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Cash-flow business taxation revisited: bankruptcy and asymmetric information

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Abstract
We study the effects of cash-flow taxation on firms' entry and investment decisions when there is bankruptcy risk and when banks face asymmetric information problems in financing heterogeneous firms. When there is moral hazard, firms underinvest, while with adverse selection too many firms enter. Cash-flow taxation applying to both real and financial cash flows corrects these distortions by inducing more investment in rent-generating projects where moral hazard exists and reducing firm entry under adverse selection. Our results in the moral hazard case depend on the tax losses of bankrupt firms accruing to the banks. If bankrupt firms retain tax losses, the cash-flow corporate tax is neutral as in Bond and Devereux (J Public Econ 87:1291–1311, 2003).

Keywords Cash-flow tax · Bankruptcy · Asymmetric information

JEL Classification H21 · H25

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1 Introduction

A classic result in the design of business taxes due to Brown (1948) concerns the neutrality of cash-flow taxation. Investment decisions undertaken in a world of full certainty will be unaffected by a tax imposed on firms' cash flows, assuming there is full loss-offsetting and the tax rate is constant (See Sandmo, 1979). In effect, a cash-flow tax will divert a share of the pure profits or rents from the firm's owners to the government. This is of obvious policy interest since it represents a non-distorting source of tax revenue.

In this paper, we revisit the use of cash-flow taxation as a rent-collecting device when firms face asymmetries of information in capital markets. We do so in a partial equilibrium model of risk-neutral entrepreneurs who vary in their productivity, so that returns to inframarginal entrepreneurs generate rents. Entrepreneurs decide both whether to enter an industry—an extensive-margin decision—and if they enter how much to invest—an intensive-margin decision. Investment outcomes are uncertain and entrepreneurs face the possibility of bankruptcy. Banks may face two types of asymmetric information in financing firms. In the first, involving moral hazard, they can only verify that firms are bankrupt by engaging in costly monitoring. In this case, entry by firms is efficient but investment is inefficiently low compared with the full-information case. In the second, involving adverse selection, banks cannot observe entrepreneurs' types. In this case, too many firms enter so there is too much investment relative to the social optimum.

We study the effects of cash-flow taxation on both the entry decision of potential entrepreneurs and the decision as to how much to borrow and invest. We assume the cash-flow tax is of what Meade (1978) referred to as the R+F type, so it is levied on real and financial cash flows and applies to both entrepreneurs and banks. Importantly, in the event of bankruptcy, the cash flows of bankrupt firms accrue to the creditor banks, and this includes tax losses which we assume are carried forward with interest and are refundable. Unlike in the full-information case, R+F cash-flow taxation is not neutral. In the moral hazard case, the tax does not affect entry, but its effect on intensive-margin decisions turns out to be especially important. It encourages rent-generating investment and improves efficiency thereby acting as a corrective device in a distorted environment. In the adverse selection case, cash-flow taxation reduces entry. Efficiency improves until the tax rate reaches the level where the number of firms entering is optimal. Thus, in either case, the R+F cash-flow tax not only taxes the rents of firms but enhances economic efficiency.

The Brown neutrality result inspired a sizable literature on neutral business tax design, much of which generalized his result to taxes that are equivalent to cash-flow taxes in present-value terms. Boadway and Bruce (1984) show that the cash-flow tax is a special case of a more general class of neutral business taxes that have the property that the present value of deductions for future capital costs (interest plus depreciation) arising from any investment just equals initial investment expenditures. A special case of this is the Capital Account Allowance (CAA) tax in which investment expenditures are added to a capital account each year, and each year, the capital account is depreciated at a rate specified for tax purposes. Annual deductions for capital costs then consist of the sum of the cost of capital plus the depreciation rate multiplied by the book value of the capital account.

More generally, neutrality can be achieved by a business tax in which the present value of future tax bases just equals the present value of cash flows. An example of a cash-flow equivalent tax system of this sort is the Resource Rent Tax (RRT) proposed by Garnaut and Clunies-Ross (1975) for the taxation of non-renewable natural resources. In their version, firms starting out are allowed to accumulate negative cash flows in an account that rises each year with the cost of capital. Once the account becomes positive, cash flows are taxed as they occur. Negative cash flows are carried forward at the interest rate to achieve the equivalent of cash-flow taxation.

These basic results continue to apply if returns to investment are uncertain and capital markets are complete. Fane (1987) considers the case where firms' owners are risk-neutral and shows that neutrality holds under uncertainty as long as tax credits and liabilities are carried forward at the risk-free nominal interest rate, and tax credits and liabilities are eventually redeemed. Bond and Devereux (1995) show that even if firms are risk-averse, the CAA tax remains neutral in the presence of uncertainty and the possibility of bankruptcy provided that a risk-free interest rate applied to the value of the capital account is used for the cost of capital deduction, that any unused negative tax credits are refunded to the firm in the event of bankruptcy, and that the valuation of risky assets satisfy the value-additivity principle.[1] The use of a risk-free discount rate reflects the assumption that there is no risk to the firm associated with postponing capital deductions into the future (i.e., no political risk). Boadway and Keen (2015) show that the same neutrality result applies to the RRT in the presence of uncertainty.

The above results focused on taxes applied to real cash flows or their equivalent, what Meade (1978) called R-base cash-flow taxation. Bond and Devereux (2003) show that neutrality can be achieved using the more general case of R+F cash-flow taxation in which both real and financial cash flows are included in the base. In their model, the size of investment by firms is fixed and the focus is on entry and exit, or extensive-margin, decisions. They also study the neutrality of the Allowance for Corporate Equity (ACE) tax, which is a version of the CAA tax that allows actual interest deductions alongside a cost of capital deduction for equity-financed investment. Notably, Bond and Devereux assume that, while tax losses are refundable, they are refunded to the firms' owners rather than the banks in the event of bankruptcy. Under this assumption, their analysis shows that cash-flow taxes reduce the value of the firm by one minus the tax rate, so firms' entry and exit decisions are not affected. The relation between our results and those of Bond and Devereux is explained later. For now, note that cash-flow taxation does not play a corrective role in their model since investment is fixed so there is no scope for inefficiency on moral hazard grounds.

[1] The value-additivity principle implies that the present value of the sum of stochastic future payoffs is equal to the sum of the present values of these payoffs and is consistent with a no-arbitrage principle in the valuation of assets. Value-additivity automatically applies for risk-neutral investors.

These neutrality results have inspired various well-known policy proposals, some of which have been implemented. A cash-flow business tax was recommended by the United States Treasury (1977), Meade (1978) in the UK and the President's Advisory Panel on Federal Tax Reform (2005) in the USA. The latter two both recommended separate cash-flow taxation on financial institutions. The Australian Treasury (2010) (the Henry Report) recommended an RRT for the mining industries in Australia. Several bodies have recommended an ACE corporate tax system, including the Institute for Fiscal Studies (1991), Mirrlees et al. (2011) and Institut d'Economia de Barcelona (2013). ACE taxes have been deployed in a few countries, including Brazil, Italy, Croatia and Belgium. Reviews of their use may be found in Klemm (2007), de Mooij (2011), Panteghini et al. (2012) and Princen (2012). Cash-flow-type taxes with full loss-offset are used in the Norwegian offshore petroleum industry (Lund 2014), and the RRT was applied temporarily in the Australian mining industry.

Our analysis is also related to other parts of the literature on business taxation and investment under asymmetric information. For example, Keuschnigg and Nielsen (2004a, b) examine the impact of business taxation on risky investment in a setting with two-sided moral hazard between entrepreneurs and venture capitalists. Their analysis shows that capital gains taxation tends to weaken the incentives of entrepreneurs and venture capitalists to contribute effort to joint projects under incomplete contracts. Keuschnigg and Ribi (2013) also consider active financial intermediaries who, in addition to providing funding, offer advice to entrepreneurs in an incomplete contracts setting. They show that both a cash-flow tax and an ACE are non-neutral as they tend to discourage advising effort and therefore reduce probabilities of success of risky investments. Hagen and Sannarnes (2007) consider the impact of profit taxation on the allocation of risk associated with investment projects financed by external equity investors and show that the neutrality of a profit tax no longer holds. A profit tax affects the allocation of risk, and in turn, distorts effort. Finally, Koethenbuerger and Stimmelmayr (2014) examine corporate taxation in the presence of agency problems between the managers and shareholders of firms. Their analysis shows that when managers can undertake both productive and unproductive investments, the deductibility of investment costs may reduce welfare, hence distorting the neutrality of the corporate tax.

Our main results are as follows. With investment undertaken by entrepreneurs of differing productivity and the possibility of bankruptcy, the (R+F)-base cash-flow tax is neutral in the absence of asymmetric information as in Bond and Devereux (1995, 2003). But, the cash-flow tax distorts investment decisions if banks must incur monitoring costs when firms declare bankruptcy, that is, banks face moral hazard. Remarkably, the cash-flow tax increases investment while leaving bankruptcy risk and firms' expected profits unchanged. Expected rents, government expected revenues and social surplus all increase with the tax rate. In effect, as well as raising revenue, the cash-flow tax serves as a corrective device for the market failure arising from moral hazard. As mentioned, these results depend on the assumption that in the event of bankruptcy, cumulated tax losses are refunded to the firms' creditors. If, as in Bond and Devereux (2003), they are refunded to the firms' owners, the tax is neutral so affects neither entry nor investment. In that sense, it is optimal to refund tax losses of bankrupt firms to the banks rather than to the firms. Alternatively, if the productivity of entrepreneurs is private information, so there is adverse selection, there tends to be too much investment in the market equilibrium. In this case, the cash-flow tax discourages entry in the risky industry and is efficiency-enhancing up to the point where the tax rate is optimal.

We begin by outlining the main elements of the model when banks cannot freely observe investment outcomes so face moral hazard, but they can observe firm types so there is no adverse selection. We show that under-investment occurs in the absence of taxes. We then study the effects of cash-flow taxation on the entry and investment decisions of entrepreneurs. Finally, we consider the effect of cash-flow taxation when there is adverse selection but no moral hazard, so the banks can observe investment outcomes but the productivity of entrepreneurs is private information.

2 The case with moral hazard

2.1 A model with asymmetric information about investment outcomes

Our model with moral hazard is designed to capture the following key features. Firms' investments are heterogeneous such that inframarginal investments generate pure profits or rents. Profit taxation taxes these rents, and in a first-best world, cash-flow profits taxation would do so in a non-distorting way. Our moral hazard case departs from the first best by assuming that firms, who rely on the banks to finance their investments, face the possibility of bankruptcy, but that banks cannot freely observe the profits of firms that declare bankruptcy. Banks can learn these profits at a cost by ex post monitoring or verification. The consequence is that relative to the full-information setting, there are too few loans. In this context, cash-flow taxation encourages lending and improves social efficiency.

