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The impact of insufficient cash flow on payment term and supply chain contracts

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ABSTRACT

Payment term is an essential part of contract in a supply chain. We consider a supply chain with a retailer who moves first to choose one of three payment terms, e.g. 'early payment' (EP), 'delayed payment' (DP) and 'punctual payment' (PP) and a manufacturer who may have a preference conflict on the payment term with the retailer. Analysing with a sufficient cash flow as a benchmark, strategic impact of insufficient cash flow on payment term and supply chain contracts is investigated to mitigate the conflict. Our study shows that the retailer with cash constraint moves from EP to DP when the initial cash is very small, otherwise holds EP either by borrowing a bank loan with a large wholesale price or using up all initial cash with a small wholesale price. Moreover, a subsidy contract with large wholesale price under the retailer's cash constraint absolutely mitigates the payment preference conflict, while the revenue-sharing contract with small wholesale price may be effective. Moreover, under the manufacturer's cash constraint, the retailer is better off under EP, while the manufacturer prefers DP with (without) bank credit related to a small (larger) interest rate when wholesale price is small, then prefers EP as cash flow decreases. Furthermore, the revenue-sharing contract is more effective.

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Payment term; conflict; insufficient cash flow; supply chain contract

1. Introduction

A routine transaction that occurs in a supply chain is that products usually flow from a manufacturer to a retailer, while cash flows from the retailer to the manufacturer (Anupindi et al., 1998). Payment term, as an important part of contract, influences cash flow and operational decisions, such as procuring, producing and ordering etc. In practice, three payment terms are usually used: 'early payment' (EP), 'delayed payment' (DP) and 'punctual payment' (PP). Generally, the retailer pays before receiving products under EP to get a discount of wholesale price (See-to & Ngai, 2018), which reduces the risk of manufacturer's cash flow (Chen, 2015b). DP means the purchased product is paid by the retailer in a fix allowed period after receiving, which is usually linked with an interest rate of procurement cost when the retailer faces financial constraint (Chen, 2015a; Jing & Seidmann, 2014). Alternatively, the retailer pays on delivery under the PP payment term, which seems fair for both manufacturer and retailer, requiring enough cash for production or procurement with neither discount nor interest rate.

In practice, retailers have their own channel power, which manufacturer is more dependent on, to select payment term. In the pharmaceutical industry, for instance, a famous pharmaceutical retailer Li'an with sufficient cash, who has many physical stores scattered in various areas, benefits from EP with a discount procurement cost paid to the Dongeejiao company, focusing on producing gelatin by donkey skin. However, Lei Yunshang, the noticeable pharmaceutical retailer brand in eastern China, often chooses DP due to limited cash endowment. Clearly, cash flow level plays an important role in a firm's payment term decision. Operation may fail when cash flow falls below a specific level (Kouvelis & Zhao, 2012). However, manufacturers may have a payment term preference conflict with retailers, which may halt the transaction. For example, the famous electric appliance manufacturing company Sanyo and the China's domestic air conditioner brand Gree leave electrical retailer Gome in 2008 and in 2011, respectively as they cannot agree with the payment term offered by Gome.

These observations motivate the research of this paper is to address the following questions: *First, how does the retailer choose the payment term* especially *under insufficient cash flow? Second, how to mitigate the preference conflict of the payment term among supply chain partners to realise the transaction?*

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To answer the above questions, we develop a basic vertical supply chain with one upstream manufacturer and one downstream retailer where the manufacturer sells via an exogenous wholesale product to an incumbent retailer who faces a linear demand of the product. In addition, the retailer possesses more channel power that he moves first to choose one of three payment terms, e.g. 'early payment' (EP), 'delayed payment' (DP) and 'punctual payment' (PP). Then, the manufacturer would decide a discount of wholesale price for EP; or an interest rate of procurement cost for DP; or neither for PP. Finally, the retailer places the orders. We divide the analysis in this study into two parts. In the first part, both supply chain members are embedded with sufficient cash flow. As a benchmark, we derive the optimal choice of the payment term for the retailer and the payment term preference conflict between the manufacturer and retailer, moreover, a designed revenue-sharing contract can alleviate the conflict. In the second part, either the retailer or manufacturer has insufficient cash flow (but not both). We examine the impact of insufficient cash flow on the payment term and preference conflict through comparisons. In addition, we investigate the effectivity of a schemed subsidy contract under the retailer's cash constraint and the revenue-sharing contract under cash constraint of each member.

The contribution of the paper is twofold. First, we study the strategic impact of insufficient cash flow on the payment term through employing ordering quantity effect, margin profit effect and cash constraint effect. Under sufficient cash flow as a benchmark, the retailer always benefits from EP that is due to both ordering quantity and margin profit advantage effect, while the manufacturer benefits from EP (DP) owing to ordering quantity (margin profit) advantage effect dominating margin profit (ordering quantity) disadvantage effect when wholesale price is large (small). Under insufficient cash flow of the retailer, ordering quantity and margin profit probably become disadvantage effect under EP for the retailer on account of her cash constraint disadvantage effect. This can explain that the retailer holds EP either by borrowing bank loan with a large wholesale price or using up all initial cash with a small wholesale price when his initial cash flow is above a certain level, otherwise moves to DP finally, and the manufacturer would prefer DP when wholesale price is large. Moreover, owing to the manufacturer's cash constraint disadvantage effect that has little influence on the payment term for the retailer, ordering quantity advantage effect would dominate margin profit disadvantage effect for the manufacturer when wholesale price is small. This is why the retailer is better off under EP, while the manufacturer would prefer EP as her cash flow decreases.

Second, we characterise the designed supply chain contract that is investigated to mitigate the payment term preference conflict. Under sufficient cash flow, we find that a designed revenue-sharing contract under EP that the corresponding procurement cost depends on not only a positive discount of wholesale price, but also a fraction of supply chain revenue the retailer shares, sheds light on the effect of alleviating payment term preference conflict when satisfying certain conditions. Under insufficient cash flow, a subsidy contract under DP, regarding the procurement cost not only depends on an interest rate, but also a unit subsidy paid by the manufacturer, absolutely mitigates the payment preference conflict in the supply chain when wholesale price is large, and the revenue-sharing contract under the retailer's cash constraint may be effective when wholesale price is small, while it is more effective under the manufacturer's cash constraint.

The rest of the paper is organised as follows. In section 2, we briefly review the related literature. In section 3, we describe the model and key assumptions. In section 4 and 5, the model analyses under sufficient cash flow and insufficient cash flow are presented, respectively. In section 6, we present a set of numerical experiments to examine the main analytical results. Finally, we summarise this paper and suggest avenues for future research. All Proofs are presented in the Appendix.

2. Literature review

Our work studies the choice of payment term for the retailer and preference conflict on the payment term for the manufacturer in a supply chain, and the viable contract to mitigate the conflict between them. Related literature includes two streams: payment terms and supply chain contracts.

In the payment terms literature, the first one was Goyal (1985) who analyses the vendor's economic order quantity with delayed payment. Following Goyal's conclusions, many researchers posed relative issues under more general conditions: Chung (2012), Teng et al. (2012) discussed the situation that demand was a non-decreasing function of time. Huang (2007) studied the economic production quantity with two levels of delayed payment. Besides delayed payment as a given setting, some papers discussed the incentive, coordinated and financial effect of delayed payment. Babich and Tang (2012) showed the delayed payment, as an incentive mechanism, could keep quality of products. Yang and Wee (2006), Luo (2007) took delayed payment, as a decision variable, to coordinate supply chains under a different setting. Jing et al. (2012), Cai et al. (2014) investigated the financing effect of delayed payment where the retailer was capital constraint, and compared it with bank credit. Some previous works also focused on early payment, such as (2009) and Thangam (2012) who developed inventory models with early payment. Furthermore, some researchers compared the different payment terms with newsvendor model. Mateut and Zanchettin (2013) and Jin et al. (2018) inspected the interaction between supplier credit sales and customer's early payment with a discount. Chen et al. (2013) further examined three payment schemes and discussed the effect on inventory decision in a laboratory study. Moreover, Jin et al. (2018) and Yang and Birge (2018) investigated the influence of the retailer's cash flow level on choice of credit strategy, while Jin et al. (2018) also assume that there exits only one payment term between a supplier and a capital-constrained retailer, but Yang and Birge (2018) provided punctual payment and delayed payment for a capital-constrained retailer.

The aforementioned literatures focus on examining the impact and effect of delayed payment and early payment, and most literatures derive the optimal operational decisions given a fixed payment term. Unlike them, we consider the retailer's best choice from three payment terms (e.g. EP, PP and DP) and investigate the impact of insufficient cash flow of the retailer or manufacturer (but not both) on the choice, reveal the preference conflict on payment term for the manufacturer, respectively. This paper would be a complementarily to previous research and give practical advices for the manager of firm on the decision of payment term.

Our paper also relates to the literatures on supply chain contracts, because vertically integrating with a downstream retailer may not be the profit-maximising strategy for a manufacturer under several settings (e.g. Gupta, 2008 Moorthy, 1988;). To alleviate the double marginalisation effect in the decentralised supply chain, many literatures proposed the different contract, including quantity flexibility (e.g. Tsay, 1999), return policies (e.g. Krishnan et al., 2004; Song et al., 2008), quantity discounts (e.g. Raju & Zhang, 2005; Weng, 1995), revenue sharing (e.g. Cachon & Lariviere, 2005), rebate (e.g. Ferguson et al., 2006) and others. These contracts are all designed in a supply chain without capital constraint, while Kouvelis and Zhao (2016) examine that the revenue-sharing contract with constrained working capital can also coordinate the decentralised supply chain. In contrast to them, we focus more on designing the contract to mitigate the preference conflict on payment term between a manufacturer and a retailer, which has received less analysis in above literatures. Moreover, the main difference in our paper is that we design a revenue-sharing contract under EP and a subsidy contract under DP. To the best of our knowledge, this contract schemed in our paper is among the first few to mitigate even resolve the happened conflict on the payment term. Furthermore, we examine the impact of insufficient cash flow on the payment term preference conflict, and investigate the effectivity of the contract, respectively. In addition, the revenue-sharing contract under EP can also coordinate the supply chain simultaneously under certain conditions, that is, our results not only enrich the performance of contract which should be more consistent with practice in a transaction, but also provide the support for the use of revenue-sharing contract or subsidy contract between firms.

3. Model description

We consider a vertical supply chain in which a retailer (he) procures products from an upstream manufacturer (she) and suppose that consumer demand is a linear, downward sloping function which is analytically tractable (e.g. Arya et al., 2007), e.g. $p(q) = a - b \times q$, where a and b are strictly positive constants, p and q represent the market clearing price and the quantity of the goods, respectively. Moreover, the manufacturer produces at a constant marginal cost c. However, we assume that the wholesale price $w(c < w < a)$ for the manufacturer is exogenous (Dong & Rudi, 2004; Ozer & Wei, 2006; Wu et al., 2019), which takes advantage to distinguish the three payment terms, and identify the supply chain members' performance under each payment term (Yang & Birge, 2012).

Furthermore, in our paper, we firstly consider the scenario that each member in the supply chain has enough cash endowment that is considered as the benchmark scenario (Figure 1(a)). And then we consider the scenario that either the retailer or manufacturer (but not both) is constrained by cash flow and can have access to bank market (Figure 1(b,c)). In addition, we suppose that both the manufacturer and the retailer are risk neutral and seek to maximise their own profits with symmetrical information.

