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# Tolerance framework for robust group multiple criteria decision making

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ABSTRACT

A tolerance framework is developed to address a group multiple criteria ranking problem with indirect preference information, which is referred to as the interaction, importance and tolerance of criteria as well as pairwise comparisons among alternatives and criteria. Choquet integral preference model is employed to capture the interaction, importance and tolerance of criteria, all of which are specific to decision makers (DMs). Some mandatory/sufficient requirements concerning criteria which are global or local, also called the tolerance attitudes of DMs, are quantified as tolerability constraints. Preference disaggregation analysis is extended to solve this type of tolerability constraints for preference elicitation. Confronted with the inconsistency issue (the feasibility of the whole preference constraints translated from indirect preference information), cause oriented strategy and consequence oriented strategy are established by regression-based mixed 0–1 integer linear programs with the objectives of the most credible minimal inconsistent preference constraints and the most credible maximal preference constraints in the context of group decision making. Considering a wide range of the minimal unsatisfied subsets of preference constraints responsible for the inconsistency and the maximum satisfied subsets of preference constraints as a consistent result, to reach a robust decision, stochastic multicriteria acceptability analysis (SMAA)-like simulation analysis is generated to examine the whole instances of compatible preference models in the modified feasible preference polyhedron and compute the result in a probabilistic form. Simulation experiment is conducted to investigate the influence of global tolerance attitudes of DMs on preference elicitation in conservative and radical scenarios. Finally, the application of the proposed approach to a credit ranking of small and medium-sized enterprises (SMEs) and the comparison analysis with objective methods are presented and discussed for the effectiveness of the proposed tolerance framework.

## 1. Introduction

Many real-world decision problems in different fields can be formulated as multiple criteria decision making (MCDM) as the performance of alternatives can be decomposed as the evaluation of alternatives on considered criteria and ranked by the comprehensive score which is a straightforward intuition for decision makers (DMs) to comprehend. The preference model in MCDM would be paid more attention to the property that is consistent with observable properties of human decision process (i.e., intuitive reasoning, common sense, and expert knowledge), otherwise it would deteriorate a justifiable decision. The theory of fuzzy measures and integrals Liginlal and Ow (2006) has emerged to characterize the preference of DMs with the realistic hypothesis about preferential dependence among criteria and give the opportunity to represent and interpret the typical human decision process, which is such a predominant character that can address numerous practical preference modelling.

The DMs have distinguished aggregation behaviors in context of group MCDM, also called the tolerance attitudes of DMs in some literatures. From a mathematical perspective, the tolerance attitudes of DMs are equivalent to the tolerance (or intolerance) of criteria represented by the aggregation operator (Li, Yao, Sun, & Wu, 2018). In a strategic investment decision problem, when searching for most qualified candidates among prospective alternatives, every candidate failing at most $k$ criteria would be rejected. It implies $k$-intolerance attitudes of the DMs which can be perceived as a selection policy. In the field of fuzzy aggregation operator field, ordered weighted averaging (OWA) operator was studied which used the global tolerance of criteria to measure the

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optimism degree of DMs (Kim & Ahn, 2018; Zeng, Li, & Gu, 2018). Based on the $k$-tolerance (or $k$-intolerance) properties of the Bonferroni mean aggregation operator (Beliakov & James, 2013), Dutta, Figueira, and Das (2019) studied orness measure to define the or-like degree of the Bonferroni mean and its variants. Considering non-additive value function, previous researchers constructed the optimization model using fuzzy measure to capture tolerance attitudes required from DMs without any post optimality analysis (Li, et al., 2018; Yao, Li, Sun, & Wu, 2018). It is doubted that the decision from the inferred preference model is robust in case of the existence of the multiple feasible near-optimal solutions. As noted by Dujmović and Larsen (2007), the requirements (or expectations) on criteria combining by preference model cannot be neglected during preference aggregation. Subsequently, concise mathematical formulas for them (Hudec & Mesiar, 2020; Marichal, 2004) were defined to express a global or local tolerance degree of the DMs. Such the mandatory or sufficient requirements of DMs can be transformed in the form of the tolerability constraints for the sake of simplicity of preference construction. Any quantified preference information may reduce the feasible polyhedron of all compatible preference model as long as inconsistent preference constraints don't exist. Therefore, the investigation of the impact of tolerability constraints on the preference constructive learning process is significant. Even though aforementioned studies prompt the development of tolerability theory, they emphasize the characterization of tolerance attitude and provide a retrospective understanding of how to make a strategic decision but not capture the tolerance attitudes of the DMs into MCDM process.

Preference elicitation is an important and fundamental component of multiple criteria decision aiding (MCDA). In order to achieve this, preference elicitation method should (a) maintain a flexible representation of the DMs' preference; (b) handle uncertainty in a principled manner; (c) rank alternatives that allow the inferred preference model to discriminate among alternatives; and (d) allow for the incorporation of prior knowledge from the DMs (Corrente, Greco, Kadziński, & Słowiński, 2013). Fuzzy measure (or capacity) and integrals can be used to capture the value system of the DM associated with every subset of criteria. Therefore, models' parameters are in exponential number relatively to the number of criteria and their elicitation is a challenging issue. Most of the works aiming at determining a suitable capacity for the Choquet integral. With distinguished objectives such as entropy maximum (Aggarwal, 2019; Beliakov & Wu, 2019), andness/orness index maximum (Beliakov & Wu, 2019) or maximum split approach (Chen & Huang, 2019; Marichal & Roubens, 2000), the "optimal" preference parameters can be acquired, which vary as the distinctive selection of the objective function. Apart from the statistic preference elicitation, through a sequence of pairwise comparisons among alternatives, Benabbou, Perny, and Viappiani (2017) investigated the incremental elicitation of the capacity using a minimax regret strategy. The consistency of the stated preference statements and the inferred preference model is implicitly enforced. Korhonen, Silvennoinen, Wallenius, and Öörni (2012) conducted an empirical study and observed that people are not very often fully consistent with any value function when making binary choices. Taking this fact into account, preference disaggregation analysis (PDA) explores indirect preference information grounded on the ordinal regression paradigm and concludes that whether the preference over pairwise alternatives induced by the whole inferred preference model instances are consistent with DMs' prior preference or not (Tomczyk & Kadziński, 2019). It is in line with the essence of value judgment of MCDA, i.e., the elicited preference model should be consistent with observable properties of human reasoning (Dujmović, 2013). Indirect preference information admitted by PDA is in a variety of categories such as pairwise comparison (i.e., one vs one among reference alternatives or criteria) (Bous, Fortemps, Glineur, & Pirlot, 2010; Liu, Liao, Kadziński, & Słowiński, 2019), intensities of preference (i.e., pair vs pair among reference alternatives or criteria) (Ma, 2019; Roszkowska & Wachowicz, 2015), rank-related requirements (Kadziński, Greco, & Słowiński, 2013) and interaction among criteria (Angilella, Corrente, & Greco, 2015). The global or local pairwise judgments specified by the DMs impose constraints on the parameters of preference model. Such extensive and accounted types of indirect preference information improve the flexibility of preference construction procedure. DMs just need to provide any decision judgments rather than explaining them without less cognitive efforts. On the other hand, the plausible set of the instances of preference model ensures a large credit for generality, plurality and interpretability, but the aforementioned frameworks are insufficient and challenging in the characterizations of the mandatory or sufficient requirements on criteria which limits the potential to represent the real-world aggregation process.

There is no compatible Choquet integral preference model if indirect preference information given by DMs cannot match the underlying preference model due to the preference controversary or preference dominance violation among the DMs or the insufficient expressive power of chosen preference model (Corrente, et al., 2013). The decision about working alongside this inconsistency or seeking and revising the pieces of preference information impeding the incompatibility is next step. The prevailing methods devoted to such inconsistency issue can be divided into two categories. Some scholars assume that the DMs are acceptable to carry out the decision analysis with this incompatibility when the margin of the misranking error is not more than artificial threshold (Corrente, Greco, Kadziński, & Słowiński, 2016). Built on this minimal change principle, Mousseau, Dias, and Figueira (2006) extended algorithms to relax (rather than to delete) assignment examples and incorporated confidence level attached to each assignment example in multiple criteria sorting model. As for pairwise comparison matrix (PCM), Bozóki, Fülöp, and Poesz (2015) established nonlinear mixed-integer optimization to determine the minimum number of judgment changes required to reach a threshold value of inconsistency. Considering a trade-off between the judgment alternation of alternatives and the inconsistency reduction, Abel, Mikhailov, and Keane (2018) constructed a multi-objective optimization model facilitating inconsistency reduction whilst also looking to minimize the amount of alteration. Fernandez, Navarro, Solares, and Coello (2020) proposed an interval-based PDA approach guided by a genetic algorithm, allowing the DM's decision policy to contain imperfect knowledge. Others try to identify the sources of inconsistency and propose correction strategies for further decision process. Recognizing the troublesome pieces from the whole preference constraints, once this set is removed, a set of the remaining preference constraints can generate a non-empty polyhedron of compatible value functions (Mousseau, Figueira, Dias, Gomes da Silva, & Climaco, 2003). Therefore, Mousseau, et al. (2003) proposed two alternative algorithms to identify such subsets of constraints which, when removed, lead to a consistent preference system. The maximal subsets of preference constraints are explored to act as input-level information in the subsequent decision process (Kadzhiski, Ghaderi, & Dabrowski, 2019; Liu, Liao, Huang, & Liao, 2019). The literature discussed above doesn't involve with the selection policy and the DMs are confused with the existence of multiple feasible solutions such as the minimal subsets of inconsistent preference constraints or maximal subsets of consistent preference constraints. The DMs suffers from a subjective selection among objectively determined solutions.

Some scholars characterize the inconsistency or conflict pieces of preference information through Dempster–Shafer Theory (DST) and resolve it by improving the combination rule or designing the discounting methods (Zhao, Xue, Dong, Tang, & Wei, 2020). The mass of the empty set, i.e., m(∅), can be interpreted as the amount of conflict in the preference information (Destercke & Burger, 2013). $m(\emptyset) > 0$ corresponds to the fact that no preference model satisfies all preferential information items at once. In the field of preference elicitation, without the assumption of the form of preference model, Destercke (2018) adopted belief functions to quantify such inconsistencies defining the selected space of preference model. To construct an efficient and robust preference elicitation, Guillot and Destercke (2019) extended the DST framework based on the minimax regret strategy considering a subset of possible preference models. To capture the extensive conflict, different conflict measures are taken into account in multi criteria decision making method. Therefore, outranking model is adopted by Silva and de Almeida-Filho (2016) and Silva and de Almeida-Filho (2018) to divide the conflict into the different levels within DST framework and then addressed prudently the pieces of preference information with high conflict.

Preference elicitation and machine learning (ML) have the same spirit in the paradigm of learning form examples (Corrente, et al., 2013). Both try to build a specified preference model from exemplary judgments delivered by the DM or decision examples from a training data set complying with the underlying preference. Th former does not allow the violation between the known holistic judgments and predicted results, while the latter converts it to the empirical error as loss function (Aggarwal & Fallah Tehrani, 2019). ML can be qualified as preference discovery without any intervention of the DM, which emphasizes the predictive precision, while preference elicitation is capable of acquiring enough knowledge and arguments for explanation of the decision (Liu, Kadziński, Liao, & Mao, 2021). This enables the DM to establish the preferences that previously had not pre-existed in her/his mind, to discover what is important for them, and to learn about their values, which contributes to the increase of the consistency between the evolution of the process and value systems of the DMs. In summary, the multi-dimensional comparisons among Preference elicitation, Evidence reasoning and ML (covering the ordinal regression, ranking and preference learning settings) are conducted in Table 1:

**Table 1**
Elicitation procedure versus other methods.

| | Preference elicitation | Evidence reasoning | ML |
|---|---|---|---|
| Preference model | Assumed a prior | Not tailored to a specific form | Flexible |
| Input oriented information | Human subjective judgments | Human subjective judgments | Observation of people`s behavior |
| Interaction with human | Yes | Yes | No |
| The number of DMs | One or group | one | – |
| Confidence levels to pieces of preference information | Yes | Yes | No |
| Typical sample size | Small | Small | Large |
| Noise/Inconsistency | Noise-free data | Noise-free data | Noise allowed |
| How to deal with noise or inconsistency? | Discover and address | Characterized as the empty set of mass function | Viewed as loss function |
| Is the preference model explicit? | Yes | No | Yes |
| Goal | Provide a ranking result for DMs | | |

Uncertainties and imperfections constitute an indispensable part of MCDA such as uncertain data linked with future outcomes or imprecise model parameters (Mavrotas, Pechak, Siskos, Doukas, & Psarras, 2015). It is acknowledged that the inferred preference model should preserve the partial order between the compared pairs of alternatives. More concretely, pairwise comparison in the form of $a \succ b$ given by DMs should be consistent with the inferred preference relation, i.e., $a$ should be ranked at least better than $b$. However, there is typically more than one minimal subset composed of inconsistent preference constraints or maximal subset composed of consistent preference information so that a wide range of the instances of preference model compatible with indirect preference information exist. From a constructive perspective, preferences are highly context-dependent (Tversky & Simonson, 1993), and the selection policy and the tolerance attitudes of the DMs may change as the information learnt by the DMs is processing (Yang, Toubia, & De Jong, 2015). Confronted with multiple optional strategies, the DMs are likely to change their selection on spot, depending on the information they encounter. Therefore, a major internal factor underlying uncertainty of preferences stems from changing the strategies and changing the revised preference constraints set used for conducting an assessment of decision alternatives, which would result in the variation of ranking order of alternatives. This uncertainty has been addressed by a Bayesian approach over the feasible utilities and an exploration of the space of the feasible utilities in the literatures (Corrente, et al., 2013). From the second view, depending on the problem setting, stochastic multicriteria acceptability analysis (SMAA) (Pelissari, Oliveira, Amor, Kandakoglu, & Helleno, 2019) determines all possible rankings or classifications for the alternatives, and quantifies the possible results in term of probabilities which considers each scenario that can occur or each set of preference parameters that can constitute a good representation of the reality (Hites, De Smet, Risse, Salazar-Neumann, & Vincke, 2006). Furthermore, SMAA could provide the DMs with the descriptive indices like the rank acceptability index of alternatives or central preference parameters. As noted by Doumpos, Zopounidis, and Grigoroudis (2016), the investigation of robustness of the provided conclusions consists in verifying whether they are valid for the set of the most plausible instances of preference model. Obviously, the greater the robustness of the decision result that can be derived after taking into account uncertainties in the elicitation of preference parameters, the better. Therefore, in this paper, a SMAA-like simulation algorithm is proposed to analyze the robustness of the decision problem with respect the uncertainty of the decision model.

