



## Research article

## Golden ratio-based capital structure as a tool for boosting firm's financial performance and market acceptance

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## ABSTRACT

In the 20<sup>th</sup> century, the golden ratio has been discovered by modern science, including economics, business, and finance. In the field of finance, the ratio is mostly applied for technical analysis, and much less attention is given to its use in solving corporate finance problems, such as capital structure decisions. In this study, 455 US and European manufacturing and service firm's data are examined from the period 2010–2019. The purpose of the investigation was to determine if there are any positive impacts of a golden ratio-based capital structure on financial performance and market acceptance. We find significant positive relationships between the deviation from the golden ratio-based capital structure and the deviations of firms' revenue, income, stock price and market value data from their historical maximum. Thus, indicating the golden ratio-based capital structure may be an efficient tool for firms to boost their performance and market acceptance. Based on our results, this relationship is more obvious in the United States than in Europe, and stronger for service firms than for manufacturing companies.

## 1. Introduction

0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987, 1597, 2584, 4181, 6765...the series that came to life as the solution of rabbits' birth rate calculations by the famous mathematician Leonardo da Pisa in the 12th century. Da Pisa was well known in his time for introducing the decimal number system to Europe and became a legend for eternity for the "Fibonacci Summation" series, named after his nickname. In this series each number will be produced by summing the previous two, starting from 0 and 1. The series creates a unique proportion by dividing any number by the previous number, the result will approach 1.618 asymptotically.

A proportion usually denotes the balance or symmetry that exists between the components of an ensemble or between a particular component and ensemble (Haylock, 2006). Various types of proportionalities exist including mathematical proportions, geometric proportions, harmonic proportions, etc. (Kotliar, 2016), but 1.618 is found in each type of proportionality. The proportion was first mentioned in Euclid's celebrated book *Elements of Geometry* written around 300 B.C. Luca Pacioli, a man of faith, a lover of mathematics, geometry, algebra, and accounting in the 16th century was determined to prove that God is one and three, and that this truth descends algebraically from the definition of the *Divine Proportion*. This is indicated by the relationship within

any segment between two unequal lengths, of which the greater is the mean proportional between the lower and the sum of the two (Biancone et al., 2017). It can be defined as point C on line AB, where  $AC/CB = AB/AC = 1.618$ . An equivalent statement to this is that point C divides the AB line into two parts, where AC represents 61.8% and CB represents 38.2% of the total length of AB.

The term 'golden ratio' was first used for the divine proportion by the German mathematician Martin Ohm in 1815 ('Goldener Schnitt' in German language). The terms golden mean (0.618) and golden ratio (1.618) are representing phi and the inverse of it. The proportion is an irrational number that, since 1914, is denoted with the Greek letter phi (common notations:  $\phi$ ,  $\phi_i$ ,  $\varphi$ ,  $\Phi$ ). The golden ratio will be an irrational number, a non-recurring decimal, the only number whose decimal part, the part following the decimal point – is identical to that of its square and its inverse as  $\varphi^2 = 2.61803398874$ , and  $1/\varphi = 0.61803398874$ .

The discovery of this irrational number caused a philosophical crisis, although it has also connected mathematicians, biologists, artists, musicians, historians, architects, psychologists and even mystics throughout the centuries, who have investigated and discussed its unexpected presence in the most diverse fields. As Urmantsev (2009) argues: "In the second half of the 20th century the Fibonacci numbers and the golden ratio find use in almost all branches of science and arts".

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Unsurprisingly, this tendency reached the field of economics and business, too. Applications ranged from economic development through management, sales, and quality control questions to the fields of stock market analysis, fraud detection, and growth forecasts. Among these possible fields of application, only little attention was given in the literature to the vital question of capital structure. Therefore, the motivation behind our study is to conduct a deeper examination of whether and how the golden ratio could be applied to find the optimal capital structure and through this, to improve the financial performance and market acceptance of companies.

## 2. Literature review

Kulis and Hodzic (2020) published a comprehensive literature review that addresses the interdisciplinary nature of research behind the number 1.618. They summarized the main findings related to the golden ratio in the field of physics, chemistry, biology, medicine, psychology, sociology, geology, engineering, fashion and design, modern technologies, economics, and business. Using their results and incorporating our own research, we summarize the economics and business-related literature that uses the golden ratio in Table 1.

In the field of economics, the most recent studies focus on economic development with the help of the golden ratio. Endovitsky et al. (2017, 2019) explore methods of managing economic theories based on the system life cycle concept. They conclude that methodological provisions of resource-based economic analysis could be used, not only to determine but also to predict, the balance of the system structure by changing the number of resources consumed and their returns.

Beyond this, several business applications of the golden ratio can be found in the literature, the most important of which are management, marketing, operations management, finance, and accounting. The management applications examine the organizational structure (Opalenko and Rudenko 2019), the organizational development (Henderson and Boje 2015), and management functions (Dimovski and Uhan 2012) through the proportion. The latter highlight in their work that by applying Fibonacci numbers in management practice, employee

satisfaction and the optimal functioning of the organization leads to better business results. The use of the golden ratio in business is most evident in the marketing field, especially with regard to sales strategies (Fischer, 1993; Thomas and Chrystal 2013). Furthermore, Nikolic et al. (2011) investigated the relationship between the golden ratio and consumer preferences by analyzing packaging designs and confirmed the priority of packaging designs based on the golden ratio over other packaging designs, in terms of consumer preference. Operations management uses the golden ratio to determine the optimal gain in inventory and the work in progress feedback loop (Disney et al., 2004). Another application is that of Pan and Jarrett (2013), who applied a so-called ‘golden ratio search’ in quality management to search for an optimum. The golden ratio search is the use of the proportions derived from the golden ratio (0.618 and 0.382) to condense the width of the range in each step. This approach becomes effective in unimodal optimization as it results in the least number of searches or trials to locate the optimum.

