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A novel portfolio optimization model via combining multi-objective optimization and multi-attribute decision making

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Abstract

In order to solve the problem of portfolio optimization, this paper proposes a method that combines multi-objective optimization and multi-attribute decision-making to solve the dual-objective portfolio optimization model with conditional value-at-risk (CVaR) measuring risk and including transaction costs. First, in the multi-objective optimization stage, a multi-population parallel NSGA-II based on sparsity strategy (SMP-NSGA-II) is proposed to obtain multiple Pareto optimal solutions of the model. Second, in the multi-attribute decision-making stage, in order to reflect different investment preferences, the Pareto optimal set obtained is clustered through the fuzzy C-means, and then the grey relational projection method is used to evaluate the solutions belonging to the same cluster to select the optimal compromise solution. Finally, a case study of 9 semiconductor stocks in China's Shanghai and Shenzhen stock markets is carried out, and the optimal compromise portfolio under different investment preferences is given. At the same time, the proposed algorithm is compared with the other six multi-objective evolutionary algorithms (MOEAs), which verifies that the algorithm in this paper has certain competitiveness.

Keywords Portfolio optimization · Sparsity strategy · Multi-population parallel NSGA-II · Fuzzy C-means · Grey relational projection

1 Introduction

Nowadays, in the field of securities investment, the indicators used to quantify risk mainly include value-at-risk (VaR) and conditional value-at-risk (CVaR). Among them, VaR is represented by nonlinear, non-convex and non-differentiable function with multiple local optima, making it difficult to calculate. To solve these problems, Rockafellar et al. [1] introduced the CVaR, which is a coherent risk measure that considers risk as the most serious loss in a given scenario, taking into account a certain degree of confidence. Since CVaR is a convex function, it can effectively solve the optimization problem that uses CVaR as a minimization goal or constraint [2, 3]. At the same time, Yu et al. [4] compared five different risk models and verified through experiments that using CVaR to measure risk is a good choice.

As the complexity of practical applications continues to increase, scholars have developed various heuristic algorithms to solve portfolio optimization problems. The application of heuristic algorithms in portfolio optimization problems is divided into two categories. The first category simplifies portfolio objectives through the setting of weight coefficients [5–7], and obtains a risk-return curve by continuously changing the risk avoidance parameters of representative investors. This method has a certain degree of subjectivity. The second type uses multi-objective evolutionary algorithm (MOEA) to directly optimize risks and benefits simultaneously [8–11], and can obtain a complete effective frontier in one operation. Obviously, it is more convenient to use MOEAs to solve portfolio optimization problems.

It is not easy for the decision maker (DM) to identify the preferred portfolio by the direct use of MOEA to solve the problem of portfolio optimization, essentially because multiple goals are competing with each other (high returns are accompanied by high risks). In this way, we can only

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get a set of Pareto optimal solutions, which contains a large number of portfolio solutions. To solve this problem, there are three methods (a priori, a posteriori or interactively) in the search process that can be used to introduce DM preferences in the optimization process. At present, most of the work that introduces the decision process into the optimization problem is a priori method [12] and an interactive method [13], and the posterior decision method has been proved to be more subjective than the other two [14]. In addition, the search and decision process of the posterior decision method are separated, so when the DM preferences change, there is no need to repeat the optimization algorithm.

At the same time, some scholars use the method of multi-attribute decision-making to determine the optimal portfolio from limited portfolio schemes [15, 16]. In reality, investment portfolios have unlimited possibilities, so we can consider combining multi-objective optimization and multi-attribute decision-making to solve the problem of portfolio optimization. First, a limited Pareto optimal set is determined from an infinite portfolio through multi-objective optimization, and then multi-attribute decision-making is adopted as a posterior decision method for the Pareto optimal set for DM preferences, and finally the optimal compromise portfolio is determined. For securities investment, DM preferences can be roughly divided into risk-averse, risk-neutral and risk-seeking. Therefore, a clustering algorithm can be used to divide the Pareto optimal set to distinguish different DM preferences. Among them, the fuzzy C-means (FCM) clustering algorithm introduces the concept of membership to measure the degree of data objects belonging to different groups [17], and has the advantages of using a minimum number of fuzzy rules and minimum computational complexity. At the same time, the grey relational projection (GRP) method [18] is constructed by the combination of the grey relational method and the projection method. It has the advantages of both and has a good application in the field of multi-attribute decision-making.

Therefore, in the multi-attribute decision-making stage, this paper adopts the FCM-GRP hybrid method. At the same time, because the multi-attribute decision-making stage is a posterior decision-making method for the multi-objective optimization stage, the pros and cons of the final selected optimal compromise portfolio will depend on the pros and cons of the Pareto optimal set obtained in the multi-objective optimization stage. Therefore, in the multi-objective optimization stage, this paper proposes a multi-population parallel NSGA-II based on sparsity strategy (SMP-NSGA-II) to obtain a better Pareto optimal set.

Finally, a case study of 9 semiconductor stocks in China's Shanghai and Shenzhen stock markets was carried out for a dual-objective portfolio optimization model with CVaR as a risk measurement and including transaction costs, which verified the effectiveness of the improved algorithm and the feasibility of the multi-attribute decision-making method. At the same time, the optimal compromise portfolio under different DM preferences is given.

The rest of the paper is organized as follows. Section 2 introduces the mathematical model of the portfolio model. Section 3 introduces the method of model solving, that is, the method of combining multi-objective optimization and multi-attribute decision making. At the same time, several comparative MOEAs introduced to verify the effectiveness of the improved algorithm and evaluation indicators for comparing the performance of MOEA are introduced. In Section 4, the dual-objective portfolio optimization model with CVaR measuring risk and including transaction costs is set with different parameters to discuss its impact on the portfolio. At the same time, Section 4 introduces the comparison results of several MOEAs in the multi-objective optimization stage to solve the model, and applies the multi-attribute decision-making method to the Pareto optimal set obtained by the best-performing MOEA to give the optimal compromise portfolio under different DM preferences. Finally, Section 5 summarizes and discusses the full paper.

2 Model building

The problem of portfolio optimization is a multi-objective problem that maximizes returns and minimizes risks. In this section, we use CVaR [3] as the risk measurement method. The return is based on Markowitz's M-V model, which is weighted by multiplying the average return by the investment ratio. At the same time, in the real securities market, the transaction costs is unavoidable. Therefore, the transaction costs is considered in the expected return, and finally a dual-objective portfolio model including the transaction costs is constructed.

