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Optimal long-run inflation rate in an open economy

Hirokazu Ishise[1]

Osaka School of International Public Policy, Osaka University, 1-31 Machikaneyama, Toyonaka, 560-0043, Osaka, Japan

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ABSTRACT

I analyze the long-run optimal inflation rate in the sticky price model with trend inflation and a two-country, two-good Ricardian trade structure. As in the closed-economy model, price stickiness effectively reduces an industry's productivity under trend inflation. Contrary to the standard closed-economy models in which the optimal inflation rate is approximately zero, the model implies that the optimal rate is positive under certain conditions. Welfare gains come from manipulations of the terms of trade, and are hence associated with the loss of the trade partner. If the partner counter-acts, the allocation can be worse than what would occur under autarky.

1. Introduction

Motivated by the recent convention that many central banks set positive target inflation rates, monetary studies analyze the consequence of trend inflation on economic welfare using various versions of sticky price models with trend inflation (e.g., King and Wolman, 1999; Ascari, 2004; Schmitt-Grohé and Uribe, 2007, 2011; Ascari and Ropele, 2007; Coibion et al., 2012; Damjanovic and Nolan, 2010; Ascari and Sbordone, 2014; Carreras et al., 2016; Kurozumi and Van Zandweghe, 2016). With few exceptions, these studies conclude that a zero-inflation rate maximizes welfare in the long run.[2] However, they focus on single-good, closed-economy settings, and the long-run consequences of trend inflation in an open economy are yet to be analyzed.[3]

I examine the long-run optimal inflation rate in a two-economy setting with a focus on the role played by the terms of trade. The terms of trade, that is the price of the exported goods relative to the price of the imported goods, are one of the key determinants of a country's welfare in an open-economy setting. Moreover, the terms of trade are empirically correlated with the inflation rate. Fig. 1 shows the relationship between the inflation rate and the terms of trade for four developed countries: Canada (CAN), Japan

E-mail address: ishise@osipp.osaka-u.ac.jp.

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[2] In the short run, the initial price dispersion induces a slightly negative rate as optimal (Yun, 1996; Damjanovic and Nolan, 2010), whereas a possibility of hitting the zero-lower bound of the nominal interest rate implies that a slight positive rate is optimal (Schmitt-Grohé and Uribe, 2011; Coibion et al., 2012). There are at least two papers showing non-zero optimal trend inflation in the closed-economy setting. Carreras et al. (2016) show that the standard New Keynesian model with a regime-switching specification of risk premium shocks implies a high optimal trend inflation rate. Kurozumi and Van Zandweghe (2016) introduce the kinked demand curve in the Calvo-model with trend inflation and show that the long-run output is positively associated with the inflation rate.

[3] The multi-country, multi-good extension literature also focuses on short-run implications (e.g., Obstfeld and Rogoff, 1996; Benigno and Benigno, 2003; Benigno, 2004; Corsetti and Pesenti, 2005; Corsetti et al., 2011; Bergin and Corsetti, 2016).

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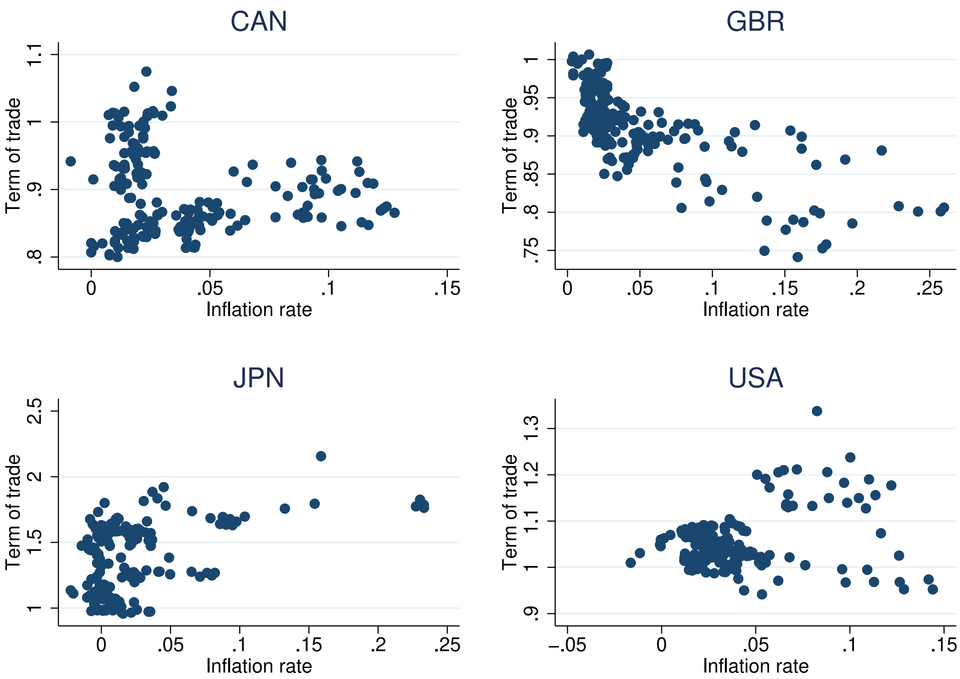

Fig. 1. Annual inflation rate and the terms of trade for four developed countries.

(JPN), the United Kingdom (GBR), and the United States (USA).[4] As shown in Fig. 1, these two are correlated. The correlation is negative for the United Kingdom, while it is positive for Japan. The cases for Canada and the United States are somewhat noisier, but the regression coefficient is still statistically significant for the United States (See Section 4). When the inflation rate affects the terms of trade, the welfare implication depends on an additional channel not considered in the closed-economy setting: the impact through trade.

The baseline model combines the textbook Calvo-style sticky-price model (Calvo, 1983; Yun, 1996) with trend inflation (Schmitt-Grohé and Uribe, 2007, 2011; Ascari and Sbordone, 2014) and the classical two-country, two-good Ricardian model (and the Ricardo–Viner model as an extension).[5] Contrary to the standard closed-economy models in which the optimal long-run inflation rate is approximately zero, the model with trade implies that the optimal rate is positive under certain conditions. Even if not positive, the welfare loss of deviating from zero inflation can be lower under the two-country models than the loss under the closed-economy or small-open models.

This happens because the inflation rate has an impact on the terms of trade. Even though trend inflation under price rigidity leads to a loss in productivity (Ascari, 2004; Ascari and Ropele, 2007; Schmitt-Grohé and Uribe, 2007, 2011; Ascari and Rossi, 2012; Ascari and Sbordone, 2014), a loss of productivity in the exporting industry may lead to an increase in terms of trade. In some cases, improving the terms of trade leads to improvements in the country's welfare. The reverse version of this welfare impact (i.e., technology improvement leads to welfare loss) is classically known as immiserizing growth (Johnson, 1955; Bhagwati, 1958).[6] In other words, technological improvement may decrease the country's welfare because the expansion of the exporting industry leads to a deterioration of terms of trade. Due to this mechanism, under a certain set of parameters, non-zero inflation effectively decreases an exporting industry's productivity, improves the terms of trade, and subsequently, the country's welfare. In this study, I show that the welfare effect can be expressed by a simple formula consisting of a country's trade share, the elasticity of the terms of trade with respect to the inflation rate, and the elasticity of the price distortion with respect to the inflation rate. Moreover, I provide the exact conditions for the non-zero optimal inflation.

[4] The inflation rate is based on the consumer price index, while the terms of trade refer to the ratio of the export price index to the import price index. I choose these four countries for two reasons. The model considers large countries with flexible exchange rate regimes. Good candidates are the countries in the group of seven. I exclude France, Germany, and Italy because they have a unified monetary policy. The quarterly data is from OECD stat database from the fourth quarter of 1973 to the fourth quarter of 2018, that is, after the introduction of the floating exchange rate. The Appendix provides further details on data and calculations.

[5] A simpler version of the model is developed by Ishise (2020), but his focus is on predicting the trade pattern and empirically testing its implication. Although the quantitative implications depend on the Calvo assumption, the main implications are qualitatively similar to the Rotemberg's adjustment cost model (Rotemberg, 1982; Ascari et al., 2011) and the endogenous duration model (Ball et al., 1988; Levin and Yun, 2007; Kurozumi, 2016). See the Appendix.

[6] Prior studies usually consider immiserizing growth in the Heckscher–Ohlin model—notable exception is Matsuyama (2000). Sawada (2009) empirically shows the plausibility of immiserizing growth.

The welfare gain comes from improvements in the terms of trade such that the above result is only true when the other country's inflation rate is fixed. A change in the home country's inflation rate naturally invokes a reaction from the trade partner; such a strategic situation resembles the short-run competitive devaluation problem (c.f., Corsetti et al., 2011) and the strategic tariff problem (c.f., Bagwell and Staiger, 1999). Each country increases the inflation rate to improve the terms of trade by reducing the exporting industry's productivity. In this sense, the structure of the game is similar to the prisoner's dilemma. Even though a cooperative solution—setting zero inflation for both countries—maximizes joint welfare, countries cannot achieve the cooperative solution without outside mechanisms such as the currency union.

