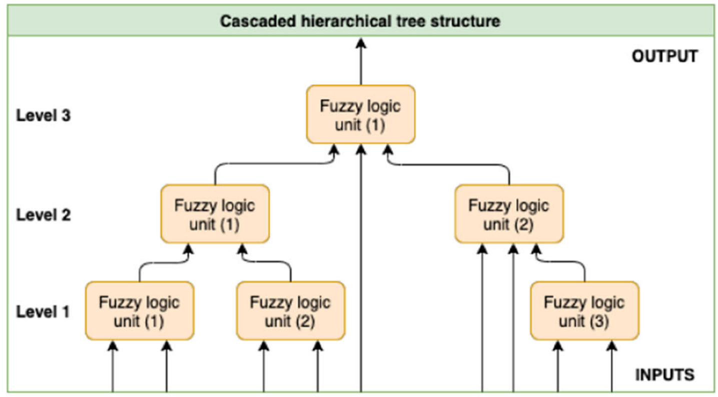
Similarly, the multi-input multi-output (MIMO) systems can be designed using hierarchical systems by connecting various distributed multi-input single-output (MISO) systems in a hierarchical manner, without losing generalization and transparency. In the literature, Kamthan and Singh [7] proposed an algorithm to design hierarchical systems for multi-input multi-output systems by removing the duplication of common subsystems among various outputs and by enhancing the overall system performance. The algorithm showcases the ability of the hierarchical system to handle and manage large rule dimensions such as images.

A. Hierarchical tree structures.

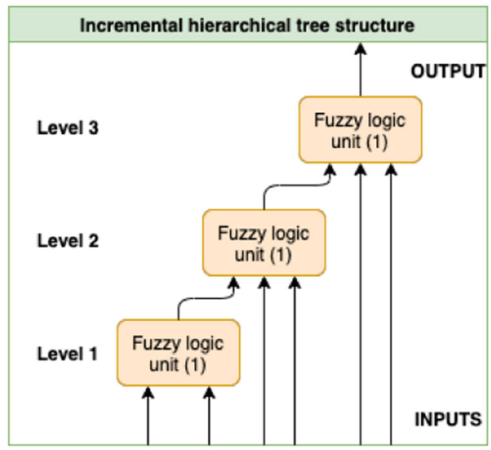
The hierarchical tree structures are described by multiple levels and each level consists of several subsystems. For the hierarchical fuzzy design, these subsystems are referred to as fuzzy units. Besides raw inputs and lowest level, the output from the previous level becomes the input to the next level and so on. Figure 1a shows the cascaded hierarchical tree structure.

Chung and Duan [7, 9] proposed the incremental hierarchical tree structure, as shown in Fig. 1b. The structure

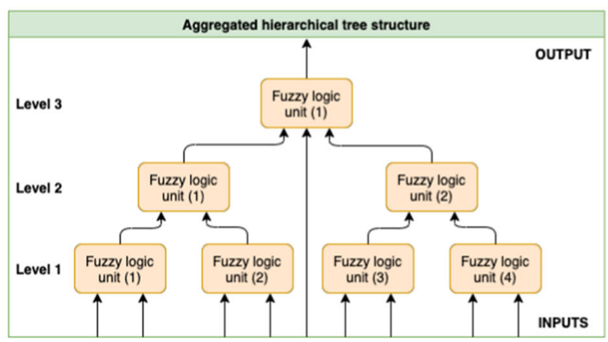
Fig. 1 Hierarchical fuzzy tree structures



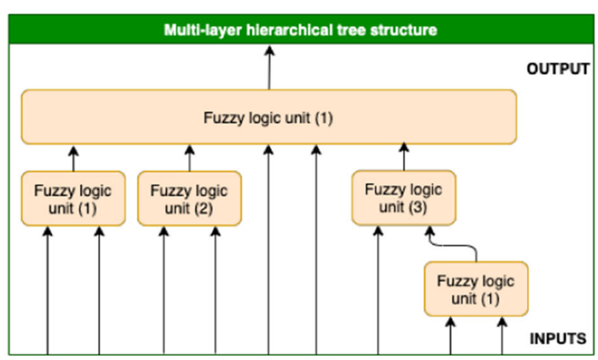
(a) *Cascaded structure of hierarchical systems*



