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A three-way multi-attribute decision making method based on regret theory and its application to medical data in fuzzy environments

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ABSTRACT

Cardiovascular disease is a global leading cause of death, and timely monitoring can determine its extent. Clinicians use these diagnostic indicators to make scientific and reasonable decisions. However, when decision-makers (DMs) encounter risks in complex environments, their limited rationality may affect decision behaviors. Therefore, the paper explores a new three-way multi-attribute decision making method based on regret theory (3W-MADM-R), which uses heart disease data to make decisions in fuzzy environments. There are three main steps in developing 3W-MADM-R, i.e., (i) we propose the notion of relative outcome functions and corresponding aggregated regret-based utility functions of each object; (ii) we estimate the conditional probability via an outranked set defined by an outranking relation based on the Preference Ranking Organization Method for Enrichment Evaluation (PROMETHEE II); (iii) we construct three-way decision rules to solve the problems of clustering and ranking of objects in data analysis. In order to demonstrate the usefulness of 3W-MADM-R, we apply it to analyze heart disease data. By comparing with results of other methods, we show the feasibility, stability and superiority of the presented 3W-MADM-R method.

1. Introduction

Multi-attribute decision making (MADM) refers to a decision problem in which the importance of different attributes is considered and the decision scheme is ranked or the optimal one is selected. There are many applications of MADM methods in management, engineering design, economics and other fields. For instance, Churchman et al. [1] first used a simple weighting method to solve the problem of "choosing the investment policy of enterprises". At present, the classic MADM methods, including the TOPSIS method [2], the ELECTRE method [3], the PROMETHEE-II method [4] and others. Meanwhile, many types of uncertainties have been considered in MADM methods, including randomness [5], fuzziness [6] and roughness [7]. According to the behavior characteristics of a DM, stochastic, fuzzy-set, and rough-set-based MADM models have been proposed and investigated [8–12].

With the rise of MADM techniques, their applications in the medical field [13] have become more and more widespread. A case in point is that the MADM method can be applied to solve cardiovascular and cerebrovascular diseases that seriously threaten human health. According to a report released by World Health Organization (WHO), 18 million people die from cardiovascular diseases each year, and 85% of which are due to heart disease and stroke. Data from the National Bureau of Statistics (https://data.stats.gov.cn/) show that the proportion of deaths due to heart disease in the total number of deaths is increasing year by year and has reached 23.65% in 2019. Therefore, the prevention and control of heart disease need joint efforts of all individuals. Early monitoring, diagnosis and treatment can effectively reduce the morbidity and mortality of heart disease and improve the life quality of patients. One of the key problems in the field of life sciences is how to quickly and effectively diagnose heart disease. At present, routine detection items for this disease mainly include blood pressure, electrocardiogram, blood routine, blood lipids, blood glucose, hemorheology, and so forth. These tests will help clinicians to determine the location and extent of cardiovascular diseases. However, in complex environments, facing with limited rationality owned by DMs and different forms of biomedical data, there are two issues that need to be addressed. One is how to effectively and reasonably deal with ambiguities in psychological and medical data along with the bounded rationality of DMs, and the other one is how DMs with limited rationality apply these data to diagnose a variety of diseases. In real world, clinical decision making and other aspects remain to be explored.

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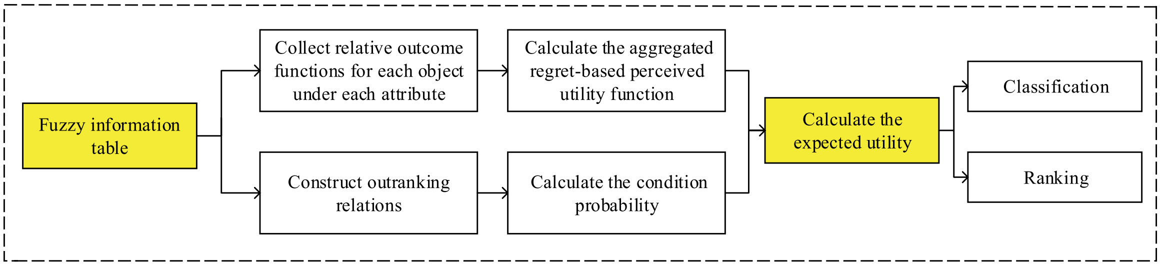

Fig. 1. The main components of the 3W-MADM-R method.

As an important method in the field of decision analysis, MADM analysis plays a unique role in the treatment of medical decision making problems. However, some existing traditional MADM methods have some shortcomings. For instance, these methods can either only rank medical plans and cannot make direct qualitative medical decisions for patients [14], or make decisions that are too subjective. That is to say, these methods can only address part of decision making problems [15]. In light of the above shortcomings, for improving the overall healthcare delivery and introducing the attitude of DMs towards risks in uncertain decision making environments, we aim to explore a new data-driven knowledge discovery and analysis method in clinical decision making via data related to heart disease diagnosis, the basic ideas of three-way decisions (3WD) and regret theory (RT). That is, the 3W-MADM-R method can be constructed. In this paper, we explore two recent trends in MADM problems. One trend is the combination of the theories of 3WD with MADM methods to classify and rank alternatives. The other trend is applications of prospect theory (PT) and RT in MADM problems.

The 3WD theory, proposed by Yao [16–18], concerns thinking, problem-solving, and information-processing in threes. There are a narrow sense and a wide sense of 3WD. The narrow sense concerns a semantic interpretation of three types of rules derived in probabilistic rough sets [16,19]. With respect to the positive, negative and boundary regions, we can construct three types of rules: positive rules for acceptance, negative rules for rejection, and non-commitment rules for neither acceptance nor rejection. The wide sense focuses on a general use of triads, i.e., a set of three items, a group of three things, a triple of three elements and others in scientific theories, methodology, and day-to-day practice. The principles and ideas of 3WD lie in dividing and representing a whole into three parts, understanding and processing by three parts, which conform to the thinking process of "divide and conquer" and "complexity simplification" in human cognition. 3WD has been explored in relation to granular computing [20], fuzzy sets [21], rough sets [22], statistical inferences [8], and many other contexts. Meanwhile, 3WD has attracted extensive attention from researchers worldwide in a broad range of different disciplines. Besides, 3WD has been successfully applied in many fields such as particle computing [23,24], machine learning [22], data mining [25], and so forth.

Many existing MADM studies assume that a DM is perfectly rational and cognitively clear. In reality, when a DM is faced with uncertainties in a decision making process, he/she often shows a limited rationality, with a behavior of risk aversion under certain gain or risk-seeking under certain loss. There exist two main psychological behavior theories: The first one is to make decisions based on PT proposed by Kahneman [26] and Tversky [27]. The second one is to make decisions based on RT proposed by Bell [28] and Loomes and Sugden [29]. Both PT and RT have been widely used for many aspects in realistic problems [30–32]. Tan et al. [33] proposed a decision making method based on the prospect random dominance degree for stochastic MADM problems where the attributes have expectation levels. Peng and Yang [34] proposed a method to consider both subjective and objective information in relation to the regret psychology of a DM. Several recent studies attempted to combine 3WD, PT, with RT for MADM. Wang et al. [35,36] combined 3WD with cumulative prospect theory (CPT), and calculated the prospect value to divide and rank alternatives. Liang et al. [37] applied RT to 3WD with project resource allocation in the context of interval numbers. Wang et al. [15] constructed a 3WD model based on RT under the interval type-2 fuzzy environment. In addition, a wide sense of 3WD emerges as a theory of thinking, problem-solving, and information-processing in threes, i.e., a triad consisting of three things.

Based on research results from these three theories, the main objective of this paper is to propose a new 3W-MADM-R method. The main components of 3W-MADM-R method are given in Fig. 1, which integrate ideas with results from the theories of 3WD, MADM and RT. We summarize the motivations of the 3W-MADM-R method as follows:

(1) Some existing 3WD approaches to MADM idealize the decision making process and ignore the fact that behaviors of a DM may have an impact on the results of alternative ranking and clustering [38].

(2) Some researchers have combined RT with 3WD. There are possibilities for improving their studies. For instance, an outcome matrix used by Wang et al. [15] may be too subjective, which does not reflect the original attribute evaluation table. Liang et al. [37] considered only a single attribute and used a subjective loss function. These subjective factors in these methods are not easy to implement in realistic problems.

(3) A key issue in 3WD is the determination of the conditional probability used for making decisions. Most of the existing studies assume that the conditional probability is subjectively given [39] or do not provide a semantic interpretation [40].

Therefore, the 3W-MADM-R method is developed to address these improvable issues. Compared with other decision making methods, our proposed method has several innovations:

(1) The 3W-MADM-R method complements behavioral psychology and provides a more realistic decision making model, rather than a purely rational MADM [8,39]. In order to avoid the influence of artificially selected reference points in PT [35], we consider the attitudinal preferences of a DM by introducing RT into 3WD, which addresses the impact of risks on the psychological behavior of a DM.

(2) 3W-MADM-R uses the original information table to construct the result function of each object under each attribute in a fuzzy environment, and combines RT with 3WD in a more objective way. This differs from Wang et al. [15] who overall assigned all objects an identical outcome matrix, and from Liang et al. [37] who constructed interval loss functions for objects.

(3) 3W-MADM-R defines a new outranking relation to estimate the conditional probability based on the ideas of a net-flow of the PROMETHEE-II method.

(4) Unlike the researches in [15,39], which use small-scale data to test the superiority of the method, the 3W-MADM-R method

Table 1
The loss functions.

	$S(P)$	$\neg S(N)$
ac_P	π_{PP}	π_{PN}
ac_B	π_{BP}	π_{BN}
ac_N	π_{NP}	π_{NN}

Table 2
The relative loss functions.

	$S(P)$	$\neg S(N)$
ac_P	0	$\tilde{\pi}_{PN}$
ac_B	$\tilde{\pi}_{BP}$	$\tilde{\pi}_{BN}$
ac_N	$\tilde{\pi}_{NP}$	0

can solve practical problems. Three realistic medical data sets are taken as examples to illustrate the feasibility of the presented method.

The rest of the paper is organized as follows. In Section 2, we review pertinent concepts from 3WD, the PROMETHEE-II method and RT. In Section 3, we combine 3WD with RT to propose a new 3W-MADM-R method. In Section 4, we apply the proposed 3W-MADM-R method to a data set "Statlog(Heart)" to confirm its effectiveness and operability. In addition, after exploring the optimal parameters of different algorithms, we compare the 3W-MADM-R method with three other types of decision making methods. In Section 5, we discuss the differences between the proposed method and other decision making methods and illustrate its advantages from two perspectives. In Section 6, the influence of parameter variations on the proposed method is investigated. The supplementary data set experiments in Section 7 illustrate the feasibility of the presented method. In Section 8, we summarize main results and comment on possible future research topics.

2. An overview of 3WD, the ROMETHEE-II method and RT

In this section, we recall basic concepts and notions of 3WD, the PROMETHEE-II method and RT.

2.1. 3WD

A narrow sense of 3WD was first introduced by Yao [16,19] within a decision-theoretic rough set model (i.e., a probabilistic rough set model based on the Bayesian decision process). It is assumed that U is a non-empty finite alternative set, $\varsigma = \{S, \neg S\}$ is a state set and $\mathcal{D} = \{ac_P, ac_B, ac_N\}$ is an action set, where S means that an element x is in a decision class S and $\neg S$ means that the element x is not in the class S, and ac_P, ac_B, and ac_N represent the action of acceptance, non-commitment, and rejection in a state. For instance, under the state S, by taking an action, an object x is put into one of the three regions, namely, $x \in Pos(S)$, $x \in Bnd(S)$, or $x \in Neg(S)$. Here, the universe of discourse U is divided into three disjoint regions: positive region ($Pos(S)$), negative region ($Neg(S)$) and boundary region ($Bnd(S)$), where $U = Pos(S)\bigcup Neg(S)\bigcup Bnd(S)$ and $Pos(S)\bigcap Neg(S) = \varnothing, Pos(S)\bigcap Bnd(S) = \varnothing, Bnd(S)\bigcap Neg(S) = \varnothing$. Taking different actions under the two states induces different risks, then the loss functions of actions are listed as follows (see Table 1):

Here, π_{PP}, π_{BP}, and π_{NP} denote the risk for taking the actions ac_P, ac_B, and ac_N when the object $x \in S$. Similarly, π_{PN}, π_{BN}, and π_{NN} represent the risk for taking the actions ac_P, ac_B, and ac_N when the object $x \in \neg S$. Based on an intuitive understanding of the severity of the risks of the three actions, we assume that $\pi_{PP} \le \pi_{BP} < \pi_{NP}$ and $\pi_{NN} \le \pi_{BN} < \pi_{PN}$.

Let $\mathcal{P}r(S|x)$ denote the conditional probability of an object x belonging to S. From the loss function and the conditional probability, the expected loss $ER(ac_i|x)$ $(i = P, B, N)$ of the three actions taken for each object x can be calculated by:

$$\begin{aligned} ER(ac_P|x) &= \pi_{PP}\mathcal{P}r(S|x) + \pi_{PN}\mathcal{P}r(\neg S|x),\\ ER(ac_B|x) &= \pi_{BP}\mathcal{P}r(S|x) + \pi_{BN}\mathcal{P}r(\neg S|x),\\ ER(ac_N|x) &= \pi_{NP}\mathcal{P}r(S|x) + \pi_{NN}\mathcal{P}r(\neg S|x). \end{aligned} \tag{1}$$

Based on the Bayesian theory, the minimum loss means the best decision, hence decision rules can be expressed as:

$$\begin{aligned} &\text{(P0) If } ER(ac_P|x) \le ER(ac_B|x) \text{ and } ER(ac_P|x) \le ER(ac_N|x),\\ &\qquad \text{decide that } x \in Pos(S);\\ &\text{(B0) If } ER(ac_B|x) \le ER(ac_P|x) \text{ and } ER(ac_B|x) \le ER(ac_N|x),\\ &\qquad \text{decide that } x \in Bnd(S);\\ &\text{(N0) If } ER(ac_N|x) \le ER(ac_P|x) \text{ and } ER(ac_N|x) \le ER(ac_B|x),\\ &\qquad \text{decide that } x \in Neg(S). \end{aligned} \tag{2}$$

Since $\mathcal{P}r(S|x)+\mathcal{P}r(\neg S|x) = 1$, the decision rules (P0)-(N0) can be re-written as:

$$\begin{aligned} &(\mathrm{P}')\ \text{If } \mathcal{P}r(S|x) \ge \alpha \text{ and } \mathcal{P}r(S|x) \ge \gamma,\ \text{then decide } x \in Pos(S),\\ &(\mathrm{B}')\ \text{If } \mathcal{P}r(S|x) \le \alpha \text{ and } \mathcal{P}r(S|x) \ge \beta,\ \text{then decide } x \in Bnd(S),\\ &(\mathrm{N}')\ \text{If } \mathcal{P}r(S|x) \le \beta \text{ and } \mathcal{P}r(S|x) \le \gamma,\ \text{then decide } x \in Neg(S), \end{aligned} \tag{3}$$

where

$$\begin{aligned} \alpha &= \frac{(\pi_{PN} - \pi_{BN})}{(\pi_{PN} - \pi_{BN}) + (\pi_{BP} - \pi_{PP})},\\ \beta &= \frac{(\pi_{BN} - \pi_{NN})}{(\pi_{BN} - \pi_{NN}) + (\pi_{NP} - \pi_{BP})},\\ \gamma &= \frac{(\pi_{PN} - \pi_{NN})}{(\pi_{PN} - \pi_{NN}) + (\pi_{NP} - \pi_{PP})}. \end{aligned} \tag{4}$$

That is, the required parameters can be computed based on the loss function.

It can be observed that the three thresholds are only related to the relative differences of the loss function instead of actual values of the loss function. Based on this notion of "relative costs", Jia and Liu [39] introduced the concept of relative loss functions (see Table 2):

Here, $\tilde{\pi}_{BP} = \pi_{BP} - \pi_{PP}$, $\tilde{\pi}_{NP} = \pi_{NP} - \pi_{PP}$, $\tilde{\pi}_{PN} = \pi_{PN} - \pi_{NN}$ and $\tilde{\pi}_{BN} = \pi_{BN} - \pi_{NN}$. The values of α, β, and γ can be similarly expressed in terms of a relative loss function.

In the past decade, 3WD has achieved fruitful research results in theory, methods, algorithms and applications. There have been many major theoretic results, algorithmic developments, and applications of 3WD. Hu [41] unified several kinds of representative 3WD models into the mathematical theoretical framework to study the spatial problems of 3WD models. Yao [17] provided a research framework for the 3WD and cognitive computing. Ciucci and Dubois [42] discussed the three-valued logics, which is related to 3WD. Liang et al. [43,44] considered 3WD in group decision making in a multi-expert decision making environment. Zhang et al. [45] proposed a 3WD model based on the utility theory for considering new loss functions. Three-way attribute reductions have also received plenty of attention [21,46]. Chen et al. [47] used three monotone measures of the conditional entropy combined with heuristic algorithm to explore the region preservation reduction method of the 3WD neighborhood system. Ma et al. [48] studied different kinds of attribute reduction methods when the decision region remained unchanged. Li et al. [20] successfully applied the sequential three-way decision strategy in the field of cost-sensitive portrait recognition. Zhou et al. [49] applied 3WD to email spam filtering. Yao and Azam [50] used 3WD to analyze uncertainties in medical decisions.

2.2. The PROMETHEE-II method

Compared with the differences in attribute values of various schemes, Brans [4] proposed the PROMETHEE method to consider the advantages of different schemes in case of differences in attribute values of different schemes. The main idea of this method is to define a preference function $\digamma$ for each attribute to describe the relationship between the attribute value and the degree of target achievement, then define the priority index between the scheme pairs according to the attribute weight and the preference function, and finally calculate the outgoing flow and incoming flow of each scheme. Moreover, the PROMETHEE-II method defines a net-flow on this basis, which is sorted according to the value of the net flow of each scheme from the largest to the smallest.

Suppose that a finite alternative set is $T = \{t_1, \ldots, t_i, \ldots, t_m\}$, m is the total number of alternatives. An attribute set is $C = \{c_1, \ldots, c_j, \ldots, c_n\}$, and n is the total number of attributes, the attribute weights of each scheme are $\omega_j, j = 1, 2, \ldots, n$ and $\sum_{j=1}^{n} \omega_j = 1$, the original decision matrix is $L = \{l_{ij}\}$. In this paper, in order to prevent the influence of a DM's subjective preferences on attributes, we use the deviation maximization method [51] to calculate the weight, and the calculation is as follows:

$$\omega_j = \frac{\sum_{i=1}^{m}\sum_{k=1}^{m} |a_{ij} - a_{kj}|}{\sum_{q=1}^{n}\sum_{i=1}^{m}\sum_{k=1}^{m} |a_{iq} - a_{kq}|}, j = 1, 2, \ldots, n, \tag{5}$$

where a_{ij} represents the element in the normalized matrix $A = \{a_{ij}\}$. The standard 0–1 transformation method is used to normalize the decision matrix [51]:

$$a_{ij} = \begin{cases} \dfrac{l_{ij} - l_j^{min}}{l_j^{max} - l_j^{min}} & \text{if } j \text{ is a benefit attribute,} \\ \dfrac{l_j^{max} - l_{ij}}{l_j^{max} - l_j^{min}} & \text{if } j \text{ is a cost attribute,} \end{cases} \tag{6}$$

where l_j^{min} is the minimum evaluation value in the original decision matrix under the attribute c_j; l_j^{max} is the maximum evaluation value in the original decision matrix under the attribute c_j. Without loss of generality, it is assumed that the attributes are of benefit type, the difference between the scheme t_i and the scheme t_k on the attribute j is denoted as $d = a_{ij} - a_{kj}$. Then the preference function κ is expressed as:

$$\kappa(t_i, t_k) = \begin{cases} 0 & a_{ij} \le a_{kj}, \\ \digamma(d) & a_{ij} > a_{kj}. \end{cases} \tag{7}$$

There are six typical evaluation criteria commonly used in the PROMETHEE-II method, and the criterion with linear preference and indifference area is adopted in this part:

$$\digamma(d) = \begin{cases} 0 & d \le q, \\ (d-q)/(p-q) & q < d \le p, \\ 1 & d > p, \end{cases} \tag{8}$$

where the function has two parameters q, p, which can be determined according to the specific data. For the schemes $t_i, t_k \in T$, the priority index Π is defined as:

$$\Pi(t_i, t_k) = \frac{\sum_{j=1}^{n} \omega_j \kappa_j(t_i, t_k)}{\sum_{j=1}^{n} \omega_j}. \tag{9}$$

The outgoing flow ϕ^+ and the incoming flow ϕ^- of the alternative t_i are:

$$\phi^+(t_i) = \sum_{t_k \in T} \Pi(t_i, t_k), \tag{10}$$

$$\phi^-(t_i) = \sum_{t_k \in T} \Pi(t_k, t_i). \tag{11}$$

The PROMETHEE-II method defines a net-flow ϕ:

$$\phi(t_i) = \phi^+(t_i) - \phi^-(t_i), \tag{12}$$

which is given by the difference of the outgoing and incoming flows.

2.3. RT

In a risk-taking environment, individuals often make irrational decisions. A DM not only cares about the outcomes of his/her choice of alternatives, but also compares the outcomes of choosing one option with the outcomes of choosing another one. If the expectation is met, a DM will be happy with the decision; otherwise, he/she will experience regret. Therefore, Bell [28] put forward RT from the perspective of psychology, and according to the expected utility theory [52], the perceived utility of a DM will change according to different options. The decision making process in RT includes two kinds of psychological perception: regret and euphoria. In other words, a DM may feel regret or rejoice at the outcomes of the choices he/she makes in the decision making process, and tries to avoid the choices that he/she regrets.