The Nash equilibrium dictates that neither countries have any incentive to manipulate the terms of trade. One example is that the relative productivity of the sectors is equalized across countries, and trade completely disappears. This situation is even worse than the Nash equilibrium in the strategic tariff setting. In the tariff case, an increase in the tariff prevents trade by changing the prices and production inefficiency due to inefficient allocation of resources across sectors. In the worst-case scenario, the tariff war leads to an allocation similar to autarky. On the contrary, in the strategic inflation case, the cost is direct: productivity declines in the exporting sectors of both countries. This direct effect is strong so welfare likely becomes worse than it would be under autarky.

The rest of this study is organized as follows: Section 2 introduces the model; Section 3 analyzes the optimal inflation rates under constant-returns-to-scale (CRS) technology; Section 4 shows the analysis under decreasing-returns-to-scale (DRS) technology; Section 5 concludes the study.

2. Model setup and the aggregation

The model is a two-country, two-good extension of Ascari and Sbordone (2014).

2.1. Environment

The model is a two-country, two-final-good, discrete-time, infinite horizon model. Time is indexed by $t = 0, 1, \ldots$. The index for the final good, $i = A, B$, is also referred to as the industry. The final goods are tradeable without any trade costs for simplicity. Both countries can produce both of the final goods, but each country uses specific intermediate inputs. The intermediate inputs are non-tradeable.[7] There is a continuum of differentiated intermediate inputs for each i. The production of intermediate inputs depends on labor. The intermediate input market is monopolistically competitive, and each intermediate producer sets the price subject to the probabilistic chance of the price resetting (Calvo friction).

The description of the model is based on the home country. The problem of the foreign country is symmetric, and the asterisk (*) indicates foreign variables. The two countries differ in (1) population size (N and N^*), (2) exogenous labor productivity (θ_i and θ_i^*), and (3) the probability of a price adjustment (ω_i and ω_i^*). In addition, the home-country's government controls the country's inflation rate (π_t). In the baseline model, the foreign country's inflation rate (π_t^*) is exogenously fixed. Section 3.7 examines the strategic situation in which two countries strategically determine the inflation rates.

2.2. Households

The home-country stand-in household maximizes the lifetime utility as follows:

$$\max_{\{\{c_{it}\}_i, l_t, b_{t+1}\}} E_0 \sum_{t=0}^{\infty} \beta^t \log\left(c_t^{\psi}(1-l_t)^{1-\psi}\right), \tag{1}$$

where

$$c_t = \left(\sum_i \alpha_i c_{it}^{\frac{\rho-1}{\rho}}\right)^{\frac{\rho}{\rho-1}}, \tag{2}$$

$\alpha_i \in [0, 1]$, $\sum_i \alpha_i = 1$, and $\rho > 0$, subject to the budget constraint

$$\sum_i \frac{P_{it}}{P_t} c_{it} + b_t + \tau_{Lt} = \frac{1 + i_{t-1}}{1 + \pi_t} b_{t-1} + w_t l_t + f_t, \tag{3}$$

and the appropriate transversality condition.[8] The period utility depends on consumption c_t, which is the composite of two final goods c_{it},[9] and the disutility of hours worked l_{it}. These variables are per person values. The nominal price of good i at period t is P_{it}. With the exception that there are multiple goods, the household problem is the standard one. Given subjective discounting $\beta \in (0, 1)$,

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[7] In the real world, the majority of international trade is in intermediate goods. By introducing new non-tradeable final good producers that combine two internationally traded intermediate goods (final goods in the original model), trade in the modified model is of intermediate goods. This modified model is isomorphic to the model presented in this study.

[8] A separable utility function between consumption and leisure can produce the same result. More fundamentally, the inclusion of the labor-leisure choice is not essential in the steady-state and hence does not affect the results. Nevertheless, the labor-leisure choice is included here to make the model comparable to the closed-economy models (Ascari and Sbordone, 2014). See the Appendix for details.

[9] As is clear from Section 3, the aggregator must be the constant elasticity of substitution (CES) function. If $\rho = 1$ (Cobb–Douglas), the positive inflation is never optimal.

utility parameter $\psi \in (0,1)$, the household supplies labor l_t, earns the wage w_t, purchases the risk-free bond b_{t+1} which pays the nominal interest rate i_t in the next period, obtains the lump-sum payment from the domestic firms f_t, and pays the lump-sum tax τ_{Lt}.

The optimal choice of consumption implies that the nominal price index P_t is a variant of the average price of final goods P_{it},

$$P^{\iota}_t = \left(\sum_i \alpha_i^{\rho} P^{\iota\,1-\rho}_{it}\right)^{\frac{1}{1-\rho}}. \tag{4}$$

Let Λ_{tt+j} denote the stochastic discount factor, that is, the ratio of the future and current marginal utilities,

$$\Lambda_{tt+j} \equiv \beta^j \frac{c_t}{c_{t+j}}. \tag{5}$$

The households work in multiple sectors

$$l_t = \sum_i l_{it}, \tag{6}$$

where l_{it} is the sum of labor in different firms in industry i (see (18) below).

2.3. Production

The final good producer produces the final good y_{it} using a continuum ($\upsilon \in (0,1)$) of intermediate inputs $y_{it}(\upsilon)$.[10]

2.3.1. Final goods producers

The profit maximization problem is static and

$$\max_{\{y_{it},\{y_{it}(\upsilon)\}_{\upsilon\in(0,1)}\}} \frac{P_{it}}{P_t} y_{it} - \int_0^1 \frac{P_{it}(\upsilon)}{P_t} y_{it}(\upsilon)\mathrm{d}\upsilon, \tag{7}$$

where

$$y_{it} = \left(\int_0^1 y_{it}(\upsilon)^{\frac{\eta-1}{\eta}}\mathrm{d}\upsilon\right)^{\frac{\eta}{\eta-1}}. \tag{8}$$

The parameter η captures the elasticity of substitution across different inputs.[11] Profit maximization implies that the demand for each intermediate input is downward sloping,

$$y_{it}(\upsilon) = \left(\frac{P_{it}}{P_{it}(\upsilon)}\right)^{\eta} y_{it}, \tag{9}$$

and that the price of the final goods is the "average" price of inputs,

$$P_{it} = \left(\int_0^1 P_{it}(\upsilon)^{1-\eta}\mathrm{d}\upsilon\right)^{\frac{1}{1-\eta}}. \tag{10}$$

2.3.2. Intermediate input producers

The intermediate input producers maximize the present value of profits over time. In each period, a firm decides how much the firm produces and how many workers the firm employs. The firm obtains an employment subsidy τ_{it+j} in period $t+j$. If a firm could adjust the price in each period, then the problem is purely static. Here the firm sets the price so that the price cannot be probabilistically revised $\omega_i \in [0,1)$ in each coming period.[12] The future profits are evaluated from the household's point of view, which is summarized by Λ_{tt+j}. The maximization problem is

$$\max_{\{\{y_{it+j}(\upsilon),l_{it+j}(\upsilon)\}_{j=0}^{\infty},P_{it}(\upsilon)\}} E_t \sum_{j=0}^{\infty} \Lambda_{tt+j}\omega_i{}^j \left[\frac{P_{it}(\upsilon)}{P_{t+j}} y_{it+j}(\upsilon) - (1-\tau_{it+j})w_{t+j} l_{it+j}(\upsilon)\right], \tag{11}$$

where the production technology is

$$y_{it+j}(\upsilon) = \theta_{it+j} l_{it+j}(\upsilon)^{\gamma}, \tag{12}$$

and the firm faces the demand function (9), but its price ($P_{it}(\upsilon)$) will be fixed over $j = 1,2,\ldots$,

$$y_{it+j}(\upsilon) = \left(\frac{P_{it+j}}{P_{it}(\upsilon)}\right)^{\eta} y_{it+j}. \tag{13}$$

[10] Many open-economy macro models introduce price rigidity in the traded final goods (Obstfeld and Rogoff, 1995; Devereux and Engel, 2003; Corsetti et al., 2011). I leave this extension for future analysis.

[11] The main results hold if the elasticity parameter η varies across countries and industries. See the Appendix.

[12] The main results are qualitatively the same even if the price is partially and automatically updated (i.e., "price indexation"). See the Appendix.

