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S.I.: STATISTICAL RELIABILITY MODELING AND OPTIMIZATION

Multiple criteria decision making with reliability of assessment

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Abstract

The weight and reliability of an individual assessment are two important concepts considered in the evidential reasoning (ER) approach. Through analyzing the existing studies on the combination of individual assessments with both their weights and reliabilities considered in the ER context, their deficiencies are identified in accordance with two principles. One principle is developed in the situation where a specific individual assessment is fully unreliable and the other is developed in the situation where all individual assessments are fully reliable. To address the deficiencies, this paper proposes a new method. In the method, a combination process that takes into account both the weights and reliabilities of individual assessments simultaneously is developed to generate the overall assessment. It is theoretically proven that the combination process satisfies the two principles. Three ways are designed to help a decision maker to flexibly provide individual assessments and determine their reliabilities. A strategic project evaluation problem for an enterprise located in Changzhou, Jiangsu, China is analyzed using the proposed method as a case study to demonstrate its validity and applicability. These are highlighted by its comparison with two existing methods.

Keywords Multiple criteria decision making · Reliabilities of individual assessments · Obtainment of individual assessments · Combination of individual assessments · Evaluation of strategic project

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1 Introduction

When facing real world problems or challenges, people usually need to take into account multiple diverse or conflicting perspectives to make satisfactory decisions. To facilitate the analysis of problems, multiple criteria decision making (MCDM) methods have been applied (Hafezalkotob et al., 2019; Liao et al., 2020; Ma et al., 2018). Representative applications include the selection of sustainable suppliers (Girubha et al., 2016), the evaluation of new product pricing strategies (Baykasoğlu et al., 2017), the selection of socially responsible investments (Bilbao-Terol et al., 2016), the evaluation of logistics performance in hospitals (Longaray et al., 2018), and the evaluation of photovoltaic cells (García-Cascales et al., 2012). In studies on MCDM, criterion weights are commonly considered although their meanings may be different. They are generally used to make trade-offs among criteria (Butler et al., 1997, 2001).

There is another important concept associated with each criterion as implied in the work of Yang and Xu (2013), which is reliability. Reliability is a very common concept usually associated with specific meanings in different contexts. For example, in the context of system, reliability is evaluated to improve system performance or safety (Wang et al., 2011); while in the context of underground structures, reliability is measured by the probability that the structures have been safe over a period of time (Wang & Fang, 2018). Differently, in the context of MCDM, the reliability of a criterion means the reliability of the individual assessment on the criterion. When all data are available and the judgments of a decision maker are not explicitly required, most MCDM methods can find a solution by comprehensively considering all data. However, when the judgments of a decision maker are necessary, many MCDM methods may be able to generate a solution that is consistent with what is anticipated by the decision maker. This paper focuses on the second type of situations. In such cases, the reliability of an individual assessment should be explicitly considered in the analysis of the MCDM problems. Unfortunately, in existing studies, criterion weights are usually considered to make tradeoffs among criteria under the assumption that all individual assessments are fully reliable and thus reliabilities of criteria are not explicitly considered in MCDM processes. This assumption may not be valid in many practical cases and may result in a solution that is not satisfactory or acceptable to a decision maker. An example of breast cancer screening is provided to demonstrate this.

A surgeon usually depends on three types of reports to screen breast cancer, which are provided by three radiologists. One radiologist provides a report by using magnetic resonance imaging (MRI), the second by using ultrasound, and the third by using mammography. The three types of reports are complementary for helping the surgeon screen breast cancer. Suppose that the weights of the three types of reports are represented by w_1, w_2, and w_3, respectively. In accordance with the different precisions and effectiveness of the three ways to give examination reports, the surgeon usually specifies that $w_1 > w_2 > w_3$. The suggestions from the three types of reports are combined to generate the screening result by considering the weights of the reports with the constraint $w_1 > w_2 > w_3$. This is normally fine only when the suggestions from the three types of reports are assumed to be fully reliable, which implies that the three radiologists who provide the reports are fully reliable. When radiologists are not fully reliable, to generate the screening result, the surgeon may choose to trust more the report provided by the radiologist with more experience and better reputation, and hence more likely higher reliability while still bearing in mind the relationship of the weights of the three reports $w_1 > w_2 > w_3$. This example indicates the necessity of considering the reliability of an individual assessment in MCDM processes.