2.1 CVaR risk measurement method

In the field of securities investment, the current indicators used to quantify risk are mainly VaR and CVaR. In 1994, a risk measurement system based on VaR was proposed by Morgan J P Investment Bank, but VaR does not satisfy subadditivity and convexity, and it cannot describe the risk situation of securities investment well. To overcome the deficiencies of VaR, Rockfellar and Uryasev proposed Conditional Value-at-Risk (CVaR) [1] to measure risk. Let $\psi(\boldsymbol{\omega}, \boldsymbol{y}) : \boldsymbol{R}^n \times \boldsymbol{R}^n \rightarrow \boldsymbol{R}$ represent the loss function of the portfolio, where $\boldsymbol{\omega}$ is the asset decision vector, which represents the proportion of assets, $\boldsymbol{y}$ is a random vector of asset portfolio losses. Let $p(\boldsymbol{y})$ be the probability density function of vector $\boldsymbol{y}$, then for any $\alpha \in \boldsymbol{R}$, the expected loss distribution function of the portfolio is defined by

$$\varphi(\boldsymbol{\omega}, \alpha) = \int_{\psi(\boldsymbol{\omega}, \boldsymbol{y}) \leq \alpha} p(\boldsymbol{y}) d\boldsymbol{y} \tag{1}$$

According to the definition of VaR model and CVaR model, for any confidence level $\beta \in (0, 1)$, we can get

$$VaR_\beta = \min\{\alpha \in \boldsymbol{R} \,|\varphi(\boldsymbol{\omega}, \alpha) \geq \beta\} \tag{2}$$

$$\begin{aligned} CVaR_\beta &= VaR_\beta + E\left[\psi(\boldsymbol{\omega}, \boldsymbol{y}) - VaR_\beta \,|\psi(\boldsymbol{\omega}, \boldsymbol{y}) \geq VaR_\beta\right] \\ &= \frac{1}{1-\beta} \int_{\psi(\boldsymbol{\omega}, \boldsymbol{y}) \geq VaR_\beta} \psi(\boldsymbol{\omega}, \boldsymbol{y})\, p(\boldsymbol{y}) d\boldsymbol{y} \end{aligned} \tag{3}$$

It is not difficult to see from (3) that VaR must be used in the process of solving CVaR, and the mathematical expression given by (3) is very complicated and difficult to calculate. Therefore, Rockfellar and Uryasev effectively connect CVaR and VaR by constructing auxiliary function $F_\beta(\boldsymbol{\omega}, \alpha)$, the auxiliary function is

$$F_\beta(\boldsymbol{\omega}, \alpha) = \alpha + \frac{1}{1-\beta} \int_{\boldsymbol{y} \in \boldsymbol{R}^n} [\psi(\boldsymbol{\omega}, \boldsymbol{y}) - \alpha]^+ p(\boldsymbol{y})\, d\boldsymbol{y} \tag{4}$$

where $[\psi(\boldsymbol{\omega}, \boldsymbol{y}) - \alpha]^+$ is $\max\{\psi(\boldsymbol{\omega}, \boldsymbol{y}) - \alpha, 0\}$. At the same time, literature [1] proves that when the function $F_\beta(\boldsymbol{\omega}, \alpha)$ is the smallest, it is the value of $CVaR_\beta$, and the obtained α^* is the value of VaR, and the corresponding $\boldsymbol{\omega}^* = (\omega_1^*, \omega_2^*, \cdots, \omega_n^*)$ is the corresponding weight of each asset in the optimal portfolio. But at the same time, we can know from (4) that solving $F_\beta(\boldsymbol{\omega}, \alpha)$ requires knowing the expected loss distribution function of the portfolio. Due to the complexity of the market, the losses caused are also uncertain, and the probability density function $p(\boldsymbol{y})$ is not easy to calculate, so historical data is used to predict the distribution of random vectors in the future. For historical trading data of T trading days for n known securities, the corresponding approximation to $F_\beta(\boldsymbol{\omega}, \alpha)$ as follows

$$\tilde{F}_\beta(\boldsymbol{\omega}, \alpha) = \alpha + \frac{1}{T(1-\beta)} \sum_{t=1}^{T} \left[\psi\left(\boldsymbol{\omega}, \boldsymbol{y}_t\right) - \alpha\right]^+ \tag{5}$$

where $\psi(\boldsymbol{\omega}, \boldsymbol{y}_t) = -\sum_{i=1}^n y_{it}\omega_i$, y_{it} represents the return rate of the asset i on the trading day t. $(i = 1, 2, \ldots, n; t = 1, 2, \ldots, T)$

2.2 Dual-objective portfolio model

Suppose an investor purchases n securities, the proportion of asset i in portfolio $\boldsymbol{\omega} = (\omega_1, \omega_2, \cdots, \omega_n)$ is ω_i, and the expected return rate is $r_i = \frac{1}{T}\sum_{t=1}^T y_{it}$, then the expected return rate of the portfolio is $E = \sum_{i=1}^n r_i\omega_i$. However, in the real stock market, one cannot avoid the transaction costs such as stamp duty and commission that must be borne during the transaction process. Especially when the amount of funds is small, the dispersion of funds will increase transaction costs. Ignoring the impact of these transaction costs may result in an inefficient portfolio. Therefore, for the convenience of calculation, the transaction costs of all securities are uniformly set to c. And the dual-objective portfolio optimization model that contains transaction costs is defined as follows

$$\begin{cases} \min \tilde{F}_\beta(\boldsymbol{\omega}, \alpha) = \alpha + \frac{1}{T(1-\beta)} \sum\limits_{t=1}^{T} \left[\psi\left(\boldsymbol{\omega}, \boldsymbol{y}_t\right) - \alpha\right]^+ \\ \max E(\boldsymbol{\omega}) = \sum\limits_{i=1}^{n} r_i\omega_i - \sum\limits_{i=1}^{n} c\omega_i \\ s.t. \sum\limits_{i=1}^{n} \omega_i = 1, 0 \leq \omega_i \leq 1, i = 1, 2, \cdots, n \end{cases} \tag{6}$$

Among them, the loss function of n risk assets becomes $\psi(\boldsymbol{\omega}, \boldsymbol{y}_t) = \sum_{i=1}^n(-y_{it}\omega_i + c\omega_i)$, and $\omega_i \geq 0$ means short selling is not allowed.

3 Model solving

In order to give the optimal compromise portfolio under different DM preferences, this paper adopts the method of combining multi-objective optimization and multi-attribute decision-making. First, in the multi-objective optimization stage, the improved algorithm proposed in this paper is applied to obtain a portfolio set, which enters the multi-attribute decision-making stage as the scope of decision-making. In this stage, the FCM clustering algorithm is first used to cluster the portfolios within the decision-making range to distinguish different DM preferences, which are divided into risk-seeking portfolio collections, risk-averse portfolio collections, and risk-neutral portfolio collections. Then, the GRP method is used to evaluate each investment portfolio in the three portfolio sets, and the optimal compromise portfolio under different DM preferences is given. The specific process is shown in Fig. 1.