The only source of the asymmetry across υ in the same industry is the timing of the price adjustment so that the optimal price, $\tilde{P}_{it}$, does not depend on υ, and

$$\left(\frac{\tilde{P}_{it}}{P_{it}}\right)^{1-\eta+\frac{\eta}{\gamma}} = \frac{\eta}{\eta-1} \frac{E_t \sum_{j=0}^{\infty} \Lambda_{tt+j}\omega_i^j \left(\frac{P_{it+j}}{P_{it}}\right)^{\frac{\eta}{\gamma}} (1-\tau_{it+j}) \frac{w_{t+j}}{\gamma} \left(\frac{y_{it+j}}{\theta_{it+j}}\right)^{\frac{1}{\gamma}}}{E_t \sum_{j=0}^{\infty} \Lambda_{tt+j}\omega_i^j \frac{P_{it+j}}{P_{t+j}} \left(\frac{P_{it+j}}{P_{it}}\right)^{\eta-1} y_{it+j}}. \tag{14}$$

2.3.3. Law of industry price motion and industry aggregate

From (10), the price of the final good can be expressed as the "average" of the previous price and the updated input price,

$$P_{it} = \left(\omega_i P_{it-1}^{1-\eta} + (1-\omega_i)\tilde{P}_{it}^{1-\eta}\right)^{\frac{1}{1-\eta}}. \tag{15}$$

By combining (9) and (12),

$$y_{it}\left(\frac{P_{it}}{P_{it}(\upsilon)}\right)^{\eta} = \theta_{it} l_{it}(\upsilon)^{\gamma}, \tag{16}$$

and then

$$y_{it} = \frac{\theta_{it}}{s_{it}{}^{\gamma}} l_{it}^{\gamma}, \tag{17}$$

where the term l_{it} is the total hours worked in industry i,

$$l_{it} = \int_0^1 l_{it}(\upsilon)\mathrm{d}\upsilon, \tag{18}$$

and s_{it}^{γ} captures the price distortion,

$$s_{it} \equiv \int_0^1 \left(\frac{P_{it}}{P_{it}(\upsilon)}\right)^{\frac{\eta}{\gamma}} \mathrm{d}\upsilon. \tag{19}$$

The final output is expressed as the product of the labor term (l_{it}^{γ}) and the effective productivity term ($\theta_{it}/s_{it}^{\gamma}$). What is shown here is an extension of the aggregation result shown in the single-good, closed-economy models (Schmitt-Grohé and Uribe, 2011; Ascari and Sbordone, 2014) to the multi-country, multi-good case.

2.4. Equilibrium and the steady state

All subsidies are financed by tax.

$$\sum_i \int_0^1 \tau_{it} w_t l_{it}(\upsilon)\mathrm{d}\upsilon = \tau_{Lt}. \tag{20}$$

The final goods markets clear

$$N c_{it} + N^* c_{it}^* = N y_{it} + N^* y_{it}^*. \tag{21}$$

The bond market clears

$$b_t = 0. \tag{22}$$

Finally, I impose the trade balance condition

$$\sum_i P_{it}(y_{it} - c_{it}) = 0. \tag{23}$$

Given the above conditions, equilibrium is defined as usual. I then focus on the steady-state properties. In the steady state, each producer may set a different price, employ a different number of workers, and produce a different amount of intermediate goods, yet the aggregate variables stay the same. The steady-state variables are expressed without t subscripts.

2.4.1. Industry-level implications in the steady state

In the steady state, the industry-level output is from (17),

$$y_i = \frac{\theta_i}{s_i^{\gamma}} l_i^{\gamma}, \tag{24}$$

where the price distortion, s_i, can be written as

$$s_i = \frac{1-\omega_i}{1-\omega_i(1+\pi)^{\frac{\eta}{\gamma}}} \left(\frac{1-\omega_i(1+\pi)^{\eta-1}}{1-\omega_i}\right)^{\frac{\eta}{\eta-1}\frac{1}{\gamma}}. \tag{25}$$

From (14) and (15), the industry-level relative price ($p_i \equiv P_i/P$) is

$$p_i = \left(1-\tau_i\right) \upsilon_i \frac{s_i^{\gamma}}{\theta_i} \frac{w}{\gamma} l_i^{1-\gamma}, \tag{26}$$

where the markup term, υ_i, is

$$\upsilon_i = \frac{\eta}{\eta-1} \frac{1-\beta\omega_i(1+\pi)^{\eta-1}}{1-\beta\omega_i(1+\pi)^{\frac{\eta}{\gamma}}} \frac{1-\omega_i(1+\pi)^{\frac{\eta}{\gamma}}}{1-\omega_i(1+\pi)^{\eta-1}}. \tag{27}$$

The calculations of s_i and p_i in the steady state apply the sum of the terms from 0 to ∞ periods in, for example, (14). I restrict the parameter space to ensure the sums are finite. I impose the following assumption throughout the study:

Assumption 1. $\beta \in (0,1)$, $\eta > 1$, $\omega_i \in [0,1)$, and $\omega_i(1+\pi)^{\frac{\eta}{\gamma}} < 1$ for all i.

In the single-good, CRS models, Schmitt-Grohé and Uribe (2006) show the properties of s_i in a dynamic setting, and King and Wolman (1999) quantitatively examine the properties of υ_i.[13] Ascari and Sbordone (2014) quantitatively examine the steady state properties. Damjanovic and Nolan (2010) analyze the properties of s_i in a single-good DRS setting. Prior studies agree that the effect of υ_i is quantitatively minor. For this reason, most of the analyses focus on the usual case in which the subsidies eliminate the markup distortion.

Assumption 2. $(1-\tau_i)\upsilon_i = 1$.

The Appendix shows some of the following results without Assumption 2. However, this assumption simplifies the analytical results. Lemma 1 summarizes the properties of s_i in the steady state. The properties are developed in the literature (Ascari, 2004; Schmitt-Grohé and Uribe, 2011; Ascari and Sbordone, 2014), but it is helpful to summarize them for later use.

Lemma 1 (*Ascari, 2004; Schmitt-Grohé and Uribe, 2006, 2011; Ascari and Sbordone, 2014*). *Under Assumption 1,*

1. *If $\omega_i = 0$ or $\pi = 0$, then $s_i^{\gamma} = 1$.*
2. *$\partial s_i^{\gamma}/\partial \pi \gtrless 0$ if $\pi \gtrless 0$.*
3. *For small $\varepsilon > 0$, $s_i^{\gamma}|_{\pi=\varepsilon} > s_i^{\gamma}|_{\pi=-\varepsilon}$ under usual parameter values.*

Proof. See the Appendix for the derivations and precise conditions. □

This lemma shows the properties of the price distortion. Property 1 says there is no distortion if the inflation rate is zero. Similarly, if prices are completely flexible (non-adjustment probability, ω_i, is zero), there is no distortion. When there is no inflation, firms do not need to change the price. All the firms set the same price, and there is no allocation inefficiency. Similarly, if firms can change the price freely, all firms set the same optimal price. Again, there is no allocation inefficiency. Property 2 says that the distortion increases in the absolute deviation from zero-inflation. Property 3 says that the price distortion is asymmetric in the sense that a 1% inflation is more costly (in terms of productivity) than a 1% deflation.[14] Under deflation, firms that change prices set lower prices than the static optimal and gradually overprice. The economy-wide cost of this initial underpricing eliminates the cost of over-pricing due to monopolistic competition. Hence, a 1% deflation is not as costly as a 1% inflation.

Even though the aggregate price level increases, some firms cannot raise prices. Adjusted prices are higher than non-adjusted prices. Accordingly, some firms sell more than others do. However, since firms are ex-ante symmetric, efficient allocation is such that all firms produce the same amount. The price distortion represents this allocation inefficiency, and the magnitude of the price distortion depends on the inflation rate, the magnitude of price rigidity, and some other parameters.

This price distortion is the key reason the optimal inflation rate is zero in the closed economy. In the standard single-good, closed-economy sticky-price models, researchers focus on this loss of effective productivity due to the impact of price distortion on the optimal inflation rate (see discussion by Benigno and Woodford, 2005).[15] Since trend inflation is associated with a loss of productivity if the industry's price is rigid, a zero-inflation rate eliminates the resource loss from the price distortion (Damjanovic and Nolan, 2010). Consequently, zero inflation maximizes welfare. In the open economy, however, productivity loss is not necessarily associated with welfare loss.

[13] King and Wolman (1999) set up price rigidity differently.

[14] The asymmetry is a specific property of the Calvo assumption. Under the Rotemberg assumption, the cost is symmetric since the loss function is symmetric. See the Appendix.

[15] More precisely, the optimal inflation rate calls for managing two distortions: markup distortion and price distortion (King and Wolman, 1999). Monopolistic competition leads to markup distortion. When the inflation rate is high, price setters choose high markup. Meanwhile, the expected average markup during the non-adjusting period is low. Thus, average markup is lowest under mild inflation, which minimizes the loss from the markup distortion. Under standard parameters, the welfare gain from reducing markup distortion is quantitatively small. Although King and Wolman (1999) show this result using the staggered price model rather than the Calvo model, the difference between these two models is not essential in the steady state. Partly due to this quantitatively minor role of the welfare gain, researchers drop the markup distortion by including a distortion-corrective subsidy, and focus on the price distortion.

3. Optimal monetary policy under Ricardian trade

This section examines the optimal monetary policy under CRS production ($\gamma = 1$). The derivations and proofs are provided in the Appendix.

As a reference, the analysis starts from the single-good autarky case, followed by the open-economy case. In the analysis, I measure welfare by the steady-state utility of the household (King and Wolman, 1999; Ascari, 2004).