(b) *Incremental hierarchical structure*



(c) *Aggregated hierarchical structure*



(d) *Distributed multi-layer hierarchical structure* contains a multistage platform, where one stage incorporates one hierarchical level. In incremental hierarchical tree architecture, each level consists of only one fuzzy unit. To enhance the accuracy, it is suggested to use the most important inputs at the lower level of hierarchical structure and rest can be added at later stages.

Wang [7, 9] proposed the aggregated hierarchical tree structure, as shown in Fig. 1c. The aggregated hierarchical structure has a multistage platform, and one stage incorporates one hierarchical level. In this structure, each level can have several fuzzy units. All actual inputs are consumed at the lowest level of hierarchical structure, and later, the outputs from the previous level are considered as inputs.

To further refine the accuracy of the hierarchical structure, Karr and Magladena [7, 9] proposed the distributed hierarchical structure where all the real inputs are distributed at all the levels. Figure 1d shows the multilayered hierarchical tree structure.

B. Real-life applications of hierarchical fuzzy logic.

A few examples of the hierarchical systems are image classification, target acquisition, mobility, tracking systems, advanced transformation networks such as WSNs, IoTs, defense and security [6, 7], and survivability of unmanned vehicles [6, 8]. Many real-life applications possess large dataset. A large dataset increases the dimensionality of rules and thus increases the complexity of the system. In recent times, this increase in complexity becomes unmanageable and uncontrollable and surpasses the capacity of the conventional systems. The hierarchical systems present a workable solution to surpass the drawbacks of conventional systems by reducing the complexity of the system [5–7].

C. Representation and design approach of hierarchical systems.

The hierarchical fuzzy systems [7], in mathematical form, can be represented by Eqs. (2a) and (2b). Consider $\{x_1, x_2, x_3, \ldots x_n\}$ as n-inputs and $\{\widehat{x_1}, \widehat{x_2}, \ldots, \widehat{x_n}\}$ as fuzzy member variables abstract from these inputs.

$$\text{IF } \left(\widehat{x_1} = U_1^j\right) \text{ AND } \left(\widehat{x_2} = U_2^j\right), \text{ THEN } \left(y_1 = O_1^j\right)$$
$$\text{IF } \left(\widehat{x_{i+1}} = U_{i+1}^j\right) \text{ AND } \left(\widehat{y_{i-1}} = O_{i-1}^j\right), \text{ THEN } \left(y_i = O_i^j\right) \tag{2a}$$

$$y_1 = \frac{\sum_{j=1}^{m_1} Q_{1,j}\mu_{U_1^j}(x_i)_{U_2^j}(x_i)}{\sum_{j=1}^{m_1} \mu_{U_1^j}(x_i)\mu_{U_2^j}(x_i)}$$
$$y_i = \frac{\sum_{j=1}^{m_i} Q_{i,j}\mu_{O_{i-1}^j}(y_{i-1})\mu_{U_{i+1}^j}(x_{i+1})}{\sum_{j=1}^{m_1} \mu_{O_{i-1}^j}(y_{i-1})\mu_{U_{i+1}^j}(x_{i+1})} \tag{2b}$$

where the parameters are defined as:

- 'j' = 1, 2, 3, …, m, with 'm' = total rules
- $\{U_1^j$ and $U_2^j\}$ and O_1^j represent the fuzzy sets for inputs and outputs, respectively
- 'i' ranges from (2, 3, 4, … $n - 1$)
- $\widehat{x_1}$ and $\widehat{x_2}$ present inputs to the first fuzzy logic unit
- $\widehat{x_{i+1}}$ and $\widehat{y_{i-1}}$ present the real input variable and output from the previous fuzzy logic unit, respectively.
- $\mu_{U_i^j}(x_i)$ and $\mu_{O^j}(y_i)$ present the input and output membership functions

