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ORIGINAL PAPER

Survey on Equivalent Continuum Modeling for Truss Structures and Their Nonlinear Dynamics and Vibration Control

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Abstract

Purpose With the development of engineering structures towards the direction of large-scale, light-weight and multi-function, truss structures are utilized widely in the aerospace and civil sectors due to their outstanding advantages, e.g. light weight, large stiffness ratio and high packaging efficiency. Meanwhile, investigating the nonlinear dynamic mechanism and developing vibration control strategies for large space truss are of practical importance and give rise to interesting scientific issues.
Methods Finite element method is a popular approach but brings great challenges to the nonlinear dynamic analysis and vibration controller design of truss structures due to the high degree of freedom of the full-scale finite element model. Therefore, the equivalent continuum modeling methodology becomes one of the most important developing trends to address these difficulties and is of high-efficiency especially for the nonlinear dynamic analysis.
Results In the present paper, the research status about the equivalent continuum modeling of truss structures is sorted out including equivalent modeling methods (the energy equivalent method, the homogenization method, the displacement equivalent criterion, etc.) together with their advantages and drawbacks. Issues on static, dynamic, and buckling analyses of various structural styles (beamlike truss, platelike truss, hoop truss, etc.) with different connection joints are discussed.
Conclusions More specifically, the research progresses on equivalent nonlinear continuum modeling and the design of vibration control law of the large truss structures are investigated, respectively, and the gap on the nonlinear analysis and vibration control is summarized for the existing researches. Additionally, comment, perspective and opportunity are proposed which could be valuable for the future developments of equivalent continuum modeling and vibration control of the large truss structures.

Keywords Equivalent continuum modeling · Truss structures · Nonlinear analysis · Analytical method · Vibration control

Introduction

Motivated by the ever-increasing demands for aerospace structures, the new opportunity to create large space deployable truss structures also call for advanced equivalent continuum modeling techniques, in particular when the number of the degrees of freedom (DOFs) increases dramatically for the large-scale truss structures [1–4]. Also, the truss structures are widely involved in civil engineering [5, 6] (e.g. buildings, bridges or transmission towers), material science [7, 8] (e.g. carbon nanotubes, graphitic materials), and other applications [9–11]. The finite element method (FEM) is one of most popular methods to calculate dynamic behaviors of truss structures due to its convenience of intuitionist result. However, the FEM requires a significant amount of computing capacity for the large truss structures and the results cannot be applied effectively for the vibration controller design, especially, it is difficult to comprehend and grasp the essential nonlinear dynamic characteristics when the nonlinear analysis needs to be carried out. Therefore, the equivalent continuum modeling techniques are becoming increasingly promising in practical engineering fields [12, 13]. The equivalent continuum modeling approaches, their advantages, the development and future directions were summarized in Ref. [14]. The equivalent continuum modeling possesses three promising advantages, as follows:

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(1) Can obtain single equivalent continuum model and further reduce DOFs significantly using the Galerkin method to achieve analytical mode shapes, which brings great convenience to the design of vibration controller for large trusses [15]. Moreover, the equivalent continuum modeling techniques cannot capture generally local vibration modes, which is precisely an advantage, because local vibration modes are not crucial greatly for the vibration control of the large truss structures.
(2) Can analyze the nonlinear dynamic characteristics in analytical algorithm for the complex and high-dimensional truss structures after the equivalent modeling process [16]. Consequently, the nonlinear behaviors of the truss structures such as frequency–response, bifurcation, internal resonance, etc., can be further studied in classical nonlinear analytical methods, for example, average method, multiscale approach and harmonic balance method.
(3) Can remarkably reduce the computational cost for nonlinear response analysis especially for complicated truss structures.

Thus, the equivalent continuum modeling techniques have revived many researchers' interest in accurately predicting the dynamic behaviors of the large truss structures.

Over the past 3 decades, a variety of techniques and methodologies have been developed for the static and dynamic equivalent continuum modeling of the truss structures. Based on the transformation matrices, Nayfeh and Hefzy [17, 18] developed an equivalent continuum modeling method of the discrete truss structures where only extensional force was taken into account for the individual rod, but the equivalent mass matrix was not deduced. Another prevalent equivalent modeling methodology based on the energy equivalence principle has been widely utilized in developing substitute continuum to the discrete structures. The concept of the equivalent continuum modeling based on the energy equivalent principle was proposed by Noor [19] in early 1978. The large repetitive beamlike and platelike trusses were established as continuum models by introducing basic assumptions. The truss structures with repeating element were pin-jointed and only the extensional strain energy of the members was calculated in Ref. [19]. Besides, the homogenization technique for periodic lattice structures was used to construct the continuum models which involved rigorous mathematical derivation [20, 21].

With progress in aerospace technology, a variety of new-type structures keep emerging to meet the needs of space missions. Therefore, different types of structures were studied to establish their equivalent continuum models, for instance, beamlike trusses [22], platelike trusses [23] and hoop trusses [24]. When dealing with the connections of the trusses in the process of equivalent modeling, there are mainly three assumptions to model the joints based on the practical engineering structures, namely pin joint, rigid joint and flexible joint. At present, plenty of researches focused on establishing the equivalent continuum models of the pin-jointed trusses [25, 26]. In contrast, the equivalent modeling studies issued on the rigid-jointed [27] and flexible-jointed trusses [28] were limited. Another aspect of the study is that the static and dynamic analysis types including static deflections [29], natural characteristics [30], dynamic responses [20], buckling analyses [21, 31], etc., were commonly investigated to check the validity of the equivalent modeling approaches. Particularly, the advantages of the equivalent continuum modeling technology in dynamics study were outstanding to the controller design, analytical solution and computational high-efficiency in nonlinear dynamic analysis [16]. In fact, most of researchers' efforts were put into the study of the equivalent linear equivalent models for the truss structures. The equivalent continuum modeling of the trusses with nonlinearity and implementing the equivalent transformation of the laws between the equivalent model and the original truss systems are important topics which would be worthy to be pursued to further study.

For the current study on equivalent continuum modeling of truss structures, we summarize and analyze the research status in categories from the aspects of the structure types, connected joint styles and modeling methods. With the gradual implementation of the major projects in aerospace engineering, the truss structures are heading towards the direction of large-scale, flexibility and diversity, which makes the nonlinear dynamic analysis and active vibration control of the truss structures become particularly important. But the FEM requires a significant amount of computing capacity when the nonlinear dynamic analysis of large truss structures needs to be carried out and it is extremely difficult to comprehend and grasp the essential nonlinear dynamic characteristics. And meanwhile, the number of degree-of-freedom of the FE model is too large to design vibration controller conveniently for truss systems. Two main purposes based on the equivalent continuum modeling strategy of truss structures are expected to be achieved:

- The geometric and joint nonlinearities can be introduced into the equivalent linear model and then classical nonlinear analytical methods can be used to study the nonlinear dynamic characteristics of the truss system.
- The vibration controller of truss system can be designed effectively using the modal approach on the basis of the equivalent continuum model.

Based on the research status and engineering requirement, we propose several aspect challenges and perspectives which deserve to be further studied for meeting the above two goals in this paper. The prospects of future research include equivalent nonlinear modeling and its solving approach, active vibration control on the truss structures and complex multi-body truss structures.

This survey aims to provide an overview and analysis on the current research status and future development of equivalent continuum modeling of truss structures. The outline of this survey is structured as follows. "Equivalent Continuum Modeling of Truss Systems (Sect. 2)" describes various equivalent continuum modeling methods of linear trusses, which are related to different structural types, connection joints, analytical methods, and their initial stress states, etc. "Equivalent Continuum Modeling of Truss Systems" introduces the equivalent continuum modeling studies about the nonlinear truss structures including geometrical, material and joint nonlinearities. "Nonlinear Dynamics and Vibration Control on Truss Structures Sect. 10" presents the current study status of vibration control of trusses and the relevant investigations on equivalent continuum modeling. "The Prospects of Future Research" indicates future research prospects and challenges in key scientific issues and new mission scenarios concerning the equivalent continuum modeling study. "Conclusions" concludes this survey.

Equivalent Continuum Modeling of Truss Systems

With the characteristics of light weight, large stiffness ratio and high packaging efficiency, large deployable trusses are widely used in different engineering applications [32–36], such as those in aerospace engineering, civil engineering, etc. (see Fig. 1a). Generally, the structural types can be classified into beamlike, platelike and hoop trusses (see Fig. 1b). According to the joints in practical trusses, the connections between the members are commonly classified as pin joint, rigid joint and flexible joint. Additionally, static or dynamic analyses need to be carried out based on the special operational environment. Owing to those characteristics, many researchers have paid attention to the equivalent continuum modeling study and extensive relevant researches have been reported. The primary categories of equivalent continuum modeling are classified into three groups in this paper, namely, the energy equivalence method, homogenization method, and other methods, such as the displacement equivalence approach. Here, the categorical equivalent continuum modeling studies are presented and discussed.