In the case of finance, investigations based on the Fibonacci numbers and the golden ratio concentrate mainly on financial markets, arguing that the stock markets act as irrationally and as randomly as the numbers themselves, since financial markets are filled with human wants and needs, mass psychology and emotion. In stock markets, an environment of chaos and a non-linear phenomenon is given, which can be measured using fractal geometry, like the Fibonacci series (Williams, 2012). Ralph Nelson Elliott developed his Wave Principle in 1935, by which he predicted stock price movements on stock exchanges based on Fibonacci numbers. Elliott's discovery changed the perception of stock market trading, which at that time was thought to be chaotic and disorganized. His theory found that trading follows repetitive cycles that are predictable and can be observed in stock price movements (Fischer 1993; Livio 2003; Brown 2010). However, a large part of the contemporary market analysts are less optimistic about predicting financial markets with Fibonacci numbers, arguing that the use of the theory in practice is limited (Lo et al., 2000; Narasimhan and Jagadeesh 2000; Bhattacharya and Kumar 2006).

Nevertheless, practitioners do not completely abandon the notion and continue to reference Fibonacci numbers as part of their analytics, with

**Table 1. Golden-ratio based economics and business-related research findings.** This table summarizes the research works in different sub-disciplines of Economics and Business dealing with the possible applications of the golden ratio. The author's summary combines the findings of Kulis and Hodzic (2020) in addition to our literature survey.

Discipline	Sub-discipline	Author(s)	Research area
Economics	Macroeconomics	Endovitsky et al. (2017)	economic development
		Endovitsky et al. (2019)	economic development
Business	Management	Henderson and Boje (2015)	organizational development
		Dimovski and Uhan (2012)	management functions
		Opalenko and Rudenko (2019)	organizational structure
	Marketing	Fischer (1993)	sales strategies
		Nikolic et al. (2011)	customer preferences
		Thomas and Chrystal (2013)	sales strategies
	Operations management	Disney et al. (2004)	production and inventory control
		Pan and Jarrett (2013)	quality control
	Finance	Fischer (1993)	trading strategies
		Frost and Prechter (2005)	stock market movements
		Livio (2003)	stock markets
		Brown (2010)	stock market analysis
		Lahutta (2016)	stock markets
		Greenblat (2007)	market conditions
		Bhattacharya and Kumar (2006)	stock market movements
Accounting		Amershi and Feroz (2000)	fraud detection
		Chapin (1957)	firm growth
	Biancone et al. (2017)	financial ratio analysis	
	Rehwinkel (2016)	capital structure analysis	

Source: Own construction based on Kulis and Hodzic (2020).

some adjustments (Bhattacharya and Kumar 2006; Frost and Prechter 2005; Greenblat 2007; Lahutta 2016).

Based on our research, the most important literature is that which applies accounting knowledge or experimenting with accounting tools or financial reporting outputs. Why accounting? Pacioli himself was an accountant, he was the author of *Summa*, containing the detailed description of the Venetian way to keep books of businesses, which made him the father of accounting. Ohm and the previously mentioned Elliot were also accountants. In fundamental analyses, financial analysts use several accounting ratios in addition to earnings data to derive information on firm value, future prospects, growth, stability, or bankruptcy. Chapin (1957) collected data on 80 churches and their two largest subgroups, membership, and Sunday-school enrolment, searching for evidence of optimal firm size, and found that organizations are bounded in growth according to the Fibonacci proportion. Amershi and Feroz (2000) examined if the Fibonacci sequence, the golden ratio, the golden mean and the frequency of them, might be able to distinguish between fraud and legitimate companies. They argue that there is very little systematic work on the probability distribution of the frequently used financial ratios, either at a point in time or several periods as stochastic processes. The purpose of their paper was to conduct an analysis of the unconditional probability distribution of the Total Debt/Total Invested Capital ratio to determine if the occurrence of the Fibonacci golden mean and the golden ratio as possible values of this ratio are random or indicative of firm survival. They found no statistical significance in either the sample of surviving companies or the randomly selected ones. Although they identified an infinity of distributions under which they could statistically discriminate the surviving companies and all others. Biancone et al. (2017) were searching for the golden ratio within the corporate balance sheets. They applied the statistical test Z in search of the golden mean. Their goal was to prove that the averages of the calculated financial ratios for each company were equal to a predetermined value (which they called the golden number). Their research provided inconclusive results. Rehwinkel (2016) applied the fundamentals of the constructal law, areas demarcated according to the golden ratio, and the second law of thermodynamics, assuming that business entities are attracted towards states of maximum debt. The author argues that the constructal law, the second law of thermodynamics, as well as the golden ratio are relevant when analyzing liabilities to assets, despite that the laws and the ratio are associated with natural sciences. In the fields of corporate financial reporting and financial risk analysis, symmetry is embedded in the basic accounting equation, making an impact on the creation and interpretation of corporate capital structures. The author used phi inverted to determine the areas within which the capital structure ratios could fall over time, and concluded that through the constructal law, the golden ratio and the second law, a transdisciplinary guideline can be developed, by which disclosing and analyzing capital structure formations of business entities and sectors becomes possible.

Our literature review demonstrates that there are many different finance and accounting-related areas where the golden ratio is used, however, these are mainly focusing on the stock market and technical analysis. Rehwinkel (2016) was the only author who tried to connect the golden ratio with capital structure, but in that study no attention was given to the relationship between the capital structure and the company's financial figures or value. In other words: Rehwinkel did not focus on the question of optimal capital structure.

On the other hand, the connection between capital structure and financial performance (or market acceptance) has been widely investigated in the international literature. Although there are a few research works using the hypothesis of a reverse causal effect (namely that performance determines the capital structure), stating that debt holders are more attracted to profitable and financially sound firms (i.e. Margaritis and Psillaki 2010), the traditional thinking (which states that the choices made about capital structure determine the firm's performance and value) is dominant in the literature. Several empirical studies showed this causality, but with mixed results. While Arbor (2005), Adair and Adaskou

(2015) and Jouida (2018) found a positive relationship, Gleason et al. (2000), Majumdar and Chhibber (1999), Qayyum and Noreen (2019), Le and Phan (2017) as well as Vo and Ellis (2017) demonstrated a negative effect of leverage on performance. Furthermore, there are studies available, such as those of Bandyopadhyay and Barua (2016) or Jaisinghani and Kanjilal (2017) that show both positive and negative impacts. Although, these studies deeply examine the relationship itself, the authors do not focus on seeking the optimal level of debt or equity.

