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Determinants of Wealth Disparities in the EU: A Multi-scale Development Accounting Investigation

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Abstract

This paper presents a development accounting framework in order to quantify the determinants of disparities in GDP per hour worked within the EU in 2016. Its originality is twofold insofar as, on the one hand, it theoretically extends the existing framework from 2 factors up to n explanatory factors and on the other, it numerically illustrates this same framework in case where $n = 3$ factors. This illustration is made macro-economically between 19 EU countries—representing 90% of its aggregate GDP—and sectorally between their market, state, and mixed sectors. The calibration data come from the latest EU-KLEMS and PWT versions. Examination of the results by decomposition shows a strong proximity of macroeconomic standard deviations ($ 13.74/h) and the market (13.38) and non-market (12.34) spheres. The differences between countries are fundamentally (around 90% according to each of the three spheres) explained by the disparities in labor quality (and around 10% by the disparities in capital deepening). The profile, however, is not at all the same in real estate activities (mixed sector) whose GDP per hour' standard deviation reaches $ 570.28/h and is completely explained by the disparities in capital deepening.

Keywords Development accounting · Living standards inequalities · Explanatory power of production factors · Variance decomposition · Market economy · State sector and mixed economy in EU

JEL Classifications C00 · D24 · O11 · O47 · O52 · O57

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Introduction

International development accounting works involve decomposing the variance of a variable of interest. They open up interesting perspectives in the identification and quantification of the sources of divergence in living standards between nations. Their application here in the GDP per hour worked (GDP/h) disparities within the framework of the theory of production is not moreover the only perspective. The application to many other relevant variables, to assess the sources of inequalities between countries, regions, sectors ... as indicators of the World Bank (2019), is also possible. More generally, it is the same for indicators other than GDP (including non-economic issues) with the condition, like here, that their variances can be decomposed.

As economic growth accounting, development accounting (DA) seeks to measure the contribution of each production factor (including that of total factor productivity (TFP)) to a chosen aggregate measuring wealth (GDP but also GDP per capita, per hour worked or per worker…). In summary, the difference is that growth accounting seeks to assess these contributions in the form of flow to flow (contribution of the variation in the volume of each production factor—including the quality associated with it when this is measurable[1]—to the evolution of GDP in volume) while that of DA is concerned with stocks (contribution of the volume of each production factor—including its quality—to GDP in volume).

In summarizing their respective objects, we can write:

Economic growth accounting:

$$\text{(A)} \quad \frac{\Delta\ \text{GDP}_{\text{real}}}{\text{GDP}_{\text{real}}} = F\left(\frac{\Delta \text{Factors of production}}{\text{Factors of production}}, \frac{\Delta \text{TFP}}{\text{TFP}}\right)$$

Economic development accounting:

$$\text{(B)} \quad \text{GDP}_{\text{real}} = F(\text{Factors of production, TFP})$$

The variation of TFP emerges as a residual between the change of GDP and that of volume production factors (A), while the TFP always appears as a residual between GDP and the volume of these inputs (B).

Both accounting exercises (A and B) can be applied to national or regional macroeconomic frameworks but also at sectoral level… (one or more countries, regions or sectors, on one or more date or over one or more periods) or to international comparisons on one or more dates or periods.

The application will concern here 19 EU countries in 2016 envisaged both at a macroeconomic level and at three sectoral levels: The global economy—EG, the sphere of market activities—SM, that of non-market or state activities—SNM, and finally the mixed sphere or semi-public—Sm.

From an international DA perspective, the dependent variable is no longer the quantity itself (like GDP in the above formulations) but its variance. Thus, the

[1] We speak more precisely of "technical bias evolution" on the concerned production factor.

so-called (international) DA method will specifically relate to the decomposition of the variance of GDP in levels for a given year. The formula for the variance of the dependent variable which is a function of n explanatory factors will obviously include the variances in these factors but also their covariances.

In this article, the evaluation framework is therefore macro-accounting and sectoral using a production function with n = 3 factors. A generalization to n factors is also proposed showing how to identify and attribute explanatory powers of factor variances to disparities in GDP in each of the four configurations.

The identification step makes it possible to know, for each explanatory factor considered in a production function, its variance as well as its co-movements with other factors. Because of these latter, there will appear a problem of breakdown of each co-movement toward the variance in each factor. In other words, the relevant amount to be attributed to variance of each explanatory factor should be extracted from each covariance: This is the question of attribution; we develop a breakdown approach for the co-movements of factors and show how it constitutes an advance compared to the usual way of proceeding. We will work here with 3 factors, but our approach will be generalized in the case of n factors. Finally, as regards the evaluation of each factor contributions variance to the variance in living standards, we apply (see Sect. 2.1) the usual growth accounting method (Solow 1956, 1957).

Summarizing these steps, we can say that the identification corresponds to the simple knowledge of the variances–covariances matrix of the explanatory factors selected, while the attribution corresponds to the transfer of the only covariances' values to the variances of each factor.

The scope therefore concerns 19 EU countries in 2016 (13 are in Eurozone). EU membership and the availability of data needed to calibrate the DA equation are the two criteria for constructing this sample. 2016 is the most recent year for which all the data are fully available. The sample is therefore currently the largest possible (19/28). It represents 9/10 of the wealth produced in EU.

Finally, with regard to the decomposition of economies according to the spheres indicated, we use the EU-KLEMS database (Stehrer et al. 2019), which is based on the latest revision of the Statistical Classification of Economic Activities in the European Community NACE Rev. 2 (Eurostat, 2008). The PPP comparability of the data is ensured by the use of the PWT 9.1 (Feenstra et al. 2015).

Among the DA works based on a decomposition analysis of the variance in GDP, in which our contribution is situated, three aspects currently raise questions. They hinder identification and precise measurement of the determinants of variations in living standards between countries.

A first aspect is that they consider the reproducible productive factors as a single aggregate of factors. This has two consequences in terms of measuring development: (i) the contribution of the variance of the other factors is not distinguished from that of the aggregate and (ii) the co-movements (covariances) between these factors are not either, since the single aggregation does not let them evolve separately. However, there can be differences between the contributions of each factor, on the one hand, and the co-movements are empirically considerable, on the other hand (the literature and our results corroborate it); there is therefore an interest in separating the factors in order to attribute more precisely the portion of each co-movement which should go back to the variance of each factor. This portion is added to the variance of each factor to form its true contribution in the total variance of living standards.

A second is linked to the quality of accounting breakdown. Indeed, when all co-movements are not taken into account, outliers such as the TFP variance and that of the single aggregate of factors exceeding the variance in the dependent variable or is less than that of the dependent variable ... can arise. The example below allows to better visualize: Suppose, without loss of generality, we can write the general production function as: y = TFP ∗ Factors (in case of Cobb–Douglas, Factors = $K^{\alpha}L^{1-\alpha}$). By taking the logarithms of this function and then its variance, we write:

$$\mathrm{varlny} = \mathrm{var}\ \ln \mathrm{TFP} + 2\mathbf{cov}(\mathbf{lnTFP}, \mathbf{lnFactors}) + \mathrm{varlnFactors}$$

Suppose that the statistical results of the 3 series (y, TFPand Factors), give:

$$\begin{cases} \mathrm{varlny} = 6 \\ \mathrm{varlnTFP} = 9 \\ 2\mathrm{cov}(\mathrm{lnTFP}, \mathrm{lnFactors}) = -4 \\ \mathrm{varlnFactors} = 1 \end{cases}$$

From these results, ignoring the third term above leads to consider that the variance in the TFP explains 150% (9/6) of that of the product, which is an aberrant result. Similarly, the sum of the variances in the TFP and the factors would explain the variance in the product up to 166.7% ((9 + 1)/6) which is also aberrant. Saying that the variance in the factors explains 1/6 of that of the product is not in itself absurd but becomes so by noting that these factors are indeed in co-movement with TFP.

A third is that the solutions to the aberrations presented in the empirical situation exposed above are not currently satisfactory. This third aspect draws attention to the fact that the solutions proposed in the literature to redistribute covariances are themselves likely to generate aberrations. It will therefore be necessary to find a redistribution key from each co-movement toward each variance concerned, without generating aberrations like those mentioned in aspect 2 and pointed out in the literature.

The purpose of the article is fourfold. We are interested in a DA framework: (i) able to take into account all the co-movements, (ii) including in dimension n > 2 production factors, (iii) which solves the possible aberrations in the explanatory powers reported in the literature and finally (iv) applied not only at the macroeconomic level (EG) but also sectoral (SM, SNM and Sm).

It seems instructive before closing this introduction, to present this goal equivalently, by including it the evolution of DA works. Indeed, by mapping 4-step evolution process, we have:

Step 1: var (Standard of living) = F (var TFP, var Factors of production)

Step 2: var (Standard of living) = F (var TFP, **cov(TFP, Factorsofproduction)**, var Factors of production)

$$\text{var (Standard of living)}$$

Step 3: $= F\left(\begin{array}{c}\text{var TFP, cov}(\mathbf{TFP}, \mathbf{factorofproduction}1), \text{cov}(\mathbf{TFP}, \mathbf{factorofproduction}2), \\ \text{cov}(\mathbf{factorofproduction}1, \mathbf{factorofproduction}2), \text{var(factor of production 1)}, \\ \text{var(factor of production 2)}\end{array}\right)$

$$\textit{Step 4}: \text{var (Standard of living } \mathbf{bysector - orsphere-}) = \text{F}\begin{pmatrix} \text{var TFP, cov}(\mathbf{TFP}, \mathbf{factorofproduction}1), \text{cov}(\mathbf{TFP}, \mathbf{factorofproduction}2), \\ \text{cov}(\mathbf{factorofproduction}1, \mathbf{factorofproduction}2), \text{var(factor of production 1)}, \\ \text{var(factor of production 2)} \end{pmatrix}$$

Our contribution consists here in the elaboration and quantification of steps 3 and 4. The difficulty they pose (we allude here to the so-called problems of identifying and attributing the explanatory powers of each factor variance to the variance in the inter-country living standards) and its solution will also be explained and applied in a framework with three production factors. A generalization of this framework to *n* factors is also proposed. It will be macroeconomic (Step 3) and sectoral (Step 4), thus focusing on global economies (EG) as well as on their SM, SNM and Sm' spheres.

The text is organized as follows: "Development Accounting, Literature in Growth and Development Accounting" section reviews the practice of DA, draws up an inventory of literature and offers an overview of changes in living standards in the EU between 1999 and 2016. "Development Accounting Framework and Statistical Sources" section presents the accounting frameworks (macroeconomic and sectoral) and all the statistical data to be assembled for the calibration of these frameworks. "Numerical Applications for Identifying and Attributing Explanatory Powers of GDP Disparities in the EU-19 (Macro-Economic Case: EG)" section shows the identification results of each factor to the disparities in GDP/h between the 19 EU countries. It then lays out the question of the attribution of their explanatory powers and deduces from it, in a generalized framework, a proposal for a solution to the aberrations mentioned. It ends by applying this solution to determine the explanatory powers of the variance in each production factor to the disparities in GDP/h. "Numerical Applications for Identifying and Attributing Explanatory Powers of GDP/h Disparities in EU-19 (Sectoral Cases: SM, SNM and Sm)" section applies this same solution but at the three sectoral levels. To do this, it first shows how the economies considered are each distributed according to these three spheres; then, it determines the explanatory powers of each production factor to the disparities of GDP/h in each sphere; finally, results are summarized and commented before concluding.

Development Accounting, Literature in Growth and Development Accounting

Illustration of the Development Accounting Exercise for a Single Country

In its last stage, where it quantifies the contributions of the variances in each factor to the variance in the dependent variable, the (international) DA exercise uses exactly the same technique as that of growth accounting. So, we illustrate the latter here for a country and a given year (the dependent variable is therefore not the variance in the quantity to be explained but the quantity itself). It is a case of (national) DA which allows us to focus only on the quantification aspect without mentioning the phases of identification and attribution which here, become irrelevant, since it is the formula of variance who justified them; let us note, respectively:

y, l, k_s, l_s and tfp the logarithms of the volume of GDP, the volume of hours worked (but this may also be the number of employees), the type s capital, the type s labor and TFP; α_s and β_s the elasticities of GDP relative to k_s and l_s (with $\sum_s \alpha_s + \sum_s \beta_s = 1$). We have the productivity decomposition equation (but it can be only GDP too), with a Cobb–Douglas production function:

$$Y = TFP \prod_s K_s^{\alpha_s} \prod_s L_s^{\beta_s}$$

$$y - l = tfp + \sum_s \alpha_s (k_s - l) + \sum_s \beta_s (l_s - l)$$

Following the right term of the equation, the logarithm of labor productivity breaks down between three contributions: (i) that of the logarithm of the TFP or Solow residual (measured as the accounting's residual between the left term and the other two terms on the right and supposed to capture any contribution not attributable to physical or qualitative changes in the other two terms of the right-hand member), (ii) that of global capital intensity (or global capital deepening (CD) equal to the sum of CD by category of capital) and (iii) that of the labor composition (or labor quality which is equal to the sum of the contributions of each category of workers)—of course, it is possible to calculate the contribution to the level of productivity, of each of the categories of capital or labor. Here is a numerical example:

$$(y - l) = 15; tfp = 7.5; \sum_s \alpha_s = \frac{1}{3}; \sum_s \beta_s = \frac{2}{3}; \sum_s \alpha_s (k_s - l) = 6; \sum_s \beta_s (l_s - l) = 1.5; (l_2 - l) = 2; \beta_2 = \frac{3}{10}$$

We will have the following relative contributions:

The TFP will have contributed to labor productivity by 50% (7.5/15); the global CD will have contributed by 40% (6/15) and the labor quality by 10% (1.5/15). If we are interested in any of the categories of hours worked, for instance s = 2, and knowing that elasticity of productivity with respect to this category $\beta_2 = \frac{3}{10}$ and that the composition effect attached to this category is: $(l_2 - l) = 2$, we calculate $\beta_2 (l_2 - l) = 0.6$ which implies a relative contribution of labor quality of category 2 of 4% (0.6/15).