For two alternatives t_1 and t_2, let O_1 and O_2 represent the results brought by the choice of these two alternatives respectively, then a DM's perceived utility $\mathscr{V}$ for the choice of the alternative t_1 is defined as follows:

$$\mathscr{V}(O_1) = \mathscr{U}(O_1) + \mathscr{R}(\mathscr{U}(O_1) - \mathscr{U}(O_2)), \tag{13}$$

where $\mathscr{U}(O_1)$ denotes the direct utility function when adopting the alternative t_1 and $\mathscr{R}(\mathscr{U}(O_1) - \mathscr{U}(O_2))$ denotes the regret-rejoice function to measure the difference in utility between choosing the alternative t_1 and the alternative t_2. Here $\triangle\mathscr{U} = \mathscr{U}(O_1) - \mathscr{U}(O_2)$. In [53], the direct utility function $\mathscr{U}(O_i)$ and the regret-rejoice function $\mathscr{R}(\triangle\mathscr{U})$ is shown as:

$$\mathscr{U}(O_i) = \frac{1 - e^{-\theta O_i}}{\theta}, \tag{14}$$

$$\mathscr{R}(\triangle\mathscr{U}) = 1 - e^{-\delta \triangle \mathscr{U}}, \tag{15}$$

where $\theta \in (0, 1)$ indicates the risk aversion parameter and $\delta \in [0, +\infty)$ is the regret aversion parameter. Based on Eqs. (14), (15), Fig. 2 illustrates the influence of θ on the direct utility function $\mathscr{U}(O_i)$ and the influence of δ on the regret-rejoice function $\mathscr{R}(\triangle\mathscr{U})$.

According to Fig. 2, we can find that when the risk aversion parameter θ is constant, the direct utility function $\mathscr{U}(O)$ monotonically increases with the increase of the variable O, and the larger the value of the parameter θ is, the more sensitive a DM is to the risk utility, which is consistent with the psychology of the increased risk aversion. Similarly, we can see that when the value of the regret aversion parameter δ is constant, the regret-rejoice function $\mathscr{R}(\triangle\mathscr{U})$ monotonically increases with the increase of the variable $\triangle\mathscr{U}$.

Through RT, $\mathscr{R}(\triangle\mathscr{U}) > 0$ indicates that a DM is rejoiced with the benefits when he/she chooses the option t_1 over the option t_2. On the contrary, $\mathscr{R}(\triangle\mathscr{U}) < 0$ implies a DM regrets the loss caused by choosing the alternative t_1 over the alternative t_2.

Realistic decision making problems often contain multiple choices. For this reason, Quiggin [54] extended the application scope of RT to multiple action sets, selected an alternative that maximizes the goal and proposed a more general formula. Let $O_1, O_2, \ldots, O_m$ be the outcomes of choosing options $t_1, t_2, \ldots, t_m$, respectively. Consequently, a DM's perceived utility for the scheme O_i is defined as:

$$\mathscr{V}(O_i) = \mathscr{U}(O_i) + \mathscr{R}(\mathscr{U}(O_i) - \mathscr{U}(O^*)), \tag{16}$$

where $O^* = max\{O_i | i = 1, 2, \ldots, m\}$. $\mathscr{R}(\mathscr{U}(O_i) - \mathscr{U}(O^*))$ denotes the regret value, which is always non-positive.

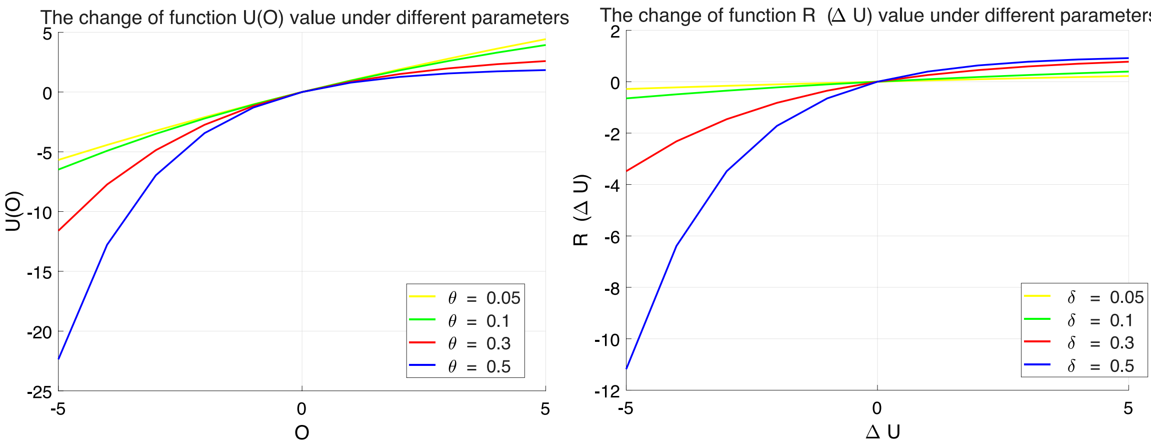

Fig. 2. The direct utility function $\mathscr{U}(O)$ and the regret-rejoice function $\mathscr{R}(\triangle \mathscr{U})$.

Table 3
The outcome function.

	$S(P)$	$\neg S(N)$
ac_P	O_{PP}	O_{PN}
ac_B	O_{BP}	O_{BN}
ac_N	O_{NP}	O_{NN}

Table 4
A transformed outcome function.

	$S(P)$	$\neg S(N)$
ac_P	$\tilde{O}_{PP}$	0
ac_B	$\tilde{O}_{BP}$	$\tilde{O}_{BN}$
ac_N	0	$\tilde{O}_{NN}$

Table 5
An example of an outcome function.

	$S(P)$	$\neg S(N)$
ac_P	5	2
ac_B	2	4
ac_N	1	7

Table 6
An example of a relative outcome function.

	$S(P)$	$\neg S(N)$
ac_P	4	0
ac_B	1	2
ac_N	0	5

3. 3WD with RT

In this section, we will analyze the relationship between the outcome function obtained when selecting an object and the attribute evaluation value, and construct the relative outcome function based on the idea of the relative loss function [39]. Moreover, the perceived utility function is calculated based on RT. However, our method does not mention how to obtain the conditional probability before proposing clustering rules. In the end of this section, a new method to calculate the conditional probability is proposed, that is, to construct a fuzzy set membership function for calculating the conditional probability via an outranking relation.

3.1. The relative outcome function

Similar to the loss function in Section 2.1, the 3W-MADM-R method considers two states $\varsigma = \{S, \neg S\}$. The three actions of the acceptance, deferment and rejection decisions all correspond to the result of this choice and constitute the outcome function, as shown in Table 3. It is not difficult to understand that they satisfy the conditions $O_{PP} \geq O_{BP} > O_{NP}$ and $O_{NN} \geq O_{BN} > O_{PN}$, that is to say, when the object t is in the state S, the outcome of the acceptance decision is greater than or equal to the outcome of the deferment decision, and both of these outcomes are strictly greater than the result of the rejection decision. The reverse order of outcomes is used for the object t which is in the state $\neg S$.

Inspired by the relativity of the loss function, the 3W-MADM-R method is only connected with the difference rather than the exact values of O. Given the above, we propose a special transformation rule for the outcome function that $O_{\diamond P}(\diamond = P, B, N)$ minus O_{NP} simultaneously and $O_{\diamond N}(\diamond = P, B, N)$ minus O_{PN} simultaneously. By this transformation, the outcome function has been converted to the form in Table 4.

Here, $\tilde{O}_{PP} = O_{PP} - O_{NP}$, $\tilde{O}_{BP} = O_{BP} - O_{NP}$, $\tilde{O}_{BN} = O_{BN} - O_{PN}$ and $\tilde{O}_{NN} = O_{NN} - O_{PN}$. Table 4 can be explained as follows: when an object x belongs to S, taking the outcome of the action ac_N as the benchmark, the outcomes for adopting the actions ac_P, ac_B and ac_N to the action ac_N are $\tilde{O}_{PP}$, $\tilde{O}_{BP}$ and 0, respectively. When an object t belongs to $\neg S$, taking the outcome of action ac_P as the benchmark, the outcomes of adopting the actions ac_P, ac_B and ac_N to the action ac_N are $\tilde{O}_{PN}$, $\tilde{O}_{BN}$ and 0, respectively. As a result, we name the transformed outcome function shown in Table 4 as the relative outcome function.

Example 3.1. Assume that there is an outcome function of 3WD as shown in Table 5, then we transform them into the relative outcome function based on Table 4, and the results are displayed in Table 6.

3.2. The relative outcome functions derived from fuzzy numbers (evaluation values of attributes)

In existing researches on the combination of RT with 3WD, the outcome function is fixed [15]. That is to say, for different alternatives, when they are in the same set of states (S or $\neg S$), the outcome function of taking the same action (ac_P, ac_B or ac_N) is the same. For instance, in the selection of investment projects, the classes S and $\neg S$ can be regarded as the profit and non-profit, respectively. Assume that the projects t_1 and t_2 belong to $\neg S$, as the result of the immobility of the outcome function, when choosing to invest in these projects, we can get the same outcome O_{PN}. However, when it comes to solving realistic problems, this

Table 7
The relative outcome function derived from a_{ij}.

	S_j	$\neg S_j$
ac_P	$a_{ij} - a^j_{min}$	0
ac_B	$\zeta \cdot (a_{ij} - a^j_{min})$	$\zeta \cdot (a^j_{max} - a_{ij})$
ac_N	0	$a^j_{max} - a_{ij}$

approach is unreasonable. In the process of an MADM, the outcomes of different objects for taking the same action in the same state should be different, which cannot be simply summarized by a single value. As the example just mentioned, the experts evaluate the profit value of two projects t_1 and t_2 as 1 and 3, 10 being the highest standard. Obviously, these two projects are not worth investing in when considering profitability, which means that t_1 and t_2 fall into $\neg S$. Therefore, the projects t_1 and t_2 have the same outcome in the fixed outcome function. Yet, considering an MADM, the project t_2 is slightly better than the project t_1 in terms of the profit value. For this reason, the outcome of choosing the investment project t_2 should be slightly greater than that of choosing the investment project t_1.

In order to eliminate the contradiction between attribute evaluation values and outcome functions and to compare the relationship between attribute evaluation values and loss functions explored by [39] on fuzzy numbers, we explore the relationship between attribute evaluation values and outcome functions in this section and eliminate the contradiction between them.

Assume that the attribute evaluation value of the alternative t_i under the attribute c_j is $a_{ij} (i = 1, 2, \ldots, m; j = 1, 2, \ldots, n)$, and the state set $\varsigma = \{S_j, \neg S_j\}$ denotes two states, where $t_i \in S_j$ represents that the alternative t_i has the attribute c_j. On the contrary, $t_i \in \neg S_j$ indicates that t_i has not the attribute c_j. The three actions ac_P, ac_B and ac_N denote the acceptance, deferment and rejection decision, respectively. Consequently, with respect to the attribute c_j, the 3WD model based on the idea of [39] can be constructed and we take $a_{ij} \in [a^j_{min}, a^j_{max}]$. Here, a^j_{min} and a^j_{max} denote the minimum value and maximum value of the attribute c_j, respectively. Table 7 shows the relative outcome function derived from the attribute evaluation value a_{ij}.

From the proposed outcome function, we can see that the erroneous action for allocating objects in the correct domain produces the smallest outcome value. As the minimum fuzzy number is 0, $O_{NP} = 0$ and $O_{PN} = 0$. Secondly, given that $t_i \in S_j$, the outcome value for which the alternative t_i makes the acceptance decision is expressed as a_{ij}, and the outcome for accepting t_i is represented as $a_{ij} - a^j_{min}$, which implies that the outcome of a DM's accepting action for the alternative t_i is $a_{ij} - a^j_{min}$ as compared to the outcome for the rejecting action; in the condition of $t_i \in \neg S_j$, the outcome for rejecting t_i is the complement of t_i, which implies that the outcome of a DM's rejecting action for the alternative t_i is $a^j_{max} - a_{ij}$ as compared to the outcome for the rejecting action. Finally, the outcome of the deferred acceptance is between the acceptance domain and the rejection domain. To this end, we need to reference the idea of parameters proposed by Li and Zhou [40] to calculate the outcome in the boundary domain. In the same mode, we introduce a parameter $\zeta \in [0, 1]$ called the utility pursuit coefficient to represent the outcome of an object for making a deferred acceptance decision. It follows that $O_{BP} = \zeta \cdot (a_{ij} - a^j_{min})$ and $O_{BN} = \zeta \cdot (a^j_{max} - a_{ij})$.

Example 3.2. A DM evaluates two investment projects t_1 and t_2 in the context of fuzzy numbers based on the attribute c_j, which means that the project evaluation value $a_{ij} \in [0, 1]$ $(i = 1, 2)$. Assume that the attribute evaluation values of the two alternatives are respectively $a_{1j} = 0.8$, $a_{2j} = 0.1$ and $\zeta = 0.3$.

Table 8
The relative outcome function of t_1.

t_1	S_j	$\neg S_j$
ac_P	0.8	0
ac_B	0.24	0.06
ac_N	0	0.2

Table 9
The relative outcome function of t_2.

t_2	S_j	$\neg S_j$
ac_P	0.1	0
ac_B	0.03	0.27
ac_N	0	0.9

Table 10
The inverse outcome function.

	$\tilde{S}_j$	$\neg \tilde{S}_j$
bc_P	$b_{ij} - b^j_{min}$	0
bc_B	$\zeta \cdot (b_{ij} - b^j_{min})$	$\zeta \cdot (b^j_{max} - b_{ij})$
bc_N	0	$b^j_{max} - b_{ij}$

Then the relative outcome functions of t_1 and t_2 for the attribute c_j can be calculated separately in Tables 8 and 9.

By comparing Tables 8 and 9, it can be found that even if t_1 and t_2 belong to the class S_j (or $\neg S_j$) at the same time, the outcome of the two alternatives for choosing the same action are also different, which is reasonable in practical problems. Further, as the evaluation values satisfy the following inequality: $a_{1j} > a_{2j}$ $(0.8 > 0.1)$, which indicates that if $t_1, t_2 \in S_j$, investing alternative t_1 achieves better outcome than t_2 in terms of the attribute c_j, thus taking the action of the acceptance or the delayed determination for t_1 has a greater outcome than t_2. Similarly, the inequality for $0.2 < 0.9$ shows that if $t_1, t_2 \in \neg S_j$, rejecting the alternative t_2 achieves a better outcome than t_1 in terms of the attribute c_j. In summary, the condition for $a_{1j} > a_{2j}$ can explain the conclusion that accepting t_1 has a greater outcome than t_2.

In order to enhance the comprehension of the relative outcome function, especially the understanding of the outcome when $t_i \in \neg S_j$, we introduce a new concept of an inverse outcome function in light of the inverse loss function. Suppose that $\tilde{c}_j$ denotes the opposite side of c_j and $b_{ij} = a^j_{max} + a^j_{min} - a_{ij}$ represents the attribute evaluation values of t_i aiming at $\tilde{c}_j$ when attribute evaluation values of t_i aiming at c_j is a_{ij}, where a^j_{max} and a^j_{min} represent the maximum and minimum values of the attribute c_j in the fuzzy environment, respectively. In other words, $a^j_{max} = 1$ and $a^j_{min} = 0$. For instance, if c_j denotes "energy protection" in the investment project problem, $\tilde{c}_j$ expresses "energy waste". If the attribute evaluation value of t_i aiming at c_j is denoted as a fuzzy number 0.8, the value of t_i on c_j is calculated as 0.2. That is to say, a DM' recognition of the $t_i's$ performance in "energy protection" is 0.8, and the recognition of the $t_i's$ performance in "energy waste" is 0.2. Therefore, $a_{ij} = 0.8$ and $b_{ij} = 0.8$ have equivalent connotations.

Considering the relationship between relative outcome functions and inverse selection outcome functions, the relative outcome function shown in Table 7 for t_i can be rewritten into the inverse relative outcome function shown in Table 10.
where the three actions bc_P, bc_B and bc_N indicate the clustering of t_i into $Pos(\tilde{S}_j)$, $Bnd(\tilde{S}_j)$ and $Neg(\tilde{S}_j)$, respectively. In view of the reason that $\tilde{S}_j = \neg S_j$, which represents a state of the attribute $\tilde{c}_j$, we know that bc_P, bc_B and bc_N correspond to ac_P, ac_B and ac_N, respectively. Thus we obtain the following properties $Pos(\tilde{S}_j) = Neg(S_j)$, $Bnd(\tilde{S}_j) = Bnd(S_j)$ and $Neg(\tilde{S}_j) = Pos(S_j)$. Meanwhile,

Table 11
The transformed inverse outcome function.

	S_j	$\neg S_j$
ac_P	$a^j_{max} - a_{ij}$	0
ac_B	$\zeta(a^j_{max} - a_{ij})$	$\zeta(a_{ij} - a^j_{min})$
ac_N	0	$a_{ij} - a^j_{min}$

$b_{ij} = a^j_{max} + a^j_{min} - a_{ij}$, $b^j_{max} = a^j_{max}$, $b^j_{min} = a^j_{min}$, hence we can acquire the following:

$$b^j_{max} - b_{ij} = a^j_{max} - (a^j_{max} + a^j_{min} - a_{ij}) = a_{ij} - a^j_{max},$$

$$b_{ij} - b^j_{min} = (a^j_{max} + a^j_{min} - a_{ij}) - a^j_{min} = a^j_{max} - a_{ij}.$$

Therefore, the inverse outcome function can be converted as Table 11, which corresponds to the original outcome function in Table 7.

In terms of the inverse outcome function shown in Tables 10 and 11, we can draw the following conclusions: when $t_i \in \tilde{S}_j$, the outcomes for putting t_i into the positive region bc_P, boundary region bc_B and negative region bc_N are $a^j_{max} - a_{ij}$, $\zeta(a^j_{max} - a_{ij})$ and 0, respectively, which are one-to-one corresponding to the results obtained when $t_i \in \neg S_j$ is placed in the positive, boundary and negative regions; when $t_i \in \neg\tilde{S}_j$, the outcomes of putting t_i into the positive region bc_P, boundary region bc_B and negative region bc_N are 0, $\zeta(a_{ij} - a^j_{min})$ and $a_{ij} - a^j_{min}$, respectively, which are one-to-one corresponding to the results obtained when $t_i \in S_j$ is placed in the positive, boundary and negative regions. According to the above special properties, the inverse outcome function can be utilized for dealing with the conversion problem between cost attributes and benefit attributes.

3.3. 3WD based on RT

The 3WD rules based on RT also contain two states $\varsigma = \{S, \neg S\}$ and three actions $\mathcal{D} = \{ac_P, ac_B, ac_N\}$. Through the relative outcome function discussed in Section 3.2, we can give the outcome function $O^{ij}_{\diamond\star}(\diamond = P, B, N; \star = P, N)$ of each object under each attribute, where O^{ij}_{PP}, O^{ij}_{BP} and O^{ij}_{NP} respectively represent the relative outcome function when the actions ac_P, ac_B and ac_N are taken when t_i belongs to S_j; O^{ij}_{PN}, O^{ij}_{BN} and O^{ij}_{NN} respectively represent the relative outcome function when the actions ac_P, ac_B and ac_N are taken when t_i belongs to $\neg S_j$. In order to specifically describe the relative outcome function in the MADM problems, we express its expression form as shown in Table 12.

Similar to Table 12, we define the direct utility function of each alternative under each attribute in the MADM that does not consider the regret values by Eq. (17), and its detailed expression form is also a $3i \times 2j$ matrix:

$$\mathscr{U}^{ij}_{\diamond\star} = \frac{1 - e^{-\theta O^{ij}_{\diamond\star}}}{\theta}. \tag{17}$$

In terms of RT, a DM will compare the results of the chosen option with those of other alternatives. Because they all contain two states and three actions, we establish a new model on the multi-action set based on RT. The utility function for each attribute considering the regret value is defined as the regret-based perceived utility function for each attribute, as shown in Table 13.

On the basis of Eq. (16), in order to compute the regret-based perceived utility function for each attribute, we need to select the decision alternative with the maximum outcome to calculate the regret values. Due to the monotony of direct utility function and regret-based perceived utility function for each attribute, when the object $t_i \in S_j$ or $t_i \in \neg S_j$, the maximum value of the six outcome functions corresponding to each attribute under

Table 12
Summarized relative outcome functions of MADM.

		c_1		c_2		$\cdots$	c_j	
		S_1	$\neg S_1$	S_2	$\neg S_2$	$\cdots$	S_j	$\neg S_j$
t_1	ac_P	O^{11}_{PP}	O^{11}_{PN}	O^{12}_{PP}	O^{12}_{PN}	$\cdots$	O^{1j}_{PP}	O^{1j}_{PN}
	ac_B	O^{11}_{BP}	O^{11}_{BN}	O^{12}_{BP}	O^{12}_{BN}	$\cdots$	O^{1j}_{BP}	O^{1j}_{BN}
	ac_N	O^{11}_{NP}	O^{11}_{NN}	O^{12}_{NP}	O^{12}_{NN}	$\cdots$	O^{1j}_{NP}	O^{1j}_{NN}
t_2	ac_P	O^{21}_{PP}	O^{21}_{PN}	O^{22}_{PP}	O^{22}_{PN}	$\cdots$	O^{2j}_{PP}	O^{2j}_{PN}
	ac_B	O^{21}_{BP}	O^{21}_{BN}	O^{22}_{BP}	O^{22}_{BN}	$\cdots$	O^{2j}_{BP}	O^{2j}_{BN}
	ac_N	O^{21}_{NP}	O^{21}_{NN}	O^{22}_{NP}	O^{22}_{NN}	$\cdots$	O^{2j}_{NP}	O^{2j}_{NN}
$\vdots$								
t_i	ac_P	O^{i1}_{PP}	O^{i1}_{PN}	O^{i2}_{PP}	O^{i2}_{PN}	$\cdots$	O^{ij}_{PP}	O^{ij}_{PN}
	ac_B	O^{i1}_{BP}	O^{i1}_{BN}	O^{i2}_{BP}	O^{i2}_{BN}	$\cdots$	O^{ij}_{BP}	O^{ij}_{BN}
	ac_N	O^{i1}_{NP}	O^{i1}_{NN}	O^{i2}_{NP}	O^{i2}_{NN}	$\cdots$	O^{ij}_{NP}	O^{ij}_{NN}

the option is selected as the outcome value of the ideal scheme, which is denoted in Eq. (18):

$$O^*(t_i) = max\{O^{ij}_{\diamond\star}\}. \tag{18}$$

At the moment, the regret-based perceived utility function for each attribute $\mathscr{V}^{ij}_{\diamond\star}$ is computed as Eq. (19) based on Eq. (16):

$$\mathscr{V}^{ij}_{\diamond\star} = \mathscr{U}^{ij}_{\diamond\star} + \mathscr{R}(\mathscr{U}^{ij}_{\diamond\star} - \mathscr{U}(O^*(t_i))). \tag{19}$$

By virtue of Eq. (19), we can get the regret-based perceived utility functions after aggregation of all attributes via the following Eq. (20).

$$V_{\diamond\star}(t_i) = \sum_{j=1}^{n} \omega_j \mathscr{V}^{ij}_{\diamond\star}. \tag{20}$$

Example 3.3. In the background of fuzzy numbers, we through the rand function in R2018b-MATLAB to generate uniformly distributed random numbers to form a 6×4 matrix as an evaluation information matrix of 6 objects with 4 attributes (all of which are assumed to be benefit attributes). The attribute weights are calculated by Eq. (5). All information is shown in Table 14.