We assume that each firm is operated by a risk-neutral entrepreneur. There is a population of potential entrepreneurs with identical endowments of wealth who can undertake an investment project. They differ in the productivity of their projects. We simplify our analysis by assuming there is a single period so we can suppress the entrepreneurs' consumption-savings decisions and focus on production decisions. At the beginning of the period, potential entrepreneurs decide whether to enter a risky industry and invest their wealth in a risky project. Those who do not enter invest their wealth in a risk-free asset and consume the proceeds at the end of the period. Entrepreneurs who enter the risky industry choose how much to borrow to leverage their own equity investment, which determines their capital stock. After investment has been undertaken, risk is resolved. Entrepreneurs with good outcomes earn profits. Those with bad outcomes go bankrupt. Their production goes to their creditors, which are risk-neutral competitive banks.

There are thus two decisions made by potential entrepreneurs. First, they decide whether to enter, which is an extensive-margin decision; and second, they decide how much to borrow to expand their capital, which is an intensive-margin decision.

For simplicity, we suppress their labor income: all income comes from profits they earn if they enter the risky sector, or from their initial wealth if they do not. Adding labor income (as in Kanniainen and Panteghini 2012) would make no substantial difference for our result on business taxation. Our purpose is to study how cash-flow corporate taxation affects the extensive- and intensive-margin decisions of firms, and as a result the efficiency of firms' behavior.

In this moral hazard case, we assume that banks know the productivity of entrepreneurs, so can offer type-specific interest rates. The interest rate offered to a given type of entrepreneur depends on the borrowing the entrepreneur chooses. More borrowing increases the risk of bankruptcy, which in turn affects the expected profit of the lending bank. Since banks are competitive, their expected profits from loans to each type of entrepreneur will be zero in equilibrium, and this zero-profit condition determines the interest rate. Entrepreneurs know how their borrowing affects their interest rate, and that influences how much they borrow.

The details of the model with asymmetric information about investment outcomes are as follows. Later we consider the consequences of entrepreneurs' types being private information resulting in adverse selection.

2.2 Details of the moral hazard model

There is a continuum of potential risk-neutral entrepreneurs, all endowed with initial wealth E. For simplicity, we assume that the production function is linear in capital K. The average product of capital, denoted R, is constant, but differs across entrepreneurs, and is distributed over $[0, R_{\max}]$ by the distribution function $H(R)$. The value of output is subject to idiosyncratic risk, and the stochastic value of a type-R entrepreneur's output is $\tilde{\varepsilon}RK$, where $\tilde{\varepsilon}$ is distributed over $[0, \varepsilon_{\max}]$ by the distribution function $G(\tilde{\varepsilon})$, assumed to be uniform with density $g = 1/\varepsilon_{\max}$. The expected value of $\tilde{\varepsilon}$ is:[2]

$$\bar{\varepsilon} \equiv \mathbb{E}[\tilde{\varepsilon}] = \frac{\varepsilon_{\max}}{2} = \frac{1}{2g}. \tag{1}$$

We assume that the distribution of $\tilde{\varepsilon}$ is the same for all entrepreneurs, so they differ only by their productivity R. Capital is financed by the entrepreneur's own equity and debt and depreciates at the proportional rate δ per period. Entrepreneurs who do not enter invest all their wealth E in a risk-free asset with rate of return ρ, so consume $(1 + \rho)E$. Since all potential entrepreneurs have the same alternative income, those with the highest productivity as entrepreneurs will enter the entrepreneurial sector. Let $\widehat{R}$ denote the productivity of the marginal entrepreneur.

Entrepreneurs who enter invest all their wealth in the risky firm, so E is the common value of own-equity of all entrepreneurs. The type-R entrepreneur who has entered borrows an amount $B(R)$ so his aggregate capital stock is $K(R) = E + B(R)$.

[2] A stochastic variable is denoted $\tilde{x}$; its expected value is $\bar{x}$; and its cutoff value where relevant is $\hat{x}$.

Denote the leverage rate by $\phi(R)$, where $\phi(R) \equiv B(R)/K(R)$. Then, $K(R)$ can be written:

$$K(R) = \frac{E}{1 - \phi(R)}. \tag{2}$$

We assume that there is a maximum size of the capital stock, such that $K(R) \leqslant K_{\max}$, and that $E < K_{\max}$ so the entrepreneur's wealth is less than the maximum capital stock. By (2), this implies that $0 \leqslant \phi(R) \leqslant 1 - E/K_{\max} < 1$. Since we assume that all the entrepreneur's wealth is invested, the minimum level of capital for entrepreneurs who enter is E. Allowing entrepreneurs to invest only part of their wealth would complicate the analysis slightly without adding any insight.[3] The entrepreneur's capital stock is therefore in the range $K(R) \in [E, K_{\max}]$. The assumption of a maximum capital stock reflects the notion that after some point additional capital is non-productive. It is like a strong concavity assumption on the production function, which precludes extreme outcomes that would otherwise occur with linear production. In most of our analysis, entrepreneurs choose an interior solution so $K(R) \leqslant K_{\max}$ is not binding.

Since we assume in this moral hazard case that banks can identify entrepreneurs by type and set a type-specific interest rate, equilibrium analysis applies separately to entrepreneurs of each type.[4] Consider a representative type-R entrepreneur and drop the identifier R from most functions for simplicity. After the shock $\tilde{\varepsilon}$ is revealed, the entrepreneur's ex post after-tax profits (or return to own-equity) evaluated at the end of the period are given by:

$$\widetilde{\Pi}(R) = \tilde{\varepsilon}RK + (1 - \delta)K - (1 + r)B - \widetilde{T} \tag{3}$$

where $\widetilde{T}$ is the tax paid and r is the interest rate, so $(1 + r)B$ is the repayment of interest and principal on the borrowing B. The term $(1 - \delta)K$ is the value of capital remaining after production, given the depreciation rate δ. We assume it is sold at its market value. The type-specific interest rate r will depend upon the leverage ϕ chosen by the entrepreneur since this affects bankruptcy risk. The manner in which ϕ affects r depends upon the behavior of the lending banks as discussed below.

We assume the government applies an (R+F)-base cash-flow tax to both the firms and the banks. As we show below, this is equivalent to ACE taxation. In the absence of market failures, these taxes are non-distorting. However, in our setting with asymmetric information and bankruptcy, cash-flow taxes can affect firms' investment

[3] If entrepreneurs were to invest part of their wealth in the safe asset, leverage would increase for any given level of investment. That would increase bankruptcy risk and the interest rate faced by the entrepreneur. If entrepreneurs face unlimited liability in the case of bankruptcy, there would be no incentive to invest less than total wealth in the risky project since the interest rate on borrowing will be higher than the rate of return on the safe asset. If there is limited liability in the case of bankruptcy, entrepreneurs may choose to hold wealth in the safe asset although that would result in higher interest costs on borrowing.

[4] If wealth differed among entrepreneurs, leverage ϕ and therefore the interest rate could vary with both R and E. This would not affect the qualitative results of our analysis.

choices and partly correct the market distortions. The effect of cash-flow taxation depends critically on how tax losses are treated in the event of bankruptcy, in particular whether they go to entrepreneurs or to the banks who receive the cash flows of bankrupt firms. Whether the entrepreneurs or the banks claim tax losses may depend on the timing of the payments. If tax losses are refundable as they occur, entrepreneurs will receive them. If, however, they are carried forward with interest and refunded when bankruptcy is declared, they are more likely to be awarded to the banks as part of bankruptcy proceedings. This will also be the case with the ACE system, which is equivalent to carrying forward tax losses with interest. In what follows, we take as our base case that in which the tax losses in the event of bankruptcy are awarded to the banks. Later we contrast this with the case where tax losses of bankrupt firms go to the firms' owners as assumed by Bond and Devereux (2003).[5]

Tax liability under cash-flow taxation, again evaluated at the end of the period after $\tilde{\varepsilon}$ is revealed, is given by:

$$\widetilde{T} = \tau\Big(\underbrace{\tilde{\varepsilon}RK - (1+\rho)K + (1-\delta)K}_{R-\text{ base}} + \underbrace{(1+\rho)B - (1+r)B}_{F-\text{ base}}\Big) \tag{4}$$

where τ is the tax rate and, as noted above, ρ is the risk-free interest rate. The real component of the cash-flow tax base (R-base) in (4) consists of three terms. The first is the revenue of the firm, $\tilde{\varepsilon}RK$. The second, $(1+\rho)K$, is the end-of-period value of the deduction for investment. Since investment K occurs at the beginning of the period, we assume that the tax savings from deducting investment are either refunded immediately or are carried over to the end of the period with interest at rate ρ. As discussed below, whether immediate refundability of tax credits or carry-forward with interest are allowed affects the efficiency properties of the tax. Our base-case approach is most consistent with tax credits being carried forward.[6] Third, the cash-flow tax is levied on selling or winding-up the depreciated value of business assets, $(1-\delta)K$, at the end of the period. Financial cash flows (F-base) include the end-of-period value of the borrowing B obtained by the firm, $(1+\rho)B$, less the principal and interest repaid, $(1+r)B$.[7]

Equation (4) applies whether $\widetilde{T}$ is positive or negative, so implicitly assumes that the tax system allows full refundability of tax losses either when they occur or at the end of the period with interest included. The tax liability $\widetilde{T}$ is incurred by the firm as long as it is not bankrupt. If the firm goes bankrupt, we assume in our base case

[5] An alternative case is where tax losses are not refunded if firms go bankrupt. In this case, a cash-flow tax would discourage entry as shown in Boadway et al. (2021).

[6] A referee notes that if tax losses are immediately refundable, the amount of bank finance an entrepreneur needs will be reduced and so will the probability of bankruptcy. This will mitigate under-investment in our model, but not eliminate it.

[7] We assume as is conventional that entrepreneurs report their income truthfully to the government. Truthful reporting is enforced by a system of random audits and penalties in the event of misreporting. This is in contrast to the banks who cannot observe the cash flows of their client entrepreneurs unless they undertake costly monitoring as discussed below.

that the bank receives the firm's cash flows and assumes liability for taxes owed or receives any tax refunds.

Using $B = E - K$, (4) may be rewritten as:

$$\widetilde{T} = \tau\Big(\tilde{\varepsilon}RK - \delta K - rB - \rho E\Big) = \widetilde{T}^{\mathrm{ACE}} \tag{5}$$

where the term in brackets is the ACE tax base. It includes revenues $\tilde{\varepsilon}RK$ less three deductions for capital at the end of the period: depreciation δK, interest rB and the cost of equity finance ρE. This verifies that the R+F and ACE bases are equivalent. Of course, the ACE does not give rise to tax losses at the beginning of the period so the issue of refundability versus carry-forward does not arise.