For any linear or nonlinear equations, the following steps below summarize the design approach [7] for a hierarchical system:

1. Identify all the input(s), represented as $\{x_1, x_2, x_3, \ldots x_n\}$, and the output, represented as 'y,' for the system to be designed.
2. Consider a system and expand it to the desired number of elements. For example: Expression $\{Y = A.(B + C)\}$ can be expanded further as $\{Y = (A.B) + (A.C)\}$, where $(A.B)$ and $(A.C)$ are different elements of 'Y.'
3. Group each element of the equation separately, i.e., consider $(A.B)$ and $(A.C)$ from the above expression.
4. Identify input(s) for each element separately and keep the output as same as desired.
5. For the first level of hierarchy, design the fuzzy unit for each element with its respective inputs and desired output using fuzzy c-mean clustering method. For example: $(A.B)$ requires input(s) as A and B only, whereas $(A.C)$ requires input(s) as A and C only.
6. From second level of hierarchy or onwards, design the fuzzy unit for each element with its respective inputs and the output from previous layer as an additional input, keeping the desired output the same using fuzzy c-mean clustering.
7. Connect all the fuzzy units in the form of hierarchy using any of the given hierarchical tree structures as shown in Fig. 1.
8. Using aggregation method, evaluate the final output of the hierarchical structure.

Figure 2 represents the design approach of a hierarchical system [7] proposed in this paper. A nonlinear equation is expanded into a desired number of expressions. Generate a fuzzy inference system for every expression separately using fuzzy c-mean clustering. In this paper, the fuzzy c-mean clustering is preferred to map center points extracted from clusters directly into a fuzzy rule base. Arrange these fuzzy subsystems in a required hierarchical tree structure shown in Fig. 1. To evaluate the final output, aggregate all the outputs at the final level of hierarchy.