3.1. Autarky

3.1.1. Single-good

Under the single-good autarky, the standard result holds that welfare is maximized when the inflation rate is zero. In particular, the steady state welfare, $u(c, l)$, is

$$u = c^{\psi}(1-l)^{1-\psi} = \left(\frac{s}{\theta}\right)^{-\psi}. \tag{28}$$

The welfare is maximized where s is minimized, that is, $\pi = 0$.

3.1.2. Two-good

When there are two goods, welfare is expressed as a generalized version of (28), which depends on s_A, s_B, θ_A, θ_B, and other parameters. Intuitively, welfare is the weighted average of the price distortion of two goods. If two goods are symmetric, the expression reduces to (28). It is straightforward that the optimal inflation rate is $\pi = 0$.

3.2. Two-country Ricardian model

Under the linear-in-labor (CRS) production function in the intermediate sectors, the aggregate productions also exhibit a linear-in-labor aggregate form. Then, in the two-country, two-(traded)-good models with linear-in-labor technology, the trade pattern is determined as the textbook Ricardian model.[16] The relative price under autarky indicates the comparative advantage. Using (26) when $\gamma = 1$ and Assumption 2, the relative price under autarky is

$$\frac{p_A}{p_B} = \frac{s_A}{s_B}\frac{\theta_B}{\theta_A}. \tag{29}$$

The relative price depends on the relative productivity of sectors. In the textbook Ricardian model, the relative productivity is exogenously given. Here, the relative price also depends on the endogenous component s_i.

3.2.1. Trade equilibrium under CRS technology

As in the textbook Ricardian model (see, for example, Ch. 1 of Feenstra, 2016), the open-economy equilibrium is illustrated in Fig. 2. The downward-sloping curve is the world demand for good A, while the stair-step shape is the world supply of good A. This figure illustrates a case in which the home country has a comparative advantage in producing A, and the foreign country has a comparative advantage in B. The situation is such that the relative price of A to B in the home country under autarky is lower than the relative price in the foreign country,

$$\frac{s_A}{s_B}\frac{\theta_B}{\theta_A} < \frac{s_A^*}{s_B^*}\frac{\theta_B^*}{\theta_A^*}. \tag{30}$$

Depending on the location of the demand curve, there are three possible locations of the intersection point (i.e., equilibrium), as illustrated by the three different intersects. If the intersection is on the lower step (between 0 and $N\theta_A/s_A$), the open-economy relative price is the same as the autarky relative price of the home country. In this case, the home country is indifferent to producing A and B and hence produces both. The foreign country produces only good B. If the intersection is on the middle vertical line (at $N\theta_A/s_A$), then the open-economy relative price is higher than the autarky relative price of the home and lower than that of the foreign country. The home country specializes in A and the foreign country B. Finally, if the intersection is on the upper step (between $N\theta_A/s_A$ and $N\theta_A/s_A + N^*\theta_A/s_A$), the home country produces only A, and the foreign country produces both A and B.

In each case, equilibrium exists and is unique:

Proposition 1. *Consider a two-country, two-good model. Under costless trade, the specialization pattern is determined by the autarky relative prices of the two countries, which depend on exogenous productivity, price rigidity, and the inflation rate. As in the Ricardian model, the world supply curve in the steady state has a stair-step pattern. There exists a unique stationary equilibrium.*

Proof. See the Appendix. □

[16] Ishise (2020) empirically confirms this implication about the trade pattern.

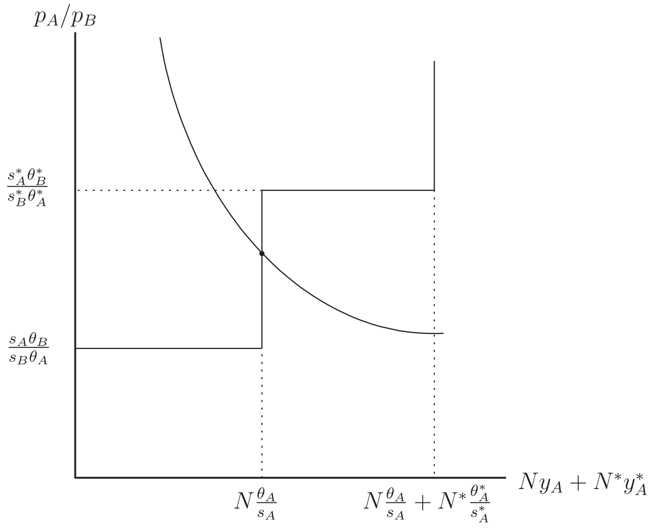

Fig. 2. Costless trade equilibrium of the two-good model.

3.2.2. Terms of trade

Suppose each country entirely specializes such that the home country produces good A, and the foreign country produces good B, as shown in Fig. 2. The demand curve crosses the middle vertical line. As shown in the Appendix, the equilibrium relative price in the open economy, p, is

$$p \equiv \frac{p_A}{p_B} = \frac{\alpha_A}{\alpha_B}\left(\frac{\theta_B^*}{\theta_A}\frac{s_A}{s_B^*}\frac{N^*}{N}\right)^{\frac{1}{\rho}}. \tag{31}$$

This relative price is the home country's terms of trade because the home country exports good A and imports good B.

Given (31), an increase in s_A through a deviation from zero inflation improves the home country's terms of trade because an increase in the price distortion (s_A) leads to a decrease in the industry-level productivity, which is associated with high prices in a perfectly competitive final goods market. Fig. 3 describes the result of high s_A (broken-line) and low s_A. The middle vertical line under high s_A is to the left of the line under low s_A. Given a downward-sloping world demand curve, this difference in the vertical line implies a higher relative price (the terms of trade).[17] In summary,

Proposition 2. *Consider a two-country, two-good model where the home country specializes in good A and the foreign country specializes in good B, and suppose that Assumption 2 is satisfied. The home country's terms of trade improve as π diverges from 0.*

3.2.3. Welfare enhancing inflation

The welfare of the home country is

$$\begin{aligned} u =& (c)^{\psi}(1-l)^{1-\psi} \\ =& \psi^{\psi}(1-\psi)^{1-\psi}\left(\frac{\theta_A}{s_A}\left(\alpha_A^{\rho}+\alpha_B\alpha_A^{\rho-1}\left(\frac{\theta_B^*}{\theta_A}\frac{s_A}{s_B^*}\frac{N^*}{N}\right)^{\frac{\rho-1}{\rho}}\right)^{\frac{1}{\rho-1}}\right)^{\psi}. \end{aligned} \tag{32}$$

Taking the derivative with respect to π, the welfare change due to a change in π can be expressed as a reduced form expression.

Proposition 3. *Consider a two-country, two-good model where the home country specializes in good A and the foreign country specializes in good B, and Assumption 2 is satisfied. Using the endogenous variables, the change in the home country's welfare caused by a change in π can be expressed as*

$$\frac{du}{d\pi}\frac{\pi}{u} \propto \underbrace{\underbrace{\frac{p_B c_B}{c}}_{\text{import ratio}}\frac{dp}{d\pi}\frac{\pi}{p}}_{\text{terms of trade effect}} - \underbrace{\frac{\partial s_A}{\partial \pi}\frac{\pi}{s_A}}_{\text{direct costs}}. \tag{33}$$

[17] If $|\partial s_A/\partial \pi| < |\partial s_B/\partial \pi|$, the lower step goes down rather than up. This difference does not affect the effect on the terms of trade.

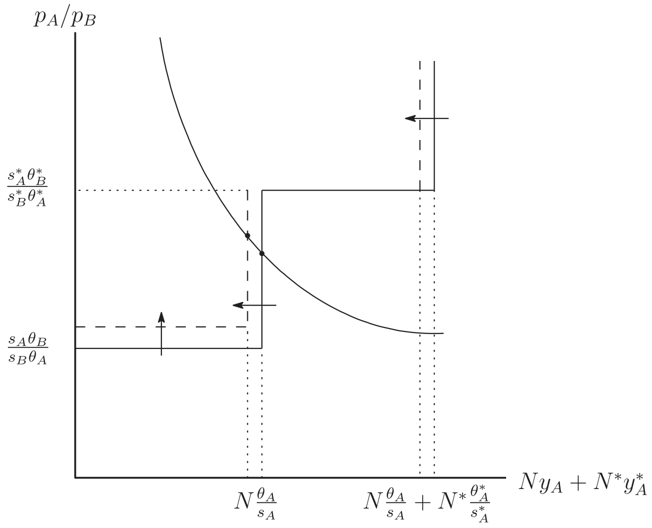

Fig. 3. Terms of trade under high- and low-inflation rates.

Proof. See the Appendix. □

This model does not include investment and government expenditures, and trade is balanced. Hence, the first term (the ratio of imports to consumption) is equivalent to the import-GDP (or export-GDP) ratio. The next term is the elasticity of the terms of trade with respect to the inflation rate—how much the terms of trade change as the inflation rate changes. The last term is a change in the price distortion due to a change in the inflation rate. Thus, the change in welfare depends on the trade-GDP ratio, inflation elasticity of the terms of trade, and inflation elasticity of the price distortion.