Energy Equivalence Method

Pin Joint Connections

In general, only the extensional strain energy of the single rod member was calculated when the truss structures were assumed to be pin-jointed, because there is no moment transmission. In 1988, Noor [37] stated the deformation characteristics of the pin-jointed truss versus the rigid-jointed truss, and pointed out that it was appropriate to employ the ordinary and micropolar continuums to imitate the pin-jointed and rigid-jointed trusses, respectively. Currently, most of existing researches focus on the equivalent modeling studies of pin-jointed trusses which can be classified as the beamlike, platelike and hoop trusses. Dow et al. [38, 39] developed a more general approach in comparison to the kinematic assumptions proposed by Noor. A third-order polynomial was used to represent approximatively the displacements at each node in three directions. The 60 coefficients can be identified in terms of the strain–displacement relations and the equivalent stiffness of the equivalent continuum models

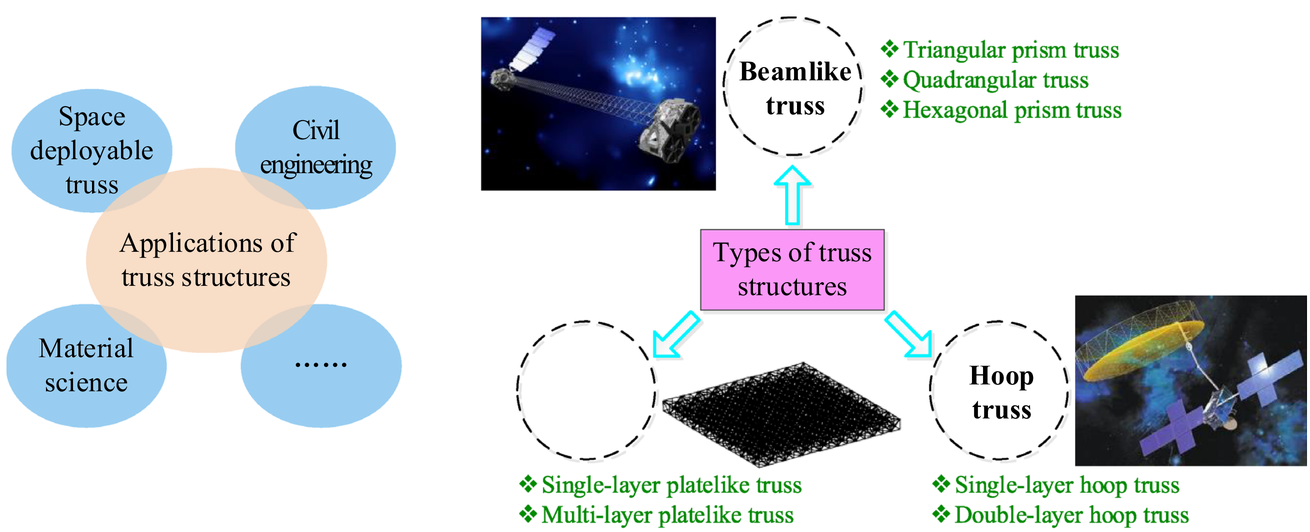

Fig. 1 Schematic view of **a** exemplary engineering applications of truss structures and **b** classification of truss structures

were achieved by four transformations. In 1990, the stiffness and mass matrices of the equivalent models of the planar truss structures were derived by Lee [30] using existing finite element matrices. However, the natural frequencies predicted by the equivalent model were slightly higher as compared with the full-scale finite element model of the truss structure, because fewer continuum degrees of freedom implied additional constraints. Subsequently, Lee [40] established an equivalent continuum extended Timoshenko beam by assembling the spectral elements for each structural members within the lattice cell. Then, the equivalent continuum structural properties and vibration characteristics of the lattice were investigated to show the validity. Besides, another equivalent modeling method was proposed by Burgardt and Cartraud [29] based on the averaging method, in which the stress and strain parameters of the equivalent beam model were determined by their average values over the continuum cell. And, the piecewise linear functions were applied to represent the displacements of the pin-jointed planar truss structure for static analysis.

(a) Beamlike Trusses

The concept of equivalent continuum modeling based on the energy equivalence principle was put forward in the 1970s and has been developing continuously. An equivalent continuum modeling approach proposed by Noor et al. [19] was applied to the periodic beamlike truss having triangular cross-section shown in Fig. 2.

First, separating out a periodic element, and the kinematic assumptions were proposed based on the linear variation in the plane of the cross-section, as

$$\begin{cases} u_1(x_1,x_2,x_3) = u_1^0 - x_2\phi_3 + x_3\phi_2 \\ u_2(x_1,x_2,x_3) = u_2^0 + x_2\varepsilon_2^0 + x_3\left[-\phi_1 + \frac{1}{2}\left(2\varepsilon_{23}^0\right)\right] \\ w(x_1,x_2,x_3) = w^0 + x_2\left[\phi_1 + \frac{1}{2}\left(2\varepsilon_{23}^0\right)\right] + x_3\varepsilon_3^0, \end{cases} \quad (1)$$

where u_1^0, u_2^0, w^0 are the displacement components; ϕ_1, ϕ_2, ϕ_3 are the rotation components; ε_2^0 and ε_3^0 are the extensional strains in the x_2 and x_3 directions; and $2\varepsilon_{23}^0$ is the shearing strain in the plane of the cross-section. Then, stiffness and mass coefficients of the equivalent continuum beam model were obtained according to the energy equivalence principle. Finally, the static and free vibration problems were analyzed to evaluate the accuracy of the equivalent beam model.

Salehian et al. [26, 41] followed Noor et al.'s work, but they applied their method to the Innovative Space-Based Radar Antenna Technology (for its detailed characterizations, the reader can refer to Refs. [26, 41]) mounting of the panel and went one step further to derive the governing partial differential equations for different degrees of freedom of vibrations explicitly, which were similar to a Timoshenko equations. The coupled PDEs of the equivalent beam model were solved to obtain structural natural frequencies so that the results can be compared with those of the FEM to check the effectiveness of the equivalent continuum model.

(b) Platelike Trusses

The platelike truss has many attractive features for application to large-area space structures. In Ref. [19], the large-area double-layered truss shown in Fig. 3 was also studied to establish its equivalent continuum plate model combining the classical plate theory and the Taylor series expansion.

The displacement field was assumed to have a linear variation in the thickness coordinate as

$$\begin{cases} u_\alpha(x_\beta,x_3) = u_\alpha^0 + x_3\phi_\alpha \\ w(x_\beta,x_3) = w^0 + x_3\varepsilon_3^0, \end{cases} \quad (2)$$

where $\alpha,\ \beta\ =\ 1,\ 2$; u_α^0 and w^0 are the displacement components; ϕ_α is the rotation component; and ε_3^0 is the transverse normal strain. Similarly, the static and free vibration analyses were presented to check the validity of the equivalent plate model proposed in Ref. [19].

Moreover, the platelike truss structures were modelled with the Mindlin plate elements which incorporate bending as well as shear deformation [42]. The direct energy approach [42] versus the method of Bur-

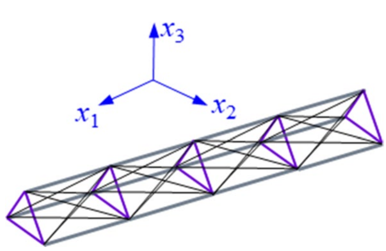

Fig. 2 Schematic view of the beamlike truss [19]

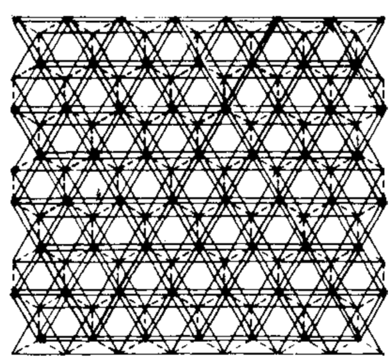

Fig. 3 Schematic view of the double-layered truss [19]

gardt–Cartraud [29] were performed through four models to reveal the effects of the two approaches on the equivalent stiffness for different configuration trusses.

(c) Hoop trusses

The hoop deployable truss possessing high folding ratio and small mass is an ideal structural form for large-aperture antenna, which enables it to keep a stable mass and avoid radical mass fluctuation when its diameter is increasing [43]. In a similar spirit to [19], Guo et al. [25] developed an equivalent hoop beam model for the double-layer hoop truss (see Fig. 4) based on the energy equivalence principle and solved the natural frequencies by finite element method according to the equivalent stiffness and mass. Simultaneously, modal test was implemented to demonstrate that the precision of the equivalent model of the double-layer hoop truss was acceptable.

Rigid Joint Connections

In special situation, it is appropriate to deal with the connection as rigid joints on the basis of the practical structural connection, working mechanism and material characteristics. Thus, the equivalent modeling study for the rigid-jointed truss has attracted many researchers' attentions. In the study of Noor and Nemeth [27, 44], the rigid-jointed planar truss and rigid-jointed beamlike truss with triangular cross-section were equivalent to micropolar beam models, which have independent microrotation and displacement fields. Differing from the ordinary beam theory used in Ref. [19], the micropolar beam theory was employed to simulate accurately the deformation of the truss structures. Besides, Wu and Chen [45] presented a structural simplification approach based on the static condensation method to reduce the degrees of freedom of the rigid-jointed beamlike lattice girders and performed the natural frequencies, dynamic responses and CPU time for comparing with the conventional FEM.