The sequence of the event is as follows. Firstly, the retailer moves first to choose one of the three payment terms early payment (EP), delayed payment (DP) and punctual payment (PP). Secondly, the manufacturer decides the discount $\theta, (\theta \geq 0)$ for EP (e.g. Kouvelis & Zhao, 2012), or the interest rate $r, (0 \leq r \leq \bar{r} < 1)$ for DP (e.g. Chen, 2015a). Thirdly, the retailer chooses an order quantity q. The retailer pays to the manufacturer after placing the order instantly under EP or after receiving the products instantly under PP, while maybe approaching a bank loan if his cash flow is not enough. The interest rate of bank credit is r_B. All the above decisions occur in the first stage at time $t = 0$. At the end of the sales season,

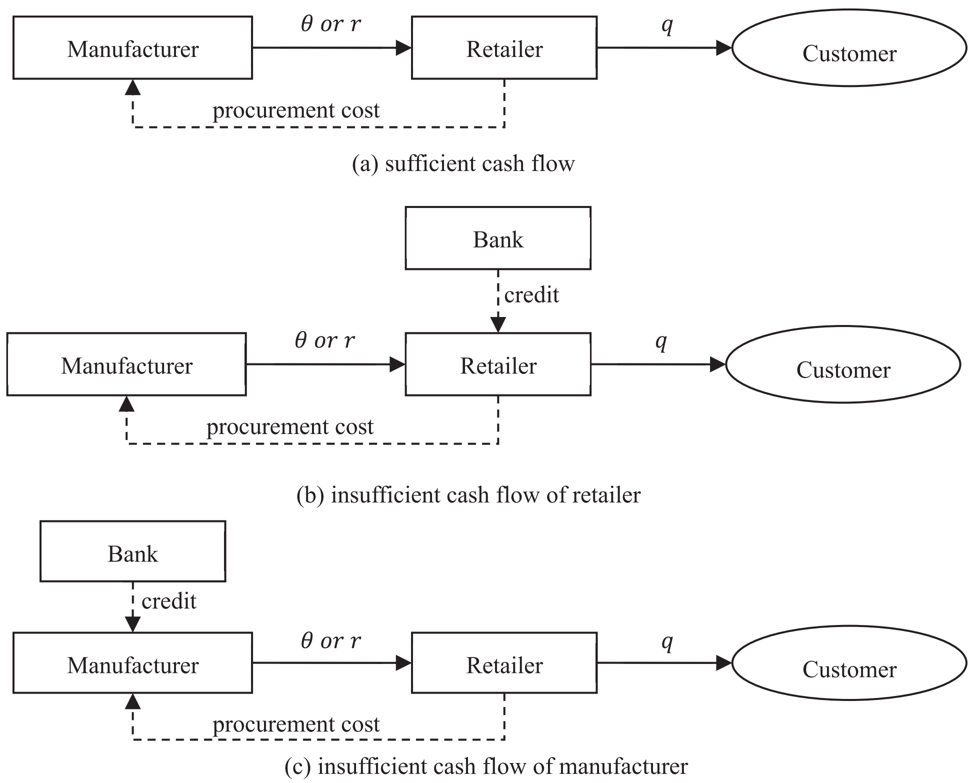

Figure 1. Cash flow and production flow under different scenarios.

$t = 1$, the retailer earns sales revenue and repays to the bank if the bank loan is used in the first stage, or pays to the manufacturer if he chooses DP in the first stage. Alternatively, if the manufacturer is capital constrained, he may get a bank loan to produce when the retailer chooses DP or PP for producing and delivering in the first stage and repays her bank loan after getting the trade credit loan from the retailer in the second stage. In addition, following the convention in interfaces of finance and operations in a supply chain, we assume that the risk-free interest rate is normalised to zero without loss of generality (e.g. Jing et al., 2012; Xu & Birge, 2004). A zero risk-free interest rate grants the convenience by permitting us to focus attention on the comparison of each channel member's operational decision and profit among all payment terms.

Table 1 lists the notations and symbols used in this paper.

Table 1. List of notations and symbols.

Notation	Description
w	The unit exogenous wholesale price
c	The unit production cost
θ	The discount of wholesale price for early payment
$f(\theta) = w/(1+\theta)$	Discounted wholesale price
r	The interest rate of procurement cost for delayed payment
$g(r) = w \times (1+r)$	The wholesale price with interest rate
$\bar{r} \in (0,1)$	The upper bound of interest rate for delayed payment
$r_B \in (0,1)$	The interest rate of bank credit
F_j	Initial cash of firm $j \in \{R, M\}$
ϕ	The fraction of supply chain revenue the retailer keeps
q_R^i	The optimal ordering quantity of retailer under the payment term $i \in \{P, E, D\}$ where P represents punctual payment, E represents early payment and D represents delayed payment
π_j^i	The optimal profit of firm $j \in \{R, M\}$ under the payment term i where R represents the retailer and M represents the manufacturer
$\Delta_j^i = \pi_j^i / q_R^i$	The margin profit of firm j under the payment term i

4. Benchmark: analysis with sufficient cash flow

To quantify the impact of insufficient cash flow on the payment term, we first establish a benchmark model in which the retailer and the manufacturer both hoard sufficient cash. We investigate the firms' equilibrium outcomes under each payment term and identify the optimal choice for the retailer and preference conflict in a supply chain, respectively. Moreover, we design a revenue-sharing contract to mitigate the happened conflict on the payment term between them.

4.1. Solution derivation under each payment term

In this part, we analyse the equilibrium results regarding operational decisions and profits for channel members through backward induction under each payment term.

(1) Under PP. When wholesale price w is exogenously given, the retailer chooses output q_R^P to maximise his profit. The problem is

$$\pi_R^P = \max_{q_R^P}[p(q_R^P) - w]q_R^P. \tag{1}$$

(2) Under EP. The retailer decides output q_R^E to maximise his profit under a given discount of wholesale price θ. The retailer's problem is

$$\pi_R^E = \max_{q_R^A}[p(q_R^E) - f(\theta)]q_R^E. \tag{2}$$

With the retailer's response to discount, the manufacturer chooses θ to maximise her profit. Her problem is

$$\pi_M^E = \max_{\theta}[f(\theta) - c]q_R^E \tag{3}$$

(3) Under DP. The retailer decides q_R^D to maximise his profit as an interest rate r of total purchasing cost is given. The retailer's problem is

$$\pi_R^D = \max_{q_R^D} p(q_R^D)q_R^D - wq_R^D(1+r). \tag{4}$$

Then, the manufacturer chooses r to maximise her profit. Her problem is

$$\pi_M^D = \max_{r}[g(r) - c]q_R^D. \tag{5}$$

The profits under each payment above are all a concave function; thus, through first-order condition for a maximum and backward induction, equilibrium results and the corresponding lowest cash flow level are shown in Table 2 to analyse the scenario in which the retailer or manufacturer suffers from insufficient cash flow in section 5. For convenience, we denote the corresponding lowest cash flow level by $\underline{F}_{j,K}^i$, $j \in \{R, M\}, K \in \{h, m, l\}, i \in \{P, E, D\}$, where h represents the high wholesale price (e.g. $w \geq \hat{w}$), m represents the medium wholesale price (e.g. $\tilde{w} \leq w < \hat{w}$), l represents the low wholesale price (e.g. $c < w < \tilde{w}$), and $\hat{w} = (a+c)/2$, $\tilde{w} = (a+c)/[2(1+\bar{r})]$; moreover, the difference in the retailer's (manufacturer's) optimal profit between EP and DP under different range of wholesale price is denoted by R_K^{ED} (M_K^{DE}) e.g. $R_h^{ED} = \pi_R^{E*} - \pi_R^{D*}$, $M_h^{DE} = \pi_M^{D*} - \pi_M^{E*}$.

4.2. Payment term preference conflict and contract design

In this part, we explore the optimal choice of the retailer and the preference conflict between the manufacturer and the retailer which are described in the Proposition 1. Then, we develop a viable supply chain contract to mitigate the conflict, as described in Proposition 2.

Proposition 1. Under the scenario of each member with sufficient cash flow, then

(i) *When $w \geq \hat{w}$, then $q_R^{E*} \geq q_R^{D*} = q_R^{P*}$, $\Delta_R^{E*} \geq \Delta_R^{D*} = \Delta_R^{P*}$, $\Delta_M^{D*} = \Delta_M^{P*} \geq \Delta_M^{E*}$; $\pi_R^{E*} \geq \pi_R^{D*} = \pi_R^{P*}$, $\pi_M^{E*} \geq \pi_M^{D*} = \pi_M^{P*}$, which means the retailer and manufacturer are all better off under EP.*

(ii) *When $c < w < \hat{w}$, then $q_R^{E*} = q_R^{P*} > q_R^{D*}$, $\Delta_R^{E*} = \Delta_R^{P*} > \Delta_R^{D*}$, $\Delta_M^{D*} > \Delta_M^{P*} = \Delta_M^{E*}$; $\pi_R^{E*} = \pi_R^{P*} > \pi_R^{D*}$, $\pi_M^{E*} = \pi_M^{P*} < \pi_M^{D*}$, which means the retailer is seldom better off under DP, while the manufacturer is better off under DP.*

Proposition 1 states that if wholesale price is larger than a threshold, the manufacturer sets a positive discount under EP and a zero interest rate under DP. The retailer orders the largest quantity to get the positive discount and gets the highest margin profit under EP. Although the manufacturer's margin profit is the lowest under EP, the ordering quantity advantage effect dominates the margin profit disadvantage effect for the manufacturer. In this case, the retailer absolutely chooses EP under which the manufacturer is also better off. On the other hand, if wholesale price is smaller than the threshold, the manufacturer sets a zero discount under EP and a positive interest rate under DP which results in the identical performances of retailer under EP and PP, the lowest quantity and profit/marginal profit of retailer under DP. While the marginal profit of the manufacturer under DP is more than that under other payment, and marginal profit advantage effect dominates the ordering quantity disadvantage effect for the manufacturer. In this case, the retailer seldom chooses DP from which the manufacturer would benefit.

Payment term preference conflict arises when wholesale price is smaller than the threshold. Thus, we would develop a viable supply chain contract to mitigate the conflict in the following proposition.

Proposition 2. *Under sufficient cash flow of each member, a revenue-sharing contract involving the procurement cost $T^{EC}(\phi, \theta)$ under EP when the wholesale price is low, can alleviate the payment term preference conflict between the retailer and the manufacturer.*

Where $T^{EC}(\phi, \theta) = f(\theta)q_R^{EC} + (1-\phi)p(q_R^{EC})q_R^{EC}$.

Proposition 2 states that when wholesale price is low (e.g. $w < \hat{w}$), we characterise the revenue-sharing contract, schemed by the manufacturer under EP, that the corresponding procurement cost depends on not only a positive discount of wholesale price, but also a fraction of supply chain revenue the retailer shares. Moreover, we find that if the fraction of supply chain revenue is always equal to one, unit procurement cost with the contract is the same as that without the contract, e.g. $T^{EC}(1,\theta)/q_R^{EC} = f(\theta)$. Therefore, to distinguish the procurement cost only depends on the discount; hereafter, EP with revenue-sharing contract is indexed by EPC.

Furthermore, if both the fraction and the discount satisfy certain conditions (see proof of Proposition 2), the output of retailer under EPC is higher than those under other payments, even equal to that of centralised supply chain. Then, the retailer extracts the highest profit under EPC; meanwhile, the manufacturer earns more profit than that under EP or PP. In other words, the optimal profit of manufacturer under EPC may approach or outweigh that under DP. Therefore, the revenue-sharing contract sheds light on the effect of mitigating the conflict of the payment term. In addition, this contract also shows the effect of coordination in a supply chain that the profit of decentralised supply chain under EPC is equal to that of centralised supply chain.

5. Analysis with insufficient cash flow

In this section, considering the scenario in which either the retailer or manufacturer (but not both) has insufficient cash flow, respectively, we derive the optimal choice for and preference of the payment term for the manufacturer to investigate the impact of insufficient cash flow on the payment term and the payment term conflict, further mitigate the appearing conflict through the designed supply chain contract. Moreover, both the retailer and manufacturer can relieve the issue of cash constraint through suitable payment term (Chen, 2015a, 2015b), or borrowing bank credit (Kouvelis & Zhao, 2012; Yan & Sun, 2013), that is, the initial cash flow of retailer (manufacturer) F_R (F_M) raises up to $F_R + F_{RB}$ ($F_M + F_{MB}$) by borrowing a loan F_{RB} (F_{MB}), then he would pay $F_{RB}(1 + r_B)$ ($F_{MB}(1 + r_B)$) with an exogenous interest rate of bank r_B at the end of the period. It well fits the situation where there is a competitive bank market. We also assume that retailer (manufacturer) has no other investment opportunity besides his (her) retail (production) business.

