JOURNAL OF STRUCTURAL INTEGRITY AND MAINTENANCE
2022, VOL. 7, NO. 2, 120–127
https://doi.org/10.1080/24705314.2021.2018890

Nonlinear healing approach for reinforced concrete beams

Ahmad Salah Edeen Nassef[a,b]

[a]Faculty of Engineering-Matria, Helwan University, Cairo, Egypt; [b]College of Engineering, University of Buraimi, Buraimi, Oman

ABSTRACT
This paper has proposed a new formula relating the damage state and effective state of the flexural member, this formula enables studying different concrete healing variables. A new relation between the effective reinforced concrete healing/damage variable and the effective healing/ damage variables of concrete and steel has been introduced to relate concrete and steel healing/ damage and the reinforced concrete healing/ damage. The proposed formula has been verified with experimental results of full-healed flexural members, it has achieved good agreement. The flexural member stiffness has been studied considering different parameters like concrete healing variables, concrete cover and percentage of steel reinforcement. The flexural member stiffness has increased as the concrete healing variable has increased, it has increased as the concrete cover has increased and the steel reinforcement percentage of 1% has given the higher flexural member stiffness.

KEYWORDS
Healing variable; damage variable; flexural; reinforced concrete; stiffness

Introduction

Self-healing is one of the most attractive research topics in the last decades, reinforced concrete is a heterogeneous material that needs the self-healing to protect the reinforcement during serviceability life. Robins et al. (2001) studied cracks progression of reinforced concrete beam with steel fibers to obtain response in the form of a load deflection. The nonlocal formulation through concepts of continuum damage mechanics was proposed by Jirásek (2004), the standard continuum theory with a stress-strain law were shown and he increased efficiency of the non-local simulation. The analytical results of the concrete behaviour under plane strain conditions were introduced by Bobinski and Tejchman (2005) by using a simple damage continuum isotropic model which was enriched by non-local terms to avoid a pathological mesh sensitivity and to get a well-posed rate boundary value problem. Voyiadjis and Kattan (2009) investigated the damage tensor that was used to link the damage state of the material with effective undamaged configuration using different paths including fabric tensors to connect the two configurations.

Harries et al. (2012) developed the current ACI and AASHTO crack control provisions using high-strength reinforcing steel to anticipate the higher service level stresses. Allam et al. (2012) investigated and verified codes of practice provisions beside some equations calculating the crack width of reinforced concrete beams, five reinforced concrete models were studied theoretically. Voyiadjis and Kattan (2012) introduced and verified new damage variables, by using higher-order strain energy to lay the theoretical results for the design of undamageable materials. Darabi et al. (2012) proposed a novel continuum damage mechanics approach to model the micro-damage/ healing state in the materials that perform self-heal. Several presented examples were used to show the powerful of the proposed model to treat micro-damage healing.

The development of engineered self-healing and self-repairing concrete was stated by Mihashi and Nishiwaki (2012), where they stated that fiber reinforced cementitious composites (FRCC) have better self-healing potential than ordinary plain concrete and geomaterial approach using mineral admixtures is an effective healing technology. Tsangouri et al. (2013) investigated the formation of damage and recovery of reinforced concrete beams through the mechanical properties by application of an encapsulated healing agent. Hongming et al. (2014) studied a type of concrete beam with adhesive that enable crack self-repair, which was verified by bending tests. It was shown that the stiffness of the beam increases after crack healing, also the effect of crack healing level on the properties of the beams was studied with the finite element analysis software ANSYS.

Mačanovskis et al. (2016) evaluated serviceability of the capillary insulation by carrying out water permeability test and frost resistance test. The self-repair effect of capillary hydro insulation was evaluated, fiber concrete flexural members were pre-cracked at the beginning, then treated with capillary insulation and retested under flexural. Nassef (2016) used nonlinear damage mechanics approach to investigate fracture mechanism in reinforced concrete beams. The proposed model was verified with experimental results, different concrete damage variables were considered with different concrete cover values and reinforcement ratios. Kuang and Goh (2017) studied new sensors that enabled crack sensing and healing. Flexural tests of two-meter reinforced concrete beams were carried out to demonstrate the capability of the sensors in detecting crack widths. Bonilla et al. (2017) developed several strategies to improve the self-healing efficiency of cementitious materials to self-repair cracks. Structural behavior and healing efficiency were studied, where they measured and compared the stiffness, maximum strength, and deformation with post healing measurements. In addition, the cracks were monitored to determine crack healing, they concluded that using microcapsules was more efficient in healing by reducing crack width.