Research works that concentrate directly on the question of optimal capital structure are, among others, those of Fruhan et al. (1992), Damodaran (1994) and Fernandez (2001). Fruhan et al. (1992) use a very simple example of a fictive company to illustrate how the firm value, the stock price, and the cost of capital changes at different levels of debt, and concluded that the optimal capital structure is at a Debt/Equity ratio of 30%. Damodaran (1994) uses a similar approach based on the figures of a real company (Boeing), and obtains the same result, namely that a 30% Debt/Equity ratio can be regarded as the optimal capital structure. Fernandez (2001) goes against Fruhan et al.'s and Damodaran's approaches, with detailed explanations of the detected conceptual problems of the cited works. A common point in these three research papers is that, even if Damodaran uses real company data, all of them interpret the optimal capital structure from a point of view of finance theory, and none of them deal with the golden ratio as a possible element of the investigation.

To summarize, to the best of our knowledge, there is no research available in the literature that investigates the question of optimal capital structure with an empirical approach and applying the golden ratio. We address the above-mentioned research gap in this study. We generally agree with Fruhan et al. (1992), Damodaran (1994) and Fernandez (2001) in that the best possible capital structure is the one that maximizes the firm value and the stock price and minimizes the cost of capital. However, in our study a purely empirical approach is used: that capital structure is optimal when the company's financial performance figures (revenue and net income) and market indicators expressing investor's opinion (stock price and enterprise value) are closest to the firm's potential, that is, the historical maximum within a given period. Our investigation is based on the hypothesis that this optimal capital structure follows the golden ratio.

Data of US and European manufacturing and service companies from a 10-year period (2010–2019) were collected to investigate if there are any advantages to companies who build up a 'golden ratio based' capital structure in terms of financial performance and/or market acceptance compared to other firms. The reason for selecting this period is to observe the above relationship under 'normal' market circumstances, therefore, the period examined contains the years after the 2008–2009 global crisis and before the outbreak of the coronavirus pandemic in 2020. Beyond the total sample, our analysis is extended to the examination of potential regional differences between US and European firms, and between different company profiles (manufacturing and service). We believe that our findings provide a real contribution to the existing literature and may be useful for financial managers for their decisions on capital structure.

### 3. Materials and methods

#### 3.1. Data

For the empirical investigation, data were retrieved from the Thomson-Reuters database. In the original search, financial data (assets, liabilities, shareholders' equity, revenue, income, stock price and the EV/EBIT valuation multiple) of all available quoted companies were collected for the period 2010–2019. This original query found 1632 companies. We then applied our relevant selection criteria in line with our research objective, and filtered the database as follows:

- The original search contained firms from 68 different countries. As our research has a regional focus, only companies that have their headquarters in the United States or in Europe were kept in the

sample. After excluding all others, US firms and companies from 27 European countries remained in the sample.

- From the 18 different industry codes in the first query (according to the North American Industry Classification System, NAICS), we filtered the data for those industries that represent a traditional manufacturing or service profile and excluded the others (for example: agriculture, mining, real estate, or utilities). As a result, 9 industry codes remained.
- Finally, we excluded all firms where data were not fully available for each year of the examined period (2010–2019).

As a result of the filtering detailed above, a final sample of 455 companies was obtained. The composition of our sample is shown by Table 1.

Our analysis uses the raw data of six different variables (financial performance indicators and market data) for each company-year, as presented in Table 2 (see Table 3).

Based on the number of firms in the sample (455) and the length of the examined period (10 years), the data collection resulted in 4550 observations for each variable.

### 3.2. Definition of the ‘golden ratio-based’ capital structure

As discussed earlier, our research objective is to investigate if there is any observable advantage in terms of financial performance and/or market acceptance for companies with a golden ratio-based capital structure, compared to other companies with a capital structure deviating from the golden ratio. However, it is not obvious how the golden ratio is interpreted in case of the capital structure. Given that  $1/1.618 = 0.618$  and  $1 - 0.618 = 0.382$ , it can be stated that the capital structure follows the golden ratio if the two components of capital represent proportions of 61.8% and 38.2%, respectively. Nevertheless, it remains a question how these proportions should be matched with equity and debt.

To make the right decision, an empirical approach was used. First, analysing the 4550 company-years in our sample, we found that the average Debt/Equity ratio was 1.38, which is identical to an Equity to Total Assets ratio of around 42%, given that  $1/(1 + 1.38) = 0.42$ . This indicates that debt typically represents the higher share of total capital.

Second, we sorted all company-years in our database according to the equity to total assets ratio (excluding extreme values below 30% and above 70%) and computed the averages of actual financial performance (revenue, net income) and market acceptance (stock price and EV/EBIT [Enterprise Value to Earnings Before Interest and Tax]) indicators in those company-years related to their historical maximum (the highest value reached by the given company in the 2010–2019 period). Our findings are presented by Figure 1:

Numbers indicate that in terms of revenue, price and EV/EBIT, companies demonstrated the highest relative performances in years when their capital structure contained equity of 41–45%, while in those company-years when this ratio was higher, relative performances were lower. Net income is an exception, as the highest relative values can be seen at higher equity to total assets ratio (46–50%). Even with this exception, our general conclusion is that a capital structure containing less equity and more debt results in better financial performance and market acceptance.

Repeating the same examination on subsamples created based on region and company profile, the picture is similar, as shown in Table 4.

**Table 2. Composition of the sample.** This table shows the number of companies in the selected sample, divided based on region (US and Europe) and company profile (manufacturing and service).

Profile/Region	US	Europe	Total
Manufacturing	120 firms	229 firms	349 firms
Service	40 firms	66 firms	106 firms
Total	160 firms	295 firms	455 firms

**Table 3. Input data collected for the empirical analysis.** The table presents the variables collected from each company years in our sample. The first two (Shareholders’ Equity and Total Assets) are used to formulate the independent variable, while the last four (Total Revenue, Net Income, Stock Price, and EV/EBIT) will serve as the dependent variables in our regression models.