5.1. Insufficient cash flow of the retailer

In this part, we consider that the retailer is endowed with cash constraint to procure, that is, his initial cash flows less than the lowest cash flow level under EP shown in Table 2 e.g. $F_R < \underline{F}^E_{R,K}$, while the manufacturer has enough cash flow. Under this situation, to avoid repetition, we will straightly focus on the optimal choice of payment term for the retailer that derived by the following proposition.

Proposition 3. *With insufficient cash flow of the retailer $F_R < \underline{F}^E_{R,K}$,*

(i) *If $w \geq \hat{w}$, the retailer chooses EP with bank credit when his initial cash is large, but not enough, e.g. $\underline{F}^E_{R,h} - R_h^{ED}/(1 + r_B) \leq F_R < \underline{F}^E_{R,h}$, otherwise chooses DP, e.g. $0 \leq F_R < \underline{F}^E_{R,h} - R_h^{ED}/(1 + r_B)$;*
(ii) *If $\tilde{w} \leq w < \hat{w}$, the retailer chooses EP or PP without bank credit when his initial cash is large, but not enough, e.g. $\hat{F}_R \leq F_R < \underline{F}^E_{R,m}$, otherwise chooses DP, e.g. $0 \leq F_R < \hat{F}_R$;*

Table 2. The optimal decisions and profits under each payment term and the corresponding lowest cash flow level

		EP		DP		
	PP	$w \geq \hat{w}$	$c < w < \hat{w}$	$w \geq \hat{w}$	$\tilde{w} \leq w < \hat{w}$	$c < w < \tilde{w}$
q_R^{i*}	$\frac{a-w}{2b}$	$\frac{a-c}{4b}$	$\frac{a-w}{2b}$	$\frac{a-w}{2b}$	$\frac{a-c}{4b}$	$\frac{a-\bar{w}}{2b}$
θ^*	—	$\frac{2w-(a+c_0)}{a+c_0}$	0	—	—	—
r^*	—	—	—	0	$\frac{a+c-2w}{2w}$	$\bar{r}$
π_R^{i*}	$\frac{(a-w)^2}{4b}$	$\frac{(a-c)^2}{16b}$	$\frac{(a-w)^2}{4b}$	$\frac{(a-w)^2}{4b}$	$\frac{(a-c)^2}{16b}$	$\frac{[a-\bar{w}]^2}{4b}$
π_M^{i*}	$\frac{(a-w)(w-c)}{2b}$	$\frac{(a-c)^2}{8b}$	$\frac{(a-w)(w-c)}{2b}$	$\frac{(a-w)(w-c)}{2b}$	$\frac{(a-c)^2}{8b}$	$\frac{(\bar{w}-c)(a-\bar{w})}{2b}$
Δ_R^{i*}	$\frac{a-w}{2}$	$\frac{a-c}{4}$	$\frac{a-w}{2}$	$\frac{a-w}{2}$	$\frac{a-c}{4}$	$\frac{a-\bar{w}}{2}$
Δ_M^{i*}	$w-c$	$\frac{a-c}{2}$	$w-c$	$w-c$	$\frac{a-c}{2}$	$\bar{w}-c$
$\underline{F}^i_{R,K}$	wq_R^{P*}	$\frac{a+c}{2}q_R^{E*}$	wq_R^{E*}	0	0	0
$\underline{F}^i_{M,K}$	cq_R^{P*}	0	0	cq_R^{D*}	cq_R^{D*}	cq_R^{D*}

Note: $\bar{w} = w \times (1 + \bar{r})$, $\hat{w} = \frac{a+c}{2}$, $\tilde{w} = \frac{a+c}{2(1+\bar{r})}$.

(iii) *If* $max\left\{c_0, \frac{2a\bar{r}}{(2+\bar{r})\bar{r}+4(1+r_B)^2}\right\} < w < \tilde{w}$, *the retailer chooses EP or PP without bank credit when his initial cash is large, but not enough, e.g.* $\tilde{F}_R \le F_R < \underline{F}^E_{R,m}$, *otherwise chooses DP, e.g.* $0 \le F_R < \tilde{F}_R$;

where $R^{ED}_h = \frac{(a-c_0)^2}{16b} - \frac{(a-w)^2}{4b}$, $R^{ED}_m = \frac{(a-w)^2}{4b} - \frac{(a-c_0)^2}{16b}$, $R^{ED}_l = \frac{(a-w)^2}{4b} - \frac{[a-\bar{w}]^2}{4b}$, *and* $\hat{F}_R = \underline{F}^E_{R,m} - \frac{w}{\sqrt{b}}\sqrt{R^{AD}_m}$, $\tilde{F}_R = \underline{F}^E_{R,l} - \frac{w}{\sqrt{b}}\sqrt{R^{ED}_l}$.

From Proposition 3, under insufficient cash flow of the retailer, if wholesale price is large and the retailer's initial cash is large, but not enough, he adopts bank credit to choose EP to get a positive discount, higher profit. In this case, the increased profit with adopting bank credit can compensate the increased bank interest cost. If wholesale price is medium or small, and the retailer's initial cash is large, but not enough, then the manufacturer set little EP discount that the retailer uses up his own cash, not borrowing money from bank, to choose EP. While the retailer would choose DP as his initial cash is not large, decreasing below the corresponding threshold, to get higher profit. It implies that the output of the retailer under EP is much less than that under DP owing to lower cash flow.

The next proposition summarises the preference of the manufacturer under the insufficient cash flow of the retailer.

Proposition 4. *Under insufficient cash flow of the retailer* $F_R < \underline{F}^E_{R,K}$,

(i) *When* $w \ge \hat{w}$,
   (a) *If the retailer's initial cash is large, but not enough, e.g.* $\underline{F}^E_{R,h} - R^{ED}_h/(1+r_B) \le F_R < \underline{F}^E_{R,h}$, *then the manufacturer is better off under EP;*
   (b) *If the retailer's initial cash is medium, e.g.* $\underline{F}^P_{R,h} \le F_R < \underline{F}^E_{R,h} - R^{ED}_h/(1+r_B)$, *then the manufacturer is better off under EP ifa* $> 3c, r_B \ge \widehat{r_B} - 1$ *or* $c < a \le 3c$; *otherwise DP if* $a > 3c$*and*$0 < r_B < \widehat{r_B} - 1$;
   (c) *If the retailer's initial cash is small, e.g.* $0 \le F_R < \underline{F}^E_{R,h} - R^{ED}_h/(1+r_B)$, *then the manufacturer is better off under DP;*

(ii) *When* $c < w < \hat{w}$, *the manufacturer is better off under DP.*

where $\widehat{r_B} = \frac{R^{ED}_h}{\underline{F}^E_{R,h}-\underline{F}^P_{R,h}}$.

Proposition 4 shows how the retailer's initial cash and bank loan interest impact the manufacturer's preference of payment term. When wholesale price and the retailer's initial cash are large, the manufacturer is better off under EP, because the retailer borrows not too much cash from the bank to realise the same order quantity with the retailer's sufficient cash. When wholesale price is large and the retailer's initial cash is medium, the manufacturer is better off under DP if the interest rate of bank is small (e.g. $r_B < \widehat{r_B} - 1$) and the production cost is small (e.g. $a > 3c$), otherwise EP, while the retailer orders the lowest quantity and gets lowest profit/marginal profit under EP. The reason is that small bank interest rate decreases the threshold of the retailer' cash flow e.g. $\underline{F}^E_{R,K} - R^{ED}_h/(1+r_B)$, in turn decreases the ordering quantity under EP, and small manufacturer's production cost increases the manufacturer's margin profit under DP, all these enhance the margin profit advantage effect for the manufacture under DP. When wholesale price is large and the retailer's initial cash is small or wholesale price is small, the manufacturer sets a zero discount under EP, a positive interest rate under DP and is better off under DP. Therefore, the payment term preference conflict arises, when wholesale price is small and the retailer's initial cash is large or wholesale price is large, the retailer's initial cash is medium and $a > 3c, r_B \ge \widehat{r_B} - 1$ *or* $c < a \le 3c$ (Proposition 3). We compare the optimal choice and preference of the payment term with or without sufficient cash flow in Section 5.3.

5.2. Insufficient cash flow of the manufacturer

In this part, we consider that the manufacturer is endowed with cash constraint to produce, that is, her initial cash flow is less than the lowest cash flow level under DP shown in Table 2 e.g. $F_M < \underline{F}^D_{M,K}$, while the retailer has sufficient cash flow. Under this situation, to avoid repetition, we directly derive the optimal choice of payment term for the retailer in the following proposition.

Proposition 5. *Under insufficient cash flow of the manufacturer* $0 \le F_M < \underline{F}^D_{M,K} \le \underline{F}^P_{M,K}$, *then*

(i) *If* $w \ge \hat{w}$, *the retailer chooses EP;*
(ii) *If* $3c < w < \hat{w}$, *the retailer chooses EP or PP with the manufacturer's borrowing from bank;*
(iii) *If* $2c < w < 3c$, *then the retailer chooses EP or PP with the manufacturer's borrowing from bank if* $0 < r_B < (w-2c)/c$, *and EP if* $(w-2c)/c < r_B < 1$;
(iv) *If* $c < w \le 2c$, *then the retailer chooses EP.*

From Proposition 5, we find that EP is always an optimal option for the retailer regardless of wholesale price, while PP would become an optimal choice only if wholesale price is medium (e.g. $3c < w < \hat{w}$), or the interest rate of bank is low (e.g. $0 < r_B < (w-2c)/c$), because PP requires the manufacturer to hold certain cash flow by borrowing from bank to satisfy the retailer's ordering, and increased profit with bank credit under PP for manufacture can compensate the interest through lower interest rate or higher profit for the manufacturer under PP with medium wholesale price.

Next proposition summarises the preference of the manufacturer with insufficient cash flow.

Proposition 6. *With insufficient cash flow of the manufacturer $F_M < \underline{F}_{M,K}^D$, then*

(i) *When $w \geq \hat{w}$, then $\forall F_M, F_M \geq 0$, the manufacturer is better off under EP;*

(ii) *When $\tilde{w} \leq w < \hat{w}$ and $0 < \bar{r} < \min\left\{\frac{2c(1+r_B)}{a-c-2cr_B}, \frac{1}{\frac{a+c}{\sqrt{2c(a-c)}}-1}\right\} \leq 1$, manufacturer is better off under EP if the manufacturer's initial cash is small, e.g. $0 \leq F_M < \hat{F}_M$, otherwise DP without bank credit if $\hat{F}_M \leq F_M < \underline{F}_{M,m}^D$;*

(iii) *When $c < w < \tilde{w}$,*

Case 1: If $c < \bar{w} \leq 2c$ or $2c < \bar{w} < 3c$ and $(\bar{w}-2c)/c < r_B < 1$, then, *the manufacturer is better off under DP without bank credit when $\tilde{F}_M \leq F_M < \underline{F}_{M,l}^D$, and EP when $0 \leq F_M < \tilde{F}_M$;*

Case 2: If $\bar{w} \geq 3c$ or $2c < \bar{w} < 3c$ and $0 < r_B \leq (\bar{w}-2c)/c$, then, the manufacturer is better off under DP with bank credit when $\check{F}_M \leq F_M < \underline{F}_{M,l}^D$, and *EP when $0 \leq F_M < \check{F}_M$.*

where

$$M_m^{DE} = \frac{(a-c)^2}{8b} - \frac{(w-c)(a-w)}{2b},$$

$$M_l^{DE} = \frac{(\bar{w}-c)(a-\bar{w})}{2b} - \frac{(w-c)(a-w)}{2b},$$

$$\hat{F}_M = \underline{F}_{M,m}^D - \frac{c\sqrt{2M_m^{DE}}}{2\sqrt{b}},$$

$$\tilde{F}_M = \underline{F}_{M,l}^D - \frac{cM_l^{DE}}{\bar{w}-c},$$

$$\check{F}_M = \underline{F}_{M,l}^D - \frac{M_l^{DE}}{1+r_B}$$

As demonstrated in Proposition 6, we find that when wholesale price is larger than the threshold, the manufacturer benefits from EP without regard to her initial cash flow. The reason is that in spite of insufficient cash flow, the manufacture can wholly satisfy the retailer's optimal ordering under EP that also generates highest profit for her. It implies that the manufacturer will fund its production from the retailer in this case. While when wholesale price is medium and the upper bound of interest rate for DP is small, the manufacturer benefits from DP without bank credit if her initial cash flow is not small (e.g.$\hat{F}_M \leq F_M < \underline{F}_{M,m}^D$), and EP if her initial cash flow is small.