Minnebo et al. (2017) presented such a vascular system, which was designed to enable providing healing material from outside of the concrete beam. Both clay and inorganic

CONTACT Ahmad Salah Edeen Nassef ahmad_nassef@hotmail.com

phosphate cement were compared as materials for the vanes of this system, 4-point bending test was conducted, in order to obtain realistic conditions. They showed that repeatable self-healing was possible, that the system was able to heal multiple cracks at the same time and the mechanical properties were restored. Luis Bonilla et al. (2018) presented an evaluation of dual self-healing mechanisms in concrete, a decrease in the flexural strength of concrete beams was found due to the microcapsules and the best crack healing efficiency could be observed after the water curing period. Ferrara (2018) presented novel self-healing cement-based materials and experimental methods for the assessment of the self-healing ability and analyzed new challenges that faced in designing reliable structures. Chahmi Oucif et al. (2018) extended linear super-healing theory to nonlinear super-healing theory to can be used in rehabilitation of structures. Oucif et al. (2019) defined super healing term depending on the change in elastic strain and applied the generalized nonlinear and quadratic super healing concepts. Voyiadjis et al. (2020) introduced several healing variables that depending on recovering material and engineering properties. They found that, by assuming the equality of the strain energy in damage case and healing case, the healing variable evaluated using reduction of the modulus of elasticity is greater than which depend on the reduction of the cross sectional area.

This paper seeks to introduce a new formula to correlate the damage/ healing of concrete, the damage/ healing of steel and the damage/healing of the reinforced concrete flexural members to control damage and design or provide suitable healing. The structure of the paper is as follows. The entitled section of "Nonlinear Damage – Healing Approach for the Beam" presents the stages of the damage and healing the reinforced concrete flexural elements. This section shows that the relation between effective healing /damage variable of concrete, effective healing /damage variable of steel and the effective healing /damage variable of the reinforced concrete beams are correlated and introduces the final formula for the effective healing /damage variable of the reinforced concrete beams. The section entitled of "Effective Reinforced Concrete Healing/Damage Variable" shows the steps that are followed to determine the effective healing /damage variable of the reinforced concrete beams. The next section discusses the results and analysis where the introduced formula is verified, the effects of the concrete healing variable, the concrete cover and the used reinforcement on the beams stiffness are studied. Finally, the conclusion is presented.

Nonlinear damage – healing approach for the beam

Consider a reinforced concrete beam has length *L*, its cross-sectional dimensions are and for width and thickness respectively, and modulus of elasticity is *E*. Figure 1 shows the beam under the bending moment *M*.

By applying the flexural moment, cracks will propagate and the damaged shape will be as shown in Figure 2. Healing agent starts to work and repairs the formed cracks as shown in Figure 3.

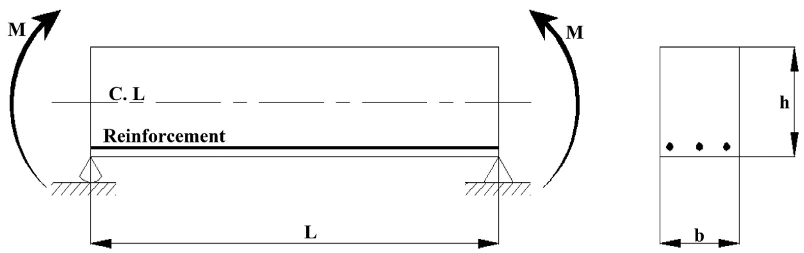

Figure 1. Dimensions of the reinforced concrete beam.