Using (3), (4) and $K = E + B$, ex post after-tax profits under cash-flow taxation can be written:

$$\widetilde{\Pi}(R) = (1 - \tau)\Big(\tilde{\varepsilon}RK + (1 - \delta)K - (1 + r)B\Big) + \tau(1 + \rho)E. \tag{6}$$

Entrepreneurs are confronted with bankruptcy when $\tilde{\varepsilon}$ is too low to meet debt repayment obligations, that is, when $\widetilde{\Pi}(R) < 0$. This occurs for entrepreneurs with $\tilde{\varepsilon} < \hat{\varepsilon}$, where $\hat{\varepsilon}$ (which is specific to type R) satisfies:

$$0 = (1 - \tau)\Big(\hat{\varepsilon}RK + (1 - \delta)K - (1 + r)B\Big) + \tau(1 + \rho)E. \tag{7}$$

In what follows, we refer to $\hat{\varepsilon}$ as *bankruptcy risk*. The higher the value of $\hat{\varepsilon}$, the greater the chances of the entrepreneur going bankrupt. Combining (6) and (7), we obtain:

$$\widetilde{\Pi}(R) = (1 - \tau)(\tilde{\varepsilon} - \hat{\varepsilon})RK \quad \text{for} \quad \tilde{\varepsilon} \geqslant \hat{\varepsilon}. \tag{8}$$

Note that the bankruptcy condition (7) assumes that the term $\tau(1 + \rho)E$ goes to the bank if the entrepreneur declares bankruptcy. This term represents the tax credit on start-of-period real and financial cash flows, $\tau(K - B)$, carried forward to the end of the period at the risk-free interest rate ρ.

Equation (7) determining $\hat{\varepsilon}$ can be rewritten, using $B = \phi K$ and $E = (1 - \phi)K$, as:

$$(1 - \tau)\Big(\hat{\varepsilon}R + (1 - \delta) - (1 + r)\phi\Big) + \tau(1 + \rho)(1 - \phi) = 0. \tag{9}$$

As this expression indicates, bankruptcy risk $\hat{\varepsilon}$ depends on both the leverage, ϕ, chosen by the entrepreneur and the interest rate, r. The latter is determined by a competitive banking sector as follows. Assume that banks are risk-neutral and can observe R and ϕ for each entrepreneur, but cannot observe $\tilde{\varepsilon}$ or ex post profits. Thus, there is no adverse selection since banks know entrepreneurs' types, but there is moral hazard since entrepreneurs may have an incentive to declare bankruptcy to avoid repaying the loan. Imperfection of the financial market due to asymmetric information is addressed by an ex post verification or monitoring cost in the event a firm declares bankruptcy. Following the financial accelerator model of Bernanke et al. (1999), we assume that the verification cost is proportional to ex post output so takes the form $c\tilde{\varepsilon}RK$, for $\tilde{\varepsilon} \leqslant \hat{\varepsilon}$, where c is fixed. This might reflect the fact that the verification cost includes the costs of seizing the firm's output in a default.[8] We assume for simplicity that there are no errors of monitoring. Then, only entrepreneurs with $\tilde{\varepsilon} < \hat{\varepsilon}$ will declare bankruptcy in equilibrium. The expected total monitoring cost incurred by a bank for a given type of entrepreneur will be:

$$\int_0^{\hat{\varepsilon}} c\tilde{\varepsilon}RKg\mathrm{d}\tilde{\varepsilon} = cRKg\frac{\hat{\varepsilon}^2}{2} \tag{10}$$

so the expected monitoring cost is increasing and convex with bankruptcy risk, $\hat{\varepsilon}$. This specific form of the monitoring cost is chosen for analytical convenience and is not critical for our results.

In the event of bankruptcy, the firm no longer repays its debt plus interest, $(1+r)B$, and its after-tax profits go to the bank. Using (6), these after-tax profits become:

$$\widetilde{\Pi}(R) = (1-\tau)\Big(\tilde{\varepsilon}RK + (1-\delta)K\Big) + \tau(1+\rho)E \quad \text{for} \quad \tilde{\varepsilon} < \hat{\varepsilon}. \tag{11}$$

Competition among banks ensures that expected profits earned from lending to the representative entrepreneur of each type are zero. We assume that banks will not go bankrupt, so they pay the risk-free interest rate ρ on their deposits. We also assume that banks incur no operating costs for simplicity.

The banks pay the (R+F)-base tax on their financial income less monitoring costs plus any cash flows they obtain from bankrupt firms. These cash flows include tax losses cumulated from the beginning of the period. The expected tax liability of a bank from a loan B to a given type of entrepreneur is:

$$\begin{aligned}\overline{T}_B =&\tau\Bigg(\int_{\hat{\varepsilon}}^{\varepsilon_{\max}} \Big((1+r)B - (1+\rho)B\Big)g\mathrm{d}\tilde{\varepsilon} \\ &+ \int_0^{\hat{\varepsilon}} \Big(\tilde{\varepsilon}RK + (1-\delta)K - (1+\rho)E - (1+\rho)B - c\tilde{\varepsilon}RK\Big)g\mathrm{d}\tilde{\varepsilon}\Bigg).\end{aligned} \tag{12}$$

The first term of the tax base is net financial cash flow when the loan is repaid. The second includes the net revenues from the bankrupt firms less the unclaimed tax credit owing to those firms, the deduction for the cost of repaying deposits and the cost of monitoring. Using (12), the bank's zero-expected profit condition can be written:

[8] Bernanke and Gertler (1989) introduced a fixed verification cost in a business cycle model where there is asymmetric information between lenders and borrowers about the realized return on risky projects, while Townsend (1979) explored the design of debt contracts with verification costs that could either be fixed or functions of realized project output. See also Bernanke et al. (1996) for an analysis of the implications of agency costs in lending contracts arising from asymmetric information about project outcome.

$$(1-\tau)(1+\rho)B = (1-\tau)\Bigg(\int_{\hat{\varepsilon}}^{\varepsilon_{\max}}(1+r)Bg\mathrm{d}\tilde{\varepsilon} + \int_0^{\hat{\varepsilon}}\left(\tilde{\varepsilon}RK + (1-\delta)K + \frac{\tau(1+\rho)}{1-\tau}E\right)g\mathrm{d}\tilde{\varepsilon} - cRKg\frac{\hat{\varepsilon}^2}{2}\Bigg). \tag{13}$$

This zero-profit condition, which applies for each type of entrepreneur, determines the interest rate the entrepreneur of a given type pays, given their bankruptcy risk, or equivalently, their leverage. Using (13) in (12), expected tax liabilities of the bank simplify to:

$$\overline{T}_B = -\int_0^{\hat{\varepsilon}}\frac{\tau}{1-\tau}(1+\rho)Eg\mathrm{d}\hat{\varepsilon}. \tag{14}$$

Though this is negative in expected terms, the bank's tax liabilities will be positive if the firm does not go bankrupt.

2.3 Leverage, bankruptcy risk and the interest rate

The bankruptcy condition (9) and the bank's zero-profit condition (13) jointly determine the relations among r, ϕ, $\hat{\varepsilon}$ and τ. By combining these equations, we can eliminate $(1+r)B$ and obtain a relationship among ϕ, $\hat{\varepsilon}$ and τ as shown in the following lemma.

Lemma 1 *The leverage rate* $\phi = B/K$, *for* $0 < \phi < 1 - E/K_{\max}$, *is given by:*

$$\phi(\hat{\varepsilon}, R, \tau, c) = \frac{1-\tau}{1+\rho}\left(\left(1 - \frac{g\hat{\varepsilon}}{2} - \frac{cg\hat{\varepsilon}}{2}\right)\hat{\varepsilon}R + (1-\delta)\right) + \tau \tag{15}$$

The proofs of all lemmas are given in the Appendix. Routine differentiation of (15) gives properties of $\phi(\hat{\varepsilon}, R, \tau, c)$ that are useful in what follows:

$$\phi_{\hat{\varepsilon}} = \frac{1-\tau}{1+\rho}(1 - g\hat{\varepsilon} - cg\hat{\varepsilon})R; \quad \phi_c = -\frac{1-\tau}{1+\rho}\frac{Rg\hat{\varepsilon}^2}{2}; \quad \phi_R = \frac{1-\tau}{1+\rho}\left(1 - \frac{g\hat{\varepsilon}}{2} - \frac{cg\hat{\varepsilon}}{2}\right)\hat{\varepsilon};$$
$$\phi_\tau = \frac{1-\phi}{1-\tau}; \quad \phi_{\hat{\varepsilon}\hat{\varepsilon}} = -\frac{1-\tau}{1+\rho}(1-c)Rg\hat{\varepsilon}; \quad \phi_{\hat{\varepsilon}\tau} = -\frac{1 - g\hat{\varepsilon} - cg\hat{\varepsilon}}{1+\rho}R. \tag{16}$$

Given τ, (9) and (15) represent two equations in three unknowns: ϕ, $\hat{\varepsilon}$ and r. In practice, an entrepreneur chooses leverage ϕ, and this determines both bankruptcy risk $\hat{\varepsilon}$ and the interest rate r through (9) and (15). Bankruptcy risk is increasing in leverage by (16), and, as the following lemma states, so is the interest rate. That is, an increase in leverage increases the probability of the entrepreneur going bankrupt, and that increases the interest rate that banks must charge if their zero-expected-profit condition is to be satisfied.

Lemma 2 *The interest rate r facing an entrepreneur is increasing in leverage ϕ.*

To interpret the role of (15) in our analysis, we assume that entrepreneurs understand how the leverage they choose affects the probability of bankruptcy and the interest rate they face through the bankruptcy condition (9) and the bank's zero-profit condition (13). Therefore, they know the relationship between ϕ and $\hat{\varepsilon}$ in (15), and how it implicitly takes account of the interest rate they face. In what follows, we take advantage of Lemma 1 to suppress the interest rate r from our analysis. While in practice, entrepreneurs choose leverage ϕ, it is convenient for us to assume that they choose bankruptcy risk $\hat{\varepsilon}$, which is related to leverage via (15). The choice of $\hat{\varepsilon}$ is equivalent to choosing leverage ϕ because, even though ϕ is not necessarily monotonic in $\hat{\varepsilon}$, $\phi_{\hat{\varepsilon}\hat{\varepsilon}} < 0$ by (16). We proceed by deriving an expression for expected profits as a function of $\hat{\varepsilon}$.[9]

2.4 Entrepreneurs' expected after-tax profits

Prior to $\tilde{\varepsilon}$ being revealed, the expected after-tax profits of a representative entrepreneur of a given type are $\overline{\Pi}(R) \equiv \int_{\hat{\varepsilon}}^{\varepsilon_{\max}} \widetilde{\Pi}(R) g\mathrm{d}\tilde{\varepsilon}$. (Recall that for $\tilde{\varepsilon} < \hat{\varepsilon}$, profits are claimed by the bank.) Given the expression for $\widetilde{\Pi}(R)$ in (6), this becomes:

$$\overline{\Pi}(R) = \int_{\hat{\varepsilon}}^{\varepsilon_{\max}} \Big((1-\tau)\Big(\tilde{\varepsilon}RK + (1-\delta)K - (1+r)B\Big) + \tau(1+\rho)E \Big) g\mathrm{d}\tilde{\varepsilon}. \tag{17}$$

The entrepreneur takes into account the fact that the interest rate r depends on the zero-profit condition of the bank, which in turn depends upon the bankruptcy risk or leverage he chooses. Using the bank's zero-profit condition (13) to eliminate $(1-\tau)\int_{\hat{\varepsilon}}^{\varepsilon_{\max}} (1+r)Bg\mathrm{d}\tilde{\varepsilon}$ from (17), and using (1), (2) and $B = K - E$, (17) may be written after integration as:

$$\overline{\Pi}(R) = \left(\frac{1-\tau}{1-\phi(\hat{\varepsilon}, R, \tau, c)} \left(\bar{\varepsilon}R - \delta - \rho - cgR\frac{\hat{\varepsilon}^2}{2} \right) + 1 + \rho \right) E \equiv \overline{\pi}(\hat{\varepsilon}, R, \tau, c)E \tag{18}$$

where $\overline{\pi}(\hat{\varepsilon}, R, \tau, c)$ is expected profit per unit of own equity.