Beyond the elementary distinction in economics between level and evolution rate (or stock and flow) which has been said to draw the line between these two accounting frameworks, the essential technique for assessing the contribution of a factor to growth (or to the level of GDP) is identical : The contribution of a factor to the growth of GDP (or of GDP in volume when reasoning in DA) is measured by the product of the growth rate in volume of this factor (or in the volume of this factor) and its share in GDP's value. The growth residual (or the level of TFP) then stands out as the difference between the evolution of GDP (or GDP itself) and the sum of the contributions of each factor, each contribution being calculated as above.

Literature in Growth and Development Accounting

We present here a set of growth and development accounting works giving greater place to these latter, in particular those proceeding from GDP variance decomposition, more in connection with this article.

The growth accounting framework is associated with Solow (1956, 1957), then Jorgenson (1966) and Hulten (1992) but also with the numerous variations works[2] related to it in the literature. The latter is much more abundant than that on DA.

The DA framework is associated with the pioneering work of Denison (1967) looking at the detailed explanatory factors of the differences in wealth between the USA and 8 European countries, in terms of level (year 1960) and rates (from 1950 to 1964) or Christensen et al. (1981) examining the differences in product, factors of production and productivity between the USA and 8 of their largest trading partners ; with econometrics approach, Mankiw et al. (1992) add to the framework of Solow's model (1956), the "human capital" variable (alongside capital and labor : known as "augmented Solow model"). The authors manage, from a regression of the product per capita on investment rates, and therefore of human capital formation, to explain 78% of the variance in the product per capita worldwide (the rest coming from TFP) in 1985. The identification question is not posited, and everything happens as if the co-movements were all zero. Furthermore, their measurement of human capital would not be perfect (Klenow and Rodriguez-Clare, 1997, p.

[2] A general and synthetic overview of the conceptual framework for growth accounting can be found for instance in the OECD manual (2001, section 2.4). Several works in the spirit of this framework can be evoked here: The works of Jorgenson (1995) for example, analyzing comparatively the differences in growth between industrialized countries post-World War II; of Young (1995) explaining that the "Asian miracle," in particular an average annual growth around 5.5% in 8 countries in Southeast Asia between 1960 and 1995, could be explained by the growth of labor and capital and less by TFP; or Barro and Sala-i-Martin (2004) showing that the "augmented Solow model" was consistent with the speeds of convergence between countries and between states or regions of the same entity (USA, Japan, Europe).

More recently, in France, we have for example the works of Cette et al. (2004, 2005a, b, 2014) and Daw (2019) ; in the USA, the works of Oliner and Sichel (2002) or Jorgenson *et al.* (2004, 2006 and 2008) ; Baier et al. (2006), show for 144 countries between 1990 and 2000, that only 14% of the evolution of the product per worker is due to the evolution of TFP (authors present a development accounting part that we describe in this section). The papers of Van Ark et al. (2008) for a comparison between the USA and the European Union or even Oulton (2002) and Marrano et al. (2009) for the UK, illustrate according to the authors or the year of publication, aggregated (macroeconomic) or more disaggregated frameworks of exogenous retrospective and prospective growth accounting without bias of technical progress (Except Daw 2019, where we retrieve these biases). In Japan, Sato and Tamaki (2009) illustrate an aggregated growth accounting exercise (exogenous long-term retrospective growth accounting with technical progress bias). The works of Madsen (2010a, b), Fernald and Jones (2014) and Bergeaud et al. (2017) for example can also be considered as variation but in which endogenous modeled mechanisms appear explaining the evolution of factors contributing to growth without, however, moving toward what is meant by general equilibrium modeling of economies that are measured.

Indeed, today, alongside the more or less "standard" growth accounting frameworks mentioned above, there is a second approach of growth accounting, but which is carried out using general equilibrium modeling (à la Uzawa, 1963) of the economy to be measured (Greenwood et al. 1997; Cummins and Violante 2002; Whelan 2003; Fisher 2006; Ngai and Samaniego 2009; Oulton 2012; Byrne and Corrado 2017…).

79). Especially the econometric method[3], which does not deal with the identification question of co-movements between explanatory variables, is not the most suitable for DA (*Ibid*.).

(International) DA works relate to the variance decomposition of GDP in level for a given period:

Klenow and Rodriguez-Clare (1997) analyze for 1985 (and from 1960 to 1985 for their growth accounting exercise) and for 98 countries, the so-called identification[4] question and therefore the role of TFP and reproducible factors (physical capital, human capital, labor) in the explanation of the differences in level (and rate) of GDP per worker between nations. The co-movement between TFP and the aggregate of reproducible factors is attributed to TFP which would boost capital productivity and therefore accumulation. Their overall result gives as main explanatory factor, TFP differences between countries (while Mankiw et al. (1992) showed that major role—80%—was due to reproducible factors)[5].

Hall and Jones (1999), for year 1988 and 127 countries, come to the conclusion according to which TFP and beyond this one, what authors call the "social infrastructures" (institutions, economic policies and social factors which explain TFP but also physical capital accumulation by productive organizations and human capital by individuals) are responsible for the disparities in product per worker between countries. The factors are disaggregated, and their respective contributions are calculated but without considering any co-movement between them or *vis-à-vis* the TFP.

Caselli (2005), for the year 1996 and 93 countries, seeks to understand to what extent the model with only an aggregate of reproducible factors explains the disparities in product per worker. The author supposes that all the countries have an identical TFP (consequently, the co-movement between the TFP and the single aggregate is here zero) in order to calculate the explanatory power of the model with an aggregated factor ("Factor-only model"). Its main result, if all TFPs are identical, is an explanatory power of 40% of the variance in the product per worker.

Baier et al. (2006), between 1990 and 2000, for 144 countries, show that on average, only 14% of the evolution of output per worker was due to the evolution of TFP for the growth accounting in time series (temporal data by country). This order of magnitude is similar to that of Turner et al. (2013) for the USA. Although this is a growth accounting job, it is also used for international comparisons. We quote it here, because as such, our reflection on the question of attribution through their

[3] With regard to growth accounting, Klenow and Rodriguez-Clare (1997, p.79), Barro and Sala-i-Martin (2004, p.441-442), Caselli (2005, p.10) or even Baier et al. (2006, p.37) highlight the inconvenience of the econometric method linked, among other things, to endogeneity of TFP with GDP as well as to co-movements between explanatory variables. Hulten (2001) sees possible synergies when, for example, econometrics manages to shed light on the content of the growth residual (therefore "our ignorance") which is previously determined by the growth accounting exercise. Although attributing merit to econometrics, OECD (2001, p.19) also highlights several limitations and indicates that the tool recommended in practice remains the usual growth accounting framework.

[4] That we have distinguished from the question of attribution to put forward more the solution adopted for the treatment of the problem than the problem itself.

[5] Even if the variable to be explained is not exactly the same: product per capita in Mankiw et al. (1992) and product per worker for Klenow and Rodriguez-Clare (1997).

Eqs. 4, 5, 6 and 7 joined one aspect of our work. The authors show how to calculate the explanatory power attributed to the aggregate of factors according to whether one adopts, in a polar way, the endogenous growth theories and at the other extreme, the share that goes to TFP in the minds of standard growth theories with exogenous technical evolution in order to distribute the correlated portion[6]; this is done for the co-movement between TFP and the single aggregate of factors since their model considers only these two factors. For the growth accounting in cross section (country data by each period), the authors arrive at the result according to which, the TFP variance explains a little more than 90% of the variance in product per worker over the entire sample. These results are in line with those of Klenow and Rodriguez-Clare (1997) or Easterly and Levine (2001).

Turner et al. (2013), in the part of their work on DA (work which includes growth accounting too), also study the roles of the single aggregate of reproducible factors and of TFP in relation to the disparities of the product by worker between the various American states over the period 1840–2000. Depending on the date of interest, they conclude that the differences in TFP between states explain between 75 and 80% of the variance in the product per worker, the rest being attributable to all of the reproducible factors. Co-movement is considered here too but only between the single aggregate and TFP. The authors apply a technique from Baier et al. (2006) to take account the attribution question linked to co-movement (see p. 346, Eq. 11). In the presence of a single co-movement or what amounts to the same, of two factors in the production function, the attribution they propose seems satisfactory to us in the current state of knowledge. But it is not, in our opinion, usable beyond, that is to say in the presence of more than two factors in the production function.

Tamura et al. (2019) examine the disparities evolutions of product per worker over 168 countries observed in the long period between 1800 and 2010 (their work also includes growth accounting). The authors use a measure of human capital different from that of Barro and Lee (2013) and therefore capable of measuring the level of workers education which is specific to each country; they also envision that human capital is transmitted between generations of the same country but also that there may be externalities of human capital between countries. They perform 22 DA exercises 10 years apart and then take the average. Their decomposition of the variance in the product per worker uses techniques of Baier et al. (2006) and Turner et al. (2013). We always have the aggregate of reproducible factors and TFP as the only co-movement. The results, for their model with intergenerational human capital (with or without externalities between countries), show an explanatory power of around 60% for reproducible factors to the variance in the product per capita; this highlights the positive role of intergenerational accumulation of capital (and therefore of the very long evaluation period) and of externalities (since the results of the model with externalities are even more favorable to reproducible factors role). The TFP role, "our ignorance," is then reduced but remains substantial.

[6] Please, see note 15 for the meaning of "correlated portion."

Living Standards in EU Economies Between 1999 and 2016: A Macro-Economic Glance

In 2016, according to our calculations from the PWT 9.1 database, the aggregated GDP of the EU-28 which is $ 18,471,588,300,000 (in PPP, US $ 2011) is the highest in the world compared to that of USA (17,327,400,000,000) and China (16,817,996,000,000). A subsequent Brexit formalized since January 31, 2020, would downgrade the EU-27 to third place in the world in 2016.

The sum of GDP in PPP (variable cgdpo, from PWT 9.1) of the 19 countries (EU-19) selected represents 90% of the aggregated GDP of the EU-28 in 2016. The aggregated GDP of the nine remaining countries (Bulgaria, Croatia, Cyprus, Estonia, Ireland, Latvia, Malta, Portugal and Romania) is around 10%.

Between 1999 and 2016 (see Table 1), aggregate GDP of EU-19 increased by 45%, while hours worked (H_EMP from EU-KLEMS) increased by 7%, which led to an increase in GDP/h of more than 35%.

Even if the GDP/h progresses of the last EU memberships are much higher than this last figure (see Hungary, Lithuania, Poland, Czech Republic, Slovakia, and Slovenia—and the profile is the same for those even more recent like Bulgaria, Croatia, and Romania), the evolution of the level differences remains nevertheless, although modestly (2%), generally increasing.

Observation in levels at a given date allows to see a standard deviation of the GDP/h of $ 13.38 between the countries of the sample in 2016 (for information, the standard deviation is $ 17.91 in the Eurozone EU-19). The DA exercise will seek to determine the contributions of the variances in each factor of production considered to the variance in the level of GDP/h.

The existence of a positive standard deviation, as is the case in this sample, is, however, not a necessary condition for implementation of the DA exercise. A zero standard deviation can also be examined with interest. The exercise issue will remain identical but just to reformulate; it will seek to study the contributions of the variances in each production factor to a same development level between countries (since their variance is zero). Although the sum of the contributions of the variances' factors is zero, the structure of the contributions may reveal (or not) various ways to reach the same level of development.

Development Accounting Framework and Statistical Sources

The Macro-Accounting Framework (global economy—EG)

The production function used, as in the literature, is a Cobb–Douglas:

$$Y_G = AK^{\alpha}(aH)^{1-\alpha} \tag{1}$$

With K representing the productive services of the capital factor which is measured at constant quality and aH the hours worked (H) adjusted for the quality of these hours (a). α and $(1-\alpha)$ are the capital and labor factors elasticities of output. A

represents the total factor productivity (TFP) measuring "our ignorance" of the precise sources explaining the total output level Y_G.
Two minor differences should be mentioned compared to the literature; the production function therefore has the same form as that used in Hall and Jones (1999) and Caselli (2005)[7] with the following difference: for these authors, the labor factor is the product of the number of workers L by a human capital index h calculated in accordance with Barro and Lee (2013) whereas in Eq. 1, it is the product of the number of hours worked H by a quality index of these hours (a) calculated in accordance with Jorgenson et al. (1987). Electronic Supplementary Material (ESM) explains how to obtain it.

Unlike Caselli (2005, p. 7 who uses $\alpha = 1/3$) or similarly Tamura et al. (2019, p. 4), we consider that α must be recalculated according to the list of our countries. Therefore, the factors' elasticities of output will be averages of the elasticities of each country, weighted by the ratio of its GDP and the aggregate GDP of the sample (19 EU countries in 2016). This information—as the previous relating to the labor quality—are not directly available in the EU-KLEMS database and ESM shows how to obtain it.

The first difference is recalled by the author himself specifying the interest and lack of data (see p.17). The second because empirically, the average elasticities are not always 1/3 and 2/3, especially when looking at their average by sector (and not just by country).

Two other important differences, related to the objective of this work this time, are also to be recalled here. The first consists in extending the calculations of factor contributions to consider more than two factors of production and their respective co-movements. Thus, we show how to generalize the accounting framework to n separate factors where the literature sticks to one or two factors. The second difference consists in the application of this framework in the case of our production function with therefore n = 3 and that, for EG as well as for SM, SNM and Sm.

The first difference offers the framework a more general scope since the factors of production considered and therefore their interactions are simply more numerous. The second, in addition to its statistical interest—breaking down the macroeconomic calibration data into sectoral data—seeks to know whether the role of the factors explaining the disparities in living standards between the spheres of the countries considered differs from that existing between their global economies.

Now writing Eq. 1 in hours worked, we get:

$$y_G = Ak^{\alpha}a^{1-\alpha} \tag{2}$$

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[7]

$$Y = AK^{\alpha}(Lh)^{1-\alpha}$$

With: $k = \frac{K}{H}$ the capital per hour worked in the global economy. We also know that[8]:

$$\text{var}\,(\text{aX} + \text{bY} + \text{cZ}) = \text{a}^2\text{varX} + \text{b}^2\text{varY} + \text{c}^2\text{varZ} + 2\mathbf{abcov(X, Y)} + 2\mathbf{accov(X, Z)} + 2\mathbf{bccov(Y, Z)} \tag{3}$$

By writing Eq. 2 in logarithms and then taking its variance[9], we come to:

$$\text{varlny}_G = \text{var lnA} + \alpha^2\text{var lnk} + (1-\alpha)^2\ \text{var lna} + 2\boldsymbol{\alpha}\mathbf{cov(lnA, lnk)} + 2(1-\boldsymbol{\alpha})\ \mathbf{cov(lnA, lna)} + 2\boldsymbol{\alpha}(1-\boldsymbol{\alpha})\mathbf{cov(lnk, lna)} \tag{4}$$

This equation, here with three factors (TFP, capital per hour worked, quality of hours worked) is that of DA, which will need to be calibrated in order to identify, attribute and quantify the origins of disparities in living standards within the EU.