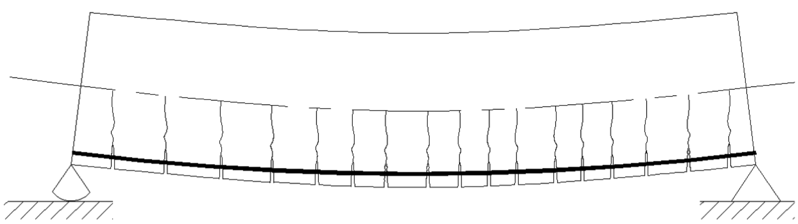

Figure 2. Damaged shape.

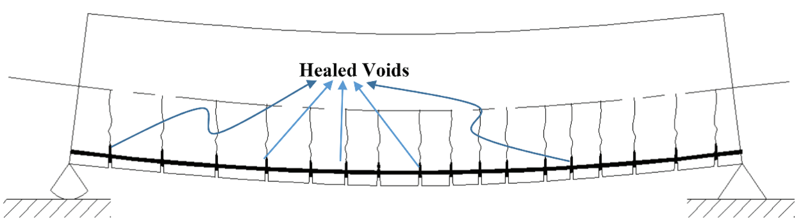

Figure 3. Damaged – healed shape.

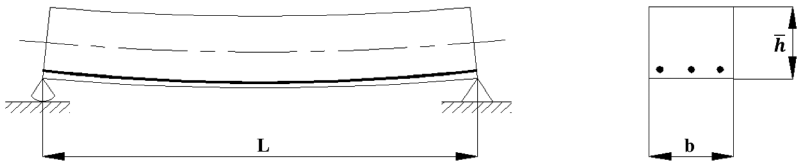

Figure 4. Effective state.

The effective undamaged shape, shown in Figure 4, can be obtained by subtracting both the healed and unhealed cracks, that leads the cross-sectional area to be. For simplicity, cracks and voids will be removed and only the depth will be modified regardless the width.

Flexural stresses in the beam at the original state can be obtained as the following:

$$\sigma = \frac{M}{S} \tag{1}$$

Where:

σ is the normal stress due to bending.

M is the moment.

S is the section modulus.

The normal stress due to bending in the damaged state is:

$$\sigma_d = \frac{M}{S_d} \tag{2}$$

σ_d is the normal stress due to bending in damaged state.

S_d is the section modulus in damaged state.

In the same way, the normal stress due to bending in damaged – healed state is:

$$\sigma_h = \frac{M}{S_h} \tag{3}$$

Where:

σ_h is the normal stress due to bending in damaged – healed state.

S_h is the section modulus in damaged – healed state.

The normal stress due to bending in the effective (fictitious) undamaged beam (after removing both healed and unhealed cracks and voids) can be determined from:

$$\bar{\sigma} = \frac{M}{\bar{S}} \tag{4}$$

Where:

$\bar{\sigma}$ is the normal stress due to bending in the effective undamaged beam.

$\bar{S}$ is the section modulus for the effective undamaged beam.

Where, the damage variable (φ) relies on the change of the modulus of elasticity, as showed by Voyiadjis and Kattan (2009):

$$\varphi = \frac{\bar{E} - E_d}{E_d} \tag{5}$$

Where:

$E_d = \frac{\sigma_d}{\in_d}$ is the modulus of elasticity in damaged state.

$\bar{E} = \frac{\bar{\sigma}}{\bar{\in}}$ is the effective modulus of elasticity.

$\in_d$ is the damage state strain.

$\bar{\in}$ is the effective state strain.

In the same manner, the healing variable h can be obtained, as showed by Voyiadjis and Kattan (2009):

$$h = \frac{E_h - E_d}{E_d} \tag{6}$$

Where:

$E_h = \frac{\sigma_h}{\in_h}$ is the elastic modulus in healed state.