Therefore, regarding the weights of DMs as credible levels attached to pieces of preference information, we undertake all these challenges together and provide a tolerance framework for robust group multiple criteria decision making to address the following research questions:

- How to model the mandatory/sufficient requirements on criteria and accommodate these tolerability constraints into group decision making process?
- How to resolve the inconsistence issue resulting from indirect preference information associated with the distinctive credible levels under tolerance framework?
- How to derive a robust decision confronted with the multiple feasible minimal unsatisfied inconsistent preference constraints or maximum satisfied consistent preference constraints?

The new features of the proposed approach are outlined as follows in the fourfold. First, the tolerance attitudes of DMs such as the mandatory/sufficient requirements on criteria for specific alternatives, are quantified as tolerability constraints. PDA is extended to solve this type of tolerability constraints for preference elicitation. Second, in case of inconsistency issue, cause oriented optimization model and consequence-oriented optimization model are established so as to identify the whole minimal inconsistent or maximal consistent subsets of preference constraints. Third, simulation experiment is conducted to investigate and verify the influence of global tolerance on the decision results in the conservative and radical scenarios. Lastly, to treat all possible sets of consistent preference constraints derived from the disposal of the minimal inconsistent preference constraints or the restoration of the maximum consistent preference constraints from a robust perspective, SMAA-like simulation algorithm is generated to handle the uncertainties in a probabilistic form by exploring the whole compatible set of instances of preference model.

Overall, the paper is organized as follows. In Section 2, we introduce fuzzy measure and integrals and the characterization of local/global tolerance of criteria. In Section 3, two strategies with tolerability constraints are developed for inconsistency management in the context of group MCDM. Then, an SMAA-like simulation algorithm is proposed for robust analysis. In Section 4, a simulation experiment is given in conservative and conservative scenarios. In Section 5, a hypothetical application of the credit ranking of small and medium-sized enterprises (SMEs) is illustrated using the proposed framework. In Section 6, conclusions and future research are discussed.

## 2. Preliminaries

Considering the advantage of fuzzy measures over additive measures in describing criteria interactions, Choquet integral preference model (Grabisch, 1997) defined on fuzzy measures is employed in this paper characterizing the human preference over alternatives with interaction criteria. Then tolerability constraints are also described.

### 2.1. Fuzzy measure

In general, fuzzy measures can be regarded as generalizations of weights during the aggregation process for MCDM. Denote $N = \{1,\ldots,n\}$ as the finite set (the label set of criteria set) and $P(N)$ as power set. Denote lower case letter $s$ as the cardinality of set $S$.

**Definition 1.** (*Grabisch, 1997*) *A fuzzy measure is a set function:*$\mu : P(N)\rightarrow[0,1]$, *satisfying the normalization and monotonic conditions respectively*:

(1) $\mu(\varnothing) = 0$ and$\mu(N) = 1$;
(2) $\forall S, T\subseteq P(N)$ and$T\subseteq S\subseteq N \Leftrightarrow \mu(T)\leqslant\mu(S)$.

A fuzzy measure assigns weights to all subsets $S\subseteq N$ to reflect relative importance of the coalitions of inputs. The Möbius representation of a fuzzy measure $\mu$ is a set function $m$ on $N$ defined by.

$$m(S) = \sum_{T\subseteq S}(-1)^{(s-t)}\mu(T), \forall S\subseteq N \tag{1}$$

Its inverse is the Zeta-transform:

$$\mu(S) = \sum_{T\subseteq S} m(T), \forall S\subseteq N \tag{2}$$

In terms of Möbius representation, properties (1) and (2) are respectively transformed into.

(1a) $m(\varnothing) = 0$ and$\sum_{T\subseteq N} m(T) = 1$;
(2a)$\forall i \in N, S\subseteq N\backslash i, m(\{ i\}) + \sum_{T\subseteq S} m(T\cup\{ i\})\geqslant 0$.

**Definition 2.** (*Grabisch, 1997*) *For a given*$x \in [0,1]^n$, *its discrete Choquet integral with respect to* $\mu$ *is defined as follows*:

$$\mathscr{CI}_\mu(\mathbf{x}) = \sum_{i=1}^{n}\left[x_{(i)} - x_{(i-1)}\right] \mu (\{(i),\ldots,(n)\}) \tag{3}$$

or equally by $\mathscr{CI}_\mu(x) = \sum_{i=1}^{n}[N_{(i)} - N_{(i+1)}] x_{(i)}$ where the parentheses used for indices represent a permutation on $N$. $x_{(i)}$ stands for a non-decreasing permutation induced by $x_i$ such that $x_{(0)}\leqslant x_{(1)}\leqslant\ldots\leqslant x_{(n)}$ and$x_{(0)} = 0. N_{(i)} = \{(i),\ldots,(n)\}$.

In Möbius representation, the above discrete Choquet integral can be explained as follows:

$$\mathscr{CI}_\mu(x) = \sum_{T\subseteq N} m(T)\min_{i\in T}\{x_i\} \tag{4}$$

**Definition 3.** (*Grabisch, 1997*) *Let* $\mu$ *be fuzzy measure on N. The Shapley index can be used to characterize the importance of individual inputs*:

$$\varphi(i) = \sum_{S\subseteq N\backslash i}\frac{(n-s-1)!s!}{n!}(\mu(S\cup\{i\}) - \mu(S)) \tag{5}$$

where $0! = 1$ as usual. Interaction index which generalized Shapley index to characterize the interactions between the inputs is defined as:

$$I(S) = \sum_{T\subseteq N\backslash S}\frac{(n-t-s)!t!}{(n-s+1)!}\sum_{R\subseteq S}(-1)^{s-r}\mu(T\cup R), \forall S\subseteq N \tag{6}$$

$I(S)$ reduces to Shapley index when $S$ is a singleton.

When $I(S)$ is negative, criteria among $S$ are substitutive or have a negative synergy. Similarly, when $I(S)$ is positive, criteria among $S$ are complementary or have a positive synergy. When $I(S)$ is equal to 0, criteria among $S$ are independent and have no interaction.

**Definition 4.** *A fuzzy measure* $\mu$ *is said to be k-additive if its Möbius transform satisfies* $m(S) = 0$ *for any* $S\subseteq N$ *such that* $s > k$ *and there exists at least one subset* $S\subseteq N$ *of exactly k elements such that* $m(S) \neq 0$.

**Definition 5.** (*Marichal, 2007*) *Fuzzy measure* $\mu$ *on N is k-tolerant* ($k = 1,\ldots,n$) *if* $\mu(S) = 1$ *for all* $S\subseteq N$ *such that* $s \geqslant k$ *and there exists a subset* $T\subseteq N$ *with* $t = k-1$ *such that*$\mu(T) \neq 1$; *Fuzzy measure* $\mu$ *on N is k-intolerant* ($k = 1,\ldots,n$) *if* $\mu(S) = 0$ *for all* $S\subseteq N$*such that* $s \leqslant n-k$ *and there exists a subset* $T\subseteq N$ *with* $t = n - k + 1$ *such that*$\mu(T) \neq 0$.

### 2.2. Tolerance attitudes of the DMs

The tolerance attitudes of the DMs measure the tolerance of criteria locally and globally, which is a type of requirement on criteria and can be modeled into human decision process under fuzzy measure decision making environment.

#### 2.2.1. Characterization of global tolerance

The concept of andness (closeness to the minimum) and orness (closeness to the maximum) is first proposed to depict conjunctive and disjunctive degrees, respectively. Based on the theoretical research, the practical implications of these concepts are considered in real decision-making problems. In the engineering field (Dujmovic, 2007), the level of satisfaction of system was interpreted as the degree to which an analyzed system completely satisfies all requirements in post-hoc analysis. Yager and Alajlan (2015) suggested that the degrees of conjunction and disjunction of operators, measured by andness/orness, can be regarded as a measure of optimism. Marichal (2004) provided a concise format of andness/orness index to depict the tolerance attitudes of the DMs on criteria for management decision.

**Definition 6.** (*Marichal, 2004*) *Let* $\mu$ *be fuzzy measure on N. The relationship between global tolerance and intolerance is deduced as follows*:

$$oreness\left(\mathscr{CI}_\mu\right) = \frac{1}{n-1}\sum_{T\subset N}\frac{(n-t)!t!}{n!}\mu(T) \tag{7}$$

$$andness\left(\mathscr{CI}_\mu\right) = 1 - \text{orness}\left(\mathscr{CI}_\mu\right) \tag{8}$$

The mathematical and behavioral interpretations of orness index correspond to the disjunction of max operator and a tolerant (rick-seeking) propensity of DMs which are willing to accept that only some criteria are satisfied. On the other side, the mathematical and behavioral interpretations of the andness correspond to a conjunction of min operator and an intolerant (rick-averse) propensity of DMs demanding that most criteria be satisfied. From an applicable perspective, $k$-intolerant and $k$-tolerant fuzzy measures can reduce preference parameters of preference elicitation from $O(2^n)$ to $O(n^{k-1})$. Moreover, when varying $k$ from 1 to $n$, all the possible fuzzy measures on $n$ criteria can be disclosed.

### 2.2.2. Characterization of local tolerance

The global tolerance attitudes of the DMs can be expressed by the andness/orness index, but sometimes local attitudes to specific criteria also play an integral role where some criteria may have a certain degree of veto or favor effect. A veto index indicates more or less the degree to which the DMs demand that a specific criterion is satisfied. A favor index indicates the degree to which the DMs consider that a good score along a specific criterion is sufficient to be satisfied.

**Definition 7.** (Marichal, 2004) *Let $\mu$ be fuzzy measure on N. The veto index and favor index for certain criteria as local tolerance are defined as follows*:

$$veto\left(\mathscr{C}\mathscr{I}_{\mu}, i\right) = 1 - \frac{n}{n-1}\sum_{T \subseteq N/i}\frac{1}{t+1}m(T) \tag{9}$$

$$favor(\mathscr{C}\mathscr{I}_{u}, i) = \frac{n}{n-1}\sum_{T \subseteq N/i}\frac{1}{t+1}[m(T \cup i) + m(T)] - \frac{1}{n-1} \tag{10}$$

Even though the veto and favor effect on criterion mentioned above are extreme cases, some criteria may have a certain degree of veto or favor in the real applications. Therefore, it is worth investigating the influence of the mandatory or sufficient requirement of criteria on the final decisive decision.

The local/global tolerance degrees of DMs for criteria can be defined in multiple forms. First, they can be derived in a direct manner rated by a numerical degree measuring the intensity (between 0 and 1) to which a given criterion behaves like a veto or a favor for Choquet integral preference model. For example, the question that "would you like to accept the performance of criterion $g_1$ as the minimum of the entire alternative?" could be provided to the DMs and they are required to assign the scale of favor/veto degree on certain criterion. The larger the score is, the more possibly the DMs would like to accept the performance of criterion $g_1$ as the minimum limit of the entire alternative and vice versa. To enhance the flexibility of the proposed approach, the various applicable indirect manners are provided to the DMs for expressing the tolerance attitudes. The detail descriptions consist in the Subsection 3.1.

## 3. Robust development of tolerance framework by Monte Carlo simulation

### 3.1. Problem formulation

We consider the following multiple criteria decision making problem. $A = \{a_1, ..., a_i, ..., a_m\}$ is a set of alternatives evaluated by a family criteria set$G = \{g_1, ..., g_j, ..., g_n\}$. Without the loss of the generality, we assume that all criteria are benefit types. The priority of each DM in the context of group decision making is differentiated by a weight vector $\lambda = \{\lambda_1, ..., \lambda_t, ..., \lambda_T\}$ such that $\sum_{t=1}^{T}\lambda_t = 1$ and$\lambda_t > 0$, which can be perceived as confidence level or credible level of preference information. According to indirect preference information such as pairwise comparison among alternatives and criteria as well as the interaction, importance and tolerance of criteria, Choquet integral preference model tends to give a range of aggregation results for the same arguments of aggregation under the condition that indirect preference information is consistent. To this end, we intend to learn the whole compatible instances of preference parameters and give the result in a probabilistic form for robust multiple criteria group decision making.