Sectoral Frameworks

The global production function is here declined in 3 versions. Each one represents the spheres considered. To lighten the notation, only the dependent variable will distinguish these spheres; M for the market, NM the non-market (state sector) and m the mixed sphere. The explanatory factors, although noted indistinctly, obviously refer to the factors which are specific to each of 4 spheres.

Market sphere: SM

$$Y_M = AK^\alpha(aH)^{1-\alpha} \text{ or } y_M = Ak^\alpha a^{1-\alpha} \tag{5}$$

$$\text{var ln y}_\text{M} = \text{var lnA} + \alpha^2\text{var lnk} + (1-\alpha)^2\ \text{var lna} + 2\boldsymbol{\alpha}\mathbf{cov(lnA, lnk)} + 2(1-\boldsymbol{\alpha})\mathbf{cov}(\ln \mathbf{A}, \mathbf{lna}) + 2\boldsymbol{\alpha}(1-\boldsymbol{\alpha})\mathbf{cov(lnk, lna)} \tag{6}$$

Non-market sphere: SNM

$$Y_{NM} = AK^\alpha(aH)^{1-\alpha} \text{ or } y_{NM} = Ak^\alpha a^{1-\alpha} \tag{7}$$

8

$$\text{var}\,(aX+bY+cZ) = E\left(([aX+bY+cZ]-E[aX+bY+cZ])^2\right) = E\left((a[X-E[X]]+b[Y-E[Y]]+c[Z-E[Z]])^2\right)$$
$$= E\left(\left(a^2(X-E[X])^2\right)\right) + \left(b^2(Y-E[Y])^2\right) + \left(c^2(Z-E[Z])^2\right) + 2ab(X-E[X](Y-E[Y]) + 2ac(X-E[X])(Z-E[Z]) + 2bc(Y-E[Y])(Z-E[Z]))$$
$$= a^2E(X-E[X])^2 + b^2E(Y-E[Y])^2 + c^2E(Z-E[Z])^2 + 2ab\,E((X-E[X])(Y-E[Y])) + 2ac\,E((X-E[X])(Z-E[Z])) + 2bc\,E((Y-E[Y])(Z-E[Z])),$$ and finally:

$$\text{var}\,(\text{aX} + \text{bY} + \text{cZ}) = \text{a}^2\text{varX} + \text{b}^2\text{varY} + \text{c}^2\text{varZ} + 2\mathbf{abcov(X, Y)} + 2\mathbf{accov(X, Z)} + 2\mathbf{bccov(Y, Z)}$$

If the interdependencies were zero: $\text{var(aX + bY + cZ)} = \text{a}^2\text{varX} + \text{b}^2\text{varY} + \text{c}^2\text{varZ}$.

9 By analogy with the variance formula, set: $\text{X} = \text{lnA}$; $\text{Y} = \text{lnk}$; $\text{Z} = \text{lna}$; $\text{a} = 1$; $\text{b} = \alpha$ and $\text{c} = (1-\alpha)$

$$\text{var}\ln y_{NM} = \text{var}\ln A + \alpha^2\text{var}\ln k + (1-\alpha)^2\ \text{var}\ln a + 2\boldsymbol{\alpha}\mathbf{cov}(\mathbf{lnA},\mathbf{lnk}) + 2(1-\boldsymbol{\alpha})\mathbf{cov}(\ln\mathbf{A},\mathbf{lna}) + 2\boldsymbol{\alpha}(1-\boldsymbol{\alpha})\mathbf{cov}(\mathbf{lnk},\mathbf{lna}) \tag{8}$$

Mixed sphere: Sm

$$Y_m = AK^{\alpha}(aH)^{1-\alpha} \text{ or } y_m = Ak^{\alpha}a^{1-\alpha} \tag{9}$$

$$\text{var}\ln y_{m} = \text{var}\ln A + \alpha^2\text{var}\ln k + (1-\alpha)^2\ \text{var}\ln a + 2\boldsymbol{\alpha}\mathbf{cov}(\mathbf{lnA},\mathbf{lnk}) + 2(1-\boldsymbol{\alpha})\mathbf{cov}(\ln\mathbf{A},\mathbf{lna}) + 2\boldsymbol{\alpha}(1-\boldsymbol{\alpha})\mathbf{cov}(\mathbf{lnk},\mathbf{lna}) \tag{10}$$

Data for Calibration of the Development Accounting Equations

For each member of EU-19 in 2016, the necessary data come from the EU-KLEMS (2019) and PWT 9.1 (2018) databases. The exchange rates of the six countries outside Eurozone are annual (ECB and INSEE). The allocation of economic activities sectors belonging to each sphere considered is based on NACE Rev. 2 (2008). Table 2 summarizes these data, their notations and sources.

Numerical Applications for Identifying and Attributing Explanatory Powers of GDP Disparities in the EU-19 (Macro-Economic Case: EG)

Identification

Before discussing the problem of the attribution of explanatory power, the validity of DA Eq. 4[10]—and therefore calculations in Tables 3 and 4—should be checked numerically. We have indeed:

$$0.092 = 0.05 + 0.0215 + 0.1482 - 0.0501 - 0.1538 + 0.0761$$

Problem of Attributing Explanatory Power to Each Production Factor

Attributions of Explanatory Powers of Factors: State-of-the-Art and Critical Discussion

In Caselli's "Factor-only model" (2005), the interdependencies are canceled out by the double assumption that the factors of production form a single entity which itself is not affected by the movements of the TFP. We can clearly see this double hypothesis by observing its Eq. 5, p. 10. However, Table 4 clearly shows that such interdependencies are not zero. They are therefore likely to play a more or less substantial role in explaining the disparities in the product per hour worked.

[10] Or what, except for the notations, is equivalent to (15) or (17).

Table 1 GDP/h (PPP $ 2011) for 19 EU countries

Countries	cgdpo 1999	H_EMP 1999	(cgdpo/H_EMP) 1999	cgdpo 2016	H_EMP 2016	(cgdpo/H_EMP) 2016
Germany (Accession: EU: 1958; € 1999)	2,704,247,000,000	57,716,000,000	46.85	3,887,404,000,000	59,477,000,000	65.36
Austria (1995;1999)	266,874,000,000	6,687,950,000	39.90	402,444,200,000	7,022,609,000	57.31
Belgium (1958;1999)	312,376,000,000	6,373,957,000	49.01	460,596,300,000	7,207,172,000	63.91
Denmark (1973)	173,941,000,000	3,983,185,000	43.67	257,640,500,000	4,055,490,000	63.53
Spain (1986;1999)	982,929,000,000	27,922,830,000	35.20	1,591,142,000,000	32,372,429,000	49.15
Finland (1995;1999)	159,920,000,000	3,952,600,000	40.46	216,680,800,000	4,111,700,000	52.70
France (1958;1999)	1,804,415,000,000	39,407,107,000	45.79	2,593,848,000,000	42,094,471,000	61.62
Greece (1981;2001)	238,605,000,000	9,048,108,000	26.37	261,900,300,000	8,288,920,000	31.60
Hungary (2004)	135,643,500,000	7,133,200,000	19.02	222,811,500,000	7,827,347,000	28.47
Italy (1958;1999)	1,833,668,000,000	42,234,536,000	43.42	2,209,241,000,000	42,742,323,000	51.69
Lithuania (2004;2015)	35,054,000,000	2,522,552,000	13.90	75,371,940,000	2,565,893,000	29.37
Luxembourg (1958;1999)	23,926,000,000	399,149,000	59.94	43,483,140,000	633,135,000	68.68
Netherlands (1958;1999)	573,257,000,000	11,898,370,000	48.18	808,651,500,000	12,804,417,000	63.15
Poland (2004)	494,955,300,000	30,219,168,000	16.38	987,992,100,000	33,028,202,000	29.91
Czech Republic (2004)	193,513,900,000	9,277,752,000	20.86	327,763,000,000	9,362,692,000	35.01
UK (1973)	1,745,226,000,000	46,515,691,000	37.52	2,540,459,000,000	53,067,778,000	47.87
Slovakia (2004;2009)	72,111,000,000	3,750,882,000	19.23	142,028,400,000	4,038,652,000	35.17
Slovenia (2004;2007)	41,093,000,000	1,557,415,000	26.39	57,953,510,000	1,585,963,000	36.54
Sweden (1995)	294,118,300,000	6,988,320,000	42.09	424,675,200,000	7,943,530,000	53.46
Total	11,791,754,700,000	310,600,452,000	37.96	17,087,411,190,000	332,286,193,000	51.42
Standard deviation			13.12			13.38

Source: Author's calculations based on EU KLEMS database, 2019 release and PWT 9.1 database, July 2018 release

Table 2 Calibration data

Data	Notations	Source
GDP at ppp (2011 US $)	cgdpo	PWT 9.1
Capital stock at ppp (2011 US $)	cn	Idem
Capital elasticity of GDP	CAP/VA	EU-KLEMS and Author[a]
Labor elasticity of GDP	(1-CAP/VA)	Idem
Total hours worked	H_EMP	EU-KLEMS
Quality of hours worked	LAB_QPH	EU-KLEMS and Author[b]
Total factor productivity (TFP)	A	Author[c]
Country exchange rates (6) outside Eurozone	n.a	ECB and INSEE
Market sphere	SM = MARKT + Sector T	EU-KLEMS - NACE Rév.2
Non-market sphere	SNM = Sectors O, P, Q + U	Idem
Mixed sphere	Sm = Sector L	Idem

[a]Technique for obtaining CAP/VA described below

[b]Idem for LAB_QPH

[c]Idem for TFP

Source: Author

Considering co-variations therefore raises the issue of power attribution. If we admit that TFP stimulates co-movement with the accumulation of capital/h, then the correlated portion between these factors should be attributed to TFP. Otherwise, it should be attributed to capital deepening.

The current literature takes into account, but only partially, co-movements (and not all of the interdependencies) and proposes a breakdown of co-movements which only applies in a framework that does not exceed two factors (TFP and single aggregate of reproducible factors).

Covariances nevertheless have an undisputed empirical reality in the literature. The quantification of Eqs. 4, 6, 8 and 10 confirms this stylized fact insofar as in absolute value, they count according to the sphere considered, between 67 and about 304% of the variance in the GDP/h. We will call this proportion, "redistributable co-movement." Its amount will be calculated for each of the spheres.

Each of the existing explanatory power attribution approaches has its merits but also shortcomings in the breakdown of these covariances. We can therefore choose not to take them into account (Caselli, 2005), but then we underestimate the explanatory power of the factors of production whose total explanatory power will actually be different from 1 which is not very suitable; Klenow and Rodriguez-Clare (1997) propose to attribute half of each covariance to each variance. Insofar as we do not really know which factor induces the variance of other, it is intuitive to consider that each is at the origin of half of their covariance and which will inflate or deflate the variance of the two factors. However, with this way of doing things, nothing excludes that the swelling of the variances is of such importance that it unfortunately

Table 3 Data for the calibration of the development accounting equation (global economy—EG) and ln A calculation

Countries	(1) cgdpo/H_EMP	(2) ln (cgdpo/ H_EMP)	(3) cn/H_EMP	(4) ln (cn/H_EMP)	(5) 0,355*4)	(6) LAB_QPH	(7) ln LAB_QPH	(8) 0,645*7	(9) ln A
Germany (Accession: EU: 1958; € 1999)	65.36	4.180	311.90	5.743	2.039	5.31	1.670	1.077	1.064
Austria (1995;1999)	57.31	4.048	349.13	5.855	2.079	4.51	1.506	0.972	0.998
Belgium (1958;1999)	63.91	4.157	415.56	6.030	2.141	5.63	1.728	1.115	0.902
Denmark (1973)	63.53	4.151	356.08	5.875	2.086	5.64	1.730	1.116	0.950
Spain (1986;1999)	49.15	3.895	313.35	5.747	2.040	3.46	1.241	0.801	1.054
Finland (1995;1999)	52.70	3.965	269.67	5.597	1.987	4.35	1.470	0.948	1.029
France (1958;1999)	61.62	4.121	339.75	5.828	2.069	5.64	1.730	1.116	0.936
Greece (1981;2001)	31.60	3.453	283.65	5.648	2.005	1.76	0.565	0.365	1.083
Hungary (2004)	28.47	3.349	181.39	5.201	1.846	0.99	-0.010	-0.006	1.509
Italy (1958;1999)	51.69	3.945	392.97	5.974	2.121	3.89	1.358	0.876	0.948
Lithuania (2004;2015)	29.37	3.380	126.12	4.837	1.717	1.87	0.626	0.404	1.259
Luxembourg (1958;1999)	68.68	4.229	344.15	5.841	2.074	4.66	1.539	0.993	1.163
Netherlands (1958;1999)	63.15	4.146	325.14	5.784	2.053	5.05	1.619	1.045	1.048
Poland (2004)	29.91	3.398	76.45	4.337	1.539	1.68	0.519	0.335	1.524
Czech Republic (2004)	35.01	3.556	235.61	5.462	1.939	1.4	0.336	0.217	1.399
UK (1973)	47.87	3.868	244.04	5.497	1.952	3.83	1.343	0.866	1.051
Slovakia (2004;2009)	35.17	3.560	164.52	5.103	1.812	1.12	0.113	0.073	1.675
Slovenia (2004;2007)	36.54	3.598	279.87	5.634	2.000	1.39	0.329	0.212	1.386
Sweden (1995)	53.46	3.979	280.88	5.638	2.001	4.21	1.437	0.927	1.050

Source: Author's calculations based on EU KLEMS database, 2019 release and PWT 9.1 database, July 2018 release3

Table 4 Global economy: Summary of results obtained: variance of the GDP/h; variance–covariance and correlations matrices; weighted average elasticities

Var ln (cgdpo/H_EMP)	0.092		
Var-cov matrix	*ln(cn/H_EMP)*	*ln(LAB_QPH)*	*ln A*
ln (cn/H_EMP)	0.1705		
ln (LAB_QPH)	0.1662	0.3562	
ln A	− 0.0706	− 0.1192	0.0500
Correlations matrix	*ln(cn/H_EMP)*	*ln(LAB_QPH)*	*ln A*
ln (cn/H_EMP)	1		
ln (LAB_QPH)	1	1	
ln A	0.4140	0.4320	1
Average factor shares	Capital factor: 0.355		Labor factor: 0.645

Source: Author's calculations

leads to a relative variance (compared to that of the product per hour worked) greater than one, which is meaningless. Likewise, deflation can be of such importance that it causes a negative variance (which is not possible, a variance cannot be negative), which also does not make sense[11].