From Eqs. (1) through (6), one can deduce that:

$$\varphi = \frac{S_d \in_d}{\bar{S} \bar{\in}} - 1 \tag{7}$$

$$h = \frac{(\varphi + 1)\bar{S}\bar{\in}}{S_h \in_h} - 1 \tag{8}$$

Where, the strain energy can be assumed unchanged in damage and effective states, one obtains:

$$\frac{1}{2}\sigma_h \in_h = \frac{1}{2}\bar{\sigma}\bar{\in} \tag{9}$$

From Eqs. (3) through (8) and by substituting in Eq. (9):

$$\varphi = \frac{{S_h}^2}{\bar{S}^2}(h + 1) - 1 \tag{10}$$

Eq. (10) shows new relation between damage and healing variables for flexural member, for reinforced concrete flexural member the damage variable will be denoted as:

$$\varphi_T = \frac{{S_h}^2}{\bar{S}^2}(h + 1) - 1 \tag{11}$$

Where:

φ_T is the reinforced concrete damage variable.

Consequently, concrete and steel damage variable can be introduced as:

$$\varphi_C = \frac{{S_{hc}}^2}{{\bar{S}_c}^2}(h_c + 1) - 1 \tag{12}$$

$$\varphi_s = \frac{{S_{hs}}^2}{{\bar{S}_s}^2}(h_s + 1) - 1 \tag{13}$$

Where:

φ_C is the concrete damage variable.

S_{hc} is section modulus for concrete section in damage – healing state.

$\bar{S}_c$ is the effective section modulus for concrete section in fictitious state.

h_c is the healing variable for concrete.

φ_s is the reinforcement steel damage variable.

S_{hs} is the section modulus for reinforcement steel in damage – healing state.

$\bar{S}_s$ is the effective section modulus for reinforcement steel.

h_s is the healing variable for steel.

Where concrete and steel are forming reinforced concrete flexural member, it can be found that:

1 -Damaged state:

$$E_d I_d = E_{dc} I_{dc} + E_{ds} I_{sd} \tag{14}$$

2 -In damaged – healed case:

$$E_h I_h = E_{hc} I_{hc} + E_{hs} I_{hs} \tag{15}$$

3 -Effective state:

$$\overline{EI} = \overline{E_c I_c} + \overline{E_s I_s} \tag{16}$$

Where:

E_{dc} and E_{ds} are the damaged state moduli of elasticity of concrete and steel.

E_{hc} and E_{hs} are the damaged – healed moduli of elasticity of concrete and steel.

$\overline{E_c}$ and $\overline{E_s}$ are the effective moduli of elasticity of concrete and steel.

I_{dc} and I_{ds} are the damaged state moment of inertia for concrete and steel.

I_{hc} and I_{hs} are the damaged – healed moment of inertia for concrete and steel.

$\overline{I_c}$ and $\overline{I_s}$ are the effective moment of inertia for concrete and steel.

By dividing Eq. (15) by Eq. (14) and ignoring small distance due to the concrete cover, one can obtain:

$$(h+1)\frac{S_h}{S_d} = \frac{S_{hc}}{\frac{S_{dc}}{(h_c+1)} + nS_{ds}} + \frac{S_{hs}}{\frac{S_{dc}}{n} + \frac{S_{ds}}{(h_s+1)}} \tag{17}$$

Where:

$n = \frac{E_s(\text{in any case})}{E_c(\text{in any case})}$ is the modular ratio.

In the same way, by dividing Eqs. (16) by (14), one can obtain:

$$(\varphi_T+1)\frac{\bar{S}}{S_d} = \frac{\bar{S}_c}{\frac{S_{dc}}{(\varphi_c+1)} + nS_{ds}} + \frac{\bar{S}_s}{\frac{S_{dc}}{n} + \frac{S_{ds}}{(\varphi_s+1)}} \tag{18}$$

Then divide Eqs. (17) by (18), on can obtain:

$$\psi_T = \frac{\frac{\frac{\bar{S}_C}{\psi_C}}{\frac{S_{dC}}{h_C+1}+nS_{ds}} + \frac{\frac{\bar{S}_s}{\psi_s}}{\frac{S_{dC}}{n}+\frac{S_{ds}}{h_s+1}}}{\frac{\bar{S}_C}{\frac{S_{dC}}{\varphi_C+1}+nS_{ds}} + \frac{\bar{S}_s}{\frac{S_{dC}}{n}+\frac{S_{ds}}{\varphi_s+1}}} \tag{19}$$

Where:

is the effective healing/damage variable for reinforced concrete.