Moreover, on the one hand, If the upper bound of interest rate for DP is small (e.g.$c < \bar{w} \leq 2c$), or if interest rate for DP is medium and interest rate of bank ($(\bar{w}-2c)/c < r_B < 1$) is higher, the manufacturer with small wholesale price benefits from DP without bank credit when the cash flow is more than a threshold (e.g. $\tilde{F}_M$), and benefits from EP when the cash flow is smaller than the threshold. The main reason is that the lower upper bound of interest rate for DP would decrease the manufacturer's profit under DP, and higher interest rate of bank would increase the interest of bank credit, thus, increased profit with bank credit under DP cannot compensate the interest of bank credit. On the other hand, if the upper bound of interest rate for DP is large (e.g. $\bar{w} \geq 3c$), or if medium interest rate for DP and interest rate of bank is small (e.g. $0 < r_B \leq (\bar{w}-2c)/c$), the manufacturer benefits from DP with bank credit when the cash flow is more than a threshold (e.g. $\check{F}_M$), and benefits from EP when the cash flow is smaller than the threshold. Because increased profit with bank credit under DP can compensate the interest of bank credit through higher margin profit and lower interest rate of bank.

In addition, we find that although the manufacturer suffers from cash constraint, she still benefits from DP if the cash flow is more than the threshold $\hat{F}_M, \tilde{F}_M, \check{F}_M$. That is, the cash flow of the manufacturer can still cover certain product cost, and margin profit advantage effect dominates the ordering quantity disadvantage effect under DP.

5.3. Impact of insufficient cash flow

We have presented the optimal choice of payment term for the retailer and the preference conflict of the payment term under sufficient cash flow scenario and insufficient cash flow scenario, as shown in Figure 2. In this part, we firstly examine the impact of insufficient cash flow on the payment term and the preference conflict through comparisons. Secondly, we design supply chain contract to mitigate the appearing conflict.

Proposition 7.

(i) ***Impact of insufficient cash flow on the payment term decisions****: under insufficient cash flow of retailer, he chooses DP instead of EP if his initial cash is smaller than the threshold, otherwise he still chooses EP by borrowing bank loan with a large wholesale price or using up all initial cash with a small wholesale price. Under insufficient cash flow of the manufacturer, the retailer chooses EP instead of EP or PP if her initial cash is smaller than the threshold with a small wholesale price.*

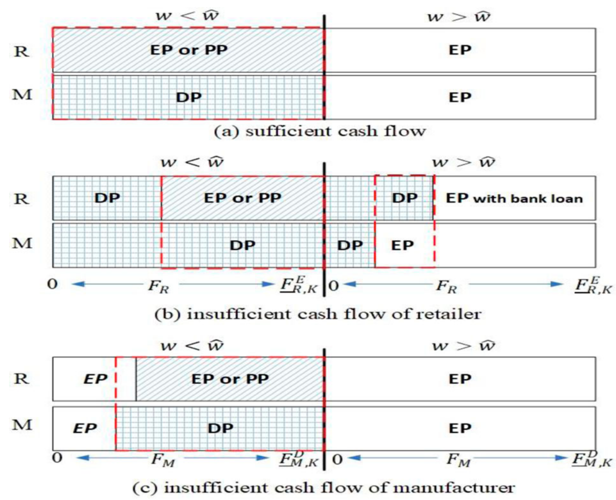

Figure 2. The optimal choice of payment term and preference of payment term with or without sufficient cash flow.

(ii) ***Impact of insufficient cash flow on payment term preference conflicts:*** *under insufficient cash flow of the retailer, there arises a new region of payment term conflict when wholesale price is large and the retailer's cash flow is medium with small production cost ($a > 3c$), large interest rate of bank ($r_B \geq \widehat{r_B} - 1$) or lager production cost ($c < a \leq 3c$). Under insufficient cash flow of each member, the region of payment term conflict becomes small when whole price is lower than the threshold.*

On the one hand, comparing the results from Figure 1(a) with that from Figure 1(b), we investigate that insufficient cash flow of the retailer would make the choice of payment that deviates to DP as his cash flow decreases regardless of wholesale price. The reason is that with the lower cash flow level, the output of the retailer under EP (or PP) is much less than that under PP, then ordering quantity and the margin profit probably become disadvantage effect under EP on account of the retailer's cash constraint disadvantage effect. Therefore, the preference of payment for the manufacturer also moves from EP to DP when wholesale price is high (e.g. $w \geq \hat{w}$). This result is consistent with that in Cai et al., 2014; Chen 2015a that DP shows the effect of financing for cash-constrained retailer, and all channel members can benefit from DP, but the assumption of retailer endowed with no cash flow in Cai et al., 2014; Chen 2015a is a special case in our study.

On the other hand, comparing the results from Figure 1(a) with that from Figure 1(c), we find that the insufficient cash flow of the manufacturer has little influence on the optimal choice of the retailer and preference of the manufacturer when wholesale price is high, otherwise partly narrows the option of optimal choice for the retailer and influences the preference for the manufacturer that would move from DP to EP as her cash flow decreases. On account of the lower cash flow and small wholesale price, the retailer can only choose EP, because PP also requires the manufacture to endow certain cash flow, in addition, the manufacturer cannot provide certain product under DP in spite of higher margin profit. That is, the order quantity advantage effect dominates the margin profit disadvantage effect under EP for the manufacturer with her cash constraint disadvantage effect when wholesale sale price is small. Moreover, whether each channel member is endowed with sufficient cash flow or cash constraint, the manufacturer rarely prefers to PP. This result can explain why the firms rarely employ PP in practice.

Next, we easily find that the payment term conflict still happened under the retailer's insufficient cash flow shown in the right part of in Figure 2(b) when wholesale price is higher than the threshold, e.g. $w \geq \hat{w}$, the retailer chooses DP, while the manufacturer still prefers to EP. In the following proposition, we would develop another contract to solve the happened conflict.

Proposition 8. *When wholesale price is larger than the threshold ($w \geq \hat{w}$) and $\underline{F}^{P}_{R,h} \leq F_R < \underline{F}^{E}_{R,h} - R^{ED}_{h}/(1 + r_B)$, a designed subsidy contract regarding procurement cost $T^{DC}(\rho, r) = w \cdot (1 + r)q^{DC}_R - \rho q^{DC}_R$ under DP can solve the happened conflict of payment term.*
where q^{DC}_R is the output for the retailer under DP with the contract, $\rho \geq 0$ is unit subsidy paid by the manufacturer to the retailer.

Under DP with the subsidy contract, the optimal profit of the retailer is more than that without the contract; moreover, the optimal profit of the manufacturer is also more than that under EP. Therefore, the retailer and manufacturer would agree with DP with the subsidy contract, that is, such a contract can not only alleviate the conflict of the payment term between the retailer and manufacturer, but also achieve Pareto improvement for each channel member.

When wholesale price is lower than the threshold, e.g. $w < \hat{w}$, there exists a payment term conflict where the manufacturer still benefits from DP and the retailer still chooses EP or PP under insufficient cash flow of the retailer in the left part of Figure 2(b) or the manufacturer in the left part of Figure 2(c). Therefore, we examine whether the revenue-sharing contract mentioned in Proposition 2 is still effective in the following proposition.

Proposition 9. *When wholesale price is lower than the threshold, $w < \hat{w}$, compared with that under sufficient cash flow, then,*

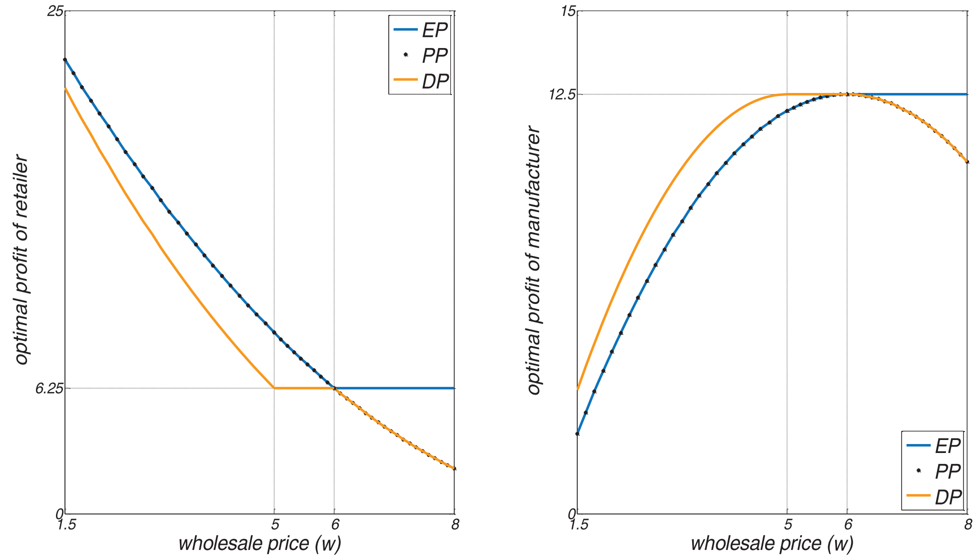

Figure 3. The optimal profit for the retailer and manufacturer with sufficient cash flow under each payment.

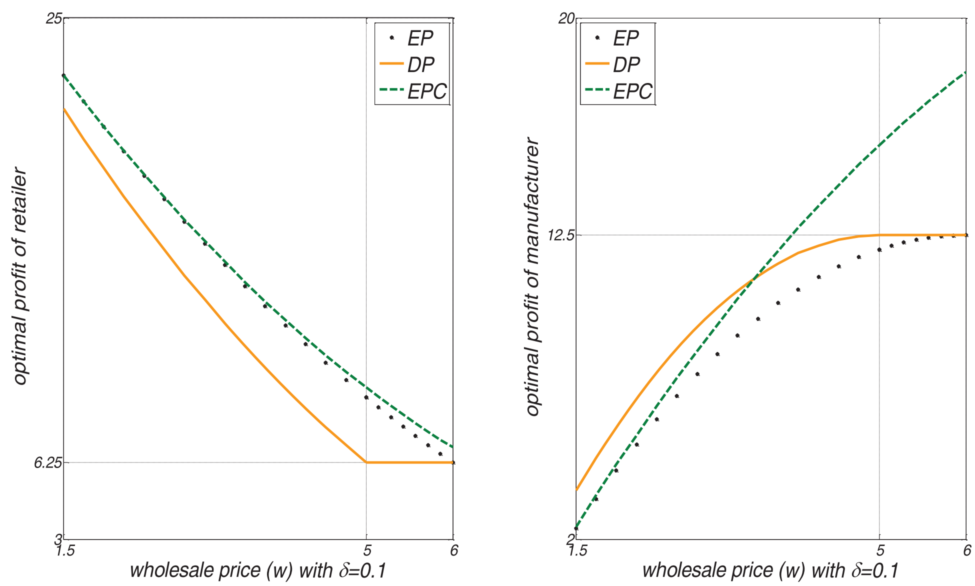

Figure 4. The optimal profit for each member under EPC, DP and EP.

(i) *Under insufficient cash flow of the retailer, the revenue-sharing contract would be effective if the retailer's cash flow is not small, otherwise would be ineffective;*

(ii) *Under insufficient cash flow of the manufacturer, the revenue-sharing contract would be more effective to mitigating the payment term conflict.*

Executing the revenue-sharing contract under EP requires retailers to be endowed with certain cash flow. Therefore, Proposition 9 shows that on the one hand, this contract under the retailer's insufficient cash flow would be ineffective if cash flow is small; on the other hand, although the manufacturer with insufficient cash flow prefers DP, the optimal profit is less than that with sufficient cash flow. That is, the optimal profit of the manufacturer with revenue-sharing contract is more likely than that under DP under insufficient cash flow of the manufacturer. Therefore, the contract would be more effective. This result provides the advice of using the revenue-sharing with capital-constrained downstream or upstream firms.