(a) Beamlike Trusses

Recently, with the development of materials science, shape memory polymer was applied to improve performance and increase versatility of the space deployable truss structures [46]. A new type of large-space beamlike truss with initial stress was studied by Liu et al. [22] to gain its equivalent continuum model utilizing the Timoshenko beam theory and the static condensation method. The beamlike truss and its spatial repeating element in the study [22] are illustrated in Fig. 5. Obviously, compared with Ref. [44] which was based on the micropolar beam theory to establish the equivalent continuum model, the equivalent modeling approach proposed [22] has a simpler derivation procedure and the dimensions of the equivalent beam model are reduced from ten to six. In addition, the configuration of the beamlike truss is more complicated and the

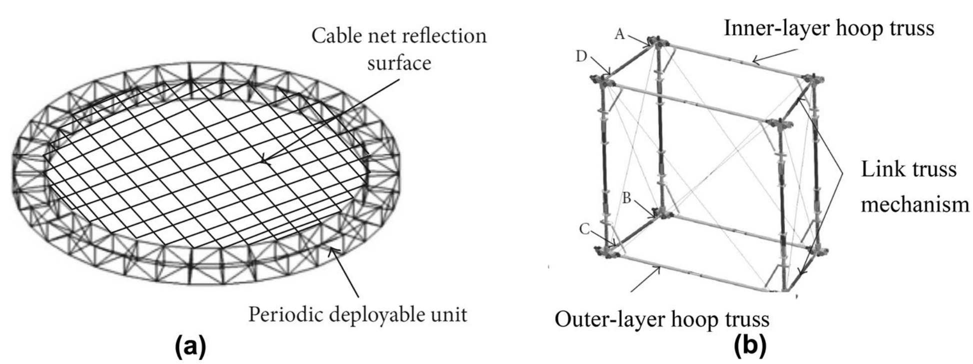

Fig. 4 Schematic view of **a** double-layer hoop truss and **b** periodic element [25]

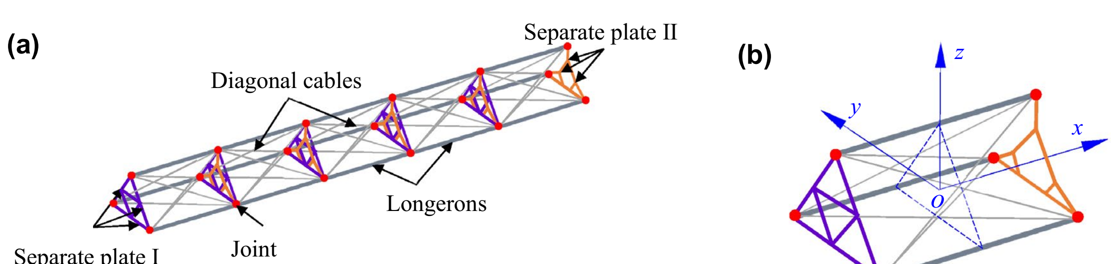

Fig. 5 Schematic view of **a** beamlike space antenna truss and **b** spatial repeating element [22]

initial stress is considered using the static condensation method.

According to the kinematic assumptions proposed by Liu et al. and the geometric equations of the Timoshenko beam, the displacement field of the cross-section of the spatial repeating element deduced from the Taylor series expansion was described as follows:

$$\begin{cases} u_x(x,y,z) \approx u_{x0} - y\theta_{z0} + z\theta_{y0} + x\varepsilon_{x0} - xy\kappa_{y0} + xz\kappa_{z0} \\ u_y(x,y,z) \approx u_{y0} + y\varepsilon_{y0} + z\left(-\theta_{x0} + \frac{1}{2}\gamma_{yz0}\right) + x\left(\theta_{z0} + \gamma_{xy0}\right) - xz\kappa_{x0} + \frac{1}{2}x^2\kappa_{y0} \\ u_z(x,y,z) \approx u_{z0} + y\left(\theta_{x0} + \frac{1}{2}\gamma_{yz0}\right) + z\varepsilon_{z0} + x\left(\gamma_{xz0} - \theta_{y0}\right) + xy\kappa_{x0} - \frac{1}{2}x^2\kappa_{z0} \\ \theta_x = \frac{1}{2}\left(\frac{\partial u_z}{\partial y} - \frac{\partial u_y}{\partial z}\right) \quad \theta_y = \frac{1}{2}\left(\frac{\partial u_x}{\partial z} - \frac{\partial u_z}{\partial x}\right) \quad \theta_z = \frac{1}{2}\left(\frac{\partial u_y}{\partial x} - \frac{\partial u_x}{\partial y}\right), \end{cases} \tag{3}$$

where u_{x0}, u_{y0}, u_{z0}, θ_{x0}, θ_{y0} and θ_{z0} are the displacement and rotation components at the center of the spatial repeating element; ε_{x0}, ε_{y0}, ε_{z0}, γ_{xy0}, γ_{xz0}, γ_{yz0}, κ_{x0}, κ_{y0} and κ_{z0} are the strain and curvature measures evaluated at the center position. To apply the energy equivalence between the beamlike truss and the equivalent beam model, the total strain and kinetic energy were computed as

$$U_e = \sum_{\text{members}} \frac{1}{2}\mathbf{w}^{(k)T}\mathbf{T}^{(k)T}\mathbf{K}^{(k)}\mathbf{T}^{(k)}\mathbf{w}^{(k)}, \tag{4}$$

$$U_g = \sum_{\text{members}} \frac{1}{2}\mathbf{w}^{(k)T}\mathbf{T}^{(k)T}\mathbf{K}_g^{(k)}\mathbf{T}^{(k)}\mathbf{w}^{(k)}, \tag{5}$$

$$T_T = \sum_{\text{members}} \frac{1}{2}\dot{\mathbf{w}}^{(k)T}\mathbf{T}^{(k)T}\mathbf{M}^{(k)}\mathbf{T}^{(k)}\dot{\mathbf{w}}^{(k)} + \sum_{i=1}^{6}\frac{1}{2}\dot{\mathbf{w}}_i^T\mathbf{m}_i\dot{\mathbf{w}}_i, \tag{6}$$

where U_e, U_g and T_T are strain energy, potential energy caused by initial stress, and kinetic energy of the spatial repeating element, respectively; $\mathbf{w}^{(k)}$ is the displacement vector of nodes in the kth member; $\mathbf{K}^{(k)}$, $\mathbf{K}_g^{(k)}$ and $\mathbf{M}^{(k)}$ are the stiffness, geometric stiffness and mass matrices of the kth member, respectively; $\mathbf{m}_i$ is the mass matrix of joint i; $\mathbf{T}^{(k)}$ is the coordinate transformation matrix of the kth member; superscript "·" stands for the derivative with respect to time. The authors went one step further to obtain the elasticity and inertia matrices of the equivalent beam model and made exhaustive comparison in terms of the natural characteristics between the equivalent beam model and the full-scale finite element model of the beamlike truss to demonstrate the effectiveness of the proposed equivalent model.

Additionally, the equivalent continuum modeling technique was also applied in the telecommunication industry [47]. The lattice mast, as shown in Fig. 6, was equivalent to a continuum model based on the given assumptions, which have a linear variation of the displacement field similar to Ref. [19]. The governing differential equations of the six degrees of freedom including two transverse and axial displacements, torsional and two bending angles were exploited according to the Hamilton's principle, subsequently, numerical results dealing with deflections, buckling loads and natural frequencies were solved to compare with those of the original structure in the finite element method.

(b) Platelike trusses

The continuum theories applied to the analysis of platelike trusses rigidly connected at the joints have been reported in the literatures (see for example [48–50]). For the large rigid-jointed single layered and double layered grids, Hefzy and Nayfeh [48] utilized the energy approach to construct equivalent continuum model with couple stresses. The stiffness coefficients and the bending rigidities of the equivalent continuum plate were given in terms of the geometrical and material properties of the lattice structures. A two-dimensional continuum model with coupled stress was established for a plane grid work [49], in which the equations

Fig. 6 Schematic view of the lattice mass [47]

were employed to investigate transverse vibrations and frequencies were calculated for comparing with the finite element method. Employing the micropolar plate continuum theory, Lamberson [50] derived a continuum plate finite element with micropolar rotations and transverse shear deformations. Currently, there are a few existing articles focused on rigid connection of platelike trusses to establish their equivalent continuum models employing the energy equivalence method.

(c) Hoop trusses

Liu et al. [24] proposed an equivalent hoop beam model for the hoop truss structure shown as Fig. 7. It can be seen that the hoop truss structure is composed of planar repeating elements different from the spatial repeating element involved in the Ref. [22]. Hence, there exists difference on the kinematic assumptions between Refs. [22] and [24]. Using the classical continuum theory instead of resorting to the micropolar elasticity theory, the planar repeating element was modeled as an anisotropic beam model, which was taken as a basis element to compose the hoop beam model. The contribution of the bending curvatures on the transverse displacements was considered in the process of the Taylor series expansion.

A typical handling approach was carried out to reduce the number of strain parameters in the beam theory, which assumes the fibers in the cross section do not interact each other. Thus, the forces associated with the strain components are set equal to zero, that is, [24]

$$\frac{\partial U_E}{\partial \varepsilon_{z0}} = 0, \tag{7}$$

where U_E and ε_{z0} are the strain energy of the repeating element and the strain in z direction, respectively. The similar assumptions can be also found in Refs. [19, 22, 25].

Flexible Joint Connections

In fact, large space truss structures consist of plenty of structural members and connection joints which could produce significant effects on the structural dynamic responses [51]. Therefore, the flexibility of joints needs to be taken into account to precisely predict the dynamic characteristics of the structures under specified situations. Webster and Velde [52] discussed a method of modeling nonlinear joints in the two-dimensional beamlike truss and conducted an experiment to verify this approach. The modeling methods and major impacts of the flexible joints on the dynamic behaviors have been reported in tremendous studies such as Refs. [53] and [54].