On the basis of the original attribute information table presented in Table 14 and making $\zeta = 0.7, \theta = 0.3, \delta = 0.3$, through Table 7, we can convert the attribute evaluation value a_{ij} into a relative outcome functions $O^{ij}_{\diamond\star}$ in Table 15.

Calculate the direct utility functions $\mathscr{U}^{ij}_{\diamond\star}$ by Eq. (17), which is presented in Table 16.

Through the above analysis, we can get the maximum outcome value of each option from Table 16, which can be used as a reference point to calculate the regret values, i.e.,

$$\mathscr{U}(O^*(t_1)) = 0.8481, \mathscr{U}(O^*(t_2)) = 0.8403, \mathscr{U}(O^*(t_3)) = 0.8268,$$
$$\mathscr{U}(O^*(t_4)) = 0.8383, \mathscr{U}(O^*(t_5)) = 0.7909, \mathscr{U}(O^*(t_6)) = 0.7338.$$

Then, based on Eqs. (15), (19), the calculation results of regret-based perceived utility functions $\mathscr{V}^{ij}_{\diamond\star}$ for each alternative under each attribute are shown in Table 17.

The values of aggregated regret-based perceived utility function obtained from Eq. (20) are shown in Table 18:

With the regret-based perceived utility function and the conditional probability of each object in different states, a DM can calculate the expected utility under the three actions $\mathcal{D} = \{ac_P, ac_B, ac_N\}$ and express it in the following form:

$$\begin{aligned} EU(ac_P|t) &= V_{PP}\mathcal{P}r(S|t) + V_{PN}\mathcal{P}r(\neg S|t), \\ EU(ac_B|t) &= V_{BP}\mathcal{P}r(S|t) + V_{BN}\mathcal{P}r(\neg S|t), \\ EU(ac_N|t) &= V_{NP}\mathcal{P}r(S|t) + V_{NN}\mathcal{P}r(\neg S|t). \end{aligned} \tag{21}$$

Table 13
The regret-based perceived utility functions for each attribute of MADM.

		c_1		c_2		$\cdots$	c_j	
		S_1	$\neg S_1$	S_2	$\neg S_2$	$\cdots$	S_j	$\neg S_j$
t_1	ac_P	$\mathcal{V}_{PP}^{11}$	$\mathcal{V}_{PN}^{11}$	$\mathcal{V}_{PP}^{12}$	$\mathcal{V}_{PN}^{12}$	$\cdots$	$\mathcal{V}_{PP}^{1j}$	$\mathcal{V}_{PN}^{1j}$
	ac_B	$\mathcal{V}_{BP}^{11}$	$\mathcal{V}_{BN}^{11}$	$\mathcal{V}_{BP}^{12}$	$\mathcal{V}_{BN}^{12}$	$\cdots$	$\mathcal{V}_{BP}^{1j}$	$\mathcal{V}_{BN}^{1j}$
	ac_N	$\mathcal{V}_{NP}^{11}$	$\mathcal{V}_{NN}^{11}$	$\mathcal{V}_{NP}^{12}$	$\mathcal{V}_{NN}^{12}$	$\cdots$	$\mathcal{V}_{NP}^{1j}$	$\mathcal{V}_{NN}^{1j}$
t_2	ac_P	$\mathcal{V}_{PP}^{21}$	$\mathcal{V}_{PN}^{21}$	$\mathcal{V}_{PP}^{22}$	$\mathcal{V}_{PN}^{22}$	$\cdots$	$\mathcal{V}_{PP}^{2j}$	$\mathcal{V}_{PN}^{2j}$
	ac_B	$\mathcal{V}_{BP}^{21}$	$\mathcal{V}_{BN}^{21}$	$\mathcal{V}_{BP}^{22}$	$\mathcal{V}_{BN}^{22}$	$\cdots$	$\mathcal{V}_{BP}^{2j}$	$\mathcal{V}_{BN}^{2j}$
	ac_N	$\mathcal{V}_{NP}^{21}$	$\mathcal{V}_{NN}^{21}$	$\mathcal{V}_{NP}^{22}$	$\mathcal{V}_{NN}^{22}$	$\cdots$	$\mathcal{V}_{NP}^{2j}$	$\mathcal{V}_{NN}^{2j}$
$\vdots$								
t_i	ac_P	$\mathcal{V}_{PP}^{i1}$	$\mathcal{V}_{PN}^{i1}$	$\mathcal{V}_{PP}^{i2}$	$\mathcal{V}_{PN}^{i2}$	$\cdots$	$\mathcal{V}_{PP}^{ij}$	$\mathcal{V}_{PN}^{ij}$
	ac_B	$\mathcal{V}_{BP}^{i1}$	$\mathcal{V}_{BN}^{i1}$	$\mathcal{V}_{BP}^{i2}$	$\mathcal{V}_{BN}^{i2}$	$\cdots$	$\mathcal{V}_{BP}^{ij}$	$\mathcal{V}_{BN}^{ij}$
	ac_N	$\mathcal{V}_{NP}^{i1}$	$\mathcal{V}_{NN}^{i1}$	$\mathcal{V}_{NP}^{i2}$	$\mathcal{V}_{NN}^{i2}$	$\cdots$	$\mathcal{V}_{NP}^{ij}$	$\mathcal{V}_{NN}^{ij}$

Table 14
Attribute evaluation information table under the MADM.

	c_1	c_2	c_3	c_4
t_1	0.9787	0.7060	0.6948	0.7655
t_2	0.7577	0.0318	0.3171	0.7952
t_3	0.7431	0.2769	0.9502	0.1869
t_4	0.3922	0.0462	0.0344	0.4898
t_5	0.6555	0.0971	0.4387	0.4456
t_6	0.1712	0.8235	0.3816	0.6463
w	0.2366	0.2828	0.2305	0.2501

RT makes a DM to choose the decision that has the maximum utility. Therefore, the 3W-MADM-R method advises to use the following optimization problem for finding the optimum action: $\arg max_{ac_\diamond \in \mathcal{D}} V(ac_P|t)$.

Thus, the decision process suggests that in the 3W-MADM-R method, the decision rules of maximum-utility are shown as:

(P1) If $EU(ac_P|t) \geq EU(ac_B|t)$ and $EU(ac_P|t) \geq EU(ac_N|t)$, decide that $t \in Pos(S)$;

(B1) If $EU(ac_B|t) \geq EU(ac_P|t)$ and $EU(ac_B|t) \geq EU(ac_N|t)$, decide that $t \in Bnd(S)$;

(N1) If $EU(ac_N|t) \geq EU(ac_P|t)$ and $EU(ac_N|t) \geq EU(ac_B|t)$, decide that $t \in Neg(S)$.

In the three decision making processes, it is vital to simplify the decision rules according to the probability $\mathcal{P}r(S|t)$ and thresholds. In our proposed model, the condition that the thresholds satisfy $\alpha^* > \beta^*$ can be computed. That is, when $\alpha^* > \beta^*$, there is $(V_{PP} - V_{BP})(V_{NN} - V_{PN}) < (V_{BP} - V_{NP})(V_{BN} - V_{PN})$. Meanwhile, on account of $\mathcal{P}r(S|t) + \mathcal{P}r(\neg S|t) = 1$. The decision rules $(P1)-(N1)$ can be simplified as follows:

(P2) If $\mathcal{P}r(S|t) \geq \alpha^*$, decide that $t \in Pos(S)$;

(B2) If $\beta^* < \mathcal{P}r(S|t) < \alpha^*$, decide that $t \in Bnd(S)$;

(N2) If $\mathcal{P}r(S|t) \leq \beta^*$, decide that $t \in Neg(S)$,

where,

$$\alpha^* = \frac{1}{1 + \frac{V_{PP} - V_{BP}}{V_{BN} - V_{PN}}},\quad \beta^* = \frac{1}{1 + \frac{V_{BP} - V_{NP}}{V_{NN} - V_{BN}}},\quad \gamma^* = \frac{1}{1 + \frac{V_{PP} - V_{NP}}{V_{NN} - V_{PN}}}. \tag{22}$$

Otherwise, decision rules $(P1)-(N1)$ can be simplified as follows:

(P3) If $\mathcal{P}r(S|t) \geq \gamma^*$, decide that $t \in Pos(S)$;

(N3) If $\mathcal{P}r(S|t) < \gamma^*$, decide that $t \in Neg(S)$.

3.4. The conditional probability based on an outranking relation

The problem of how to get the conditional probability of an object in 3WD is a crucial one. In most of the previous 3WD models, either the conditional probability is given objectively by a DM, which is not appropriate if there are too many objects in a realistic problem, or the conditional probability calculation method proposed in the model has no reasonable semantic interpretation. Inspired by [55], we utilize the net-flow of the PROMETHEE-II method to construct an outranking relation and figure out the conditional probability in the fuzzy information system.

Through the introduction in Section 2.2, we can first construct the preference function between scheme pairs in the fuzzy environment in accordance with the attribute evaluation of the object combined with the linear preference and the indifference area criterion. Then, the priority index can be calculated after the attribute weight is obtained by the deviation maximization method. Finally, the outgoing flow, incoming flow and net-flow of the alternative are calculated. As described above, we define a new outranking relation.

Definition 3.1 (*[56]*)**.** The fuzzy set χ represents a concept of "good state", which is denoted as $\chi = \frac{\chi(t_1)}{t_1} + \frac{\chi(t_2)}{t_2} + \cdots + \frac{\chi(t_m)}{t_m}$, thereinto the membership function χ about t_i can be calculated from the following formula: $\chi(t_i) = \sum_{j=1}^{n} \omega_j \cdot a_{ij}$.

Remark 3.1. Another "fuzzy concept" can be defined as $\neg\chi = \frac{\neg\chi(t_1)}{t_1} + \frac{\neg\chi(t_2)}{t_2} + \cdots + \frac{\neg\chi(t_m)}{t_m}$, here $\neg\chi(t_i) = 1 - \chi(t_i)$.

The conditional probability consists of a state set and a binary relation. In subsequent studies, the state set consists of a fuzzy set χ and the complement of the fuzzy set χ, where the fuzzy set χ represents a "good state" and the complement of the fuzzy set χ represents a "bad state". In order to calculate the conditional probability of each alternative in different states more reasonably, Definition 3.2 gives an outranking relation based on net-flow of each alternative.

Definition 3.2. Assume that for all $c \in C$, $t_i, t_k \in T$, if the net-flow of the alternative t_i is not less than that of t_k, we believe that t_i is superior to t_k. Thus a new outranking relation $\Phi_{\succeq}$ is defined as $t_i \Phi_{\succeq} t_k$. i.e.,

$$\Phi_{\succeq} = \{(t_i, t_k) \in T \times T \mid \phi(t_i) \geq \phi(t_k)\}. \tag{23}$$

Table 15
The relative outcome functions $O^{ij}_{\diamond\star}$ of each alternative.

		c_1		c_2		c_3		c_4	
		S_1	$\neg S_1$	S_2	$\neg S_2$	S_3	$\neg S_3$	S_4	$\neg S_4$
	ac_P	0.9787	0	0.7060	0	0.6948	0	0.7655	0
t_1	ac_B	0.6851	0.0149	0.4942	0.2058	0.4864	0.2136	0.5358	0.1642
	ac_N	0	0.0213	0	0.2940	0	0.3052	0	0.2345
	ac_P	0.7577	0	0.0318	0	0.3171	0	0.7952	0
t_2	ac_B	0.5304	0.1696	0.0223	0.6777	0.2220	0.4780	0.5566	0.1434
	ac_N	0	0.2423	0	0.9682	0	0.6829	0	0.2048
	ac_P	0.7431	0	0.2769	0	0.9502	0	0.1869	0
t_3	ac_B	0.5202	0.1798	0.1938	0.5062	0.6651	0.0349	0.1308	0.5692
	ac_N	0	0.2569	0	0.7231	0	0.0498	0	0.8131
	ac_P	0.3922	0	0.0462	0	0.0344	0	0.4898	0
t_4	ac_B	0.2745	0.4255	0.0323	0.6677	0.0241	0.6759	0.3429	0.3571
	ac_N	0	0.6078	0	0.9538	0	0.9565	0	0.5102
	ac_P	0.6555	0	0.0971	0	0.4387	0	0.4456	0
t_5	ac_B	0.4588	0.2412	0.0680	0.6320	0.3071	0.3929	0.3119	0.3881
	ac_N	0	0.3445	0	0.9029	0	0.5613	0	0.5544
	ac_P	0.1712	0	0.8235	0	0.3816	0	0.6463	0
t_6	ac_B	0.1198	0.5802	0.5765	0.1235	0.2671	0.4329	0.4524	0.2476
	ac_N	0	0.8288	0	0.1765	0	0.6184	0	0.3537

Table 16
The direct utility functions $\mathscr{U}^{ij}_{\diamond\star}$ of each alternative.

		c_1		c_2		c_3		c_4	
		S_1	$\neg S_1$	S_2	$\neg S_2$	S_3	$\neg S_3$	S_4	$\neg S_4$
	ac_P	0.8481	0	0.6362	0	0.6272	0	0.6840	0
t_1	ac_B	0.6193	0.0149	0.4593	0.1996	0.4525	0.2069	0.4950	0.1602
	ac_N	0	0.0212	0	0.2814	0	0.2916	0	0.2264
	ac_P	0.6778	0	0.0316	0	0.3025	0	0.7075	0
t_2	ac_B	0.4903	0.1654	0.0222	0.6133	0.2147	0.4453	0.5126	0.1403
	ac_N	0	0.2337	0	0.8403	0	0.6175	0	0.1986
	ac_P	0.6661	0	0.2657	0	0.8268	0	0.1818	0
t_3	ac_B	0.4816	0.1751	0.1883	0.4696	0.6030	0.0347	0.1283	0.5232
	ac_N	0	0.2472	0	0.6500	0	0.0494	0	0.7215
	ac_P	0.3700	0	0.0459	0	0.0342	0	0.4555	0
t_4	ac_B	0.2635	0.3994	0.0322	0.6050	0.0240	0.6118	0.3258	0.3387
	ac_N	0	0.5556	0	0.8295	0	0.8383	0	0.4731
	ac_P	0.5951	0	0.0957	0	0.4111	0	0.4171	0
t_5	ac_B	0.4287	0.2326	0.0673	0.5757	0.2934	0.3706	0.2978	0.3663
	ac_N	0	0.3273	0	0.7909	0	0.5166	0	0.5107
	ac_P	0.1669	0	0.7297	0	0.3606	0	0.5875	0
t_6	ac_B	0.1177	0.5325	0.5294	0.1213	0.2567	0.4060	0.4231	0.2386
	ac_N	0	0.7338	0	0.1719	0	0.5644	0	0.3356

Definition 3.3. By means of the outranking relation $\Phi_{\succeq}$ based on the net-flow definition in the PROMETHEE-II method, an outranked set of any alternative $t_i \in T$ is defined as follows:

$$[t_i]_{\Phi_{\succeq}} = \{t_k \mid t_i \Phi_{\succeq} t_k \wedge t_k \in T\}. \quad (24)$$

For instance, if $t_k \in [t_i]_{\Phi_{\succeq}}$, we have $t_i \Phi_{\succeq} t_k$. In other words, the alternative t_i is superior to t_k.

Example 3.4 (*Continued from Example 3.3*)**.** Referring to the PROMETHEE-II method [4], we take the parameter $q = 0.1$ and $p = 0.9$ in the fuzzy environment. The priority index Π among the six alternatives and outgoing flow ϕ^+, incoming flow ϕ^- are represented in Table 19.

Though $\phi(t_i) = \phi^+(t_i) - \phi^-(t_i)$, the net-flows of the six alternatives are:

$$\phi(t_1) = 1.7683, \phi(t_2) = -0.1823, \phi(t_3) = 0.1737,$$
$$\phi(t_4) = -1.4695, \phi(t_5) = -0.5325, \phi(t_6) = 0.2423.$$

In light of Definitions 3.2 and 3.3, we have an outranked set for six objects:

$$[t_1]_{\Phi_{\succeq}} = \{t_1, t_2, t_3, t_4, t_5, t_6\},$$
$$[t_2]_{\Phi_{\succeq}} = \{t_2, t_4, t_5\},$$
$$[t_3]_{\Phi_{\succeq}} = \{t_2, t_3, t_4, t_5\},$$
$$[t_4]_{\Phi_{\succeq}} = \{t_4\},$$
$$[t_5]_{\Phi_{\succeq}} = \{t_4, t_5\},$$
$$[t_6]_{\Phi_{\succeq}} = \{t_2, t_3, t_4, t_5, t_6\}.$$

Proposition 3.1. *In an MADM matrix, suppose that $[t_i]_{\Phi_{\succeq}}$ is an outranked set of $t_i \in T$. The following properties hold:*

(1) *For any $t_i \in T$, there is $t_i \in [t_i]_{\Phi_{\succeq}}$, namely, $[t_i]_{\Phi_{\succeq}}$ satisfies the reflexivity;*

(2) *For any $t_i, t_k, t_r \in T$, if $t_k \in [t_i]_{\Phi_{\succeq}}$ and $t_r \in [t_k]_{\Phi_{\succeq}}$, then we have $t_r \in [t_i]_{\Phi_{\succeq}}$.*

Proof. (1) Obviously, we can directly verify this property by Definitions 3.2 and 3.3.

Table 17
The regret-based perceived utility functions $\mathscr{V}^{ij}_{\diamond\star}$ of each alternative.

		c_1		c_2		c_3		c_4	
		S_1	$\neg S_1$	S_2	$\neg S_2$	S_3	$\neg S_3$	S_4	$\neg S_4$
t_1	ac_P	0.8481	−0.2897	0.5706	−0.2897	0.5586	−0.2897	0.6335	−0.2897
	ac_B	0.5482	−0.2691	0.3356	−0.0152	0.3265	−0.0052	0.3832	−0.0690
	ac_N	−0.2897	−0.2603	−0.2897	0.0961	−0.2897	0.1100	−0.2897	0.0214
t_2	ac_P	0.6278	−0.2867	−0.2429	−0.2867	0.1274	−0.2867	0.6668	−0.2867
	ac_B	0.3797	−0.0591	−0.2560	0.5428	0.0083	0.3195	0.4094	−0.0933
	ac_N	−0.2867	0.0341	−0.2867	0.8403	−0.2867	0.5484	−0.2867	−0.0136
t_3	ac_P	0.6167	−0.2815	0.0824	−0.2815	0.8268	−0.2815	−0.0317	−0.2815
	ac_B	0.3725	−0.0409	−0.0228	0.3565	0.5335	−0.2336	−0.1048	0.4279
	ac_N	−0.2815	0.0574	−0.2815	0.5956	−0.2815	−0.2132	−0.2815	0.6894
t_4	ac_P	0.2192	−0.2859	−0.2225	−0.2859	−0.2386	−0.2859	0.3338	−0.2859
	ac_B	0.0753	0.2587	−0.2414	0.5326	−0.2527	0.5415	0.1596	0.1770
	ac_N	−0.2859	0.4671	−0.2859	0.8268	−0.2859	0.8383	−0.2859	0.3573
t_5	ac_P	0.5345	−0.2678	−0.1362	−0.2678	0.2903	−0.2678	0.2984	−0.2678
	ac_B	0.3139	0.0503	−0.1752	0.5090	0.1324	0.2362	0.1383	0.2305
	ac_N	−0.2678	0.1781	−0.2678	0.7909	−0.2678	0.4308	−0.2678	0.4231
t_6	ac_P	−0.0185	−0.2463	0.7284	−0.2463	0.2421	−0.2463	0.5426	−0.2463
	ac_B	−0.0853	0.4702	0.4661	−0.0804	0.1028	0.3026	0.3253	0.0785
	ac_N	−0.2463	0.7338	−0.2463	−0.0117	−0.2463	0.5123	−0.2463	0.2087

Table 18
The aggregated regret-based perceived utility functions $V_{\diamond\star}(t_i)$ of each alternative.

	V_{PP}	V_{PN}	V_{BP}	V_{BN}	V_{NP}	V_{NN}
t_1	0.6492	−0.2897	0.3957	−0.0864	−0.2897	−0.0037
t_2	0.2760	−0.2867	0.1217	0.1898	−0.2867	0.3687
t_3	0.3518	−0.2815	0.1785	0.1443	−0.2815	0.3053
t_4	0.0174	−0.2859	−0.0688	0.3809	−0.2859	0.6269
t_5	0.2295	−0.2678	0.0898	0.2680	−0.2678	0.4709
t_6	0.3931	−0.2463	0.2167	0.1779	−0.2463	0.3406

Table 19
The priority index among the six alternatives.