3.1 Multi-objective optimization

Compared with traditional techniques, MOEAs incorporate the concept of Pareto optimality and modified selection schemes to simultaneously evolve a set of solutions at multiple points along the trade-off surface, Such features provide the MOEAs with a global perspective of the multi-objective problem, as well as the capability of identifying a set of Pareto-optimal solutions in a single run. At the same time, literature [19] compares the computing power of several popular multi-objective evolutionary algorithms in solving portfolio optimization problems, and believes that nondominated sorting genetic algorithm II (NSGA-II) [20] and strength pareto evolutionary algorithm 2 (SPEA2) [21]

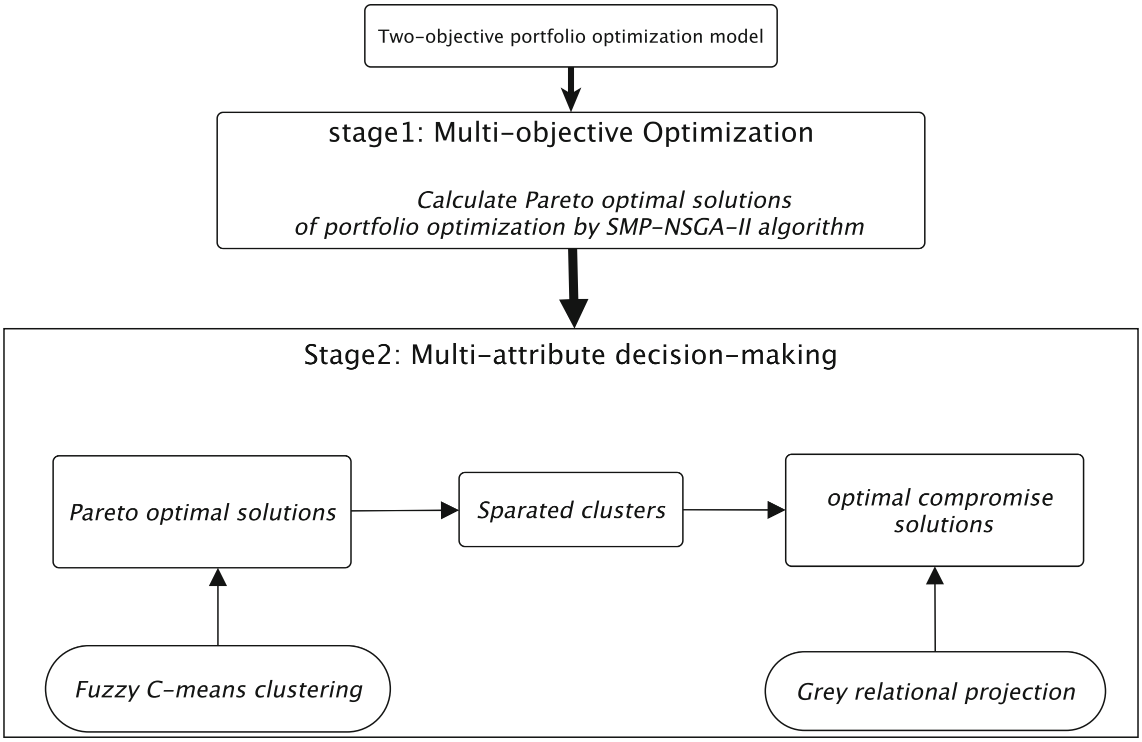

Fig. 1 Solution framework

have certain advantages in them. Based on NSGA-II, this paper proposes a multi-population parallel NSGA-II based on sparsity strategy, namely SMP-NSGA-II.

The normal distribution crossover (NDX) operator [22] improves the exploration and development capabilities of the solution space by introducing the normal distribution on the premise of solving accuracy, and uses the discrete reorganization operation to further expand the search space. If this operator is introduced into the NSGA-II, the global search performance and convergence of the algorithm can be improved, thereby obtaining a high-quality Pareto optimal solution set.

Therefore, the main ideas of the improved algorithm in this paper are as follows: First, a single NSGA-II population is decomposed into multiple subpopulations, the main population using the simulated binary crossover (SBX) operator and the auxiliary exploration population using the NDX operator are constructed to carry out a multi-population parallel strategy which can make the individuals close to the true Pareto front. Secondly, in order to promote the evolution of various groups, a parallel evolution cycle is set, and the non-dominated solutions obtained in each parallel evolution cycle are externally archived. Finally, the sparsity strategy is used to select the optimal solution set with a relatively uniform overall distribution from the candidate solution set to achieve the purpose of improving the distribution of the solution set.

3.1.1 Crossover operator

The crossover operator in real-coded NSGA-II is simulated binary crossover (SBX) [23] by

$$\begin{cases} V_{1,i}^{*} = 0.5\left[(1-\theta)\,V_{1,i} + (1+\theta)\,V_{2,i}\right] \\ V_{2,i}^{*} = 0.5\left[(1+\theta)\,V_{1,i} + (1-\theta)\,V_{2,i}\right] \end{cases}, 1 \leq i \leq m \quad (7)$$

where $V_{1,i}$ and $V_{2,i}$ are the i-th decision variables corresponding to the two parent individuals, $V_{1,i}^{*}$ and $V_{2,i}^{*}$ are the i-th decision variables corresponding to the two child individuals, m is the number of decision variables, and θ is a random variable, each decision variable i is regenerated as follows

$$\theta = \begin{cases} (2u)^{\frac{1}{1+\eta_c}}, u \leq 0.5 \\ (2\,(1-u))^{-\frac{1}{1+\eta_c}}, u > 0.5 \end{cases} \quad (8)$$

where u is a random value between (0, 1), η_c is the cross operators.

In order to enhance the algorithm's ability to search for a feasible solution space, Zhang et al. [22] introduced the normal distribution and the discrete recombination operation in the evolution strategy into the SBX operation, and proposed the normal distribution cross (NDX) operator:

$$\begin{cases} V_{1/2,i}^{*} = \frac{V_{1,i}+V_{2,i}}{2} \pm A\,|N\,(0,1)|\,\frac{V_{1,i}-V_{2,i}}{2}, u \leq 0.5 \\ V_{1/2,i}^{*} = \frac{V_{1,i}+V_{2,i}}{2} \mp A\,|N\,(0,1)|\,\frac{V_{1,i}-V_{2,i}}{2}, u > 0.5 \end{cases} \quad (9)$$

Among them, $N(0, 1)$ represents a normally distributed random variable, A is the exploration coefficient. Literature

[22] gives an experimental comparison between the SBX operator and the NDX operator. It can be seen that due to the introduction of normal distribution and discrete recombination, the NDX operator has a stronger ability to explore feasible solution spaces than the SBX operator. Therefore, by using the NDX operator, the algorithm can further expand the search range of the feasible solution space and make the searched solution space wider.