Salehian and Inman [28, 55] intended to develop an equivalent continuum model for a large antenna space truss with flexible joints. A micropolar beam rather than an ordinary beam was found, but the microrotations were expressed in terms of the ordinary strain terms while retaining the effects of these micropolar based on some assumptions. For the sake of intuition, Fig. 8 shows a comparison between the ordinary continuum (used in Ref. [56]) and the micropolar continuum (used in Ref. [55]).

Moreover, applying extensively the energy equivalence method dealing with rigid jointed truss in Ref. [24] to the flexible-jointed truss, Liu and coworkers [57] modeled an equivalent beam segment for the planar repeating element with flexible joints using the model condensation technique. Then, the equations of motion for the equivalent anisotropic

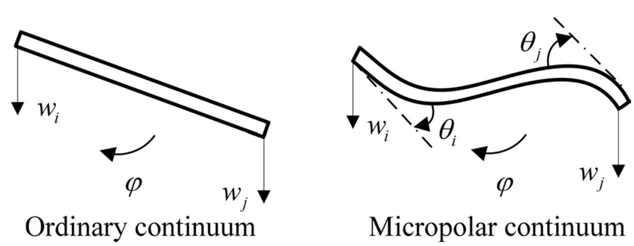

Fig. 8 A comparison between ordinary and micropolar continuums (θ_i and θ_j microrotations) and (φ macrorotation) [55]

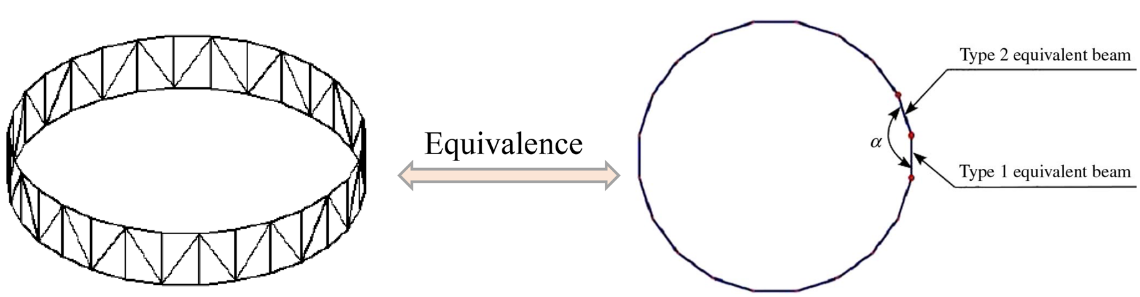

Fig. 7 The equivalent hoop beam model for the truss structure [24]

beam model were solved by the Green's function method to obtain natural frequencies. The equivalent modeling approach [57] was simpler than the method based on the micropolar theory. Subsequently, Liu et al. [58] further established an equivalent micropolar beam model for the flexible-jointed truss which was similar to the structure studied in Ref. [57] and is shown in Fig. 9. A comparison of the frequency responses of the equivalent micropolar beam model, the equivalent classical beam model and full finite element model was carried out and the authors pointed out that the equivalent micropolar beam model owns higher precision. A 1-D micropolar Timoshenko beam finite element model of a web-core lattice was developed by Karttunen et al. [59] where each joint has a rotational stiffness. Moreover, Gesualdo et al. [60] investigated the static deformations in-plane bending of the equivalent continuum model of a Vierendeel girder with linear elastic torsional joints based on the transfer matrix method and the micropolar Timoshenko beam theory. It is noticed that these investigations and applications on the equivalent continuum modeling studies mainly focused on the beamlike trusses. Here, it should be mentioned that the equivalent continuum modeling techniques cannot capture generally the local deformations of the truss structures and only mimic the global dynamic behaviors, such as the Refs. [19, 22, 24–26]. Just right, this is an advantage, because the local modes are not of major importance and concern for vibration control of the large truss structures [15].

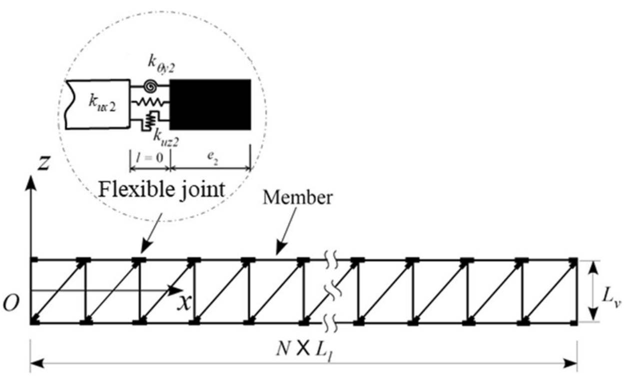

Fig. 9 Planar repetitive truss structure with flexible joints [58]

Homogenization Method

Except for the category of equivalent continuum modeling methods based on the energy equivalence principle, the homogenization method is also widely used for dealing with the establishment of the equivalent continuum models of periodic truss structures. This method has been studied extensively in the materials discipline and mainly applied to microscale research. In 1998, for a two-dimensional quasi-repetitive lattice structures with pin joints, Tollenaere and Caillerie [61] derived the equivalent continuum constitutive relation according to the homogenization technique; only the case of static deformations was presented. Penta and coworkers [62] presented a homogenizing procedure for a large repetitive beamlike structures and deduced the polar character of the equivalent Timoshenko couple-stress beam model. Also, a Timoshenko couple-stress beam was employed as substitute continuum of the beamlike truss via a homogenization method in Ref. [66]. Additionally, Piccardo et al. [12] constructed a three-dimensional Timoshenko beam model involving three displacements and three rotations to study the static and dynamic behaviors of the tower buildings; they applied the homogenization method in the building structures. Dos Reis and coworkers [23, 64] used an asymptotic homogenization method to construct the micropolar equivalent models for the lattice structures. Recently, Glaesener et al. [65] introduced a powerful homogenized continuum description of truss structures to significantly reduce computational costs while accurately capturing the dominant deformation mechanisms and discussed the first- and second-gradient homogenization results according to several numerical example analysis. For an intuitive awareness of

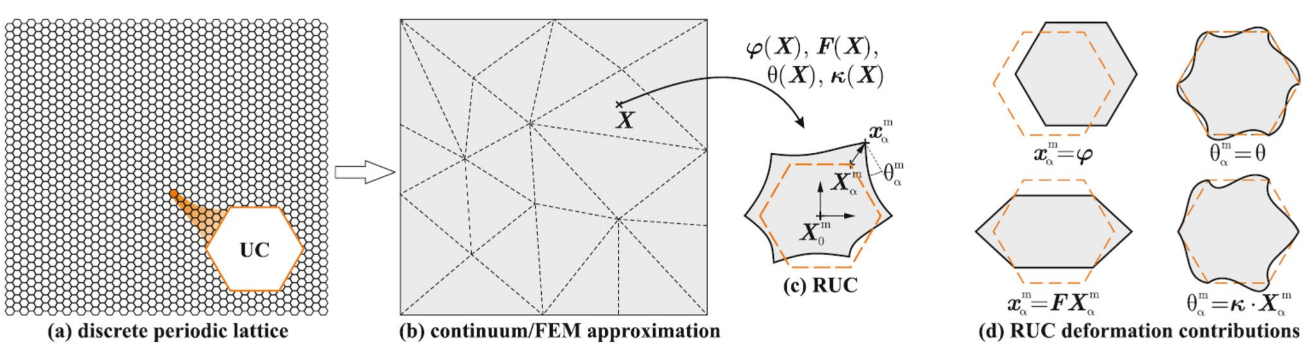

Fig. 10 Schematic view of the homogenization procedure: **a** discrete truss, **b** continuum model, **c** representative unit cell and **d** superposition of the deformation contributions [65]

the homogenized modeling process, the authors replace the discrete assembly of periodic truss unit cell by an effective continuum, as described in Fig. 10. Besides, the numerical implementation asymptotic homogenization method was adopted in the stiffened shells which was often used in the aerospace scope to reduce the computational efforts of buckling analysis by groups of Hao [66, 67] and Wang [68]. Other relevant investigations on the asymptotic homogenization methods have been reported in considerable literatures, such as the work by Kalamkarov et al. [69], Zhuang et al. [70], Luongo et al. [71], and Desmoulins et al. [72], and so forth. The homogenization process requires rigorous mathematical derivation techniques and becomes involved when dealing with the complicated structures. In addition, the homogenization method is generally used to deal with the equivalent continuum modeling on microscale.

Other Methods

Another equivalent continuum modeling method is based on the displacement equivalence criterion which is used for determining the mechanical parameters. Studies using this method can be found in Refs. [73–75]. A direct approach was applied to find the equivalent stiffness of the lattice structures [73], where the periodic character of the structure was not used so that the computational cost would generally be expensive and parameter analysis also became difficult for complicated, large truss systems. The displacement equations for multiple types of lattice trusses were derived and the equivalent stiffness were obtained from the resulting displacements [74]. Moreover, Sun and Liebbe [76, 77] employed the displacement equivalent method to gain the stiffness matrix of the equivalent continuum model and investigated the natural characteristic and transient response of the planar truss structures. Stephen and Zhang [78, 79] used the state variable transfer matrix technique to calculate the equivalent continuum properties of the pin-jointed beamlike trusses. The Poisson's ratio, cross-sectional area, torsion constant and the tension–torsion coupling coefficient of the equivalent model can be determined [79]. Necib and Sun [80] established a high order Timoshenko beam for planar beamlike truss, introducing the force–displacement relations of the repeat element; subsequently, the equivalent stiffness and mass were presented and used to study the vibration characteristics. Employing this equivalent modeling approach needs to confirm the situations of the nodal force, its distribution and boundary conditions. A generic set of symbolic equations for the effective continuum stiffness of several pin-jointed beamlike trusses was derived by Murphey [81] under the condition that the diagonals must be soft relative to the longerons and battens. And, the author investigated the influences of the structural parameters on equivalent accuracy of the equations. The representative models were studied in Ref. [81] as shown in Fig. 11.