To consider the reliability of an individual assessment, two challenges need to be handled. One is how to obtain the reliabilities of individual assessments and the other is how to correctly consider assessments' reliabilities and weights.

To address the above two challenges, a new MCDM method is proposed based on the evidential reasoning (ER) approach (Fu & Xu, 2016; Fu et al., 2016; Kong et al., 2018; Yang & Xu, 2002; Yang et al., 2006) to explicitly consider the reliability of an individual assessment. The deficiencies of existing studies on conducting the combination of individual assessments by considering their weights and reliabilities are analyzed in two extreme situations and in general situations. To overcome the deficiencies, a new combination process that considers the weights and reliabilities of individual assessments is developed. To make the combination process applicable in MCDM, the obtainment of the reliabilities of the individual assessments is analyzed. The process of generating a solution is finally presented, in which it is demonstrated that the solution is consistent with what is anticipated by a decision maker.

The rest of this paper is organized as follows. Section 2 presents the necessity of proposing a new MCDM method. Section 3 presents the proposed method. A strategic project evaluation problem is analyzed by the proposed method in Sect. 4 to demonstrate its validity and applicability. Finally, this paper is concluded in Sect. 5.

2 Analysis of the necessity of proposing a new MCDM method

2.1 Modeling of MCDM problems with belief distributions

Suppose that an MCDM problem includes M alternatives a_l ($l = 1, \ldots, M$) and L criteria e_i ($i = 1, \ldots, L$). The relative weights of the L criteria are represented by $w = (w_1, w_2, \ldots, w_L)$ such that $0 \leq w_i \leq 1$ and $\sum_{i=1}^{L} w_i = 1$. Assume that $\Omega = \{H_1, H_2, \ldots, H_N\}$ denotes a set of grades that are increasingly ordered from worst to best. The difference among grades is reflected by the utilities of the grades $u(H_n)$ ($n = 1, \ldots, N$) that satisfy the constraint $0 = u(H_1) < u(H_2) < \ldots < u(H_N) = 1$. All alternatives are assessed on the L criteria by using H_n ($n = 1,\ldots, N$). Let $\beta_{n,i}(a_l)$ denote the belief degree assigned to grade H_n when alternative a_l is assessed on criterion e_i. Then, the assessment can be profiled by a belief distribution $B(e_i(a_l)) = \{(H_n, \beta_{n,i}(a_l)), n = 1, \ldots, N; (\Omega, \beta_{\Omega,i}(a_l))\}$, where $\beta_{n,i}(a_l) \geq 0$, $\sum_{n=1}^{N} \beta_{n,i}(a_l) \leq 1$, and $\beta_{\Omega,i}(a_l) = 1 - \sum_{n=1}^{N} \beta_{n,i}(a_l)$ represents the degree of global ignorance (Xu, 2012). If $\beta_{\Omega,i}(a_l) = 0$, the assessment is complete, and otherwise it is incomplete. When $B(e_i(a_l))$ ($i = 1, \ldots, L$, $l = 1, \ldots, M$) is given, a belief decision matrix $S_{L\times M}$ is obtained. Note that because the degree of global ignorance could be assigned to any grade, its impact needs to be analyzed in the MCDM.

2.2 Deficiency of existing studies

In the framework of Dempster-Shafer theory (Denoeux & Masson, 2012; Shafer, 1976), some attempts have been made to combine belief distributions with the consideration of both their reliabilities and their weights. The representative combination methods are proposed by Jiao et al. (2016) and Yang and Xu (2013). To examine the correctness of the two combination methods, the extreme situations and the general situations related to the reliabilities of belief distributions are considered respectively.

(1) The extreme situations

Two principles are developed to examine the correctness of the two combination methods in extreme situations.

Proposition 1 Given the individual assessments $B(e_i(a_l))$ ($i = 1, \ldots, L$), their reliabilities $r_{i,l}$, and their weights w_i, if the individual assessment $B(e_{\bar{i}}(a_l))$ is fully unreliable, which means that $r_{\bar{i},l} = 0$, then the combination result of the L individual assessments is unrelated to $B(e_{\bar{i}}(a_l))$ regardless of what $w_{\bar{i}}$ is equal to.

Proposition 2 Given the individual assessments $B(e_i(a_l))$ ($i = 1, \ldots, L$), their reliabilities $r_{i,l}$ and their weights w_i, if all individual assessments are fully reliable, which means that $r_{i,l} = 1$ for $i = 1, \ldots, L$, then the contributions of the individual assessments $B(e_i(a_l))$ to the overall assessment are determined by w_i.