Using the relationship between leverage and bankruptcy risk in (15), $\overline{\Pi}(R) = \overline{\pi}(\hat{\varepsilon}, R, \tau, c)E$ satisfies the following lemma.

[9] We could instead have used (15) to determine $\hat{\varepsilon}$ as a function of ϕ, and obtained derivatives of $\hat{\varepsilon}$ with respect to ϕ and the other variables. We could then use ϕ as the choice variable of entrepreneurs. While this would more accurately reflect entrepreneurial choices, it would make the analysis more complicated and would not change the results.

$$\overline{\Pi}(R) = \overline{\pi}(\hat{\varepsilon}, R, \tau, c)E = \frac{1-\tau}{1-\phi(\cdot)}\frac{Rg}{2}(\varepsilon_{\max} - \hat{\varepsilon})^2 E. \tag{19}$$

Lemma 3

The expression for expected profits in (19) takes into account both the endogeneity of the interest rate r facing the entrepreneur through the bank's zero-profit condition (13) and the relationship between leverage and bankruptcy risk through (15). The expected utility of entrepreneurs, and thus their objective function, is given by $\overline{\Pi}(R) = \overline{\pi}(\hat{\varepsilon}, R, \tau, c)E$ in (18) or (19). We make use of both of these versions below.

It is useful in what follows to adopt the following definition:

$$D(\hat{\varepsilon}, R, \tau, c) \equiv \frac{1-\tau}{1-\phi(\hat{\varepsilon}, T, \tau, c)}. \tag{20}$$

The function $D(\cdot)$ satisfies the following property.[10]

$$\frac{\partial D}{\partial \tau} \equiv D_\tau = 0$$

Lemma 4

Since D, and therefore $(1-\tau)/(1-\phi)$, is independent of τ, the effect of changes in $\hat{\varepsilon}$ on expected profits $\overline{\Pi}(R)$ in (18) do not depend on τ. The implication is that a change in the tax rate τ does not affect bankruptcy risk $\hat{\varepsilon}$. However, it does affect $\phi(\hat{\varepsilon}, R, \tau, c)$ as shown by (16) and therefore the amount of investment undertaken. We return to this below.

To further understand the implications of D and Lemma 4, rewrite (18) as:

$$\overline{\Pi}(R) - (1+\rho)E = (1-\tau)\left(\overline{\varepsilon}R - \delta - \rho - cgR\frac{\hat{\varepsilon}^2}{2}\right)K.$$

This is the net after-tax expected return to the entrepreneur. The right-hand side is the after-tax expected profit on K invested in the firm, where the cost of monitoring in the event of bankruptcy is borne by the entrepreneur. The right-hand side can be converted into an after-tax expected profit on equity using (2) and (20):

$$\overline{\Pi}(R) - (1+\rho)E = D\underbrace{\left(\overline{\varepsilon}R - \delta - \rho - cgR\frac{\hat{\varepsilon}^2}{2}\right)E}_{\text{pre-tax profit on equity}}$$

So, D converts pre-tax expected profits on equity into after-tax expected profits on K.

[10] Proof:

$$\frac{\partial D}{\partial \tau} \equiv D_\tau = -\frac{1}{1-\phi} + \frac{1-\tau}{(1-\phi)^2}\phi_\tau = -\frac{1}{1-\phi} + \frac{1-\tau}{(1-\phi)^2}\frac{1-\phi}{1-\tau} = 0$$

□

3 Behavior of entrepreneurs

Recall that entrepreneurs are risk-neutral and make two decisions in sequence. First, they decide whether to undertake risky investments, given their productivity R. This is the extensive-margin decision. Then, if they enter, they decide how much to borrow to acquire more capital over and above their own equity, E. This is the intensive-margin decision. Once their shock $\tilde{\varepsilon}$ is revealed, their after-tax profits are determined. We consider the extensive and intensive decisions in reverse order for an entrepreneur of a given type and continue to suppress the type identifier R for simplicity. Our analysis focuses on the base case where tax loss refunds go to the banks in the event of bankruptcy.

3.1 Choice of leverage: intensive margin

Consider a type-R entrepreneur who decides to enter. As mentioned, given (15), the choice of $\hat{\varepsilon}$ is equivalent to the choice of leverage ϕ, and we use the former as the entrepreneur's choice variable. Differentiating (19) with respect to $\hat{\varepsilon}$, we obtain:

$$\frac{d\overline{\Pi}(R)}{\mathrm{d}\hat{\varepsilon}} = \overline{\pi}_{\hat{\varepsilon}}E = \left(\Delta(\hat{\varepsilon}, R, \tau, c) - \frac{2}{\varepsilon_{\max} - \hat{\varepsilon}}\right)\frac{(1-\tau)Rg}{1-\phi}(\varepsilon_{\max} - \hat{\varepsilon})^2 E \tag{21}$$

where

$$\Delta(\hat{\varepsilon}, R, \tau, c) \equiv \frac{\phi_{\hat{\varepsilon}}(\hat{\varepsilon}, R, \tau, c)}{1 - \phi(\hat{\varepsilon}, R, \tau, c)}. \tag{22}$$

Let $\hat{\varepsilon}^*$ be the optimal choice of $\hat{\varepsilon}$. It could be in the interior or it could take on corner solutions at the top or bottom. From (15), $\hat{\varepsilon}^*$ takes on a minimum value of $\hat{\varepsilon}^* = 0$ when $\phi \leqslant \phi(0, R, \tau, c) = (1-\tau)(1-\delta)/(1+\rho) + \tau$. The maximum value of $\hat{\varepsilon}^*$ satisfies $\phi(\hat{\varepsilon}, R, \tau, c) = 1 - E/K_{\max}$, which is assumed to be smaller than $\varepsilon_{\max}$ for all entrepreneurs.

If $\hat{\varepsilon}^*$ is in the interior, $\mathrm{d}\overline{\Pi}(R)/\mathrm{d}\hat{\varepsilon} = 0$, so by (21) the first-order condition on $\hat{\varepsilon}$ can be written:

$$\Delta(\hat{\varepsilon}^*, R, \tau, c) = \frac{2}{\varepsilon_{\max} - \hat{\varepsilon}^*} > 0. \tag{23}$$

Using this, we obtain the following lemma.

Lemma 5 *For $\hat{\varepsilon}^*$ in the interior, $\phi_{\hat{\varepsilon}} > 0$, $\phi_R > 0$, $\phi_{\hat{\varepsilon}R} > 0$ and, if the second-order condition is satisfied,* $\mathrm{d}\hat{\varepsilon}^*/\mathrm{d}R > 0$.

Thus, the probability of bankruptcy increases with the productivity R of the entrepreneur. This occurs because entrepreneurs with higher productivity choose higher leverage. Although a higher value of R tends to reduce bankruptcy risk directly through the bankruptcy condition (7), this direct impact on bankruptcy risk is more than offset by the increase in leverage, $\phi_R > 0$.

3.2 Decision to undertake risky investment: extensive margin

Ex ante, entrepreneurs decide whether to undertake the risky investment or to opt for the risk-free option. In the risk-free option, they invest their wealth E at a risk-free return ρ, leading to consumption of $(1+\rho)E$. They enter if their expected after-tax income as an entrepreneur, given by $\overline{\Pi}(R)$ in (18) or (19), is at least as great as their certain income if they invest their wealth in a safe asset and obtain consumption of $(1+\rho)E$, that is,

$$\overline{\Pi}(R) = \overline{\pi}(\hat{\varepsilon}, R, \tau, c)E \geqslant (1+\rho)E \quad \text{or} \quad \overline{\pi}(\hat{\varepsilon}, R, \tau, c) \geqslant 1+\rho. \tag{24}$$

Differentiating $\overline{\pi}(\cdot)$ in (19) by R and using $\phi_R > 0$ by Lemma 5, we obtain that $\overline{\pi}(\cdot)$ is increasing in R. Given that $\hat{\varepsilon}$ is being optimized, the cutoff value of R, denoted $\widehat{R}$, will be uniquely determined by $\overline{\pi}(\hat{\varepsilon}, \widehat{R}, \tau, c) = 1+\rho$. Using the expression for $\overline{\pi}$ in (18), the following lemma is apparent.

Lemma 6 *The cutoff value of R is determined by:*

$$\overline{\varepsilon}\widehat{R} - \delta - \rho - \frac{c\widehat{R}g\hat{\varepsilon}^2}{2} = 0. \tag{25}$$

Entrepreneurs with $R > \widehat{R}$ enter the risky sector and earn a rent. Those with $R < \widehat{R}$ invest their wealth in a risk-free asset, so earn no rent. As the following lemma shows, bankruptcy risk is zero for marginal entrepreneurs, and positive for all inframarginal ones.

Lemma 7 *The marginal entrepreneur* $\widehat{R}$ *chooses* $\hat{\varepsilon}^* = 0$, *while all entrepreneurs* $R > \widehat{R}$ *choose* $\hat{\varepsilon}^* > 0$.

This has implications for the effect of the cash-flow tax in what follows. To study this, consider first the social optimum as a benchmark.

4 The social optimum

To study the efficiency properties of cash-flow business taxation, it is useful to characterize the full-information social optimum when banks can observe both entrepreneurs' types and the output of bankrupt firms (i.e., $c = 0$). Social surplus includes only the surplus of projects of entrepreneurs who invest in the risky sector since no surplus is generated either by the banks, which earn zero expected profits, or by potential entrepreneurs who invest in the safe outcome and earn $(1+\rho)E$. Expected social surplus can be defined as the expected value of production by entrepreneurs less the opportunity cost of financing their capital. Financing costs include the cost of both debt and equity finance, so are given by $(1+\rho)B + (1+\rho)E = (1+\rho)K$.

For the representative entrepreneur of type-R, end-of-period expected social surplus $S(R)$ can be written as follows, using $K = E/(1-\phi)$:

$$S(R) = \int_0^{\varepsilon_{\max}} \Big(\tilde{\varepsilon}RK + (1-\delta)K\Big)g\mathrm{d}\tilde{\varepsilon} - (1+\rho)K = \Big(\overline{\varepsilon}R - \delta - \rho\Big)\frac{E}{1-\phi(\cdot)} \tag{26}$$

where $\phi(\cdot)$ satisfies (15) with $c = 0$ and $\tau = 0$, or:

$$\phi(\hat{\varepsilon}, R, 0, 0) = \frac{1}{1+\rho}\left(\left(1 - \frac{g\hat{\varepsilon}}{2}\right)\hat{\varepsilon}R + (1-\delta)\right) \tag{27}$$

and $\hat{\varepsilon}$ satisfies the bankruptcy condition, (7). Note that $S(R)$ includes the surplus earned by the investments that go bankrupt since this accrues to the banks. This expression for $S(R)$ applies whether taxes are in place or not.