It is worth noting that the DMs' tolerance attitudes are not mandatory at all. They can express neutral tolerance attitudes with no additional information. At this time, the problem is reduced to the objective preference elicitation. The basic constraints related to boundary and monotonicity conditions for the identification of fuzzy measures in Möbius representations are denoted as$E^{Base}$:

$$E^{Base}\begin{cases} m(\varnothing) = 0, \quad \sum_{g_j \in G} m(g_j) + \sum_{g_j, g_k \in G} m(\{g_j, g_k\}) = 1 \\ m(g_j) \geqslant 0, \forall g_j \in G \\ \sum_{i \in T, T \subseteq S} m(T) \geqslant 0, \forall S \in 2^G \\ m(g_j) + \sum_{g_k \in T} m(\{g_j, g_k\}) \geqslant 0, \quad \forall g_j \in G, \quad T \subseteq G \backslash g_j, \quad T \neq \varnothing \end{cases}$$

Some criteria may be decisive in the sense that the global score (of any alternative) obtained by aggregation is bounded by the partial score along one of them. With this consideration, borrowing the idea from (Li et al., 2018) to resolve the first research question, veto/favor effects are employed to depict the tolerance to individual criterion. Given this, we model tolerances attitudes fully into MCDA in fuzzy measures and enrich the literature about preference elicitation.

The DMs' indirect preference information such as pairwise comparison among alternatives and criteria as well as the interaction, importance and tolerance of criteria is translated into the linear constraints, denoted as $E^{PI}$:

Indirect preference information on criteria:

- Comparisons related to importance and interaction of criteria:

Criterion $j$ is more important than criterion $k$ ($g_j \succ g_k$): $\varphi(g_j) \geqslant \varphi(g_k) + \varepsilon$

Criteria $j$ and $k$ exhibit the same importance ($g_j \widetilde{g_k}$): $\varphi(g_j) = \varphi(g_k)$

Criteria $j$ and $k$ are synergic (or redundant): $\varphi(\{g_j, g_k\}) \geqslant \varepsilon (\leqslant -\varepsilon)$.

- Comparisons related to local tolerance attitude of criteria:

Criterion $j$ is more intolerant than criteria $k$ ($g_j \succ_{veto} g_k$): $veto(\mathscr{C}\mathscr{I}_{\mu}, g_j) > veto(\mathscr{C}\mathscr{I}_{\mu}, g_k)$.

Criteria and $k$ exhibit the same intolerance ($g_j \sim_{veto} g_k$): $veto(\mathscr{C}\mathscr{I}_{\mu}, g_j) = veto(\mathscr{C}\mathscr{I}_{\mu}, g_k)$.

Criterion $j$ is more tolerant to criteria $k$ ($g_j \succ_{fa\ vor} g_k$): $favor(\mathscr{C}\mathscr{I}_{\mu}, g_j) > favor(\mathscr{C}\mathscr{I}_{\mu}, g_k)$.

Criteria $j$ and $k$ exhibit the same tolerance ($g_j \sim_{favor} g_k$): $favor(\mathscr{C}\mathscr{I}_{\mu}, g_j) = favor(\mathscr{C}\mathscr{I}_{\mu}, g_k)$.

Indirect preference information on reference alternatives.

Alternative $a_i$ is preferred to $a_h$ ($a_i \succ a_h$): $\mathscr{C}\mathscr{I}_{\mu}(a_i) \geqslant \mathscr{C}\mathscr{I}_{\mu}(a_h) + \varepsilon$

Alternative $a_i$ is indifferent to $a_h$ ($a_i \tilde{a}_h$): $\mathscr{C}\mathscr{I}_{\mu}(a_i) = \mathscr{C}\mathscr{I}_{\mu}(a_h)$

Note that the above statements quantify the veto/favor effect on criterion in a preference relation. More expression can be referred to Subsection 3.2.

When indirect preference information provided by the DMs is consistent, there exist at least one set of preference parameters satisfying the constraints in$E^{Base} \cup E^{PI}$.

### 3.2. Strategies for inconsistency issues under tolerance framework

DMs can benefit from the available and various preference information when articulating decision judgments, while the inconsistency issue impedes the further analysis. To assure the feasibility of polyhedron transformed from the DMs' indirect preference information, a linear program (named P0) is constructed to check the feasibility of the constraints in $E^{Base} \cup E^{PI}$ and decide whether at least one set of Möbius parameters compatible with indirect preference information of DMs or exists or not. An instrumental variable$\varepsilon$, a small arbitrary positive value, is introduced to transform strict inequalities into weak inequalities.

$$(P0)\quad \max \ \varepsilon \quad s.t.\ E^{Basic} \cup E^{PI}$$

If $\varepsilon^* = \max\ \varepsilon > 0$, the linear system is consistent and at least one Choquet integral preference model compatible with indirect preference information derived from the DMs. In this case, $\boldsymbol{m}^*$ is the optimal solution of this linear program. If $\varepsilon^* = \max\ \varepsilon \leqslant 0$, there is no feasible solution.

Considering that the DMs may drop two constraints they consider unimportant rather than drop a single important one, sets of constraints with minimum cardinality that restore the consistency if removed need to be discovered. If one of the constraints caused the inconsistency is recognized, DMs removing them is a trivial manner, while the question here is what preference constraints with minimum cardinality exist. If the DMs cannot decide to remove any piece of preference information, decision analyst needs to provide possible subsets of the maximal consistent preference information. Therefore, to resolve the second research question, in case of preference inconsistency, we propose two strategies based on 0–1 mixed integer linear program to resolve it.

### 3.2.1. Cause oriented strategy

In case of the incompatibility (i.e., the linear system translated from the ordinal preference information is infeasible), the identification of the minimal subset of inconsistent preference constraints can decipher the reason of the preference contradiction for the DMs. The preference constraints in $E^{PI}$ introduce the binary variables to find the inconsistent constraints, which is denoted as $\widetilde{E}^{PI}$:

$\mathscr{CI}_\mu(a_i) + M\ v(a_i,a_h) \geqslant \mathscr{CI}_\mu(a_h) + \varepsilon^*$ if $a_i \succ a_h$.
$\mathscr{CI}_\mu(a_i) = \mathscr{CI}_\mu(a_h)$ if $a_i \tilde{a}_h$.
$\varphi(g_j) + M\ v_\varphi(g_j,g_k) \geqslant \varphi(g_k) + \varepsilon^*$ if $g_j \succ g_k$.
$\varphi(g_j) = \varphi(g_k)$ if $g_j \widetilde{g_k}$.
$\varphi(\{g_j,g_k\}) \geqslant \varepsilon$ if $g_j$ and $g_k$ are synergic.
$\varphi(\{g_j,g_k\}) + \varepsilon \leqslant 0$ if $g_j$ and $g_k$ are redundant.
$veto(\varphi_\mu,g_j) + M\ v_\mu^{veto}(j,k) \geqslant veto(\varphi_\mu,g_k) + \varepsilon$ if $g_j \succ_{veto} g_k$.
$veto(\varphi_\mu,g_j) = veto(\varphi_\mu,g_k)$ if $g_j \sim_{veto} g_k$.
$favor(\varphi_\mu,g_j) + M\ v_\mu^{favor}(j,k) \geqslant favor(\varphi_\mu,g_k) + \varepsilon$ if $g_j \succ_{fa\ vor} g_k$.
$favor(\varphi_\mu,g_j) = favor(\varphi_\mu,g_k)$ if $g_j \sim_{favor} g_k$.

The weight of each DM in the group can be regarded as the confidence level or credible level of preference information. According to this, preference constraint with the higher priority occurs more frequently in minimal inconsistent subsets of preference information than that with a lower priority. Therefore, the goal aim to find the most credible minimal set of inconsistent preference information.

$$(MIP\ 1)\quad \min \sum_{\lambda_t, t=1,\dots,T} \lambda_t \Big( \sum_{a_i,a_h} v(i,h) + \sum_{g_j,g_k} v_\varphi(j,k) + v_\mu^{veto}(j,k) + v_\mu^{favor}(j,k) \Big)$$
$$s.t.\quad E^{Basic} \cup \widetilde{E}^{PI}$$
$$v(i,h) \in \{0,1\}$$
$$v_\varphi(j,k),\quad v_\mu^{veto}(j,k), v_\mu^{favor}(j,k) \in \{0,1\}$$

After solving the model MIP1, $v(i,h)=1, v_\varphi(j,k)=1$, $v_\mu^{veto}(j,k) = 1, v_\mu^{favor}(j,k)=1$ for any $(a_i,a_h) \in A \times A$ and $(g_j,g_k) \in G \times G$ associated with indirect preference information represent inconsistent preference information which would be discarded. Let $f_1^*$ and $v(i,h)^*, v_\varphi(j,k)^*, v_\mu^{veto}(j,k)^*, v_\mu^{favor}(j,k)^*$ be the optimum of the objective function and the optimal solution of model MIP 1. Let $S^{Alt} = \{(a_i,a_h) | v(i,j)^* \neq 0\}$ and $S^{Cri} = \{(g_j,\ g_k) \big| v_\varphi(j,\ k)^* \neq 0,\ v^{veto}(j,\ k)^* \neq 0,\ v^{favor}(j,\ k)^* \neq 0\}$. Note that there is more than one minimal subset of inconsistent preference constraints. In some cases, there are multiple possible solutions corresponding to the minimum number of preference information. We can search for more solutions by solving MIP1 with one additional constraint $|\sum_{(a_i,a_h) \in S^{Alt}} v(i,h) + \sum_{(g_j,g_k) \in S^{Cri}} v_\varphi(j,k) + v_\mu^{veto}(j,k) + v_\mu^{favor}(j,k) \leqslant f_1^* - 1$. Model MIP1 is solved continuedly with the extended constraint set until the optimum of the objective function is less than $f_1^*$ The optimal solution at each iteration indicates different solutions.

### 3.2.2. Consequence oriented strategy

When the group is not willing to learn and understand where and why their judgements do not comply with the consistency principle, finding the consequence that restoring the maximal consistent preference information is a straightforward way. The constraints in $E^{PI}$ introduce the binary variables to find the whole consistent constraints, which is denoted as $\widetilde{E}^{PI}$:

$\mathscr{CI}_\mu(a_i) + M(1 - v(a_i,a_h)) \geqslant \mathscr{CI}_\mu(a_h) + \varepsilon^*$ if $a_i \succ a_h$.
$\mathscr{CI}_\mu(a_i) = \mathscr{CI}_\mu(a_h)$ if $a_i \sim a_h$.
$\varphi(g_j) + M(1 - v_\varphi(g_j,g_k)) \geqslant \varphi(g_k) + \varepsilon^*$ if $g_j \succ g_k$.
$\varphi(g_j) = \varphi(g_k)$ if $g_j \sim g_k$.
$\varphi(\{g_j,g_k\}) \geqslant \varepsilon$ if $g_j$ and $g_k$ are synergic.
$\varphi(\{g_j,g_k\}) + \varepsilon \leqslant 0$ if $g_j$ and $g_k$ are redundant.
$veto(\varphi_\mu,g_j) + M(1 - v_\mu^{veto}(j,k)) \geqslant veto(\varphi_\mu,g_k) + \varepsilon$ if $g_j \succ_{veto} g_k$.
$veto(\varphi_\mu,g_j) = veto(\varphi_\mu,g_k)$ if $g_j \sim_{veto} g_k$.
$favor(\varphi_\mu,g_j) + M(1 - v_\mu^{favor}(j,k)) \geqslant favor(\varphi_\mu,g_k) + \varepsilon$ if $g_j \succ_{fa\ vor} g_k$.
$favor(\varphi_\mu,g_j) = favor(\varphi_\mu,g_k)$ if $g_j \sim_{favor} g_k$.