Π	t_1	t_2	t_3	t_4	t_5	t_6	$\phi^+(t)$
t_1	0	0.3188	0.3061	0.5582	0.3596	0.2767	1.8193
t_2	0	0	0.1589	0.1954	0.0787	0.1592	0.5921
t_3	0.0448	0.2049	0	0.3509	0.1468	0.2746	1.0219
t_4	0	0	0.0634	0	0	0.0358	0.0992
t_5	0	0.0062	0.0496	0.1360	0	0.1137	0.3055
t_6	0.0062	0.2445	0.2702	0.3283	0.2529	0	1.1022
$\phi^-(t)$	0.0510	0.7744	0.8482	1.5687	0.8380	0.8599	–

(2) For any $t_i, t_k, t_r \in T$, we have

$$t_k \in [t_i]_{\Phi_{\succeq}} \Leftrightarrow t_i \Phi_{\succeq} t_k \Leftrightarrow \phi(t_i) \geq \phi(t_k),$$
$$t_r \in [t_k]_{\Phi_{\succeq}} \Leftrightarrow t_k \Phi_{\succeq} t_r \Leftrightarrow \phi(t_k) \geq \phi(t_r).$$

Then, $\phi(t_i) \geq \phi(t_r) \Leftrightarrow t_i \Phi_{\succeq} t_r \Leftrightarrow t_r \in [t_i]_{\Phi_{\succeq}}$

As described previous, $[t_i]_{\Phi_{\succeq}}$ satisfies the transitivity. □

Through the above definitions, we give the calculation method of the conditional probability of each alternative.

Definition 3.4. For any alternative t_i in $[t_i]_{\Phi_{\succeq}}$ belonging to the state χ, based on the defined outranking relation, the conditional probability calculation for each alternative t_i is derived as follows:

$$\mathcal{P}r(\chi|[t_i]_{\Phi_{\succeq}}) = \frac{\sum_{v\in[t_i]_{\Phi_{\succeq}}} \chi(v)}{|[t_i]_{\Phi_{\succeq}}|}, \tag{25}$$

where |✠| represents the cardinality of the set ✠.

Remark 3.2. Similarly, $\mathcal{P}r(\neg\chi|[t_i]_{\Phi_{\succeq}}) = \frac{\sum_{v\in[t_i]_{\Phi_{\succeq}}} \neg\chi(v)}{|[t_i]_{\Phi_{\succeq}}|}$. According to Remark 3.1, we have $\mathcal{P}r(\chi|[t_i]_{\Phi_{\succeq}}) + \mathcal{P}r(\neg\chi|[t_i]_{\Phi_{\succeq}}) = 1$.

Therefore, for each object $t_i \in T$, we can use $\mathcal{P}r(\chi|[t_i]_{\Phi_{\succeq}})$ to indicate the conditional probability of the object t_i belonging to the state S, which means $\mathcal{P}r(\chi|[t_i]_{\Phi_{\succeq}}) = \mathcal{P}r(S|t)$.

Example 3.5 (*Continued with Example 3.4*)**.** Based on Example 3.4, the outranked sets corresponding to the six objects and raw attribute evaluation Table 12 are obtained. The membership function of the fuzzy set is calculated by Definition 3.1:

$$\chi = \frac{0.7828}{t_1} + \frac{0.4602}{t_2} + \frac{0.5199}{t_3} + \frac{0.2363}{t_4} + \frac{0.3951}{t_5} + \frac{0.5230}{t_6}.$$

According to Eq. (25), we have:

$\mathcal{P}r(\chi|[t_1]_{\Phi_{\succeq}}) = 0.7828$, $\mathcal{P}r(\chi|[t_2]_{\Phi_{\succeq}}) = 0.5715$,
$\mathcal{P}r(\chi|[t_3]_{\Phi_{\succeq}}) = 0.6086$,
$\mathcal{P}r(\chi|[t_4]_{\Phi_{\succeq}}) = 0.4868$, $\mathcal{P}r(\chi|[t_5]_{\Phi_{\succeq}}) = 0.5362$,
$\mathcal{P}r(\chi|[t_6]_{\Phi_{\succeq}}) = 0.6529$.

In addition, based on the final results $\mathscr{V}_{\diamond\star}(t_i)$ of Example 3.3 and Eq. (22), we calculate the three thresholds α^*, β^* and γ^* in the 3W-MADM-R method below.

$\alpha^*(t_1) = 0.4451; \alpha^*(t_2) = 0.7555; \alpha^*(t_3) = 0.7106;$
$\alpha^*(t_4) = 0.8855; \alpha^*(t_5) = 0.7932; \alpha^*(t_6) = 0.7062.$
$\beta^*(t_1) = 0.1077; \beta^*(t_2) = 0.3045; \beta^*(t_3) = 0.2592;$
$\beta^*(t_4) = 0.5312; \beta^*(t_5) = 0.3621; \beta^*(t_6) = 0.2601.$
$\gamma^*(t_1) = 0.2335; \gamma^*(t_2) = 0.5381; \gamma^*(t_3) = 0.4809;$
$\gamma^*(t_4) = 0.7506; \gamma^*(t_5) = 0.5977; \gamma^*(t_6) = 0.4786.$

Then, the conditional probability $\mathcal{P}r(\chi|[t_i]_{\Phi_{\succeq}})$ is compared with the values of $\alpha^*(t_i)$ and $\beta^*(t_i)$, six objects can be divided into the following three domains based on the simplified decision rules (P2)-(N2) of maximum-utility built from 3W-MADM-R:

$Pos(\chi) = \{t_1\}, Bnd(\chi) = \{t_2, t_3, t_5, t_6\}, Neg(\chi) = \{t_4\}.$

Meanwhile, Eq. (21) is used to calculate the expected utility of each object under three actions $\mathcal{D} = \{ac_P, ac_B, ac_N\}$ in Table 20.

Finally, based on the ranking rules in the 3WD [56], the higher the expected utility value in the same domain is, the better the

Table 20
The expected utility of each object.

	t_1	t_2	t_3	t_4	t_5	t_6
$EU(ac_P\|t)$	0.4453	0.0349	0.1039	−0.1384	−0.0011	0.1712
$EU(ac_B\|t)$	0.2910	0.1509	0.1651	0.1622	0.1724	0.2032
$EU(ac_N\|t)$	−0.2276	−0.0059	−0.0518	0.1831	0.0748	−0.0426

object is on the premise that $Pos(\chi) \succ Bnd(\chi) \succ Neg(\chi)$. According to Table 20, the sorting result of the six objects is $t_1 \succ t_6 \succ t_5 \succ t_3 \succ t_2 \succ t_4$.

3.5. The core steps of the 3W-MADM-R algorithm

Algorithm 1 displays the detailed process of our proposed 3W-MADM-R method, let m denote the number of all objects and n denote the number of all attributes. Then we analyze the time complexity of the algorithm. It is worth noting that the complexity of Algorithm 1 is determined by the highest power of the polynomial.

Remark 3.3. The time complexity of Step 1 (Lines 1–8 in 1) is $O(m^2n)$. The time complexity of Step 2 (Lines 9–14 in Algorithm 1) is $O(mn)$. The time complexity of Step 3 (Lines 15–17 in Algorithm 1) is $O(m)$. The time complexity of Step 4 (Lines 18–20 in Algorithm 1) is $O(mn)$. The time complexity of Step 5 (Lines 21–23 in Algorithm 1) is $O(nm^3+3m^2+2m)$. The time complexity of Step 6 (Lines 24–26 in Algorithm 1) is $O(1)$. The time complexity of Step 7 (Lines 27–32 in Algorithm 1) is $O(m)$. The time complexity of Step 8 (Lines 33–35 in Algorithm 1)is $O(1)$. The time complexity of Step 9 (Lines 36–41 in Algorithm 1) is $O(m)$. Hence, the overall time complexity of Algorithm 1 is $O(nm^3)$.

4. An illustrative example

In the current section, our proposed 3W-MADM-R method addresses the problem of heart disease in a fuzzy environment, and the effectiveness of the method is verified by comparing with other counterparts.

4.1. Problem descriptions based on 3W-MADM-R

Heart disease is the top killer of human health. According to WHO, nearly 17 million people die of cardiovascular disease each year. Thus how to quickly and effectively diagnose heart disease has always been one of the key issues in the field of life sciences. As the world enters the era of precision medicine, the diagnosis and prevention of heart disease have also entered a new stage of immunotherapy. At present, it will play a crucial role in preventing heart disease by extracting relevant body measurement indicators and analyzing the influence of different characteristics on heart disease via data mining.

We choose an example of a data set "Statlog (Heart)" from the University of California at Irvine (UCI) database (https://archive.ics.uci.edu/ml/datasets/Statlog+%28Heart%29/20220423.html), consisting of 270 objects and 13 attributes. In specific, 270 patients with cardiac examination are represented as $T = \{t_1, t_2, \ldots, t_{270}\}$. In terms of 13 attributes, class attributes and bipolar attributes are removed. Finally, we select 7 attributes, which are expressed as ages (c_1), rest blood pressure(c_2), serum cholesterol (c_3), max heart rate (c_4), old peak (c_5), slope (c_6) and thal (c_7). By using the deviation maximization weight method mentioned in Section 2.2, we obtain the attribute weight of the example in the lymphogram domain as $w = \{0.1266, 0.1086, 0.0742, 0.1163, 0.1055, 0.1829, 0.2859\}$.

In this paper, we use the proposed 3W-MADM-R method to infer the presence of heart disease from physical examination data collected from cardiac data sets. At the medical level, the Statlog (Heart) database gives the physical characteristics of 270 subjects, but it is up to the clinician to make the final decision on whether the objects have the disease and determine the next treatment plan. For each patient, there are two states $\chi, \neg\chi$ that the presence of heart disease and the absence of heart disease, respectively. At the same time, there are three actions $\mathcal{D} = \{ac_P, ac_B, ac_N\}$, which correspond to the treatment, the delayed treatment and no treatment. In the PROMETHEE-II method, the values of the parameters q and p are 0.1 and 0.9, respectively.

4.2. Optimal selection of parameters in 3W-MADM-R method

In MADM problems, the choice of parameters endows DMs with certain initiative. However, different parameters get different decision making results, and there may be a phenomenon that the results are not comparable under the worst conditions. Based on this, in order to make it more convenient for a DM to make a choice and select the appropriate parameters, this section will explore how to select the outcome utility coefficient ζ, the risk aversion parameter θ and the regret aversion parameter δ in the data set "Statlog(Heart)" that have a key influence on the results of 3W-MADM-R method.

3WD pays attention to both sorting and classification. Therefore, the principle of optimal parameter selection is based on the existing TOPSIS method [12]) without parameters. On the premise of maintaining a high Spearman rank correlation coefficient [57] (SRCC) between the sorting results of 3W-MADM-R method and that of the TOPSIS method, several common clustering indexes are better. In fact, in the data set "Statlog(Heart)" the ranking results obtained from Algorithm 1 show that when the parameters ζ, θ and δ increase, the SRCCs decrease gradually to some extent, but the minimum is 97.98%, which is highly correlated with the ranking results of the TOPSIS method. In light of this, we select the optimal parameters according to the principle that the lower the error rate [15] is, the better the Rand index (RI) and F1 [15,58] are.

In particular, we find that the error rate equation in [15] ignores the fact that different decision making methods will produce different object numbers in the boundary domain, which cannot reflect the validity of the boundary and reasonably compare the error rates in different decision making methods. Therefore, we have corrected the original error rate equation. The corrected error rate (CE) equation and equations RI, F1 are shown as follows.

$$\text{CE} = \frac{n_{\chi\to Neg(\chi)} + n_{\neg\chi\to Pos(\chi)}}{|U| - n_{Bnd(\chi)}} \times 100\%, \tag{26}$$

$$\text{RI} = \frac{n_{\chi\to Pos(\chi)} + n_{\neg\chi\to Neg(\chi)}}{n_{\chi\to Pos(\chi)} + n_{\chi\to Neg(\chi)} + n_{\neg\chi\to Pos(\chi)} + n_{\neg\chi\to Neg(\chi)}} \times 100\%, \tag{27}$$

$$\text{F1} = \frac{2 \times Precision \times Recall}{Precision + Recall}, \tag{28}$$

where $|U|$ represents the number of all objects, meanwhile $n_{\chi\to Neg(\chi)}$ and $n_{\neg\chi\to Pos(\chi)}$ denote the number of objects belonging to χ and $\neg\chi$ that are divided into the negative and positive domains, respectively. $n_{\chi\to Pos(\chi)}$ and $n_{\neg\chi\to Neg(\chi)}$ denote the number of objects belonging to χ and $\neg\chi$ that are divided into the positive and negative domains, respectively.

Remark 4.1. The smaller the corrected error rate, the stronger the validity of the boundary. This is due to the fact that the

Algorithm 1: The algorithm of 3W-MADM-R.

Input: An information system (T, C), and the values of the parameters q, p, ζ, θ and δ.
Output: The clustering and ranking results of all objects.

begin
  for $j = 1$ *to* n **do**
    Normalize: if c_j is a benefit attribute, $a_{ij} = \frac{l_{ij}-l_j^{min}}{l_j^{max}-l_j^{min}}$,
      if c_j is a cost attribute, $a_{ij} = \frac{l_j^{max}-l_{ij}}{l_j^{max}-l_j^{min}}$. Here l_{ij} is the attribute evaluation value in the original decision matrix.
  end
  for $j = 1$ *to* n **do**
    compute: calculate attribute weights ω_j according to Eq. (5).
  end
  for $i = 1$ *to* m, $j = 1$ *to* n **do**
    compute: Based on the standardized attribute evaluation value construct the relative outcome functions O^{ij}_{PP}, O^{ij}_{BP},O^{ij}_{NP}, O^{ij}_{PN}, O^{ij}_{BN},O^{ij}_{NN}
  end
  for $i = 1$ *to* m, $j = 1$ *to* n **do**
    compute: calculate the direct utility function $\mathscr{U}_{\diamond\star}$ for each object under each attribute via Eq. (17) .
  end
  for $i = 1$ *to* m, $j = 1$ *to* n **do**
    compute: select the maximum outcome value corresponding to each alternative and calculate its direct utility functions: $\mathscr{U}(O^*(t_i)) = \frac{1-e^{-\theta O^*(t_i)}}{\theta}$, where $\theta \in (0, 1)$.
  end
  for $i = 1$ *to* m, $j = 1$ *to* n **do**
    compute: calculate the regret-based perceived utility function $\mathscr{V}^{ij}_{\diamond\star}$ by the light of Eq. (15), then get the aggregated regret-based perceived utility function: $V_{\diamond\star}(t_i) = \sum_{j=1}^{n} \omega_j \mathscr{V}^{ij}_{\diamond\star}$.
  end
  for $i = 1$ *to* m,$j = 1$ *to* n **do**
    compute: calculate the outranked sets $[t_i]_{\Phi_\succeq}$ and fuzzy membership function $\chi = \frac{\sum_{j=1}^{n} \omega_j \cdot a_{ij}}{t_i}$.
  end
  for $i = 1$ *to* m **do**
    compute: compute the condition probability $\mathcal{P}r(\chi|[t_i]_{\Phi_\succeq}) = \frac{\sum_{v\in[t_i]_{\Phi_\succeq}} \chi(v)}{|[t_i]_{\Phi_\succeq}|}$ and $\mathcal{P}r(\neg\chi|[t_i]_{\Phi_\succeq}) = \frac{\sum_{v\in[t_i]_{\Phi_\succeq}} \neg\chi(v)}{|[t_i]_{\Phi_\succeq}|}$.
  end
  for $i = 1$ *to* m **do**
    compute: calculate the thresholds α^*, β^*, γ^* via Eq. (22):
  end
  for $i = 1$ *to* m **do**
    decision: Obtain a 3WD model decision clustering rules based on RT with (P2)-(N2).
  end
  for $i = 1$ *to* m **do**
    compute: calculate the expected utility $EU(ac_\diamond|t_i)$ with respect to three actions $\mathcal{D} = \{ac_P, ac_B, ac_N\}$ by Eq. (21):
  end
  for $i = 1$ *to* m, $k = 1$ *to* m *and* $i \neq k$ **do**
    determine: rank all objects by following these rules:
      (1) if two objects t_i, t_k in the same region and $EU(t_i) > EU(t_k)$, then $t_i \succ t_k$.
      (2) according to the rules of $Pos(\chi) \succ Bnd(\chi) \succ Neg(\chi)$.
  end
end

positive and negative domains are the two sets of objects divided into two groups after removing objects in the boundary domain. The smaller the numerator of Eq. (26) means that the proportion of objects misclassified after removing the boundary domain is smaller; the larger denominator means that fewer objects need further diagnosis and fewer resources are wasted. In summary, when both are satisfied, a smaller corrected error rate reflects a stronger effective selection of boundary domains.

Based on the above analysis on optimal parameter selection, the parameters ζ, θ and δ are changed simultaneously in the data set "Statlog(Heart)", and corresponding changes of the cluster index CE, RI and F1 values are shown in Figs. 3–5.

Remark 4.2. Figs. 3–5 shows how different indicators change with three parameters at the same time. The 4D map and several 2D contour maps can be clearly observed when $\zeta = 0.9$, $\theta = 0.5$ and $\delta = 0.9$, error rate = 0%, RI = 100%, F1 = 100%. Through the above analysis, $\zeta = 0.9$, $\theta = 0.5$ and $\delta = 0.9$ are the optimal parameters of the data set "Statlog(Heart)".

4.3. Case results under 3W-MADM-R method

The values of key parameters involved in 3W-MADM-R method have been discussed in detail in Section 4.2. According to the detailed process presented in Algorithm 1, the parameters $\zeta = 0.9$, $\theta = 0.5$ and $\delta = 0.9$ and the collected heart-related data are selected, the ranking results of 270 objects are shown in Fig. 6:

Remark 4.3. As shown in Fig. 6, when we consider the seven attributes mentioned above, the optimal object for this database

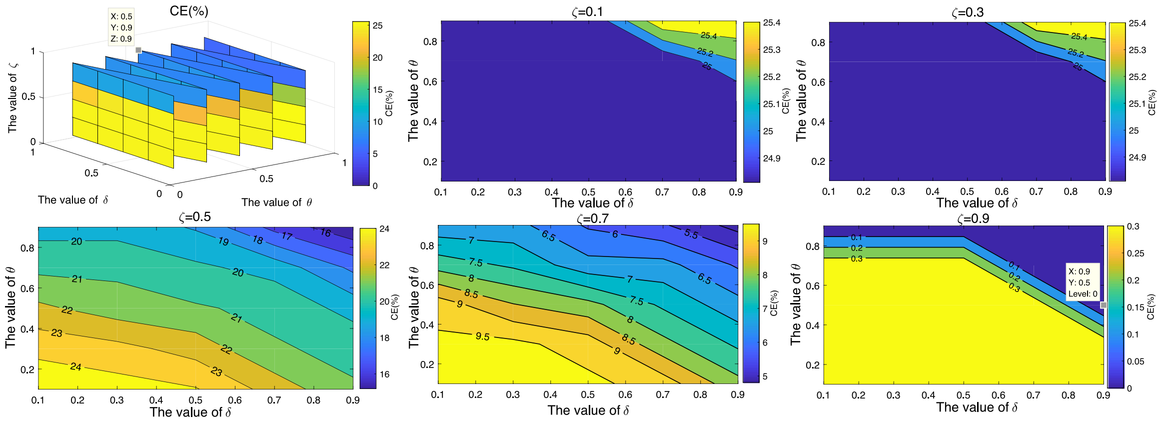

Fig. 3. CE index under three parameter changes.

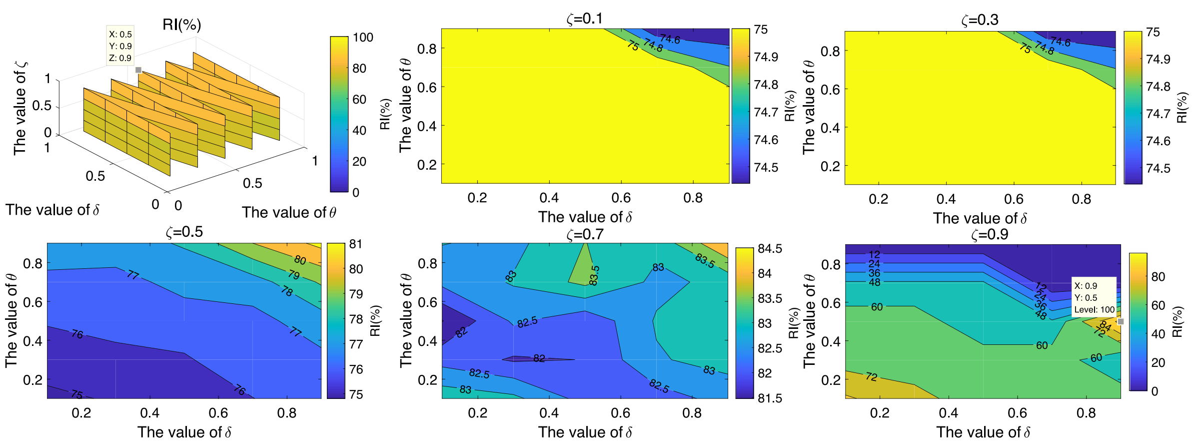

Fig. 4. RI index under three parameters change.