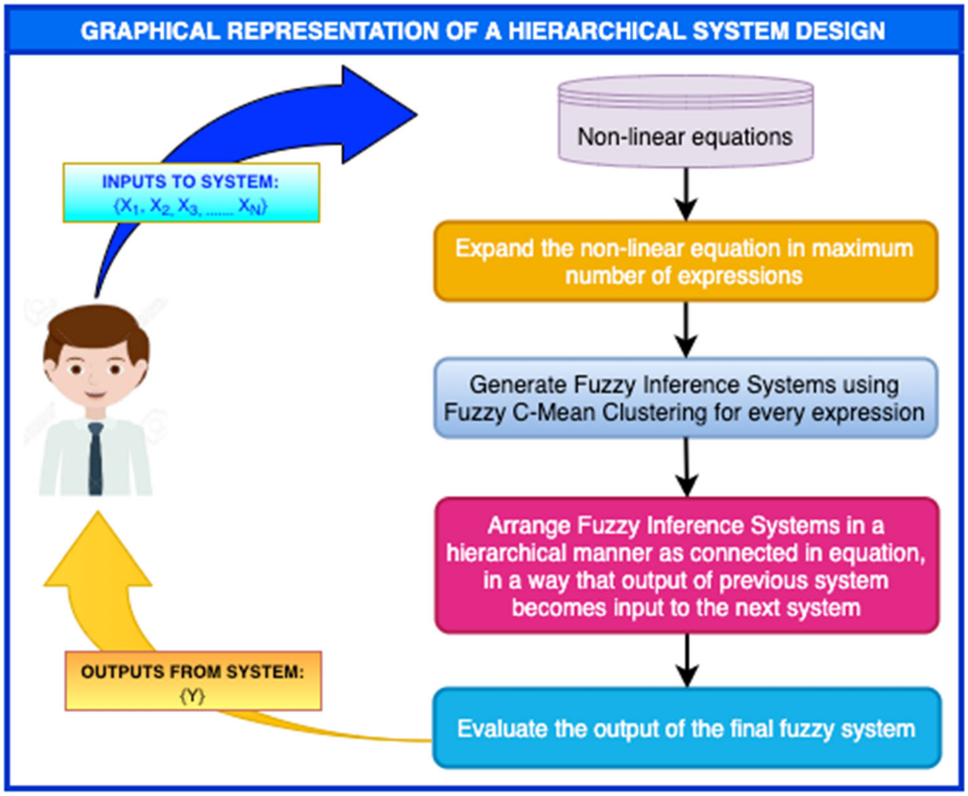


Fig. 2 Graphical representation of hierarchical system design

D. Complexity of hierarchical system.

It is an established fact that the conventional systems possess a constraint to the data dimensions [6, 7]. The constraints can be presented in the form of the complexity of the systems. The complexity has been best displayed in conjunction with the representation of the rule base. Rise in the number of inputs prompt a boost in total rules and thus escalates the complexity of the system. The increment in number of rules is proportional to the complexity. For resolving the complex issues with large dataset, these constraints narrow down the usage of conventional fuzzy systems. The hierarchical fuzzy system has become a most viable solution to surpass the constraints of conventional systems.

E. Dimensionality of rules.

In a conventional fuzzy system, the membership function creates input segments and structures all feasible interconnections between input and output parameters. The interconnection is referred to as 'rule.' The rise in the number of inputs causes the exponential growth in total rules. For large input parameters, it is very challenging to construct the conventional fuzzy systems that can handle large numbers of rules, whereas the hierarchical fuzzy system is favored as it provides solutions with reduced rule base. To understand better, assume an '$n \times 1$' system with 'm' membership functions for each input. A conventional fuzzy system has a total of 'm^n' rules. A hierarchical fuzzy system has a total number of rules defined as: '$(n - 1).m^2$.' It is assumed that every fuzzy unit in hierarchical structure has two inputs, thus '$(n - 1)$' fuzzy units are required for the system.

Radek Sindelar [10] discussed the approach to design hierarchical fuzzy systems for a simple nonlinear equation. This paper describes how rule base explosion can be controlled by converting the system into a layer-by-layer hierarchical model. The aim is to maintain the same or better accuracy than conventional systems. The flowchart of the given approach is shown in Fig. 3. A 3×1 fuzzy system has been considered with inputs (X_1, X_2, X_3) and output (Y). The system is divided into two subsystems. The two inputs X_1 and X_2 are used for subsystem S_1, and the rule base can be expressed by Eq. (3a). In the next layer, the output of S_1 and the input X_3 are used for subsystem S_2 and the rule base for the subsystem S_2 can be expressed by Eq. (3b).

$$\text{For subsystem } S_1 : R_1 \\ \Rightarrow \text{ IF } X_1 \text{ is } A_i \text{ AND } X_2 \text{ is } B_i, \text{ THEN } Z \text{ is } U_i \tag{3a}$$

$$\text{For subsystem } S_2 : R_2 \\ \Rightarrow \text{ IF } X_3 \text{ is } C_i \text{ AND } Z \text{ is } U_i, \text{ THEN } Y \text{ is } P_i \tag{3b}$$

In another example, let us take the Mackey–Glass (MG) time delay differential model [11] represented in Eq. (4)