It is assumed that each piece of preference information with a higher priority occurs more frequently in maximal subsets of consistent preference information than that with a lower priority. Therefore, the goal aim to find the most credible set composed of maximal consistent preference information.

$$(MIP\ 2)\quad \max \sum_{\lambda_t, t=1,\dots,T} \lambda_t \left( \sum_{a_i,a_h} v(i,j) + \sum_{g_j,g_k} v_\varphi(i,j) + v_\mu^{veto}(i,j) + v_\mu^{favor}(i,j) \right)$$
$$s.t.\quad E^{Basic} \cup \widetilde{E}^{PI}$$
$$v(i,h) \in \{0,1\}$$
$$v_\varphi(j,k),\quad v_\mu^{veto}(j,k), v_\mu^{favor}(j,k) \in \{0,1\}$$

After solving the model MIP2, $v(i,h)=1, v_\varphi(j,k)=1$, $v_\mu^{veto}(j,k) = 1, v_\mu^{favor}(j,k)=1$ for any $(a_i,a_h) \in A \times A$ and $(g_j,g_k) \in G \times G$ associated with indirect preference information represents the consistent preference information which would be restored. Let $f_2^*$ and $v(i,h)^*, v_\varphi(j,k)^*, v_\mu^{veto}(j,k)^*, v_\mu^{favor}(j,k)^*$ be the optimum of the objective function and the optimal solution of model MIP 2. Let $S_2^{Alt} = \{(a_i,a_h) | v(i,j)^* \neq 0\}$ and $S_2^{Cri} = \{(g_j,\ g_k) \big| v_\varphi(j,\ k)^* \neq 0,\ v^{veto}(j,\ k)^* \neq 0,\ v^{favor}(j,\ k)^* \neq 0\}$. Note that there is more than one maximal subset of consistent pairwise comparisons. In some cases, there are multiple possible solutions corresponding to the maximum number of preference information. We can search for more solutions by solving MIP2 with one additional constraint $|\sum_{(a_i,a_h) \in S_2^{Alt}} v(i,h) + \sum_{(g_j,g_k) \in S_2^{Cri}} v_\varphi(j,k) + v_\mu^{veto}(j,k) + v_\mu^{favor}(j,k) \leqslant f_2^* - 1$. Model MP2 is solved continuedly with the extended constraint set until the optimum of the objective function is less than $f_2^*$ The optimal solution at each iteration indicates different restoration solutions.

Other constraints on $veto\ (\mathscr{CI}_\mu, g_j)$ (or $favor\ (\mathscr{CI}_\mu, g_j)$) $(j = 1,\dots,n)$ can also be incorporated into the optimization models. Five types of constraints on $veto\ (\mathscr{CI}_\mu, g_j)$ (or $favor\ (\mathscr{CI}_\mu, g_j)$) $(j = 1,\dots,n)$ are also handled under tolerance framework including: (1) fuzzy preference constraint, such as $veto(\mathscr{CI}_\mu,\ g_1)/(\mathrm{veto}(\mathscr{CI}_\mu,\ g_1) + \mathrm{veto}(\mathscr{CI}_\mu,\ g_2)) = \mathrm{p}_{12}$, where $p_{12}$ denotes a reciprocal fuzzy preference relation with values in the interval [0,1] (Al Salem & Awasthi, 2018); (2) multiplicative preference constraint, such as $veto(\mathscr{CI}_\mu,\ g_1)/\mathrm{veto}(\mathscr{CI}_\mu,\ g_2) = \mathrm{r}_{12}$, where $r_{12}$ denotes a reciprocal multiplicative preference relation with values in the interval scale [1/9,9] (Herrera-Viedma, Herrera, Chiclana, & Luque, 2004); (3) linear inequality constraint, such as $veto(\mathscr{CI}_\mu, g_1) + \mathrm{veto}(\mathscr{CI}_\mu,\ g_2) \leqslant \mathrm{veto}(\mathscr{CI}_\mu,\ g_3)$, $veto(\mathscr{CI}_\mu,\ g_1) \geqslant 0.1$, $veto(\mathscr{CI}_\mu,$

$g_1) - \text{veto}(\mathscr{CI}_\mu, g_2) \geqslant 0.1$, and $\textit{veto}(\mathscr{CI}_\mu, g_1) - \text{veto}(\mathscr{CI}_\mu, g_2) \leqslant \text{veto}(\mathscr{CI}_\mu, g_3) - \text{veto}(\mathscr{CI}_\mu, g_4)$; (4) fuzzypreference inequality constraint, such as $\textit{veto}(\mathscr{CI}_\mu, g_1)/(\text{veto}(\mathscr{CI}_\mu, g_1) + \text{veto}(\mathscr{CI}_\mu, g_2)) \geqslant p_{12}$; and (5) multiplicative preference inequality constraint, such as $\textit{veto}(\mathscr{CI}_\mu, g_1)/\text{veto}(\mathscr{CI}_\mu, g_2)) \geqslant r_{12}$. The constraints (1) and (2) are stricter than (4) and (5). All specified constraints on $\textit{veto}(\mathscr{CI}_\mu, g_j)$ (or $\textit{favor}(\mathscr{CI}_\mu, g_j)$) ($j = 1, \dots, n$) should be consistent with the intrinsic constraint $E^{Base}$. If more than one constraint on $\textit{veto}(\mathscr{CI}_\mu, g_j)$ (or $\textit{favor}(\mathscr{CI}_\mu, g_j)$) ($j = 1, \dots, n$) needs to be handled, the constraints must form a feasible region. Otherwise, the relevant optimization models and problems will have no feasible solutions. For the sake of simplification, tolerability constraints can be simplified as the linear constraints $\{Am \leqslant b\}$ where $A$ is the coefficient matrix and $b$ is constant factor vector related to the preference parameters $\boldsymbol{m}$.

### 3.3. Simulation concern

Cause oriented strategy and consequence oriented strategy are available to DMs simultaneously. When DMs select any aforementioned one for inconsistency management, the obtained preference parameters in Möbius representations may generate a corresponding but different solution. Confronted with the incompatible indirect preference information, guaranteeing necessary consistency (i.e., the removal of inconsistent preference information or the restoration of the consistent preference information) are obviously achievable but not affirmable for DMs, which results in the third research question. To address this obstacle, we concatenate the strategies to resolve the inconsistency of indirect preference information into a Monte Carlo simulation used as a supplement for uncertainty. In the tolerance framework, the primary step is to produce the set of the whole preference parameters compatible with indirect preference information of DMs and further generate a solution in a probabilistic form for robustness concern. In the following we analyze how much the rank acceptability indices and the lexicographic rankings of alternatives change with the tolerance attitudes of DMs when the whole set of the feasible preference parameters is taken into account.

Choquet integral preference model can capture the interaction, importance and tolerance degree of criteria as well as pairwise comparison among alternatives and criteria in this study. The Monte Carlo simulation for robust decision is developed to.

(1) randomly and feasibly generate preference parameters in Möbius representations based on the subjective human judgments;

(2) randomly select the resolution strategies for inconsistent management;

(3) randomly select a possible minimal inconsistent or maximal consistent set of preference constraints.

This simulation is similar to SMAA and can undertake the decision mission under highly uncertain environment. It must be noted that under the term “sensitivity analysis” we change one parameter at a time. On the contrary, using Monte Carlo simulation (like e.g. in SMAA) we can simultaneously change the required parameters (the three random factors in our case) in a systematic way (Mavrotas, et al., 2015). The proposed methodology suggests that all above random variables are subject to the uniform distribution and independent and identically distributed when preferential information is available but uncertainty.

Random generation of the preference parameters is performed with the use of continue uniform distribution and defined on a convex polyhedron constrained in $E^{Base} \cup \widetilde{E}^{PI}$. The application of the Choquet integral defined on fuzzy measures is used as preference aggregation. On the other hand, random generation of the selection strategy is performed with the use of discrete uniform distribution because there are two inconsistency management strategies. On the same note, random generation of the application of the concordant preference information set disclosed by the MIP1 or MIP2 is conducted with the use of discrete uniform distribution. In each Monte Carlo iteration one independent random variable from [1,2] is generated to adopt the corresponding strategy:

$$S^k \overset{i.i.d.}{\sim} U\{1,2\} \tag{11}$$

where random variable $S^k$ generated in the $k$-th iteration determines the strategy selection (1 –cause oriented strategy, 2 –consequence oriented strategy).

$$PI^k \overset{i.i.d.}{\sim} U\{1,2,\dots\} \tag{12}$$

where random variable $PI^k$ generated in the $k$-th iteration determines the concordant preference information set. The number of the concordant preference information is associated the selection of strategies and the intra nature of inconsistency preference information.

Mathematically, Choquet integral preference model $\mathscr{CI}(a_i, \mathbf{m})$ is a map function defined on the basic feasible preference parameter space in Möbius representations denoted as $M = \left\{ \boldsymbol{m} \in R^{n+C_n^2} : \quad m(\varnothing) = 0 \quad ; \quad \forall g_i \in G, \quad T \subsetneqq G/\{g_i\}, \sum_{g_i \in G} m(g_i) + \sum_{\{g_i, g_j\} \in G} m(\{g_i, g_j\}) = 1 \quad \}; \forall i \in N, \quad m(i) \geqslant 0 \quad ; \quad T \neq \varnothing, m(g_i) + \sum_{j \in T} m(\{g_i, g_j\}) \geqslant 0 \right\}$ where the preference parameters follow a continuous uniform distribution with a density function $f_M(\boldsymbol{m}) = 1/vol(M)$. Rank function for counting the rank of alternative $a_i$ is defined as $rank(a_i) = 1 + \sum_{k=1, i \neq k}^{m} \rho(\mathscr{CI}(a_i, \boldsymbol{m}) > \mathscr{CI}(a_k, \mathbf{m}))$ where $\rho(true) = 1$, and $\rho(false) = 0$.

Similar to SMAA, the primary measure, named as the rank acceptability index (RAI) $b_i^r$ is defined which measure the share of different preference parameters to determine preference model and give the $i$-th alternative the $r$-th position in the ranking. More concretely, it is the proportion of all possible configurations of the preference parameters in favorable preference space that makes $a_i$ acceptable to a certain rank $r$. During the subsequent iterations $B_{ir}$ statistic collects the numbers of alternative $a_i$ obtaineing the $r$-th position in the ranking. The second descriptive measure named as central preference parameter vector in Möbius representations is defined which is the barycenter of the feasible preference parameter space assuring that alternative $a_i$ ranks 1. The algorithm of SMAA-like generic simulation analysis is given as following.

Algorithm. Generic simulation analysis

**Input**: Assume a preference model $\mathscr{CI}(a_i, \mathbf{m})$ for ranking with indirect preference information, i.e., the interaction, importance and tolerance of criteria as well as pairwise comparison among alternatives and criteria.
Use mathematical programming to check whether there exist the compatible preference model with DMs' indirect preference information.
***If*** the result is true
  **Compute** $\mathscr{CI}(a_i, \mathbf{m}^*)$ and **Rank** the alternatives according to $\mathscr{CI}(a_i, \mathbf{m}^*)$ where $\boldsymbol{m}^*$ is the optimal preference parameters.
***Else***
Repeat $K$ times {
  **Select** randomly strategies for the inconsistency issues and concordant preference information disclosed by MIP1 or MIP2.
  **Treat** stochastic preference parameters and **Draw** $\mathscr{CI}(a_i, \mathbf{m})$ from their distributions
  **Rank** the alternatives using $\mathscr{CI}(a_i, \mathbf{m})$
  **Update** statistics about alternatives
}
  **Output RAI and central preference parameters** based on the collected statistics

## 4. Simulation experiment

In this section, the illustration from (Angilella, Corrente, & Greco, 2012) is adopted across two extreme scenarios to verify the proposed approach and investigate the influence of the global tolerance of criteria (e.g., tolerability constraints) on the final decision. In the conservative scenario, DMs are prudent to make a decisive decision because each criterion acts as a block role where the performance on any criterion is such a dissatisfactory-one that the corresponding alternative would be

 

**Table 2**
RAIs in the conservative scenario with different $k$-intolerance ($k = 1,2,3,4$).

| | $k$ | $b_i^1$ | $b_i^2$ | $b_i^3$ | $b_i^4$ | $b_i^5$ | $b_i^6$ | $b_i^7$ | $b_i^8$ | $b_i^9$ |
|---|---|---|---|---|---|---|---|---|---|---|
| $a_1$ | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| | 2 | 0 | 0.80 | 0.20 | 0 | 0 | 0 | 0 | 0 | 0 |
| | 3 | 0 | 0.30 | 0.70 | 0 | 0 | 0 | 0 | 0 | 0 |
| | 4 | 0 | 0.14 | 0.83 | 0.03 | 0 | 0 | 0 | 0 | 0 |
| $a_2$ | 1 | 0 | 0 | 0.01 | 0.01 | 0.01 | 0.16 | 0.14 | 0.66 | 0 |
| | 2 | 0 | 0 | 0 | 0 | 0.01 | 0.08 | 0.32 | 0.60 | 0 |
| | 3 | 0 | 0 | 0 | 0 | 0.03 | 0.07 | 0.29 | 0.61 | 0 |
| | 4 | 0 | 0 | 0.01 | 0.02 | 0.08 | 0.07 | 0.33 | 0.49 | 0 |
| $a_3$ | 1 | 0 | 0 | 0.20 | 0.25 | 0.45 | 0.10 | 0 | 0 | 0 |
| | 2 | 0.01 | 0.17 | 0.26 | 0.35 | 0.21 | 0 | 0 | 0 | 0 |
| | 3 | 0.19 | 0.49 | 0.15 | 0.16 | 0 | 0 | 0 | 0 | 0 |
| | 4 | 0.34 | 0.51 | 0.09 | 0.06 | 0 | 0 | 0 | 0 | 0 |
| $a_4$ | 1 | 0 | 0 | 0 | 0 | 0.09 | 0.62 | 0.28 | 0.01 | 0 |
| | 2 | 0 | 0 | 0 | 0 | 0 | 0.87 | 0.13 | 0.01 | 0 |
| | 3 | 0 | 0 | 0 | 0 | 0.01 | 0.71 | 0.27 | 0.01 | 0 |
| | 4 | 0 | 0 | 0 | 0 | 0.10 | 0.61 | 0.28 | 0.01 | 0 |
| $a_5$ | 1 | 0 | 0 | 0 | 0 | 0 | 0.09 | 0.59 | 0.33 | 0 |
| | 2 | 0 | 0 | 0 | 0 | 0 | 0.05 | 0.56 | 0.39 | 0 |
| | 3 | 0 | 0 | 0 | 0.02 | 0.06 | 0.09 | 0.44 | 0.38 | 0 |
| | 4 | 0 | 0 | 0 | 0.01 | 0.07 | 0.05 | 0.36 | 0.50 | 0 |
| $a_6$ | 1 | 0 | 0 | 0 | 0.55 | 0.42 | 0.04 | 0 | 0 | 0 |
| | 2 | 0 | 0 | 0 | 0.22 | 0.77 | 0.01 | 0 | 0 | 0 |
| | 3 | 0 | 0 | 0 | 0.01 | 0.86 | 0.13 | 0 | 0 | 0 |
| | 4 | 0 | 0 | 0 | 0 | 0.70 | 0.27 | 0.03 | 0 | 0 |
| $a_7$ | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| | 2 | 0.99 | 0.01 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| | 3 | 0.81 | 0.19 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| | 4 | 0.66 | 0.34 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $a_8$ | 1 | 0 | 0 | 0.79 | 0.19 | 0.02 | 0 | 0 | 0 | 0 |
| | 2 | 0 | 0.03 | 0.54 | 0.43 | 0 | 0 | 0 | 0 | 0 |
| | 3 | 0 | 0.01 | 0.15 | 0.80 | 0.03 | 0 | 0 | 0 | 0 |
| | 4 | 0 | 0.02 | 0.07 | 0.86 | 0.05 | 0 | 0 | 0 | 0 |
| $a_9$ | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 |
| | 2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 |
| | 3 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 |
| | 4 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 |