By introducing the correlation coefficients between the factors[12], Baier et al. (2006) then Turner et al. (2013) and Tamura et al. (2019), remain in the idea of the average in front of the impossibility to know which factor triggers the movement of the other. The correlated part is precisely measured thanks to the correlation coefficient. The authors place themselves, on the one hand, in the situation where all the correlated part is due to one factor. On the other hand, they take the symmetrical situation. From these two polar situations, the average power of each factor is the sum of what it returns to it in the favorable situation and in the unfavorable one, the whole divided by two variances (since there have been two situations) of the variable to be explained.

[11] A negative variance (!) is synonymous with a negative explanatory power (!); a relative variance greater than unity (!) is synonymous with an explanatory power greater than 100% (!).

[12] By dividing on both sides, the equation 4 by $var\ln y_G$, one can easily have an equivalent writing but with correlation coefficients, here represented in the expressions in bold:

$$1 = \frac{\mathrm{varlnA}}{\mathrm{varlny_G}} + \alpha^2 \frac{\mathrm{varlnk}}{\mathrm{varlny_G}} + (1-\alpha)^2 \frac{\mathrm{varlna}}{\mathrm{varlny_G}} + 2\boldsymbol{\alpha\rho_{\mathrm{lnA,lnk}}} \frac{\boldsymbol{\sigma_{\mathrm{lnA}}\sigma_{\mathrm{lnk}}}}{\mathbf{varlny_G}}$$

$$+2(1-\boldsymbol{\alpha})\boldsymbol{\rho_{\mathrm{lnA,lna}}} \frac{\boldsymbol{\sigma_{\mathrm{lnA}}\sigma_{\mathrm{lna}}}}{\mathbf{varlny_G}} + 2\boldsymbol{\alpha}(1-\boldsymbol{\alpha})\boldsymbol{\rho_{\mathrm{lnk,lna}}} \frac{\boldsymbol{\sigma_{\mathrm{lnk}}\sigma_{\mathrm{lna}}}}{\mathbf{varlny_G}}$$

With: $\rho_{\mathrm{lni,lnj}} = \frac{\mathrm{cov(lni,lnj)}}{\sigma_{\mathrm{lni}}\sigma_{\mathrm{lnj}}}$ the correlation coefficient, and: $\mathrm{i,j = lnA, lnk\ orlna}$ and: $-1 \le \rho_{\mathrm{lni,lnj}} \le 1$
The object of the game is to allocate the correlated portions of the last three expressions to the numerators of the first three in accordance with the principle of averaging. By correlated portion, we mean, for example between TFP and capital per hour worked, the value of: $2\boldsymbol{\alpha\rho_{\mathrm{lnA,lnk}}\sigma_{\mathrm{lnA}}\sigma_{\mathrm{lnk}}}$. Furthermore, the "all or nothing" technique would consist in assigning all this value to the TFP variance and to that of capital/h in a symmetrical configuration.

Table 5 Explanatory powers (EP) of factors to disparities of the product per hour worked in the EU-19

Variables *(in variance)*	Initial inherent and common EP	Redistributions of common powers	EP before correction (absolute)	Corrected final EP	Final EP (relative)
ln(cn/H_EMP)	0.0215 − 0.0501 ; 0.0761	− 0.02505 + 0.03805	0.0345	0.0105 (0.345–0.024)	11.43%
ln(LAB_QPH)	0,1482 − 0.1538; 0.0761	− 0.0769 + 0.03805	0.1094	0.0814 (0.1094–0.028)	88.57%
ln$A$	0.05 − 0.0501; − 0.1538	− 0.02505–0.0769	− 0.052	0 (− 0.052 + 0.022 + 0.0222)	0%

Source: Author

This attribution strategy is the one that seems the most appropriate to us in the current state of knowledge on the subject. But it nevertheless has two failures: the first is that it is an "all or nothing" strategy: the correlated part is, according to it, either totally the fact of one factor or totally that of another. It cannot consider that a portion of the correlation could be partially due to one factor and partially to another. But this first failure is linked to the problem itself: since we do not know the factor behind the co-movement, it would be surprising to know the precise responsibilities assigned to each factor. The second is that it is no longer applicable in the presence of more than two production factors. Indeed, with two factors, x and y, we are sure that if the whole correlated portion goes to x, the latter portion will go to y in the symmetrical configuration, which guarantees a variance in the factors equal to that of the dependent variable as well in the version "all" than in "nothing" one. With a third factor z, we have several configurations of co-movements of "all or nothing" type (and not only two configurations of a single couple) precisely according to the formula: $2C_n^2$ or 6 configurations for which the property of perfect[13] symmetry no longer holds. On the other hand, in the presence of only two factors (e.g., TFP and all factors as single aggregate) it is currently the most consistent. However, it can be instructive to go beyond two factors, and, in this case, their strategy is no longer operational.

These strategies (Caselli 2005; Klenow and Rodriguez-Clare 1997; Baier et al. 2006; Turner et al. 2013; Tamura et al. 2019) are all useful but therefore have limits. Attribution Strategy in the General Case of a Function with *n* Factors section tries to remedy.

Attribution Strategy in the General Case of a Function with *n* Factors

The proposed method differs from Caselli (2005) by taking into account more factors and all of the co-movements between these factors; it is in agreement with the principle of the mean of Klenow and Rodriguez-Clare (1997) but remedies the possible aberrations that this principle is likely to generate; finally, it can be generalized to *n* factors unlike the technique used by Baier et al. (2006), Turner et al. (2013) and Tamura *et al.* (2019) and which seems satisfactory to us if $n \leq 2$.

The underlying idea of the proposed strategy is that the drawbacks that appear from a direct application of the average principle of the covariance between each pair of factors are exploited as signaling aberrations. For example, the variance in a

[13] If we materialize the "all" of the correlated portion by "+" and the "nothing" by "-", in the case of two factors, the only configuration and its perfect symmetry :x^+y^- (the whole correlated portion goes to the variance in x and nothing to the variance in y) and x^-y^+ (the whole correlated portion goes to the variance in y and nothing to the variance in x) and then we take the mean covariance which comes back to variance x and that of y in the two situations. In the case of three factors, an example configuration is: $x^+y^-;x^-z^+;y^+z^-$ (and there are 5 others, playing on the signs). We have seen in the case of two factors that the "all" is understood by factor, which generates two perfectly symmetrical configurations, which is inapplicable here. In the example given (or among the other 5 combinations), what is the "all"? There is no "all" by configuration since each factor is involved in several configurations (three configurations and their three symmetrical) and there is therefore no "nothing" perfectly symmetrical in the sense of unique. Violation of this property of perfect symmetry of configurations invalidates the "all or nothing" technique when $n > 2$.

production factor—more precisely the “augmented variance” i.e., the sum of the initial variance and the share of the covariance redistributed—which, following a breakdown according to these authors becomes negative, poses a problem. This poses a problem while signifying at the same time, that over the interval $[\text{var x}, 0]$ the responsibility for this factor is completely released and must be sought in the comovement with at least one other factor. It is the same if, following this breakdown, the relative variance (variance in a factor divided by variance in the dependent variable y) becomes greater than unity. In this eventuality, the responsibility for this factor would be totally released on the interval $[1, \frac{\text{var x}}{\text{var y}}]$.

The objective now is to show, in the general case, that is to say a production function—which is not necessarily a Cobb–Douglas—with n factors, how this solution applies (see Step 4 in particular). The transition from *n* factors to our case with n = 3 is finally numerically illustrated.

So, let x_i be a production factor, $i \in [1, n]$ and α_i the factor *i* elasticity of GDP/h; since we have n factors, the variance in GDP/h is written by analogy to Eq. 4 of DA (where we had only 3 factors) and by omitting the “ln” for simplicity, without, however, any restriction of generality:

$$\begin{aligned}\text{var y} &= \text{var}\left(A + \alpha_1 x_1 + \alpha_2 x_2 + \cdots + \alpha_i x_i + \ldots \alpha_n x_n\right)\\ &= \text{var A} + \alpha_1^2 \text{ var } x_1 + \alpha_2^2 \text{ var } x_2 + \cdots + \alpha_i^2 \text{ var } x_i + \cdots + \alpha_n^2 \text{ var } x_n\\ &+ \mathbf{2\alpha_1 cov(A, x_1) + 2\alpha_2 cov(A, x_2) + \cdots + 2\alpha_i cov(A, x_i) + \ldots}\\ &+ \mathbf{2\alpha_n cov(A, x_n) + 2\alpha_1\alpha_2 cov(x_1, x_2) + \cdots + 2\alpha_1\alpha_i cov(x_1, x_i) + \ldots}\\ &+ \mathbf{2\alpha_1\alpha_n cov(x_1, x_n) + 2\alpha_2\alpha_i cov(x_2, x_i) + \cdots + 2\alpha_2\alpha_n cov(x_2, x_n)}\\ &+ \mathbf{\cdots + 2\alpha_i\alpha_n cov(x_i, x_n)}\end{aligned} \tag{11}$$

From the general identification equation above, let us in a first step, group together the specific explanatory powers (variances) and the commons (covariances) for each of the factors:

$$\begin{cases}\mathbf{A)}\ \text{var A} + \mathbf{2\alpha_1\ cov\ (A, x_1)} + \mathbf{2\alpha_2\ cov\ (A, x_2)} + \cdots + \mathbf{2\alpha_i\ cov\ (A, x_i)} + \cdots + \mathbf{2\alpha_n\ cov\ (A, x_n)}\\ \mathbf{x_1)}\ \alpha_1^2 \text{var } x_1 + \mathbf{2\alpha_1\ cov\ (A, x_1)} + \mathbf{2\alpha_1\alpha_2\ cov(x_1, x_2)} + \cdots + \mathbf{2\alpha_1\alpha_i\ cov(x_1, x_i)} + \cdots + \mathbf{2\alpha_1\alpha_n\ cov(x_1, x_n)}\\ \mathbf{x_2)}\ \alpha_2^2 \text{var } x_2 + \mathbf{2\alpha_2\ cov\ (A, x_2)} + \mathbf{2\alpha_1\alpha_2\ cov(x_1, x_2)} + \cdots + \mathbf{2\alpha_2\alpha_i\ cov\ (x_2, x_i)} + \cdots + \mathbf{2\alpha_2\alpha_n\ cov(x_2, x_n)}\\ \vdots\\ \mathbf{x_i)}\ \alpha_i^2 \text{var } x_i + \mathbf{2\alpha_i\ cov\ (A, x_i)} + \mathbf{2\alpha_1\alpha_i\ cov(x_1, x_i)} + \mathbf{2\alpha_2\alpha_i\ cov\ (x_2, x_i)} + \cdots + \mathbf{2\alpha_i\alpha_n\ cov(x_i, x_n)}\\ \vdots\\ \mathbf{x_n)}\ \alpha_n^2 \text{var } x_n + \mathbf{2\alpha_n\ cov\ (A, x_n)} + \mathbf{2\alpha_1\alpha_n\ cov(x_1, x_n)} + \mathbf{2\alpha_2\alpha_n\ cov(x_2, x_n)} + \mathbf{2\alpha_i\alpha_n\ cov(x_i, x_n)}\end{cases} \tag{12}$$

In a second step, we redistribute the common explanatory powers by applying to the only terms in bold of each factor the mean, which is equivalent to rewriting the terms in bold without their “2”; in a third step, we sum with the first terms of each equation which allows us to know the absolute explanatory powers of each factor (before correction, because this sum may possibly have an aberration). Steps 2 and 3 thus lead to the following system:

$$\begin{cases} \mathbf{A})\text{var}\, A + \boldsymbol{\alpha}_1\, \mathbf{cov}\, (\mathbf{A}, \mathbf{x}_1) + \boldsymbol{\alpha}_2\, \mathbf{cov}\, (\mathbf{A}, \mathbf{x}_2) + \cdots + \boldsymbol{\alpha}_\mathbf{i}\, \mathbf{cov}\, (\mathbf{A}, \mathbf{x}_\mathbf{i}) + \cdots + \boldsymbol{\alpha}_\mathbf{n}\, \mathbf{cov}\, (\mathbf{A}, \mathbf{x}_\mathbf{n}) \\ \mathbf{x}_1)\, \alpha_1^2 \text{var}\, x_1 + \boldsymbol{\alpha}_1\, \mathbf{cov}\, (\mathbf{A}, \mathbf{x}_1) + \boldsymbol{\alpha}_1\boldsymbol{\alpha}_2\, \mathbf{cov}(\mathbf{x}_1, \mathbf{x}_2) + \cdots + \boldsymbol{\alpha}_1\boldsymbol{\alpha}_\mathbf{i}\, \mathbf{cov}(\mathbf{x}_1, \mathbf{x}_\mathbf{i}) + \cdots + \boldsymbol{\alpha}_1\boldsymbol{\alpha}_\mathbf{n}\, \mathbf{cov}(\mathbf{x}_1, \mathbf{x}_\mathbf{n}) \\ \mathbf{x}_2)\, \alpha_2^2 \text{var}\, x_2 + \boldsymbol{\alpha}_2\, \mathbf{cov}\, (\mathbf{A}, \mathbf{x}_2) + \boldsymbol{\alpha}_1\boldsymbol{\alpha}_2\, \mathbf{cov}(\mathbf{x}_1, \mathbf{x}_2) + \cdots + \boldsymbol{\alpha}_2\boldsymbol{\alpha}_\mathbf{i}\, \mathbf{cov}\, (\mathbf{x}_2, \mathbf{x}_\mathbf{i}) + \cdots + \boldsymbol{\alpha}_2\boldsymbol{\alpha}_\mathbf{n}\, \mathbf{cov}(\mathbf{x}_2, \mathbf{x}_\mathbf{n}) \\ \vdots \\ \mathbf{x}_\mathbf{i})\, \alpha_\text{i}^2 \text{var}\, x_\text{i} + \boldsymbol{\alpha}_\mathbf{i}\, \mathbf{cov}\, (\mathbf{A}, \mathbf{x}_\mathbf{i}) + \boldsymbol{\alpha}_1\boldsymbol{\alpha}_\mathbf{i}\, \mathbf{cov}(\mathbf{x}_1, \mathbf{x}_\mathbf{i}) + \boldsymbol{\alpha}_2\boldsymbol{\alpha}_\mathbf{i}\, \mathbf{cov}\, (\mathbf{x}_2, \mathbf{x}_\mathbf{i}) + \cdots + \boldsymbol{\alpha}_\mathbf{i}\boldsymbol{\alpha}_\mathbf{n}\, \mathbf{cov}(\mathbf{x}_\mathbf{i}, \mathbf{x}_\mathbf{n}) \\ \vdots \\ \mathbf{x}_\mathbf{n})\, \alpha_\text{n}^2 \text{var}\, x_\text{n} + \boldsymbol{\alpha}_\mathbf{n}\, \mathbf{cov}\, (\mathbf{A}, \mathbf{x}_\mathbf{n}) + \boldsymbol{\alpha}_1\boldsymbol{\alpha}_\mathbf{n}\, \mathbf{cov}(\mathbf{x}_1, \mathbf{x}_\mathbf{n}) + \boldsymbol{\alpha}_2\boldsymbol{\alpha}_\mathbf{n}\, \mathbf{cov}(\mathbf{x}_2, \mathbf{x}_\mathbf{n}) + \boldsymbol{\alpha}_\mathbf{i}\boldsymbol{\alpha}_\mathbf{n}\, \mathbf{cov}(\mathbf{x}_\mathbf{i}, \mathbf{x}_\mathbf{n}) \end{cases} \tag{13}$$

The purpose of the fourth step is precisely to correct possible aberrations of each absolute power found in step 3 and therefore to obtain the corrected explanatory powers of each factor. An aberration is detected if at the end of step 3, the variance in a factor was < 0 (Type 1) or if the relative variance (ratio of a factor's variance to dependent variable's variance) was > 1 (Type 2).

If there is no aberration, the principle of average is satisfactory[14] for lack of better, on the exact attributions—unknown—of each co-movement.

Otherwise, in case of aberrations, it is necessary to show here the proposed procedure. For that, let us therefore suppose that at the end of step 3, we detect an aberration for example on the factor x_1 and which is Type 1 (the most common type). This redistribution will concern the whole value of the negative variance (or only on the part that exceeds the variance in the dependent variable, if Type 2 aberration); it will be done in the direction of the non-aberrant variances in A, $x_2 \ldots x_i \ldots x_n$ in proportion to the correlation coefficients $\left(\rho_{x_{1,x_i}}\ i \neq 1\right)$ between x_1 and each of other factors.

The supposed negative value which is put in full redistribution is:

$$\alpha_1^2 \text{ var } x_1 + \boldsymbol{\alpha}_1 \mathbf{cov}(\mathbf{A}, \mathbf{x}_1) + \boldsymbol{\alpha}_1\boldsymbol{\alpha}_2 \mathbf{cov}(\mathbf{x}_1, \mathbf{x}_2) + \cdots + \boldsymbol{\alpha}_1\boldsymbol{\alpha}_\mathbf{i} \mathbf{cov}(\mathbf{x}_1, \mathbf{x}_\mathbf{i}) + \cdots + \boldsymbol{\alpha}_1\boldsymbol{\alpha}_\mathbf{n} \mathbf{cov}(\mathbf{x}_1, \mathbf{x}_\mathbf{n}) \tag{14}$$

Redistribution of the variance value in x_1 (which therefore stands out < 0) toward that of *A*:

$$\left[\frac{\rho_{x_{1,A}}}{\rho_{x_{1,A}} + \rho_{x_{1,x_2}} + \cdots + \rho_{x_{1,x_2}} + \cdots + \rho_{x_{1,x_n}}}\right] * \left[\alpha_1^2 \text{ var } x_1 + \boldsymbol{\alpha}_1 \mathbf{cov}(\mathbf{A}, \mathbf{x}_1) + \boldsymbol{\alpha}_1\boldsymbol{\alpha}_2 \mathbf{cov}(\mathbf{x}_1, \mathbf{x}_2) + \cdots + \boldsymbol{\alpha}_1\boldsymbol{\alpha}_\mathbf{i} \mathbf{cov}(\mathbf{x}_1, \mathbf{x}_\mathbf{i}) + \cdots + \boldsymbol{\alpha}_1\boldsymbol{\alpha}_\mathbf{n} \mathbf{cov}(\mathbf{x}_1, \mathbf{x}_\mathbf{n})\right] \tag{15}$$

Redistribution of the variance value in x_1 (which therefore stands out < 0) toward that of x_2:

[14] To be more precise, this solution would be satisfactory in the context of n > 2 factors. With 2 factors for example, the satisfactory solution is that of Baier et al. (2006), Turner et al. (2013) or Tamura *et al.* (2019).

$$\left[\frac{\rho_{x_{1,x_2}}}{\rho_{x_{1,A}}+\rho_{x_{1,x_2}}+\cdots+\rho_{x_{1,x_2}}+\cdots+\rho_{x_{1,x_n}}}\right]*\left[\alpha_1^2\ \text{var}\ x_1+\boldsymbol{\alpha}_1\mathbf{cov}(\mathbf{A},\mathbf{x}_1)+\boldsymbol{\alpha}_1\boldsymbol{\alpha}_2\mathbf{cov}(\mathbf{x}_1,\mathbf{x}_2)\right.$$
$$\left.+\cdots+\boldsymbol{\alpha}_1\boldsymbol{\alpha}_\mathbf{i}\mathbf{cov}(\mathbf{x}_1,\mathbf{x}_\mathbf{i})+\cdots+\boldsymbol{\alpha}_1\boldsymbol{\alpha}_\mathbf{n}\mathbf{cov}(\mathbf{x}_1,\mathbf{x}_\mathbf{n})\right] \tag{16}$$

Redistribution of the variance value in x_1 (which therefore stands out < 0) toward that of x_i:

$$\left[\frac{\rho_{x_{1,x_i}}}{\rho_{x_{1,A}}+\rho_{x_{1,x_2}}+\cdots+\rho_{x_{1,x_2}}+\cdots+\rho_{x_{1,x_n}}}\right]*\left[\alpha_1^2\ \text{var}\ x_1+\boldsymbol{\alpha}_1\mathbf{cov}(\mathbf{A},\mathbf{x}_1)+\boldsymbol{\alpha}_1\boldsymbol{\alpha}_2\mathbf{cov}(\mathbf{x}_1,\mathbf{x}_2)\right.$$
$$\left.+\cdots+\boldsymbol{\alpha}_1\boldsymbol{\alpha}_\mathbf{i}\mathbf{cov}(\mathbf{x}_1,\mathbf{x}_\mathbf{i})+\cdots+\boldsymbol{\alpha}_1\boldsymbol{\alpha}_\mathbf{n}\mathbf{cov}(\mathbf{x}_1,\mathbf{x}_\mathbf{n})\right] \tag{17}$$

Redistribution of the variance value in x_1 (which therefore stands out < 0) toward that of x_n:

$$\left[\frac{\rho_{x_{1,x_n}}}{\rho_{x_{1,A}}+\rho_{x_{1,x_2}}+\cdots+\rho_{x_{1,x_2}}+\cdots+\rho_{x_{1,x_n}}}\right]*\left[\alpha_1^2\ \text{var}\ x_1+\boldsymbol{\alpha}_1\mathbf{cov}(\mathbf{A},\mathbf{x}_1)+\boldsymbol{\alpha}_1\boldsymbol{\alpha}_2\mathbf{cov}(\mathbf{x}_1,\mathbf{x}_2)\right.$$
$$\left.+\cdots+\boldsymbol{\alpha}_1\boldsymbol{\alpha}_\mathbf{i}\mathbf{cov}(\mathbf{x}_1,\mathbf{x}_\mathbf{i})+\cdots+\boldsymbol{\alpha}_1\boldsymbol{\alpha}_\mathbf{n}\mathbf{cov}(\mathbf{x}_1,\mathbf{x}_\mathbf{n})\right] \tag{18}$$

Since the sum of the first brackets is 1, it follows that the negative value of the variance in x_1 is canceled when it is deducted from the sum of expressions (23)–(26).

A final step will simply consist in measuring the contribution of each variance from the previous step by relating it to the variance in the dependent variable (var y). In this example, the final contributions will be those of the variances in: A, $x_2 \ldots x_i \ldots x_n$ (var x_1 therefore becoming zero). It is necessary to add to the value of each variance (non-aberrant) obtained at the end of step 3, the corrective redistribution made in the previous step and to report the total to *vary* :

Final contribution of the variance in A:

$$\frac{\left(\begin{array}{c} varA+\boldsymbol{\alpha}_1 cov(\mathbf{A},\mathbf{x}_1)+\boldsymbol{\alpha}_2 cov(\mathbf{A},\mathbf{x}_2)+\cdots+\boldsymbol{\alpha}_\mathbf{i} cov(\mathbf{A},\mathbf{x}_\mathbf{i})+\cdots+\boldsymbol{\alpha}_\mathbf{n} cov(\mathbf{A},\mathbf{x}_\mathbf{n})+ \\ \left[\frac{\rho_{x_{1,A}}}{\rho_{x_{1,A}}+\rho_{x_{1,x_2}}+\cdots+\rho_{x_{1,x_2}}+\cdots+\rho_{x_{1,x_n}}}\right]*\left[\alpha_1^2\ \text{var}\ x_1+\boldsymbol{\alpha}_1\mathbf{cov}(\mathbf{A},\mathbf{x}_1)+\boldsymbol{\alpha}_1\boldsymbol{\alpha}_2\mathbf{cov}(\mathbf{x}_1,\mathbf{x}_2)\right. \\ \left.+\cdots+\boldsymbol{\alpha}_1\boldsymbol{\alpha}_\mathbf{i}\mathbf{cov}(\mathbf{x}_1,\mathbf{x}_\mathbf{i})+\cdots+\boldsymbol{\alpha}_1\boldsymbol{\alpha}_\mathbf{n}\mathbf{cov}(\mathbf{x}_1,\mathbf{x}_\mathbf{n})\right] \end{array}\right)}{vary} \tag{19}$$

Final contribution of the variance in x_2:

$$\frac{\left(\begin{array}{c} \alpha_2^2 varx_2+\boldsymbol{\alpha}_2 cov(\mathbf{A},\mathbf{x}_2)+\boldsymbol{\alpha}_1\boldsymbol{\alpha}_2 cov(\mathbf{x}_1,\mathbf{x}_2)+\cdots+\boldsymbol{\alpha}_2\boldsymbol{\alpha}_\mathbf{i} cov(\mathbf{x}_2,\mathbf{x}_\mathbf{i})+\cdots+\boldsymbol{\alpha}_2\boldsymbol{\alpha}_\mathbf{n} cov(\mathbf{x}_2,\mathbf{x}_\mathbf{n})+ \\ \left[\frac{\rho_{x_{1,A}}}{\rho_{x_{1,A}}+\rho_{x_{1,x_2}}+\cdots+\rho_{x_{1,x_2}}+\cdots+\rho_{x_{1,x_n}}}\right]*\left[\alpha_1^2\ \text{var}\ x_1+\boldsymbol{\alpha}_1\mathbf{cov}(\mathbf{A},\mathbf{x}_1)+\boldsymbol{\alpha}_1\boldsymbol{\alpha}_2\mathbf{cov}(\mathbf{x}_1,\mathbf{x}_2)\right. \\ \left.+\cdots+\boldsymbol{\alpha}_1\boldsymbol{\alpha}_\mathbf{i}\mathbf{cov}(\mathbf{x}_1,\mathbf{x}_\mathbf{i})+\cdots+\boldsymbol{\alpha}_1\boldsymbol{\alpha}_\mathbf{n}\mathbf{cov}(\mathbf{x}_1,\mathbf{x}_\mathbf{n})\right] \end{array}\right)}{vary} \tag{20}$$

Final contribution of the variance in x_i:

$$\frac{\left(\begin{array}{c}\alpha_i^2 var x_i + \boldsymbol{\alpha}_\mathbf{i} cov(\mathbf{A}, \mathbf{x_i}) + \boldsymbol{\alpha}_1 \boldsymbol{\alpha}_\mathbf{i} cov(\mathbf{x}_1, \mathbf{x_i}) + \boldsymbol{\alpha}_2 \boldsymbol{\alpha}_\mathbf{i} cov(\mathbf{x}_2, \mathbf{x_i}) + \cdots + \boldsymbol{\alpha}_\mathbf{i} \boldsymbol{\alpha}_\mathbf{n} cov(\mathbf{x_i}, \mathbf{x_n}) + \\ \left[\frac{\rho_{x_{1,x_i}}}{\rho_{x_{1,A}} + \rho_{x_{1,x_2}} + \cdots + \rho_{x_{1,x_2}} + \cdots + \rho_{x_{1,x_n}}}\right] * \left[\alpha_1^2 \text{ var } x_1 + \boldsymbol{\alpha}_1 \mathbf{cov}(\mathbf{A}, \mathbf{x}_1) + \boldsymbol{\alpha}_1 \boldsymbol{\alpha}_2 \mathbf{cov}(\mathbf{x}_1, \mathbf{x}_2) \right. \\ \left. + \cdots + \boldsymbol{\alpha}_1 \boldsymbol{\alpha}_\mathbf{i} \mathbf{cov}(\mathbf{x}_1, \mathbf{x_i}) + \cdots + \boldsymbol{\alpha}_1 \boldsymbol{\alpha}_\mathbf{n} \mathbf{cov}(\mathbf{x}_1, \mathbf{x_n})\right]\end{array}\right)}{vary} \tag{21}$$