First, to examine the correctness of Jiao et al.'s method, suppose that $r_{1,l} = 0$ and $r_{2,l} > 0$ for two individual assessments $B(e_i(a_l))$ ($i = 1, 2$). Under this assumption, the combination of $B(e_i(a_l))$ ($i = 1, 2$) by using Jiao et al.'s method is presented by $B(a_l) = \{(H_n, \beta_{n,b(2)}(a_l)), n = 1, \ldots, N; (\Omega, \beta_{\Omega,b(2)}(a_l))\}$ with

$$\beta_{n,b(2)}(a_l) = \frac{r_{2,l} \cdot w_2 \cdot \beta_{n,2}(a_l)}{w_1 + w_2 - w_1 \cdot w_2} \tag{1}$$

and

$$\beta_{\Omega,b(2)}(a_l) = 1 - \frac{r_{2,l} \cdot w_2 \cdot (1 - \beta_{\Omega,2}(a_l))}{w_1 + w_2 - w_1 \cdot w_2}. \tag{2}$$

It can be found from Eqs. (1) and (2) that $\beta_{n,b(2)}(a_l)$ and $\beta_{\Omega,b(2)}(a_l)$ are clearly related to the value of w_1. As a result, the principle presented in Proposition 1 is violated by the combination result derived from Jiao et al.'s method.

On the other hand, suppose that $r_{1,l} = 1$ and $r_{2,l} = 1$. Under this assumption, in Jiao et al.'s method, discounting $B(e_i(a_l))$ ($i = 1, 2$) by using $r_{1,l}$ and $r_{2,l}$ leads to no change in $B(e_i(a_l))$ ($i = 1, 2$) and then the assessments derived from discounting $B(e_i(a_l))$ ($i = 1, 2$) by using w_i ($i = 1, 2$) are combined using Dempster's rule (Shafer, 1976) to generate $B(a_l)$. This indicates that the principle presented in Proposition 2 is satisfied by Jiao et al.'s method.

Second, to examine the correctness of Yang and Xu's method, suppose that $r_{1,l} = 0$ and $r_{2,l} > 0$. Then, it can be obtained from Yang and Xu's method that

$$\tilde{w}_1 = w_1/(1 + w_1) \text{ and } \tilde{w}_2 = w_2/(1 + w_2 - r_{2,l}), \tag{3}$$

$$\begin{aligned}\beta_{n,b(2)}(a_l) = &\frac{[(1-\tilde{w}_2)\cdot\tilde{w}_1\cdot\beta_{n,1}(a_l) + (1-\tilde{w}_1)\cdot\tilde{w}_2\cdot\beta_{n,2}(a_l)]}{\beta_D} + \frac{\tilde{w}_1\beta_{n,1}(a_l)\cdot\tilde{w}_2\beta_{n,2}(a_l)}{\beta_D} \\ &+ \frac{\tilde{w}_1\beta_{n,1}(a_l)\cdot\tilde{w}_2\beta_{\Omega,2}(a_l)}{\beta_D} + \frac{\tilde{w}_1\beta_{\Omega,1}(a_l)\cdot\tilde{w}_2\beta_{n,2}(a_l)}{\beta_D}\end{aligned}, \tag{4}$$

and

$$\beta_{\Omega,b(2)}(a_l) = \frac{[(1-\tilde{w}_2)\cdot\tilde{w}_1\cdot\beta_{\Omega,1}(a_l) + (1-\tilde{w}_1)\cdot\tilde{w}_2\cdot\beta_{\Omega,2}(a_l)]}{\beta_D},$$

where

$$\begin{aligned}\beta_D = &\tilde{w}_1 + \tilde{w}_2 + \tilde{w}_1\cdot\tilde{w}_2(\textstyle\sum_{n=1}^{N}\beta_{n,1}(a_l)\cdot\beta_{n,2}(a_l) + \beta_{\Omega,1}(a_l) \\ &+ \beta_{\Omega,2}(a_l) - \beta_{\Omega,1}(a_l)\cdot\beta_{\Omega,2}(a_l) - 2)\end{aligned}. \tag{6}$$

Equations (3)–(6) indicate that the combination result is related to $B(e_1(a_l))$ and w_1. As a result, the principle presented in Proposition 1 is violated by Yang and Xu's method.