denied. On the contrary, in the radical scenario, partial satisfactory performances of the alternative on certain criteria regarded as push roles are aggregated to an adequate overall assessment. For the sake of simplicity, the tolerance attitudes of DMs are given by an ordinal assessment in two scenarios.

The conservative scenario considers a global tolerance with a $k$-intolerant fuzzy measures ($k = 1,\ldots,4$). The DMs can deny an alternative that performs poorly on any $k$ criteria, even though they are likely to perform very well on others. Let $veto\ (\mathscr{CI}_\mu, g_1) > veto\ (\mathscr{CI}_\mu, g_3) > veto\ (\mathscr{CI}_\mu, g_4) > veto\ (\mathscr{CI}_\mu, g_2)$. Jointly with the boundary and monotonicity conditions, the consistency of these preference constraints is checked, and it is verified that there is no inconsistency issue. When the veto index for each criterion is determined, the preference parameters are not to identify the most representative or general one (Kadziński, Ghaderi, Wąsikowski, & Agell, 2017) while the robust one is the target in this study. At the end of all the iterations, RAIs in the conservative scenario are obtained in Table 2.

There are some interesting results. Firstly, as shown in Table 2, with the limitation of tolerance attitude of DMs, the RAIs of alternatives drop slightly especially on the top rank with the increase of $k$ (the criteria number of intolerance), whereas RAIs increase on the bottom rank. Take an example of $a_7$. In 1-intolerance case, the first RAI of $a_7$ is 1, indicating $a_7$ is the best alternative based on the DMs' prior knowledge and global tolerance attitude. 1-tolerance is not so restrictive requirement that the assessment of the alternative $a_7$ would be slightly liberal. An alternative has more possibility to get a higher rank if it outperforms the others but only fails at most one criterion. With the increase of $k$, the first RAI of $a_7$ decreases from 1 to 0.66. The probability of $a_7$ obtaining the best rank deduces. Confronted with the more mandatory requirements on criteria, the advantageous assessment of an alternative is more difficult to acquire than before. It is the solid evidence that the tolerance framework corresponds to the human logic. Secondly, the dispersion of the ordinary alternatives whose the peak RAIs are located in the middle of rank position grows with the increase of $k$. Take examples of $a_3$ and $a_6$. In 1-intolerance case, the greatest RAIs of $a_6$ and $a_3$ are $b_6^4$ (=0.55) and $b_3^5$(=0.45), respectively, indicating that $a_6$ and $a_3$ are the most credible fourth and fifth alternatives based on the DMs' prior knowledge and global tolerance attitude. In 4-intolerance case, the possible ranks of $a_6$ and $a_3$ increase, implying the conservative attitude of DMs. They cannot affirm which is the most predominant alternative.

The radical scenario is with a $k$-tolerance fuzzy measures ($k = 1,2,3,4$). The DMs can prefer alternatives who perform predominantly on any $k$ criteria of four criteria, even though they perform poorly compared with other alternatives on others. Let $favor\ (\mathscr{CI}_\mu, g_1) > favor\ (\mathscr{CI}_\mu, g_3) > favor\ (\mathscr{CI}_\mu, g_4) > favor\ (\mathscr{CI}_\mu, g_2)$. Jointly with the boundary and monotonicity conditions, the consistency of these preference constraints is checked and it is verified that there is no inconsistency issue. At the end of all the iterations, RAIs in the radical scenario are obtained in Table 3.

There are some interesting results. Firstly, as shown in Table 3, with the alleviation of tolerance attitude of DMs, the RAIs of alternatives increase remarkably especially on the top rank with the increase of $k$ (the criteria number of tolerance). By contrast, RAIs decrease on the bottom rank. Take an example of $a_7$. In 1-tolerance case, the first RAI of $a_7$ is 0.66 and the second RAI of $a_7$ is 0.34. Note that 1-tolerance case is relatively restrictive that the assessment of the alternative would not be as liberal as 1- intolerance case. Therefore, with the increase of $k$, the first RAI of $a_7$ increases from 0.66 to 1. The probability of $a_7$ obtaining the first rank grows indicating that the advantageous assessment of any alternative from the DMs is easy compare with the 1- intolerance case. It is obvious that the DMs ease the global tolerance attitudes towards criteria where the alternative with partial satisfactions on some criteria

**Table 3**
RAIs in the radical scenario with different $k$-tolerance ($k = 1,2,3,4$).

| | $k$ | $b_i^1$ | $b_i^2$ | $b_i^3$ | $b_i^4$ | $b_i^5$ | $b_i^6$ | $b_i^7$ | $b_i^8$ | $b_i^9$ |
|---|---|---|---|---|---|---|---|---|---|---|
| $a_1$ | 1 | 0 | 0.14 | 0.83 | 0.03 | 0 | 0 | 0 | 0 | 0 |
| | 2 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 |
| | 3 | 0 | 0.62 | 0.35 | 0.03 | 0 | 0 | 0 | 0 | 0 |
| | 4 | 0 | 0.95 | 0.05 | 0 | 0 | 0 | 0 | 0 | 0 |
| $a_2$ | 1 | 0 | 0 | 0.01 | 0.02 | 0.08 | 0.07 | 0.33 | 0.49 | 0 |
| | 2 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| | 3 | 0 | 0.01 | 0 | 0.01 | 0.05 | 0.12 | 0.27 | 0.48 | 0.05 |
| | 4 | 0 | 0 | 0 | 0 | 0.01 | 0.03 | 0.06 | 0.85 | 0.04 |
| $a_3$ | 1 | 0.34 | 0.51 | 0.09 | 0.06 | 0 | 0 | 0 | 0 | 0 |
| | 2 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| | 3 | 0 | 0.32 | 0.15 | 0.20 | 0.18 | 0.09 | 0.04 | 0.02 | 0 |
| | 4 | 0 | 0.04 | 0.08 | 0.06 | 0.32 | 0.16 | 0.29 | 0.04 | 0 |
| $a_4$ | 1 | 0 | 0 | 0 | 0 | 0.10 | 0.61 | 0.28 | 0.01 | 0 |
| | 2 | 0 | 0 | 0 | 0 | 0 | 0.04 | 0.67 | 0.28 | 0 |
| | 3 | 0 | 0 | 0 | 0.02 | 0.06 | 0.66 | 0.16 | 0.10 | 0 |
| | 4 | 0 | 0 | 0 | 0 | 0.24 | 0.61 | 0.13 | 0.02 | 0 |
| $a_5$ | 1 | 0 | 0 | 0 | 0.01 | 0.07 | 0.05 | 0.36 | 0.50 | 0 |
| | 2 | 0 | 0 | 0 | 0 | 0 | 0.01 | 0.27 | 0.72 | 0 |
| | 3 | 0 | 0 | 0 | 0 | 0.05 | 0.08 | 0.52 | 0.35 | 0 |
| | 4 | 0 | 0 | 0.01 | 0.02 | 0.22 | 0.19 | 0.51 | 0.05 | 0 |
| $a_6$ | 1 | 0 | 0 | 0 | 0 | 0.70 | 0.27 | 0.03 | 0 | 0 |
| | 2 | 0 | 0 | 0 | 0 | 0 | 0.95 | 0.05 | 0 | 0 |
| | 3 | 0 | 0 | 0.07 | 0.30 | 0.58 | 0.04 | 0 | 0 | 0 |
| | 4 | 0 | 0 | 0.33 | 0.57 | 0.11 | 0 | 0 | 0 | 0 |
| $a_7$ | 1 | 0.66 | 0.34 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| | 2 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| | 3 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| | 4 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $a_8$ | 1 | 0 | 0.02 | 0.07 | 0.86 | 0.05 | 0 | 0 | 0 | 0 |
| | 2 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 |
| | 3 | 0 | 0.05 | 0.43 | 0.44 | 0.07 | 0.01 | 0 | 0 | 0 |
| | 4 | 0 | 0.01 | 0.53 | 0.36 | 0.10 | 0 | 0 | 0 | 0 |
| $a_9$ | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 |
| | 2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 |
| | 3 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.05 | 0.95 |
| | 4 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.04 | 0.96 |

**Table 4**
SMEs' criteria evaluations.

| Alternative | $g_1$ | $g_2$ | $g_3$ | $g_4$ | $g_5$ | $g_6$ |
|---|---|---|---|---|---|---|
| $a_1$ | 14 | 18 | 10 | 15 | 16 | 10 |
| $a_2$ | 17 | 15 | 11 | 17 | 14 | 11 |
| $a_3$ | 14 | 18 | 13 | 15 | 16 | 14 |
| $a_4$ | 15 | 19 | 18 | 12 | 17 | 15 |
| $a_5$ | 19 | 19 | 19 | 14 | 15 | 17 |
| $a_6$ | 15 | 12 | 13 | 11 | 14 | 16 |
| $a_7$ | 19 | 11 | 11 | 18 | 13 | 12 |
| $a_8$ | 11 | 14 | 17 | 12 | 16 | 18 |
| $a_9$ | 20 | 20 | 10 | 15 | 12 | 12 |
| $a_{10}$ | 16 | 18 | 13 | 14 | 18 | 20 |
| $a_{11}$ | 11 | 20 | 10 | 17 | 12 | 13 |
| $a_{12}$ | 13 | 17 | 11 | 17 | 15 | 12 |
| $a_{13}$ | 16 | 10 | 19 | 18 | 17 | 12 |
| $a_{14}$ | 20 | 19 | 17 | 13 | 19 | 16 |
| $a_{15}$ | 20 | 20 | 13 | 17 | 20 | 15 |
| $a_{16}$ | 11 | 17 | 20 | 17 | 16 | 13 |
| $a_{17}$ | 20 | 18 | 10 | 11 | 11 | 19 |
| $a_{18}$ | 20 | 18 | 14 | 11 | 11 | 16 |
| $a_{19}$ | 15 | 14 | 14 | 15 | 12 | 16 |
| $a_{20}$ | 18 | 17 | 18 | 20 | 19 | 20 |

are adequate, which is consistent with the human logic. Secondly, alternatives whose RAIs are equal to 1 regardless of the rank position increase, indicating the DMs' radical attitude towards criteria. Those alternatives with adequate performance on certain criteria consistent with the tolerance attitudes of the DMs have chance to rank better.

## 5. Illustrative example

### 5.1. Application in the project selection problem

A hypothetical 20 SMEs (small and medium-sized enterprises) credit ranking problem on considered six criteria is analyzed in this section, which is adopted from (Angilella & Mazzù, 2015). There are six criteria denoted as development risk ($g_1$), technological risk ($g_2$), market risk ($g_3$), production risk ($g_4$), innovation capability ($g_5$) and finance capability ($g_6$). The evaluations on criteria of SMEs have been normalized on a common scale in Table 4.