$\psi_C = \sqrt{\frac{h_C+1}{\varphi_C+1}}$ is the effective healing/damage variable for concrete.

is the effective healing/damage variable for steel.

Eq. (19) provides a new formula that relates the effective healing/damage variable for reinforced concrete ψ_T with the effective healing/damage variable for concrete (ψ_C) and the effective healing/damage variable for steel (ψ_s). This novel formula enables determining and controlling the damage and healing behavior of the reinforced concrete flexural members considering the damage and healing that can be contributed from either concrete or steel or from both of them.

Effective reinforced concrete healing/damage variable

The effective healing/damage variable for reinforced concrete is the solution key in the Equation (19), once this variable is determined, the section height at effective state $\bar{h}$ can be obtained then all responses can be determined, as shown in the following steps:

1 -In damage – healing state, calculate the first moment of area for concrete and reinforcement about the axis passing through the center of gravity considering the crack height.

2 -For easiness, both of the damage and healing in reinforcement are ignored (No damage – No healing, $\varphi_s = 0$, $h_s = 0$ then $\psi_s = 1$). On the other hand, the concrete healing and damage variables, consequently the effective concrete healing/damage variable, are obtained regarding the crack height.

3 -The modular ratio is calculated at the yielding point (crack progression point).

4 -Where the effective reinforced concrete healing/damage variable depends on the effective concrete and steel healing/damage variables, so it can be obtained from the following proposed relation:

$$\psi_T = \psi_s\left(0.45 - \frac{h_C}{10}\right) + \psi_c\left(0.55 - \frac{h_C}{10}\right) \tag{20}$$

5 -The effective section height can be determined to get beam responses.

Results and analysis

The proposed nonlinear healing approach, Eqs. (19) and (20), is verified with experimental results obtained by Hongming et al. (2014), where they considered a reinforced concrete beam of 600 mm length and cross-section dimensions are 120 mm width and 200 mm thickness, the section reinforcement is shown in Figure 5. The beam is under bending test considering four specimens L_1, L_2, L_3 and L_4 where specimen L_1 was formed without repair fibers. Repair fibers (5 cm in length) were symmetrically distributed within specimen L_2 at an interval of 100 mm about mid-span. Repair fibers (5 cm in

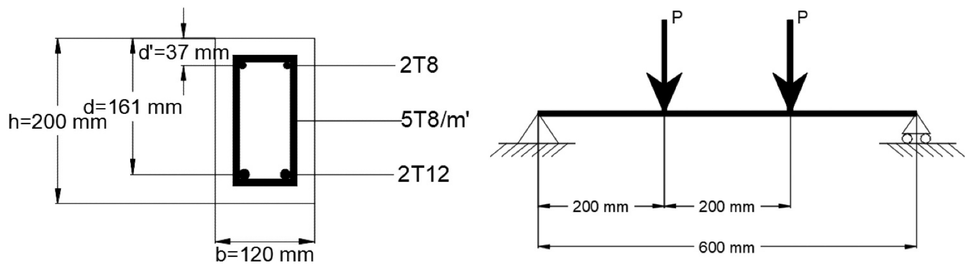

Figure 5. Loading, geometry and cross-section details of the verification beam.