Note that the expression is intuitive, but involves several endogenous variables. The change in welfare is expressed only by exogenous variables.[18]

$$\frac{\partial u}{\partial \pi}\frac{\pi}{u} = -\frac{\psi \alpha_A^{\rho}}{\alpha_A^{\rho} + \alpha_B \alpha_A^{\rho-1}\left(\frac{\theta_A}{\theta_B^*}\frac{s_B^*}{s_A}\frac{N}{N^*}\right)^{\frac{1-\rho}{\rho}}}\left(1 - \frac{1-\rho}{\rho}\frac{\alpha_B}{\alpha_A}\left(\frac{\theta_A}{\theta_B^*}\frac{s_B^*}{s_A}\frac{N}{N^*}\right)^{\frac{1-\rho}{\rho}}\right)\frac{\partial s_A}{\partial \pi}\frac{\pi}{s_A} \tag{34}$$

The first term before the parentheses is always positive. The second term (inside the parentheses) can be positive or negative depending on the parameter values. The last term is, from Lemma 1, positive if $\pi > 0$ and negative if $\pi < 0$. Hence, if the second term is positive, the shape of utility against π is the inverted-U shape, and zero-inflation maximizes welfare as in the autarky case. However, if the second term is negative, welfare increases as π diverges from zero. This contrasts with the autarky case, where the optimal inflation rate is zero. In summary,

Proposition 4. *Consider a two-country, two-good model where the home country specializes in good A and the foreign country specializes in good B, and Assumption 2 is satisfied. The home country's welfare increases as π deviates from 0 if*

$$\frac{1-\rho}{\rho}\frac{\alpha_B}{\alpha_A}\left(\frac{\theta_A}{\theta_B^*}\frac{s_B^*}{s_A}\frac{N}{N^*}\right)^{\frac{1-\rho}{\rho}} > 1. \tag{35}$$

Proof. See the Appendix. □

The condition can be decomposed into two parts. First, a necessary condition is $\rho < 1$, that is, the magnitude of complementarity of the two final goods is stronger than that in the Cobb–Douglas ($\rho = 1$) situation.[19] Second, the condition more likely holds if (1) ρ is smaller, (2) α_B is larger, (3) θ_A/θ_B^* is larger, (4) s_B^*/s_A is larger, and (5) N/N^* is larger. In words, if a large country exports essential goods using highly productive technology but the goods are not large in the budget share, then a welfare gain is more likely.

The change in welfare depends on two effects. A change in the terms of trade is positive for welfare. At the same time, a reduction in productivity reduces the country's output, resulting in a welfare loss in the home country. Hence, the home country's welfare

[18] Remember that s_i is rewritten by exogenous parameters in (25).

[19] The importance of the role of complementarity is the same as the original immiserizing growth argument (Johnson, 1955; Bhagwati, 1958).

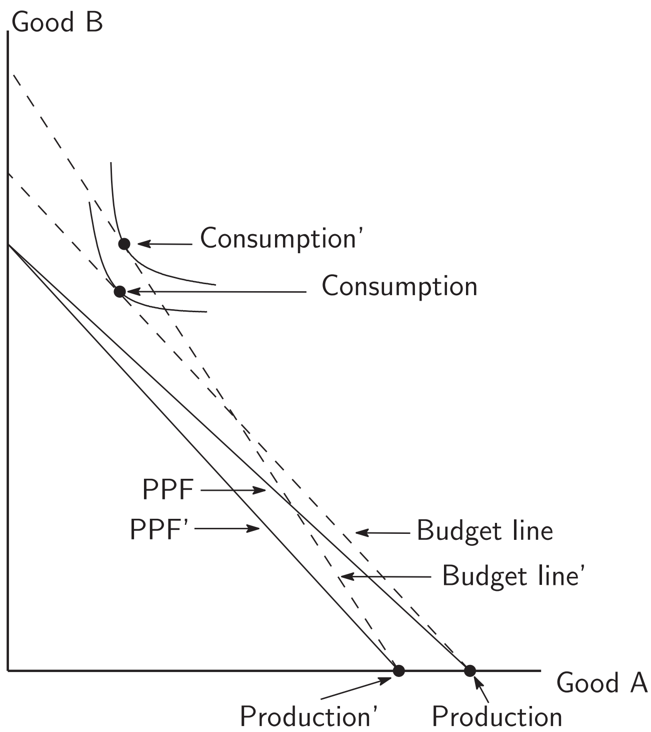

Fig. 4. The effect of non-zero inflation.

depends on which effect dominates. Under the above condition, the gains from the terms of trade overcome the loss from the output reduction.

Fig. 4 describes the result by showing the home country's production possibility frontiers (straight lines), budget constraints (broken lines), and indifference curves under different inflation rates. Suppose that $\pi = 0$ (shown without an apostrophe) or $\pi > 0$ (with an apostrophe). As Lemma 1 shows, deviating from $\pi = 0$ leads to a reduction in the industry-level productivity by increasing the price distortion. The production possibility frontier (PPF) captures the difference in the industry-level productivity: PPF' is inside PPF. When the specialization pattern is the same, the country produces good A only. At the same time, lower industry-level productivity is associated with higher terms of trade. The budget line (which captures the relative price of A in terms of B, i.e., the terms of trade) is steeper for the non-zero inflation rate case than for the zero-inflation case. The steeper slope of the budget line may not necessarily lead to higher utility. Under a certain condition (notably, in the figure, $\alpha_B \gg \alpha_A$), a higher utility is attained under the non-zero inflation rate.

In Fig. 4, the home country's productivity of good B is unchanged. This is not a required assumption. Instead, the productivity of the other industry (which is not producing) can also be decreasing in π. As long as the specialization pattern is unchanged, the welfare result holds. Moreover, since a parallel shift in the home country's PPF ensures no change in the specialization pattern, a reduction in the industry-level productivity of the non-producing industry helps sustain the main result.

The reverse version, a welfare loss through technology improvement, is classically known as immiserizing growth (Johnson, 1955; Bhagwati, 1958). The literature usually describes immiserizing growth as a result of exogenous technological growth. Here, manipulation of the terms of trade is the result of the policy change.

3.3. Optimal inflation rate

Given $\rho \in (0,1)$, the left-hand side of (35) is decreasing in s_A. Hence, as π deviates from zero, the condition (35) may not be satisfied. The optimal inflation rate is to hold the equality (not inequality) of the equation, or to choose π such that s_A satisfies the following condition

$$s_A(\pi) = \left(\frac{1-\rho}{\rho} \frac{\alpha_B}{\alpha_A} \right)^{\frac{\rho}{1-\rho}} \frac{\theta_A}{\theta_B^*} \frac{N}{N^*} s_B^*. \tag{36}$$

However, there is another bound. If the productivity drop is significant enough to break (30), the trade pattern changes. The welfare calculation from (34) does not apply. Specifically, as is clear from Fig. 5, a change in the home inflation rate raises the terms of trade to the upper step; as the vertical line moves to the left, the terms of trade increase. However, once the terms of trade reach the upper step, the relative price is determined by the autarky price of the foreign country. At this level, manipulating the home

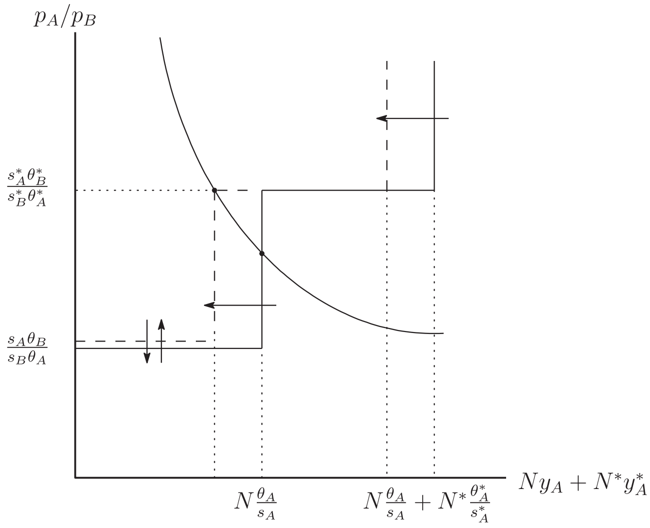

Fig. 5. Optimal inflation rate.

country's supply has no impact on the terms of trade, and reducing the effective productivity through inflation (or deflation) has a negative effect only. The upper bound of the optimal inflation rate ensures that the terms of trade are at the level of the autarky relative price in the foreign country:

$$\frac{\alpha_A}{\alpha_B}\left(\frac{\theta_B^*}{\theta_A}\frac{s_A}{s_B^*}\frac{N^*}{N}\right)^{\frac{1}{\rho}}=\frac{s_A^*}{s_B^*}\frac{\theta_B^*}{\theta_A^*}. \tag{37}$$