In a social optimum, both the extensive and intensive margins are optimized. Entry is optimized if $S(R) = 0$ for the marginal entrepreneur, or by (26),

$$\overline{\varepsilon}\widehat{R}^o - \delta - \rho = 0 \tag{28}$$

where $\widehat{R}^o$ is the marginal entrepreneur in the social optimum. All entrepreneurs $R \geq \widehat{R}^o$ enter in the social optimum. Changes in leverage, or equivalently in bankruptcy risk, affect $S(R)$ in (26) as follows:

$$\frac{\mathrm{d}S(R)}{\mathrm{d}\hat{\varepsilon}} = \frac{\phi_{\hat{\varepsilon}}}{(1-\phi)^2}\big(\overline{\varepsilon}R - \delta - \rho\big)E. \tag{29}$$

This implies by (28) that $\widehat{R}^o$, $\mathrm{d}S(\widehat{R}^o)/\mathrm{d}\hat{\varepsilon} = 0$ for the marginal entrepreneur so social surplus is independent of leverage and K. For $R > \widehat{R}^o$, $\mathrm{d}S(R)/\mathrm{d}\hat{\varepsilon} > 0$ for all $\hat{\varepsilon}$ since $\phi_{\hat{\varepsilon}} > 0$ by Lemma 5. Inframarginal entrepreneurs will therefore maximize leverage and choose $K = K_{\max}$ when $c = 0$.

As expected, when $c = 0$ so the full-information social optimum is achieved, the cash-flow tax has no effect on market outcomes. It simply diverts rents from inframarginal entrepreneurs to the government. To see this, consider first the extensive-margin decision. When $c = 0$, (25) implies that $\overline{\varepsilon}\widehat{R} - \delta - \rho = 0$ so $\widehat{R}$ is independent of τ. Thus, $\widehat{R} = \widehat{R}^o$ by (28) so entry is socially optimal. Next, consider the effect of the cash-flow tax on leverage. Differentiate $\overline{\Pi}(R)$ in (18) with respect to $\hat{\varepsilon}$ and set $c = 0$ to obtain:

$$\frac{\mathrm{d}\overline{\Pi}(R)}{\mathrm{d}\hat{\varepsilon}} = \overline{\pi}_{\hat{\varepsilon}}E = \frac{1-\tau}{1-\phi}\Delta(\hat{\varepsilon}, R, \tau, c)(\overline{\varepsilon}R - \delta - \rho)E.$$

For the marginal entrepreneur, (28) implies that $\overline{\pi}_{\hat{\varepsilon}} = 0$, so $\mathrm{d}\overline{\Pi}(R)/\mathrm{d}\hat{\varepsilon}|_{R=\widehat{R}} = 0$. Therefore, leverage ϕ and thus K are indeterminate for the marginal entrepreneur and independent of τ. For inframarginal entrepreneurs, $\overline{\varepsilon}R - \delta - \rho > 0$ since $R > \widehat{R}$, so $\overline{\pi}_{\hat{\varepsilon}}$ has the same sign as $\Delta(\cdot) = \phi_{\hat{\varepsilon}}/(1-\phi)$, which is positive by Lemma 5. Therefore, $\hat{\varepsilon}$ takes its maximum value with $\phi(\hat{\varepsilon}^*, R, \tau, c) = 1 - E/K_{\max}$. Since $\phi_{\hat{\varepsilon}} > 0$ and $\phi_\tau > 0$ by (16), we have that $\mathrm{d}\hat{\varepsilon}^*/\mathrm{d}\tau < 0$ to keep ϕ constant. While $\hat{\varepsilon}^*$ changes, τ does not distort ϕ or the capital stock $K = K_{\max} = E/(1-\phi)$.

When banks must incur a monitoring cost c to observe the profits of bankrupt firms, the social optimum will not be achieved. We saw in Lemma 7 that $\hat{\varepsilon}^* = 0$

for the marginal entrepreneur. Therefore by (25), the productivity of the marginal entrepreneur satisfies $\bar{\varepsilon}\widehat{R} - \delta - \rho = 0$, which implies that $\widehat{R} = \widehat{R}^o$ so entry is optimal. At the same time, since $\hat{\varepsilon}^* > 0$ for inframarginal entrepreneurs by Lemma 7, $\hat{\varepsilon}^*$ will be in the interior for large enough values of R, so leverage and therefore investment will be below the maximum level obtained in the social optimum. Thus, while entry is socially optimal in the presence of moral hazard, there is too little investment for entrepreneurs that incur bankruptcy risk. (We return to the adverse selection case later.)

Before turning to the implications of cash-flow taxation in the moral hazard setting, it is useful to define *constrained social surplus* as social surplus less the costs of monitoring incurred by the banks since the government confronts the same information problem that the banks do. For a type-R entrepreneur, constrained social surplus can be expressed as follows, analogous to (26):

$$\overline{S}(R) = \left(\bar{\varepsilon}R - \delta - \rho - cRg\frac{\hat{\varepsilon}^2}{2}\right)\frac{E}{1 - \phi(\cdot)}. \tag{30}$$

Combining the entrepreneur's expected profits in (17) with the bank's zero-profits expression (13), we obtain:

$$\begin{aligned}\overline{\Pi}(R) - (1+\rho)E = &\int_0^{\varepsilon_{\max}} \Big((1-\tau)\big(\tilde{\varepsilon}RK + (1-\delta)K\big) \\ &- (1+\rho)(B+E)\Big)g\mathrm{d}\tilde{\varepsilon} - (1-\tau)cRKg\frac{\hat{\varepsilon}^2}{2} \\ =&(1-\tau)\overline{S}(R).\end{aligned} \tag{31}$$

In the absence of taxation, maximizing private surplus $\overline{\Pi}(R) - (1+\rho)E$ maximizes constrained social surplus, but that will no longer be so with $\tau > 0$ in our base case where tax losses in the event of bankruptcy go to the banks. We use (31) to interpret the efficiency consequences of cash-flow taxation in a moral hazard setting.

To summarize, in the full-information social optimum, inframarginal entrepreneurs maximize leverage and choose $K = K_{\max}$, while marginal entrepreneurs are indifferent to the level of K. The cash-flow tax has no effect on entry or leverage, but diverts to the government the rents of inframarginal entrepreneurs. If there are monitoring costs, entry remains optimal and marginal entrepreneurs assume no bankruptcy risk, while some inframarginal entrepreneurs under-invest.

5 Cash-flow taxation

The model discussed in the previous sections includes both bankruptcy, when entrepreneurs are unable to repay their loans fully, and asymmetric information, in the sense that banks can only verify bankruptcy with costly ex post monitoring. In this section, we consider the effect of (R+F)-base cash-flow taxation on leverage and entry as well as on after-tax profits, tax revenue and social surplus. Again, our analysis focuses on the base case where tax losses are awarded to the banks in the event of bankruptcy.

5.1 Cash-flow taxation and leverage

The leverage decision for an inframarginal type-R entrepreneur is governed by (21), where $\mathrm{d}\overline{\Pi}(R)/\mathrm{d}\hat{\varepsilon} = 0$ if $\hat{\varepsilon}^*$ is in the interior. To determine the effect of taxes on leverage, differentiate $\phi\big(\hat{\varepsilon}(\cdot), R, \tau, c\big)$ to obtain:

$$\frac{\mathrm{d}\phi}{\mathrm{d}\tau} = \phi_{\hat{\varepsilon}}\frac{\mathrm{d}\hat{\varepsilon}^*}{\mathrm{d}\tau} + \phi_\tau \tag{32}$$

where $\phi_\tau = (1-\phi)/(1-\tau)$ by (16). To evaluate (32), we can use the first-order condition on $\hat{\varepsilon}$, (23), and the fact that $D_\tau = 0$ by Lemma 4, to obtain the following lemma.

Lemma 8 *Assume $\hat{\varepsilon}^*$ is in the interior. Then,*

$$\frac{\mathrm{d}\hat{\varepsilon}^*}{\mathrm{d}\tau} = 0.$$

This confirms as mentioned earlier that the cash-flow tax does not affect the bankruptcy risk of firms.

Using Lemma 8 and (16), (32) reduces to

$$\frac{\mathrm{d}\phi}{\mathrm{d}\tau} = \phi_\tau = \frac{1-\phi}{1-\tau} > 0. \tag{33}$$

While the tax does not affect bankruptcy risk, it does increase leverage and therefore investment. Some explanation for this comes from the following lemma.

Lemma 9 *For $\hat{\varepsilon}$ in the interior,*

$$\frac{\mathrm{d}r}{\mathrm{d}\tau} < 0. \tag{34}$$

The intuition is that the cash-flow tax allows the banks to claim a refund of the opportunity cost of investment, $\tau(1+\rho)K$, on bankrupt projects. An increase in τ increases the gain that the bank can collect from the bankrupt entrepreneurs, which improves its expected profits and thus leads to a reduction in r. By reducing r, the increase in τ induces entrepreneurs to borrow and therefore invest more.

5.2 Cash-flow taxation and entry

Consider now the extensive-margin decision. The productivity of the marginal entrepreneur $\widehat{R}$ is determined by (25). Since $\hat{\varepsilon}^*$ is independent of τ by Lemma 8, so is $\widehat{R}$ and therefore entry. Therefore, the cash-flow tax is neutral with respect to entry.

5.3 Cash-flow taxation and expected profits

Next, consider the effect of the cash-flow tax on expected profits of a type-R firm. Lemma 3 applies, so $\overline{\pi}$ is given by:

$$\overline{\pi} = \frac{1-\tau}{1-\phi}\frac{Rg}{2}(\varepsilon_{\max} - \hat{\varepsilon})^2 \equiv D\frac{Rg}{2}(\varepsilon_{\max} - \hat{\varepsilon})^2. \tag{35}$$

Since $D \equiv (1-\tau)/(1-\phi)$ is independent of the tax rate by Lemma 4, expected profits $\overline{\pi}E$ are as well. Therefore, while the tax increases leverage and therefore investment, it leaves expected after-tax profits unchanged. This is analogous to the Domar and Musgrave (1944) result albeit for a different reason in this context since entrepreneurs are risk-neutral. Here, an increase in τ leads to an increase in ϕ such that D is unaffected by Lemma 4. The increase in leverage increases investment and before-tax expected profits, but since D does not change after-tax expected profits remain the same.

5.4 Cash-flow taxation and expected tax revenue

Expected government revenue from the cash-flow tax can be written, using (4), as:

$$\overline{T} = \tau \int_{\hat{R}}^{R_{\max}} \left(\overline{\varepsilon}R - \rho - \delta - cRg\frac{\hat{\varepsilon}^2}{2}\right)\frac{E}{1-\phi(\cdot)}\mathrm{d}H(R) \equiv \tau\overline{Y} \tag{36}$$

where $\overline{Y}$ is the aggregate expected tax base and $H(R)$ has been defined as the distribution of entrepreneur types. Given from above that neither $\widehat{R}$ nor $\hat{\varepsilon}$ are affected by the tax, differentiating $\overline{T}$ yields:

$$\frac{\mathrm{d}\overline{T}}{\mathrm{d}\tau} = \overline{Y} + \tau \int_{\hat{R}}^{R_{\max}} \left(\overline{\varepsilon}R - \rho - \delta - cRg\frac{\hat{\varepsilon}^2}{2}\right)\frac{E}{(1-\phi)^2}\frac{\mathrm{d}\phi}{\mathrm{d}\tau}\mathrm{d}H(R) > 0 \tag{37}$$

where the inequality follows from (33). The first term is the mechanical effect of an increase in the tax rate on revenues which is positive. The second term is also positive given that leverage increases with the tax rate as shown above. Since leverage and therefore investment increase with τ, more rents are created and this induces an increase in the tax base $\overline{Y}$.