Variable	Description
$SHE_{i,t}$	Shareholders’ Equity of company $i$ in year $t$
$TA_{i,t}$	Total Assets of company $i$ in year $t$
$TR_{i,t}$	Total Revenue of company $i$ in year $t$
$NI_{i,t}$	Net Income of company $i$ in year $t$
$P_{i,t}$	Closing stock price of company $i$ in year $t$
$EV\_EBIT_{i,t}$	The ratio between Enterprise Value and Earnings Before Interest and Tax of company $i$ in year $t$

There are only two cells in the table (net income and stock price in the European manufacturing subsample) where the equity level generating the highest relative values of the examined indicators is higher than 50%, and there are three more cases (EV/EBIT for US service firms, net income for European firms and revenue for European service firms) when an equity level of 46–50% is attached to the best relative value of the given indicator. In all other cases, the ‘optimal’ equity proportion falls between 31% and 45%, and most frequently between 36% and 40%. Therefore, the basis of our hypothesis is that for the best-performing companies the proportion of equity within the capital structure converges to one minus the golden mean, that is, 38.2%

Using the above findings, we define the ‘golden ratio-based’ capital structure as one consisting of 38.2% equity and 61.8% debt. To check if the conclusions drawn from the above descriptive statistics for the total sample are valid, and if there are any significant differences between US and European, or manufacturing and servicing firms, regression analysis is used in the next sections.

### 3.3. Independent and dependent variables

In our regression models, the independent variable is defined as the shareholders’ equity to total assets ratio’s absolute deviation from the golden ratio-based 0.382 (which is one minus the invert of phi):

$$SHE / TA\_d_{i,t} = \left| \frac{SHE_{i,t}}{TA_{i,t}} - 0.382 \right| \tag{1}$$

It is important to note that by computing the absolute deviation, the ratio will only measure the difference from the 0.382 value, while it does not indicate the direction of the deviation. This is in line with our hypothesis, which states that either a lower or a higher equity ratio leads to a weaker financial performance and/or market acceptance.

We investigate the effect of this variable on four dependent variables, which express the absolute deviation of the already mentioned financial performance (revenue, income) and market acceptance (stock price, market value) indicators from their historical maximum (the highest value realized between 2010 and 2019 by the given company). These dependent variables are defined as follows:

Dependent variable 1: Deviation of actual total revenues from the ten-year maximum:

$$TR\_d_{i,t} = |TR_{i,t} - Max(TR_i)| \tag{2}$$

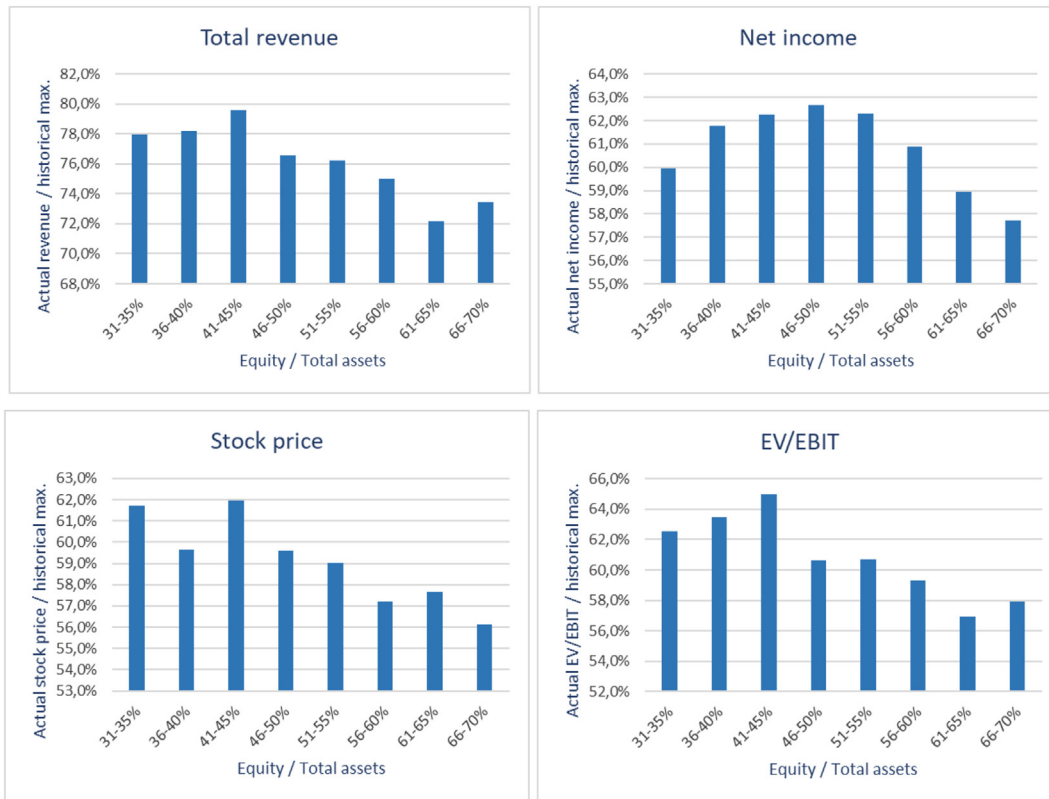
Dependent variable 2: Deviation of actual net income from the ten-year maximum:

$$NI\_d_{i,t} = |NI_{i,t} - Max(NI_i)| \tag{3}$$

Dependent variable 3: Deviation of the actual stock price from the ten-year maximum:

$$P\_d_{i,t} = |P_{i,t} - Max(P_i)| \tag{4}$$





**Figure 1.** Average percentage of firms' actual revenue, net income, stock price and EV/EBIT data related to their historical maximum at different capital structures. This figure expresses the average ratios between actual total revenue, net income, stock price and Enterprise Value to Earnings Before Interest and Tax (EV/EBIT) values and their highest values in the 2010–2019 period, expressed in percentages and shown at Equity levels from 31% to 70% with 5 percentage point increments.

**Table 4. Equity level categories of the highest actual value/historical maximum ratios for the revenue, income, price, and value indicators on subsamples based on regions and company types.** The table shows the equity level categories (5 percentage point intervals of Equity to Total assets ratio) in different subsamples in which the examined indicators (revenue, net income, stock price, EV/EBIT) are closest to their historical maximums. The percentages written with *italics* highlight the few exceptional cases when the highest values of the given indicator are attached to an equity level outside the general 31–45% interval.