Finally, as is clear from Lemma 1, there is a lower bound, $s_A \geq 1$. Thus, the optimal inflation rate is the interior solution implied by the first-order condition (36) or the corner solutions implied by the upper bound implied by (37) or the lower bound level:

Proposition 5. *Consider a two-country, two-good model where the home country specializes in good A and the foreign country specializes in good B, and Assumption 2 is satisfied. Taking the foreign country's inflation rate as given, the optimal inflation rate of the home country is to choose π such that $s_A(\pi)$ satisfies*

$$s_A(\pi)=\max\left\{\min\left\{\left(\frac{1-\rho}{\rho}\frac{\alpha_B}{\alpha_A}\right)^{\frac{\rho}{1-\rho}}\frac{\theta_A}{\theta_B^*}\frac{N}{N^*}s_B^*,\quad\left(\frac{\alpha_B}{\alpha_A}\right)^{\rho}\left(\frac{\theta_B^*}{\theta_A^*}\right)^{\rho}\frac{\theta_A}{\theta_B^*}\frac{N}{N^*}{s_A^*}^{\rho}{s_B^*}^{1-\rho}\right\},\quad 1\right\}. \tag{38}$$

3.4. Small-open economy

In the two-country model, the source of welfare improvement is a change in the terms of trade. The channel is specific to a two-country model. In fact, the welfare effect of inflation is always negative in closed-economy and small-open economy models. In a small-open setting, the world price (in particular, the relative price of goods) is exogenously given and constant. Under the small-open setting, any changes in the home country do not affect the terms of trade. Hence, the effect of π on the home country's welfare resembles the closed-economy setting. In particular, the welfare loss of the closed and small-open models is just the price distortion. Eq. (33) in Proposition 3 encompasses the autarky and small-open economy cases, in which the terms of trade do not respond; hence, the first term is zero. Welfare definitely decreases.

3.5. Empirical relevance of the key assumption

A key assumption that inflation leads to a welfare gain is $\rho < 1$. That is, the exports and imports are complements. Is this assumption empirically plausible? The parameter has two closely-related but different empirical interpretations. The first interpretation is that the parameter controls the elasticity of substitution across the variety. The second interpretation is that the parameter is the price elasticity of home or foreign goods.

In trade literature, the elasticity of substitution across the variety tends to be large, and the value is larger if the goods are more disaggregated. For example, using US import goods data from 1990 to 2001, Broda and Weinstein (2006) estimate that the median elasticity is 2.2 for three-digit, 2.7 for five-digit, and 3.1 for ten-digit classifications.[20]

[20] The high elasticity seems to be reasonable for a highly disaggregated case. In the case of ten-digit, a comparison is, for example, "Meat of Lamb Legs" (ten-digit harmonized system: 0204.22.20.20) from two different countries. For ordinal people, lamb legs from Australia are highly substitutable for lamb legs

In my model, the aggregation level is even larger: exported goods vs. imported goods. International macro literature uses this framework (usually labeled as the home goods and foreign goods), and the studies commonly employ lower elasticity of substitution than the values mentioned above. For example, a seminal work by Backus et al. (1994) uses 1.5 as their benchmark value. In the quantitative analysis, Backus et al. (1994) find that a smaller elasticity, 0.5, performs better in explaining various data characteristics. As a follow-up to Backus et al. (1994) and Heathcote and Perri (2002) employ 0.9 as their benchmark value and find that the models they examine perform better if the elasticity is small.

These lower values are also empirically plausible. An alternative way to estimate the parameter is to use the demand function derived from (2). Under the CES function, ρ is the absolute value of the price elasticity of the home (exporting) good as well as that of the foreign (importing) good. Taylor (1993) and Hooper et al. (2000) comprehensively estimate the short-run and long-run price elasticities of the import and export goods for G-7 countries. For the long-run elasticity, the overall median estimated by Taylor (1993) is 0.45, while Hooper et al. (2000) find that the median is 0.5. Motivated by these values, Corsetti et al. (2008) set the price elasticity to be 0.425.

In this sense, the assumption $\rho < 1$ is empirically plausible for considering this level of aggregation. In the following quantitative sections, I use the value $\rho = 0.45$, which is in the range of the above-mentioned price elasticity estimates.

3.6. Comparison

Fig. 6 shows the impact of the terms of trade using a parameterized version of the model. Although the two-country, two-good setting is highly stylized, and a more empirically relevant calculation is provided in Section 4, I employ empirically relevant parameter values as much as possible. Following Ascari and Sbordone (2014), I set $\eta = 10$, $\beta = (1/1.04)^{(1/12)}$, and $\psi = 1/3$.

For the remaining parameters, I mimic the basic properties of the goods trade between the United States, the home country, and Canada, the foreign country. I set $N = 1$ and $N^* = 0.1$ to roughly match the economic size of these two countries. I then divide economic sectors into three categories: Canada's importing sector, Canada's exporting sector, and the service sector. Assuming that the service sector is separable in the utility so that it does not appear in the model, I focus on goods sectors. Based on the World Input–Output Database in 2006, sectors are considered to be Canada's importing (US exporting) sector if Canada's use of value from the US is larger than 35% of the sum of the use of values from the US and Canada.[21] The remaining sectors are considered Canada's exporting (US importing) sectors. I then calculate the labor productivity: the total value added of the sectors divided by the total number of workers. The number of workers is taken from KLEMS' 2006 data. After normalizing the home's productivity $\theta_A = 1$, the remainders are $\theta_B = 0.44$, $\theta^*_A = 0.76$, and $\theta^*_B = 1.26$. Thus, in this setting, both countries have an absolute advantage in the exporting sector. The parameters affecting price rigidity for home's exporting sector $\omega_A = 11/12$ follow Ascari and Sbordone (2014). The remaining cases are $\omega_B = \omega^*_A = \omega^*_B = 0$; this exercise does not depend on these parameters. Finally I choose $\alpha_A = 0.825$, which led to Canada's importing ratio of goods, 0.58, in the World Input–Output Database in 2006.[22]

Fig. 6 compares the effects of π on welfare in a single-good autarky, a two-good small-open economy, and a two-country model with two alternative ρ values, 0.45 and 1.1. The other parameter values are fixed.

If good A is the only good in the single-good closed-economy model, the welfare impact of π is the same for the single-good closed economy and the small-open economy models. The loss is the real resource loss caused by price distortion. In the two-country setting, the welfare loss is generally smaller due to the effect of the terms of trade. The loss in production improves the terms of trade, which, in turn, attenuates the loss from the distortion. The positive impact of the terms of trade exists regardless of the parameter value for ρ. Even if two goods are substitutes, a reduction in the effective productivity of the exporting industry leads to an improvement in terms of trade. Thus, the welfare cost of inflation in a two-country model is qualitatively smaller than that in a closed-economy or small-open economy model. However, under the current parameters, the quantitative impact is small. As Proposition 4 shows, the positive impact of the terms of trade outperforms the direct negative impact of the price distortion if $\rho = 0.5$.

Contrary to the single-good autarky case, welfare is locally *minimized* at $\Pi = 1$. The effect is not large for 1% to 2% inflation or deflation, but is non-negligible when the inflation rate is large. In terms of deflation/inflation, the welfare gain is larger for 5% inflation than for 5% deflation, reflecting the asymmetry of the price distortion.[23]

However, the positive impact of the inflation rate is not a global property due to the upper bound. As the inflation rate increases, welfare increases. Once the rate reaches the critical point, welfare decreases, as the decreasing portion of the curve shows.[24]

Finally, the inflation rate cannot be higher than the level constrained by Assumption 1. Under the current parameterization, the optimal inflation rate depends on (36). If the inflation rate is higher than this level, the negative effect of reducing productivity

from New Zealand. Lamb legs fall into the five-digit, "Meat of sheep, fresh or chilled" (five-digit standard international trade classification: 01211) and in three-digit, "Other meat and edible meat offal, fresh, chilled or frozen" (three-digit standard international trade classification: 012). This category, 012, includes the meat of sheep and swine. For some religions, the distinction between these two is critically important so that substitutability between lamb and pork can be lower.

[21] I choose 2006 for two reasons. First, I exclude the period affected by the Global financial crisis. Second, the industry category and the available variables are mostly consistent for the Input–Output database, KLEMS for the US, and KLEMS for Canada.

[22] See the Appendix for additional details.

[23] As footnote 14 discusses, this asymmetry is again due to the Calvo assumption. Under the Rotemberg assumption, the cost is symmetric; hence the effect of inflation on welfare is also symmetric.

[24] If Assumption 2 is not imposed, that is, the subsidy is not included, a slightly higher inflation rate mitigates the markup distortion as pointed out by King and Wolman (1999). As a result, the entire graph slightly moves to the right.