6. Numerical study

In this section, firstly, we conduct a set of numerical computation for demonstrating the optimal profit of each member with sufficient cash flow under each payment. Furthermore, we illustrate the optimal profit of each member under EP with revenue-sharing contract to show

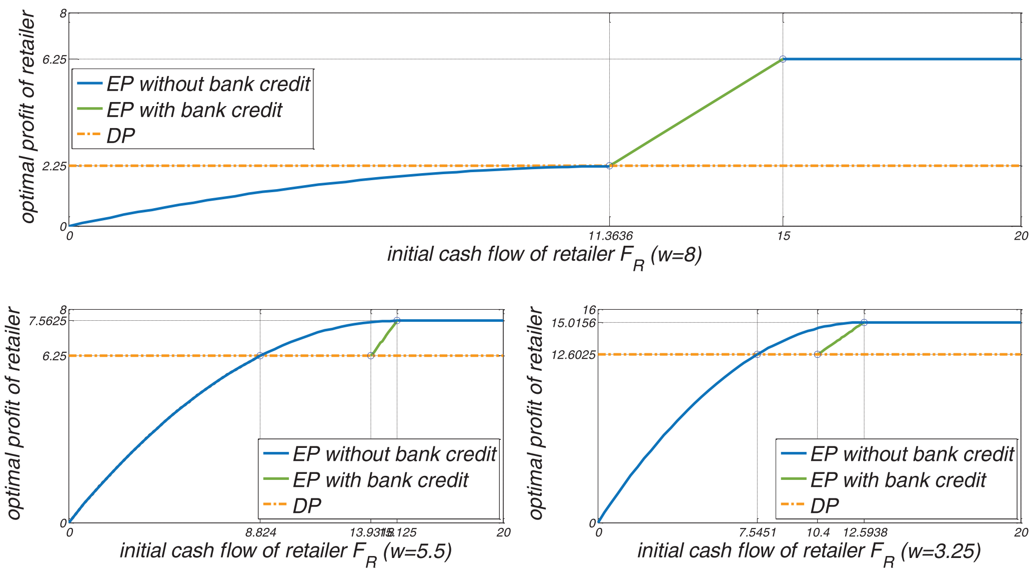

Figure 5. The optimal profit of the retailer with his initial cash flow under different payments.

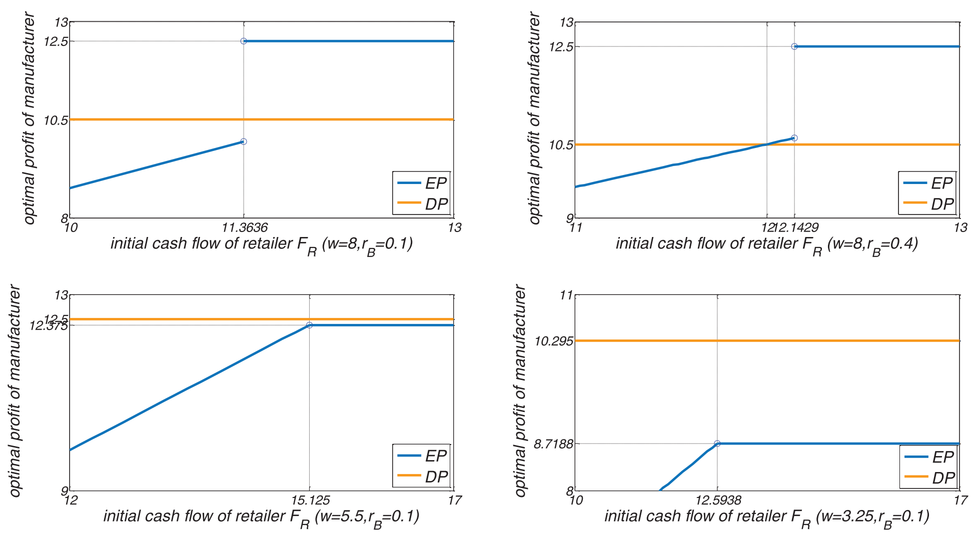

Figure 6. The optimal profit of the manufacturer with the retailer's initial cash flow under different payments.

the effect of contract on mitigating the payment term conflict. To perform these experiments, we assume that $a = 11, c = 1, b = 1, \bar{r} = 0.2, r_B = 0.1$.

As shown in Figure 3, on the one hand, both the retailer and the manufacturer extract the highest profit under EP when wholesale price is larger than a threshold e.g. $w = 6$, otherwise, the retailer earns the lowest profit under DP, while the manufacturer earns the highest profit. Thus, there exists a payment term conflict.

Then, by employing the revenue-sharing contract from Proposition 2, we define $\delta = \frac{\phi\Pi^I - \pi_R^{A*}}{\Pi^I - \pi_R^{A*} - \pi_M^{A*}} \in (0, 1)$, which denote the rate of the retailer's increased profit, compared with increased profit for decentralised channel with the contract. Thus, $\pi_M^{EC*} = (1-\phi)\Pi^I = \pi_M^{E*} + (1-\delta)(\Pi^I - \pi_R^{E*} - \pi_M^{E*})$ and $\pi_R^{EC*} = \phi\Pi^I = \pi_R^{E*} + \delta(\Pi^I - \pi_R^{E*} - \pi_M^{E*})$. Moreover, without contract, the performance of PP is the same as EP or DP, thus, we do not plot the optimal profit for each member under DP, hereafter. As depicted in Figure 4 e.g. $\delta = 0.1$, the optimal profit of the retailer under EPC is the highest, and the optimal profit of the manufacturer under EPC is more than that under EP, sometimes even higher than that under DP. Therefore, the tailored contract shows the effect of mitigating the payment term conflict, even achieving the total agreement on EPC.

Secondly, under the insufficient cash flow of retailer, as shown in Figure 5, we find that when wholesale price is larger (smaller) than the threshold, e.g. $w = 8$ (e.g. $w = 5.5$ or $w = 3.25$), the retailer would choose EP with (without) bank credit if cash flow is lower than $\underline{F}_{R,K}^E$, but more than a threshold e.g. $\underline{F}_{R,K}^E - R_h^{ED}/(1 + r_B) = 11.3636$

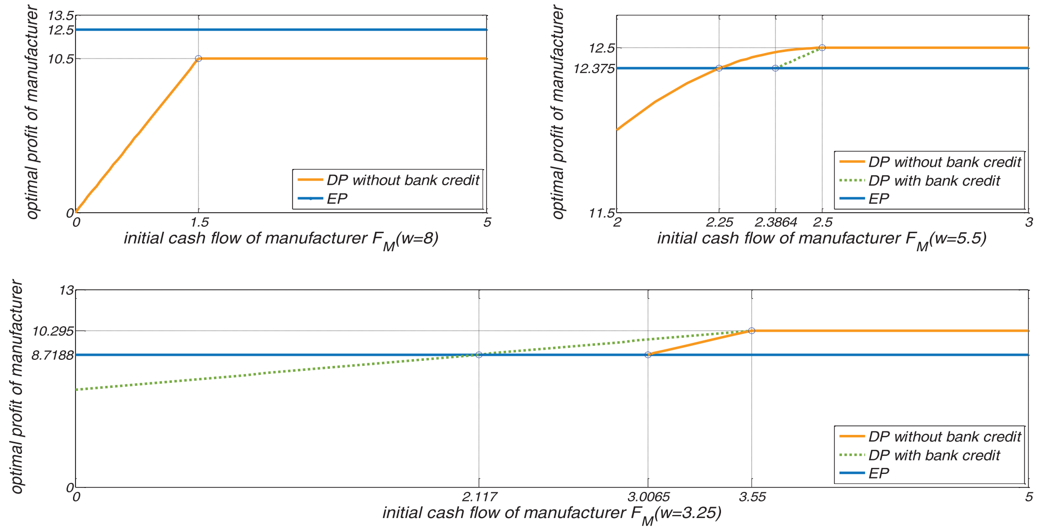

Figure 7. The optimal profit of the manufacturer with her initial cash flow under different payments.

(e.g. $\hat{F}_R = 8.824$ or $\tilde{F}_R = 7.5451$), otherwise chooses DP. For the manufacturer under the retailer's insufficient cash flow, as illustrated in Figure 6, when wholesale price is larger than the threshold and interest rate from bank is low, e.g. $w = 8, r_B = 0.1$, she benefits from EP if $11.3636 = \underline{F}^E_{R,K} - R^{ED}_h/(1+r_B) \le F_R < \underline{F}^E_{R,h}$, otherwise DP. When the interest rate from bank is high, e.g. $w = 8, r_B = 0.4$, the manufacture obviously benefits from EP obviously if $12.1429 = \underline{F}^E_{R,K} - R^{ED}_h/(1+r_B) \le F_R < \underline{F}^E_{R,h}$, then there exists a zone that the manufacturer still benefits from EP if $12 = \underline{F}^P_{R,h} \le F_R < 12.1429$, finally the manufacturer benefits from DP if $0 \le F_R < 12$. In addition, when wholesale price is small, e.g. $w = 5.5$ or $w = 3.25$, the manufacturer always benefits from DP.

Thirdly, as Figure 7 shows the preference of the manufacturer depends on her initial cash flow, when wholesale price is larger than the threshold, e.g. $w = 8$, the manufacturer always benefits from EP; when the wholesale price is smaller than the threshold, e.g. $w = 5.5$ ($w = 3.25$), the manufacturer still benefits from DP without (with) bank credit if cash flow is lower than the lowest cash flow under DP, e.g. $\underline{F}^D_{M,h} = 2.5$ ($\underline{F}^D_{M,l} = 3.55$), but higher than a threshold e.g. $\hat{F}_M = 2.25$ ($\underline{F}^D_{M,l} - M^{DE}_l/(1+r_B) = 2.117$). Finally, she would benefit from EP if cash flow is lower than the threshold.

7. Conclusion

We derive several interesting results. Under the situation of each member having sufficient cash flow, the retailer would choose EP from which the manufacturer benefits when wholesale price is higher than a threshold, otherwise, the retailer seldom chooses DP from which the manufacturer benefits. Therefore, a payment term conflict between them exactly arises, thus a revenue-sharing contract schemed by the manufacturer under EP, regarding procurement cost depends on not only a discount but also a fraction of supply chain revenue, demonstrates the effect of mitigating the conflict and coordinating the supply chain simultaneously.

Furthermore, on the one hand, it is noteworthy that insufficient cash flow of the retailer has deep effect on his optimal choice that would move to DP finally regardless of wholesale price because of cash constraint disadvantage effect. In addition, the preference of the payment term for the manufacturer would also deviate from EP to DP when wholesale price is higher than a threshold; furthermore, we design a subsidy contract to solve the happened conflict in this situation. While the manufacturer still benefits from DP when wholesale price is smaller than the threshold, and the revenue-sharing contract may be effective when the retailer's cash flow is not small. On the other hand, the manufacturer's insufficient cash flow partly narrows the option of optimal choice for the retailer. While, the insufficient cash flow has little impact on her preference when wholesale price is higher than the threshold, otherwise, causes her preference moving from DP to EP owing to cash constraint disadvantage effect. Furthermore, we find that the revenue-sharing contract is more effective on mitigating the conflict under insufficient cash flow of the manufacturer. Our results provide several advices of choosing payment term for the retailer and support of using supply chain contract to alleviate the payment term conflict under insufficient cash flow, such as revenue-sharing contract and subsidy contract.

In our paper, we assume the linear demand relationships in a supply chain. However, more general demand relationships that include different forms and/or uncertainty parameters can be considered, and then other contracts in aforementioned literatures can be examined regarding the effect of mitigating the conflict of payment. Secondly, based on the different payment timings, we investigate the three payments and focus more attention on the impact of insufficient cash flow on the payment term, but we consider little the time value of cash flow. In addition, the exiting competition between retailers or manufacturers deserves careful consideration. Moreover, we do not consider that both the manufacturer and retailer suffer insufficient cash flow. These can present new interesting research which we leave for future study.

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Appendices

Proof of Proposition 1: It is easily proved by comparing, thus omits.