On the other hand, when $r_{1,l} = 1$ and $r_{2,l} = 1$, it can be known from Yang and Xu's method that $\tilde{w}_1 = \tilde{w}_2 = 1$. In this situation, it is derived from Eqs. (4)–(6) that

$$\beta_{n,b(2)}(a_l) = \frac{\beta_{n,1}(a_l)\cdot\beta_{n,2}(a_l)}{\beta_D} + \frac{\beta_{n,1}(a_l)\cdot\beta_{\Omega,2}(a_l)}{\beta_D} + \frac{\beta_{\Omega,1}(a_l)\cdot\beta_{n,2}(a_l)}{\beta_D} \tag{7}$$

and

$$\beta_{\Omega,b(2)}(a_l) = \frac{\beta_{\Omega,1}(a_l)\cdot\beta_{\Omega,2}(a_l)}{\beta_D}, \tag{8}$$

where

$$\beta_D = 2 + \left(\sum\nolimits_{n=1}^{N} \beta_{n,1}(a_l)\cdot\beta_{n,2}(a_l) + \beta_{\Omega,1}(a_l) + \beta_{\Omega,2}(a_l) - \beta_{\Omega,1}(a_l)\cdot\beta_{\Omega,2}(a_l) - 2\right). \tag{9}$$

Because w_1 and w_2 do not appear in the combination result shown in Eqs. (7)–(9), the principle presented in Proposition 2 is violated by Yang and Xu's method.

The above analyses show that Jiao et al.'s method and Yang and Xu's method cannot satisfy the two principles shown in Propositions 1 and 2 simultaneously.

(2) The general situations

The remaining issue is whether the combination results derived from Jiao et al.'s method and Yang and Xu's method are correct in general situations. An example is provided to address this issue.

Example 1 Suppose that five individual assessments on $\Omega = \{H_1, H_2, \ldots, H_5\}$ are given as $B(e_i(a_l))$ $(i = 1, \ldots, 5) = (\{(H_2, 0.2), (H_3, 0.6), (\Omega, 0.2)\}, \{(H_3, 0.5), (H_5, 0.4), (\Omega, 0.1)\}, \{(H_1, 0.3), (H_4, 0.5), (\Omega, 0.2)\}, \{(H_4, 0.5), (H_5, 0.4), (\Omega, 0.1)\}, \{(H_1, 0.4), (H_2, 0.2), (H_3, 0.2), (\Omega, 0.2)\})$ and two different sets of criterion weights are given as $w^1 = (0.5, 0.15, 0.15, 0.1, 0.1)$ and $w^2 = (0.3, 0.3, 0.15, 0.1, 0.15)$. Assume that $r_{i,l}$ $(i = 1, \ldots, 5)$ changes from 0.1 to 0.9 with a step of 0.2. Under the conditions, Jiao et al.'s method and Yang and Xu's method are iteratively applied to generate 20 sets of combination results. The combination result associated with w^1 is denoted by $B^1(a_l)$ and the one associated with w^2 is denoted by $B^2(a_l)$. All results are presented in Table 1.

In general, the larger the influence of criterion weights to the combination result, the larger the difference between $B^1(a_l)$ and $B^2(a_l)$. By following this idea, the difference between $B^1(a_l)$ and $B^2(a_l)$ can be used to indicate the influence of different criterion weights on the combination results. For this purpose, the difference between $B^1(a_l)$ and $B^2(a_l)$ is defined as. $\Delta B^{12}(a_l) = \sum_{n=1}^{5} \left|\beta_n^1(a_l) - \beta_n^2(a_l)\right| + \left|\beta_\Omega^1(a_l) - \beta_\Omega^2(a_l)\right|$.
The relevant results of $\Delta B^{12}(a_l)$ generated using Jiao et al.'s method and Yang and Xu's method with the consideration of five sets of $r_{i,l}$ $(i = 1, \ldots, 5)$ are also presented in Table 1.