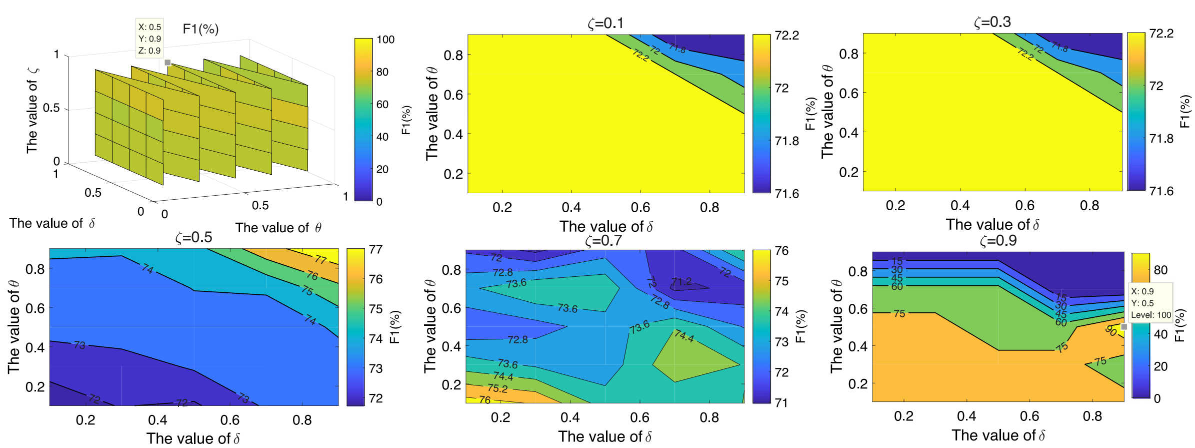

Fig. 5. F1 index under three parameters change.

is the object 48. In other words, the 48th patient in the Statlog (Heart) data set has the highest expected utility and the best treatment efficacy. The other objects are treated in an ascending order with diminishing effects. At the bottom of the list is the patient 130, who has the lowest expected utility of the treatment, which is not ideal. Meanwhile, Table 21 displays the clustering results that based on the three thresholds and the $(P2)-(N2)$ decision rules of the presented 3W-MADM-R method.

Remark 4.4. It can be seen from Table 21 that the 270 objects are divided into three domains, among which 7 objects are divided into the positive domains, 257 objects into the boundary domains and 6 objects into the negative domains. We explain the division of the three domains in a practical context, that is, by looking at the results of the 7 attributes of the heart test, the physician can conclude that there are 7 patients have been diagnosed with heart disease and needed treatment; 257 patients require further testing to be conclusive; 6 patients are normal and had no disease.

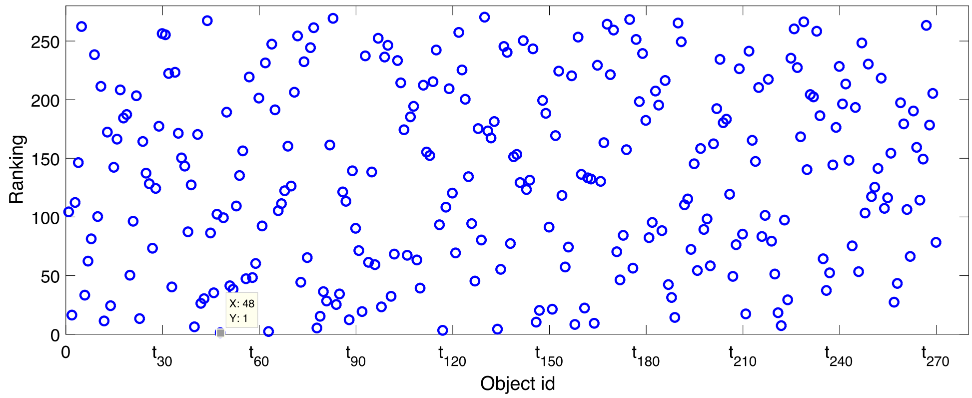

Fig. 6. The ranking results of all objects.

Table 21
The clustering results of all objects.

Domain	Objects	Numbers
$Pos(\chi)$	$t_{40}, t_{48}, t_{63}, t_{78}, t_{117}, t_{134}, t_{222}$	7
$Bnd(\chi)$	$t_1, t_2, t_3, t_4, t_5, t_6, t_7, t_9, t_8, t_{10}, t_{11}, t_{12}, t_{13}, t_{14}, t_{15}, t_{16}, t_{17}, t_{18}, t_{19}, t_{20}, t_{21}, t_{22}, t_{23}, t_{24}, t_{25}, t_{26}, t_{27}, t_{28}, t_{29}, t_{30}, t_{31}, t_{32}, t_{33}, t_{34}, t_{35}, t_{36}, t_{37}, t_{38}, t_{39}, , t_{41}, t_{42}, t_{43}, t_{45}, t_{46}, t_{47}, t_{49}, t_{50}, t_{51}, t_{52}, t_{53}, t_{54}, t_{55}, t_{56}, t_{57}, t_{58}, t_{59}, t_{60}, t_{61}, t_{62}, t_{64}, t_{65}, t_{66}, t_{67}, t_{68}, t_{69}, t_{70}, t_{71}, t_{72}, t_{73}, t_{74}, t_{75}, t_{76}, t_{77}, t_{79}, t_{80}, t_{81}, t_{82}, t_{84}, t_{85}, t_{86}, t_{87}, t_{88}, t_{89}, t_{90}, t_{91}, t_{92}, t_{93}, t_{94}, t_{95}, t_{96}, t_{97}, t_{98}, t_{99}, t_{100}, t_{101}, t_{102}, t_{103}, t_{104}, t_{105}, t_{106}, t_{107}, t_{108}, t_{109}, t_{110}, t_{111}, t_{112}, t_{113}, t_{114}, t_{115}, t_{116}, t_{118}, t_{119}, t_{120}, t_{121}, t_{122}, t_{123}, t_{124}, t_{125}, t_{126}, t_{127}, t_{128}, t_{129}, , t_{131}, t_{132}, t_{133}, t_{135}, t_{136}, t_{137}, t_{138}, t_{139}, t_{140}, t_{141}, t_{142}, t_{143}, t_{144}, t_{145}, t_{146}, t_{147}, t_{148}, t_{149}, t_{150}, , t_{151}, t_{152}, t_{153}, t_{154}, t_{155}, t_{156}, t_{157}, t_{158}, t_{159}, t_{160}, t_{161}, t_{162}, t_{163}, t_{164}, t_{165}, t_{166}, t_{167}, t_{168}, t_{169}, t_{170}, t_{171}, t_{172}, t_{173}, t_{174}, t_{176}, t_{177}, t_{178}, t_{179}, t_{180}, t_{181}, t_{182}, t_{183}, t_{184}, t_{185}, t_{186}, t_{187}, t_{188}, t_{189}, t_{191}, t_{192}, t_{193}, t_{194}, t_{195}, t_{196}, t_{197}, t_{198}, t_{199}, t_{200}, t_{201}, t_{202}, t_{203}, t_{204}, t_{205}, t_{206}, t_{207}, t_{208}, t_{209}, t_{210}, t_{211}, t_{212}, t_{213}, t_{214}, t_{215}, t_{216}, t_{217}, t_{218}, t_{219}, t_{220}, t_{221}, t_{223}, t_{224}, t_{225}, t_{226}, t_{227}, t_{228}, t_{230}, t_{231}, t_{232}, t_{233}, t_{234}, t_{235}, t_{236}, t_{237}, t_{238}, t_{239}, t_{240}, t_{241}, t_{242}, t_{243}, t_{244}, t_{245}, t_{246}, , t_{247}, t_{248}, t_{249}, t_{250}, t_{251}, t_{252}, t_{253}, t_{254}, t_{255}, t_{256}, t_{257}, t_{258}, t_{259}, t_{260}, t_{261}, t_{262}, t_{263}, t_{264}, t_{265}, t_{266}, t_{267}, t_{268}, t_{269}, t_{270}$	257
$Neg(\chi)$	$t_{44}, t_{83}, t_{130}, t_{175}, t_{190}, t_{229}$	6

4.4. Comparative analysis

In order to verify the effectiveness and rationality of the above deduced method, this section compares six methods with the 3W-MADM-R method in sorting or clustering from three aspects: the general RT method, the classic 3WD (C3WD) methods and the regret behavior. In addition, ranking decision has no an objective "ground truth". Therefore, in order to give a "ground truth", we use the ranking results obtained by the existing methods (such as the classic TOPSIS method [12]) as the "ground truth" for the comparison of sorting decisions in this article, as shown in Fig. 7, Meanwhile, we know that the 3WD clustering method is a clustering method in unsupervised learning. Therefore, there should be no "ground truth" about clustering decision making. In order to give an objective "ground truth", this article uses the clustering results of all objects under the decision attributes of the data set as the "ground truth" of the clustering decision in this paper.

Remark 4.5. In comparative analysis, it should be noted that in order to avoid the deviation of different comparison methods, it is better to select all the optimal parameters to obtain the decision results. In practice, however, some of the comparison methods may already give fixed parameter values based on existing experience, or may focus on the DM's preference in MADM, where the DM gives the parameters. For the latter, this paper uses the method of selecting optimal parameters in Section 4.2 to keep the consistency of parameters to a certain extent.

4.4.1. Comparative analysis with the general RT

Bell [28] first proposed RT and gave the calculation method. Later, with the continuous development of RT, they began to build regret models under different environments by combining with various backgrounds. This section will compare with Bell's classic regret method [28][1] and the model constructed by Liu et al. [14][2] under the hesitant fuzzy environment.

Remark 4.6. Based on Bell's theory of regret [28], Tversky et al. [27] specified the utility function as $\mathscr{U}(O) = O^{\rho}$, and the parameter $\rho = 0.88$. Meanwhile, Peng et al. [53] determined δ in the regret-rejoice function to be 0.3. In Liu et al.'s method [14], group satisfaction is used to replace the utility value under the background of hesitant fuzzy information. Due to the application background, hesitation number could not be used in this paper, so the utility function proposed by Tversky et al. [27] is adopted, with the parameter $\rho = 0.88$.

Since the two models mentioned above can only be ranked, the results of comparing the ranking of the proposed method with the general RT method. When parameters are known, the ranking results are shown in Fig. 8.

As can be seen from the ranking results in Fig. 8, our designed method has a high degree of similarity with the two ones of general RT, SRCCs are 0.7991 and 0.7990, respectively. Meanwhile, our method and Bell's method have the same optimal object at the 48th position, and the TOPSIS method, as the ranking benchmark, also has the same optimal object, our method has the same optimal object as Bell's method in the 48th place, and the TOPSIS method has the same optimal object as the ranking benchmark, indicating that our method conforms to the basic principle of MADM problems. However, the optimal object of Liu et al.'s method is the 63rd position and the 48th position, which has some deviation compared with the TOPSIS method.

1 $\rho = 0.88$, $\delta = 0.3$.
2 $\rho = 0.88$, $\delta = 0.3$, the probability of the two states is $P = (1, 0)^T$.

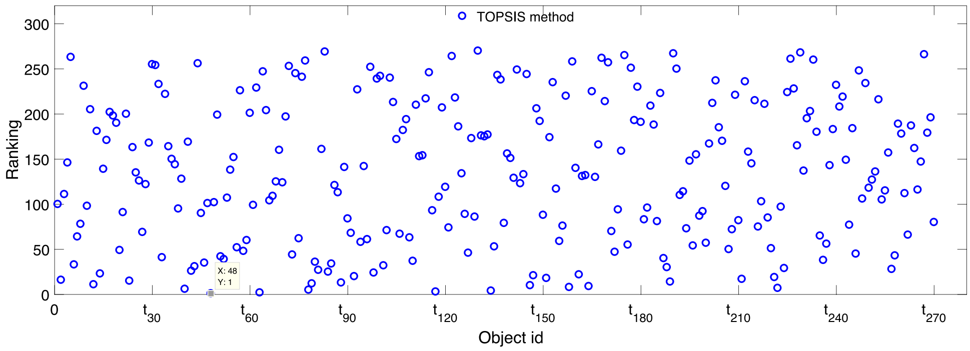

Fig. 7. The ranking result under the TOPSIS method.

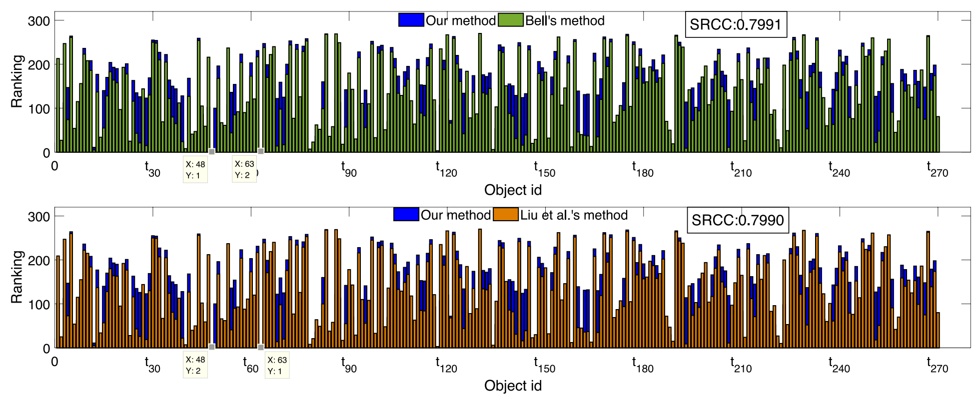

Fig. 8. Comparison of the ranking results obtained with the general RT method.

More importantly, our method can not only rank objects but also categorize them, which is not possible with the general RT.

4.4.2. Comparative analysis with the C3WD

Ever since the introduction of 3WD, it has been developed rapidly and many effective models can be used to solve the clustering problems. In order to better explain the effectiveness and rationality of the constructed method, three new 3WD-MADM models are selected in this section. Jia and Liu's method [39],[3] Zhan et al.'s method [55][4] and Wang et al.'s method [59][5] are compared with our method.

Remark 4.7. According to the method of selecting optimal parameters in the data set "Statlog(Heart)" mentioned in Section 4.2, when the risk avoidance coefficient $\sigma = 0.2$ and the conditional probability $\mathcal{P}r(\chi|[t_i]_{\Phi_{\succeq}}) = 0.6$ in Jia and Liu's method [39], the lowest CE = 8.33%, the highest RI = 91.67% and F1 = 90.91% can be obtained simultaneously. When the concordance level $\delta = 0.9$ and the veto threshold $p = 0.1$, the lowest CE = 41.96%, the highest RI = 58.04% and F1 = 73.45% is achieved simultaneously in the method of Zhan et al. [55]. When the confidence interval $\sigma = 0.5$ in Wang et al.'s method [59], the lowest CE = 20.15%, the highest RI = 97.70% and F1 = 73.59% can be obtained simultaneously.

After the optimal parameter is determined, the ranking and clustering results of 3W-MADM-R method and C3WD method are shown in Figs. 9 and 10, respectively.

It can be seen from Fig. 9 that the proposed method can not only sort objects, but also classify them. From the overall ranking results, the SRCCs between us and these three methods are 0.9995, 0.3214 and 0.8176. The optimal target of our proposed method is the same as the optimal object calculated by Jia and Liu's method and Zhan et al.'s method in C3WD, which proves that the 3W-MADM-R method can withstand the classic test. Wang et al.'s method ranks the 48th patient at 16, which is different from the results of the TOPSIS and other ranking methods.

Fig. 10 shows the reader that using the 3W-MADM-R method, 7 objects would be diagnosed as healthy, 6 as having heart disease and 257 as needing further diagnosis. Using Jia and Liu's method [39], 6 objects would have been diagnosed as healthy, 6 would have had heart disease and 258 would have required further diagnosis by a doctor. Using Zhan et al.'s method [55], 143 test objects would have been diagnosed as healthy and 127 would have required further diagnosis by doctors. Using Wang et al.'s method [59], 171 objects would have been diagnosed as healthy, 11 would have had heart disease and 88 would have required further diagnosis by a doctor.

4.4.3. Comparative analysis with the regret behavior psychology based on 3WD

In the past two years, 3WD based on behavioral psychology have been gradually developed. In order to better explain the effectiveness and rationality of our constructed method, we should not only compare it with the general RT and the C3WD, more important is to compare with the method with both RT and 3WD. Therefore, we compare the proposed method with that of Wang et al. [15].[6] After determining the optimal parameters, the

[3] $\sigma = 0.2$, $\mathcal{P}r(\chi|[t_i]_{\Phi_{\succeq}}) = 0.6$.

[4] The parameters in ELECTRE-I method $\delta = 0.9$ and $p = 0.1$. Choose a pessimistic strategy.

[5] Confidence level $\sigma = 0.5$.

[6] $\theta = 0.15$, $\delta = 0.1$.

Table 22
Comparison of different methods.

Different methods	Risk attitude	Conditional probability	Outcome or loss functions	Parameter number	Ranking	Classification
Our method	✓	*Objective*	*Objective*	3	✓	✓
Bell's method [28]	×	×	×	2	✓	×
Liu et al.'s method [14]	×	×	×	2	✓	×
Jia et al.'s method [39]	×	*Subjective*	*Objective*	1	✓	✓
Zhan et al.'s method [55]	×	*Objective*	*Subjective*	2	✓	✓
Wang et al.'s method [59]	×	*Objective*	*Subjective*	1	✓	✓
Wang et al.'s method [15]	✓	*Objective*	*Subjective*	2	✓	✓

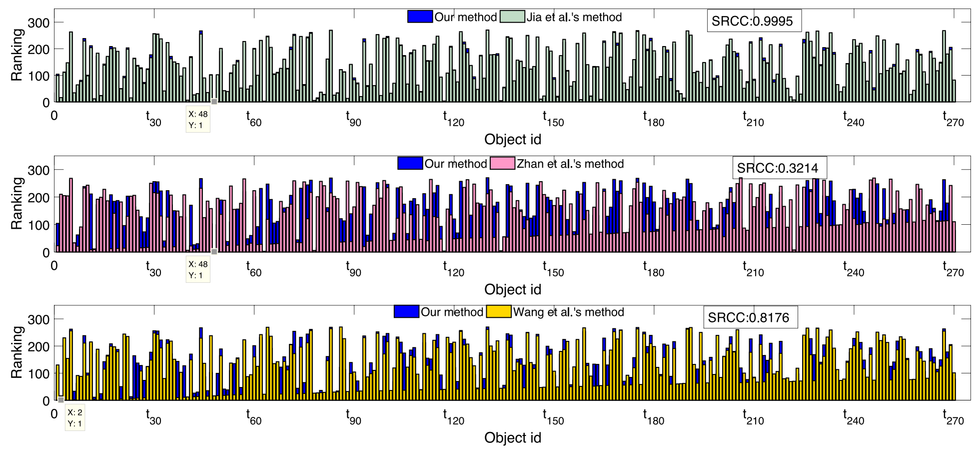

Fig. 9. Comparison of the ranking results obtained with the C3WD method.

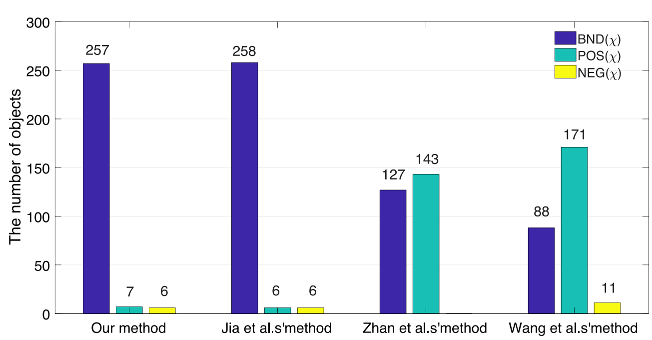

Fig. 10. Comparison of the clustering results obtained with the C3WD method.

comparison results of ranking and clustering are shown in Figs. 11 and 12, respectively.

Remark 4.8. According to the method of selecting optimal parameters in the data set "Statlog(Heart)" mentioned in Section 4.2, when the risk aversion coefficient $\theta = 0.15$ and the regret aversion coefficient $\delta = 0.1$ in Wang et al.'s method [15], the lowest CE $= 16.00\%$, the highest RI $= 84.00\%$ and F1 $= 91.30\%$ can be obtained simultaneously.

Fig. 11 exhibits the SRCC between our method and Wang et al.'s one is 0.9935 and the optimal object is consistent, both of them take regret psychology into account to classify and rank objects. The clustering results in Fig. 12 show that under Wang et al.'s method, 25 objects are healthy and 245 objects require further diagnosis and treatment.

Remark 4.9. Figs. 8, 9 and 11 show the comparison of ranking results between the 3W-MADM-R method and other decision making methods, which can only reflect the principle of MADM problems, that is, the results of the proposed method are close to the verified method and the optimal object is consistent. The advantages of the proposed method such as better stability and better clustering effect can be detailed in Sections 5.2 and 5.3.

5. Discussion

In the previous section, we have demonstrated the ranking or clustering results of the designed method and other methods. Meanwhile, we show that our method satisfies the principle of MADM methods, that is, the ranking results are close to those of the tested method under the premise of keeping the optimal object consistent. In what follows, we will exhibit the similarities and differences between the proposed approach and the classic methods and sum up the advantages of our method.

5.1. The similarities and differences between the proposed method and other MADM ones

It can be clearly seen from Table 22:

• The similarities and differences between our method and the general RT methods

(1) Our method, Bell's method [28] and Liu et al.'s method [14] add RT to decision making problems to compensate for the "limited rationality" of DMs. In particular, in the field of medical decision making, due to the complexity of the human body, the practical nature of medicine and the individual nature of diseases, the ability of clinicians is limited in some cases and subjective judgments can be misdiagnosed. Therefore, it is necessary to apply the behavioral psychology of regret to medical decision making.

(2) The difference is that we apply the 3WD method to triple-classify the objects based on the ranking, which can improve the efficiency of diagnosis.

• The similarities and differences between our method and the C3WD method

(1) The calculation of conditional probabilities is objectively obtained by defining new superorder classes or dominance classes, just as Zhan et al.'s idea [55] and Wang et al.'s idea [59].

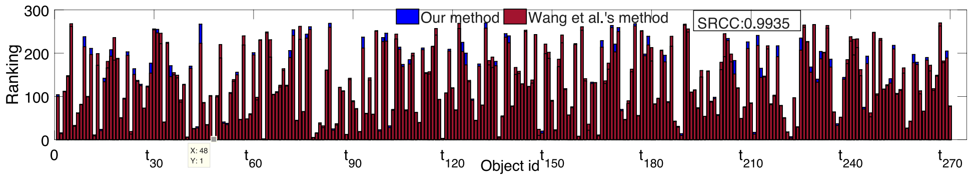

Fig. 11. Comparison of the ranking results obtained with the regret behavior psychology based on 3WD.