Several Concerned Issues

In this subsection, several aspects of the studies related to the equivalent continuum modeling are presented and summarized.

(a) Tensegrity Trusses

The tensegrity trusses commonly composed of the tensile members such as cable or membrane were developed with the enhancement of the space deployable truss structures due to the promising characteristics that can improve packaging efficiency, ease deployment, serve structures' stability, and decrease the number of joints with complicated mechanisms [82–85]. In recent years, with the development of material science, the shape memory polymer was used in space deployable trusses which were controlled by the temperature. In some cases, these trusses were under the initial stress state to increase the structural performance and versatility [46]. In the study of Kebiche and coworkers [86], the continuum equivalent properties of truss structures in selfstress state were deduced after four matrix transformations based on the approach proposed by Dow et al. [38], and the influence of the initial stress on the equivalent stiffness was investigated. Afterwards, Yildiz and Lesieutre [87] extended the works in Refs. [38, 86] to the equivalent modeling study of n-strut cylindrical tensegrity towers shown in Fig. 12 and only compared the equivalent stiffness with those of the finite element models. Furthermore, Liu et al. [22] studied the equivalent modeling method of a new space antenna truss with initial stress which was

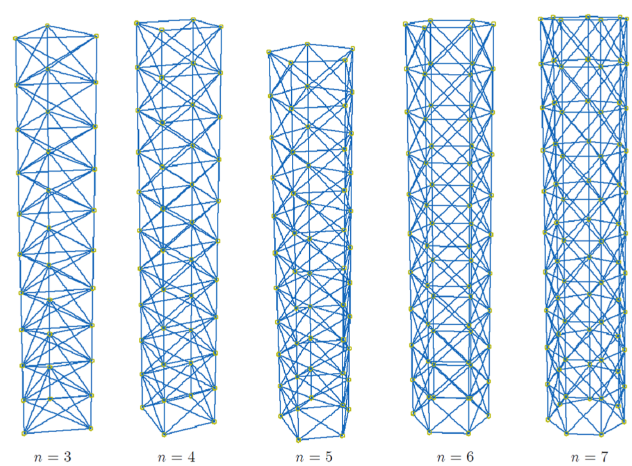

Fig. 11 Truss models for 3–7 longeron trusses [81]

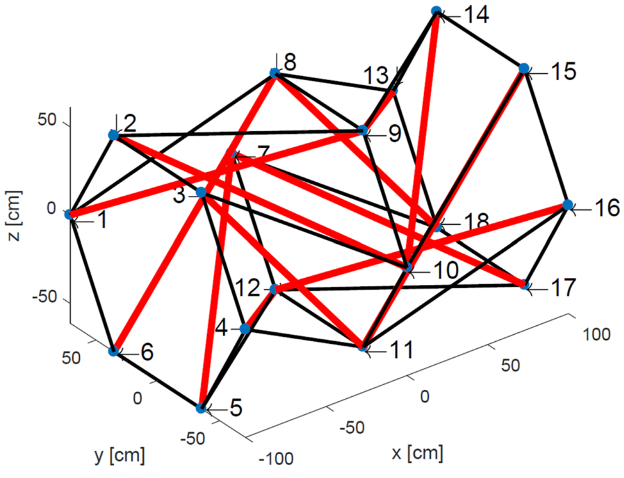

Fig. 12 Six strut tensegrity tower [87]

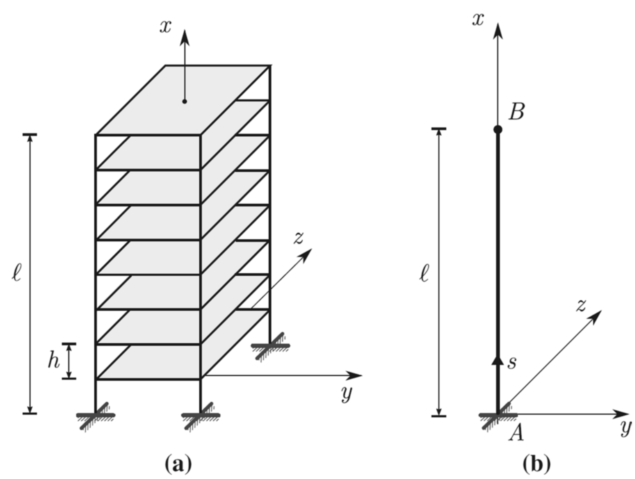

Fig. 13 Schematic view of **a** multi-store building and **b** equivalent Timoshenko beam [6]

handled with the static condensation and the energy equivalence principle, and also investigated the natural characteristics of systems. Also, the authors discussed comprehensively the influence of the initial stress on the frequencies and equivalent modeling accuracy. For a multi-store building accounting for prestress forces described in Fig. 13, Ferretti [6] developed a Timoshenko beam model via a homogenization procedure for buckling analysis of the multi-store building. Moreover, the tensegrity truss structures with selfstress were studied to establish their equivalent continuum models by group of Al Sabouni-Zawadzka and the detailed procedure can be found in literatures such as Refs. [88–90]. On the other hand, Zhang et al. [91] used a modal equivalent method to make the natural frequencies and vibration modes of the original full-area membrane with pre-tension and the alternative grid membrane identical, thereby reducing the influence of the air on the structures for the dynamic analysis in the ground experiment.

(b) Solution Methods

In general, the equivalent continuum models for the truss structures are anisotropic and exhibit more complex dynamic behaviors, unlike the regular beam or plate models. For example, the governing partial differential equations of the equivalent beam model for a space antenna truss including six degrees of freedom display bending-torsional and bending-extensional couplings due to the asymmetry of the truss [92]. For the special case reported in Ref. [92], the governing equations were divided into two sets of PDEs which were described in the following as

$$\begin{cases} \overline{EI}_z\theta_z'' - \overline{GA}_{xy}\theta_z + \overline{GA}_{xy}u_y' - \overline{J}_z\ddot{\theta}_z - m_{26}\ddot{u}_y - m_{46}\ddot{\theta}_x = 0 \\ \overline{GA}_{xy}u_y'' - \overline{GA}_{xy}\theta_z' - \overline{\rho A}\ddot{u}_y - m_{24}\ddot{\theta}_x - m_{26}\ddot{\theta}_z = 0 \\ \overline{GJ}\theta_x'' - \overline{J}_x\ddot{\theta}_x - m_{24}\ddot{u}_y - m_{46}\ddot{\theta}_z = 0, \end{cases} \tag{8a}$$

$$\begin{cases} \overline{EI}_y\theta_y'' + \eta_{13}u_x'' - \overline{GA}_{xz}\theta_y - \overline{GA}_{xz}u_z' - \overline{J}_y\ddot{\theta}_y - m_{15}\ddot{u}_x - m_{35}\ddot{u}_z = 0 \\ \overline{GA}_{xz}u_z'' + \overline{GA}_{xz}\theta_y' - \overline{\rho A}\ddot{u}_z - m_{35}\ddot{\theta}_y = 0 \\ \overline{EA}u_x'' + \eta_{13}\theta_y'' - \overline{\rho A}\ddot{u}_x - m_{15}\ddot{\theta}_y = 0, \end{cases} \tag{8b}$$

where $\overline{EA}$, $\overline{EI}_z$, $\overline{EI}_y$, $\overline{GJ}$, $\overline{GA}_{xz}$ and $\overline{GA}_{xy}$ are equivalent rigidities; $\overline{\rho A}$, m_{15}, m_{24}, m_{26}, m_{35} and m_{46} are equivalent mass; $\overline{J}_x$, $\overline{J}_y$, $\overline{J}_z$ denote equivalent rotational inertia per unit length; u_x, u_y, u_z, θ_x, θ_y and θ_z denote degrees of freedom of equivalent beam model. The denotations of the parameters also can refer to the Ref. [92]. It can be seen that the bending-torsion vibration and bending-extension vibration of the equivalent beam model are coupled from Eqs. (8a, 8b), respectively. To solve the two sets of PDEs in exact analytical method, Liu and coworkers extended successfully the approaches proposed by Banerjee and Su [93, 94] to the dynamic analysis of the equivalent beam model. Then, the natural characteristics for the cantilevered and free-free trusses were achieved and compared with those results calculated from the finite element method. In like manner, Stephen and Zhang [95] presented the tension–torsion coupled vibration behavior of the equivalent beam model for the repetitive truss structure and the coupled governed dynamic equations were solved by (co)sinusoidal form according to the corresponding boundary conditions. Moreover, the Green's function method was used for the solution of the coupled governing equations in Ref. [57] where an equivalent continuum beam model was constructed and its natural frequency was exhibited. For the studies of the coupled

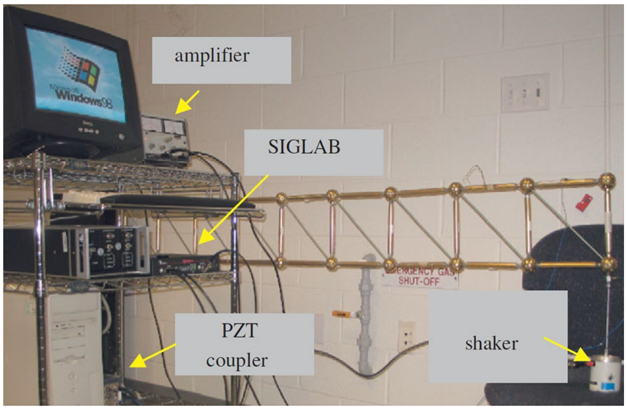

Fig. 14 Experimental setup of the planar truss [56]

Table 1 Theoretical and experimental natural frequencies [56]

Mode	Theoretical frequency (Hz)	Experimental frequency (Hz)	Error (%)
1	6.88	6.875	0.07
2	154	153	0.65
3	319	324	1.57
4	491	444	9.57

vibration beams, a variety of techniques and methodologies have been developed, for instance, the dynamic stiffness method [96, 97], the analytical method of the coupled Timoshenko beams considering the warping effects [98], the Green's function method [99], and the transfer matrix method [100], etc. In addition to aforementioned analytic solution method for the equivalent beam models, the finite element method is also one alternative means to solve the coupled governing equations, such as the related references [22, 24]. To some extent, the analytical solution is convenient to the subsequent control law design of the truss structures.