Final contribution of the variance in x_n:

$$\frac{\left(\begin{array}{c}\alpha_n^2 var x_n + \boldsymbol{\alpha}_\mathbf{n} cov(\mathbf{A}, \mathbf{x_n}) + \boldsymbol{\alpha}_1 \boldsymbol{\alpha}_\mathbf{n} cov(\mathbf{x}_1, \mathbf{x_n}) + \boldsymbol{\alpha}_2 \boldsymbol{\alpha}_\mathbf{n} cov(\mathbf{x}_2, \mathbf{x_n}) + \boldsymbol{\alpha}_\mathbf{i} \boldsymbol{\alpha}_\mathbf{n} cov(\mathbf{x_i}, \mathbf{x_n}) + \\ \left[\frac{\rho_{x_{1,x_n}}}{\rho_{x_{1,A}} + \rho_{x_{1,x_2}} + \cdots + \rho_{x_{1,x_2}} + \cdots + \rho_{x_{1,x_n}}}\right] * \left[\alpha_1^2 \text{ var } x_1 + \boldsymbol{\alpha}_1 \mathbf{cov}(\mathbf{A}, \mathbf{x}_1) + \boldsymbol{\alpha}_1 \boldsymbol{\alpha}_2 \mathbf{cov}(\mathbf{x}_1, \mathbf{x}_2) \right. \\ \left. + \cdots + \boldsymbol{\alpha}_1 \boldsymbol{\alpha}_\mathbf{i} \mathbf{cov}(\mathbf{x}_1, \mathbf{x_i}) + \cdots + \boldsymbol{\alpha}_1 \boldsymbol{\alpha}_\mathbf{n} \mathbf{cov}(\mathbf{x}_1, \mathbf{x_n})\right]\end{array}\right)}{vary} \tag{22}$$

The sum of the numerators of the different contributions above is obviously equal to *vary* (since the sum of the products of the square brackets of the numerators of the above expressions is equal to the second of the two square brackets. However, the second square bracket is the missing equation from step 1 or the DA equation). The aberration therefore only concerns the distribution of explanatory powers and not their total, which is always equal to var y because the DA equation is always verified.

We will illustrate numerically through four examples (EG, SM, SNM and Sm) each of the stages leading to the relative contributions of the variance in each explanatory factor. Our production function has now three explanatory factors ($n = 3$); the number of co-movements in the presence of n factors is given by the number of combinations of 2 elements taken in a set of n :

$$C_n^2 = \frac{n!}{(n-2)!2!} = \frac{n^2 - n}{2} \tag{23}$$

Replacing, n by 3 in (23), implies that three co-movements must therefore be considered.

Explanatory Powers of Production Factors to the Disparities of GDP/h in EU-19

We apply now the strategy proposed in the previous section in order to assign explanatory powers to each factor of production in explaining disparities in GDP/h between our 19 countries. The attribution calculations also use the identification results from Sect. 4.1.

In the first column of Table 5, the figures of the inherent powers are not exactly equal to the respective variances in the three factors that we see in the variances–covariances matrix of Table 4. Indeed, as shown in Eq. 4, the inherent explanatory power of a factor must also take into account the elasticity of the dependent variable (the variance in the GDP/h here and which is found in Table 4) relative to this factor; these elasticities are always shown in Table 4. For example, the inherent explanatory power of capital per hour worked, 0.0215, is obtained by multiplying the figure of the variance in capital per hour worked (0.1705) which is found in Table 4 by the square of capital elasticity (0.126) according to Eq. 4 (Table 5).

In the second column, we apply to the terms in bold in Eq. 4—and which are presented by couple under the figure of proper power in column 1—an egalitarian division; for example, the number − 0.02505 is obtained either from the first bold term of Eq. 4: ((2*0.355*(− 0.0706))/2) or directly from column 1 (− 0.0501/2), same for the other two terms.

For information, the global redistributable co-movement here represents 304.35% of the dependent variable's variance, namely: ((|− 0.0501 + |− 0.1538| + 0.0761)/0.092).

The third is simply the sum of the inherent explanatory power of each factor (column 1) and the previous distribution of power in covariance or common explanatory power (column 2).

The purpose of the fourth column is to correct any aberration in the results, namely a negative explanatory power (i.e., a negative variance) or greater than unity (i.e., a variance which is greater than that of the dependent variable, i.e., 0.092).

Here TFP has a negative variance (− 0.052). We apply the strategy proposed to redistribute the latter; we first calculate, from correlations matrix (Table 4), the portion that should go to LAB_QPH variance:

$$\left[\frac{\rho_{\ln A,\ln a}}{\left(\rho_{\ln A,\ln a}+\rho_{\ln k,\ln a}\right)}\right]=\frac{-0.8932}{(-0.8932-0.7653)}=53.86\%\ \text{and}\ 0.5386*(-0.052)=-0.028 \tag{24}$$

We would then deduct that the remainder (− 0.0222) would go to the variance in capital per hour worked. The egalitarian attribution of negative co-movements between TFP and the other two factors will therefore have been excessive in this example; there is reason to think that a coefficient less than ½ during the passage from column 1–2 would have avoided the aberration of having a negative explanatory power (− 0.052). In other words, applying a coefficient of ½ to the supposedly negative co-movements of the first column when switching to column 2 overestimates the TFP role in these co-movements (and underestimates the role of the other two factors).

The last column divides the corrected final explanatory powers by the variance in the product per hour worked. Thus, the labor quality variance with 88.57% (0.0814/0.092) and the variance in capital per hour worked (11.43%) explain GDP/h variance.

Numerical Applications for Identifying and Attributing Explanatory Powers of GDP/h Disparities in EU-19 (Sectoral Cases: SM, SNM and Sm)

Breakdown of Global Economies: Data Source and Statistical Results

Economies' Structure from the EU-KLEMS Database

Before proceeding according to the same strategy to the calculations of identification and attribution of explanatory powers to disparities in standard of living between the spheres of the countries, it was necessary to decompose the global economies according to the sphere considered (SM, SNM and Sm).

The decomposition of each of the 19 economies in the sample is practically that carried out by EU-KLEMS (Stehrer et al. 2019) on the basis of the Statistical Classification of Economic Activities in the European Community—NACE Rev. 2 (Eurostat (2008)). Below, summary this decomposition:

The decomposition is therefore practically or even exactly the same as the latest version of EU-KLEMS; here, we simply describe some nuances aimed at delimiting the 3 spheres considered, which is not done in EU-KLEMS who, as Table 6 shows, only codes the global economy (TOT) and the market sphere (MARKT). This delimitation is necessary for consistency, for each variable examined (GDP, Capital, hours worked) between its EG recorded amount and the sum of its sectoral amounts (SM, SNM and Sm).

For the global economy coded therefore "TOT" in EU-KLEMS, the same scope is used.

For the market sphere, coded "MARKT," we also use the same perimeter with a difference which is statistically extremely negligible and relates precisely to section T of NACE Rev.2 which is entitled: "Activities of households as employers; undifferentiated activities of households as producers of goods and services for own use." We do not exclude this section from the market sphere; the private nature of households also argues in this direction. Likewise, this avoids creating a special sphere, not elsewhere classified (nec), for a statistically particularly negligible section. EU-KLEMS excludes it from SM, but since it does not offer SNM, Sm or any other section to fit it in, it did not cover our breakdown.

For the non-market sphere, there is no assigned coding in EU-KLEMS but by deduction (Table 6 and NACE Rev. 2), its perimeter is made up of sections[15] O, P and Q. We add to O, P and Q the section U "Extra-territorial activities." This is statistically even more negligible than section T with amounts reported by EU-KLEMS which are almost always zero. The public nature of its actors (activities of the United Nations, the IMF, the World Bank, the OECD, diplomatic and consular missions, etc.) militates in favor of this type of addition. Moreover, there still more than for the

[15] The sections of NACE Rév. 2: O "Public administration," P "Education" and Q "Human health and social action," group together activities usually carried out on a non-profit basis.

Table 6 Decomposition of global economies into market, non-market, and mixed spheres

EU-KLEMS/NACE Rév.2	Author
Global economy	Global economy
TOT (A-U)	Idem
Market sphere	Market sphere
MARKT Sections [A to U]—Sections [L,O,P,Q, T, U]	Sections [A to U]—Sections [L,O,P,Q]
Non market sphere	Non market sphere
Sections O, P and Q	Sections O, P, Q and T
Mixed sphere	Mixed sphere
Not delimited	Section L

The detailed content of each section is described in EUROSTAT (2008), cf. Bibliographic references

Source: Author

section T, it is not statistically useful to create a special sphere, nec, to put section U there.

For the mixed sphere[16], there is again no coding in EU-KLEMS, but by deduction, it can be considered that this is section L "Real estate activities." In fact, EU-KLEMS excludes it from the market sphere while it is not conceivable either to register it in the non-market sphere whose perimeter is O, P and Q (and T), which implies that section L is assimilated to the mixed sphere.

Decomposition Results of the Global EU Economies: Sphere Structure of GDP and Factors.

The results of this decomposition are visible on the previous figures (Figs. 1, 2, 3) which successively show, for each EU country, the statistics for the distribution of its GDP, its capital stock and its hours worked between the three spheres.

The statistics in these three charts, by prior application of their percentages to GDP and the other production factor of an economy (capital per hour worked), allow knowing their distribution in absolute terms between the spheres and which we use now—with statistics on the quality of hours worked—for identification calculations. Data relating to the quality of hours worked are shown in Tables 3, 7, 10, and 13 (column 6).

16 The mixed sphere—Section L "Real estate activities" includes the purchases, sales, rentals of private or public real estate, related activities such as the valuation and management-for own account or for others- of these goods.

Fig. 1 Distribution of **GDP** according to the market, non-market, and mixed spheres (%). *Source*: Author's calculations based on EU KLEMS database, 2019 release

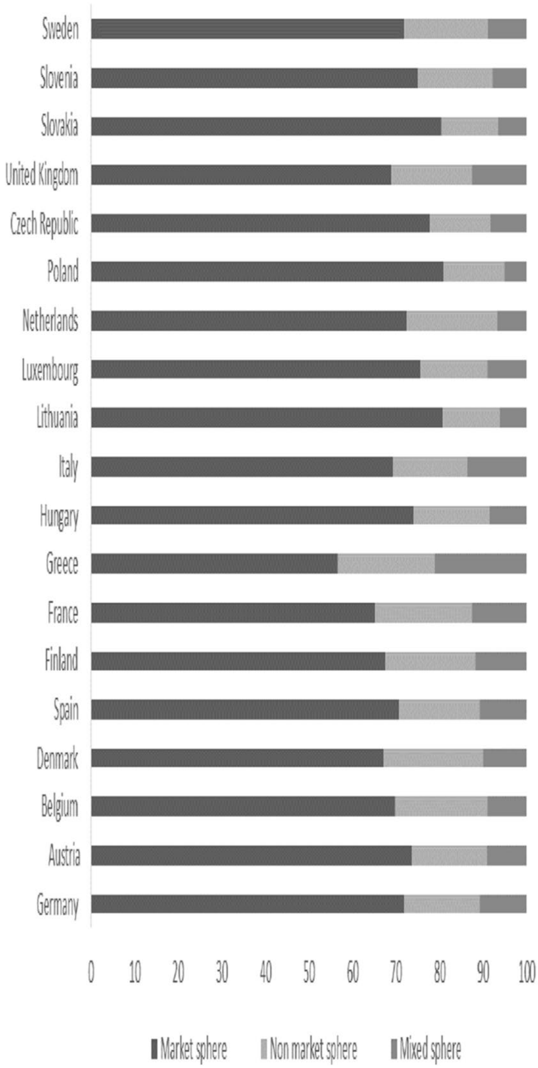


Fig. 2 Distribution of **capital** according to the market, non-market, and mixed spheres (%). *Source*: Author's calculations based on EU KLEMS database, 2019 release

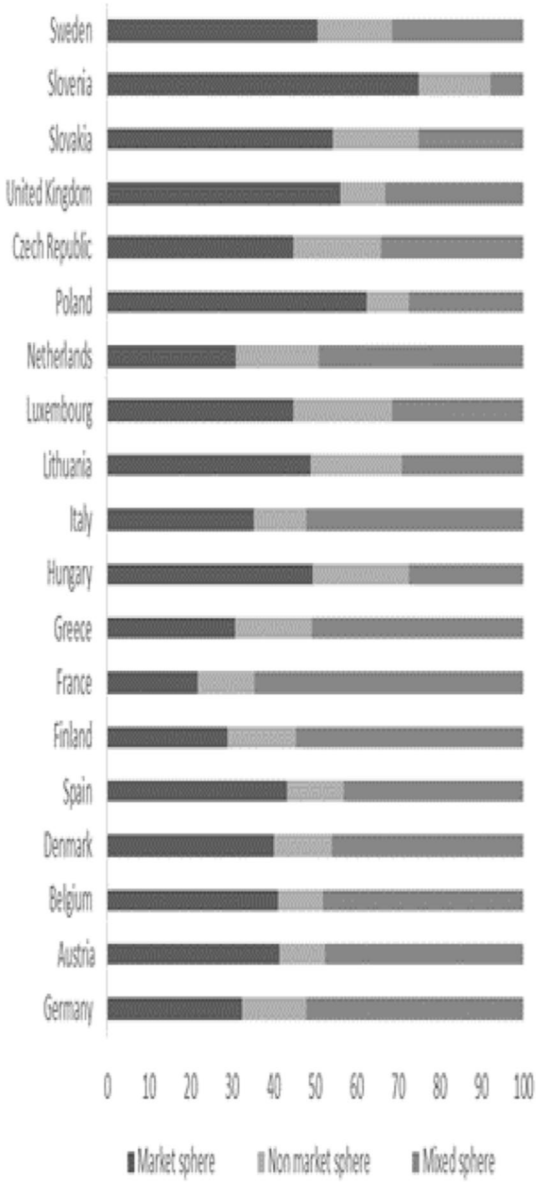
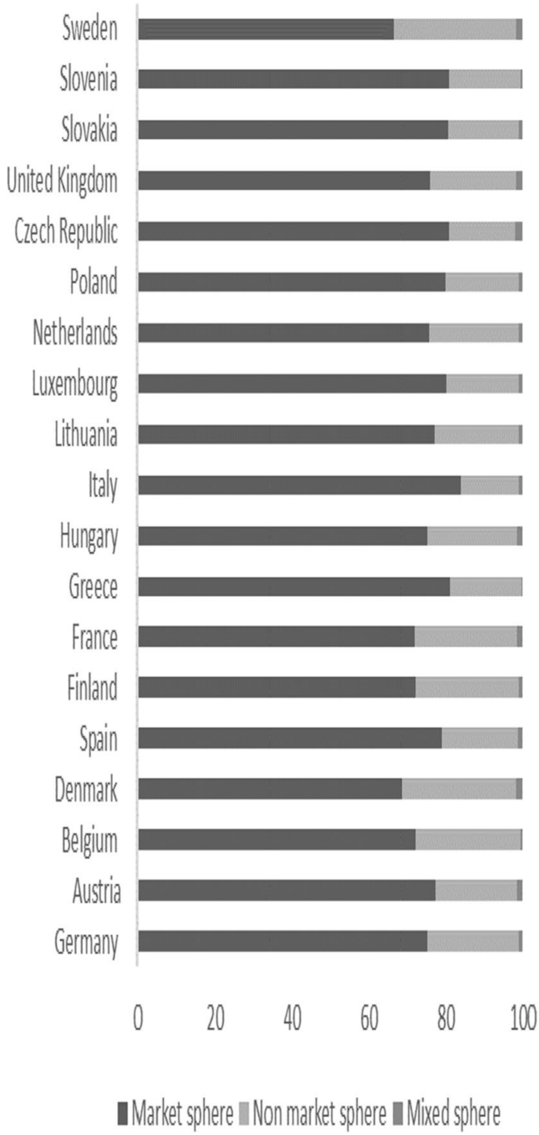