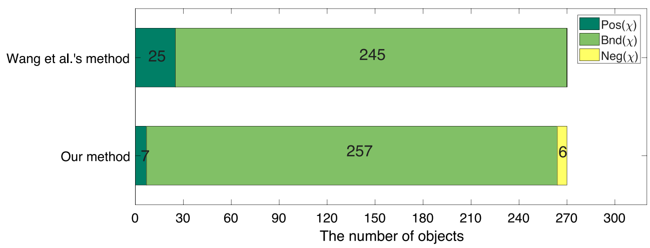

Fig. 12. Comparison of the clustering results obtained with the regret behavior psychology based on 3WD.

Outcome functions are consistent with Jia and Liu's idea [39], which is objectively constructed by using "relative" idea.

(2) Compared with C3WD methods, we use the 3WD method as a tool to make up the deficiency of sorting only in MADM problems. The difference is that our method adds RT to reflect the irrational psychology of people in complex circumstances. Meanwhile, Jia and Liu [39] subjectively gave the conditional probability to the object, whereas we define a new objective relation. The methods proposed in [55,59] subjectively designed the loss functions. This paper constructs the relative outcome functions of different objects under different attributes based on the attribute evaluation value of the object itself.

• The similarities and differences between our method and the regret behavior psychology based on 3WD methods

(1) The method designed in this paper is consistent with the 3WD method and RT adopted by [15], and both of us use objective methods to calculate the conditional probability.

(2) The difference lies in the different application background proposed firstly. We apply in fuzzy environments, whereas Wang et al. applied in interval type-2 fuzzy environments. Secondly, the 3W-MADM-R method constructs the outcome functions for each object in two states, the method in [15] is to construct the interval type-2 fuzzy outcome matrix according to the two states.

5.2. The advantages of our method on ranking

Section 5.1 discusses the similarities and differences between our method and other existing methods in various aspects. The 3W-MADM-R method integrates the highlights of the above methods, thus this section will analyze the advantages of our proposed method from both qualitative and quantitative perspectives, and summarize as follows.

5.2.1. Qualitative analysis

(1) In reality, individuals often make irrational decisions. However, the C3WD method does not consider a DM's risk preference attitude, and only constructs an intellectualized model that uses loss functions for domain segmentation and sorting. Our method considers a DM's psychological behavior under the uncertain environment, and describes the risk attitude from both the point of view of acquisition and loss. Based on this, our method has more advantages in application when solving practical problems. Fig. 13 shows the effect of RT on 3WD.

We randomly generate attribute evaluation values for 100 objects and 500 objects. Set $\zeta = 0.7$, $\theta = 0.3$, δ increases in the range from 0.1 to 0.9 with a step size of 0.1, and the conditional probability of each object is assumed to be 0.5. It can be seen that the positive and negative domains gradually become smaller with the value of the parameter δ, whereas the boundary domain gradually becomes larger. This suggests that regret aversion increases and a DM needs to gather more information to make a decision. Nevertheless, the 3WD method without RT has the same number of objects in the three domains, which cannot reflect the psychological changes of a DM.

(2) Compared with the defects of the method proposed by Bell [28] and Liu et al. [14], which can only do ranking, our method can not only select the optimal object, but also divide all objects into three domains to speed up the decision making process and reduce the waste of resources. If a DM is looking for an exact solution, he/she should directly look for it from the positive domain $Pos(\chi)$ and the negative domain $Neg(\chi)$; if a DM wants to make a further judgment on the indecisive objects, he/she should focus on the objects in the boundary domain $Bnd(\chi)$. Consequently, compared to the above two methods, our method has stronger superiority and practicability.

(3) In addition, there is no effective calculation method and reasonable semantic interpretation of the conditional probability in the existing 3WD works. In Jia and Liu's method [39], the conditional probabilities are subjectively assigned and the conditional probabilities of each object in the same state are equal, which can cause significant errors in the decision making process. Our method is objective and reasonable in terms of constructing outranking relation to calculate conditional probabilities. From this point of view, our method is superior to the one [39].

(4) In Zhan et al.'s method [55] and Wang et al.'s method [59], the loss functions are both too subjective given by DMs. Furthermore, the amount of data processed by their methods is too small to explain the effectiveness of the method. Wang et al.'s method [15] uses the outcome matrix, but it is also subjectively assigned and does not have the original information table to explain. Wang et al.'s method [15] directly gives the final 3×2 outcome matrix, which is not conducive to a DM's implementation. However, our method starts from the attribute values of the original information table, and each object constructs an objective relative outcome function. From this perspective, our approach is more realistic.

5.2.2. Quantitative analysis

In this section, we will use different methods to quantitatively evaluate the ranking results and clustering performance of the proposed method.

5.2.2.1. SRCC. SRCC is used to examine the degree of direct correlation between two variables, with a value between −1 and +1. The larger the absolute value of the correlation coefficient is, the stronger the correlation is.

In general, the correlation coefficient is between 0.8–1.0, indicating that the variables are extremely strongly correlated. The correlation coefficient is between 0.6–0.8, indicating strong correlation between variables. The correlation coefficient is between

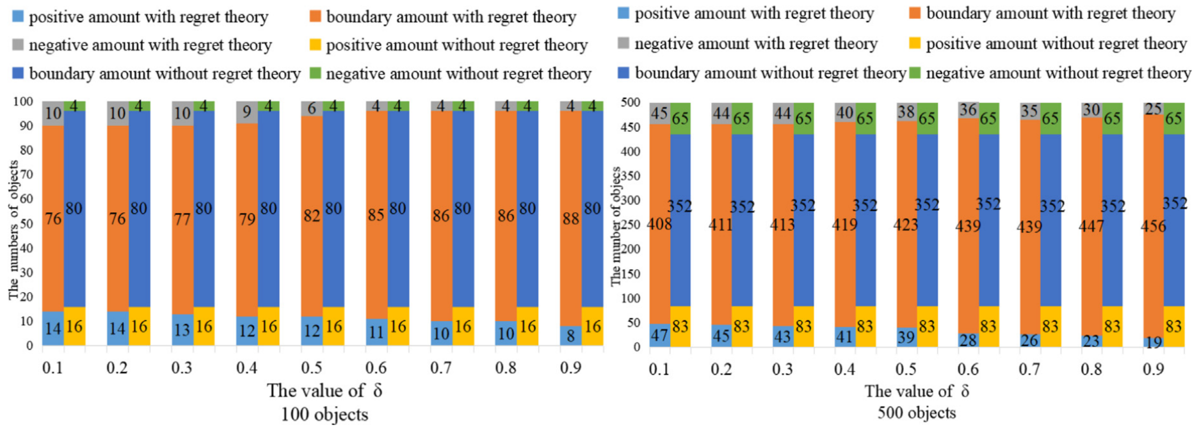

Fig. 13. Comparison results of two objects groups between 3W-MADM-R and C3WD.

Table 23
The SRCCs between different methods.

	Our method	TOPSIS	Bell	Liu et al.	Jia and Liu	Zhan et al.	Wang et al. [59]	Wang et al. [15]
Our method	1.0000	0.9883	0.7991	0.7990	0.9995	0.3214	0.8176	0.9935
TOPSIS [12]		1.0000	0.7133	0.7129	0.9912	0.2897	0.7853	0.9927
Bell [28]			1.0000	0.9997	0.7521	0.2935	0.8172	0.8041
Liu et al. [14]				1.0000	0.7520	0.3001	0.8170	0.8041
Jia and Liu [39]					1.0000	0.3306	0.8117	0.9954
Zhan et al. [55]						1.0000	0.3854	0.3047
Wang et al. [59]							1.0000	0.8143
Wang et al. [15]								1.0000

0.4–0.6, indicating that the variables are moderately correlated. The correlation coefficient is between 0.2 and 0.4, indicating that the variables are weakly correlated. The correlation coefficient is between 0.0–0.2, indicating a very weak correlation between variables. SRCC is used to test the correlation between different ranking methods, which has been verified in [55,59].

Figs. 7, 8, 9 and 11 all only superficially observe the similarity of sorting results between the proposed method and other methods. Table 23 uses the SRCCs between our method and the ranking results of other decision making methods in the paper to quantify the similarity.

Remark 5.1. Table 23 indicates that our method is extremely strongly correlated with the TOPSIS method, which acts as a ranking benchmark, and also with the methods of Jia and Liu [39], Wang et al. [59] and Wang et al. [15]; it is strongly correlated with the methods of Bell [28] and Liu et al. [14]; it is weakly correlated with the methods of Zhan et al. [55]. This implies that the 3W-MADM-R method is consistent with the basic principles of MADM problems.

5.2.2.2. Ordered similarity degree.

Definition 5.1 (*[60]*). Assume that $Q = l_1, l_2, \dots, l_m$ is an object set, an optimal ranking alternative $M : l_{i1} \succ \cdots \succ l_{ig} \succ \cdots \succ l_{if} \succ \cdots \succ l_{im}$ is obtained by a ranking decision making method, where $l_{i1}, \dots, l_{ig}, \dots, l_{if}, \dots, l_{im}$ is an m-level permutation. For any two objects l_{ig} and l_{if}, the relation $l_{ig} \succ l_{if}$ is called an ordered relation. Therefore, the number of ordered relations of M is $(m-1)+(m-2)+\cdots+2+1 = \frac{m(m-1)}{2}$.

A family of objects $A = Q_1, Q_2, \dots, Q_k$, where $Q_1 \subseteq Q_2 \subseteq \cdots \subseteq Q_k$. Suppose that the number of ordered relations of $Q_t (t = 1, 2, \dots, k)$ is c. If there are b $(b \le c)$ same ordered relations of Q_t and Q_h $(h \le t$ and $h = 1, 2, \dots, k)$, then the ranking similarity between Q_t and Q_h is $\frac{b}{c}$ (see Table 24).

Remark 5.2. According to Definition 5.1, we divide the data set "Statlog (Heart)" into ten groups $Q_1, Q_2, \dots, Q_{10}$. From Table 21, the ordered similarity degrees of Q_1 and Q_2, Q_1 and Q_3, Q_1 and Q_4, Q_1 and Q_5, Q_1 and Q_6, Q_1 and Q_7, Q_1 and Q_8, Q_1 and Q_9, Q_1 and Q_{10} can be calculated as 98.77%, 98.46%, 98.77%, 98.77%, 98.77%, 98.46%, 98.46%, 98.46%, 98.46%, respectively. When the number of objects increases gradually, the number of order pairs with the same dominance relationship among the ranking methods proposed by us does not decrease significantly. In this way, it reflects the stability of the 3W-MADM-R method in evaluating the rankings.

Inspired by Zhang et al. [60], we define the ordered similarity degree of objects under different methods.

Definition 5.2. Assume that $Q = l_1, l_2, \dots, l_m$ is an object set, an optimal ranking alternative $M : l_{i1} \succ \cdots \succ l_{ig} \succ \cdots \succ l_{if} \succ \cdots \succ l_{im}$ is obtained by a ranking decision making method, where $l_{i1}, \dots, l_{ig}, \dots, l_{if}, \dots, l_{im}$ is an m-level permutation. For any two objects l_{ig} and l_{if}, the relation $l_{ig} \succ l_{if}$ is called an ordered relation. Therefore, the number of ordered relations of M is $(m-1)+(m-2)+\cdots+2+1 = \frac{m(m-1)}{2}$.

The ranking result sets of all objects under different methods are $G_1, G_2, \cdots, G_k$. Suppose that the number of ordered relations of $G_t (t = 1, 2, \dots, k)$ is f. If there are e $(e \le f)$ same ordered relations of G_t and G_h $(h \le t$ and $h = 1, 2, \dots, k)$, then the ordered similarity degree between G_t and G_h is $\frac{e}{f}$.

Remark 5.3. The ranking result set obtained by our proposed method is denoted as G_1, the methods [12,14,28,39,55,59], and in [15] are denoted as $G_2, \dots, G_8$, respectively. According to Definition 5.2, the ordered similarity degrees of G_1 and G_2, G_1 and G_3, G_1 and G_4, G_1 and G_5, G_1 and G_6, G_1 and G_7, G_1 and G_8 can be calculated as 95.30%, 84.15%, 84.08%, 98.53%, 54.87%, 82.34%, 96.67% respectively. On the whole, our method has a great ordered similarity degree with most decision making methods.

Table 24
The ranking results of objects in Q_1 on different object sets.

Different object sets	Ranking of objects in Q_1
$Q_1 = \{t_1, t_2, \ldots, t_{27}\}$	$t_{12} \succ t_2 \succ t_{23} \succ t_{14} \succ t_6 \succ t_{20} \succ t_7 \succ t_{27} \succ t_8 \succ t_{21} \succ t_{10} \succ t_1 \succ t_3 \succ t_{26} \succ t_{15} \succ t_{25} \succ t_{24} \succ t_4 \succ t_{16} \succ t_{13} \succ t_{19} \succ t_{22} \succ t_{17} \succ t_{11} \succ t_{18} \succ t_9 \succ t_5$
$Q_2 = \{t_1, t_2, \ldots, t_{54}\}$	$t_{12} \succ t_2 \succ t_{23} \succ t_{14} \succ t_6 \succ t_{20} \succ t_7 \succ t_{27} \succ t_8 \succ t_{21} \succ t_{10} \succ t_1 \succ t_3 \succ t_{26} \succ t_{15} \succ t_{25} \succ t_4 \succ t_{24} \succ t_{16} \succ t_{13} \succ t_{18} \succ t_{19} \succ t_{22} \succ t_{17} \succ t_{11} \succ t_9 \succ t_5$
$Q_3 = \{t_1, t_2, \ldots, t_{81}\}$	$t_{12} \succ t_2 \succ t_{23} \succ t_{14} \succ t_6 \succ t_{20} \succ t_7 \succ t_{27} \succ t_8 \succ t_{21} \succ t_{10} \succ t_1 \succ t_3 \succ t_{26} \succ t_{25} \succ t_{15} \succ t_4 \succ t_{24} \succ t_{16} \succ t_{13} \succ t_{18} \succ t_{19} \succ t_{22} \succ t_{17} \succ t_{11} \succ t_9 \succ t_5$
$Q_4 = \{t_1, t_2, \ldots, t_{108}\}$	$t_{12} \succ t_2 \succ t_{23} \succ t_{14} \succ t_6 \succ t_{20} \succ t_7 \succ t_{27} \succ t_8 \succ t_{21} \succ t_{10} \succ t_1 \succ t_3 \succ t_{26} \succ t_{25} \succ t_{15} \succ t_4 \succ t_{24} \succ t_{16} \succ t_{13} \succ t_{18} \succ t_{19} \succ t_{22} \succ t_{17} \succ t_{11} \succ t_9 \succ t_5$
$Q_5 = \{t_1, t_2, \ldots, t_{135}\}$	$t_{12} \succ t_2 \succ t_{23} \succ t_{14} \succ t_6 \succ t_{20} \succ t_7 \succ t_{27} \succ t_8 \succ t_{21} \succ t_{10} \succ t_1 \succ t_3 \succ t_{26} \succ t_{25} \succ t_{15} \succ t_4 \succ t_{24} \succ t_{16} \succ t_{13} \succ t_{18} \succ t_{19} \succ t_{22} \succ t_{17} \succ t_{11} \succ t_9 \succ t_5$
$Q_6 = \{t_1, t_2, \ldots, t_{162}\}$	$t_{12} \succ t_2 \succ t_{23} \succ t_{14} \succ t_6 \succ t_{20} \succ t_7 \succ t_{27} \succ t_8 \succ t_{21} \succ t_{10} \succ t_1 \succ t_3 \succ t_{26} \succ t_{25} \succ t_{15} \succ t_4 \succ t_{24} \succ t_{16} \succ t_{13} \succ t_{18} \succ t_{19} \succ t_{22} \succ t_{17} \succ t_{11} \succ t_9 \succ t_5$
$Q_7 = \{t_1, t_2, \ldots, t_{189}\}$	$t_{12} \succ t_{23} \succ t_2 \succ t_{14} \succ t_6 \succ t_{20} \succ t_7 \succ t_{27} \succ t_8 \succ t_{21} \succ t_{10} \succ t_1 \succ t_3 \succ t_{26} \succ t_{25} \succ t_{15} \succ t_4 \succ t_{24} \succ t_{16} \succ t_{13} \succ t_{18} \succ t_{19} \succ t_{22} \succ t_{17} \succ t_{11} \succ t_9 \succ t_5$
$Q_8 = \{t_1, t_2, \ldots, t_{216}\}$	$t_{12} \succ t_{23} \succ t_2 \succ t_{14} \succ t_6 \succ t_{20} \succ t_7 \succ t_{27} \succ t_8 \succ t_{21} \succ t_{10} \succ t_1 \succ t_3 \succ t_{26} \succ t_{25} \succ t_{15} \succ t_4 \succ t_{24} \succ t_{16} \succ t_{13} \succ t_{18} \succ t_{19} \succ t_{22} \succ t_{17} \succ t_{11} \succ t_9 \succ t_5$
$Q_9 = \{t_1, t_2, \ldots, t_{243}\}$	$t_{12} \succ t_{23} \succ t_2 \succ t_{14} \succ t_6 \succ t_{20} \succ t_7 \succ t_{27} \succ t_8 \succ t_{21} \succ t_{10} \succ t_1 \succ t_3 \succ t_{26} \succ t_{25} \succ t_{15} \succ t_4 \succ t_{24} \succ t_{16} \succ t_{13} \succ t_{18} \succ t_{19} \succ t_{22} \succ t_{17} \succ t_{11} \succ t_9 \succ t_5$
$Q_{10} = \{t_1, t_2, \ldots, t_{270}\}$	$t_{12} \succ t_{23} \succ t_2 \succ t_{14} \succ t_6 \succ t_{20} \succ t_7 \succ t_{27} \succ t_8 \succ t_{21} \succ t_{10} \succ t_1 \succ t_3 \succ t_{26} \succ t_{25} \succ t_{15} \succ t_4 \succ t_{24} \succ t_{16} \succ t_{13} \succ t_{18} \succ t_{19} \succ t_{22} \succ t_{17} \succ t_{11} \succ t_9 \succ t_5$

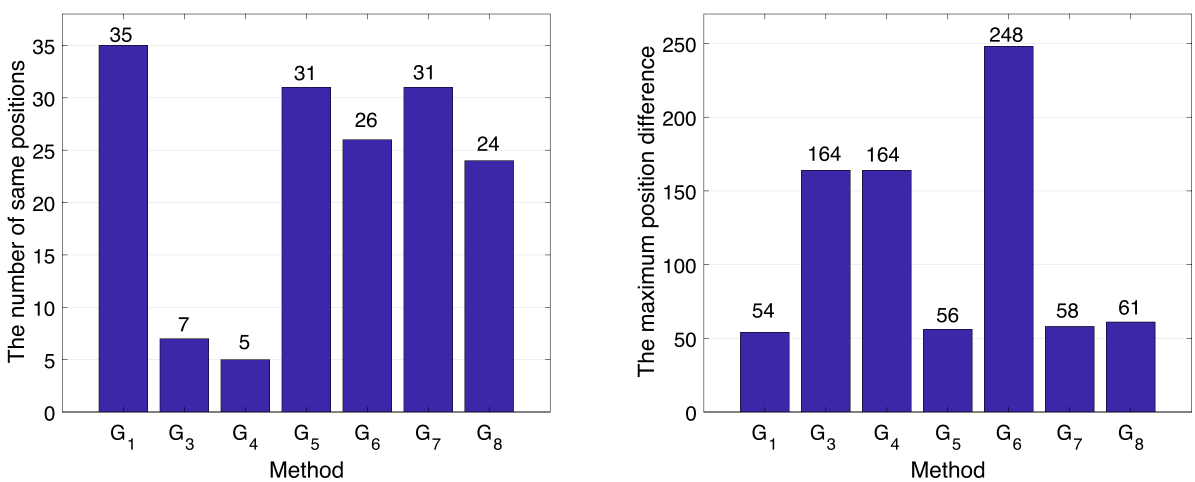

Fig. 14. The number of difference between the same position and the maximum position under different methods.

5.2.2.3. Position difference analysis. As shown in Fig. 12, in order to identify the differences between difficult objects in the data set in a more detailed way, by using the TOPSIS method as the ranking benchmark, we calculate the number of the objects with the same position for each method and the TOPSIS method, and exactly analyze which objects' positions are the same for the proposed method and the TOPSIS method in Remark 5.4. In addition, we calculate the objects with the largest differences between the sorting results of each method and the TOPSIS method and the number of differences, explaining them in detail in Remark 5.5. The comparison results of the data set "Statlog (Heart)" are shown in Fig. 14:

Remark 5.4. Our proposed method has a higher number of identical positions compared to other methods. In the data set "Statlog (Heart)", there are 35 objects with exactly the same ranking position as the TOPSIS method, namely, t_2, t_8, t_{14}, t_{22}, t_{30}, t_{33}, t_{48}, t_{58}, t_{63}, t_{70}, t_{79}, t_{80}, t_{92}, t_{106}, t_{107}, t_{117}, t_{128}, t_{134}, t_{135}, t_{147}, t_{176}, t_{177}, t_{181}, t_{183}, t_{187}, t_{189}, t_{190}, t_{196}, t_{208}, t_{210}, t_{211}, t_{222}, t_{224}, t_{247}, t_{262}.

Remark 5.5. From Fig. 14, compared with the ranking results of the TOPSIS method, t_{44} has the maximum position difference calculated by our method, and the difference number is 54. Meanwhile, the objects with the largest position difference of the methods of [14,28,39,55,59], and [15] are t_{86}, t_{86}, t_{44}, t_{146}, t_{44}, t_{121}, the maximum position difference of the methods are 164, 164, 56, 248, 58 and 61, indicating that our method owns less variation in ranking floating. Our method has the same maximum position difference with other decision making methods under the same object, which shows that our method is still persuasive in finding objects with the greatest location differences.

5.3. The advantages of our method on clustering

While achieving better results in ranking, the clustering results of the proposed method are compared with the "ground truth" (i.e., the clustering results obtain by decision attributes). We take five metrics to verify the clustering effectiveness of the other 3WD methods.