5.5 Cash-flow taxation and expected social surplus

Finally, consider the effect of the tax on constrained expected social surplus $\overline{S}(R)$ for any value of R. Using $\overline{\Pi}(R) - (1+\rho)E = (1-\tau)\overline{S}(R)$ from (31), we have:

$$\overline{S}(R) = \frac{\overline{\Pi}(R) - (1+\rho)E}{1-\tau}.$$

Since a tax increase leaves $\overline{\Pi}(R) = \overline{\pi}E$ and therefore the numerator unchanged, it will increase $\overline{S}(R)$. In effect, the tax induces the firms to increase leverage while keeping $\hat{\varepsilon}$ constant. Since investment satisfies $K = E/(1-\phi)$ by (2), introducing the tax increases K, and as can be seen from (30), $\overline{S}(R)$ increases. Equivalently, the increase in K holding $\hat{\varepsilon}$ constant generates more pre-tax profits or rents. The government taxes away those profits, leaving after-tax expected profits unchanged and improving constrained expected social surplus. Thus, while the no-tax outcome replicates the constrained social optimum, implementing a cash-flow tax improves social outcomes without changing firms' expected profits. It does so by breaking the connection between leverage and bankruptcy risk. This reflects the fact that levels of K in the absence of the tax are less than in the unconstrained social optimum for some entrepreneurs as discussed above.

An implication of this is that setting the tax rate sufficiently high might lead to a socially efficient outcome. Given that increases in τ cause ϕ and thus K to rise without affecting $\hat{\varepsilon}$, if τ can be increased until $K = K_{\max}$ for the marginal entrepreneur $\widehat{R}$, the social optimum will be achieved. That is because $\phi_R > 0$ by Lemma 5, so $K = K_{\max}$ for all inframarginal entrepreneurs. To achieve this requires setting the tax rate τ such $\phi = 1 - E/K_{\max}$ for the marginal entrepreneur. Increasing τ further beyond the level at which $K = K_{\max}$ for all firms will transfer profits from entrepreneurs to the government with no effect on efficiency.

5.6 The effect of cash-flow taxation in the Bond–Devereux case

Suppose the tax credit on start-of-period cash flows, $\tau(1+\rho)E$, remains with entrepreneurs (or the firms' shareholders in the case of an S-base cash-flow tax as noted by Bond and Devereux). Consider first the intensive-margin decision. The bankruptcy condition (7) and the bank zero-profit condition (13) become:

$$\begin{aligned} 0 =& \hat{\varepsilon}RK + (1-\delta)K - (1+r)B, \quad \text{and} \\ (1+\rho)B =& \left(\int_{\hat{\varepsilon}}^{\varepsilon_{\max}} (1+r)Bg\mathrm{d}\tilde{\varepsilon} + \int_0^{\hat{\varepsilon}} \left(\tilde{\varepsilon}RK + (1-\delta)K \right) g\mathrm{d}\tilde{\varepsilon} - cRKg\frac{\hat{\varepsilon}^2}{2} \right). \end{aligned}$$

Combining these, we obtain an expression for leverage:

$$\phi(\hat{\varepsilon}, R, c) = \frac{1}{1+\rho} \left(\left(1 - \frac{g\hat{\varepsilon}}{2} - \frac{cg\hat{\varepsilon}}{2} \right) \hat{\varepsilon}R + (1-\delta) \right).$$

Since leverage is independent of τ, so is B, and therefore $\hat{\varepsilon}$ from the bankruptcy condition above. Therefore, the corporate tax has no effect on either bankruptcy risk or K.

Next, consider the extensive-margin decision. When entrepreneurs retain the tax credit $\tau(1+\rho)E$, the entrepreneur's expected-profit condition (17) becomes:

$$\overline{\Pi}(R) = \int_{\hat{\varepsilon}}^{\varepsilon_{\max}} \left((1-\tau)\Big(\tilde{\varepsilon}RK + (1-\delta)K - (1+r)B \Big) \right) g\mathrm{d}\tilde{\varepsilon} + \int_{0}^{\varepsilon_{\max}} \tau(1+\rho)Eg\mathrm{d}\tilde{\varepsilon}.$$

Combining this with the bank zero-profit condition above, we obtain (18). The entry condition is again (24) and Lemma 6 applies. The extensive-margin decision is independent of τ as in our base case. This confirms the Bond and Devereux (2003) result that the corporate tax is neutral if tax losses are refundable to entrepreneurs in the event of bankruptcy.[11]

The main results of the analysis in the moral hazard model are summarized as follows.

Proposition 1 *With asymmetric information about investment outcomes, equilibrium has the following properties.*

i. Entrepreneurs with productivity R above some threshold level $\widehat{R}$ enter the risky industry and earn a rent. For those with K in the interior, leverage ϕ and bankruptcy risk $\hat{\varepsilon}^*$ are increasing with R.

ii. In the absence of taxation, entry is socially efficient in equilibrium, but leverage and therefore investment are below the full information socially optimal levels.

iii. In the base case where tax losses in the event of bankruptcy go the banks, (R+F)-base cash-flow taxation increases leverage, while bankruptcy risk and expected profits remain unchanged, and expected tax revenue increases. Expected rents and expected social surplus both increase, so the cash-flow tax acts as a corrective device. The social optimal outcome might be achieved by a sufficiently high tax rate.

iv. In the Bond–Devereux case where tax losses in the event of bankruptcy stay with the entrepreneurs, the (R+F)-base cash-flow tax is neutral and transfers profits from inframarginal entrepreneurs to the government without affecting investment.

We assumed in this section that the government deployed an (R+F)-base cash-flow tax, or equivalently an ACE corporate tax. In a previous version of this paper (Boadway et al., 2016), we considered the case of an R-base cash-flow tax where financial cash flows are not taxed. If banks are exempt from the tax, marginal entrepreneurs still assume no bankruptcy risk and entry is not affected by the tax. However, bankruptcy risk falls with the tax and the change in leverage is smaller than in the (R+F)-base case. Leverage may even fall in which case the tax would move the equilibrium away from the social optimum. On the other hand, if the real cash flows of banks were taxable, the same results as in the (R+F)-base cash-flow tax are obtained.

[11] Bond and Devereux note that in their setting with fixed investment, a cash-flow tax will also be neutral if the tax losses in the event of bankruptcy are refunded to the bank provided they are grossed up by $1-\tau$. Grossing-up is required since banks do not pay taxes in their model.

6 The case with adverse selection

So far we have assumed that the banks can observe entrepreneurs' types (R), but need to engage in costly monitoring to verify bankruptcy. We now explore the consequences of entrepreneurial types being private information following De Meza and Webb (1987). In this case, the interest rate r is the same for all types and is taken as given by entrepreneurs. To focus on adverse selection, we assume that $c = 0$ so there is no moral hazard. Eqs. (1)–(8), (11) and (17) continue to apply.

Recall that for an entrepreneur of productivity R, expected profit is given by (17). Since $K = B + E$ with E is fixed, and each entrepreneur takes r as given, (17) implies:

$$\frac{\mathrm{d}\overline{\Pi}(R)}{\mathrm{d}K} = \int_{\hat{\varepsilon}}^{\varepsilon_{\max}} (1-\tau)\big(\tilde{\varepsilon}R - \delta - r\big)\mathrm{d}G(\tilde{\varepsilon}).$$

If this expression is positive, the level of capital will be set at its maximum value, $K = K_{\max}$. We assume that this is the case for all active entrepreneurs, which we can show will be the case in equilibrium.

From (7), $\hat{\varepsilon}$ is determined as a function of R, r and τ such that:

$$\hat{\varepsilon}(\cdot) = \frac{(1+r)B}{RK} - \frac{\tau(1+\rho)E}{(1-\tau)RK} - \frac{(1-\delta)}{R}, \quad \text{with} \ \ \hat{\varepsilon}_r = \frac{B}{RK} > 0, \ \ \hat{\varepsilon}_\tau = \frac{-(1+\rho)E}{(1-\tau)^2 RK} < 0. \tag{38}$$

So, given r, an increase in the tax rate tends to lower bankruptcy risk.

6.1 Equilibrium interest rate

Banks cannot observe R, so must offer the same interest rate to all firms. The banks' zero-profit condition (13), with $c = 0$, aggregated over all entrepreneur types gives:

$$\begin{aligned}
&\int_{\widehat{R}}^{R_{\max}} (1+\rho)B\mathrm{d}H(R)\\
&\quad = \int_{\widehat{R}}^{R_{\max}} \left[\int_{\hat{\varepsilon}}^{\varepsilon_{\max}} (1+r)B\mathrm{d}G(\tilde{\varepsilon}) + \int_0^{\hat{\varepsilon}} \left(\tilde{\varepsilon}RK + (1-\delta)K\right.\right.\\
&\qquad \left.\left. + \frac{\tau}{(1-\tau)}(1+\rho)E\right)\mathrm{d}G(\tilde{\varepsilon})\right]\mathrm{d}H(R).
\end{aligned}$$

Using (7) and rearranging, we can rewrite this equation as

$$r = \rho + \frac{K}{B}\Phi(\widehat{R}, r, \tau) \tag{39}$$

where

$$\Phi(\widehat{R}, r, \tau) \equiv \frac{1}{1 - H(\widehat{R})} \int_{\widehat{R}}^{R_{\max}} \left(\int_0^{\hat{\varepsilon}} R(\hat{\varepsilon} - \tilde{\varepsilon}) \mathrm{d}G(\tilde{\varepsilon}) \right) \mathrm{d}H(R) \; > \; 0. \tag{40}$$

Note that Φ depends on r since $\hat{\varepsilon}$ depends on r through (38). Equation (39) implicitly determines the interest rate as a function of the productivity of the marginal entrepreneur and the tax rate. Let $r(\widehat{R}, \tau)$ denote this solution. The following lemma indicates that $r(\widehat{R}, \tau)$ is decreasing in both the productivity of the marginal entrepreneur and the tax rate.

Lemma 10 *At the banking sector equilibrium, the interest rate satisfies*

$$r_{\widehat{R}}(\widehat{R}, \tau) < 0; \qquad r_{\tau}(\widehat{R}, \tau) < 0.$$

6.2 Entry decisions

Entrepreneurs choose to enter the risky industry if their expected profit is at least as large as the risk-free return on their initial wealth, that is, if $\overline{\Pi}(R) \geqslant (1 + \rho)E$, with equality applying for the marginal entrepreneur $\widehat{R}$. The condition characterizing equilibrium entry is given by the following lemma.