Indicator	Equity level category of the highest average actual value/historical maximum ratio					
	US			Europe		
	all firms	manufacturing	service	all firms	manufacturing	service
Revenue	36–40%	31–35%	36–40%	41–45%	41–45%	<i>46–50%</i>
Net income	36–40%	41–45%	36–40%	<i>46–50%</i>	51–55%	36–40%
Stock price	41–45%	31–35%	36–40%	41–45%	51–55%	36–40%
EV/EBIT	41–45%	41–45%	<i>46–50%</i>	36–40%	36–40%	36–40%

Dependent variable 4: Deviation of the actual EV/EBIT multiple from the ten-year maximum:

$$EV / EBIT_{i,t} \cdot d_{i,t} = |EV / EBIT_{i,t} - Max(EV / EBIT_i)| \tag{5}$$

Similar to the independent variable, the four dependent variables will have only positive values that indicate how far the actual company-year's performance is from the highest performance of the 2010–2019 period.

**3.4. Model development**

Our concept is that, if the assumption about the positive effect of a golden ratio-based capital structure on financial performance and/or market acceptance is valid, then there should be a significant positive relationship between our dependent variable and one or more dependent variables. To obtain insight into each of the possible relationships, four regression models were developed, which we call *Revenue*, *Income*, *Price*, and *Value* models. In case of the latter, the word ‘value’ refers to Enterprise Value (EV), which is generally defined as the aggregate market

value of equity and debt, computed as the sum of market capitalization and the market value of debt. Based on the above, the four models tested are the following:

$$Revenue \text{ model: } TR_{i,t} \cdot d_{i,t} = \alpha_i + \beta \times SHE/TA_{i,t} \cdot d_{i,t} + u_{i,t} \tag{6}$$

$$Income \text{ model: } NI_{i,t} \cdot d_{i,t} = \alpha_i + \beta \times SHE/TA_{i,t} \cdot d_{i,t} + u_{i,t} \tag{7}$$

$$Price \text{ model: } P_{i,t} \cdot d_{i,t} = \alpha_i + \beta \times SHE/TA_{i,t} \cdot d_{i,t} + u_{i,t} \tag{8}$$

$$Value \text{ model: } EV / EBIT_{i,t} \cdot d_{i,t} = \alpha_i + \beta \times SHE/TA_{i,t} \cdot d_{i,t} + u_{i,t} \tag{9}$$

The *gretl* software was used for the statistical analysis, by setting a significance level of 5%. In order to select the appropriate methodology, the three widely known panel models were considered, which are the pooled OLS (ordinary least squares), the fixed effect and the random effect models. Pooled OLS could be the appropriate method if there are no unique attributes of individuals and no time-effect is present in the

sample, which in our case would mean that there is no unique effect for particular companies or years. On the other hand, the fixed effect model is the efficient method if there are unique effects for companies, but these do not vary over time. Finally, if there are unique company-attributes which vary over time, then the random effect model is the best choice. To make the right selection between the three models, panel diagnostics were conducted on the models. First, in case of each model, the F-test and the Breusch-Pagan test were conducted, both of which counted against the null hypothesis (with  $p < 0.05$ ) that the pooled OLS is adequate, in favor of the fixed effect and the random effect alternative, respectively. Then a Hausmann test was conducted for each model, which (with  $p < 0.05$ ) lead to the rejection of the null hypothesis that the random effect model is adequate. Therefore, we concluded that the fixed effect model is the adequate choice to test the four regression models.

#### 4. Results and discussion

First, our four models were tested on the total sample, which includes all companies (US and Europe, manufacturing, and service). Our findings are present in Table 5.

Results confirm the relationships shown earlier in Figure 1. In the case of Revenue, Price and Value models, a significant positive relationship was found between the Equity to Total Assets ratio's absolute deviation from the golden ratio-based 38.2% proportion, and the deviations of the firms' realized revenue, stock price and value data from their historical maximum. In other words, in those company-years when the firms' equity ratio was closer to the golden ratio-based 38.2% level, their revenue, stock price and value was closer to the potential maximum. This indicates that the golden ratio-based capital structure may boost the companies' revenue-generating ability and their market acceptance. In the case of the Income model, this relationship could not be confirmed, which is consistent with the findings presented in Figure 1, indicating that the highest relative income is realized by companies with a slightly higher equity ratio.

To confirm the robustness of these results, control tests were made, in which two alternative versions of the independent variable were applied. While the original  $SHE/TA_{i,t}$  variable measured the Equity to Total Assets ratio's deviation from 0.382, our alternative  $SHE/TA_{i,t}^{0.5}$  and  $SHE/TA_{i,t}^{0.618}$  variables express the deviation of the ratio observed for company  $i$  in year  $t$  from 0.5 (assuming a balanced capital structure of 50-50% of equity and debt) and 0.618 (assuming a reverse golden ratio-based capital structure, containing 61.8% equity and 38.2% debt). The same tests were then run as those presented in Table 5, and examined the changes in p-values, the explanatory powers ( $R^2$ ) and the coefficients ( $\beta$ ). Our conclusion is that in case of the Revenue, Price and Value models, using the alternative versions of the independent variable, the p value generally increased, while R squared decreased. This is a confirmation of our earlier statement that the highest level of revenue, stock price and market acceptance can be observed in those company-years when the firm's equity to total assets ratio is closest to the 38.2% level. However, for the Income model, the modified  $SHE/TA_{i,t}^{0.5}$  variable led to a lower p-value and a higher explanatory power compared to the original model, which indicates that net income was closer to its historical maximum for

companies that had a capital structure converging to a balanced 50-50% of equity and debt. Beyond p-values and  $R^2$ , this phenomenon may be better illustrated by the independent variable's coefficients, as shown by Figure 2. By checking the definition of our variables again, it becomes clear that the optimal capital structure is the one where the coefficient of the SHE/TA variable reaches its highest positive value, as the deviation from the given capital structure leads to the biggest fallback compared to the historical best performance of the firm. A negative coefficient indicates that it is advantageous for the company to deviate from the given capital structure. Examining the figure, it is evident that for the Revenue, Price and Value models, the highest coefficients of the independent variable are attached to the originally defined golden ratio-based capital structure (equity 38.2%, debt 61.8%), while for the Income model, SHE/TA has the highest coefficient in case of the balanced capital structure (50-50% of equity and debt). The results of these control tests confirm the robustness of our results presented in Table 5.