These five indicators include CE,RI, and F1 as expressed in Eqs. (26)–(28) mentioned in Section 4.2. At the same time, the

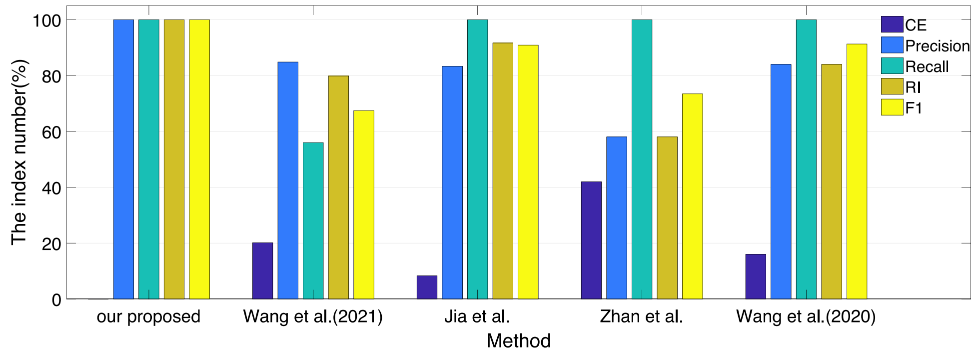

Fig. 15. Indicators under different methods.

Table 25
The SRCCs between ranking results when values of ζ change.

$\theta = 0.5$, $\delta = 0.9$	$\zeta = 0.1$	$\zeta = 0.2$	$\zeta = 0.3$	$\zeta = 0.4$	$\zeta = 0.5$	$\zeta = 0.6$	$\zeta = 0.7$	$\zeta = 0.8$	$\zeta = 0.9$
$\zeta = 0.1$	1.0000	1.0000	1.0000	0.9999	0.9999	0.9998	0.9998	0.9997	0.9997
$\zeta = 0.2$		1.0000	1.0000	1.0000	0.9999	0.9999	0.9999	0.9998	0.9998
$\zeta = 0.3$			1.0000	1.0000	1.0000	0.9999	0.9999	0.9998	0.9998
$\zeta = 0.4$				1.0000	1.0000	1.0000	0.9999	0.9999	0.9999
$\zeta = 0.5$					1.0000	1.0000	1.0000	0.9999	0.9999
$\zeta = 0.6$						1.0000	1.0000	1.0000	0.9999
$\zeta = 0.7$							1.0000	1.0000	1.0000
$\zeta = 0.8$								1.0000	1.0000
$\zeta = 0.9$									1.0000

clustering index Precision and Recall [15,58] are also added and the expressions are as follows:

$$\text{Precision} = \frac{n_{\chi \to Pos(\chi)}}{n_{\chi \to Pos(\chi)} + n_{\neg\chi \to Pos(\chi)}} \times 100\%, \tag{29}$$

$$\text{Recall} = \frac{n_{\chi \to Pos(\chi)}}{n_{\chi \to Pos(\chi)} + n_{\chi \to Neg(\chi)}} \times 100\%, \tag{30}$$

Remark 5.6. In all comparison methods, the optimal parameters are selected according to the lowest CE value, the highest RI and F1 values, and then performed calculations to obtain the clustering results. It can be seen from Fig. 15 that the 5 indexes of 3W-MADM-R method are 0%, 100%,100%, 100% and 100% respectively after clustering according to the same standard, which is the best clustering effect among the methods involved.

6. Experimental evaluations

In this section, we conduct some experiments based on the instance in Section 4 to evaluate the performance capability of our proposed method. Based on the above discussions, it turns out that the utility pursuit coefficient ζ, the risk aversion coefficient θ and the regret aversion coefficient δ play a crucial role in the 3W-MADM-R method. Among them, ζ exists in the relative outcome functions of the 3WD model and controls whether the final clustering is 2WD or 3WD; the coefficients θ and δ exist in RT and reflect the mental attitude of DMs when making decisions. Therefore, the parameter ζ is separately considered, θ and δ are comprehensively taken into account, and are divided into two groups for sensitivity analysis. Then, we observe the changes of the ranking result and clustering by using the method of the controlled variables.

6.1. Sensitivity analysis of the parameter ζ

Section 6.1 discusses the influence of the change of the parameter ζ involved in this paper on the results of object ranking and clustering. For 270 patients, we have experimented with 9 different values of ζ from 0.1 to 0.9 in steps of 0.1 with the parameters in RT set to the optimal parameters of 0.5 and 0.9 explored in Remark 4.2.

6.1.1. Sensitivity analysis of the parameter ζ on ranking

As can be seen in Fig. 16, the value of the parameter ζ is changing, and the 48th position of the optimal target remains unchanged, indicating that the 3W-MADM-R method is consistent with the principles of the MADM problem and is not affected by the parameters, and the ranking results are almost the same for the 9 subplots as a whole without any change.

Remark 6.1. Table 25 shows SRCCs between different ranking results when the parameters θ and δ remain constant and the parameter ζ increases gradually. The data in the table more clearly reflect the stability of sorted objects when the parameters change.

Remark 6.2. Even so, we explore the subtle ranking changes of the objects in Fig. 17. By using the TOPSIS method as the reference standard for ranking, the change of the parameter ζ has a slight effect on the number of identical positions and the maximum position difference, meanwhile the object with the maximum number of position differences is still 44th. Combined with Fig. 16 and Table 25, it can be seen that the proposed method has good stability, that is, parameter changes hardly change the optimal object and the overall ranking result.

6.1.2. Sensitivity analysis of the parameter ζ on clustering

This section analyzes the influence of the parameter ζ-value changes on the clustering results of the proposed method. The clustering results and 5 cluster evaluation indicators when the parameter ζ changes are shown in Fig. 18.

Remark 6.3. According to the first subgraph of Fig. 18, we can find that with the value of ζ increases, the $Pos(\chi)$ and $Neg(\chi)$ get smaller, and the $Bnd(\chi)$ gets larger. When the utility pursuit coefficient is less than 0.5, a DM makes a deterministic

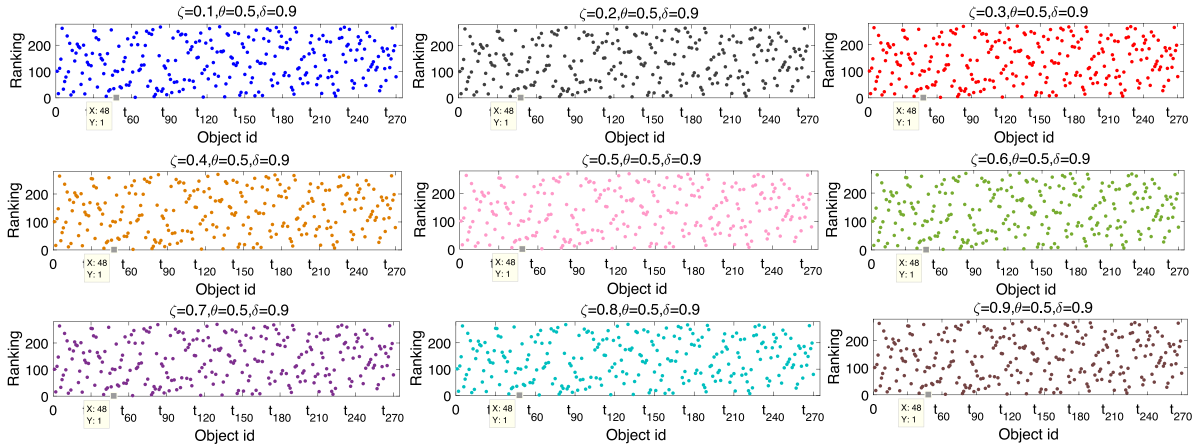

Fig. 16. The ranking results of different values of ζ.

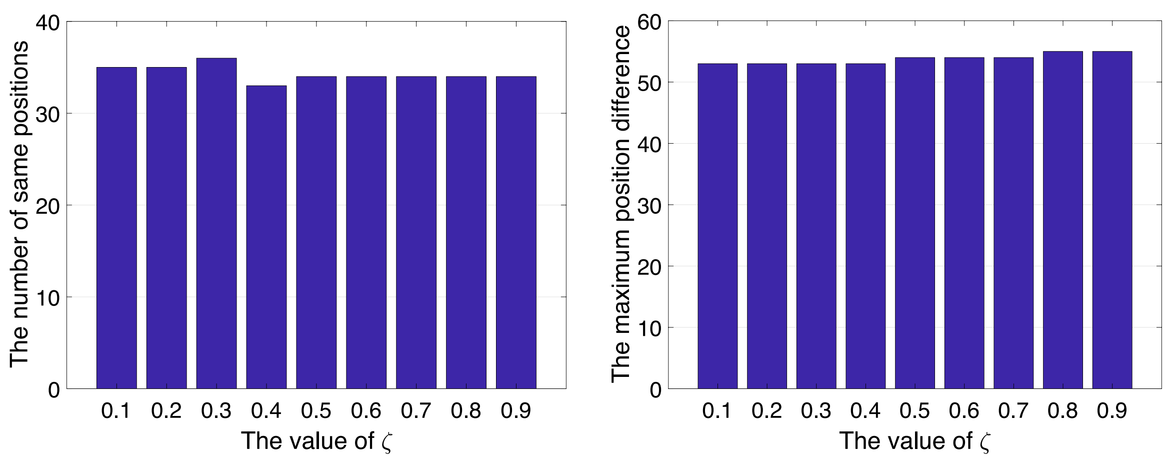

Fig. 17. The number of same position and maximum position under different values of ζ.

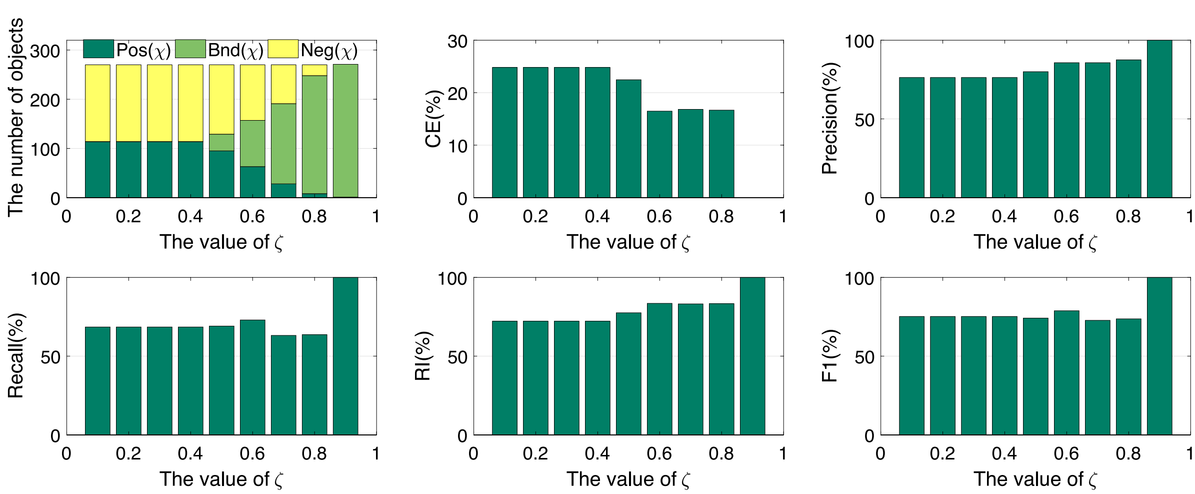

Fig. 18. Classification results and indexes under different values of ζ.

decision; when pursuing utility, a DM makes more and more uncertain decisions and his/her hesitant psychology gradually emerges. Meanwhile, with the change of the ζ value, the CE gradually decreases from 24.81% to 0.00%. Precision gradually increases from 76.32% to 100%. Recall, RI and F1 remain stable at 68.50%–100%, 72.20%–100% and 72.73%–100%, respectively.

The change of parameters representing the preference of a DM will affect the final clustering result, but the clustering index always floats within a reasonable range, which also reflects the subjectivity of the DM in MADM.

6.2. Sensitivity analysis of the parameters θ and δ

Section 6.2 discusses the influence of the change of the parameters θ and δ involved in this paper on the results of object ranking and clustering. Our experiment takes the optimal utility

Table 26
The SRCCs between ranking results when values of θ and δ change.

$\zeta = 0.9$	$\theta = 0.1$ $\delta = 0.1$	$\theta = 0.1$ $\delta = 0.5$	$\theta = 0.1$ $\delta = 0.9$	$\theta = 0.5$ $\delta = 0.1$	$\theta = 0.5$ $\delta = 0.5$	$\theta = 0.5$ $\delta = 0.9$	$\theta = 0.9$ $\delta = 0.1$	$\theta = 0.9$ $\delta = 0.5$	$\theta = 0.9$ $\delta = 0.9$
$\theta = 0.1$, $\delta = 0.1$	1.0000	0.9999	0.9993	0.9994	0.9992	0.9984	0.9982	0.9976	0.9966
$\theta = 0.1$, $\delta = 0.5$		1.0000	0.9997	0.9998	0.9996	0.9990	0.9988	0.9983	0.9974
$\theta = 0.1$, $\delta = 0.9$			1.0000	1.0000	1.0000	0.9997	0.9996	0.9993	0.9987
$\theta = 0.5$, $\delta = 0.1$				1.0000	0.9999	0.9996	0.9995	0.9991	0.9985
$\theta = 0.5$, $\delta = 0.5$					1.0000	0.9998	0.9997	0.9994	0.9988
$\theta = 0.5$, $\delta = 0.9$						1.0000	1.0000	0.9999	0.9995
$\theta = 0.9$, $\delta = 0.1$							1.0000	0.9999	0.9996
$\theta = 0.9$, $\delta = 0.5$								1.0000	0.9998
$\theta = 0.9$, $\delta = 0.9$									1.0000

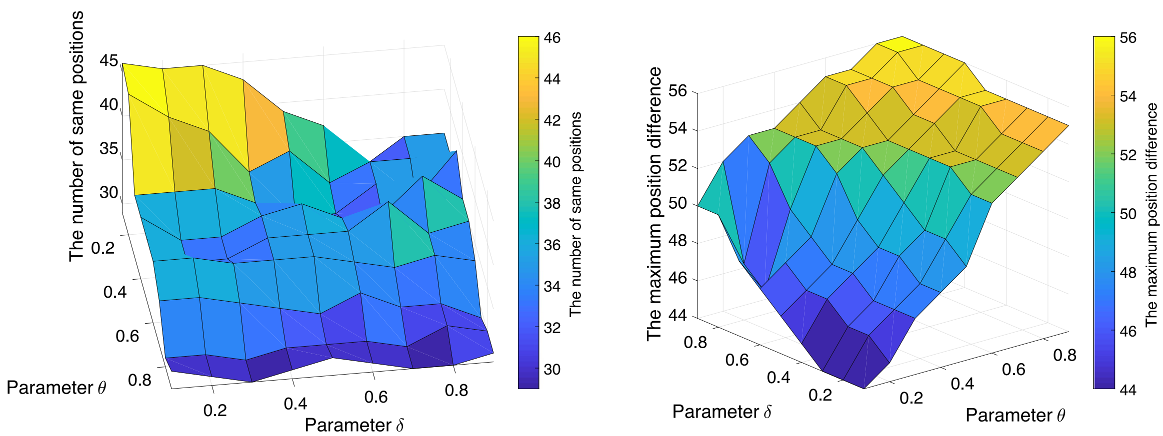

Fig. 19. The number of same position and maximum position under different values of θ and δ.

pursuit coefficient $\zeta = 0.9$ unchanged, and the values of the parameters θ and δ change from 0.1 to 0.9 with a step size of 0.1, respectively.

6.2.1. Sensitivity analysis of the parameters θ and δ on ranking

For a fixed parameter ζ, the parameters θ and δ change simultaneously, SRCCs between partial ranking results is shown in Table 26.

Remark 6.4. It can be observed from the values in Table 26 that the ranking results under different parameters have extremely strong correlation, that is to say, the change of the parameters θ and δ values will not have a great impact on the overall ranking results. At the same time, the ranking results obtained by each set of different parameters show that the optimal object in the data set "Statlog (Heart)" is still t_{48}, which indicates that our proposed method adheres to the optimal consistency of the MADM problem.

When the parameters θ and δ values change, the same number of positions and the maximum number of position differences obtained by the 3W-MADM-R method compared with the TOPSIS method are shown in Fig. 19.

Remark 6.5. Fig. 19 explores the effect of changes in parameter values on individuals. It can be seen that the number of objects with the same number of positions fluctuates between 29 and 46; the maximum position difference in ranking fluctuates between 44 and 56. It is normal and reasonable for parameters change to cause the position of an individual object to vary within a certain range.

6.2.2. Sensitivity analysis of the parameters θ and δ on clustering

This section analyzes the influence of the parameters θ and δ value change on the clustering results of the proposed method. When the parameters θ and δ change, the clustering results are shown in Fig. 20. In addition, the evaluation index of clustering performance when the parameters change is described in Remark 6.7.

Remark 6.6. According to Fig. 20, the increments of θ and δ will decrease the number of objects in the positive and negative domains, while the number of objects in the boundary domain will increase. It also implies that the values of the parameters θ and δ need to be reduced if a DM wants more deterministic decisions.

Remark 6.7. Meanwhile, the five indexes mentioned above have little change. Among them, the CE decreases with the increase of the values of the parameters θ and δ, Precision varies from 66.67%–100%, Recall remains at 100%, RI varies from 85.71%–100%, and F1 varies from 80%–100%. In conjunction with Fig. 19, when analyzing the test subjects one by one, different parameters do result in a slightly different order of position of individuals and changes in clustering results and clustering performance. Therefore, it is essential to select the optimal parameters and carry out a comparative analysis.

7. Data set experiments

In this part, we will conduct two experiments to evaluate the performance of our proposed method. The data sets of our experiments are downloaded from the machine learning data repository, UCI (http://archive.ics.uci.edu/). The detailed description of the experimental data sets is shown in Table 27 and Remark 7.1. The parameters are the same as those in the case. All attributes in the two data sets are reserved and all are beneficial attributes. Meanwhile, our experiences are achieved by using MATLAB R2018b on a personal computer with Microsoft Windows 10, Intel (R) Core (TM) i5-9400U CPU @ 2.90 GHz and 8.00 GB memory.

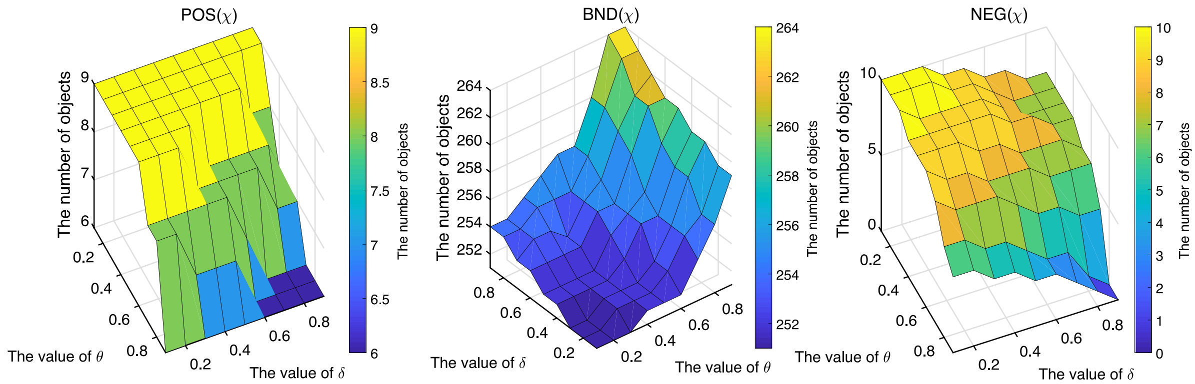

Fig. 20. The clustering results of different values of θ and δ.

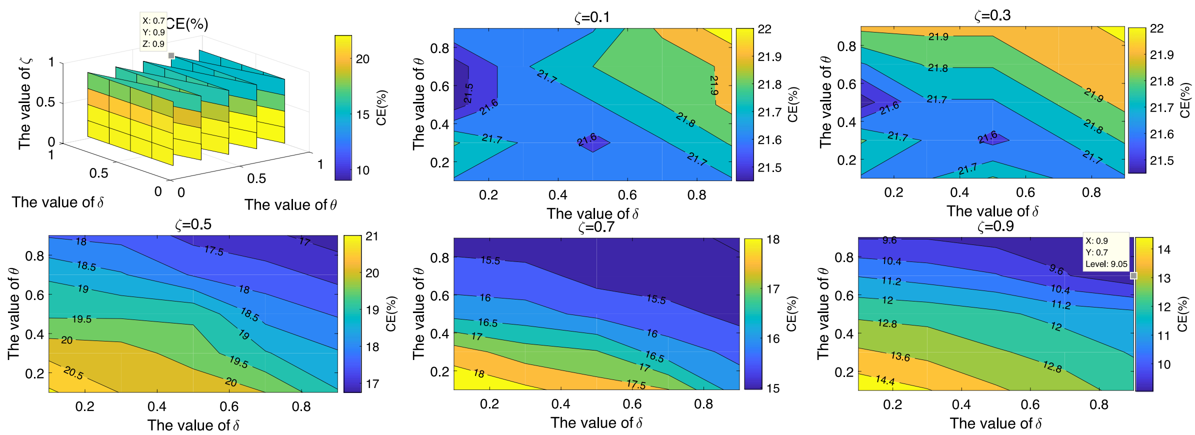

Fig. 21. CE index under three parameters change in the data set "Mammographic Mass".

Table 27
The description of the data sets.

Data sets	Alternatives	Attributes	$\chi : \neg\chi$
Mammographic Mass	961	$5 + 1$	403 : 427
Cervical Cancer Behavior Risk	72	$19 + 1$	51 : 21

Remark 7.1. The 5 attributes in the data set "Mammographic Mass" are BI-RAD assessment, age, shape, margin and density, respectively, and all of them are benefit attributes. In addition, this data set contains missing values, but our method cannot deal with missing values. Therefore, the number of subjects actually participating in the experiment is 830. The 19 benefit attributes in the data set "Cervical Cancer Behavior Risk" are behavior sexual risk, behavior eating, behavior personal Hygiene intention aggregation, intention commitment, attitude consistency, attitude spontaneity, norm fulfillment, perception vulnerability, perception severity, motivation strength, motivation willingness, norm significant person, social support emotionality, social support, appreciation, social support instrumental, empowerment knowledge, empowerment abilities and empowerment desires.