3.1.2 Multi-population parallel evolution strategy

Based on the idea of a parallel evolution strategy for multiple populations, SMP-NSGA-II set up two populations (total number of individuals is N) instead of a single population of the original population size of N. The two populations use different evolution strategies to search in the feasible solution space in parallel. Among them, the subpopulation 1 is the main population, adopts the SBX operator, and carries out certain explorations while taking into account exploitation, and the number of individuals is τN, where $\tau \in (0, 1)$ is population size division coefficient. At the same time, it can be seen from Section 3.1.1 that the NDX operator can improve the algorithm's ability to search for a feasible solution space, so in order to expand the searchable space of the algorithm and avoid premature convergence, this paper sets an subpopulation 2 with $(1 - \tau)N$ individuals as auxiliary exploration population. At the same time, in order to promote the evolution of each subpopulation, this paper assumes that each parallel evolution cycle is g generation, that is, after each parallel evolution cycle, the subpopulations are merged, and the optimal N individuals are selected through elitism approach. Then the initial population is assigned to the subpopulation again, and the next generation of parallel evolutionary cycles is searched, and repeated until the maximum generation number is reached.

3.1.3 Sparsity strategy

During evolution, each individual in the population has two attributes, that is

a. non-domination rank (i_{rank}) and b. crowding distance ($i_{distance}$).

The crowded-comparison operator relies on the above two attributes to guide the selection process to a uniformly spread-out Pareto optimal front at different stages of the algorithm. That is, between two individuals with differing non-domination ranks, we prefer the individual with the lower rank (smaller i_{rank} value). Otherwise, if two individuals belong to the same front, then we prefer the individual that is located in a lesser crowded region (higher $i_{distance}$ value).

Generally speaking, for a population with sufficient evolution, the individuals in the late evolution stage are almost all located in the first non-dominated front $F_1(i_{rank} = 1)$, that is, the uniformity of the final solution set depends on the uniformity of the individuals in F_1. When selecting N uniformly distributed individuals from $|F_1|$ individuals, regions with sparsity solution set distributions cannot be uniformly distributed by adding individuals, but for regions with dense solution set distributions, individuals can be reduced, so we can consider increasing the number of candidate solutions.

It can be seen from Section 3.1.2 that every time a parallel evolutionary cycle needs to merge subpopulations, an external archive set is established at this time to store individuals in the first non-dominated front of the merged population, and when the maximum generation is reached, the external archive set is re-executed non-dominated sorting process, from which the individuals in the first non-dominated front after deduplication are selected as candidate solutions (because these individuals may be retained during evolution, so there is duplication).

In order to see the effect of the sparsity strategy intuitively, here is the classic dual-objective test function ZDT4 [24] as an example. Figure 2 shows the distribution of individuals in the candidate solution set after 500 iterations. Then, the candidate solution set is processed by the sparsity strategy, and then a uniformly distributed solution set is obtained. The sparsity strategy is as follows:

Let the number of individuals in the candidate solution set be N'. First sort the candidate solution set, calculate the Euclidean distance between the two consecutive candidate solution sets dx_i, and then accumulate to get

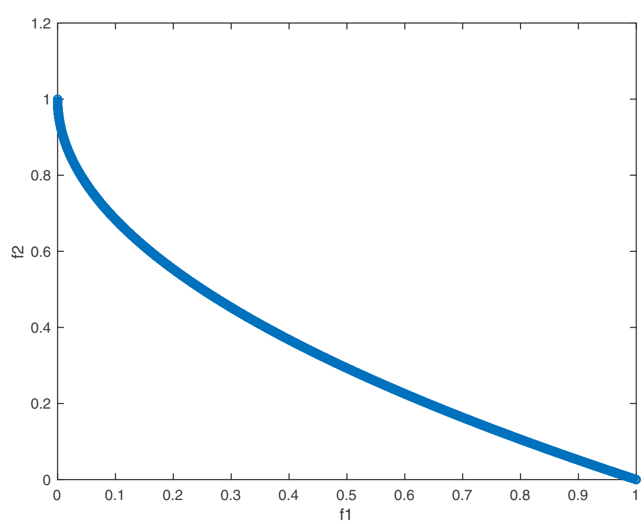

Fig. 2 The distribution of individuals in the candidate solution set of ZDT4 problem

$Dx = \sum_{i=1}^{N'-1} dx_i$. Since there are many individuals in the candidate solution set, if N uniformly distributed individuals are selected from this time, the distance between each individual is approximately $d_{int} = \frac{Dx}{N-1}$, and the first and last individuals in the candidate solution set are taken as the head and end individuals of the final solution set, then $N-2$ individuals in the middle are distributed in the interval with the length of $d_{area} = \frac{Dx}{N-2}$.

Then, starting from the first interval, using the greedy strategy, traverse all the individuals in the interval, and keep the individuals with the closest distance to d_{int} from the previous one in this interval, and delete the rest individuals, then traverse all the intervals in turn. Finally, the remaining individuals are used as the final solution set. Figures 3 and 4 respectively show the individual distributions obtained by using two different methods for the candidate solution set in Fig. 2: the elitism strategy and the sparsity strategy. It can be seen that a well-distributed solution set is obtained through the sparsity strategy.

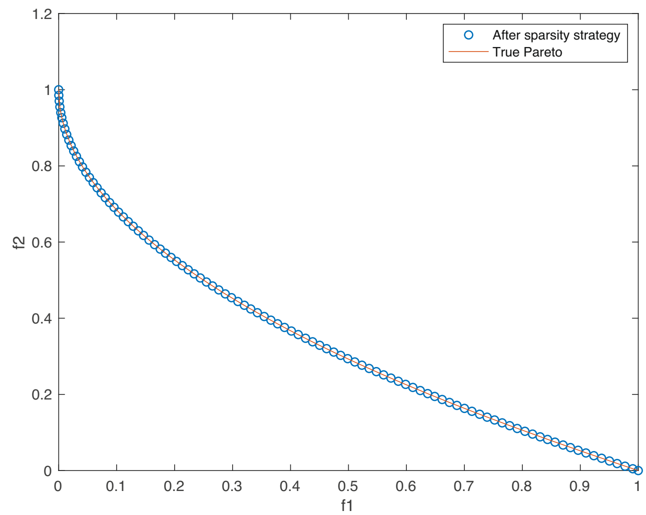

Fig. 4 The distribution of individuals obtained by the sparsity strategy

3.1.4 Algorithm flow

Through the above introduction, we proposed the SMP-NSGA-II, and then applied it to the multi-objective optimization stage of the portfolio optimization model solution, where the maximum evolutionary generation number of the population be gen, and each parallel evolutionary cycle be g generations. The specific steps as follows:

Step 1: Initialize the population, using real coded, for any chromosome $\boldsymbol{V}_k = (\boldsymbol{X}_k, \alpha)$, where $\boldsymbol{X}_k = (x_1, x_2, \cdots, x_i)$, $k = 1, 2, \cdots, N$, x_i $(1 \le i \le n, \sum_{i=1}^{n} x_i = 1)$ is a random value generated in [0, 1], which represents the weight of each assets, And α represents the VaR value, N represents the population size and n represents the number of securities invested;

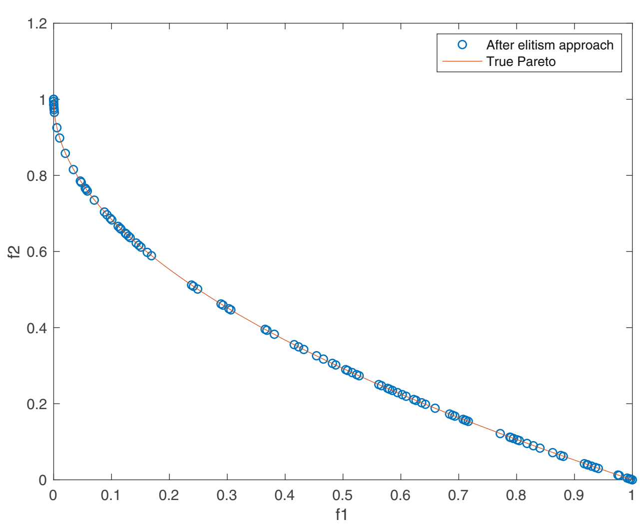

Fig. 3 The distribution of individuals processed by elitism approach

Step 2: Pass all the individuals in the initial population to the subpopulation and start parallel evolution;

Step 3: The objective function value is calculated for all individuals in the subpopulation according to the risk measurement function and expected return function in (6), where the expected return function is minimized by taking a negative number;

Step 4: Perform the fast nondominated sorting method for each subpopulation in parallel and calculate the crowding distance;

Step 5: Each subpopulation is selected through a binary tournament mechanism;

Step 6: Set different crossover methods for the subpopulations, expand the search range by exploration population while the main population is searching normally, and generate new portfolios;

Step 7: Respectively use the elitism approach for the subpopulations separately to obtain new populations, that is, select the optimal N^* group portfolios;

Step 8: When the independent evolution generation of the subpopulation is g, merge the population, and use the elitism approach to select the new initial population from it, and at the same time keep the solution in the first non-dominated front to the external archive set and return to Step 2, otherwise return to Step 3; When the maximum evolution generation is gen, the candidate solution set is processed by sparsity strategy, and the currently obtained solution set is the Pareto optimal solution set, which enters the following multi-attribute decision-making stage as a decision range.

3.1.5 Comparison algorithm

In the problem of portfolio optimization, literature [19] compared five kinds of MOEAs, and the experiment proved that NSGA-II [20] and SPEA2 [21] are better than the other three algorithms. Multiobjective optimization framework based on nondominated sorting and local search (NSLS) [25] has been proposed in recent years as an effective algorithm to solve multi-objective optimization problems. Different from NSGA-II's crowding-comparison mechanism, it uses a farthest-candidate mechanism to maintain the diversity of non-dominated solutions. At the same time, based on the non-dominated sorting suggested in NSGA-II, NSLS presents a new local search schema for convergence. Different from GA-based MOEAs such as NSGA-II, SPEA2, and NSLS, multi-objective particle swarm optimization (MOPSO) algorithm [26] is a PSO-based MOEA. Due to its high efficiency and speed, it is also widely used in multi-objective Optimization problem.

Furthermore, in addition to the above-mentioned MOEAs based on the Pareto dominance framework, multiobjective evolutionary algorithm based on decomposition (MOEA/D) as a decomposition-based MOEA was proposed by Zhang et al. [27]. In MOEA/D, the traditional decomposition method is used to decompose the multi-objective optimization problem into a set of single-objective sub-problems, and the neighbor relationalship between the sub-problems is defined by the distance relationalship between the weight vectors, and then the evolutionary algorithm is used to solve these sub-problems at the same time. Meanwhile, when optimizing a sub-problem, a new solution is generated through the evolution process of cross-mutation between adjacent sub-problems, and the new solution is used to update the solution of the current sub-problem. MOEA/D also introduces a method of information sharing between neighbor sub-problems, that is, a new solution updates its neighbor sub-problems while updating the corresponding sub-problems. At the same time, their research team further improved MOEA/D and proposed MOEA/D based on differential evolution (MOEA/D-DE) [28] and MOEA/D with covariance matrix adaptation evolution strategy (MOEA/D-CMA) [29] as new versions of MOEA/D, which further improved the performance of the algorithm.

Therefore, in this paper, the performance of the SMP-NSGA-II will be compared with NSGA-II, SPEA2, NSLS, MOPSO, MOEA/D-DE and MOEA/D-CMA.

3.1.6 Performance evaluation indicators

In this paper, we choose the spacing (SP) [30] and hypervolume (HV) [31] to verify the effectiveness of the improved algorithm compared with other algorithms in terms of convergence and diversity through the above indicators. The SP and HV are computed as follows:

$$SP = \sqrt{\frac{1}{|C|}\sum_{k=1}^{|C|}\left(\bar{d} - dx_k\right)^2} \tag{10}$$

$$HV\left(C, R\right) = volume\left(\bigcup_{k=1}^{|C|} a_k\left(c_k, r_k\right)\right) \tag{11}$$

where C is the solution set to be evaluated, dx_k is the Euclidean distance between the k-th solution in the solution set C and its nearest solution, and $\bar{d}$ is the average value of all dx_k; R is the reference set, for each solution $c_k \in C$, a hypercube $a_k\left(c_k, r_k\right)$ is constructed with c_k and $r_k \in R$ as diagonal corners. Among them, smaller values of SP indicate the solutions distribute more uniform, larger values of HV indicate the solutions have better convergence as well as diversity.