(c) Experimental studies

To validate the proposed equivalent beam model, a planar truss was fabricated shown in Fig. 14 to find its frequency response functions and natural frequencies [56, 101]. The longerons and diagonals members were made of aluminum tubes and steel rods, respectively. The aluminum ball joints were used to connect the bar members and the truss structure was hung on wires from the ceiling at its ends. The errors of the first four natural frequencies between the equivalent model and the truss structure were estimated by the experiment. For an intuitive display of the state, the results for theory of the equivalent model and the experiment are listed in Table 1. The maximum error was 9.57% in the 4th mode, which demonstrated that the accuracy of the equivalent model can be accepted and the equivalent model can capture well the dynamic behaviors of the truss structure. And, the authors demonstrated that the effects of the strain component terms on the equivalent accuracy became strong with increase of mode order. Also, the group of Guo implemented dynamic vibration experiments for the beamlike truss with rectangular section-cross [102] and double-layer hoop truss [25] to compare with the theory results calculated from the equivalent continuum models. Note that the experiment study on one unit separated from the hoop truss was performed, see Fig. 15. The frequency results [25] obtained through equivalent model, finite element model, and experimental system are illustrated in Table 2, which indicated that the fractional errors of the first and second order frequencies of equivalent model to the FEM were 8.82% and 5.30% respectively.

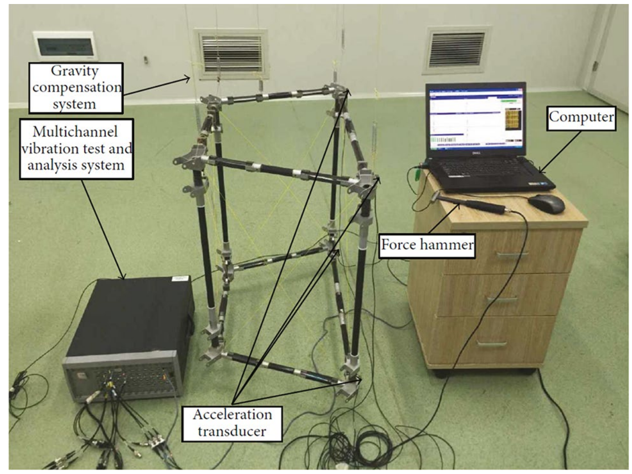

Fig. 15 Modal test system [25]

Table 2 Modal frequency of the unit of double-layer hoop truss [25]

Mode	Equivalent model (Hz)	Finite element model (Hz)	Experimental system (Hz)
1	43.358	47.454	40.113
2	54.396	51.658	42.845

Nonlinear Dynamics and Vibration Control on Truss Structures

In this section, the equivalent nonlinear modeling and vibration control studies of the truss structures are surveyed and relevant potential issues are analyzed.

Equivalent Continuum Modeling of Nonlinear Truss Structures

Truss structures are a special type of novel flexible structures and the nonlinearity is a critical factor which should be considered in the studies of the dynamic responses [103]. Variety of approaches can be found for the nonlinear analysis of truss structures, for example, the Newton's method [104, 105], the point iterative method [106], the nonlinear FEM [107–109] and the co-rotational method [110], etc. Currently, the FEM is one of the most popular strategy to deal with the nonlinear dynamic problems of the truss structures. However, in the studies of Li et al. [111] and Rezaiee-Pajand et al. [112], it has been stated that nonlinear analysis of the truss structures is an extremely computationally intensive task, especially for large-scale structural systems. More importantly, the FEM is not convenient to the controller design and cannot be utilized effectively for the analytical solution of the systems. To this end, investigations on equivalent continuum modeling considering the nonlinearity of the trusses are desired urgently. Comparing with the researches on the equivalent modeling in linear range, the nonlinear equivalent modeling is limited. Generally, for the space deployable truss structures, the geometric, material and joint nonlinearities are primary concerns for the researchers. Early, based on the energy equivalence principle, McCallen and Romstad [113, 114] introduced the geometric, as well as material nonlinearities into the equivalent modeling procedure of the planar trusses and developed a nonlinear solution algorithm to solve the static deformations and dynamic responses. In Ref. [115], a circular truss composed of repetitive beamlike units was equivalent to a continuum circular cylindrical short shell to study the dynamic responses of the truss with geometrical nonlinearity, but the results comparisons between the equivalent model and the circular truss were not carried out so that it cannot demonstrate adequately their consistency on the nonlinear behaviors. Also, Zhang et al. [116] analyzed the nonlinear responses of the equivalent beam-ring model of the circular truss. Subsequently, Liu et al. [117] studied the nonlinear dynamic behaviors of the equivalent cylindrical shell employing the equivalent modeling approach proposed by Zhang et al. [118]. Similar to [115, 116], comparisons between the equivalent model and the original truss were not discussed. More recently, Liu et al. [16] proposed an equivalent nonlinear dynamic model for beamlike truss through introducing Karman nonlinear strain–displacement relationship into the linear equivalent beam model. In addition, differing from the equivalent modeling studies [115, 116], the nonlinear dynamic responses and computational time were compared comprehensively with the results obtained from the nonlinear full-scale FE model of the beamlike truss to verify the effectiveness and high-efficiency of the proposed equivalent nonlinear model. An extended continuum model of the planar beamlike truss with geometrical nonlinearity was presented by the group of Qi [119] by introducing a co-rotational coordinate system. In addition, to reduce the order of the models, an equivalent continuum model of the 2D truss was built by combining the extended multiscale finite element method and the co-rotational approach, and the geometrical nonlinear analysis was investigated by Liu and Lv [120]. For the same purpose, the group of Tiso [121] developed a modal derivatives approach based on the enrichment of reduction basis constituted of vibration modes for the dynamic analysis of flexible multibody systems featuring the nonlinear elastic deflection. Tiso and coworkers also developed other several equivalent modeling approaches Refs. [122, 123]. Based on the energy equivalence principle, Piccardo et al. [124] and D'Annibale et al. [125] introduced the nonlinear equivalent beam models for the nonlinear analyses of shear-type beamlike buildings accounting for nonlinearities generated by the stretching of the columns. Pal et al. [126] developed a continuum model exploiting the non-convex strain energy function for hexagonal lattices composed of a set of masses connected by linear axial and angular springs, with nonlinearity arising solely from geometric effects. On the other hand, the equivalent continuum modeling methodology was used in the geometrically nonlinear periodic sandwich structures, e.g. sandwich beams [127–129]. The study on equivalent continuum modeling of truss structures with nonlinear joint is really lacking comparing with those of linear truss systems. Based on an improved finite element, Zhang et al. [53] presented the equivalent modeling of a planar truss with axial cubic stiffness joint using the dynamic stiffness and mass matrices and evaluated the dynamic influences of nonlinear joint on the truss structure. Recently, Liu et al. [130] exhibited an equivalent continuum modeling method for the spatial lattice structure with bilinear hysteretic nonlinear joints employing the describing function method. The validity of the equivalent model was checked and the effect of joint parameters on the nonlinear dynamic response of the lattice structure was investigated. The joint nonlinearity also was considered in Refs. [52, 131] to construct the order reduction models, however, their works focused on the equivalence of the joints. Other applications of the equivalent modeling for the nonlinear analyses can be found in Refs. [132–135]. Continuous efforts have been devoted to developing equivalent continuum modeling methods for the static and dynamic analysis of truss structures, as summarized in Fig. 16. Detailed surveys for each category have been discussed in "Equivalent Continuum Modeling of Truss Systems" and "Nonlinear Dynamics and Vibration Control on Truss Structures".

It is important to consider the nonlinear characteristics in the procedure of the equivalent continuum modeling, since this can reduce significantly computational time, as well as can analyze the nonlinear dynamic behaviors in a few degrees of freedom so that the analytical solution methods (i.e., the multiscale method, the average method, and the harmonic balance method, etc.) can be utilized adequately in the nonlinear investigations of the large truss structures. The large flexible spacecraft are mainly composed of the link, beam, plate and shell members, thereby constituting the complicated structure systems, which have complex nonlinear phenomena, for example, the bifurcation, chaos, inter-resonance, and so forth. From the numerical analysis for the discrete truss structures to the approximate analytical or semi-analytical methods, the simplification of the models is of great necessity. Hence, the equivalent continuum modeling methodology is a feasible way and deserves further study.