7.1. The optimal parameter selection of data set experiments

In the experiment of supplementary data set, the method of selecting optimal parameters in Section 4.2 is also utilized. The experiment finds that when the parameters ζ, θ and δ all change, SRCCs of our method on the two data sets and the ranking results of the TOPSIS method always remain above 99.5%. Therefore, the optimal parameters are also studied according to the principle that the lower the CE is, the better the RI and F1 are. The results are shown in Figs. 21–26.

Remark 7.2. From Figs. 21–23, in the data set "Mammographic Mass", when $\zeta = 0.9$, $\theta = 0.7$ and $\delta = 0.9$, the lowest error rate is 2.17%, the highest RI is 91.95%, and the highest F1 is 81.19%. At this point, $\zeta = 0.9$, $\theta = 0.7$ and $\delta = 0.9$ are the optimal parameters of the data set. From Figs. 24–26, in the data set "Cervical Cancer Behavior Risk", when $\zeta \in (0.7, 0.9)$, $\theta \in (0.3, 0.9)$ and $\delta \in (0.5, 0.9)$, the error rate = 0%, RI = 100%, F1 = 100%. At this point, $\zeta \in (0.7, 0.9)$, $\theta \in (0.3, 0.9)$ and $\delta \in (0.5, 0.9)$ are the optimal parameters of the data set.

Remark 7.3. Through the exploration in Sections 4.2 and 7.1, it can be found that in MADM problems, the optimal parameters of different data sets are not consistent or the optimal parameters of the same data set cannot be uniquely determined. This is reasonable and normal. As long as the optimal object is consistent, the parameter selection is subjective, which is convenient for DMs to regulate their risk attitudes.

7.2. Data set experiments on ranking

From the analysis in Section 4.2 and Algorithm 1, in the data set "Mammographic Mass", we select the parameters $\zeta = 0.9$, $\theta = 0.7$ and $\delta = 0.9$. In the data set "Cervical Cancer Behavior Risk", given that the optimal parameter is a range of values, we select the average value instead of the deterministic value. That is, $\zeta = 0.8$, $\theta = 0.6$ and $\delta = 0.7$.

Due to the large amount of data in data set "Mammographic Mass", its results are poorly presented. Therefore, taking data set "Cervical Cancer Behavior Risk" as an example, the ranking and comparison results of different methods are shown in Fig. 27, which shows that the ranking of objects in the supplementary

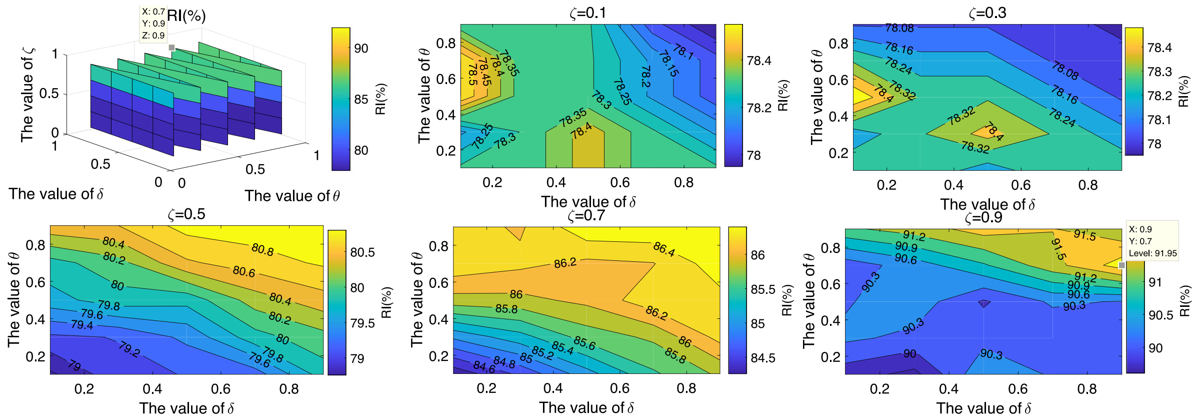

Fig. 22. RI index under three parameter changes in the data set "Mammographic Mass".

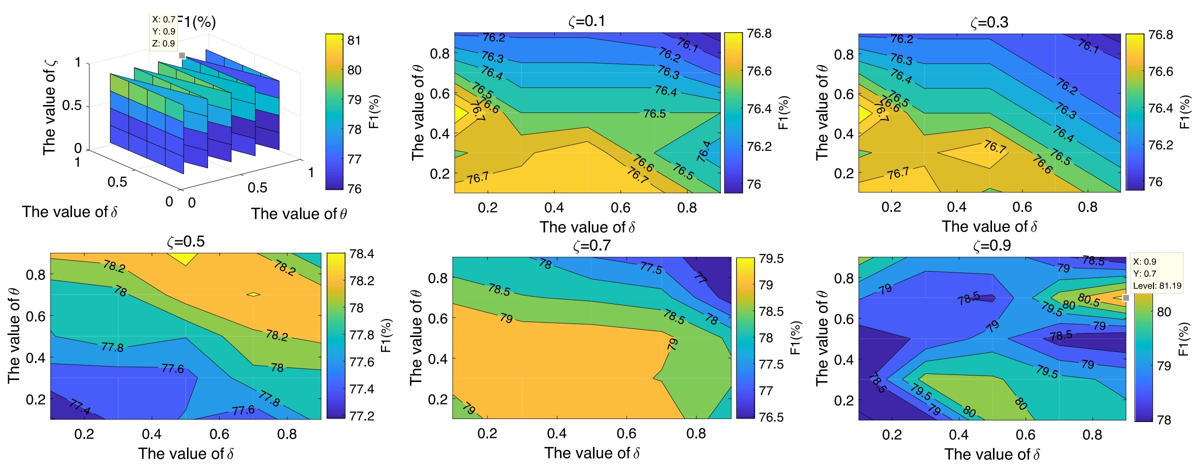

Fig. 23. F1 index under three parameters change in the data set "Mammographic Mass".

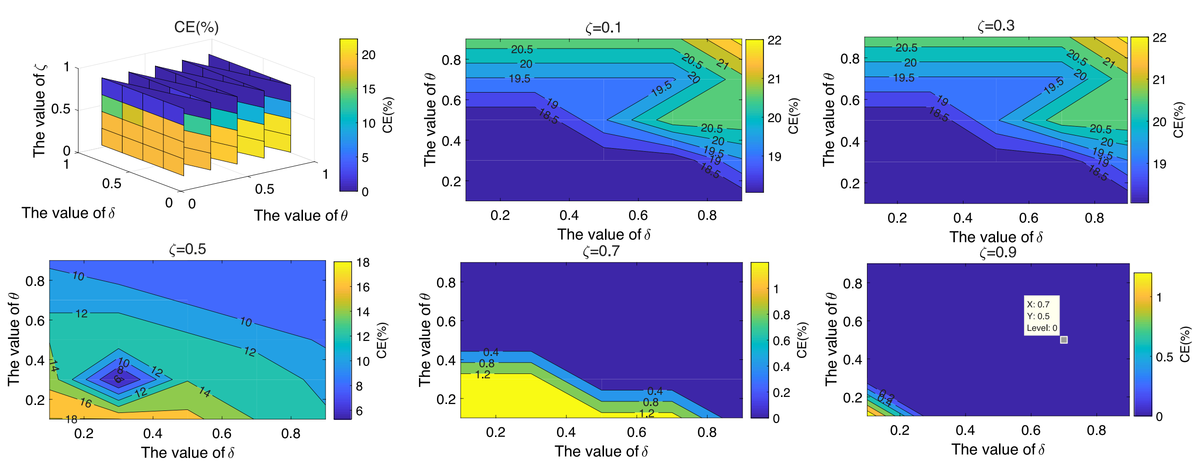

Fig. 24. CE index under three parameters change in the data set "Cervical Cancer Behavior Risk".

dataset roughly matches under different decision methods, and the optimal objects remain largely consistent. This suggests that our method is equally applicable to other datasets.

Remark 7.4. For other decision-making methods, the same method is adopted to select the optimal parameters. In data set "Mammographic Mass", the optimal parameters of Jia and Liu's method [39] are $\sigma = 0.2$, $\mathcal{P}r(\chi|[t_i]_{\Phi_{\succeq}}) = 0.6$, the optimal parameters of Zhan et al.'s method [55] are $\delta = 0.7$ and $p = 0.3$, the optimal parameter of Wang et al.'s method [59] is $\sigma = 0.6$, the optimal parameters of Wang et al.'s method [15] are $\theta = 0.1$, $\delta = 0.2$. In data set "Cervical Cancer Behavior Risk", the optimal parameters of Jia and Liu's method [39] are $\sigma \in (0.2, 0.4)$, $\mathcal{P}r(\chi|[t_i]_{\Phi_{\succeq}}) \in (0.3, 0.5)$, the optimal parameters of Zhan et al.'s method [55] are $\delta = 0.6$ and $p = 0.25$, the optimal parameter of Wang et al.'s method [59] is $\sigma = 0.6$, the optimal parameters of Wang et al.'s method [15] are $\theta = 0.25$, $\delta = 0.3$.

In order to test the indistinguishability of intermediate objects in the ranking results of supplementary data sets and verify the ranking performance of this method, SRCCs are added to

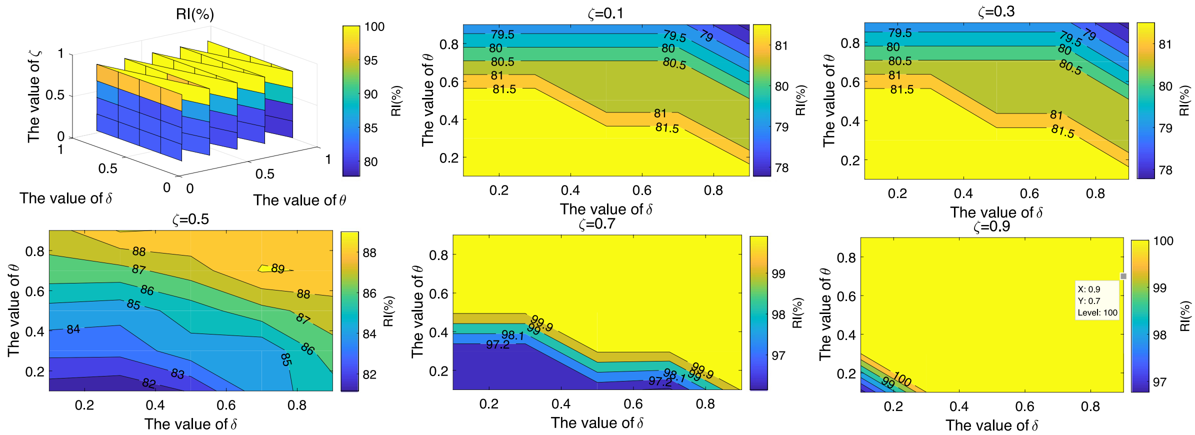

Fig. 25. RI index under three parameters change in the data set "Cervical Cancer Behavior Risk".

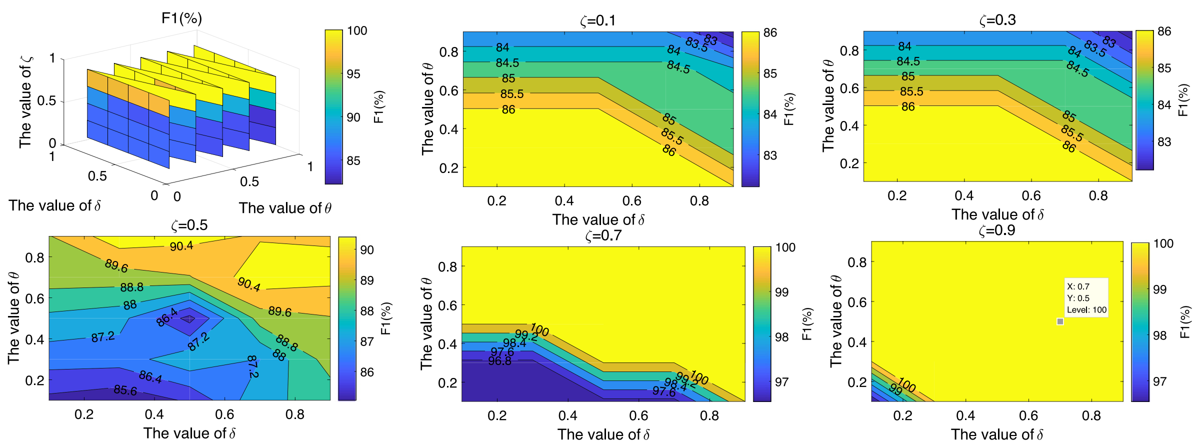

Fig. 26. F1 index under three parameters change in the data set "Cervical Cancer Behavior Risk".

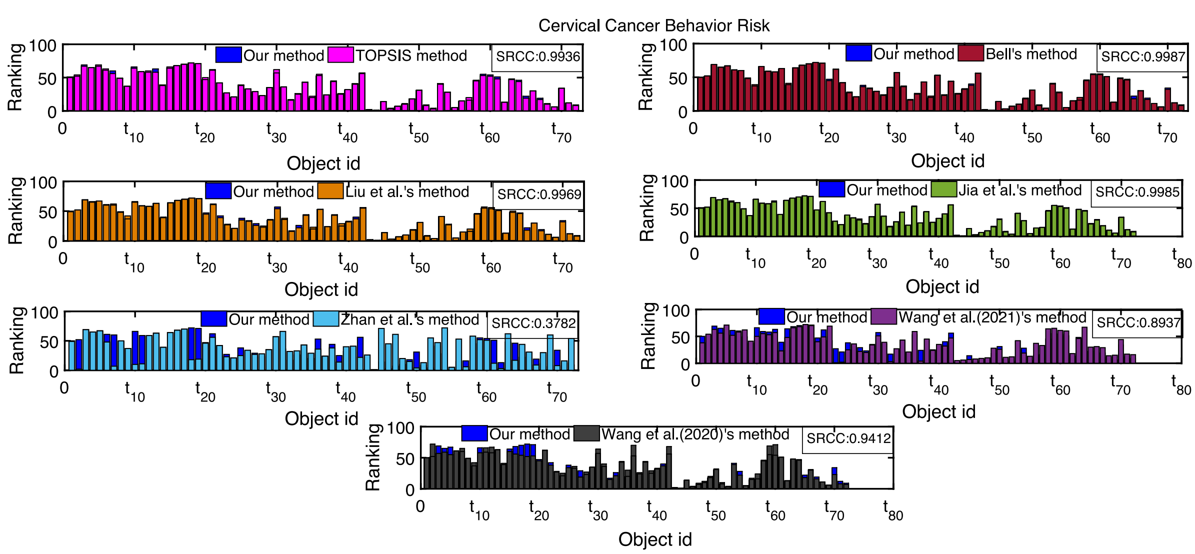

Fig. 27. Comparison of ranking results between different methods and our method in the data set "Cervical Cancer Behavior Risk".

measure the correlation between the ranking results and other decision making methods. This coefficient can reflect the overall comparison between the sorting of objects and the ground truth value, and the specific results are shown in Tables 28 and 29.

Remark 7.5. According to previous studies, when the SRCC between the two decision making methods is greater than 0.8, it indicates that the two methods have strong similarities in ranking performance. Based on the results of SRCCs obtained from the three data sets, the 3W-MADM-R method has a strong similarity with most existing decision making methods, which shows the effectiveness and rationality of the method proposed in this paper.

Table 28
The SRCCs comparison of different methods in the data set "Mammographic Mass".

	Our method	TOPSIS	Bell	Liu et al.	Jia and Liu	Zhan et al.	Wang et al. [59]	Wang et al. [15]
Our method	1.0000	0.9969	0.9998	0.9996	0.9978	0.1874	0.7818	0.9965
TOPSIS [12]		1.0000	0.9976	0.9977	0.9994	0.1854	0.7753	0.9933
Bell [28]			1.0000	0.9999	0.9993	0.1875	0.7764	0.9948
Liu et al. [14]				1.0000	0.9992	0.1867	0.7763	0.9954
Jia and Liu [39]					1.0000	0.1865	0.7751	0.9939
Zhan et al. [55]						1.0000	0.0867	0.1886
Wang et al. [59]							1.0000	0.7822
Wang et al. [15]								1.0000

Table 29
The SRCCs comparison of different methods in the data set "Cervical Cancer Behavior Risk".

	Our method	TOPSIS	Bell	Liu et al.	Jia and Liu	Zhan et al.	Wang et al. [59]	Wang et al. [15]
Our method	1.0000	0.9936	0.9987	0.9969	0.9985	0.3782	0.8937	0.9412
TOPSIS [12]		1.0000	0.9912	0.9886	0.9938	0.3573	0.7823	0.9368
Bell [28]			1.0000	0.9986	0.9978	0.3662	0.8957	0.9514
Liu et al. [14]				1.0000	0.9954	0.3658	0.8958	0.9521
Jia and Liu [39]					1.0000	0.3794	0.8821	0.6032
Zhan et al. [55]						1.0000	0.3672	0.9487
Wang et al. [59]							1.0000	0.3383
Wang et al. [15]								1.0000

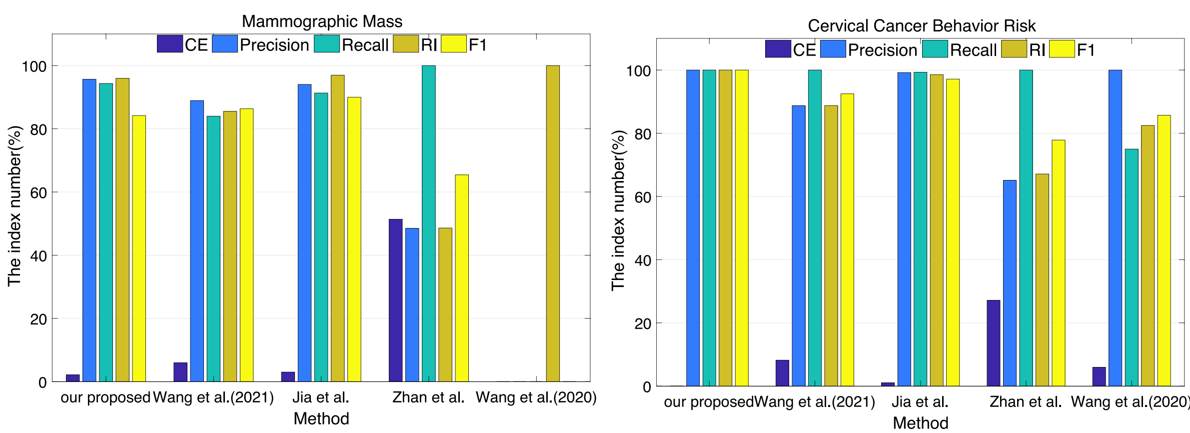

Fig. 28. Comparison of indexes between different methods and our method under two supplementary data sets.

7.3. Data set experiments on clustering

On this basis, the clustering results of the proposed method are compared with the "ground truth" (i.e., the clustering results obtain by decision attributes). We use the following categorical indicators: CE, Precision, Recall, RI, F1, as shown in Fig. 28. Fig. 28 represents the clustering performance evaluation of the two sets of data when all decision-making methods select the optimal parameter.

Remark 7.6. With the data set "Mammographic Mass", although a few methods have slightly better partial indices than those with 3W-MADM-R, the proposed method performs better overall at five cluster indices. Similarly, in the data set "Cervical Cancer Behavior Risk", the indicators under our method are in the best state.

8. Conclusions

In recent years, the researches on MADM problems and 3WD models have become the mainstream. On this basis, we have combined RT with the traditional PROMETHEE-II method to construct a new 3W-MADM-R method to solve the decision making problems with behavioral psychology in real life. The effectiveness and superiority of our proposed method have been verified by an illustrative example and comparative analysis of other methods. In addition, in order to demonstrate the stability and feasibility of the 3W-MADM-R method, parameter analysis has been carried out and another three data sets have been added for further verification. The main contributions of this paper are summarized as follows:

(1) Based on the net-flow of the traditional PROMETHEE-II method, we have constructed a new outranking relation and objectively given the membership degree of the membership function to calculate the conditional probability, which solves the deficiency of the existing methods [38,39].

(2) Considering the uncertainty of the environment, we have used the combination of regret psychology to describe the risk attitude and preference of DMs. Meanwhile, utility pursuit coefficients, risk aversion and regret aversion coefficients give DMs great maneuverability.

(3) The 3WD model combined with RT [15] under existing MADM problems is the last outcome matrix given subjectively, without intermediate process. However, we have used the attribute evaluation value of the original information table to objectively calculate the relative outcome function of each scheme, which is more objective and reasonable.

(4) Analytic expressions of three thresholds have been calculated to simplify the decision rules.

(5) The proposed new 3W-MADM-R method can solve some practical problems, and is no longer confined to the construction

of small data cases [40,61]. Meanwhile, the theories of 3WD method and MADM problems have been further enriched.

Considering the complexity and diversity of representations of big data in practical problems, the following aspects are worth exploring in the future: (1) The proposed 3W-MADM-R method will be extended to different information environments, such as the applications of intuitionistic fuzzy numbers [62], hesitant fuzzy numbers [58] and incomplete environments [63]. (2) The analytical formula with the parameter ζ in the threshold can be solved and the sufficient and necessary conditions that satisfy the 3WD model can be studied. (3) We will combine the proposed method with the fields of dynamic 3WD [61], machine learning [22], and so forth.

CRediT authorship contribution statement

Jinxing Zhu: Conceptualization, Methodology, Investigation, Writing – original draft. **Xueling Ma:** Methodology, Writing – original draft. **Jianming Zhan:** Writing – review & editing. **Yiyu Yao:** Model construction, Writing – review & editing.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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