For the sake of simplicity, 2- tolerant or 2-intolerant fuzzy measures are considered here. For example, a SME performs well on financial indicator ($g_6$) and innovation indicators ($g_5$) but poorly in other indicators. In this regard, the DMs (credit officers in the bank) think it qualified well in the long term. By contrast, a SME is perceived as unqualified because it performs deficiently on technological risk ($g_2$) and market risk ($g_3$). In the former case, the DMs regards finance capability ($g_6$) and innovation capability ($g_5$) as favor criteria, which work more like a pusher than other criteria. In other words, the DMs exert more expectation on these two criteria (e.g., $g_5$ and $g_6$) than other criteria. However, technological risk ($g_2$) and market risk ($g_3$) act as veto criteria, indicating a block effect than other criteria. Namely, the DMs exert more restrictions on these two criteria (e.g. $g_2$ and $g_3$) than any other criteria. Assume that the weights of four DMs in the group are equal. Indirect preference information is expressed as follows:

- $\varphi\ (1) > \varphi\ (2) > \varphi\ (4) > \varphi\ (5) > \varphi\ (3) > \varphi\ (6)$
- $veto\ (5) > veto(3) > veto(4)$
- $favor\ (6) > favor(1) > favor(2) > favor(4)$
- $a_1 \succ a_2 \succ a_3 \succ a_4 \succ a_5 \succ a_6$

**Table 5**
The partial result when adopting the cause-oriented strategy.

| $\varphi_1 >$ $\varphi_2$ | $\varphi_2 >$ $\varphi_4$ | $\varphi_4 >$ $\varphi_5$ | $\varphi_5 >$ $\varphi_3$ | $\varphi_3 >$ $\varphi_6$ | $veto_5 >$ $veto_3$ | $veto_3 >$ $veto_4$ | $favor_6 >$ $favor_1$ | $favor_1 >$ $favor_2$ | $favor_2 >$ $favor_4$ | $U_1 >$ $U_2$ | $U_2 >$ $U_3$ | $U_3 >$ $U_4$ | $U_4 >$ $U_5$ | $U_5 >$ $U_6$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| √[a*] | √ | ×[b*] | √ | √ | √ | √ | √ | √ | √ | √ | × | × | √ | √ |
| √ | √ | √ | √ | √ | √ | √ | √ | √ | √ | × | √ | × | × | √ |
| √ | √ | √ | √ | √ | √ | × | √ | √ | √ | × | √ | √ | × | √ |

a*: the piece of preference information is feasible.
b*: the piece of preference information is not feasible.

Based on transformation rule in Section 3.1, the above pieces of preference information can be converted into the linear constraints. Taking the preference information concerning the veto effect on criteria as an example, the tolerability constrains are converted as follows:

$veto(5) > veto(3) \Rightarrow -3/5^*m_3 + 3/5^*m_5 - 2/5^*m_{\{1,3\}} + 2/5^*\, m_{\{1,5\}} - 2/5^*m_{\{2,3\}} + 2/5^*\, m_{\{2,5\}}\text{-}\ 2/5^*m_{\{3,4\}} - 2/5^*m_{\{3,6\}} + 2/5^*\, m_{\{4,5\}} + 2/5^*m_{\{5,6\}} > 0;$

$veto(3) > veto(4) \Rightarrow 3/5^*m_3 - 3/5^*m_4 + 2/5^*m_{\{1,3\}}\text{-}\ 2/5^*m_{\{1,4\}} + 2/5^*m_{\{2,3\}} - 2/5^*m_{\{2,4\}} + 2/5^*m_{\{3,5\}} + 2/5^*m_{\{3,6\}} - 2/5^*m_{\{4,5\}} - 2/5^*m_{\{4,6\}} > 0.$

At the beginning of group decision making process, the DMs provide indirect preference information, whereas some controversy opinion may exist. The resolution is to disclose and conserve the consistent information to proceed the decision. For example, the cause-oriented strategy is adopted and the minimal subsets resulting in the inconsistency are recognized by MIP1. The first three of them are showed in Table 5.

For robust concern, we propose and run the SMAA-like simulation algorithm to compute RAIs and central preference parameters in Möbius representations though the favorable preference parameter space. The RAIs of alternatives are showed and illustrated in Table 6 and Fig. 1. Fig. 1**.** visually shows the probabilities that each alternative obtains the rank from 1 to 20. Table 6 indicates that RAIs of alternatives $a_{20}$, $a_{15}$ and $a_{14}$ are pretty large for best ranks and extremely small for worst ranks. For example, $a_{20}$ has quite large RAIs for the best ranks (61.1 % for rank 1, 31.3 % for rank 2, and they add up to 92.4 %) and zero RAIs for the worst ranks (0 % for rank 10–20). $a_{15}$ and $a_{14}$ have similar RAI distributions where the sum of the 2nd and 3rd RAI of $a_{15}$.

and $a_{14}$ are respectively up to 60.2 % and 60.0 % while the 1st RAI of $a_{15}$ (equal to 38.6 %) is far greater than.

$a_{14}$'s (close to 0.3 %) so that $a_{15}$ is superior to $a_{14}$. We can conclude that $a_{20}$, $a_{15}$ and $a_{14}$ are the most credible SMEs, since they are almost impossible to obtain the worst credit rank through the favorable preference parameter space. The last three RAIs of alternatives $a_{17}$, $a_{11}$ and $a_6$ are almost equal to 50 %, indicating that these alternatives have the great higher possibility of defaulting. According to the holistic RAI (Lahdelma & Salminen, 2001) measuring the overall performance of alternatives, the full ranking sequence of the 20 alternatives determined by simulation analysis are:

$a_{20} \succ a_{15} \succ a_{14} \succ a_5 \succ a_{10} \succ a_4 \succ a_3 \succ a_{16} \succ a_{13} \succ a_2 \succ a_9 \succ a_{12} \succ a_1 \succ a_7 \succ a_{19} \succ a_{18} \succ a_8 \succ a_{17} \succ a_6.$

Fuzzy measures, defined over subsets of criteria in the decision model, provide a natural basis for analyzing criteria interaction and importance. The central preference parameters for projects $a_{20}$, $a_{15}$ and $a_{14}$ are given in Fig. 2. According to the definition of central preference parameters, an alternative will have a larger probability to get the best rank if the sampled fuzzy measures in Möbius representations are closer to its central one. It suggests that the arbitrary selection of the optimal preference parameters may make an alternative the best. In the case study, all of the criteria are benefit type (i.e. the larger the better), hence, those criteria with large criteria assessments should be assigned with relatively large weights to guarantee the best rank for a specific alternative. Overall, the shapely indices Table 7 computed by central preference parameters in rationally reflect the weighting rule when each alternative gets the first rank. Due to the irreversibility of decision making, DMs usually face huge psychological pressure about indirect preference information because MCDA results depend greatly on it and will have significant impacts on the credit risk. In conclusion, central preference parameters are useful to help the DMs express their preferences considering various situations.

As shown in Fig. 2, taking an example of $a_{14}$, the interaction between criterion $g_4$ and any other criteria are negative. That implies disjunctive behavior (subadditivity) of criterion $g_4$ across all other criteria when assessing alternative $a_{14}$. More concretely, the DMs are willing to compensate for the weakness of one criterion with the strength of the other. In fact, it may be interpreted that they seem to consider the satisfaction of one of the two criteria as sufficient. When the sign of the interaction index is positive for two criteria, it means that considering them together is much more important for the decision problem than considering them separately; that is, they are synergistic or complementary. With respect to $a_{20}$, all the interaction indices of $g_4$ across the other criteria except $g_2$ are close to zero, which implies a certain degree of neutral attitudes.

As shown in Table 8, the veto indices are greater than favor indices across all criteria. This confirms that risk-averse DMs in the group demand that all criteria be satisfied to a relatively higher degree than their risk-seeking counterparts. We note from Table 8 that andness index of these three promising SMEs are quite higher than orness index, implying the robustness of result. A higher andness degree indicates a conjunctive aggregation behavior with the comprehensive score near to the minimum of the criteria evaluations. DMs would deny those the evaluations of alternatives failing on at most $k$ criteria. Only one of the criteria not doing well would bring the aggregated value down with high andness. An alternative tends to have a high comprehensive score only when it has all the criteria evaluations with high values. The most credible SMEs are selected with the mandatory requirements. Overall, DMs are conservative to decide which is consist with the problem background. Such information regarding andness or orness in aggregation behavior provides vital information for modelling DMs behavior.

### 5.2. Compassion with the sole selection strategy for inconsistency issue

What has been analyzed above is on the condition that the unified tolerance framework is adopted without any arbitrary selection for strategy and concordant preference information. It is an interesting question to find out what will happen if the sole strategy and concordant preference information is adopted. Answering this question helps to highlight the significant influence of tolerance framework on solutions to the credit ranking problem.

To examine the influence of the sole strategy on preference elicitation to the SMEs credit ranking problem, we find the solution to the SMEs credit ranking problem under the conditions where the cause oriented strategy is adopted and compare it with the solutions generated in Section 5.1. As explained in Section 3, we need to check whether the preference constraints are compatible with the indirect preference information provided by the DMs. We solve$\varepsilon^* = \max\varepsilon$, s.t. $E^{Basic} \cup E^{PI}$and find that$\varepsilon^* < 0$, indicating that there is no Choquet integral preference model that can restore indirect preference information. Specifically, there is no admissible solutions in the linear system$E^{Basic} \cup E^{PI}$. Thus let us identify the minimal subsets of inconsistent pairwise comparisons for DMs. Set $\varepsilon = 0.0001$ and solve MIP1. In the first iteration, the minimal

**Table 6**
The RAI of the whole SMEs.

| | $b^1$ | $b^2$ | $b^3$ | $b^4$ | $b^5$ | $b^6$ | $b^7$ | $b^8$ | $b^9$ | $b^{10}$ | $b^{11}$ | $b^{12}$ | $b^{13}$ | $b^{14}$ | $b^{15}$ | $b^{16}$ | $b^{17}$ | $b^{18}$ | $b^{19}$ | $b^{20}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $a_1$ | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.001 | 0.014 | 0.053 | 0.126 | 0.069 | 0.091 | 0.083 | 0.127 | 0.133 | 0.076 | 0.107 | 0.102 | 0.016 | 0.002 | 0.000 |
| $a_2$ | 0.000 | 0.000 | 0.000 | 0.000 | 0.148 | 0.011 | 0.029 | 0.081 | 0.116 | 0.072 | 0.099 | 0.077 | 0.069 | 0.168 | 0.055 | 0.033 | 0.025 | 0.015 | 0.000 | 0.000 |
| $a_3$ | 0.000 | 0.000 | 0.000 | 0.000 | 0.040 | 0.320 | 0.114 | 0.072 | 0.085 | 0.111 | 0.114 | 0.089 | 0.039 | 0.013 | 0.003 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| $a_4$ | 0.000 | 0.000 | 0.003 | 0.106 | 0.105 | 0.086 | 0.313 | 0.131 | 0.060 | 0.036 | 0.098 | 0.060 | 0.001 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| $a_5$ | 0.000 | 0.280 | 0.073 | 0.134 | 0.092 | 0.094 | 0.047 | 0.149 | 0.041 | 0.011 | 0.002 | 0.023 | 0.050 | 0.004 | 0.001 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| $a_6$ | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.003 | 0.002 | 0.005 | 0.107 | 0.467 | 0.113 | 0.302 |
| $a_7$ | 0.000 | 0.000 | 0.002 | 0.010 | 0.023 | 0.022 | 0.022 | 0.001 | 0.033 | 0.084 | 0.046 | 0.106 | 0.111 | 0.048 | 0.090 | 0.097 | 0.066 | 0.035 | 0.154 | 0.050 |
| $a_8$ | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.011 | 0.008 | 0.005 | 0.005 | 0.114 | 0.038 | 0.045 | 0.031 | 0.070 | 0.054 | 0.147 | 0.278 | 0.043 | 0.064 | 0.086 |
| $a_9$ | 0.000 | 0.000 | 0.000 | 0.000 | 0.066 | 0.055 | 0.043 | 0.033 | 0.082 | 0.048 | 0.029 | 0.074 | 0.120 | 0.124 | 0.161 | 0.073 | 0.049 | 0.015 | 0.027 | 0.000 |
| $a_{10}$ | 0.000 | 0.000 | 0.000 | 0.287 | 0.303 | 0.206 | 0.162 | 0.038 | 0.004 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| $a_{11}$ | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.002 | 0.009 | 0.053 | 0.053 | 0.048 | 0.079 | 0.090 | 0.093 | 0.164 | 0.080 | 0.170 | 0.140 | 0.019 |
| $a_{12}$ | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.007 | 0.008 | 0.026 | 0.179 | 0.189 | 0.162 | 0.050 | 0.104 | 0.064 | 0.085 | 0.044 | 0.070 | 0.011 | 0.000 | 0.000 |
| $a_{13}$ | 0.000 | 0.000 | 0.049 | 0.053 | 0.049 | 0.026 | 0.086 | 0.087 | 0.128 | 0.090 | 0.053 | 0.078 | 0.093 | 0.095 | 0.029 | 0.030 | 0.013 | 0.000 | 0.000 | 0.040 |
| $a_{14}$ | 0.003 | 0.200 | 0.401 | 0.385 | 0.011 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| $a_{15}$ | 0.386 | 0.207 | 0.397 | 0.009 | 0.001 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| $a_{16}$ | 0.000 | 0.000 | 0.000 | 0.015 | 0.080 | 0.116 | 0.131 | 0.203 | 0.075 | 0.077 | 0.091 | 0.095 | 0.045 | 0.014 | 0.009 | 0.043 | 0.005 | 0.000 | 0.000 | 0.000 |
| $a_{17}$ | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.001 | 0.004 | 0.010 | 0.034 | 0.020 | 0.015 | 0.013 | 0.015 | 0.022 | 0.053 | 0.053 | 0.095 | 0.086 | 0.138 | 0.441 |
| $a_{18}$ | 0.000 | 0.000 | 0.000 | 0.000 | 0.005 | 0.043 | 0.018 | 0.070 | 0.018 | 0.012 | 0.045 | 0.054 | 0.067 | 0.101 | 0.109 | 0.070 | 0.043 | 0.078 | 0.217 | 0.050 |
| $a_{19}$ | 0.000 | 0.000 | 0.000 | 0.000 | 0.079 | 0.000 | 0.000 | 0.039 | 0.003 | 0.013 | 0.064 | 0.105 | 0.048 | 0.050 | 0.178 | 0.134 | 0.066 | 0.063 | 0.145 | 0.012 |
| $a_{20}$ | 0.611 | 0.313 | 0.076 | 0.001 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

set of inconsistent preference constraints is $\{\varphi(4) > \varphi(5),\ a_2 \succ a_3,\ a_3 \succ a_4\}$. Without the loss of generality, it is assumed that the DMs discard this one. According to the holistic RAI measuring the overall performance of alternatives, the full ranking sequence of the 20 alternatives determined by simulation analysis are:

$$a_{20} \succ a_{14} \succ a_{15} \succ a_4 \succ a_5 \succ a_{10} \succ a_{13} \succ a_{16} \succ a_8 \succ a_3 \succ a_{12} \succ a_6 \succ a_1 \succ a_2 \succ a_{19} \succ a_{18} \succ a_7 \succ a_9 \succ a_{11} \succ a_{17}$$