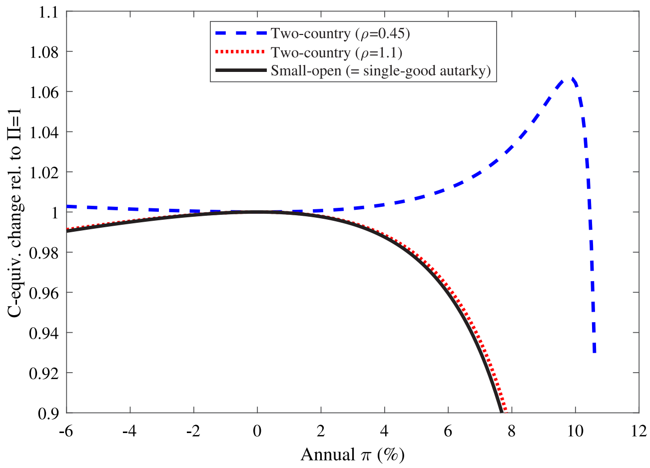

Fig. 6. Consumption-equivalent changes in welfare under single-good autarky, small-open, and two-country models with various parameterizations.

dominates the positive effect of improving terms of trade. Depending on the parameter values, the optimal rate is constrained by (37). In that case, an inflation rate higher than the upper bound implies that the home country specializes in good A, while the foreign country produces both A and B, and the terms of trade are determined by the ratio of the effective productivity of A and B in the foreign country. A change in the inflation rate of the home country decreases effective productivity and the supply of good A, but the change does not affect the terms of trade. Such a reduction in supply only hurts the home country.

3.7. Nash equilibrium

Manipulations in the terms of trade are the channel through which non-zero inflation is optimal, yet these manipulations sacrifice the trading partner's welfare. This trade-off of home and foreign welfare is similar to the effects of manipulation of the nominal exchange rate in the short-run models (e.g., Corsetti et al., 2011) or of the tariff rate in the strategic tariff models (e.g., Bagwell and Staiger, 1999). A natural consequence is a counter-response by the foreign country.

Depending on the parameter values, there are several types of Nash equilibria. Instead of analyzing all possible equilibria, I focus on a parameterized version of a particular symmetric equilibrium that illustrates the worst case.[25] The parameterization has two features. First, the world is CRS and symmetric. The home country specializes in A and the foreign country in B. The specialization is driven by both comparative and absolute advantages. Second, the non-operating sector (B in the home country and A in the foreign country) faces slightly less sticky input prices. In addition, I assume $\Pi, \Pi^* \geq 1$. This last assumption excludes an obvious source of multiple equilibria, that is, a 1% change in s_i is driven by either an increase or decrease in the inflation rate. Although the parameters are modified, these parameter values are closely related to the previous parameterization. Thus, the exercise can be considered a stylized version of the inflation rate setting problem between two developed countries.

Fig. 7 shows the cooperative optimum, best response functions, and Nash equilibrium under the particular set of parameters. The inner straight lines show the lower ($\Pi, \Pi^* \geq 1$) and upper ($\Pi, \Pi^* < \omega^{-1/\eta}$ implied by Assumption 1) bounds. The square mark is the point that maximizes the joint welfare (that is, zero inflation). The Nash equilibrium is the point at which two best response curves intersect, as shown by the circle mark.[26]

Table 1 shows the inflation rates and the welfare values in various cases. The inflation rate is the annualized value, and the welfare value is relative to welfare under autarky.

Under symmetry, the cooperative optimum is such that both countries choose zero inflation rates. The choice maximizes the effective productivity of both countries. The choice of the inflation rate is the same as under autarky, but cooperative optimum allocation achieves higher welfare for both the home and foreign countries than autarky does, thanks to the gains from specialization and trade. However, the cooperative optimum is not an equilibrium, except for the case under the unified monetary policy such as the Euro area.

If the foreign country sticks to zero inflation regardless of the home country's choice, the home country's optimal inflation rate is positive. The choice and its effects are illustrated in Fig. 5. At this inflation level, the foreign country is worse off than the autarky situation. However, here I assume that the foreign country does not go back to autarky. The home country captures all gains from

[25] Compared with the previous parameterization, I choose the parameters to make the countries symmetric: $\alpha_A = \alpha_B = 0.5$, $N = N^* = 1$, $\theta_A = \theta_B^* = 1$, $\theta_B = \theta_A^* = 0.75$, $\omega_A = \omega_B^* = 11/12$, and $\omega_B = \omega_A^* = 10/12$. Other parameters are the same as before.

[26] In addition to meeting at the marked circle (Nash), the two curves seem to intersect at the upper right corner, but that intersection point is outside the upper bound.

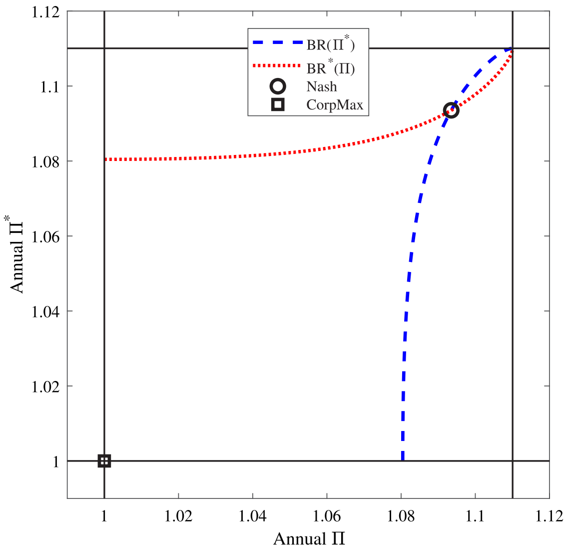

Fig. 7. Nash equilibrium. The figure shows the best response function of the home country (the optimal choice of Π for given Π^*, BR(Π^*)), best response function of the foreign country (the choice of Π^* for given Π, BR*(Π)), Nash equilibrium (Nash), and cooperative optimum point (CorpMax). See footnote 25 for the parameter values.

Table 1
Comparison of autarky, cooperative optimal, one-sided optimal, and Nash equilibrium.

	Inflation rate		Welfare	
	Home	Foreign	Home	Foreign
Autarky	0%	0%	1.00	1.00
Cooperative max	0%	0%	1.16	1.16
Home's one-sided optimum	8.0%	0%	1.17	0.96
Nash equilibrium	9.4%	9.4%	0.86	0.86

The table shows the inflation rates and welfare values relative to welfare under autarky. See Footnote 25 for the parameter values.

trade. As a result, even with the loss of effective productivity, the home country achieves higher welfare than the cooperative optimal allocation.

However, if the foreign country reacts and to raise the inflation rate, the home country is then motivated to choose an even higher inflation rate to improve the terms of trade. Both countries are incentivized to manipulate the terms of trade when they engage in it. The equilibrium is such that both countries lose the comparative (and absolute) advantage, and trade collapses. Under the Nash equilibrium, trade collapses, and the inflation rates are higher than they would be in the one-sided optimum case. Moreover, due to the huge loss of the effective productivities of both countries, the welfare values are lower than those under autarky.

4. Decreasing-returns-to-scale: Ricardo-Viner trade model

When the intermediate production shows DRS ($\gamma < 1$), the industry-level aggregate production is also DRS. The Ricardo-Viner (also known as the specific factors) model describes the situation. In this case, both countries produce both goods, and export the goods if production exceeds domestic consumption. As shown in the Appendix, stationary equilibrium exists and is unique, even without Assumption 2.

Proposition 1′. *Consider a Ricardo-Viner model (i.e., $0 < \gamma < 1$, two-country, two-good costless trade model) where the home country exports good A and the foreign country exports good B. There exists a unique stationary equilibrium.*

Proof. See the Appendix. □

As in the CRS case, a change in Π (Π's deviation from one) leads to a change in the terms of trade. In the CRS, Proposition 2 shows that Π's deviation from one always improves the home country's terms of trade when both countries specialize. In the DRS, since the home country produces both A and B, the direction of the change is generally not determined. The relative price depends on the parameters of both the home and foreign countries. A simple, sufficient condition to improve the terms of trade is that A is more sticky than B in the home country. That is,

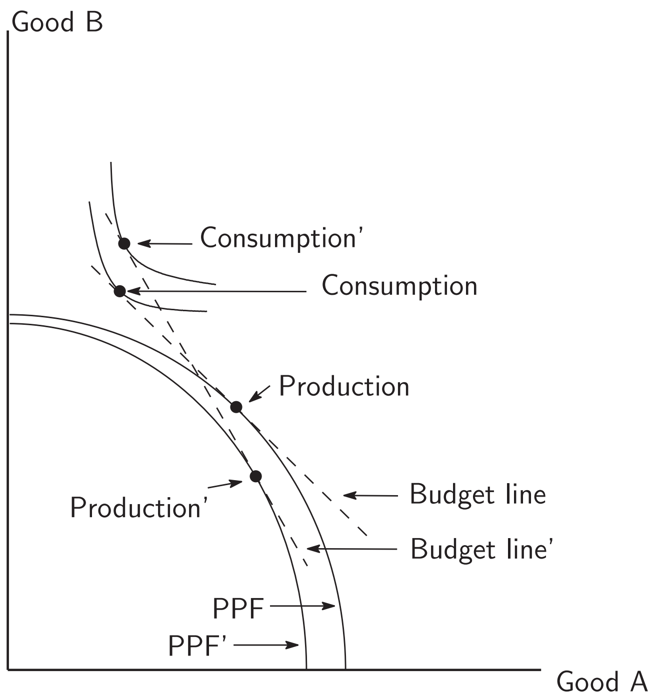

Fig. 8. The effect of non-zero inflation in the Ricardo-Viner model.