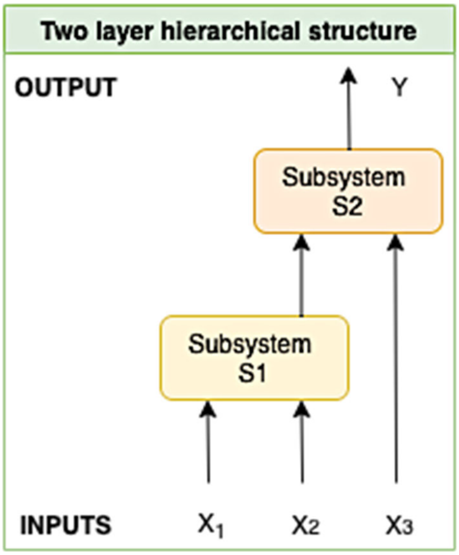


Fig. 3 Flowchart of two-level hierarchical structure

and determine the time series values until 't' time to estimate subsequent values at ($t + P$). A typical approach is to depict Z sample points for 'u' units in time ($x(t - (Z - 1)\ u)$, …, $x(t - u)$, $x(t)$) to an estimated value $x(t + P)$ in future. The input training data with $Z = 4$, $P = 6$ and $u = 6$ can be represented in vector form as shown in Eq. (5) as:

$$\dot{x}(t) = \frac{0.2x(t-T)}{1 + x^{10}(t-T)} - 0.1x(t) \tag{4}$$

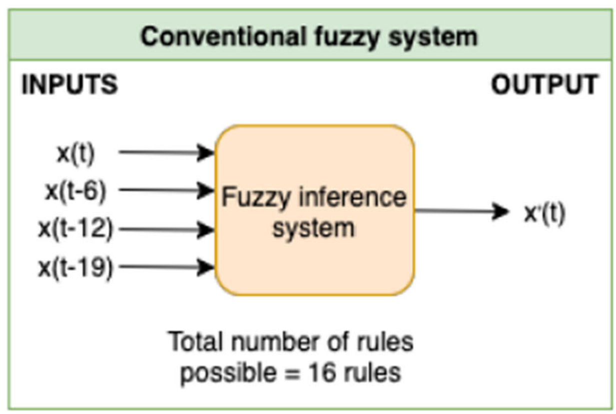
$$Y(t) = [x(t-19), x(t-12), x(t-6), x(t)] \tag{5}$$

To calculate the maximum rules possible via conventional fuzzy, design a fuzzy inference system for the Mackey–Glass time delay differential equation. In this paper, fuzzy c-mean clustering method [5, 7] with 16 cluster points has been considered. Each input is classified into two Gaussian membership functions. The maximum number of rules in such a case is $2^4 = 16$ rules.

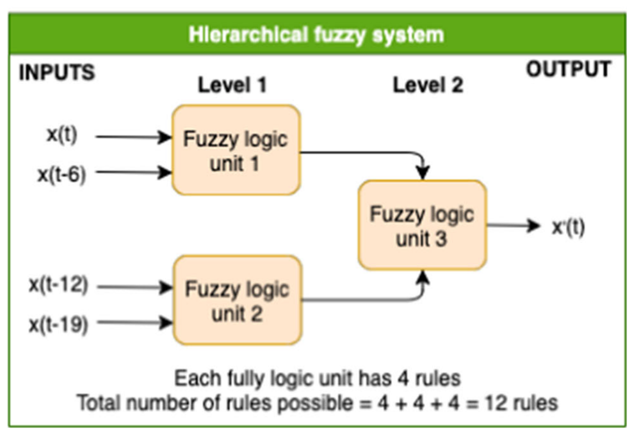
Similarly, a two-layer hierarchical fuzzy system has been designed for the Mackey–Glass time delay differential equation to estimate the maximum number of rules. First level has two fuzzy units, and each unit consists of two inputs. The next level has one unit. The outputs from the first level become inputs to the second level. The final output is represented by the output of the second level. Like conventional system design, each real input has been classified into two Gaussian membership functions.

The maximum number of rules in such a case is $(2^2 + 2^2 + 2^2) = 12$ rules, where every unit has $2^2 = 4$ rules. Figure 4a, b represents the design of the Mackey–Glass time delay differential model, shown by Eq. (5), with a conventional fuzzy system and a hierarchical fuzzy system, respectively.