It can be observed from Table 1 that $\Delta B^{12}(a_l)$ generated using Jiao et al.'s method has increased with the increase in $r_{i,l}$, which means that the increase in $r_{i,l}$ results in more significant influence of criterion weights on the combination results. This observation is consistent with the assumption that the individual assessments on all criteria are fully reliable when only criterion weights are used to make trade-offs among all criteria. The assumption implies that the influence of criterion weights on the aggregated assessment should have gradually become larger with the increase in the individual assessments' reliabilities. From this perspective, Jiao et al.'s method seems correct to some extent. On the other hand, Table 1 also shows that $\beta_\Omega^1(a_l)$ and $\beta_\Omega^2(a_l)$ have become larger with the decrease in $r_{i,l}$. When $r_{i,l}$ decreases to 0.1, $\beta_\Omega^1(a_l)$ and $\beta_\Omega^2(a_l)$ are close to 0.9. Large $\beta_\Omega^1(a_l)$ and $\beta_\Omega^2(a_l)$ significantly

Table 1 Combination results generated using the method of Jiao et al. and the method of Yang and Xu with the consideration of different reliabilities of individual assessments

$r_{i,l}$ $(i = 1, \ldots, 5)$	Weights	Methods	Combination results	$\Delta B^{12}(a_l)$
0.1	w^1	Jiao et al	$B^1(a_l) = \{(H_1, 0.0112), (H_2, 0.0162), (H_3, 0.0537), (H_4, 0.0165), (H_5, 0.0132), (\Omega, 0.8891)\}$	0.0256
0.1	w^2	Jiao et al	$B^2(a_l) = \{(H_1, 0.0144), (H_2, 0.0124), (H_3, 0.0504), (H_4, 0.0171), (H_5, 0.0222), (\Omega, 0.8834)\}$	
0.3	w^1	Jiao et al	$B^1(a_l) = \{(H_1, 0.0296), (H_2, 0.0453), (H_3, 0.1511), (H_4, 0.0438), (H_5, 0.035), (\Omega, 0.6952)\}$	0.0716
0.3	w^2	Jiao et al	$B^2(a_l) = \{(H_1, 0.0383), (H_2, 0.0336), (H_3, 0.1399), (H_4, 0.0455), (H_5, 0.0604), (\Omega, 0.6822)\}$	
0.5	w^1	Jiao et al	$B^1(a_l) = \{(H_1, 0.0438), (H_2, 0.0712), (H_3, 0.2403), (H_4, 0.065), (H_5, 0.0521), (\Omega, 0.5276)\}$	0.1148
0.5	w^2	Jiao et al	$B^2(a_l) = \{(H_1, 0.0574), (H_2, 0.0512), (H_3, 0.2198), (H_4, 0.068), (H_5, 0.093), (\Omega, 0.5106)\}$	
0.7	w^1	Jiao et al	$B^1(a_l) = \{(H_1, 0.0549), (H_2, 0.0957), (H_3, 0.3262), (H_4, 0.0816), (H_5, 0.0655), (\Omega, 0.3761)\}$	0.1591
0.7	w^2	Jiao et al	$B^2(a_l) = \{(H_1, 0.0731), (H_2, 0.0666), (H_3, 0.2951), (H_4, 0.0865), (H_5, 0.122), (\Omega, 0.3567)\}$	
0.9	w^1	Jiao et al	$B^1(a_l) = \{(H_1, 0.0633), (H_2, 0.12), (H_3, 0.4133), (H_4, 0.0946), (H_5, 0.076), (\Omega, 0.2328)\}$	0.2079
0.9	w^2	Jiao et al	$B^2(a_l) = \{(H_1, 0.0865), (H_2, 0.0806), (H_3, 0.37), (H_4, 0.1021), (H_5, 0.1492), (\Omega, 0.2116)\}$	
0.1	w^1	Yang and Xu	$B^1(a_l) = \{(H_1, 0.0871), (H_2, 0.1149), (H_3, 0.4053), (H_4, 0.1298), (H_5, 0.1042), (\Omega, 0.1587)\}$	0.1461
0.1	w^2	Yang and Xu	$B^2(a_l) = \{(H_1, 0.1041), (H_2, 0.0868), (H_3, 0.38), (H_4, 0.1256), ((H_5, 0.1603), (\Omega, 0.1432)\}$	
0.3	w^1	Yang and Xu	$B^1(a_l) = \{(H_1, 0.0876), (H_2, 0.1135), (H_3, 0.4079), (H_4, 0.1311), (H_5, 0.1053), (\Omega, 0.1545)\}$	0.1424
0.3	w^2	Yang and Xu	$B^2(a_l) = \{(H_1, 0.1038), (H_2, 0.086), (H_3, 0.3851), (H_4, 0.1258), (H_5, 0.1603), (\Omega, 0.139)\}$	
0.5	w^1	Yang and Xu	$B^1(a_l) = \{(H_1, 0.0885), (H_2, 0.1112), (H_3, 0.4124), (H_4, 0.1332), (H_5, 0.1073), (\Omega, 0.1474)\}$	0.1362
0.5	w^2	Yang and Xu	$B^2(a_l) = \{(H_1, 0.1033), (H_2, 0.0845), (H_3, 0.3937), (H_4, 0.1261), (H_5, 0.1605), (\Omega, 0.1319)\}$	
0.7	w^1	Yang and Xu	$B^1(a_l) = \{(H_1, 0.0901), (H_2, 0.1062), (H_3, 0.4216), (H_4, 0.1377), (H_5, 0.1114), (\Omega, 0.133)\}$	0.1231