Lemma 11 *The productivity of the marginal entrepreneur $\widehat{R}$ satisfies*

$$\overline{\varepsilon}\widehat{R} - \delta - \rho + \int_0^{\hat{\varepsilon}} \widehat{R}(\hat{\varepsilon} - \tilde{\varepsilon}) \mathrm{d}G(\tilde{\varepsilon}) - \Phi(\widehat{R}, r, \tau) \equiv \Omega(\widehat{R}, r, \tau) = 0. \tag{41}$$

The solution to (41), denoted by $\widehat{R}(r, \tau)$, gives the productivity of the marginal entrepreneur as a function of the interest rate and the tax rate. The following lemma indicates its properties.

Lemma 12 *Equilibrium entry in the risky industry is such that the productivity of the marginal entrepreneur satisfies*

$$\widehat{R}_r(r, \tau) < 0; \qquad \widehat{R}_{\tau}(r, \tau) > 0.$$

Thus, the productivity of the marginal entrepreneur decreases with the interest rate and increases with the tax rate.

The equilibrium interest rate and number of entrepreneurs in the risky industry simultaneously solve $r(\widehat{R}, \tau)$ and $\widehat{R}(r, \tau)$. Let the solutions be $r(\tau)$ and $\widehat{R}(\tau)$ with a slight abuse of notation. The properties of these equilibrium solutions are given in the following lemma.

Lemma 13 *The equilibrium values of $r(\tau)$ and $\widehat{R}(\tau)$ satisfy*

$$\frac{\mathrm{d}r(\tau)}{\mathrm{d}\tau} < 0; \qquad \frac{\mathrm{d}\widehat{R}(\tau)}{\mathrm{d}\tau} > 0.$$

That is, an increase in the tax rate lowers the equilibrium interest rate and increases the productivity of the marginal entrepreneur thereby discouraging entry in the risky industry.

Finally, the following lemma indicates that there is excessive entry in this adverse selection case. This applies regardless of the tax rate.

Lemma 14 *The social surplus obtained by the marginal entrepreneur is negative, so there is excessive entry.*

This corresponds with the results found in De Meza and Webb (1987). Since all firms pay the same interest rate, high-R entrepreneurs are cross-subsidizing low-R entrepreneurs. Too many low-R entrepreneurs enter because they are paying an interest rate below the actuarial value.

6.3 Corrective tax rate

Denote the corrective tax rate by τ^*. It is the tax rate such that $\hat{\varepsilon} = 0$ for the marginal entrepreneur. To see this, note that when $\hat{\varepsilon} = 0$, we have $r = \rho$. In that case, by (40) and (41), we have $\overline{\varepsilon}\widehat{R} - \delta - \rho = 0$ which characterizes the productivity of the marginal entrepreneur in the social optimum as in (28).[12] Intuitively, when $r = \rho$ and $\hat{\varepsilon} = 0$, there is no cross-subsidization of low-R by high-R entrepreneurs so the adverse selection problem disappears.

By (7), the tax rate required to achieve $\hat{\varepsilon} = 0$ satisfies

$$(1-\delta)K - (1+\rho)B + \frac{\tau^*}{1-\tau^*}(1+\rho)E = 0$$

Using $B = K - E$ and solving for τ^*, we obtain

$$\tau^* = 1 - \frac{(1+\rho)E}{(\rho+\delta)K}, \quad \text{where} \quad 1 > \tau^* > 0. \tag{42}$$

The inequality $\tau^* > 0$ follows from noting that, in the social optimum, $\rho + \delta = \overline{\varepsilon}\widehat{R}$ and $\overline{\varepsilon}\widehat{R}K > (1+\rho)E$, while $\tau^* < 1$ follows from (42).

Since the tax rate that achieves the optimum is positive and as shown $\widehat{R}$ increases with the tax rate, this confirms that there is too much entry in the risky industry in the absence of taxation. Introducing $\tau *$ discourages entry and achieves full efficiency. As in the moral hazard case, increasing τ beyond τ^* transfers profits from inframarginal entrepreneurs to the government without affecting resource allocation.

[12] Note that $\hat{\varepsilon} = 0$ is also necessary for $\overline{\varepsilon}\widehat{R} - \delta - \rho = 0$ in (41). Since $R\hat{\varepsilon}$ is constant by (7) when K and B are constant, the last term in (41), $\Phi(\widehat{R}, r, \tau)$, is decreasing in R. Therefore, the last two terms are positive for $\hat{\varepsilon} > 0$.

The findings of the analysis with adverse selection are as follows.

Proposition 2 *If banks cannot observe entrepreneurial types but can observe investment profits, equilibrium has the following properties:*

i. The same interest rate applies to all types and too many entrepreneurs undertake risky investments at the credit market equilibrium relative to the efficient level;
ii. The cash-flow tax reduces the equilibrium interest rate and reduces the number of entrepreneurs who undertake risky investments relative to the no-tax equilibrium;
iii. The optimal cash-flow tax rate eliminates the risk of bankruptcy.

As in the moral hazard case, the cash-flow tax acts as a corrective device when there is adverse selection. In the moral hazard case, entry is efficient, but there is too little investment by each entrepreneur. By encouraging investment, the cash-flow tax corrects the inefficiency in investment. In contrast, with adverse selection there is excessive entry because all entrepreneurs face the same interest rate regardless of productivity. A cash-flow tax discourages entry and thereby reduces the inefficiency of entry.

7 Concluding remarks

We have analyzed the impact of cash-flow business taxation on entrepreneurs' decision of whether to enter a risky industry and, if so, how much to borrow when entrepreneurs face bankruptcy risk, and when there is asymmetric information between entrepreneurs and financial intermediaries. We assume that cash-flow taxation is of the (R+F)-base sort so it applies both to entrepreneurs and banks. The cash flows of firms that go bankrupt are taken over by the creditor banks. Under the cash-flow tax, losses incurred by entrepreneurs are carried forward with interest and are fully refunded. When firms go bankrupt, the losses are refunded to the banks along with the firms' cash flows. We investigate the efficiency properties of cash-flow taxation when asymmetric information takes the form of moral hazard and of adverse selection.

In the moral hazard case, banks must undertake costly monitoring of firms that declare bankruptcy. In equilibrium, the entry of firms is optimal, but they invest too little relative to the social optimum. The cash-flow tax does not affect entry decisions, given that the marginal entrepreneur earns no rent, but it does encourage leverage and therefore investment. As a result, the tax will actually increase social welfare. By inducing firms to increase leverage, and therefore investment, the cash-flow tax leads to higher pre-tax profits or rents, without affecting bankruptcy risk. These additional pre-tax profits are taxed away by the government leading to a higher constrained social surplus. By inducing more investment, the cash-flow tax is implicitly correcting for the inefficiencies of the information-constrained outcome. These results differ from those of Bond and Devereux (2003) who find that an R+F-base cash-flow tax is neutral. The reasons for the difference are that they assume that the level of investment is fixed and that tax losses of bankrupt firms are refunded to the owners of the firms rather than to creditors.

If banks cannot observe the productivity of entrepreneurs ex ante, there will be an adverse selection problem as in De Meza and Webb (1987), among others. In this case, if banks cannot offer separating contracts, all entrepreneurs face the same interest rate. The equilibrium without taxation is inefficient along the extensive margin and cash-flow taxation is not neutral. In particular, there is excessive entry by the least-productive entrepreneurs to take advantage of the favorable interest rate. A cash-flow tax discourages entry, thereby improving efficiency.

In Boadway et al. (2016), we considered the case of risk-averse entrepreneurs. In this case, cash-flow taxation taxes both rents and return to risk. The cash-flow tax does not affect the entry decision and increases leverage, while leaving bankruptcy risk unchanged as in the risk-neutral case. In addition, neither expected profits nor expected utility are affected. Expected tax revenues increase, and the government will incur greater risk if it cannot pool risk better than the private sector. This is analogous to the results of Domar and Musgrave (1944) and Atkinson and Stiglitz (1980).

Appendix

Proof of Lemma 1 Using (7), the right-hand side of (13) can be written:

$$
\begin{aligned}
&\big((1-\tau)(\hat{\varepsilon}R+(1-\delta))K+\tau(1+\rho)E\big)(1-g\hat{\varepsilon})+(1-\tau)RK\frac{g\hat{\varepsilon}^2}{2}\\
&\quad+\big((1-\tau)(1-\delta)K+\tau(1+\rho)E\big)g\hat{\varepsilon}-(1-\tau)\frac{cRKg\hat{\varepsilon}^2}{2}\\
&=(1-\tau)\Big(\hat{\varepsilon}(1-g\hat{\varepsilon})+\frac{g\hat{\varepsilon}^2}{2}\Big)RK+(1-\tau)(1-\delta)K+\tau(1+\rho)E\\
&\quad-(1-\tau)\frac{cRKg\hat{\varepsilon}^2}{2}\\
&=(1-\tau)\Big(1-\frac{g\hat{\varepsilon}}{2}-\frac{cg\hat{\varepsilon}}{2}\Big)\hat{\varepsilon}RK+(1-\tau)(1-\delta)K+\tau(1+\rho)E.
\end{aligned}
$$

Setting this equal to the left-hand side of (13), we can obtain Lemma 1. □

Proof of Lemma 2 Differentiate (9) with respect to $\hat{\varepsilon}$ and r, and use $\phi_r = 0$ to obtain:

$$
0 = R\mathrm{d}\hat{\varepsilon} - \phi dr - \frac{1}{\phi}\left[\hat{\varepsilon}R + (1-\delta) + \frac{\tau(1+\rho)}{1-\tau}\right]\phi_{\hat{\varepsilon}}.
$$

Using $\phi_{\hat{\varepsilon}}$ from (16) gives:

$$0 = R\mathrm{d}\hat{\varepsilon} - \phi dr - \frac{1}{\phi}\left[\hat{\varepsilon}R + (1-\delta) + \frac{\tau(1+\rho)}{1-\tau}\right]\frac{1-\tau}{1+\rho}\big(1-(1+c)g\hat{\varepsilon}\big)R.$$

This can be rewritten using (15) as:

$$\begin{aligned}\frac{\phi^2}{R}\frac{\mathrm{d}r}{\mathrm{d}\hat{\varepsilon}} =& \frac{1-\tau}{1+\rho}\left(\left(1-\frac{1+c}{2}g\hat{\varepsilon}\right)\hat{\varepsilon}R + (1-\delta)\right) + \tau \\ &- \left[\hat{\varepsilon}R + (1-\delta) + \frac{\tau(1+\rho)}{1-\tau}\right]\frac{1-\tau}{1+\rho}\big(1-(1+c)g\hat{\varepsilon}\big)\end{aligned}$$

which simplifies to:

$$\frac{\phi^2}{R}\frac{\mathrm{d}r}{\mathrm{d}\hat{\varepsilon}} = (1+c)g\hat{\varepsilon}\left(\tau + \frac{1-\tau}{1+\rho}\left((1-\delta) + \frac{\hat{\varepsilon}}{2}R\right)\right) \ > \ 0.$$

□

Proof of Lemma 3 Rewrite $\overline{\pi}$ in (18) as

$$\overline{\pi}(\hat{\varepsilon}, R, \tau, c) = \frac{1}{1-\phi}\left((1-\tau)\left(R\overline{\varepsilon} - \delta - \rho - \frac{cRg\hat{\varepsilon}^2}{2}\right) + (1+\rho)(1-\phi)\right).$$