The conclusions drawn to this point are valid for the entire sample (which includes all US and European, and both manufacturing and service companies). To discover if there are any geographical and profile-related differences, testing of our models continued on manufacturing and service subsamples of US and European firms.

First, by dividing the total sample based on the region of headquarters (US versus Europe), results presented in Table 6 were obtained.

The statistical results shown in the table reveal that there are notable differences in the relationship between US and European firms. In case of companies located in the US, all models (Revenue, Income, Price, Value) are significant, with a positive coefficient for the independent variable, and with remarkable explanatory powers ranging between 24% and 41%. This indicates that in the US financial markets, a capital structure in which equity represents a proportion close to the golden ratio-based 38.2% (and as a consequence, the debt represents a proportion of close to 61.8%) provides a clear advantage for the firms in terms of financial performance (revenue- and profit-generating ability), and market acceptance (stock price and enterprise value), compared to other companies that have different proportions of equity and debt in their capital structure. By doing the same examination on European firms only, remarkably different results are obtained. On this subsample, results indicate that at the 5% significant level set, only the Revenue and the Value models are significant with positive coefficients of the independent variables. Furthermore, the explanatory powers of these models are lower than in the case of US firms. As for income and stock price, the assumed relationship could not be proven statistically on the European subsample. These findings may be an indication of differences between US and European financial markets in investor attitude and in financing strategies chosen by firms.

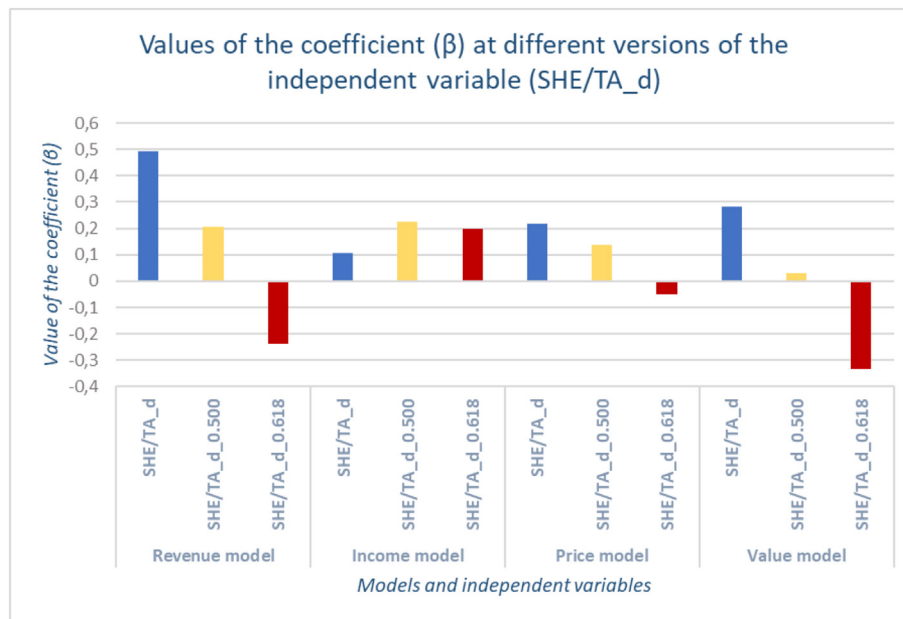
The second specific investigation concentrated on different company profiles, in addition to the regional division of the total sample. In this step our models were tested separately on manufacturing and service firms within the regionally split subsamples. Table 7 presents the testing results obtained for US manufacturing and US service companies.

The numbers indicate that, regarding the positive impact of a golden ratio-based capital structure on financial performance and market acceptance, there is no significant difference between manufacturing and service companies within the US, as all the models are significant for both

**Table 5. Results on the total sample.** This table summarizes the regression result obtained for the four models (Revenue, Income, Price, and Value) on the total sample containing all US and European manufacturing and service companies. Beyond the dependent and independent variable, the coefficient of the independent variable, the standard error, the t and p values and the explanatory power (R squared) are presented. ‘\*’ denotes that the model is significant at 5% level ( $p < 0.05$ ).

Sample: all companies (n = 455), 2010–2019

Model	Dependent variable	Independent variable	Coefficient ( $\beta$ )	Std. error	t-ratio	p-value	$R^2$
Revenue	$TR_{i,t}$	$SHE/TA_{i,t}$	0.4924	0.0402	12.230	0.0000*	0.3648
Income	$NI_{i,t}$	$SHE/TA_{i,t}$	0.1049	0.0593	1.768	0.0771	0.2258
Price	$P_{i,t}$	$SHE/TA_{i,t}$	0.2159	0.0571	3.782	0.0002*	0.2271
Value	$EV/EBIT_{i,t}$	$SHE/TA_{i,t}$	0.2825	0.0520	5.437	0.0000*	0.3769



**Figure 2.** Values of the  $\beta$  coefficients in case of the different versions of the independent variable. This figure summarizes how the independent variable's coefficient ( $\beta$ ) changes using the alternative versions of the independent variable ( $SHE/TA_{d,t}^{0.5}$  and  $SHE/TA_{d,t}^{0.618}$ ) compared to the original models in case of the four models (Revenue, Income, Price, and Value).

**Table 6. Results on the USA and Europe subsamples.** The table presents the regression result obtained for the Revenue, Income, Price and Value models on the US and Europe subsamples. In each line, the dependent and independent variable, the coefficient of the independent variable, the standard error, the  $t$  and  $p$  values and the explanatory power ( $R$  squared) are presented. **\*\*** denotes that the model is significant at 5% level ( $p < 0.05$ ).