Fig. 3 Distribution of **hours worked** according to the market, non-market, and mixed spheres (%). *Source*: Author's calculations based on EU KLEMS database, 2019 release



Identification and Attribution of the Explanatory Powers of Each Factor of Production to the Disparities of GDP/h in the market sphere: SM

Identification

Numerical verification of the validity of DA Eq. 6---and therefore calculations in Tables 7 and 8:

Table 7 Data needed to calibrate the development accounting equation (SM) and ln A calculation

Countries	(1) cgdpo/H_EMP	(2) ln (cgdpo/ H_EMP)	(3) cn/H_EMP	(4) ln (cn/H_EMP)	(5) 0,3111*4)	(6) LAB_QPH	(7) ln LAB_QPH	(8) 0,6889*7)	(9) ln A
Germany (Accession: EU: 1958; € 1999)	62.45	4.134	134.83	4.904	1.526	5.31	1.670	1.150	1.459
Austria (1995;1999)	54.60	4.000	187.18	5.232	1.628	4.51	1.506	1.038	1.335
Belgium (1958;1999)	61.85	4.125	236.33	5.465	1.700	5.63	1.728	1.190	1.234
Denmark (1973)	62.16	4.130	208.57	5.340	1.661	5.64	1.730	1.192	1.277
Spain (1986;1999)	44.06	3.785	171.57	5.145	1.601	3.46	1.241	0.855	1.330
Finland (1995;1999)	49.35	3.899	108.44	4.686	1.458	4.35	1.470	1.013	1.428
France (1958;1999)	40.11	3.692	74.17	4.306	1.340	5.64	1.730	1.192	1.160
Greece (1981;2001)	22.08	3.095	107.92	4.681	1.456	1.76	0.565	0.389	1.249
Hungary (2004)	27.98	3.332	119.16	4.780	1.487	0.99	-0.010	-0.007	1.851
Italy (1958;1999)	42.64	3.753	165.60	5.110	1.590	3.89	1.358	0.936	1.227
Lithuania (2004;2015)	30.74	3.425	80.03	4.382	1.363	1.87	0.626	0.431	1.631
Luxembourg (1958;1999)	64.75	4.171	192.34	5.259	1.636	4.66	1.539	1.060	1.474
Netherlands (1958;1999)	60.58	4.104	132.75	4.888	1.521	5.05	1.619	1.116	1.468
Poland (2004)	30.28	3.410	59.68	4.089	1.272	1.68	0.519	0.357	1.781
Czech Republic (2004)	33.66	3.516	130.21	4.869	1.515	1.4	0.336	0.232	1.770
UK (1973)	43.52	3.773	180.25	5.194	1.616	3.83	1.343	0.925	1.232
Slovakia (2004;2009)	35.09	3.558	110.80	4.708	1.465	1.12	0.113	0.078	2.015
Slovenia (2004;2007)	33.88	3.523	259.46	5.559	1.729	1.39	0.329	0.227	1.567
Sweden (1995)	57.59	4.053	213.04	5.361	1.668	4.21	1.437	0.990	1.395

Source: Author's calculations based on EU KLEMS database, 2019 release and PWT 9.1 database, July 2018 release

Table 8 Market sphere: summary of results obtained: variance of the GDP/h; variance–covariance and correlations matrices; weighted average elasticities

Var ln (cgdpo/H_EMP)	0.0991		
Var-cov matrix	*ln(cn/H_EMP)*	*ln(LAB_QPH)*	*ln A*
ln (cn/H_EMP)	0.1566		
ln (LAB_QPH)	0.0768	0.3562	
ln A	− 0.0337	− 0.1111	0.0560
Correlations matrix	*ln(cn/H_EMP)*	*ln(LAB_QPH)*	*ln A*
ln (cn/H_EMP)	1		
ln (LAB_QPH)	0.3250	1	
ln A	− 0.3599	− 0.7869	1
Average factor shares	Capital factor: 0.3111		Labor factor: 0.6889

Source: Author's calculations

$$0.0991 = 0.056 + 0.0152 + 0.169 - 0.021 - 0.153 + 0.0329 \tag{25}$$

Attribution

Calculations in Table 9 are carried out exactly as in Table 5. The observation in column 3 shows here a variance in the TFP, after redistribution of common explanatory powers according to average principle, which is slightly negative (− 0.031). The proposed correction should therefore be applied. This is done in column 4. With regard to the other factors, TFP is in negative co-movement. We will redistribute the amount of − 0.0027 with the correlations' matrix (Table 8); portion that will go to LAB_QPH variance is:

$$\left[\frac{\rho_{\ln A,\ln a}}{\left(\rho_{\ln A,\ln a} + \rho_{\ln k,\ln a}\right)}\right] = \frac{-0.7869}{(-0.7869 - 0.3599)} = 68.62\% \text{ and } 0.6862 * (-0.031) = -0.021 \tag{26}$$

We deduce that the remainder (-0.01) will go to the variance in the capital/h. The egalitarian attribution of negative co-movements between TFP and the other two factors was therefore very slightly excessive; there is reason to think that a coefficient lower than ½ would have avoided the aberration of having negative explanatory power for the variance in the TFP (− 0.031).

In the SM, the redistributable co-movement is at 208.78% of the variance in GDP/h:

((|− 0.021| + |− 0.153|+ 0.0329)/0.0991)). The variance of capital/h explains 11.25% (0.01115/0.0991) of the GDP/h variance while the LAB_QPH variance

Table 9 Explanatory powers (EP) of factors to disparities of the product per hour worked in the EU-19

Variables *(in variance)*	Initial inherent and common EP	Redistributions of common powers	EP before correction (absolute)	Corrected final EP	Final EP (relative)
ln(*cn*/*H_EMP*)	0.0152 − 0.021; 0.0329	− 0.0105 + 0.01645	0.02115	0.01115 (0.02115–0.01)	11.25%
ln(*LAB_QPH*)	0.169 − 0.153; 0.0329	− 0.0765 + 0.01645	0.10895	0.08795 (0.10895–0. 021)	88.75%
ln*A*	0.056 − 0.021; − 0.153	− 0.0105–0.0765	− 0.031	0 (− 0 .031 + 0.01+ 0.021)	0%

Source: Author

Table 10 Data needed to calibrate the development accounting equation (SNM) and ln A calculation

Countries	(1) cgdpo/H_EMP	(2) ln (cgdpo/H_EMP)	(3) cn/H_EMP	(4) ln (cn/H_EMP)	(5) 0,1675*4)	(6) LAB_QPH	(7) ln LAB_QPH	(8) 0,8325*7)	(9) ln A
Germany (Accession: EU: 1958; € 1999)	47.48	3.860	201.84	5.307	0.889	5.31	1.670	1.390	1.581
Austria (1995;1999)	46.32	3.836	180.66	5.197	0.870	4.51	1.506	1.254	1.711
Belgium (1958;1999)	49.73	3.907	165.03	5.106	0.855	5.63	1.728	1.439	1.613
Denmark (1973)	49.13	3.895	166.96	5.118	0.857	5.64	1.730	1.440	1.597
Spain (1986;1999)	45.44	3.816	213.92	5.366	0.899	3.46	1.241	1.033	1.884
Finland (1995;1999)	40.60	3.704	164.37	5.102	0.855	4.35	1.470	1.224	1.625
France (1958;1999)	51.29	3.938	171.83	5.147	0.862	5.64	1.730	1.440	1.635
Greece (1981;2001)	37.47	3.624	278.44	5.629	0.943	1.76	0.565	0.471	2.210
Hungary (2004)	21.44	3.065	179.50	5.190	0.869	0.99	-0.010	− 0.008	2.204
Italy (1958;1999)	58.32	4.066	321.75	5.774	0.967	3.89	1.358	1.131	1.968
Lithuania (2004;2015)	17.77	2.878	126.34	4.839	0.811	1.87	0.626	0.521	1.546
Luxembourg (1958;1999)	55.99	4.025	431.80	6.068	1.016	4.66	1.539	1.281	1.727
Netherlands (1958;1999)	55.62	4.019	275.86	5.620	0.941	5.05	1.619	1.348	1.729
Poland (2004)	21.87	3.085	40.64	3.705	0.621	1.68	0.519	0.432	2.033
Czech Republic (2004)	28.36	3.345	289.49	5.668	0.949	1.40	0.336	0.280	2.115
UK (1973)	39.18	3.668	117.61	4.767	0.799	3.83	1.343	1.118	1.752
Slovakia (2004;2009)	24.87	3.214	183.84	5.214	0.873	1.12	0.113	0.094	2.246
Slovenia (2004;2007)	34.02	3.527	260.56	5.563	0.932	1.39	0.329	0.274	2.321
Sweden (1995)	32.62	3.485	160.55	5.079	0.851	4.21	1.437	1.197	1.438

Source: Author's calculations based on EU KLEMS database, 2019 release and PWT 9.1 database, July 2018 release

Table 11 Non-market sphere: summary of results obtained: Variance of the GDP/h; variance-covariance and correlations matrices; weighted average elasticities

Var ln (cgdpo/H_EMP)	0.1233		
Var-cov matrix	*ln(cn/H_EMP)*	*ln(LAB_QPH)*	*ln A*
ln (cn/H_EMP)	0.2319		
ln (LAB_QPH)	0.0356	0.3562	
ln A	0.0186	− 0.1308	0.0716
Correlations matrix	*ln(cn/H_EMP)*	*ln(LAB_QPH)*	*ln A*
ln (cn/H_EMP)	1		
ln (LAB_QPH)	0.1238	1	
ln A	0.1441	− 0.8190	1
Average factor shares	Capital factor: 0.1675		Labor factor: 0.8325

explains 88.75% (0.08795/0.0991. The TFP variance did not have any explanatory power for disparities in GDP/h in 2016. These results show that the disparities in GDP/h in SM are completely explained.

Identification and Attribution of the Explanatory Powers of Each Factor of Production to the Disparities of GDP/h in the Non-market Sphere: SNM

Identification

Numerical verification of the validity of DA Eq. 8—and therefore calculations in Tables 10 and 11:

$$0.1233 = 0.0716 + 0.0065 + 0.2469 + 0.0062 - 0.2178 + 0.0099 \tag{27}$$

Attribution

In SNM, according to Table 12 (column 1), the redistributable co-movement is 189.7% of the variance in the dependent variable: ((0.0062 + |− 0.2178| + 0.0099)/0.1233)). As in the two previous situations, it is the co-movement between TFP and labor quality that is predominant (93% of global co-movement: (|− 0.2178/0.2339)). The variance in capital/h explains around 8% (0.00945/0.1233) of the variance in GDP/h, while the TFP variance played no role. These results show that the disparities in GDP/h in the SNM are completely explained.

Table 12 Explanatory powers (EP) of factors to disparities of the product per hour worked in the EU-19

Variables *(in variance)*	Initial inherent and common EP	Redistributions of common powers	EP before correction (absolute)		Corrected final EP	Final EP (relative)
$\ln(cn/H\_EMP)$	0.0065 0.0062; 0.0099	+ 0.0031 + 0.00495	0.01455	0.00945 (0.01455–0.0051)		7.66%
$\ln(LAB\_QPH)$	0.2469 − 0.2178; 0.0099	− 0.1089 + 0.00495	0.14295	0.11385 (0.14295–0.0291)		92.34%
$\ln A$	0.0716 0.0062; − 0.2178	+ 0.0031 − 0.1089	− 0.0342	0 (− 0.032 + 0.0051 + 0.0291)		0%

Source: Author

Identification and Attribution of the Explanatory Powers of Each Factor of Production to the Disparities of GDP/h in the Mixed Sphere: Sm

Identification

Numerical verification of the validity of DA Eq. 10—and therefore calculations in Tables 13 and 14:

$$0.5208 = 0.1285 + 0.6709 + 0.0014 - 0.3018 - 0.0144 + 0.0362 \quad (28)$$

Attribution

The observation in column 3 of Table 15 shows a variance in the labor quality and in TFP, after equally redistribution of common powers, which are, respectively, positive and negative (0.0123 and − 0.0296). The correction should therefore be applied to the negative variance. This is the subject of column 4. With respect to the other factors, TFP is in negative co-movement. The redistribution of explanatory powers can be done only in the direction of factors presenting an explanatory power, before correction, which is positive[17] (because the transfers of covariance are here negative), namely capital/h and LAB_QPH. We proceed as usual:

$$\left[\frac{\rho_{\ln A,\ln a}}{\left(\rho_{\ln A,\ln a} + \rho_{\ln k,\ln a}\right)}\right] = \frac{-0.5273}{(-0.5273 - 0.514)} = 50.64\%\,\text{et}\,0.5064 * (-0.0296) = -0.015 \quad (29)$$

This amount will go to LAB_QPH and the remainder, i.e., (− 0.0146) will go to the variance in capital/h. However, the variance in LAB_QPH (0.0123) cannot absorb the amount of − 0.015. Assigning this amount makes it negative by an amount of − 0.0027 (0.0123–0.015). Since the variance in the TFP (0) cannot absorb this amount without it also becoming negative, only the variance in capital/h is able to absorb it in addition to the negative co-movement with the TFP (− 0.0146) as shown in column 4.