In order to compare the performance of different algorithms fairly, before using performance indicators, this paper uses the following normalization method:

$$f_q = \frac{f_q - f_q^{\min}}{f_q^{\max} - f_q^{\min}} \tag{12}$$

Among them, $f_q^{\max}$ and $f_q^{\min}$ are the maximum and minmum values of the q-th objective function. At the same time, the reference point r in the HV-metric is set to (1, 1).

3.2 Multi-attribute decision making

Through the introduction of Section 3.1.4, we have obtained the Pareto optimal set. In order to further give the optimal compromise portfolio of different DM preferences, in this section, we adopt the FCM-GRP hybrid method. First, use the FCM clustering algorithm to cluster the Pareto optimal set to distinguish different DM preferences, and then run the grey relational projection method on the Pareto optimal set under different DM preferences to determine the optimal compromise portfolio.

3.2.1 Fuzzy C-means clustering

Fuzzy C-means algorithm [17] is an unsupervised fuzzy clustering algorithm, and its mathematical model is:

$$\begin{cases} \min J = \sum\limits_{k=1}^{N}\sum\limits_{j=1}^{N_{clu}} u_{kj}^{h} D_{kj}^{2} \\ s.t. \sum\limits_{j=1}^{N_{clu}} u_{kj} = 1 \end{cases} \tag{13}$$

Where J is the clustering loss function, N is the number of solutions in the Pareto optimal solution set, that is, the number of portfolios in the portfolio set, and N_{clu} is the number of clusters. To reflect different DM preferences, the number of clusters $N_{clu} = 3$, that is, the clustering results reflect different investment preferences. Where u_{kj} is the degree of membership of portfolio k that belongs to DM preference j, D_{kj} is the Euclidean distance between the portfolio k and the center of DM preference j. And in the application of FCM, fuzzifier $h = 2$ is usually selected [17]. Through the minimum similarity between portfolios under different DM preferences and the largest similarity between portfolios under the same DM preference, the division of portfolio sets is realized.

3.2.2 Grey relational projection method

GRP [18] method is constructed by the combination of the grey relational method and the projection method. It has the advantages of both and has a good application in the field of multi-attribute decision-making. According to the fuzzy C-means algorithm in Section 3.2.1, the pareto optimal set obtained in the multi-objective stage is divided into three categories, reflecting different DM preferences. In order to further give the optimal portfolio under different DM preferences, this section evaluates each portfolio under different DM preferences by the GRP method, and selects the optimal compromised portfolio based on the value of the priority membership. The process of calculating the priority membership is given below.

Suppose there are N^* portfolios under the DM preference, which are composed of Q evaluation indicators(Q in this section is 2, that is, risk and benefit). It can be seen from the characteristics of the indicators that the risk (CVaR) is a "cost-type" indicator, and the expected return is a "benefit-type" indicator. Standardize the two indicators separately, that is, both types of indicators are converted into "benefit-type" indicators and dimensionless processing is performed. And the grey relational coefficient $\xi_{kq}^{+(-)}$ of the q-th indicator of the k-th portfolio and the positive ideal (negative ideal) portfolio is calculated:

$$\xi_{kq}^{+(-)} = \frac{\min\limits_{N^*}\min\limits_{Q}\left|L_{0q}^{+(-)} - L_{kq}\right| + \lambda \max\limits_{N^*}\max\limits_{Q}\left|L_{0q}^{+(-)} - L_{kq}\right|}{\left|L_{0q}^{+(-)} - L_{kq}\right| + \lambda \max\limits_{N^*}\max\limits_{Q}\left|L_{0q}^{+(-)} - L_{kq}\right|} \tag{14}$$

Among them, L_{kq} is the q-th evaluation indicators of the k-th portfolio after standardization, $L_{0q}^{+(-)}$ is the evaluation indicators under the positive ideal (negative ideal) portfolio, $|L_{0q}^{+(-)} - L_{kq}|$ is the absolute difference between the portfolio k and the corresponding indicator of the positive ideal (negative ideal) portfolio, λ is the discrimination coefficient, usually takes 0.5 [32].

According to the grey relational coefficient $\xi_{kq}^{+(-)}$, the projection value $S_k^{+(-)}$ of the portfolio k on the positive ideal (negative ideal) portfolio is obtained as:

$$S_k^{+(-)} = \sum_{q=1}^{Q} \xi_{kq}^{+(-)} \frac{w_q^2}{\sqrt{\sum\limits_{q=1}^{Q} w_q^2}} \tag{15}$$

In particular, the modulus values of the positive ideal portfolio and the negative ideal portfolio (Where w_q is the weight of each indicator of the portfolio, which represents the relative importance of each indicator. Since returns and risks are equally important under the same DM preference, this paper takes $w_q = 0.5, q = 1, 2$):

$$S_0 = \sum_{q=1}^{Q} \frac{w_q^2}{\sqrt{\sum\limits_{q=1}^{Q} w_q^2}} \tag{16}$$

The priority membership d_k of the k-th portfolio is defined as:

$$d_k = \frac{\left(S_0 - S_k^-\right)^2}{\left(S_0 - S_k^-\right)^2 + \left(S_0 - S_k^+\right)^2}, 0 \leq d_k \leq 1 \tag{17}$$

It can be seen from (17) that the larger the value of the priority membership, the closer the portfolio is to the positive ideal portfolio, and the farther from the negative ideal portfolio, and vice versa. Therefore, under different DM preferences, the portfolio with the largest value of the priority membership is the optimal compromise portfolio under this DM preference.

4 Empirical analysis

In this section, the dual-objective portfolio model with CVaR as a measure of risk and including transaction costs is set with different parameters to discuss its impact on the portfolio. At the same time, the proposed SMP-NSGA-II is compared with the other six MOEAs, and the FCM-GRP hybrid method is used to make a posteriori decision on the Pareto optimal set obtained by the SMP-NSGA-II, then the optimal compromise portfolio under different DM preferences are given.

4.1 Sample selection

In recent years, the rise of new applications such as 5G mobile phones, car electrification and the Internet of Things has pushed the chip industry out of the trough. With the stock price of the semiconductor industry plummeted in the fall of 2018 and the gradual recovery in 2019, we are all reminded of the cyclicality of the industry and the long-term positive factors that we believe will bring income growth for investors in the next few years. Therefore, this paper selects 6 stocks from the Shanghai stock market and 3 stocks from the Shenzhen stock market from several stocks in the semiconductor industry. Two trading years from January 2, 2019 to January 29, 2021 are selected (The above data comes from the NetEase Finance website). Based on the data of each trading day, the average daily return rate of each stock is calculated as shown in Table 1.

4.2 Model evaluation

In this section, we set different parameters for the dual-objective portfolio model constructed in Section 2.2, and solve the models with different parameters through SMP-NSGA-II to discuss the impact of different parameters on the portfolio.