*Subsample: US companies (n = 160), 2010–2019*

Model	Dependent variable	Independent variable	Coefficient ( $\beta$ )	Std. error	t-ratio	p-value	$R^2$
Revenue	$TR_{d,t}$	$SHE/TA_{d,t}$	0.8054	0.0633	12.730	0.0000*	0.3998
Income	$NI_{d,t}$	$SHE/TA_{d,t}$	0.3780	0.0884	4.275	0.0000*	0.2451
Price	$P_{d,t}$	$SHE/TA_{d,t}$	0.6083	0.0852	7.141	0.0000*	0.2354
Value	$EV/EBIT_{d,t}$	$SHE/TA_{d,t}$	0.4154	0.0752	5.522	0.0000*	0.4147

*Subsample: European companies (n = 295), 2010–2019*

Model	Dependent variable	Independent variable	Coefficient ( $\beta$ )	Std. error	t-ratio	p-value	$R^2$
Revenue	$TR_{d,t}$	$SHE/TA_{d,t}$	0.2383	0.0521	4.574	0.0000*	0.3493
Income	$NI_{d,t}$	$SHE/TA_{d,t}$	-0.1169	0.0797	-1.467	0.1424	0.2191
Price	$P_{d,t}$	$SHE/TA_{d,t}$	-0.1027	0.0763	-1.345	0.1786	0.2334
Value	$EV/EBIT_{d,t}$	$SHE/TA_{d,t}$	0.1746	0.0710	2.458	0.0140*	0.3563

**Table 7. Results on the US manufacturing and US service subsamples.** The table presents the regression result obtained for the Revenue, Income, Price and Value models on the US manufacturing and US service subsamples. In each line, the dependent and independent variable, the coefficient of the independent variable, the standard error, the  $t$  and  $p$  values and the explanatory power ( $R$  squared) are presented. **\*\*** denotes that the model is significant at 5% level ( $p < 0.05$ ).

*Subsample: US manufacturing companies (n = 120), 2010–2019*

Model	Dependent variable	Independent variable	Coefficient ( $\beta$ )	Std. error	t-ratio	p-value	$R^2$
Revenue	$TR_{d,t}$	$SHE/TA_{d,t}$	0.6924	0.0731	9.476	0.0000*	0.3962
Income	$NI_{d,t}$	$SHE/TA_{d,t}$	0.2499	0.1052	2.376	0.0177*	0.2451
Price	$P_{d,t}$	$SHE/TA_{d,t}$	0.4543	0.0992	4.581	0.0000*	0.2191
Value	$EV/EBIT_{d,t}$	$SHE/TA_{d,t}$	0.3811	0.0885	4.308	0.0000*	0.4170

*Subsample: US service companies (n = 40), 2010–2019*

Model	Dependent variable	Independent variable	Coefficient ( $\beta$ )	Std. error	t-ratio	p-value	$R^2$
Revenue	$TR_{d,t}$	$SHE/TA_{d,t}$	1.1306	0.1252	9.031	0.0000*	0.4194
Income	$NI_{d,t}$	$SHE/TA_{d,t}$	0.7466	0.1592	4.691	0.0000*	0.2596
Price	$P_{d,t}$	$SHE/TA_{d,t}$	1.0514	0.1643	6.397	0.0000*	0.3001
Value	$EV/EBIT_{d,t}$	$SHE/TA_{d,t}$	0.5137	0.1422	3.613	0.0003*	0.4076

**Table 8. Results on the Europe manufacturing and Europe service subsamples.** This table presents the regression result obtained for the Revenue, Income, Price and Value models on the European manufacturing and European service subsamples. In each line, the dependent and independent variable, the coefficient of the independent variable, the standard error, the t and p values and the explanatory power (R squared) are presented. ‘\*\*’ denotes that the model is significant at 5% level (p < 0.05).

*Subsample: European manufacturing companies (n = 229), 2010–2019*

Model	Dependent variable	Independent variable	Coefficient ( $\beta$ )	Std. error	t-ratio	p-value	R <sup>2</sup>
Revenue	TR <sub>d<sub>it</sub></sub>	SHE/TA <sub>d<sub>it</sub></sub>	0.2120	0.0555	3.821	0.0001*	0.3217
Income	NI <sub>d<sub>it</sub></sub>	SHE/TA <sub>d<sub>it</sub></sub>	-0.2759	0.0869	-3.174	0.0015*	0.2302
Price	P <sub>d<sub>it</sub></sub>	SHE/TA <sub>d<sub>it</sub></sub>	-0.2530	0.0833	-3.036	0.0024*	0.2314
Value	EV/EBIT <sub>d<sub>it</sub></sub>	SHE/TA <sub>d<sub>it</sub></sub>	0.1561	0.0775	2.014	0.0441*	0.3681

*Subsample: European service companies (n = 66), 2010–2019*

Model	Dependent variable	Independent variable	Coefficient ( $\beta$ )	Std. error	t-ratio	p-value	R <sup>2</sup>
Revenue	TR <sub>d<sub>it</sub></sub>	SHE/TA <sub>d<sub>it</sub></sub>	0.3504	0.1353	2.590	0.0098*	0.3785
Income	NI <sub>d<sub>it</sub></sub>	SHE/TA <sub>d<sub>it</sub></sub>	0.5618	0.1913	2.936	0.0034*	0.2028
Price	P <sub>d<sub>it</sub></sub>	SHE/TA <sub>d<sub>it</sub></sub>	0.5387	0.1831	2.941	0.0034*	0.2468
Value	EV/EBIT <sub>d<sub>it</sub></sub>	SHE/TA <sub>d<sub>it</sub></sub>	0.2533	0.1726	1.467	0.1429	0.3185

**Table 9. Summary of the results.** This table presents a summary of regression results obtained for the Revenue, Income, Price and Value models on the total sample and on subsamples selected based on region (US, Europe) and company profile (manufacturing, service). The grey background highlights the cases when the assumed positive relationship between the golden ratio-based capital structure and the financial performance or market acceptance indicator was statistically proven, while cells with white background refer to cases when the model was not significant at 5% or the relationship found was negative.