The egalitarian attribution of negative co-movements between TFP and the other 2 factors was therefore slightly excessive (a coefficient less than ½ would have avoided aberration in column 3); the egalitarian attribution of the positive co-movement between the labor quality and the capital/h, on the other hand, was unfavorable to labor quality (a coefficient greater than ½ would have increased variance in LAB_QPH in column 3 and avoided the aberration of − 0.027 in column 4).

In Sm, the redistributable co-movement is the lowest of all the configurations but remains important with 66.7% of dependent variable's variance: ((|− 0.3018 | + |− 0.0144 | + 0.0362)/0.5208). Covariance between capital/h and TFP is responsible for almost 86% (0.3018/0.3524) of this overall co-movement.

The variances in labor quality and in TFP therefore ultimately have no explanatory power for the disparities in GDP/h. Consequently, for Sm, the disparities in GDP/h are, unlike the other three configurations, fully explained by the variance in capital/h.

[17] There is necessarily at least one, by definition of the developed formula of the variance.

Table 13 Data needed to calibrate the development accounting equation (Sm) and ln A calculation

Countries	(1) cgdpo/H_EMP	(2) ln (cgdpo/ H_EMP)	(3) cn/H_EMP	(4) ln (cn/H_EMP)	(5) 0,9362*4)	(6) LAB_QPH	(7) ln LAB_QPH	(8) 0,0638*7)	(9) ln A
Germany (Accession: EU: 1958; € 1999)	763.37	6.638	17,517.39	9.771	9.148	5.31	1.670	0.107	– 2.616
Austria (1995;1999)	355.32	5.873	11,183.17	9.322	8.727	4.51	1.506	0.096	– 2.950
Belgium (1958;1999)	970.65	6.878	33,911.27	10.432	9.766	5.63	1.728	0.110	– 2.998
Denmark (1973)	376.09	5.930	9.761.23	9.186	8.600	5.64	1.730	0.110	– 2.781
Spain (1986;1999)	478.98	6.172	12.202.97	9.409	8.809	3.46	1.241	0.079	– 2.717
Finland (1995;1999)	620.98	6.431	14.768.87	9.600	8.988	4.35	1.470	0.094	– 2.650
France (1958;1999)	575.93	6.356	16.400.62	9.705	9.086	5.64	1.730	0.110	– 2.840
Greece (1981;2001)	2783.60	7.931	60.014.12	11.002	10.300	1.76	0.565	0.036	– 2.405
Hungary (2004)	167.06	5.118	3,452.22	8.147	7.627	0.99	– 0.010	– 0.001	– 2.508
Italy (1958;1999)	797.50	6.681	23,271.26	10.055	9.413	3.89	1.358	0.087	– 2.819
Lithuania (2004;2015)	170.12	5.137	3,489.54	8.158	7.637	1.87	0.626	0.040	– 2.540
Luxembourg (1958;1999)	688.23	6.534	12,057.02	9.397	8.798	4.66	1.539	0.098	– 2.362
Netherlands (1958;1999)	499.24	6.213	18,743.17	9.839	9.211	5.05	1.619	0.103	– 3.101
Poland (2004)	158.78	5.068	2.178.10	7.686	7.196	1.68	0.519	0.033	-2.161
Czech Republic (2004)	151.24	5.019	4,195.59	8.342	7.810	1.40	0.336	0.021	– 2.812
UK (1973)	380.41	5.941	5,103.42	8.538	7.993	3.83	1.343	0.086	– 2.137
Slovakia (2004;2009)	222.96	5.407	3.973.26	8.287	7.759	1.12	0.113	0.007	– 2.359
Slovenia (2004;2007)	492.91	6.200	3.775.14	8.236	7.711	1.39	0.329	0.021	– 1.531
Sweden (1995)	274.62	5.615	5.075.67	8.532	7.988	4.21	1.437	0.092	– 2.464

Source: Author's calculations based on EU KLEMS database, 2019 release and PWT 9.1 database, July 2018 release

Table 14 Mixed Sphere: Summary of results obtained: Variance of the GDP/h; variance–covariance and correlation matrices; weighted average elasticities

Var ln (cgdpo/H_EMP)	0.5208		
Var-cov matrix	*ln(cn/H_EMP)*	*ln(LAB_QPH)*	*ln A*
ln (cn/H_EMP)	0.7654		
ln (LAB_QPH)	0.3034	0.3562	
ln A	− 0.1612	− 0.1128	0.1285
Correlations matrix	*ln(cn/H_EMP)*	*ln(LAB_QPH)*	*ln A*
ln (cn/H_EMP)	1		
ln (LAB_QPH)	0.5810	1	
ln A	− 0.5140	− 0.5273	1
Average factor shares	Capital factor: 0.9362		Labor factor: 0.0638

Source: Author's calculations

Origins of Disparities in Global and Sectoral Living Standards: Summary and Discussion

Summary of the Results

Discussion

Concerning the variable to be explained, namely disparities in GDP/h, the standard deviation in the Sm is, respectively, around 42, 43 and 46 times higher than that observed in EG, SM and SNM. Before even evoking factorial determinants of the GDP/h variances in each sphere, it should therefore be mentioned that the Sm, which is equivalent to section L of NACE Rev.2 "Real estate activities" (purchases, sales, rentals of private property or public, related activities such as the evaluation and management—for own account or for others—of these goods…) participates relatively much more than other spheres in GDP/h disparities noted between the EG of the 19 EU countries. This is one of the lessons in Table 16. Another is that the SNM is the one that contributes relatively the least (although incommensurately with the contribution to the widening of these disparities of Sm) to these same disparities.

The factorial contributions to these disparities are examined in EG and then in each of the spheres.

Table 15 Explanatory powers (EP) of factors to disparities of the product per hour worked in the EU-19

Variables *(in variance)*	Initial inherent and common EP	Redistributions of common powers	EP before correction (absolute)	Corrected final EP	Final EP (relative)
ln(cn/H_EMP)	0.6709 − 0.3018 ; 0.0362	− 0.1509 + 0.0181	0.5381	0.5208 (0.5381–0.0146–0.0027)	100%
ln(LAB_QPH)	0.0014 − 0.0144; 0.0362	− 0.0072 + 0.0181	0.0123	0 (0.0123–0.015 + 0.0027)	0%
ln$A$	0.1285 − 0.3018; − 0.0144	− 0.1509–0.0072	− 0.0296	0 (− 0.0296 + 0.0146 + 0.015)	0%

Source: Author

Table 16 Production factors' contributions to disparities in GDP/h in EU-19 and at their SM, SNM and Sm in 2016

In PPP (US $ 2011)	Global economy—EG	Market sphere	Non market sphere	Mixed sphere
		SM	SNM	Sm
	(Taille T: 100%)	(T: 70.43%)	(T: 18.54%)	(T: 11.03)
Dependant variable				
Var (cgdpo/H_EMP)	188.06	179.1	152.39	325,213.98
S.Dev.(cgdpo/H_EMP)	13.71	13.38	12.34	570.28
Redistributable co-movement				
(In % of var. GDP/h or var.cgdpo/H_EMP))	304.35%	208.78%	189.7%	66.7%
Explanatory contributions				
Var (cn/H_EMP)	11.43%/21.5	11.25% /20.15	7.66%/11.67	100% /325,213.98
Var (LAB_QPH)	88.57% /166.56	88.75%/158.95	92.34%/140.72	0%/0
Var TFP	0% /0	0%/0	0% /0	0% /0

Example: Column 1: In 2016, the disparities in GDP per hour worked between the global economies of the 19 EU countries (i.e., $ 188.06—standard deviation: $ 13.71) are explained at 11.43 % (or $ 21.5) by those of capital per hour worked in these countries, at 88.5% (or $ 166.56) by those of labor quality and for 0% by those of the TFP. The total co-movement that was redistributed between the variances in the 3 explanatory factors weighed 304.35% of the variance in GDP/h between countries. Source: Author

In EG, the contribution of LAB_QPH variance is 88.57%, while the remainder relates to variance in the capital/h. TFP variance does not contribute to explanation of disparities in GDP/h this year.

In SM (whose size, the sum of merchant GDP of the 19 countries compared to the sum of their GDP, exceeds 70%), the role of LAB_QPH variance is very similar to that of EG and therefore the role of capital deepening as well, since TFP plays no role here either.

In SNM, the standard deviation of GDP/h is slightly lower than for the EG (as well as the SM). The profile of the contributions shows that the role of the labor quality variance is very high (92.34%) and much higher compared to that of the EG while the variance in capital/h is quite small (7.66%) and lower relative to EG; as here, the variance in the GDP/h differs a little from that of EG, a comparison in relative terms is instructive. The ratio of the two variances is near 81% (152.39/188.06). The ratio of the variances in capital/h is 54.28% (11.67/21.5). The latter does not reach the first ratio; as TFP variance is identical between these two spheres, it follows from the DA equation that the contribution of LAB_QPH variance in SNM is relatively higher than in EG compensating for the relative insufficiency of the variance in capital/h—and therefore able to bring the ratio of total contributions to 81%. Thus, the ratio of LAB_QPH variance between SNM and EG is 84.5% (140.72/166.56) exceeding 81%. The ratio of total contributions ((11.67 + 140.72)/(21.5 + 166.56)) then reaches 81%.

In Sm, on the other hand, it is the variance in capital/h that completely explains disparities in GDP/h; the same previous reasoning *vis-à-vis* the EG applies. The ratio of variances in GDP/h between the two spheres is around 1730 (325,213.98/188.06). However, unlike the SNM case, the ratio of the variances in capital/h between the two spheres is higher and even maximum, reaching approximately 15,126 (325,213.98/21.5). As the contribution of TFP variance is 0 in EG, variance in labor quality compensates for the relative excess of variance in capital/h of Sm compared to EG. The ratio of total contributions (325,213.98/(21.5 + 166.56)) reaches well that of variances in GDP/h between these spheres (1730).

This same reasoning in relative terms could have been conducted between EG and SM but the variances in the dependent variable being practically the same there, reasoning in absolute terms is sufficient. It can be run between SM and SNM, SM and Sm or even between SNM and Sm but the EG is the natural reference for our comparisons; furthermore, reasoning in relative terms can now be more easily carried out by proceeding analogously to the last two cases above (Table 13).

Finally, it is important to point out that the redistributable co-movements related to the variances in GDP/h—and which have therefore been redistributed to the variances in the explanatory variables in this table—are everywhere considerable. In EG, the ratio exceeds 300% and remains remarkably high in the other three situations. Under these conditions, when the variances in the explanatory variables before redistribution are not large enough, a negative co-movement is likely to relieve them—not only partially, but completely—of any claim to explain the variance in GDP/h. This scenario typically occurred with regard to the TFP variance whose value before redistribution did not manage to control the importance of its co-movements with the other two variables.

Conclusion

The subject studied consisted of quantifying the contributions of production factors to disparities in living standards for 19 countries representing 9/10 of the EU wealth in 2016. The processing framework is that of development accounting, precisely by decomposition of the GDP/hour's variance. The results are methodological and empirical. Methodologically, we note three advances:

We have generalized the development accounting framework which is currently 2-factors in the literature toward a *n*-factors one. It also becomes possible to count for all the interactions between factors explaining the disparities in GDP/h. Four numerical illustrations were displayed;

A second contribution was to show how to carry out the statistical breakdown of these economies into three sectoral levels (SM, SNM and Sm) from the most recent statistical sources;

A third relates to the accounting decomposition of these economies—that is to say, the application of the development accounting framework and our strategy of redistribution of covariances—both macro-economically (between the 19 EG) and between their sectoral levels (SM, SNM and Sm).

Finally, it is worth mentioning that this approach would obviously extend to the variance decomposition of growth as by replacing variables in level with the same variables in rate of change.

Empirically, the highlights are:

The GDP/h disparities between the EU countries are practically on the same order of magnitude, except for the Sm: $ 13.71 between their EG, $ 13.38 between their SM, $ 12.34 between their SNM and $ 570.28 between their Sm;

The calibration of the development accounting equation with three production factors (TFP, capital/h and labor quality) shows factor co-movements of very considerable magnitude. This justifies taking them into account in the accounting exercise and therefore the construction of a method capable of realizing their redistribution toward the variance in each of the factors considered under the constraint of not generating aberrations (negative explanatory variance or greater than that of the dependent variable). Between the EU global economies, the redistributable co-movement represents more than 300% of the variance in GDP/h. For their SM, SNM and Sm, it represented, respectively, 209, 190 and 67%. Within each total co-movement by sphere, it is that between labor quality and TFP which is regularly preponderant reaching about 54, 74 and 93% in EG, SM and SNM; on the other hand, in Sm, the preponderant co-movement is that between capital deepening and TFP (around 85%);

The disparities in GDP/h between countries are very largely explained at 89% by those of labor quality and capital/h (11%). The disparities in "our ignorance" (TFP) seem to be absorbed by that of labor quality and capital deepening due both to the negative co-movement that TFP maintains with these two factors and to a greater variability of these latter before redistribution of these negative co-movements;

The disparities in GDP/h in the SM and SNM of the EU countries are again explained, but much more markedly by the disparities in labor quality (around 89 and 92%, respectively), giving the disparities in capital/h a positive but secondary role (11 and 8%, respectively);

The disparities in GDP/h in Sm can be explained entirely by that of capital/h. The disparities in TFP and in the quality of hours worked have no explanatory power here. The absence of a role for TFP contrasts here with the other three results, due to remarkably high capital deepening characterizing this sphere.

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