4.2.1 The impact of the confidence level on the portfolio

In order to study the impact of the confidence level on the portfolio, the confidence level $\beta = 0.90$, 0.95 and 0.99 are selected respectively, and the transaction costs c=0.00185 is set. It can be seen from Fig. 5 that when the expected return is constant, as the confidence level increases, the CVaR value increases, which means that the degree of risk aversion increases.

4.2.2 The impact of transaction costs on portfolio

For comparison, when the same confidence level $\beta = 0.90$, different transaction costs $c = 0.00185$, 0.001 and 0.0005 are selected. It can be seen from Figure 6 that under the same CVaR value, the increase in transaction costs reduces the expected return. Since transaction costs are incorporated into the optimization problem, they also affect the choice of stocks.

Through the above discussion, we can see that the selection of different parameters on the model will affect the final investment portfolio. Literature [4] through experimental comparison believes that if the confidence level β higher than 0.90, it will decrease the effectiveness of CVaR portfolios. Therefore, for the portfolio optimization model constructed in Section 2.2, we set the confidence level $\beta = 0.90$. At the same time, for the transaction costs, in order to facilitate the calculation, refer to the Shanghai and Shenzhen Stock Exchange regulations, and set the transaction costs of all securities as $c = 0.00185$.

Table 1 Experimental stock and the average daily rate of return

Stock code	The average daily rate of return
600360	0.320451%
603005	0.533416%
600703	0.309700%
600171	0.220730%
603328	0.071419%
600206	0.225541%
002129	0.218171%
002156	0.348161%
002185	0.422847%

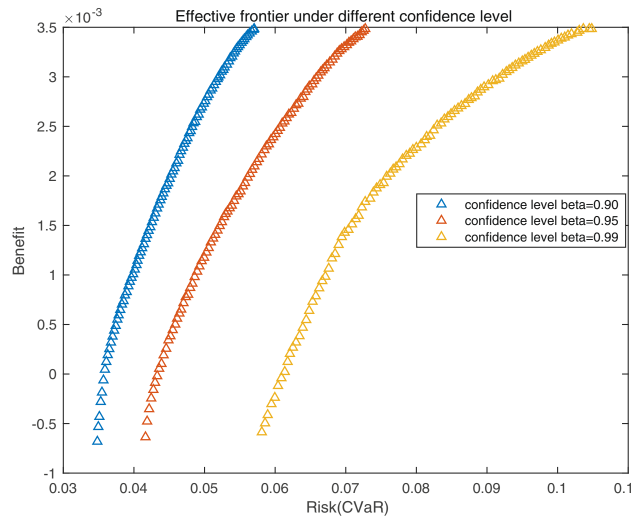

Fig. 5 Impact of different confidence levels on the portfolio

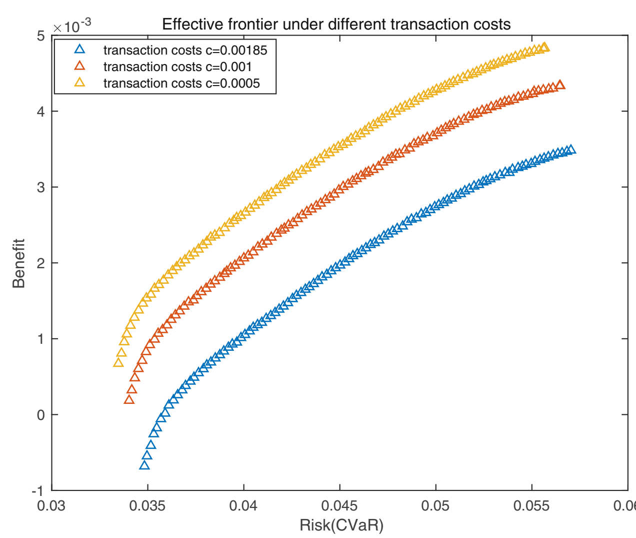

Fig. 6 Impact of different transaction costs on the portfolio

4.3 Performance comparison of MOEAs

4.3.1 Parameter settings

In order to compare the performance of SMP-NSGA-II with the other six algorithms, experiments with different parameters were carried out. Except that the two parameters (the population size is 100 and the maximum number of generations is 500) common to each algorithms. SMP-NSGA-II, NSGA-II, SPEA2, NSLS, MOPSO, MOEA/D-DE, MOEA/D-CMA also need some other parameters. For other parameters in NSGA-II, SPEA2, NSLS, MOPSO, MOEA/D-DE, MOEA/D-CMA, they have remained the same with their original studies [20, 21, 25, 26, 28, 29]. For SMP-NSGA-II, three additional parameters need to be set, that is, the parallel evolution cycle g is 25, the population size division coefficient τ is 0.8, and the exploration coefficient A is 1.481 [22]. Among them, g and τ were empirically set during previous experiments.

4.3.2 Results discussion

According to the portfolio optimization model in Section 2.2 and the SMP-NSGA-II proposed in Section 3.1, we perform 30 independent repeated experiments on all algorithms in the same computing environment and set the same initial population for each algorithm to achieve fairness. At the same time, in addition to reflecting the performance of each algorithm through the results of the performance evaluation indicators in Section 3.1.6, the time required for the algorithm to run is also compared here.

Table 2 shows the average values, standard deviations, best values and worst values of the two indicators and running time. It can be seen from Table 2 that although SMP-NSGA-II is lower than SPEA2 and MOPSO in the best value of spacing, it has the best performance in average value and standard deviation. At the same time, SMP-NSGA-II performs better than the other six algorithms on hypervolume, so it can be considered that the Pareto solution obtained by our proposed algorithm has good convergence and diversity. In terms of running time, although the performance in standard deviation is poor, it can be seen from the reflection of the average value of running time that SMP-NSGA-II still has a certain degree of competitiveness compared to other algorithms. Further analysis of other algorithms shows that NSLS performs the worst in terms of running time. The results also show that, in the case of this paper, although MOPSO performs quite good on running time and spacing, its worst performance on hypervolume shows that its ability to approximate the true frontier is the worst. At the same time, the overall performance of MOEA/D-DE and MOEA/D-CMA is also poor.

4.4 Selection and analysis of the optimal compromise portfolio

After the solution of the multi-objective optimization stage, the Pareto frontier of the portfolio under the confidence level $\beta = 0.90$ is obtained, as shown in Figure 7. We know that the optimal Pareto means that the risk cannot be reduced without reducing the return. For these non-dominated solutions, it is still difficult for DMs to determine the preferred portfolio. Therefore, this paper adopts the FCM-GRP hybrid method to automatically identify portfolios representing different DM preferences.