Active Vibration Control of the Truss Structures

During the operation, the large space structures are subjected to external incentive such as unbalance inertial force comes from the attitude adjustment, periodic thermal load when the spacecraft pass in or out the earth's shadow, space debris, and so on in space environment, which produces the complicated dynamic responses [136–138]. The space truss structures have the characteristics of large size, great flexibility, light weight, weak damping and high precision demand, which results in low frequencies and weak ability

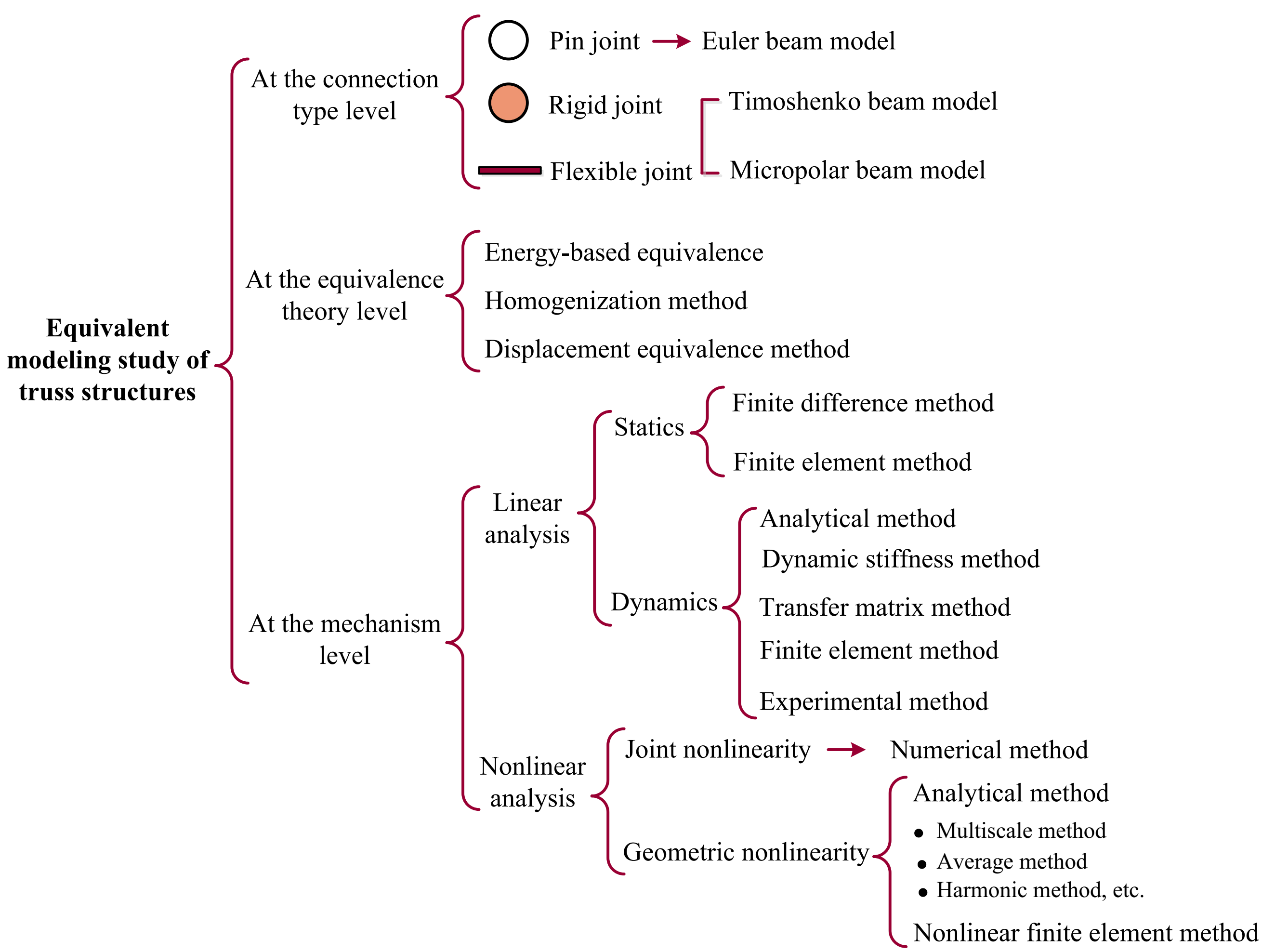

Fig. 16 Category of the static and dynamic equivalent modeling approaches for truss structures

of resisting deformation of the systems. Thus, to improve the service quality, extend structural lifetime and consume low energy, the study of vibration suppression quickly becomes an important and hot research topic [139]. Contrary to the large order FE models for truss structures, the existing methods for the control law design for distributed parameter models can be applied effectively on the equivalent continuum models. Another particular advantage is that the transfer function can be established between the sensing and actuation points of the structure employing a closed form solution [140]. As shown in Fig. 17, three vibration control strategies of large truss structures are listed and compared, as well as their advantages and disadvantages.

Numerous works have been published on active vibration suppression of the large space structures [141]. Based on the friction damping in semi-active joints, Gaul et al. [34] presented a vibration suppression approach for the large space quadrangular truss. To obtain a low-order model, a reduction method was proposed which was a combination of balanced reduction and matching moments method. For the space quadrangular truss, Park and Kim [142] also proposed a semi-active control method using a dry friction damper, in which the most development was the normal force can be explicitly obtained in terms of modal parameters and estimated amplitude of the modal displacement. And, the numerical simulations and experiments were presented to show the reduction of the transient vibration in the proposed control approach. Preumont et al. [143] examined the active damping used an integral force feedback and passive damping of a piezoelectric tri-prism truss and the results demonstrated the active control was very effective and able to achieve critical damping. Gosiewski and Koszewnik [144] proposed a fast prototyping method for the active vibration control system of a space quadrangular truss. The mathematical model was reduced and decoupled to change the TITO system into SISO system. Such an approach allowed simple control laws to be designed with the aid of a computer simulation procedure. The effectiveness of the proposed active vibration damping system was verified via simulation and experiment. In recent years, the intelligent control of distributed parameter system using neural networks, fuzzy logic, etc. has achieved some progress. For example, Li et al. [145] presented a decentralized adaptive fuzzy control method which was improved via sliding control method, to solve the vibration control problem of one T-typed truss structure. Besides, a robust vibration control scheme, namely, one degree-of-freedom fuzzy active force control was utilized to a nonlinear electromagnetic-actuated flexible plate with CFCF boundary conditions in Ref. [146]. Then, an experimental rig was developed and tested to investigate the performance of the proposed control scheme. Luo et al. [147] proposed a hybrid control algorithm and designed a fuzzy logic controller for a large hoop truss to improve structural vibration attenuation performance. Through the experimental verification, the decay time of the hoop truss was reduced when using the hybrid control algorithm in comparison with the PD control algorithm only. Although, the intelligent control is an alternative approach to suppress the vibration of the distributed parameter systems, there exist many issues in such designs (like selection of basis function, appropriate choice of fuzzification, etc.), which need further attention [148].