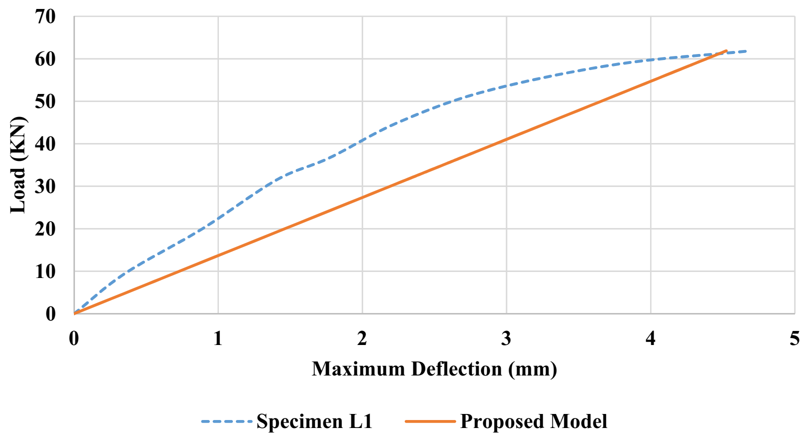

Figure 6. Maximum deflection versus load for specimen L_1 obtained experimentally and by proposed model.

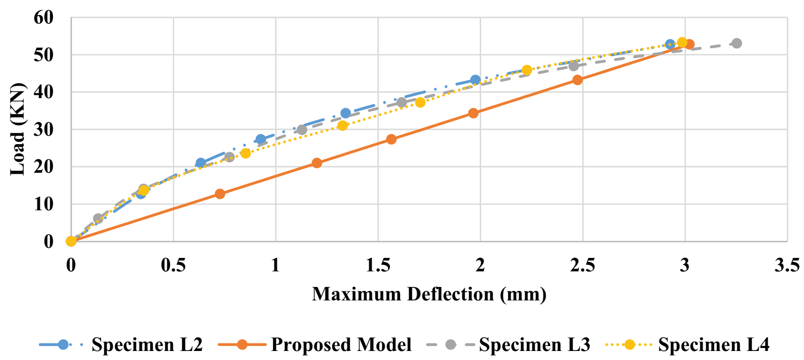

Figure 7. Maximum deflection versus load for specimens L_2, L_3 and L_4 obtained experimentally and by proposed model.

length) symmetrically distributed within specimen L_3 at an interval of 80 mm about mid-span. Repair fibers (30 cm in length) running the length of the tension zone between two loading points within specimen L_4 contained.

Figure 6 shows the stiffness of the beam obtained by the proposed model and that obtained experimentally for specimen L_1, good agreement can be noted. The obtained maximum deflections are close to each other, especially as the load approaches the bearing capacity of the beam. Ignoring the damage of steel leads to obtain linear stiffness that shows the best agreement between the proposed model and experimental results as the beam reaches its loading capacity.

Figure 7 shows the stiffness of the beam obtained by the proposed model and that obtained experimentally for specimens L_2, L_3 and L_4, good agreement can be noted. The proposed model results are closer to the experimental results for the specimen L_4, which consider continuous healing in the maximum tension zone, it can be referred to that the proposed model consider both of the damage and healing are equally distributed through the material. So, the proposed model will be more efficient as the healing points increase. In addition, the results of the proposed model become closer to experimental results as the load approaches beam bearing capacity load.

The stiffness of the considered beam is studied for different concrete healing variables, where $h_c = 0$, $h_c = \frac{1}{3}$, $h_c = \frac{2}{3}$ and $h_c = 1$ are considered. Figure 8 shows that the beam stiffness increases as the concrete healing variables (h_c) increases, because as the healing variable increases the effective cross-section dimensions increase. It can be noted that the healing variable $h_c = \frac{2}{3}$ is the effective healing ratio, where the beam gains the majority of its stiffness.

The concrete cover controls the first moment of area of steel reinforcement so the effective healing/damage variable for reinforced concrete is affected as can be shown in Eq. (19). The flexural rigidity of a beam decreases as the concrete cover increases where the distance between the center of gravity and the reinforcement decreases, so the effect of concrete cover is studied at the concrete healing variable $h_c = \frac{2}{3}$. Figure 9 shows that the maximum deflection of the beam increases as the concrete cover decreases, consequently the beam stiffness increases as the concrete cover increases at the constant healing variable for concrete, so it is recommended to increase the concrete cover to enhance healing of the beam.