This indicates that $a_{20}$, $a_{14}$, $a_{15}$, $a_4$ and $a_5$ are the most appropriate three options with the preferential order of$a_{20} \succ a_{14} \succ a_{15} \succ a_4 \succ a_5$. This solution is clearly different from the solutions generated from a robust perspective when tolerance framework is adopted because $a_{14}$, $a_{15}$ and $a_5$ have changed from the third, second, and fourth best choices to the second, third, and five best choices, respectively. $a_{10}$ enters the top five SMEs and $a_4$ falls out of the top five rank. The observation indicates the significant influence of the sole strategy on the credit ranking order of the SMEs.

It can be seen that the rank of alternatives changes slightly. However, the difference mainly occurs in the middle of rank position. When the DMs focus on the promising alternatives, the two methods would not have a big influence. However, when the DMs focus on the ordinary alternatives, in this credit ranking problem, it is difficult to make a convincible decision. Then, we select three RAIs (i.e.,$b_{20}^1$,$b_4^5$, $b_9^{15}$) with low, medium and high values to perform the robustness analysis concerning sampling variability. Fig. 3 illustrates the box plot of these RAIs and holistic RAIs under the 200 replicates. As can be seen, the three RAIs vary within small ranges and holistic RAI vary very small ranges where the box plots almost narrow into lines. The results indicate that the proposed simulation algorithm is robust with respect to sampling variability in terms of the RAI.

### 5.3. Comparison with objective methods

To validate the proposed tolerance framework, it is compared with three existing methods for preference elicitation by addressing the credit ranking problem, including the maximum entropy method (Li, et al., 2018), the maximum deviation method (Lo & Guo, 2010) and the maximum orness/andness method (Beliakov & Wu, 2019; Chen, Yang, Wang, Chin, & Tsui, 2019). Three methods are objective to handle preference elicitation in the context of fuzzy measure decision making. For this reason, the three methods are selected to be compared with the tolerance framework. The brief introduction of each method is shown as following.

$$(LP\ \ 4)\qquad \max \sum_{g_j\in G} \sum_{S\subseteq G\backslash g_j} \frac{(n-s-1)!s!}{n!}\, h\,[\mu(S\cup i)-\mu(S)],$$

$$h(x)=\begin{cases} -x\ln x, & x>0 \\ 0, & x=0 \end{cases}$$

$$s.t.\qquad E^{Basic}\cup \widetilde{E}^{PI}$$

$$(LP\ \ 5)\qquad \max \sum_{\substack{i,h=1,\ldots,m \\ i\neq h}} (U(a_i)-U(a_h))^2$$

$$s.t.\qquad E^{Basic}\cup \widetilde{E}^{PI}$$

$$(LP\ \ 6)\qquad \max/\min \quad Orness(\mathscr{CI}_\mu)/Andness(\mathscr{CI}_\mu)$$

$$s.t.\qquad E^{Basic}\cup \widetilde{E}^{PI}$$

The three methods are used to determine preference parameters in Möbius representations. Based on the three sets of the optimal preference parameters, the overall assessment and ranking order of SEMs are obtained by using Choquet integral preference model as presented in

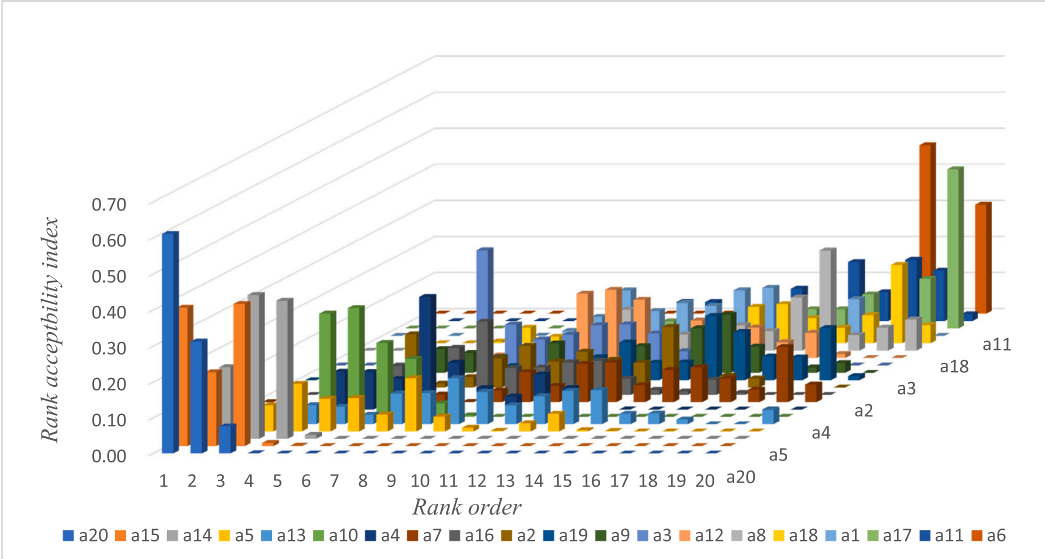


**Fig. 1.** RAIs for twenty SMEs with the DMs' tolerance attitudes on criteria.

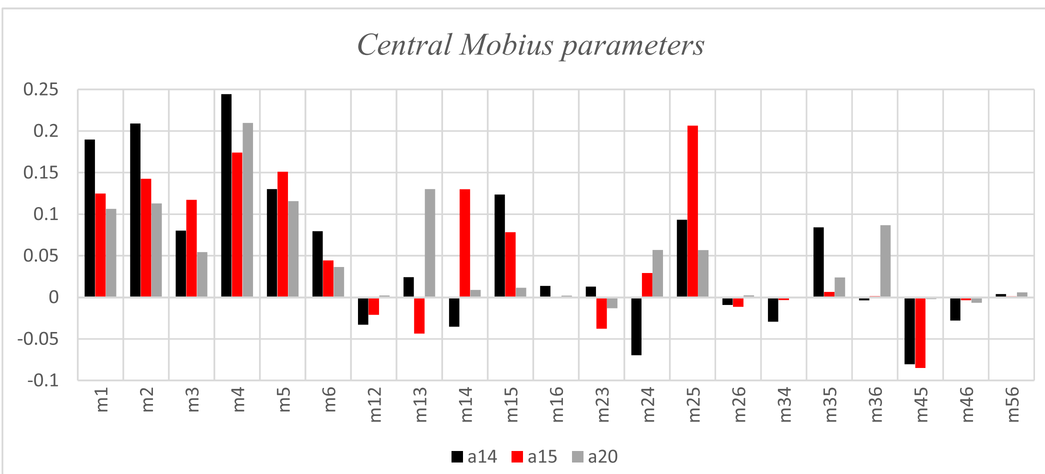


**Fig. 2.** Central preference parameters for three SMEs with non-zero first RAI.

**Table 7**
The shapely index of alternatives with central preference parameters.

| | $\varphi$ (1) | $\varphi$ (2) | $\varphi$ (3) | $\varphi$ (4) | $\varphi$ (5) | $\varphi$ (6) |
|---|---|---|---|---|---|---|
| $a_{14}$ | 0.236 | 0.206 | 0.124 | 0.123 | 0.242 | 0.068 |
| $a_{15}$ | 0.196 | 0.225 | 0.078 | 0.208 | 0.255 | 0.038 |
| $a_{20}$ | 0.184 | 0.165 | 0.168 | 0.238 | 0.164 | 0.082 |

**Table 8**
The global/local tolerance of criteria of alternatives with non-zero first RAI.

| | | $g_1$ | $g_2$ | $g_3$ | $g_4$ | $g_5$ | $g_6$ | | |
|---|---|---|---|---|---|---|---|---|---|
| $a_{14}$ | veto | 0.565 | 0.537 | 0.497 | 0.463 | 0.581 | 0.452 | andness | 0.516 |
| | favor | 0.291 | 0.260 | 0.245 | 0.191 | 0.353 | 0.334 | orness | 0.279 |
| $a_{15}$ | veto | 0.581 | 0.601 | 0.489 | 0.580 | 0.623 | 0.471 | andness | 0.557 |
| | favor | 0.304 | 0.298 | 0.263 | 0.260 | 0.302 | 0.321 | orness | 0.292 |
| $a_{20}$ | veto | 0.599 | 0.583 | 0.596 | 0.621 | 0.581 | 0.531 | andness | 0.585 |
| | favor | 0.294 | 0.280 | 0.228 | 0.213 | 0.277 | 0.323 | orness | 0.269 |

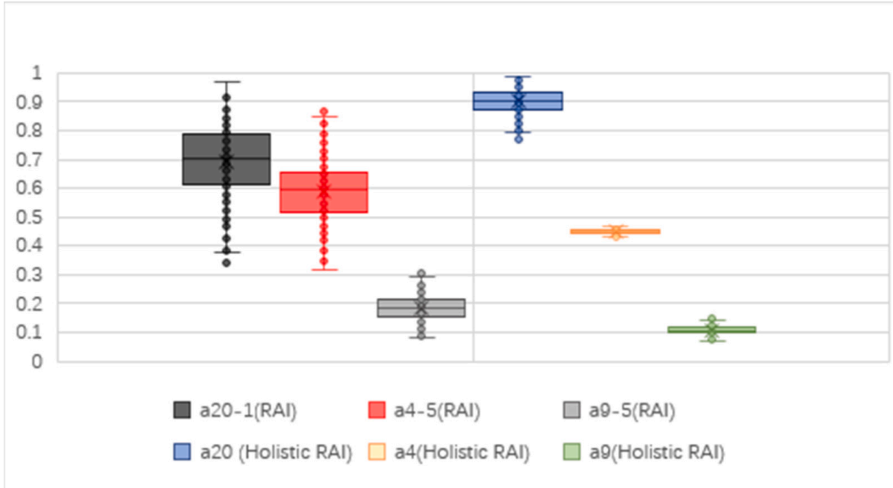


**Fig. 3.** Box plots of three RAI ($b_{20}^{1}$, $b_{4}^{5}$,$b_{9}^{15}$) and holistic RAI ($a_{20}$, $a_4$, $a_9$) under the 200 replicates.