Proof of Proposition 2: when the wholesale price is low e.g. $w < (a+c)/2$, firstly if the manufacturer and retailer are integrated, the centralised supply chain generates the optimal demand $q^I = \frac{a-c}{2b}$, and the optimal profit $\Pi^I = \frac{(a-c)^2}{4b}$.

Secondly, with the contract that involves the procurement cost $T^{EC}(\phi,\theta)$, the retailer decides q_R^{EC} to maximise profit. The retailer's problem becomes

$$\pi_R^{EC} = \max_{q_R^{EC}}[p(q_R^{EC})q_R^{EC} - T^{EC}(\phi,\theta)]$$

$$= \max_{q_R^{EC}}\left[\phi * p(q_R^{EC}) - \frac{w}{1+\theta}\right]q_R^{EC} \tag{A-1}$$

where $T^{EC}(\phi,\theta) = f(\theta)q_R^{EC} + (1-\phi)p(q_R^{EC})q_R^{EC}$. With the retailer's response, the manufacturer's problem becomes

$$\pi_M^{EC} = \max_{\phi,\theta}[T^{EC}(\phi,\theta) - cq_R^{EC}] \tag{A-2}$$

Thus, if $\phi(1+\theta) = w/c$

(i) We can easily yield optimal output $q_R^{EC*} = q^I = \frac{a-c}{2b}$, and we have $q_R^{EC*} > \frac{a-w}{2b} = q_R^{E*} = q_R^{P*} > q_R^{D*}$.
(ii) The optimal profits for each member with the contract are denoted by $\pi_M^{EC*} = (1-\phi)\,\Pi^I$, $\pi_R^{EC*} = \phi\,\Pi^I$, then $\pi_M^{EC*} + \pi_R^{EC*} = \Pi^I$. In addition, without the contract, $\pi_R^{E*} + \pi_M^{E*} < \Pi^I \Rightarrow \frac{\pi_R^{E*}}{\Pi^I} + \frac{\pi_M^{E*}}{\Pi^I} < 1 \Rightarrow \frac{\pi_R^{E*}}{\Pi^I} < 1 - \frac{\pi_M^{E*}}{\Pi^I}$. Therefore, if $\frac{\pi_R^{E*}}{\Pi^I} < \phi < 1 - \frac{\pi_M^{E*}}{\Pi^I}$, then $\pi_R^{EC*} \geq \pi_R^{E*} = \pi_R^{P*} > \pi_R^{D*}$, and $\pi_M^{EC*} > \pi_M^{P*} = \pi_M^{E*}$.

Proof proposition 3: With insufficient cash flow of the retailer F_R,

(i) If $w \geq \hat{w}$ and $0 \leq F_R < \underline{F}_{R,h}^E$, then

On the one hand, the retailer's problem under EP without bank credit becomes:

$$\pi_R^E = \max_{q_R^A}[p(q_R^E)q_R^E - f(\theta)q_R^E] \tag{A-3}$$

$$\text{s.t.}\quad 0 \leq f(\theta)q_R^E \leq F_R < \underline{F}_{R,h}^E \tag{A-4}$$

After the retailer's response to discount, the manufacturer chooses θ to maximise its profit. The problem is:

$$\pi_M^E = \max_{\theta}[f(\theta) - c]q_R^E \tag{A-5}$$

Thus, the optimal output is $q_R^E(\theta) = \frac{F_R}{f(\theta)}$, substituting $q_R^E(\theta)$ into A-5 yields the optimal discount of wholesale price, that is $\theta^* = 0$. Then, the optimal profit of the retailer is dependent on his cash flow, while no more than that under DP, that is $\pi_R^E(F_R) = -b\frac{F_R^2}{w^2} + \frac{a-w}{w}F_R \leq \frac{(a-w)^2}{4b} = \pi_R^{D*}$.

On the other hand, the retailer would borrow from the bank, so the cash flow with bank credit for the retailer would touch the lowest cash flow level under EP (e.g. $\underline{F}_{R,h}^E$) according to the optimal profit of the retailer without bank credit. Therefore, the optimal profit of the retailer with bank credit under EP also depends on his cash flow, that is $\pi_{RB}^E(F_R) = \frac{(a-c)^2}{16b} - [\underline{F}_{R,h}^E - F_R](1+r_B)$.

Consequently, if $\underline{F}_{R,h}^E - \frac{R_h^{ED}}{1+r_B} \leq F_R < \underline{F}_{R,h}^E$, then $\pi_{RB}^E(F_R) \geq \pi_R^{D*} \geq \pi_R^E(F_R)$, the retailer chooses EP with bank credit. If $0 \leq F_R < \underline{F}_{R,h}^E - \frac{R_h^{ED}}{1+r_B}$, then $\pi_R^{D*} \geq \max\{\pi_{RB}^E(F_R), \pi_R^E(F_R)\}$, the retailer chooses DP, where the proof of the inequality, that is $R_h^{ED} < \underline{F}_{R,h}^E$, is provided from below **B-1** to keep a representation more than zero, that is $\underline{F}_{R,h}^E - \frac{R_h^{ED}}{1+r_B} > 0$.

(ii) If $\tilde{w} \le w < \hat{w}$ and $0 \le F_R < \underline{F}^E_{R,m}$, then

In the same way, the optimal profit of the retailer without bank credit under EP (or PP) depends on his initial cash flow, that is $\pi^E_R(F_R) = -b\frac{F_R^2}{w^2} + \frac{a-w}{w}F_R$, furthermore, we denote $\hat{F}_R = \underline{F}^E_{R,m} - \frac{w}{\sqrt{b}}\sqrt{R^{ED}_m} > 0$, then $\pi^E_R(\hat{F}_R) = \pi^{D*}_R = \frac{(a-c)^2}{16b}$. Therefore, $\pi^{D*}_R \le \pi^E_R(F_R)$ when $\hat{F}_R \le F_R < \underline{F}^E_{R,M}$, and $\pi^E_R(F_R) < \pi^{D*}_R$ when $0 \le F_R < \hat{F}_R$.

Alternatively, the retailer borrows $F_{RB} = \underline{F}^E_{R,m} - F_R$, then the optimal profit under EP is $\pi^E_{RB}(F_R) = \frac{(a-w)^2}{4b} - [\underline{F}^E_{R,m} - F_R](1+r_B)$ and $\pi^E_{RB}(F_R) \ge \pi^{D*}_R$ when $0 < \underline{F}^E_{R,m} - \frac{R^{ED}_m}{1+r_B} \le F_R < \underline{F}^E_{R,m}$, $\pi^E_{RB}(F_R) < \pi^{D*}_R$ when $F_R < \underline{F}^E_{R,m} - \frac{R^{ED}_m}{1+r_B}$, where the proof of $R^{ED}_m < \underline{F}^E_{R,m}$ is provided from below **B-2.** Next, we will prove that $\forall F_R \in [\hat{F}_R, \underline{F}^E_{R,m})$, $\pi^E_R(F_R) > \pi^E_{RB}(F_R)$.

Firstly, we define $\Lambda(F_R) = \pi^E_R(F_R) - \pi^E_{RB}(F_R), F_R \in \left[\underline{F}^E_{R,m} - \frac{R^{ED}_m}{1+r_B}, \underline{F}^E_{R,m}\right)$, and easily find that $\Lambda(F_R) > 0$ when $F_R = \underline{F}^E_{R,m} - \frac{R^{ED}_m}{1+r_B}$, $\lim\limits_{F_R \to \underline{F}^E_{R,m}} \Lambda(F_R) = 0$, and $\lim\limits_{F_R \to \underline{F}^E_{R,m}} \frac{d\Lambda(F_R)}{dF_R} < 0$. There are two cases to consider, (1) if $\frac{d\Lambda(F_R)}{dF_R}\Big|_{F_R = \underline{F}^E_{R,m} - \frac{R^{ED}_m}{1+r_B}} \le 0$, then $\frac{d\Lambda(F_R)}{dF_R} < 0$ for $\hat{F}_R < \underline{F}^E_{R,m} - \frac{R^{ED}_m}{1+r_B} < F_R < \underline{F}^E_{R,m}$. Thus if $\exists F_{R0} \in \left(\underline{F}^E_{R,m} - \frac{R^{ED}_m}{1+r_B}, \underline{F}^E_{R,m}\right)$, $\Lambda(F_{R0}) < 0$, then $\exists F_{R1} \in \left(\underline{F}^E_{R,m} - \frac{R^{ED}_m}{1+r_B}, F_{R0}\right)$, $\Lambda(F_{R1}) = 0$; therefore, $\exists \epsilon_1 \in (F_{R1}, \underline{F}^E_{R,m})$, $\frac{d\Lambda(F_R)}{dF_R}\Big|_{F_R=\epsilon_1} = 0$, which leads to a contradiction. (2) if $\frac{d\Lambda(F_R)}{dF_R}\Big|_{F_R = \underline{F}^E_{R,m} - \frac{R^{ED}_m}{1+r_B}} > 0$, then there exits an unique point $\epsilon_0 \in \left(\underline{F}^E_{R,m} - \frac{R^{ED}_m}{1+r_B}, \underline{F}^E_{R,m}\right)$, then $\frac{d\Lambda(F_R)}{dF_R}\Big|_{F_R=\epsilon_0} = 0$. Thus if $\exists F_{R2} \in \left(\underline{F}^E_{R,m} - \frac{R^{ED}_m}{1+r_B}, \underline{F}^E_{R,m}\right)$, $\Lambda(F_{R2}) < 0$, then $\exists F_{R3} \in \left(\underline{F}^E_{R,m} - \frac{R^{ED}_m}{1+r_B}, F_{R2}\right)$, $\Lambda(F_{R3}) = 0$; therefore, $\exists \epsilon_2 \in (F_{R3}, \underline{F}^E_{R,m})$, $\frac{d\Lambda(F_R)}{dF_R}\Big|_{F_R=\epsilon_2} = 0$. Furthermore, if $\Lambda'(F_{R3}) = 0$, a contradiction; if $\Lambda'(F_{R3}) < 0$, $\exists \epsilon_3 \in \left(\underline{F}^E_{R,m} - \frac{R^{ED}_m}{1+r_B}, F_{R3}\right)$, $\frac{d\Lambda(F_R)}{dF_R}\Big|_{F_R=\epsilon_3} = 0$. Again, a contradiction if $\Lambda'(F_{R3}) > 0$, $\Lambda(F_R)$ is strictly increasing in $F_R \in \left[\underline{F}^E_{R,m} - \frac{R^{ED}_m}{1+r_B}, F_{R3}\right]$, thus $\Lambda(F_{R3}) > \Lambda(F_R)|_{F_R = \underline{F}^E_{R,m} - \frac{R^{ED}_m}{1+r_B}} > 0$, a contradiction with $\Lambda(F_{R3}) = 0$ above. Therefore $\pi^E_R(F_R) > \pi^E_{RB}(F_R)$ must hold when $\underline{F}^E_{R,m} - \frac{R^{ED}_m}{1+r_B} \le F_R < \underline{F}^E_{R,m}$.

Therefore, if $\hat{F}_R \le F_R < \underline{F}^E_{R,m}$, $\pi^E_R(F_R) > \pi^E_{RB}(F_R)$ and $\pi^E_R(F_R) \ge \pi^{D*}_R$, the retailer prefers EP or PP without bank credit. If $0 \le F_R < \hat{F}_R < \underline{F}^E_{R,m} - \frac{R^{ED}_m}{1+r_B}$, $\pi^E_R(F_R) < \pi^{D*}_R$, and $\pi^E_{RB}(F_R) < \pi^{D*}_R$, the retailer chooses DP.

(iii) if $max\left\{c, \frac{2a\bar{r}}{(2+\bar{r})\bar{r}+4(1+r_B)^2}\right\} < w < \tilde{w}$ and $0 \le F_R < \underline{F}^E_{R,l}$, we easily find that $0 < R^{ED}_l < \underline{F}^E_{R,l}$. Then,

Following the proof of (ii), if $\tilde{F}_R \le F_R < \underline{F}^E_{R,l}$, $\pi^E_R(F_R) > \pi^E_{RB}(F_R)$ and $\pi^E_R(F_R) \ge \pi^{D*}_R$, the retailer chooses EP or PP without bank credit; if $0 \le F_R < \tilde{F}_R < \underline{F}^E_{R,l} - \frac{R^{ED}_l}{1+r_B}$, $\pi^E_R(F_R) < \pi^{D*}_R$, and $\frac{(a-w)^2}{4b} - [\underline{F}^E_{R,l} - F_R](1+r_B) < \pi^{D*}_R$, the retailer chooses DP, where $\tilde{F}_R = \underline{F}^E_{R,l} - \frac{w}{\sqrt{b}}\sqrt{R^{ED}_l} > 0$.