Also, the percentage of main reinforcement is studied at the concrete healing variable $h_c = \frac{2}{3}$ and at 25 mm concrete cover, Figure 10 shows that the beam stiffness decreases as the percentage of reinforcement increase from 1% to 2%, then it increases again as the percentage of reinforcement increase from 2% to 3%. It can be referred to increase of the steel reinforcement in concrete affect the behavior of reinforced concrete, because in flexural members as the percentage of reinforcement approaches 3% the concrete and steel behave individually as a composite material. In addition, ignoring the damage of steel may not be used for all percentages of the steel reinforcement.

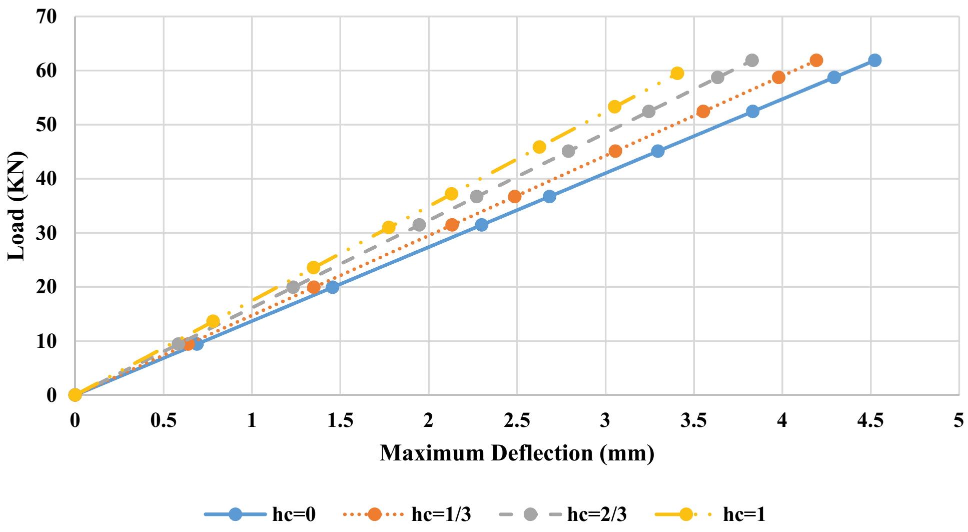

Figure 8. Beam stiffness at different concrete healing variables.

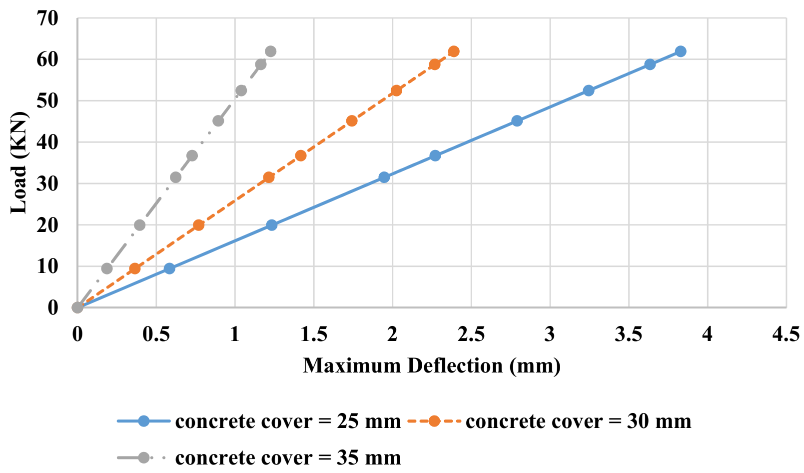

Figure 9. Beam stiffness at different concrete cover.