**Table 9**
The top ten overall assessments and ranking orders by using the objective methods.

| The top 10 SMEs | | | | | | | |
|---|---|---|---|---|---|---|---|
| Maximum Entropy | | Maximum Deviation | | Maximum Orness | | Maximum Andness | |
| Rank | score | Rank | score | Rank | score | Rank | score |
| $a_{20}$ | 18.54 | $a_{20}$ | 18.36 | $a_{15}$ | 19.99 | $a_{20}$ | 18.19 |
| $a_{14}$ | 17.45 | $a_{14}$ | 17.46 | $a_{20}$ | 19.00 | $a_{14}$ | 17.44 |
| $a_{15}$ | 17.30 | $a_4$ | 17.03 | $a_{14}$ | 19.00 | $a_{10}$ | 16.33 |
| $a_{10}$ | 16.40 | $a_5$ | 17.03 | $a_{16}$ | 18.66 | $a_{15}$ | 16.09 |
| $a_4$ | 16.37 | $a_{16}$ | 16.59 | $a_{13}$ | 18.32 | $a_4$ | 16.05 |
| $a_5$ | 16.37 | $a_{13}$ | 15.81 | $a_{10}$ | 18.00 | $a_5$ | 16.05 |
| $a_{16}$ | 15.39 | $a_{15}$ | 15.80 | $a_4$ | 17.66 | $a_3$ | 14.47 |
| $a_{13}$ | 15.27 | $a_{10}$ | 15.64 | $a_5$ | 17.66 | $a_8$ | 14.42 |
| $a_3$ | 15.08 | $a_8$ | 15.54 | $a_8$ | 16.66 | $a_{16}$ | 14.05 |
| $a_8$ | 14.69 | $a_3$ | 14.23 | $a_3$ | 16.00 | $a_{13}$ | 13.69 |

**Table 10**
The last ten overall assessments and ranking orders by using the objective methods.

| The last 10 SMEs | | | | | | | |
|---|---|---|---|---|---|---|---|
| Maximum Entropy | | Maximum Deviation | | Maximum Orness | | Maximum Andness | |
| Rank | score | Rank | score | Rank | score | Rank | score |
| $a_{12}$ | 13.96 | $a_6$ | 13.61 | $a_1$ | 15.99 | $a_6$ | 13.30 |
| $a_{19}$ | 13.66 | $a_{19}$ | 13.29 | $a_{12}$ | 15.00 | $a_{12}$ | 12.82 |
| $a_1$ | 13.53 | $a_{18}$ | 12.98 | 6 | 14.00 | $a_{19}$ | 12.58 |
| $a_2$ | 13.53 | $a_{12}$ | 12.57 | $a_2$ | 14.00 | $a_1$ | 12.21 |
| $a_9$ | 13.5 | $a_1$ | 12 | $a_{19}$ | 13.33 | $a_2$ | 12.21 |
| $a_6$ | 13.45 | $a_2$ | 12 | $a_{18}$ | 13.00 | $a_{18}$ | 12.10 |
| $a_{11}$ | 13.43 | $a_7$ | 11.9 | $a_7$ | 13.00 | $a_7$ | 11.91 |
| $a_{18}$ | 13.41 | $a_{17}$ | 11.68 | $a_9$ | 12.00 | $a_{11}$ | 11.75 |
| $a_7$ | 13.39 | $a_9$ | 11.13 | $a_{11}$ | 12.00 | $a_9$ | 11.75 |
| $a_{17}$ | 13.17 | $a_{11}$ | 11.04 | $a_{17}$ | 11.01 | $a_{17}$ | 11.14 |

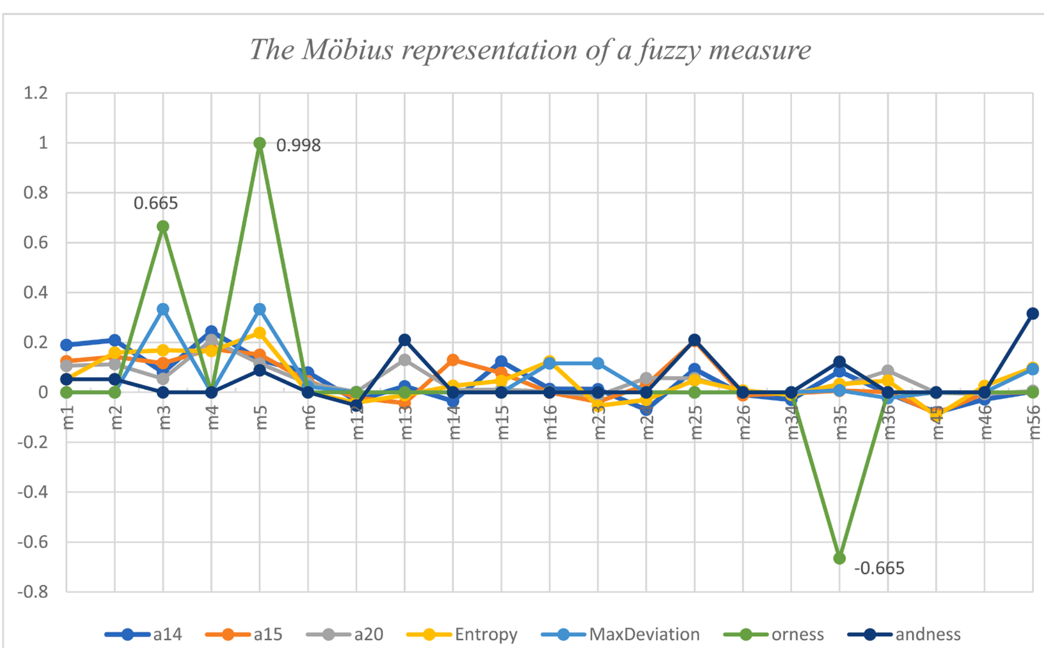


**Fig. 4.** The comparison of fuzzy measure in Möbius presentation.

Tables 8 and 9. And the optimal fuzzy measures in Möbius representations using three objective method are also shown in Fig. 4.

As shown in Table 9, when the objective methods of entropy to determine preference parameters, the most appropriate four SMEs are $a_{20}$, $a_{14}$, $a_{15}$ and $a_{10}$ with the preferential order of$a_{20} \succ a_{14} \succ a_{15} \succ a_{10}$. When the objective methods of deviation to determine preference parameters, the most appropriate four SMEs are $a_{20}$, $a_{14}$, $a_4$ and $a_5$ with the preferential order of$a_{20} \succ a_{14} \succ a_4 \succ a_5$. When the objective methods of orness index to determine preference parameters, the most appropriate four SMEs are $a_{15}$, $a_{20}$, $a_{14}$, and $a_{16}$ with the preferential order of$a_{15} \succ a_{20} \succ a_{14} \succ a_{16}$. When the objective methods of andness index to determine preference parameters, the most appropriate four SMEs are $a_{20}$, $a_{14}$, $a_{10}$ and $a_{15}$ with the preferential order of$a_{20} \succ a_{14} \succ a_{15} \succ a_{10}$. Computed by the proposed tolerance framework, the most appropriate six SMEs are $a_{20}$, $a_{15}$, $a_{14}$, $a_5$, $a_{10}$ and $a_4$ with the preferential order of$a_{20} \succ a_{15} \succ a_{14} \succ a_5 \succ a_{10} \succ a_4$. To sum up, the top four alternatives obtained by the objection methods are the same as the tolerance framework's but the rank order of them is dissimilar from each other. The highest overall score of the best alternative ($a_{20}$ or $a_{15}$) by objective methods is 19.99 which is computed by the orness index. And the lowest overall score of the best alternative ($a_{20}$ or $a_{15}$) by objective methods is 18.19 which is computed by the andness index. The overall score of the best alternative ($a_{20}$ or $a_{15}$) by other two methods consists in the interval of these two extreme values. Maximum orness index refers to an aggregation behavior approaching the maximum comprehensive score of alternatives, whereas maximum andness index refers an aggregation behavior approaching the minimum comprehensive score of alternatives. Simulation experimental and numerical results, well consistent with the theoretical predictions, have validated the proposed scheme.

As shown in Table 10, when the objective methods of entropy to determine preference parameters, the most inappropriate four SMEs are $a_{11}$, $a_{18}$, $a_7$ and $a_{17}$ with the preferential order of$a_{11} \succ a_{18} \succ a_7 \succ a_{17}$. When the objective methods of deviation to determine preference parameters, the most inappropriate four SMEs are $a_7$, $a_{17}$, $a_9$ and $a_{11}$ with the preferential order of$a_7 \succ a_{17} \succ a_9 \succ a_{11}$. When the objective methods of orness index to determine preference parameters, the most appropriate four SMEs are $a_7$, $a_9$, $a_{11}$, and $a_{17}$ with the preferential order of$a_7 \succ a_9 \succ a_{11} \succ a_{17}$. When the objective methods of andness index to determine preference parameters, the most inappropriate four SMEs are $a_7$, $a_{11}$, $a_9$ and $a_{17}$ with the preferential order of$a_7 \succ a_{11} \succ a_9 \succ a_{17}$. Computed by the proposed tolerance framework, the most inappropriate five SMEs are $a_{19}$, $a_{18}$, $a_8$, $a_{17}$ and $a_6$ with the preferential order of$a_{19} \succ a_{18} \succ a_8 \succ a_{17} \succ a_6$. To sum up, although the last four alternatives obtained by the objection methods are slightly dissimilar from each other, they all have one thing in common: the least SME is $a_{11}$ or $a_{17}$. The highest overall score of the worst alternative ($a_{17}$ or $a_{11}$) by objective methods is 13.17 which is computed by maximum entropy method. The lowest overall score of the worst alternative ($a_{17}$ or $a_{11}$) by objective methods is 11.01 which is computed by the orness index. Other comprehensive scores of inappropriate alternatives computed by the maximum entropy method and maximum orness index are higher than those by the maximum deviation method and maximum orness index. The two extreme aggregation behaviors have been manifested.

Tables 9 and 10 show that the overall assessments and ranking orders of alternatives obtained by using SMAA-like simulation analysis are slightly different from those obtained by using the objective methods such as maximum entropy, maximum deviation and maximum orness/andness index. Different from the three methods, the proposed tolerance framework does not intend to create a unique set of preference parameters but to consider set of the whole preference parameters in Möbius representations that are generated compatibly with DMs' indirect preference information. This supports the fact that tolerance framework with the provisions to consider the DMs' tolerance attitudes, models the real-world human decision-making processes with a reasonable accuracy. The different propensity of tolerance of criteria guarantees the distinguished contributions of six criteria to twenty SMEs. Extreme preference parameters that are beneficial to any SMEs are effectively avoided. Or else, some criteria may contribute almost nothing to the solution, which may not be what the DMs anticipates in general. Compared with using one optimal set of preference parameters to generate a solution, considering the set of the whole preference parameters compatible with indirect preference information seems more reliable and convinced.

As shown in Fig. 4, preference parameters in Möbius representations determined by the three methods have certain differences. Furthermore, there are certain differences in the credit ranking of SMEs, which also reflects that the optimal solution under the single objective is not the best in reality. The arbitrary choice of consistent preference information also invalidates the comprehensive evaluation. In particular, the optimal solutions based on the objectives of maximum deviation and orness index fluctuate strongly. Maximum deviation method only emphasizes the generation of preference parameters through the distances of various alternatives, as far as possible. Maximum orness index method only emphasizes the generation of preference parameters through the disjunctive aggregation behavior. The single-objective optimization method may not achieve the pareto optimal even if a certain scheme under this goal has a good performance. In the near neighborhood of the optimal preference parameters or even in the feasible space, the integrated score for some alternatives can be improved. However, the method in this paper is more robust and the results are given in the form of probability distribution. It can be seen intuitively to see the pros and cons of the alternatives, and it has a better advantage in resisting instability.

## 6. Conclusion

In this paper, under a robust perspective, we present a tolerance framework to address a group multiple criteria ranking problem with indirect preference information such as the interaction, importance and tolerance of criteria as well as pairwise comparison among alternatives and criteria. Based on indirect preference information, Choquet integral preference model is elicited to capture the interaction, importance and tolerance of criteria for the final decision. With respect to the tolerance attitudes of DMs such as the mandatory/sufficient requirements on criteria for specific alternatives, they are quantified as tolerability constraints for preference elicitation. Once inconsistency issue (the feasibility of the whole preference constraints) in preference constructive learning process occurs, in the tolerance framework, two developed strategies including cause oriented strategy and consequence oriented strategy are applied. Then identify the minimal unsatisfied subsets of preference constraints responsible for the inconsistency or the maximum satisfied subsets of preference constraints as a specific set of consistent preference information by regression-based mixed 0-1 integer linear programs. By following the idea of treating all possible sets of preference parameters in line with indirect preference information, SMAA-like simulation algorithm is constructed to generate the rank result in the probabilistic form, which is used to generate a solution considered by exploring the whole instances of the compatible preference model. The proposed method can deal with indirect preference information involved with not only less cognitive effort but also the tolerance attitudes on criteria for the DMs, which has been paid insufficient attention in the literatures. From the illustration and application, it suggested that the proposed unified framework for MCDA has two characteristics: expressiveness of the underlying preference model which can reconstruct indirect preference information provided by the DMs and robustness of the final result which address uncertainties and imprecision observed in the actual decision support processes.

What we investigate in this study is a new attempt for preference elicitation capturing both pairwise comparison among alternatives criteria and the tolerance, importance and interaction of criteria, which are not considered in existing studies on how to get a robust solution in the context of group MCDM. The restriction on the consistency of indirect preference information is significant in the process of preference elicitation which are used to generate a solution as reported in this study, but a trade-off between the number of the removal of certain preference information and the feasibility of preference constraints are not investigated with the tolerance framework, which drives more research to be conducted. Besides, if the form of prefer model is too sophisticated to set, preference handling with the assistance of evidence theory defining on the selected preference model space is worth exploring. Note that indirect preference information in the proposed framework can be perceived as training set compared with preference learning, which is a subfield of machine learning. It is still an open question that how to learn and interpret the preference from massive preference information and how to accelerate algorithms to reduce the computation time and cost, especially considering the tolerability constraints on criteria.

## CRediT authorship contribution statement

**Yu Yang:** Conceptualization, Methodology, Data curation, Formal analysis, Writing – original draft, Visualization. **Jun Lin:** Supervision, Writing – review & editing, Funding acquisition. **Yelin Fu:** Investigation, Methodology, Validation, Funding acquisition. **George Q. Huang:** Supervision, Investigation, Validation, Funding acquisition. **Weihao Huang:** Investigation, Data curation, Validation. **Chao Fang:** Investigation, Validation.

## Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

## Data availability

Data will be made available on request.

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