From (15), we obtain

$$(1+\rho)(1-\phi) = -(1-\tau)\left(1 - \frac{g\hat{\varepsilon}}{2} - \frac{cg\hat{\varepsilon}}{2}\right)\hat{\varepsilon}R + (1-\tau)(\rho+\delta).$$

Substituting this in the expression for $\overline{\pi}$ gives, using $\overline{\varepsilon} = \varepsilon_{\max}/2$ and $\varepsilon_{\max} = 1/g$:

$$\overline{\pi}(\hat{\varepsilon}, R, \tau, c) = \frac{1-\tau}{1-\phi}R\left(\overline{\varepsilon} - \hat{\varepsilon}\left(1 - \frac{g}{2}\hat{\varepsilon}\right)\right) = \frac{1-\tau}{1-\phi}\frac{Rg}{2}(\hat{\varepsilon} - \varepsilon_{\max})^2.$$

□

Proof of Lemma 5 Since $\Delta(\hat{\varepsilon}^*, R, \tau, c) > 0$ for $\hat{\varepsilon}^*$ in the interior by (23), $\phi_{\hat{\varepsilon}} > 0$ by (22). Therefore, $\phi_R > 0$ and $\phi_{\hat{\varepsilon}R} > 0$ by (16). The second-order condition on $\hat{\varepsilon}$ is:

$$\Delta_{\hat{\varepsilon}}(\hat{\varepsilon}, R, \tau, c) - \frac{2}{(\varepsilon_{\max} - \hat{\varepsilon})^2} \ < \ 0.$$

Differentiate the first-order condition (23), and use $\Delta_R = \phi_{\hat{\varepsilon}R}/(1-\phi) + \phi_{\hat{\varepsilon}}\phi_R/(1-\phi)^2 > 0$ and the second order-conditions on $\hat{\varepsilon}$ to obtain $\mathrm{d}\hat{\varepsilon}^*/\mathrm{d}R > 0$.. □

Proof of Lemma 7 Differentiate $\overline{\pi}(\cdot)$ in (18) with respect to $\hat{\varepsilon}$ to obtain:

$$\overline{\pi}_{\hat{\varepsilon}} = \frac{1-\tau}{1-\phi(\cdot)}\left(\Delta(\hat{\varepsilon}, R, \tau, c)\left(\overline{\varepsilon}R - \delta - \rho - \frac{cRg\hat{\varepsilon}^2}{2}\right) - c\hat{\varepsilon}gR\right).$$

This implies by Lemma 6 that for the marginal entrepreneur $\overline{\pi}_{\hat{\varepsilon}} < 0$ for $\hat{\varepsilon} > 0$. Therefore, the marginal entrepreneur chooses $\hat{\varepsilon}^* = 0$ and incurs no bankruptcy risk. Then, with $\hat{\varepsilon} = 0$, $\overline{\pi}_{\hat{\varepsilon}}$ in the above becomes zero. Increasing R above $\widehat{R}$ than causes $\overline{\pi}_{\hat{\varepsilon}}$ to become positive, so $\hat{\varepsilon}^*$ increases above zero. □

Proof of Lemma 8 Differentiating $\Delta(\hat{\varepsilon}, \tau, c) = \phi_{\hat{\varepsilon}}/(1-\phi)$, we have $\Delta_\tau = \phi_{\hat{\varepsilon}\tau}/(1-\phi) + \phi_\tau \phi_{\hat{\varepsilon}}/(1-\phi)^2$. Using (16) for ϕ_τ, $\phi_{\hat{\varepsilon}}$ and $\phi_{\hat{\varepsilon}\tau}$,

$$\Delta_\tau = -\frac{1 - g\hat{\varepsilon} - cg\hat{\varepsilon}}{(1+\rho)(1-\phi)} R + \frac{1-\phi}{1-\tau}\frac{1-\tau}{1+\rho}(1 - g\hat{\varepsilon} - cg\hat{\varepsilon})R\frac{1}{(1-\phi)^2} = 0. \tag{43}$$

Since $\Delta_\tau = 0$, the solution of first-order condition (23) for $\hat{\varepsilon}^*$ is independent of τ. □

Proof of Lemma 9 Differentiate (9) with respect to τ and use $\mathrm{d}\hat{\varepsilon}/\mathrm{d}\tau = 0$ by Lemma 8 to obtain:

$$-\Big(\hat{\varepsilon}R + (1-\delta) - (1+\rho) + (1+r)\phi_\tau\Big)\mathrm{d}\tau - \phi dr = 0.$$

Since $\phi_\tau = (1-\phi)/(1-\tau)$ by (16), this becomes:

$$\frac{\mathrm{d}r}{\mathrm{d}\tau} = -\frac{1}{\phi}\Big(\hat{\varepsilon}R - (\delta+\rho) + (1+r)\frac{1-\phi}{1-\tau}\Big).$$

Using (9), this can be written:

$$\frac{\mathrm{d}r}{\mathrm{d}\tau} = -\frac{1}{\phi}\frac{r-\rho}{1-\tau}$$

which is negative since $r > \rho$ by the bank's zero-profit condition. □

Proof of Lemma 10 Differentiating (40) and using the properties of $\hat{\varepsilon}$ in (38), we obtain:

$$\Phi_r = \frac{B}{K(1-H(\widehat{R}))}\int_{\widehat{R}}^{R_{\max}} G(\hat{\varepsilon}(R, r, \tau))\mathrm{d}H(R) > 0 \tag{44}$$

$$\Phi_\tau = -\frac{1}{(1-H(\widehat{R}))}\int_0^{\hat{\varepsilon}} \frac{(1+\rho)E}{(1-\tau)^2 K}\mathrm{d}G(\tilde{\varepsilon}) < 0 \tag{45}$$

and

$$\Phi_{\widehat{R}} = \frac{H(\widehat{R})}{(1-H(\widehat{R}))}\left(\Phi - \int_0^{\hat{\varepsilon}} \widehat{R}(\hat{\varepsilon} - \tilde{\varepsilon})\mathrm{d}G(\tilde{\varepsilon})\right) < 0 \tag{46}$$

where the inequality in (46) comes from noting that

$$R\hat{\varepsilon} = (1+r)\frac{B}{K} - (1-\delta) - \frac{\tau}{(1-\tau)}(1+\rho)\frac{E}{K}$$

is constant, and therefore $\int_0^{\hat{\varepsilon}} R(\hat{\varepsilon} - \tilde{\varepsilon})\mathrm{d}G(\tilde{\varepsilon})$ is decreasing in R. Differentiating (39) and using (44)–(46) immediately yields Lemma 10. □

Proof of Lemma 11 Using (17) for the entrepreneur's expected profit, we can write the condition characterizing $\widehat{R}$ as

$$\overline{\Pi}(\widehat{R}) - (1+\rho)E = \int_{\hat{\varepsilon}}^{\varepsilon_{\max}} \Big((1-\tau)\Big(\tilde{\varepsilon}\widehat{R}K + (1-\delta)K - (1+r)B\Big) + \tau(1+\rho)E\Big)\mathrm{d}G(\tilde{\varepsilon}) - (1+\rho)E = 0.$$

This equation can be rewritten as follows

$$\begin{aligned}&(1-\tau)\Big(\overline{\varepsilon}\widehat{R}K + (1-\delta)K - (1+r)B\Big) + \tau(1+\rho)E - (1+\rho)E\\ &\quad - \int_0^{\hat{\varepsilon}} \Big((1-\tau)\Big(\tilde{\varepsilon}\widehat{R}K + (1-\delta)K - (1+r)B\Big) + \tau(1+\rho)E\Big)\mathrm{d}G(\tilde{\varepsilon}) = 0.\end{aligned}$$

Using (6) and (7) and rearranging, we can rewrite the equation above as

$$\overline{\varepsilon}\widehat{R}K + (1-\delta)K - (1+r)B - (1+\rho)E + \widehat{R}K\int_0^{\hat{\varepsilon}} (\hat{\varepsilon} - \tilde{\varepsilon})\mathrm{d}G(\tilde{\varepsilon}) = 0.$$

Substituting $K = B + E$ and (39) in the equation above and rearranging gives (41). □

Proof of Lemma 12 From (41), and using (38) as well as (44)–(45), we can derive the following properties

$$\Omega_{\widehat{R}} = \left(\overline{\varepsilon} - \int_0^{\hat{\varepsilon}} \tilde{\varepsilon}\mathrm{d}G(\hat{\varepsilon})\right) - \Phi_{\widehat{R}} > 0 \tag{47}$$

$$\Omega_{\tau} = \frac{(1+\rho)E}{(1-\tau)^2 K}\left(-G(\hat{\varepsilon}) + \frac{1}{1-H(\widehat{R})}\int_{\widehat{R}}^{R_{max}} G(\hat{\varepsilon})\mathrm{d}H(R)\right) < 0 \tag{48}$$

$$\Omega_{r} = \frac{B}{K}\left(G(\hat{\varepsilon}) - \frac{1}{1-H(\widehat{R})}\int_{\widehat{R}}^{R_{max}} G(\hat{\varepsilon})\mathrm{d}H(R)\right) > 0. \tag{49}$$

The lemma follows from differentiating (41) and using (47)–(49). □

Proof of Lemma 13 We obtain $r(\tau)$ and $\widehat{R}(\tau)$ by simultaneously solving $r = r(\widehat{R}, \tau)$ and $\widehat{R} = \widehat{R}(r, \tau)$. Differentiating these yields

$$\mathrm{d}r - r_{\widehat{R}}\mathrm{d}\widehat{R} = r_{\tau}\mathrm{d}\tau, \quad \text{and} \quad \mathrm{d}\widehat{R} - \widehat{R}_r\mathrm{d}r = \widehat{R}_{\tau}\mathrm{d}\tau.$$

Using Lemmas (10) and (12) and assuming stability of this set of linear equations, we obtain $\mathrm{d}r/\mathrm{d}\tau < 0$ and $\mathrm{d}\widehat{R}/\mathrm{d}\tau > 0$.. □

Proof of Lemma 14 From (26), social surplus in this case becomes:

$$S(R) = (\overline{\varepsilon}R - \delta - \rho)K_{\max}.$$

Equation (41) for the marginal entrepreneur may be written:

$$\overline{\varepsilon}\widehat{R} - \delta - \rho + \int_0^{\hat{\varepsilon}} \widehat{R}(\hat{\varepsilon} - \tilde{\varepsilon})\mathrm{d}G(\tilde{\varepsilon}) - \frac{1}{1 - H(\widehat{R})} \int_{\widehat{R}}^{R_{\max}} \left(\int_0^{\hat{\varepsilon}} R(\hat{\varepsilon} - \tilde{\varepsilon})\mathrm{d}G(\tilde{\varepsilon}) \right) \mathrm{d}H(R) = 0. \tag{50}$$

Therefore, the last two terms are positive, implying that $\overline{\varepsilon}\widehat{R} - \delta - \rho < 0$.. Therefore, the social profit of the marginal entrepreneur is negative, so entry is excessive. □

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