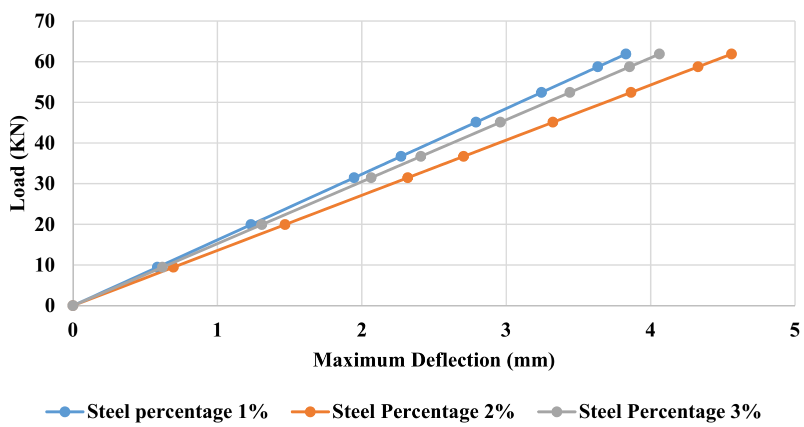

Figure 10. Beam stiffness at different reinforcement percentages.

As per the previous analyses, it can be noted that the new formula can contribute practically in controlling the damage and healing of the reinforced concrete flexural members. The controlling of the damage and healing behavior of the reinforced concrete beam can occur by controlling the healing variable of concrete, it means that the healing of the reinforced concrete beams can be designed. Many design parameters can be considered like concrete cover, steel percentage and provided healing of concrete.

Conclusion

A new nonlinear healing/ damage formula was introduced to interrelate the damage state and effective state that enables determining the effective state geometric properties to help in finding the response of the flexural member in healing / damage state. In addition, a new relation between the effective reinforced concrete healing/damage variable and the effective healing/ damage variable for concrete and steel was introduced. These new formulas enable controlling the damage of flexural members and design the healing of these members where the reinforcement, concrete dimensions, healing variables, damage variables are correlated in these formulas. Good agreement between the new proposed formula and experimental results was achieved. The proposed model's efficiency increases as the spacing of healing points decreases, the proposed model becomes very close to experimental results when the healing becomes continuous. The beam's stiffness increased by increasing the concrete healing variables (h_c) and the concrete healing variable $h_c = \frac{2}{3}$ was the reasonable healing variable that achieved the majority of the beam stiffness. Effective healing can be obtained by increasing concrete cover to increase the beam stiffness. Also, the proposed model shows that the percentage of steel reinforcement affects the behavior of reinforced concrete in healing, where the percentage of steel reinforcement affected the efficiency of the healing, where 1% gave high beam stiffness, but 2% of steel reinforcement reduced beam stiffness and 3% increased beam stiffness again.

Disclosure statement

No potential conflict of interest was reported by the author(s).

Notations

E_d is the modulus of elasticity in damaged state.
E_h is the elastic modulus in healed state.
$\bar{E}$ is the effective modulus of elasticity.
E_{dc} and E_{ds} are the damaged state moduli of elasticity of concrete and steel.
E_{hc} and E_{hs} are the damaged - healed moduli of elasticity of concrete and steel.
$\overline{E_c}$ and $\overline{E_s}$ are the effective moduli of elasticity of concrete and steel.
$\in_d$ is the damage state strain.
$\bar{\in}$ is the effective state strain.
h_c is the healing variable for concrete.
h_s is the healing variable for steel.
I_{dc} and I_{ds} are the damaged state moment of inertia for concrete and steel.
I_{hc} and I_{hs} are the damaged – healed moment of inertia for concrete and steel.
$\overline{I_c}$ and $\overline{I_s}$ are the effective moment of inertia for concrete and steel.
M is the moment.
n is the modular ratio.
S is the section modulus.
S_d is the section modulus in damaged state.
S_h is the section modulus in damaged – healed state.
is section modulus for concrete section in damage – healing state.
$\bar{S}$ is the section modulus for the effective undamaged beam.
σ is the normal stress due to bending.
σ_d is the normal stress due to bending in damaged state.
σ_h is the normal stress due to bending in damaged – healed state.
$\bar{\sigma}$ is the normal stress due to bending in the effective undamaged beam.
φ_T is the reinforced concrete damage variable.
φ_C is the concrete damage variable.
ψ_T is the effective healing/damage variable for reinforced concrete.
ψ_C is the effective healing/damage variable for concrete.
ψ_s is the effective healing/damage variable for steel.

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