Table 1 (continued)

$r_{i,l}$ ($i = 1, \ldots, 5$)	Weights	Methods	Combination results	$\Delta B^{12}(a_l)$
0.7	w^2	Yang and Xu	$B^2(a_l) = \{(H_1, 0.1022), (H_2, 0.0813), (H_3, 0.4114), (H_4, 0.1268), (H_5, 0.1608), (\Omega, 0.1175)\}$	
0.9	w^1	Yang and Xu	$B^1(a_l) = \{(H_1, 0.0937), (H_2, 0.0885), (H_3, 0.4504), (H_4, 0.1526), (H_5, 0.1268), (\Omega, 0.088)\}$	0.11
0.9	w^2	Yang and Xu	$B^2(a_l) = \{(H_1, 0.0972), (H_2, 0.0696), (H_3, 0.4656), (H_4, 0.13), (H_5, 0.163), (\Omega, 0.0745)\}$	

increase the information that is ineffective in MCDM and thus are unbeneficial for generating solutions to the MCDM. This indicates that Jiao et al.'s method is not appropriate to be applied in the MCDM in general situations.

As for Yang and Xu's method, Table 1 shows that $\Delta B^{12}(a_l)$ generated using it has decreased with the increase in $r_{i,l}$. Large $r_{i,l}$ weakens the influence of criterion weights on the combination results and small $r_{i,l}$ strengthens the influence, which results in the incompatibility between criterion weights and the aggregated assessment. This violates the assumption that the individual assessments on all criteria are fully reliable when only criterion weights are used to make trade-offs among all criteria. As a result, Yang and Xu's method is not appropriate to be applied in the MCDM in general situations.

As a whole, both Jiao et al.'s method and Yang and Xu's method cannot be correctly applied in the MCDM in both extreme and general situations. To address this issue, a new MCDM method is proposed, in which the weights and reliabilities of individual assessments are considered to implement their correct combination.

3 Proposed method

3.1 Combination of individual assessments

To make the combination of individual assessments correct in all situations, a new way to process individual assessments by using their weights and reliabilities is defined.

Definition 1 Given the individual assessments $B(e_i(a_l))$ ($i = 1, \ldots, L$), through being handled by their reliabilities $r_{i,l}$ and their weights w_i, they become

$$\left\{\left(H_n,\ r_{i,l} \cdot w_i \cdot \beta_{n,i}(a_l)\right),\ n = 1, \ldots, N;\ \left(\Omega,\ r_{i,l} \cdot w_i \cdot \beta_{\Omega,i}(a_l)\right);\ \left(P(\Omega),\ 1 - r_{i,l} \cdot w_i\right)\right\}. \tag{10}$$

As indicated by Butler et al. (1997), criterion weights are generally used to make trade-offs among all criteria in MCDM. This conclusion is under the assumption that the individual assessments on all criteria are fully reliable. The assumption implicitly means that different reliabilities of the individual assessments will influence trade-offs among all criteria made by criterion weights. As presented in the example of screening breast cancer in Sect. 1, the reliabilities of the MRI report, the ultrasound report, and the mammography report, $r_{i,l}$ ($i = 1$, 2, 3), positively amends the function of the weight of any report w_i to control the contribution of the suggestions derived from the three types of reports to the screening result.

By following the above understanding of $r_{i,l}$, reliability $r_{i,l}$ and weight w_i are viewed as a unit and used to handle the individual assessments $B(e_i(a_l))$, as shown in Definition 1. According to Definition 1, the combination of two individual assessments is presented.