Figure 5 presents three outputs extracted from the Mackey–Glass time delay differential equation shown in Eq. (5), hierarchical fuzzy system shown in Fig. 4b and conventional fuzzy system shown in Fig. 4a, where 'Mackey–Glass time delay differential equation output is the mathematical output extracted directly from Eq. (5); 'conventional fuzzy' graph is the fuzzy output from conventional fuzzy system, shown in Fig. 4a, designed for Mackey–Glass time delay differential equation shown in Eq. (5); and 'hierarchical fuzzy' graph is fuzzy output from a hierarchical fuzzy system, shown in Fig. 4b, designed for Mackey–Glass time differential equation shown in Eq. (5).



(a) *Conventional fuzzy system*



(b) *Hierarchical fuzzy system – Aggregated tree structure*

Fig. 4 Mackey–Glass time delay differentials model

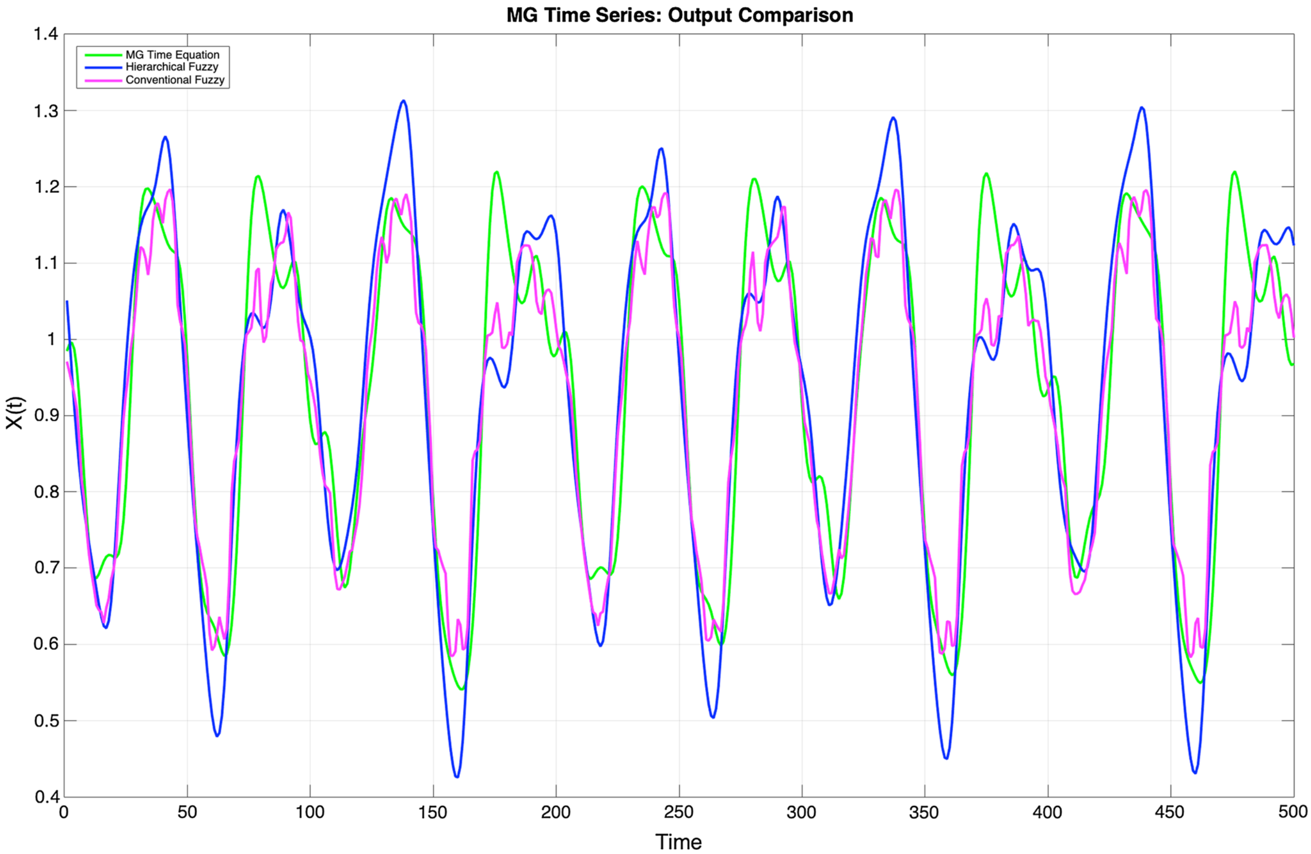


Fig. 5 Output comparison between MG time differential equation, conventional fuzzy and hierarchical fuzzy logic