Proposition 2′. *Consider a Ricardo-Viner model where the home country exports good A and the foreign country exports good B. Suppose Assumption 2 holds. A change in Π affects the terms of trade. A sufficient condition that an increase in the terms of trade by Π's deviation from one is*

$$\left|\frac{\partial s_A^\gamma}{\partial \Pi}\frac{\Pi}{s_A^\gamma}\right| \geq \left|\frac{\partial s_B^\gamma}{\partial \Pi}\frac{\Pi}{s_B^\gamma}\right| \tag{39}$$

Proof. See the Appendix. □

In other words, the condition requires that the exporting sector is at least more price sticky than the importing sector. As in the Ricardian case, immiserizing growth is possible in this model. Fig. 8 illustrates the situation. Compared with Fig. 4, the main difference is that the production possibility frontiers are now bow-shaped curves rather than straight lines.

Contrary to the CRS case, the conditions for immiserizing growth take a complicated form, and the condition is not readily interpretable. However, using some endogenous variables, an extended version of Proposition 3 still holds.

Proposition 3′. *Consider a Ricardo-Viner model where the home country exports good A and the foreign country exports good B. Suppose Assumption 2 holds. The change in welfare caused by a change in Π is expressed as*

$$\frac{du}{d\Pi}\frac{\Pi}{u} \propto \frac{dw}{d\Pi}\frac{\Pi}{w} = \underbrace{\underbrace{\frac{-p_B(y_B - c_B)}{c}}_{\text{import ratio}}\frac{dp}{d\Pi}\frac{\Pi}{p}}_{\text{terms-of-trade effect}} - \underbrace{\sum_{i=A,B}\frac{l_i}{l}\frac{\partial s_i^\gamma}{\partial \Pi}\frac{\Pi}{s_i^\gamma}}_{\text{direct costs}} \tag{40}$$

In the autarky or small-open economy models, the welfare change is captured by the second term ("direct costs") in the above expression.

Proof. See the Appendix. □

The expression now includes both exporting and importing sectors; the change in welfare is captured by the trade-GDP ratio, the inflation elasticity of the terms of trade, labor shares of import and export sectors, and inflation elasticity of the price distortion. Note that $y_B - c_B < 0$ because B is an imported good, the first term follows the sign of $dp/d\Pi$. Under the condition that $dp/d\Pi > 0$ (as Proposition 2′), the first term attenuates the loss from the second term.

Table 2
Inflation rate elasticity of welfare.

	CAN	GBR	JPN	USA
ω	0.60	0.74	0.79	0.68
Inflation rate	2.3%	1.9%	−0.6%	3.1%
Terms-of-trade elasticity	−0.20	−0.97	1.91	0.39
Trade share	0.36	0.26	0.10	0.11
Direct cost	−0.12	−0.30	−0.12	−0.30
Terms-of-trade effect	−0.07	−0.25	0.19	0.04
Overall	−0.19	−0.55	0.07	−0.26

The values are the author's calculation using (40) except for ω. The parameter value ω is from Klenow and Malin (2011). I impose $s_A = s_B$ for simplicity. The import ratio and $(dp/d\Pi)(\Pi/p)$ are from the data. The direct cost term, $(\partial s_i^\gamma/\partial\Pi)/(\Pi/s_i^\gamma)$, is calculated based on (25). The parameter values other than ω are $\eta = 10$ and $\gamma = 0.95$.

4.1. Quantitative impact

A back-of-the-envelope calculation using (40) for CAN, JPN, GBR, and USA gauges the quantitative sense of the expression, and the calculation shows a small but non-negligible impact of the terms-of-trade effect. There are two methods to show the quantitative impacts. The first method is to set all parameter values and calculate all model variables. For this, I need to feed parameter values of the hypothetical foreign country, including the price rigidity and inflation rate. Given the gap between the stylized two-country, two-good model and actual multi-country, multi-good real data, I do not employ this strategy. Instead, I use an alternative method: some summary statistics from the empirical estimates and a minimal set of parameter values to derive key quantitative implications. The cost of using this second strategy is that I cannot calculate the optimal inflation rate for the country.

Here, I show a summary of the methods and report the results. The Appendix explains the details of the data and calculations. I start with the term "direct costs". I consider the symmetric case ($s_A = s_B$). This assumption may not be empirically accurate, but it is hard to obtain various parameters of multiple countries for exporting and importing sectors separately.[27] Under this assumption, the "direct costs" term in (40) simply becomes the inflation elasticity of the price distortion, and the value depends on the price rigidity and other model parameters. The first row in Table 2 reports the values of ω, the probability that the firms cannot change the price. These values are from Klenow and Malin (2011).[28]

The average monthly inflation rate is the inflation rate of the CPI during the sample period for the calculations of ω. The second row in Table 2 reports the annual average values. Substituting these values, together with $\eta = 10$ (10% markup rate) and $\gamma = 0.95$ (small magnitude of DRS) to (25), gives the value for the "direct costs" term reported in the sixth row of Table 2. For the US, a one-percent increase in the inflation rate directly reduces welfare by 0.30%. The direct costs are in the range of a similar magnitude across countries.

The "terms-of-trade effect" term has two components: the trade share and the terms-of-trade elasticity. The trade share is the average of the import and export shares to GDP during the sample period.[29] The third row in Table 2 reports the values. I estimate the terms-of-trade elasticity using consumer, export, and import price indices. The data is from the OECD data, and the sample period is the post-Bretton-Woods period.[30] A regression fitting the log of the terms of trade on the log of the inflation rate gives the elasticity. The fourth row in Table 2 reports the results. The elasticity is 0.39 for the US.[31] With the trade share of 0.11, the terms-of-trade effect term is approximately $0.04 (\approx 0.11 \times 0.39)$. A one-percent increase in the inflation rate indirectly raises welfare by 0.04% by improving the terms of trade. The fifth row in Table 2 reports the value. Thus, the overall inflation elasticity of welfare ($= \frac{dw}{d\Pi}\frac{\Pi}{w}$) is $-0.26 (\approx -0.30 + 0.04)$, as shown in the bottom row in Table 2. That is, a one-percent increase in the inflation rate reduces welfare by 0.26%. The contribution of the terms-of-trade effect is small, but the terms of trade have a non-negligible ($15\% (\approx 0.04/0.26)$ overall) positive impact on the elasticity of welfare.

These values vary across countries. For Canada and the UK, the terms-of-trade elasticity is a negative value; hence, the overall loss is larger than the direct cost. According to Proposition 2′, the negative elasticity means that the assumption of symmetry across sectors ($s_A = s_B$) is not true for these two countries. Hence, the direct costs are also not calculated accurately. On the contrary, for Japan, a sizeable terms-of-trade elasticity dominates the direct cost such that the deviations from zero inflation improve welfare.

[27] Ishise (2020) calculate the input price rigidity for different sectors for the US.

[28] The original sources are Amirault et al. (2006) for Canada, Higo and Saita (2007) for Japan, Bunn and Ellis (2009) for the UK, and Goldberg and Hellerstein (2011) for the US. I use updated versions of the original sources for Japan and the US.

[29] The values of exports, imports, and GDP are from OECD data.

[30] The following estimation uses an extended sample period to have a reasonable sample size.

[31] This elasticity is an OLS estimate. The methodology may have several issues with time series econometrics. For a reference, I also employ the dynamic OLS fitting log of the terms of trade on the log of the inflation rate, the first difference of the log of the inflation rate, and one and two periods leads and lags of the first difference of the log of the inflation rate. In this case, the estimated elasticity for the US is 0.68. See the Appendix for additional details.

5. Concluding remarks

This paper introduces the classical Ricardian and Ricardo-Viner international trade models into a stylized model of nominal rigidity with trend inflation. In the long-run steady state, the model implies that the magnitude of price rigidity and the inflation rate affect the terms of trade and that the optimal inflation rate can be non-zero. The welfare loss of non-zero inflation is smaller in a two-country model than in a closed or small-open model if the terms of trade lead to gains. The main driver of welfare gain is the manipulation of the terms of trade. The terms-of-trade effect may have a sizeable quantitative impact depending on the country.

I focus only on the steady state. However, a dynamic consideration may lead to a different value. By extending the existing quantitative studies of the optimal inflation rate by, for example, Schmitt-Grohé and Uribe (2011), Coibion et al. (2012), and Carreras et al. (2016) who examine dynamic closed-economy settings, future studies considering a dynamic open-economy extensions can make interesting findings.

Appendix A. Supplementary data

Supplementary material related to this article can be found online at https://doi.org/10.1016/j.euroecorev.2022.104223.

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