Sample Model	Total sample	US			Europe		
		All	Manufacturing	Service	All	Manufacturing	Service
Revenue	significant positive (R <sup>2</sup> =0.3648)	significant positive (R <sup>2</sup> =0.3998)	significant positive (R <sup>2</sup> =0.3962)	significant positive (R <sup>2</sup> =0.4194)	significant positive (R <sup>2</sup> =0.3493)	significant positive (R <sup>2</sup> =0.3217)	significant positive (R <sup>2</sup> =0.3785)
Income	not significant	significant positive (R <sup>2</sup> =0.2451)	significant positive (R <sup>2</sup> =0.2451)	significant positive (R <sup>2</sup> =0.2596)	not significant	significant negative (R <sup>2</sup> =0.2302)	significant positive (R <sup>2</sup> =0.2028)
Price	significant positive (R <sup>2</sup> =0.2271)	significant positive (R <sup>2</sup> =0.2354)	significant positive (R <sup>2</sup> =0.2191)	significant positive (R <sup>2</sup> =0.3001)	not significant	significant negative (R <sup>2</sup> =0.2314)	significant positive (R <sup>2</sup> =0.2468)
Value	significant positive (R <sup>2</sup> =0.2972)	significant positive (R <sup>2</sup> =0.4147)	significant positive (R <sup>2</sup> =0.3161)	significant positive (R <sup>2</sup> =0.3408)	significant positive (R <sup>2</sup> =0.3563)	significant positive (R <sup>2</sup> =0.3681)	not significant

company profiles. Coefficients (that are positive in each case) as well as the explanatory powers are somewhat higher in case of service firms, which means that the proven relationships are even stronger for companies with a service profile.

In the last step, regression models were run separately on the manufacturing and service firm's data within the European subsample, the results of which are listed in Table 8.

It is evident that in contrast to the unambiguous results obtained for US firms, the European subsample's results are much more mixed. For the European manufacturing companies, all models turn significant, however with lower explanatory powers, and with negative coefficients for the independent variables in the case of the Income and Price models. This indicates that for this region and company profile, a golden ratio-based equity proportion is not advantageous and makes an adverse effect on the firm's net income and stock price, which reach their historical maximums at higher percentages of equity. This is in line with the descriptive statistics seen earlier in Table 4. Finally, for European service companies the assumed positive relationship can be statistically proven for all models except for the Value model, although with remarkably lower R squared values compared to the US service companies' subsample.

In Table 9, the empirical results obtained on the total sample and on different subsamples are summarized.

For the interpretation of our results, it must be noted again that the examination was conducted based on the hypothesis following the traditional thinking, meaning that the decisions on capital structure determine the firm's performance and value, while we avoided the assumption of a reverse causality represented by Margaritis and Psillaki (2010). Our empirical investigation adds a new aspect to the question of

how an increasing leverage may affect the firm's performance. Results obtained indicate that there is no obvious positive (as stated by Arbor (2005), Adair and Adaskou (2015) and Jouida (2018)) or negative (found by Gleason et al. (2000), Majumdar and Chhibber (1999), Qayyum and Noreen (2019), Le and Phan (2017) and Vo and Ellis (2017)) effect of a higher debt ratio on financial performance and value. Instead, this effect may be both positive or negative, in line with the findings of Bandyopadhyay and Barua (2016) or Jaisinghani and Kanjilal (2017). We found that there is a turning point at the golden ratio-based debt to total assets ratio of 61.8%. Up to this level, increasing the leverage is advantageous for the company, while a debt ratio beyond this level influences the financial performance and market acceptance negatively. In other words, our research confirms the idea of the existence of an optimal capital structure, in line with Fruhan et al. (1992) or Damodaran (1994). However, while Fruhan et al. and Damodaran concluded that a 30% debt/equity ratio is optimal, we found that (although with differences between regions and company profiles) this optimum is generally at a debt to total assets ratio of 61.8%, according to the golden ratio. Our results might be considered by financial managers as an important new aspect when working out the firms' financing strategies and policies.

### 5. Conclusions

The golden ratio (1.618, also named as the divine proportion or phi) is present in almost all fields of life and science, including economics and business. The business-related applications found in the literature mainly focus on technical analysis, and much less attention has been



given to the use of this ratio for fundamental analysis of companies. This study investigated whether considering the golden ratio in capital structure decisions can make any positive impact on a firm's financial performance (revenue and profit-generating ability) and/or market acceptance (that is, the investor's opinion about the firm in this context, which is manifested in stock prices and the market value of the company).

Using phi, the golden ratio-based capital structure was defined as one consisting of 38.2% (0.618/1.618) equity and 61.8% (1/1.618) debt. Our empirical research was based on the detailed examination of a 10-year data series of 455 firms headquartered in the United States and Europe, and with manufacturing or service profiles. Testing four regression models (Revenue, Income, Price and Value) remarkable differences were discovered between the two regions and also between the two company types. On one hand, we obtained mixed and sometimes contradictory results in the case of European companies. A positive effect of a golden ratio-based capital structure was only partly proven (not for all models and rather for service companies). Furthermore, in some cases (such as for the Income and Price models for manufacturing companies) the relationship turned out to be negative, showing that the highest values of these indicators are realized at different capital structures. On the other hand, results are unambiguous in the US subsample, where for both manufacturing and service firms clear statistical evidence found that the financial performance and market acceptance indicators are better in those company-years when the equity to total assets proportion is closer to the phi-based level of 38.2%.

In sum, these findings confirm that, similarly to all other areas of life, the order of nature is present in both company and investor decisions, too. Although the phenomenon is more observable in the US than in Europe, and seems to be stronger for service firms than for manufacturing companies, the general conclusion can be drawn that financial managers should apply the golden ratio in their capital structure decisions, as this may give a boost to the firm's financial performance and market acceptance.

## Declarations

### Author contribution statement

József ULBERT: Conceived and designed the experiments.

András TAKÁCS: Performed the experiments; Analyzed and interpreted the data; Wrote the paper.

Vivien CSAPI: Contributed reagents, materials, analysis tools or data; Wrote the paper.

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### Data availability statement

Data will be made available on request.

### Declaration of interests statement

The authors declare no conflict of interest.

### Additional information

No additional information is available for this paper.

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