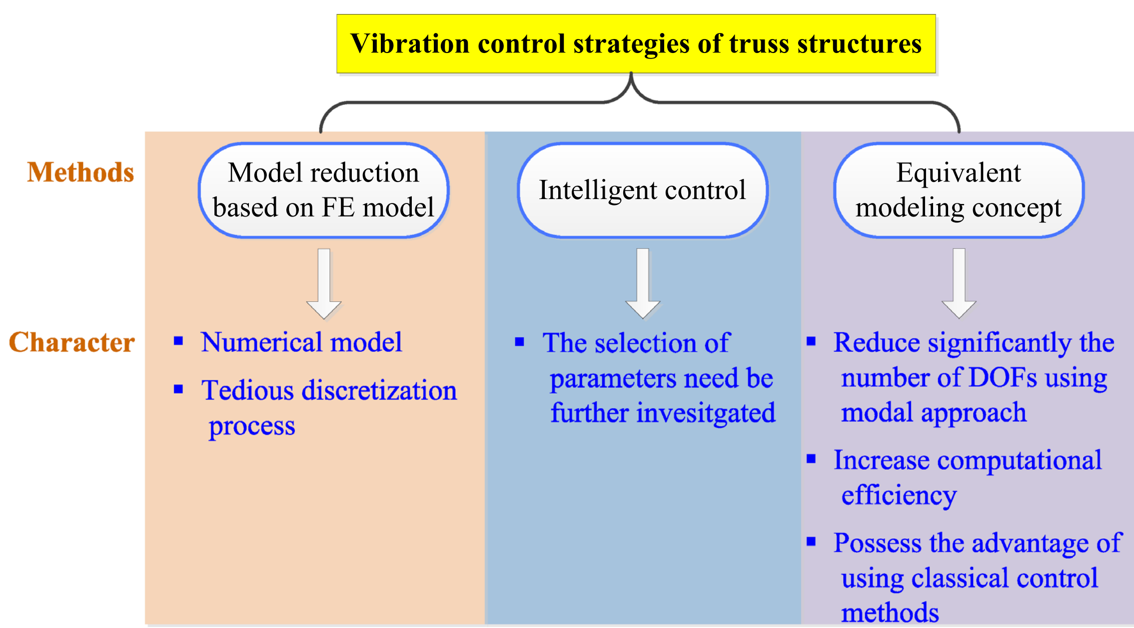

Fig. 17 Comparisons of three vibration control strategies for truss structures

In the survey by Nurre et al. [149], the authors presented important development of large space structure dynamics and control analysis and pointed out that the modal frequencies were typically closely spaced and the need for precise date dictated many degrees of freedom in the finite element representation. One of the challenges in the control law design of the truss structures is the requirement of model order reduction to deal with such high degrees of freedom systems. Based on the established equivalent dynamic model, the existing vibration control methods can be used effectively into the distribute parameter system of the large truss structure, because the system transfer function can be precisely evaluated in closed form between the sensing and actuation points [150]. This is, particularly advantageous for the purpose of the control of the large trusses and can be applied more effectively than the FEM. In view of the train of thought, Lamberson and Yang [151] designed feedback controller for the space platelike truss structure using its equivalent continuum plate model derived from the modal results and investigated the natural frequencies of the systems. In addition, based on the proposed equivalent beam model, Salehian et al. [140] employed the classical LQR method to find the optimal solution for feedback controller of the space truss. However, the presented control results obtained from the equivalent beam model were not compared with those using the original truss structure. Bennett and Kwatny [152] gave a systematic procedure for obtaining the required transfer functions and Green's functions for the hybrid model dynamics consisting of one-dimensional elastic elements such as beams, rods and cables together with rigid-body models to construct state space models for these flexible structures. Closed-form analytic expressions in terms of structural parameters can be obtained with the aid of which it is possible to reduce dramatically the computational complexity, thereby gaining valuable insight on the interaction between the truss structures and controls. The advantages of dynamic equivalent modeling were fully used to deal with the control of an Earth Pointing Satellite truss structure using distributed parameter method in Refs. [153, 154]. The vibration control of curved beam structure used in deployable mesh antenna was studied based on distributed parameter model [155], which can be extended to the study of vibration control for hoop truss. More recently, Liu and coworkers [156] applied the distributed cooperative control to the flexible structure according to an equivalent multiple autonomous substructure model and each autonomous substructure has computation and communication capability.

It is pointed out that the existing control strategies (e.g. linear quadratic regulator, proportion integration differentiation, distributed cooperative control, etc.) were used on the equivalent models of the space trusses. However, the challenge of this application lies in the validity check that how to demonstrate the consistency of the vibration suppression between the equivalent model and the truss structure, as well as the placement of the sensor, actuator, and controller on the truss structure.

The Prospects of Future Research

The equivalent continuum modeling methodology has emerged as an increasingly used technique in large space flexible trusses, civil engineering structures, and other engineering applications. Despite the advantageous properties of equivalent continuum modeling study, it is still meeting two main aspect challenges which can be addressed: nonlinearity issues and vibration control issues involved in the equivalent modeling study. They are discussed in this section.

Equivalent Nonlinear Modeling Strategy and Its Effectiveness

For the large space flexible truss structures composed of members (e.g. link, beam, cable, plate, etc.), these structures possess complicate nonlinear dynamic behaviors with low and close frequency characters. In particular situations, to accurately and efficiently determine the nonlinear dynamic responses, the crucial issue is to establish an effective equivalent dynamic model considering the geometrical and material nonlinearities of the original trusses. However, considering directly the nonlinearities in the equivalent continuum modeling process is extremely rare according to the literature review and it is a challenging research task. Therefore, for space flexible structures, such a problem should be investigated further.

In addition, the space trusses involving plenty of the nonlinear joints are complicate and high-dimensional nonlinear systems which lead to sharply increase of computational time and produce more colorful dynamic phenomena than actually needed. It is disadvantageous to understand clearly the nonlinear mechanisms of truss structures, thus, the nonlinearities of joints should be investigated by combining the aforementioned equivalent modeling methods. This issue would be intractable for complex configurations, and continuing to explore accurate equivalent modeling approaches for the space trusses with numerous joints is still an ongoing problem.

Coupled Three-Dimensional Nonlinear Model and Its Solving Approach

Due to the high degrees of freedom for truss structures, it is difficult to investigate the nonlinear dynamic characteristics in an analytical way. Fortunately, using the proposed equivalent modeling method, the nonlinear beamlike trusses can be equivalent to a single beam model (equivalent nonlinear beam model), so that the effective equivalent continuum nonlinear dynamic models can be obtained and investigated more easily in an analytical way in comparison with the traditional FE models. Consequently, the developed solution methodologies (including multiscale method, average method, harmonic balance method, etc.) for nonlinear beams, e.g. Timoshenko beam [157], microbeam [158], functionally grades beam [159], etc. can be adequately utilized in the analysis of the equivalent models. However, the challenges for their applications of the equivalent nonlinear beam model arise in asymmetrical and three-dimensional truss structures, because the governing equations are coupled in terms of linear and nonlinear parts as a result of the asymmetry of the truss structures, especially for the 3D trusses. For the coupled three-dimensional nonlinear model and its solving approach, the first step is generally linearization and then the natural frequencies and modes of the full coupled linearized model need to be achieved. Based on the mode shape orthogonality, the second step is to derive the high-dimensional coupled nonlinear ordinary differential equations using the Galerkin method. To investigate nonlinear dynamic characteristics of truss structures analytically, there exists two aspects of challenge issues. The first one is how to get the natural modes of the full coupled linearized model in which bending about two principal axes, extension, shear and torsion are considered. For a specific space antenna truss, the governing partial differential equations of the equivalent beam model can be decoupled to two coupled subsystems (namely bending-torsional model and bending-extensional model) and an effective approach is developed to get the natural frequencies and the corresponding mode functions in Ref. [92]. In spite of this, how to extend the approach adopted in Ref. [92] to obtain the natural mode functions of a full coupled linearized model of the equivalent beam model of a space truss is still an open problem. Another challenge is how to solve the high-dimensional coupled nonlinear ordinary differential equations which are derived from the full coupled nonlinear partial differential equations of the equivalent nonlinear beam model of the complex space trusses. This is also a key step to analyze the nonlinear dynamic phenomena of the space trusses in a qualitatively way.

Active Vibration Control of Truss Structures

For the active vibration control of the complex truss structures, the most critical step is to construct the state space model with an appropriate dimension. Remarkable achievements have been obtained in active vibration control of the large truss structures, nevertheless, the vibration control on line is still a challenging and difficult task. Currently, most of researches focused on the active vibration control of single member (e.g. cable, beam, plate, etc.). Actually, the effectiveness of the control law design and the vibration control online is determined by the scale and accuracy of the state space model of the system. However, in order to obtain the state space equations of the complex system, it faces enormous difficulties via the analytical method, FEM and experiment approach. Therefore, it is important to put efforts on establishing low dimensional and valid equivalent dynamic models of the complex truss structures, go on to construct their state space model with an appropriate dimension, and to design an effective control law consequently.

Moreover, the governing partial differential equations of motion can be obtained based on the established equivalent dynamic model of the truss structure. Subsequently, the controllability and observability can be arrived based on the state space model to put forward effective control law. To test the availability of the control law for the original truss system, one important issue would be interesting to carry out the vibration suppression uniformity between the equivalent dynamic model and the original truss system, in which theoretical analysis and experimental study should be devoted to the related investigation further.

Dynamic Analysis and Vibration Control of Flexible Spacecraft with Space Trusses

Specifically, modern spacecraft structure composed of rigid and flexible multi-body is often used in large space station [160]. For instance, the complex space assembly structure can be composed of central rigid, extremely flexible antenna truss and solar array, and the large and light-weight design results in the structural low-frequency vibration modes, which are coupled with the rigid body motion of the spacecraft and may be excited by the attitude maneuver or other orbital operations [161]. Therefore, to study the dynamic characteristics and design efficient vibration controller for spacecraft, how to obtain their global rigid-flexible coupling modes and establish dynamic models using the equivalent modeling concept? As described above, the number of degree-of-freedom of the FEM model is too large to design control system conveniently. On the contrary, the modal approach can significantly reduce the number of degree-of-freedom for the system and thus enhance computational efficiency. However, it is greatly difficult to obtain the analytical global mode shapes for these complex flexible spacecraft especially when the space trusses are parts of the spacecraft systems. Fortunately, the space truss structures can be equivalent to a single beam or plate models and then achieve their simple equivalent multi-body systems, such as multi-flexile-beam [162]. Furthermore, employing the developed solving methods, the structural global modes can be obtained in analytical expressions, which provides convenience for dynamic analysis and vibration control law design. The quality of the mode shapes used in the simulations, that is, on how accurate the mode shapes can represent the real deformations. It should be noted that the global mode method proposed by group of Cao [163, 164] can be effectively utilized in the study of the dynamic analysis and vibration control of the flexible spacecraft with space trusses. Further efforts should concentrate on achieving the global analytical modes and effective dynamic models using equivalent modeling concept for the flexible spacecraft with space trusses combining the equivalent modeling method with the global analytical modes approach.

Conclusions

The spacecraft and civil engineering structures are developing towards to the directions of large-size, light-weight and flexibility, which motivates the studies about the equivalent dynamic modeling of the large space truss structures. In this survey, the status of the equivalent continuum modeling on the truss structures is firstly presented. Then, the equivalent continuum modeling work for linear truss systems is described, together with different modeling approaches, structural styles and connection joint. In addition, the solution methods and experimental studies are also discussed. Afterwards, the equivalent continuum modeling of nonlinear truss systems is investigated and classified into geometrical, material and jointed nonlinearities. The vibration control researches involving the truss structures and their equivalent continuum models are performed, which are analyzed to have great potential and advantages of applications in large truss structures. Further research efforts, e.g. equivalent nonlinear modeling strategy for truss structures, solving approach of coupled three-dimensional nonlinear model, active vibration control of space trusses, as well as dynamic analysis and vibration control of flexible spacecraft with truss structures, are put forward for reference.

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Declarations

Conflict of interest The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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