Additional proof of several inequalities for Proposition 3:

$$R^{ED}_h < \underline{F}^E_{R,h} \tag{B-1}$$

$$R^{ED}_h - \underline{F}^E_{R,h} = \frac{(a-c)^2}{16b} - \frac{(a-w)^2}{4b} - \frac{a+c}{2} \times \frac{a-c}{4b}$$
$$= -\frac{1}{16b}[4(a-w)^2 + (a-c)(a+3c)] < 0.$$

$$0 < R^{ED}_m < \underline{F}^E_{R,m}, \tag{B-2}$$

Firstly, $R^{ED}_m = \frac{(a-w)^2}{4b} - \frac{(a-c)^2}{16b} > 0$; secondly, we define $f_1(w) = 12w^2 - 16aw + 4a^2 - (a-c)^2$, then $R^{ED}_m - \underline{F}^E_{R,m} = \frac{f_1(w)}{16b}$. We have $\triangle_{f1} = 64a^2 + 48(a-c)^2 > 0$, thus $f_1(w) = 0$ has two real roots: $w_1 = \frac{16a - \sqrt{\triangle_{f1}}}{24} < \frac{a+c}{2}$, $w_2 = \frac{16a+\sqrt{\triangle_{f1}}}{24} > a$. We denote $\bar{r}^*_1 = \frac{\sqrt{\triangle_{f1}/16} - (a-3c)}{4a - \sqrt{\triangle_{f1}/16}}$, and find $\bar{r}^*_1 > 1$(see proof **B-3**), then $w_1 = \frac{a+c}{2(1+\bar{r}^*_1)} < \frac{a+c}{2(1+\bar{r})}$; therefore, $\forall w$, $\frac{a+c}{2(1+\bar{r})} \le w < \frac{a+c}{2}$, we have $f_1(w) < 0$, then $0 < R^{ED}_m < \underline{F}^E_{R,m}$

$$\frac{\sqrt{\triangle_{f1}/16} - (a-3c)}{4a - \sqrt{\triangle_{f1}/16}} > 1 \tag{B-3}$$

We have $\frac{\triangle_{f1}}{4} - (5a-3c)^2 = 3a^2 + 3c^2 + 6ac > 0$ where $\triangle_{f1} = 64a^2 + 48(a-c)^2$, then $2\sqrt{\frac{\triangle_{f1}}{16}} > (5a-3c) > 0 \Rightarrow \sqrt{\triangle_{f1}/16} - (a-c) > 4a - \sqrt{\triangle_{f1}/16} > 0 \Rightarrow \frac{\sqrt{\triangle_{f1}/16}-(a-3c)}{4a-\sqrt{\triangle_{f1}/16}} > 1$.

Proof of Proposition 4: (i) If $w \ge \hat{w}$, then

(1) if $\underline{F}^E_{R,h} - \frac{R^{ED}_h}{1+r_B} \le F_R < \underline{F}^E_{R,h}$. From the result of Proposition 3(i), the manufacturer is also better off under EP according to the results of Proposition 1(i).

(2) if $0 \le F_R < \underline{F}^E_{R,h} - \frac{R^{ED}_h}{1+r_B}$.

Firstly, we define $f_4(w) = 12w^2 - 16aw + 4a^2 + (a-c)(a+3c)$, then $\underline{F}^E_{R,h} - R^{ED}_1 - \underline{F}^P_{R,h} = \frac{f_4(w)}{16b}$ and $\triangle_{f4} = 16(a-3c)^2 \ge 0$, thus $f_4(w) = 0$ has one or two real roots: $w_5 = \frac{4a-|a-3c|}{6}$, $w_6 = \frac{4a+|a-3c|}{6}$. Therefore, if $a > 3c$, then $w_5 = \frac{a+c}{2}$, $w_6 = \frac{5a-3c}{6}$, then $\forall w$, $\frac{a+c}{2} \le w < w_6$, $f_4(w) \le 0$, thus $\underline{F}^E_{R,h} - R^{ED}_h \le \underline{F}^P_{R,h}$; If $c < a \le 3c$, then $\forall w$, $\frac{a+c}{2} \le w$, $f_4(w) \ge 0$, thus $\underline{F}^E_{R,h} - R^{ED}_h \ge \underline{F}^P_{R,h} \Rightarrow \underline{F}^E_{R,h} - \frac{R^{ED}_h}{1+r_B} > \underline{F}^E_{R,h} - R^{ED}_h \ge \underline{F}^P_{R,h}$.

Secondly, the optimal profit for the manufacturer under EP is $\pi^E_M(F_R) = \frac{w-c}{w}F_R$, while the profit of the manufacturer under DP is $\pi^{D*}_M = \frac{(w-c)(a-w)}{2b}$, then $\pi^E_M(F_R) \ge \pi^{D*}_M$ when $F_R \ge \underline{F}^P_{R,h}$, and $\pi^E_M(F_R) < \pi^{D*}_M$ when $F_R < \underline{F}^P_{R,h}$. Therefore, we consider three cases:

Case 1: If $a > 3c$, $r_B \ge \widehat{r_B} - 1$, then $\underline{F}^E_{R,h} - \frac{R^{ED}_h}{1+r_B} \ge \underline{F}^P_{R,h}$, the manufacturer benefits from EP when $\underline{F}^P_{R,h} \le F_R < \underline{F}^E_{R,h} - \frac{R^{ED}_h}{1+r_B}$, and DP when $0 \le F_R < \underline{F}^P_{R,h}$;

Case 2: If $a > 3c$, $0 < r_B < \widehat{r_B} - 1$, then $\underline{F}^E_{R,h} - \frac{R^{ED}_h}{1+r_B} < \underline{F}^P_{R,h}$, the manufacturer is always better off under DP when $0 \le F_R < \underline{F}^E_{R,h} - \frac{R^{ED}_h}{1+r_B}$;

Case 3: If $c < a \le 3c$, then $\underline{F}^E_{R,h} - \frac{R^{ED}_h}{1+r_B} \ge \underline{F}^P_{R,h}$, then the manufacturer benefits from EP when $\underline{F}^P_{R,h} \le F_R < \underline{F}^E_{R,h} - \frac{R^{ED}_h}{1+r_B}$, and DP when $0 \le F_R < \underline{F}^P_{R,h}$.

(ii) If $c_0 < w < \hat{w}$. The manufacturer is always better off under DP.

where $\widehat{r_B} = \frac{R^{ED}_h}{\underline{F}^E_{R,h} - \underline{F}^P_{R,h}}$

Proof of proposition 5: With cash flow of the manufacturer F_M, EP is always an optimal option for the retailer, then

(i) If $w \ge \hat{w}$, the retailer only chooses EP.

(ii) If $3c \le w < \hat{w}$ and $0 \le F_M < \underline{F}^D_{M,K} < \underline{F}^P_{M,K}$, that is, the cash flow cannot cover the procurement cost under PP. So, without bank credit, the optimal profits of the manufacturer and the retailer under PP are $\pi^P_M(F_M) = (w-c)\frac{F_M}{c}$, $\pi^P_R(F_M) = -\frac{b}{c^2}F_M^2 + (a-w)\frac{F_M}{c} < \frac{(a-w)^2}{4b}$; alternatively, with bank credit, the optimal profits of the manufacturer and retailer are $\pi^P_{MB}(F_M) = (w-c)\frac{a-w}{2b} - \left(c \times \frac{a-w}{2b} - F_M\right)(1+r_B)$, $\pi^P_R = \frac{(a-w)^2}{4b}$. In addition, $\pi^P_{MB}(F_M) - \pi^P_M(F_M) = \left(c \times \frac{a-w}{2b} - F_M\right)\left(\frac{w-c}{c} - (1+r_B)\right) > 0$; therefore, PP is also an optimal choice for the retailer.

(iii) If $2c < w < 3c$ and $0 \le F_M < \underline{F}^D_{M,K} < \underline{F}^P_{M,K}$, then, the retailer chooses EP or PP when $0 < r_B < \frac{w-2c}{c}$, because $\pi^P_{MB}(F_M) - \pi^P_M(F_M) > 0$, otherwise chooses EP when $\frac{w-2c}{c} < r_B < 1$.

(iv) If $c < w \le 2c$, and $0 \le F_M < \underline{F}^D_{M,K} < \underline{F}^P_{M,K}$, the retailer chooses EP, because $\pi^P_{MB}(F_M) - \pi^P_M(F_M) < 0$.

Proof of Proposition 6: With insufficient initial cash flow of the manufacturer F_M

(i) If $w \ge \hat{w}$, then $M^{DE}_h < 0 < \underline{F}^D_{M,h}$, that is, the manufacturer benefits from EP regardless of her cash flow;

(ii) If $\tilde{w} \le w < \hat{w}$ and $0 < \bar{r} \le \frac{1}{\frac{a+c}{\sqrt{2c(a-c)}} - 1} \le 1$, then $0 < M^{EA}_m < \underline{F}^D_{M,m}$ (see proof **B-4**). Thus, if $0 \le F_M < \underline{F}^D_{M,m}$, firstly, the manufacturer would not adopt bank credit, then her optimal profit under DP depends on her cash flow, that is $\pi^D_M(F_M) = -\frac{2bF_M^2}{c^2} + \frac{a-c}{c}F_M$. Furthermore, we denote $\hat{F}_M = \underline{F}^D_{M,m} - \frac{c\sqrt{2M^{DE}_m}}{2\sqrt{b}} > 0$, then $\pi^D_M(\hat{F}_M) = \frac{(w-c)(a-w)}{2b} = \pi^{E*}_M$, thus $\pi^{E*}_M \le \pi^D_M(F_M)$ when $\hat{F}_M \le F_M < \underline{F}^D_{M,m}$, otherwise $\pi^D_M(F_M) < \pi^{E*}_M$;

Secondly, the manufacturer can borrow $F_{MB} = \underline{F}^D_{M,m} - F_M$, the optimal profit for the manufacturer with bank credit also depends on her cash flow, that is $\pi^D_{MB}(F_M) = \frac{(a-c)^2}{8b} - [\underline{F}^D_{M,m} - F_M](1+r_B)$. Furthermore, $\pi^D_{MB}(F_M) \ge \pi^{E*}_M$ when $\underline{F}^D_{M,m} - \frac{M^{DE}_m}{1+r_B} \le F_M < \underline{F}^D_{M,m}$, otherwise $\pi^D_{MB}(F_M) < \pi^{E*}_M$. Moreover, if $0 < \bar{r} < min\left\{1, \frac{2c(1+r_B)}{a-c-2cr_B}\right\}$, then $\hat{F}_M < \underline{F}^D_{M,m} - \frac{M^{DE}_m}{1+r_B}$ (see proof **B-5**). Following the proof of Proposition 3, we also find that if $\hat{F}_M \le F_M < \underline{F}^D_{M,m} - \frac{M^{DE}_m}{1+r_B}$, $\pi^D_M(F_M) > \pi^D_{MB}(F_M)$, and $\pi^D_M(F_M) \ge \pi^{E*}_M$, thus the manufacturer is better off under DP without bank credit; If $0 \le F_M < \hat{F}_M$, then $\pi^D_M(F_M) < \pi^{E*}_M$ and $\pi^D_{MB}(F_M) < \pi^{E*}_M$, the manufacturer is better off under EP.