First, clustering the portfolio set through the FCM algorithm. From Fig. 8, you can intuitively see that the Pareto solution set is divided into three categories, representing different DM preferences (the red part in the figure represents the risk-averse type, and the orange part represents the risk-neutral type, the blue part represents risk-seeking type). Then, the GRP method is used to evaluate each portfolio under different DM preferences. And the priority membership of each portfolio under different DM preferences is calculated according to the introduction in Section 3.2.2. We already know that the larger the value of the priority membership, the closer the portfolio is to the

Table 2 Statistical information on spacing, hypervolume metric and running time

variables	Average			Standard deviation			Best			Worst		
	SP	HV	Time(s)	SP	HV	Time(s)	SP	HV	Time(s)	SP	HV	Time(s)
SMP-NSGA-II	**0.0056**	**0.6943**	**5.1244**	**0.0003**	**0.0004**	1.0547	0.0050	**0.6953**	**4.8134**	**0.0063**	**0.6933**	10.698
NSGA-II	0.0077	0.6916	5.3696	0.0021	0.0006	0.0928	0.0064	0.6927	5.2330	0.0158	0.6898	**5.7326**
SPEA2	0.0291	0.6935	14.377	0.0797	0.0005	0.1181	0.0040	0.6947	14.224	0.3994	0.6925	14.651
NSLS	0.0073	0.6915	277.24	0.0010	0.0008	2.7113	0.0059	0.6929	275.79	0.0096	0.6897	291.20
MOPSO	0.0074	0.5771	6.2959	0.0042	0.0282	**0.0530**	**0.0035**	0.6331	6.2141	0.0241	0.5174	6.4298
MOEA/D-DE	0.8997	0.6856	28.842	1.3746	0.0042	0.1158	0.0236	0.6918	28.697	4.2609	0.6707	29.353
MOEA/D-CMA	0.8405	0.6866	38.422	1.3707	0.0036	0.1503	0.0237	0.6912	38.129	4.2610	0.6737	38.813

Bold represents the optimal value of each column

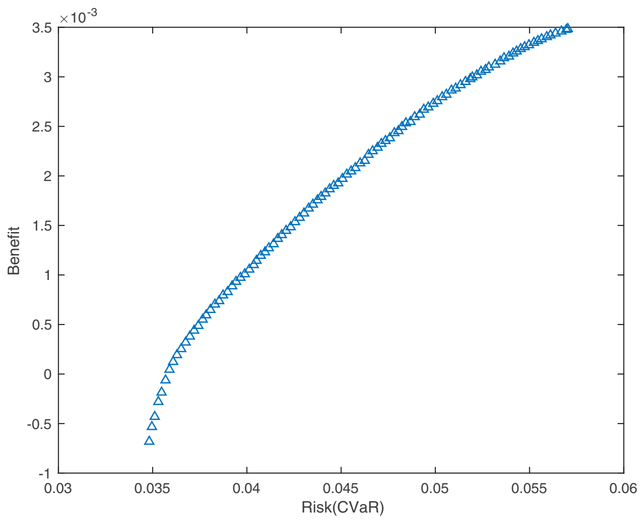

Fig. 7 Efficient frontier obtained by SMP-NSGA-II

positive ideal portfolio, so the optimal compromise portfolio under different DMs are finally given, as shown in Table 3.

It can be seen from Table 3 that for risk-seeking investors, the investment ratio will be mainly concentrated in the stock with highest yield(stock code is 603005), and the number of stocks invested will be smaller, and the actual return at this time will be higher, but the risk is also high, in line with the investment theory of high risk and high return. At the same time, for risk-averse investors, there are more stocks invested, and the investment ratio will be dispersed among other stocks with lower expected returns. Although the returns are lower, the risks will also decrease, which is also in line with the investment theory of diversified investment can reduce risks. And it can be seen from Fig. 7 that there is a positive correlational between return and risk. Obviously, this model is reasonable and effective.

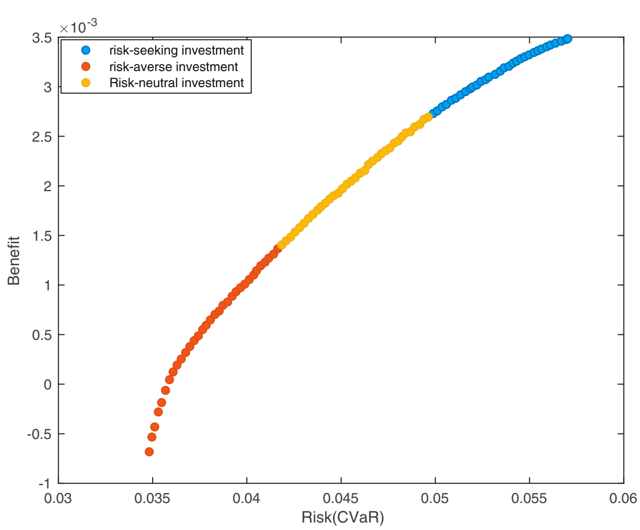

Fig. 8 Efficient frontier reflecting DM preference

Table 3 Optimal compromise portfolio under different DM preferences

DM preference	Risk seeking	Risk averse	Risk neutral
600360	0.0488	0.0047	0.2002
603005	0.8639	0.1637	0.5645
600703	0	0	0.1207
600171	0	0.1126	0
603328	0	0.5907	0.0299
600206	0	0.1042	0.0114
002129	0	0	0
002156	0	0	0
002185	0.0866	0.0237	0.0730
CVaR(Risk)	0.0545	0.0359	0.0484
Benefit	0.003282	0.000045	0.002533
The priority membership	0.5479	0.6279	0.5255

5 Conclusion

In order to solve the dual-objective portfolio optimization model with conditional value-at-risk (CVaR) as a measure of risk and including transaction costs, this paper proposes a method combining multi-objective optimization and multi-attribute decision-making. In the multi-objective optimization stage, this paper proposes a multi-population parallel NSGA-II based on sparsity strategy (SMP-NSGA-II). In the case studies of 9 stocks in the semiconductor industry, we compared SMP-NSGA-II with the other six MOEAs through two performance evaluation indicators (HV and SP) and running time, then verified the feasibility of the SMP-NSGA-II algorithm. In the multi-attribute decision-making stage, this paper adopts the FCM-GRP hybrid method to give the optimal compromise investment portfolio under different DM preferences.

There are a couple of drawbacks in the present study. Firstly, the model studied in this paper is relatively simple and does not take into account the many unstable factors of the real securities market; Secondly, the space complexity of the proposed SMP-NSGA-II algorithm is also high.

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Compliance with Ethical Standards

Conflict of Interests The authors declare that they have no conflict of interest.

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