Table 1 Comparison between conventional systems and hierarchical systems

Correlation variables	Correlation value
Mackey–Glass time differential equation and conventional fuzzy system	0.93
Mackey–Glass time differential equation and hierarchical fuzzy system	0.91

In another detailed comparison, Table 1 presents the correlation [7] between different outputs, i.e., output from Mackey–Glass time differential equation, output from conventional fuzzy and output from hierarchical fuzzy systems. Table 2 presents statistical analysis between these three outputs.

From the graph, it is clearly visible that the correlation between the outputs from the conventional fuzzy and the hierarchical fuzzy systems and the output of the Mackey–Glass time delay differential equation is similar in nature. The statistical analysis between output from Mackey–Glass time delay differential equation and hierarchical system presents a similar behavior. Nevertheless, the hierarchical fuzzy system is designed with a smaller number of rules compared to a conventional fuzzy logic as described and shown in Fig. 4 and thus provides less system complexity. This analysis showcases the hierarchical fuzzy system is an effective option especially with reduced rule base.

Comparison of hierarchical logic with conventional logic approaches

Table 3 provides the descriptive analysis between conventional system and hierarchical system approach.

Table 2 Statistical analysis between Mackey–Glass time differential equation, conventional fuzzy system and hierarchical fuzzy system

Statistical parameters	Output from 'Mackey–Glass time differential equation'	Output from 'conventional fuzzy system'	Output from 'hierarchical fuzzy system'
Variance	0.04	0.03	0.05
Standard deviation	0.2	0.18	0.23
Median	0.98	0.98	0.97
Mean	0.93	0.92	0.93
Skewness	− 0.33	− 0.33	− 0.43
Interquartile range	0.38	0.34	0.38

Table 3 Comparison between conventional systems and hierarchical systems

Conventional system (fuzzy/neuro-fuzzy)	Hierarchical fuzzy approach
Time-consuming due to processing of large rule base	Less time-consuming due to processing of reduced rule base
Does not interpret solutions in linguistic form very easily	Easily interpret and optimize due to reduced rules dimensions
Reduction in system performance in case of large number of rules derived from training data	Enhanced system performance due to reduced rule dimensions
High CPU load for a system of large input–output dimension	Low CPU load for distributed systems that consists of several subsystems with low input–output dimension
Maximum possible rules are: m^n, where 'm' is the number of membership functions for each input and 'n' is the total inputs	Maximum possible rules are: $(n - 1).m^2$, where 'm' is the number of membership functions for each input and 'n' is the total number of inputs. It is assumed that every fuzzy logic unit has two inputs
Highly reliant on data availability	Ability to develop hierarchical model for large data with reduced rule base and complexity

Conclusion

Fuzzy logic has extensively been used for several decades in a variety of applications. Because of the explosion in data, the future of fuzzy logic research work will look for more advanced approaches for big data applications. Hierarchical systems offer the most viable option to overcome the limitation of dimensions possessed by conventional systems. Hierarchical systems can handle imprecise, uncertain and vague data. Hierarchical systems provide enhanced performance and efficiency due to reduced rule base and complexity. In this paper, an approach has been discussed by which any system can be designed in the form of desired hierarchical structure. The procedure is illustrated with the help of examples and the results of hierarchical systems match with the conventional systems. This paper will help researchers in exploring design in hierarchical systems and their applications in real-life and big data applications such as WSNs and IoTs.

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Declarations

Conflict of interest The authors declare that they have no conflict of interests.

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