(iii) If $c < w < \tilde{w}$ and $0 \le F_M < \underline{F}^D_{M,l}$, then $0 < M^{DE}_l < \underline{F}^D_{M,l}$ (see proof **B-6**), then the manufacturer would not adopt bank credit, then her optimal profit is $\pi^D_M(F_M) = \frac{\bar{w}-c}{c}F_M$. And we denote $\tilde{F}_M = \underline{F}^D_{M,l} - \frac{cM^{DE}_l}{\bar{w}-c} > 0$, $\pi^D_M(\tilde{F}_M) = (w-c)\frac{a-w}{2b} = \pi^{E*}_M$, so $\pi^D_M(F_M) \ge \pi^{E*}_M$ when $F_M \ge \tilde{F}_M$, otherwise $\pi^D_M(F_M) < \pi^D_M(\tilde{F}_M)$. Alternatively, the manufacturer can borrow $F_{MB} = \underline{F}^D_{M,l} - F_M$, the optimal profit is $\pi^D_{MB}(F_M) = [\bar{w} - c]\frac{a-\bar{w}}{2b} - (\underline{F}^D_{M,l} - F_M)(1+r_B)$, and $\pi^D_{MB}(F_M) \ge \pi^{E*}_M$ when $F_M \ge \check{F}_M = \underline{F}^D_{M,l} - \frac{M^{DE}_l}{1+r_B} > 0$, otherwise $0 \le \pi^D_{MB}(F_M) < \pi^{E*}_M$. Therefore,

(a) If $c < \bar{w} \le 2c$, then $\check{F}_M > \tilde{F}_M$. Then, the manufacturer is better off under DP without bank credit when $\tilde{F}_M \le F_M < \underline{F}^D_{M,l}$, and EP when $0 \le F_M < \tilde{F}_M$;

(b) If $2c < \bar{w} < 3c$ and $\frac{\bar{w}-2c}{c} < r_B < 1$, then $\check{F}_M > \tilde{F}_M$. Then, the manufacturer is better off under DP without bank credit when $\tilde{F}_M \le F_M < \underline{F}^D_{M,l}$, and EP when $0 \le F_M < \tilde{F}_M$;

(c) If $2c < \bar{w} < 3c$ and $0 < r_B \le \frac{\bar{w}-2c}{c}$, then $\check{F}_M \le \tilde{F}_M$. Then the manufacturer is better off under DP with bank credit when $\check{F}_M \le F_M < \underline{F}^D_{M,l}$, and EP when $0 \le F_M < \check{F}_M$;

(d) If $\bar{w} \ge 3c$, then $\check{F}_M < \tilde{F}_M$. Then the manufacturer is better off under DP with bank credit when $\check{F}_M \le F_M < \underline{F}^D_{M,l}$, and EP when $0 \le F_M < \check{F}_M$.

Additional proof several inequalities for Proposition 8:

$$\text{If } \bar{r} < \frac{1}{\frac{a+c}{\sqrt{2c(a-c)}} - 1} \le 1, \text{ then } M^{DE}_m < \underline{F}^D_{M,m}, \qquad \text{(B-4)}$$

We define $f_5(w) = 4w^2 - 4(a+c)w + (a+c)^2 - 2c(a-c)$, then $M^{DE}_m - \underline{F}^D_{M,m} = \frac{f_5(w)}{8}$ and $\triangle_{f5} = 32c(a-c) > 0$, thus the equation $f_5(w) = 0$ has two real roots: $w_7 = \frac{a+c}{2} - \frac{\sqrt{2c(a-c)}}{2}$, $w_8 = \frac{a+c}{2} + \frac{\sqrt{2c(a-c)}}{2}$. In this case, if $\bar{r} < \frac{1}{\frac{a+c}{\sqrt{2c(a-c)}} - 1} \le 1$, then $w_7 < \frac{a+c_0}{2(1+\bar{r})}$: therefore, $\forall w$, $\frac{a+c_0}{2(1+\bar{r})} \le w < \frac{a+c_0}{2}$, $f_5(w) < 0$, then $M^{DE}_m < \underline{F}^D_{M,m}$.

$$\text{If } 0 < \bar{r} < min\left\{1, \frac{2c(1+r_B)}{a-c-2cr_B}\right\}, \text{ then} \frac{\sqrt{2bM^{DE}_m}}{c_0} < 1 + r_B \qquad \text{(B-5)}$$

we define $f_6(w) = 4w^2 - 4(a+c)w + (a+c)^2 - 4c^2(1+r_B)^2$, then $2bM^{DE}_m - c^2(1+r_B)^2 = \frac{f_6(w)}{4}$, $\triangle_{f6} = 64c^2(1+r_B)^2 > 0$, thus $f_6(w) = 0$ has two real roots: $w_9 = \frac{a+c}{2} - c(1+r_B)$, $w_{10} = \frac{a+c}{2} + c(1+r_B)$. In this case, if $0 < \bar{r} < min\left\{1, \frac{2c(1+r_B)}{a-c-2cr_B}\right\}$, $w_9 < \frac{a+c}{2(1+\bar{r})}$, then $\forall w$, $\frac{a+c}{2(1+\bar{r})} \le w < \frac{a+c}{2}$, $f_6(w) < 0$, then $\frac{\sqrt{2bM^{DE}_m}}{c_0} < 1 + r_B$.

$$M^{DE}_l < \underline{F}^D_{M,l} \qquad \text{(B-6)}$$

We define $f_7(w) = -(2+\bar{r})\bar{r}w^2 + [c(1+\bar{r}) + (a+c)\bar{r}]w - ac$, then $M^{DE}_l - \underline{F}^D_{M,l} = \frac{f_7(w)}{2}$, we find that $f_7(0) = -ac < 0$, $f_7(w)$ is decreasing on $\left(0, \frac{c(1+\bar{r})+(a+c)\bar{r}}{2(2+\bar{r})\bar{r}}\right]$, $\triangle_{f7} = [c(1+\bar{r}) + (a+c)\bar{r}]^2 - 4(2+\bar{r})\bar{r}ac$. If $\triangle_{f7} > 0$, the equation $f_7(w) = 0$ has two real roots: $w_{11} = \frac{-[c(1+\bar{r})+(a+c)\bar{r}] + \sqrt{\triangle_{f7}}}{-2(2+\bar{r})\bar{r}} < 0$, $w_{12} = \frac{[c(1+\bar{r})+(a+c)\bar{r}] + \sqrt{\triangle_{f7}}}{2(2+\bar{r})\bar{r}} > 0$, in this case $f_7(w_{11}) < f_7(0) < 0$, this leads to a contradiction. Therefore, $\triangle_{f7} \leq 0$, then $\forall w,\ c < w < \frac{a+c}{2(1+\bar{r})}$, $f_7(w) < 0$, then $M_l^{DE} < \underline{F}_{M,l}^D$.

Proof of Proposition 7

(i) The result is easily proved through comparing the results in Proposition 1 with those in Proposition 3 or 5.

(ii) On the one hand, comparing the results in Proposition 3 with those in Proposition 4, we can get the payment term conflict under insufficient cash flow of the retailer; On the other hand, we also get the payment term conflict under insufficient cash flow of the manufacturer by comparing the results in Proposition 5with those in Proposition 6. Then, we can derive the impact of insufficient cash flow on the payment term conflict through comparing the conflict under sufficient cash flow in Proposition 1 with that under insufficient cash flow of each member.

Proof of Proposition 8: From the results in Proposition 3 and 4, we have shown that if $\underline{F}_{R,h}^P \leq F_R < \underline{F}_{R,h}^E - R_h^{AD}/(1+r_B)$, the retailer chooses DP, his optimal profit is $\pi_R^{D*} = \frac{(a-w)^2}{4b}$; while the manufacturer is better off under EP under certain condition when wholesale price is higher than the threshold, her optimal profit is $\pi_M^E(F_R) = \frac{w-c}{w}F_R$.

Next, we present the problem of retailer and manufacturer under DP with the subsidy contract, that is

$$\pi_R^{DC} = \max_{q_R^{DC}} p(q_R^{DC})q_R^{DC} - T^{DC}(\rho, r) \qquad \text{(A-6)}$$

$$\pi_M^{DC} = \max_{r,\rho} T^{DC}(\rho, r) - c_0 q_R^{DC}(\rho, r) \qquad \text{(A-7)}$$

where $T^{DC}(\rho, r) = w \cdot (1+r)q_R^{DC} - \rho q_R^{DC}$.

By generating the optimal profit for each member, as $\pi_M^{DC*} = \frac{(a-c)^2}{8b}$, $\pi_R^{DC*} = \frac{(a-c)^2}{16b}$, we can easily have $\pi_M^{DC*} > \pi_M^A(F_R)$, $\pi_R^{DC*} > \pi_R^{D*}$. Therefore, the retailer and manufacturer would agree with DP with the subsidy contract, that is, such a contract can not only alleviate the conflict of payment term between the retailer and manufacturer, but also achieve Pareto improvement for each channel member.

Proof of Proposition 9: From the results of Proposition 2, we find that the purchasing cost of the retailer under EPC depends on ϕ, while it decreases in ϕ, that is $\hat{T}^{EC}(\phi) = \frac{a^2-c^2}{4b} - \phi\frac{(a-c)^2}{4b}$, $\underline{T} < \hat{T}^{EC}(\phi) < \bar{T}$, where $\underline{T} = \frac{a^2-c^2}{4b} - (\Pi^I - \pi_M^{E*})$ and $\bar{T} = \frac{a^2-c^2}{4b} - \pi_R^{E*}$. Then,

(1) *If* $\tilde{w} \leq w < \hat{w}$ *and* $\hat{F}_R \leq F_R < \underline{F}_{R,m}^E$, then

(a) The revenue-sharing contract is effective if $max\{\hat{F}_R, \hat{T}^{EC}(\phi)\} \leq F_R < \underline{F}_{R,m}^E$;

(b) The revenue-sharing contract is effective under borrowing $\hat{T}^{EC}(\phi) - F_R$ *by the retailer if* $max\{\hat{T}^{EC}(\phi) - \Omega(F_R), \hat{F}_R\} \leq F_R < \hat{T}^{EC}(\phi)$. Becaus*e* $\pi_R^{EC*} - \pi_R^E(F_R) - (\hat{T}^{EC}(\phi) - F_R)(1+r_B) > 0$;

(c) The revenue-sharing contract is ineffective *if* $\hat{F}_R \leq F_R < \hat{T}^{EC}(\phi) - \Omega(F_R) < \hat{T}^{EC}(\phi)$. Because the retailer's initial cash flow highly falls below the purchasing cost under EPC, and the retailer would not borrow from bank. Because $\pi_R^{EC*} - \pi_R^E(F_R) - (\hat{T}^{EC}(\phi) - F_R)(1+r_B) < 0$.

(2) if $max\left\{c_0, \frac{2a\bar{r}}{(2+\bar{r})\bar{r}+4(1+r_B)^2}\right\} < w < \tilde{w}$ *and* $\tilde{F}_R \leq F_R < \underline{F}_{R,l}^E$, then there are also three cases to consider for given ϕ

(a) The revenue-sharing contract is effective if $max\{\tilde{F}_R, \hat{T}^{EC}(\phi)\} \leq F_R < \underline{F}_{R,l}^E$;

(b) The revenue-sharing contract is effective under borrowing $\hat{T}^{EC}(\phi) - F_R$ by the retailer if $max\{\hat{T}^{EC}(\phi) - \Omega(F_R), \tilde{F}_R\} \leq F_R < \hat{T}^{EC}(\phi)$;

(c) The revenue-sharing contract is rarely effective *if* $\tilde{F}_R \leq F_R < \hat{T}^{EC}(\phi) - \Omega(F_R) < \hat{T}^{EC}(\phi)$.

(ii) Firstly, with enough cash endowment, the retailer would accept the revenue-sharing contract from Proposition 2; in addition, the optimal profit of the manufacturer with insufficient cash flow under DP is less than that with sufficient cash flow, that is, the optimal profit of the manufacturer under EP with revenue-sharing contract is more likely than that under DP. Therefore, the contract is more effective on mitigating the conflict of payment term.

where $\Omega(F_R) = [\pi_R^{EC*} - \pi_R^E(F_R)]/(1+r_B)$ and $\pi_R^E(F_R)$ is the optimal profit of the retailer under EP that depends on his insufficient cash flow, that is $\pi_R^E(F_R) = -b\frac{F_R^2}{w^2} + \frac{a-w}{w}F_R$ if $\hat{F}_R(\tilde{F}_R) \leq F_R < \underline{F}_{R,m}^E(\underline{F}_{R,l}^E)$.