Definition 2 Given the individual assessments $B(e_i(a_l))$ ($i = 1, 2$), their reliabilities $r_{i,l}$, and their weights w_i, the combination result of the two assessments is defined as

$$\left\{\left(H_n,\ \beta_{n,b(2)}(a_l)\right),\ n = 1, \ldots, N;\ \left(\Omega,\ \beta_{\Omega,b(2)}(a_l)\right)\right\}, \tag{11}$$

where

$$\beta_{n,b(2)}(a_l) = \frac{\hat{\beta}_{n,b(2)}(a_l)}{\sum_{n=1}^{N} \hat{\beta}_{n,b(2)}(a_l) + \hat{\beta}_{\Omega,b(2)}(a_l)}, \tag{12}$$

$$\beta_{\Omega,b(2)}(a_l) = \frac{\hat{\beta}_{\Omega,b(2)}(a_l)}{\sum_{n=1}^{N} \hat{\beta}_{n,b(2)}(a_l) + \hat{\beta}_{\Omega,b(2)}(a_l)}, \tag{13}$$

$$\begin{aligned}\hat{\beta}_{n,b(2)}(a_l) =& \left[\left(1-r_{2,l}w_2\right)r_{1,l}w_1\beta_{n,1}(a_l) + \left(1-r_{1,l}w_1\right)r_{2,l}w_2\beta_{n,2}(a_l)\right] \\ &+ r_{1,l}w_1\beta_{n,1}(a_l)\cdot r_{2,l}w_2\beta_{n,2}(a_l) \\ &+ r_{1,l}w_1\beta_{n,1}(a_l)\cdot r_{2,l}w_2\beta_{\Omega,2}(a_l) + r_{1,l}w_1\beta_{\Omega,1}(a_l)\cdot r_{2,l}w_2\beta_{n,2}(a_l)\end{aligned}, \tag{14}$$

and

$$\begin{aligned}\hat{\beta}_{\Omega,b(2)}(a_l) =& \left[\left(1-r_{2,l}w_2\right)r_{1,l}w_1\beta_{\Omega,1}(a_l) + \left(1-r_{1,l}w_1\right)r_{2,l}w_2\beta_{\Omega,2}(a_l)\right] \\ &+ r_{1,l}w_1\beta_{\Omega,1}(a_l)\cdot r_{2,l}w_2\beta_{\Omega,2}(a_l)\end{aligned}. \tag{15}$$

In Definition 2, the belief degree assigned to $P(\Omega)$, which is $\beta_{P(\Omega),b(2)}(a_l) = (1-r_{1,l}w_1)\cdot(1-r_{2,l}w_2)$, is reassigned back to all elements of Ω and thus is not explicitly included. When the L individual assessments $B(e_i(a_l))$ $(i = 1, \ldots, L)$ are expected to be combined using their reliabilities $r_{i,l}$ and their weights w_i, the recursive combination process is defined in the following.

Definition 3 Given the individual assessments $B(e_i(a_l))$ $(i = 1, \ldots, L)$, their reliabilities $r_{i,l}$ and their weights w_i, the combination result of the first i assessments is defined as

$$\left\{\left(H_n,\ \beta_{n,b(i)}(a_l)\right),\ n = 1, \ldots, N;\ \left(\Omega,\ \beta_{\Omega,b(i)}(a_l)\right)\right\}, \tag{16}$$

where

$$\beta_{n,b(i)}(a_l) = \frac{\hat{\beta}_{n,b(i)}(a_l)}{\sum_{n=1}^{N}\hat{\beta}_{n,b(i)}(a_l) + \hat{\beta}_{\Omega,b(i)}(a_l)}, \tag{17}$$

$$\beta_{\Omega,b(i)}(a_l) = \frac{\hat{\beta}_{\Omega,b(i)}(a_l)}{\sum_{n=1}^{N}\hat{\beta}_{n,b(i)}(a_l) + \hat{\beta}_{\Omega,b(i)}(a_l)}, \tag{18}$$

$$\vec{\beta}_{n,b(i)}(a_l) = \frac{\hat{\beta}_{n,b(i)}(a_l)}{\sum_{n=1}^{N}\hat{\beta}_{n,b(i)}(a_l) + \hat{\beta}_{\Omega,b(i)}(a_l) + \hat{\beta}_{P(\Omega),b(i)}(a_l)}, \tag{19}$$

$$\vec{\beta}_{\Omega,b(i)}(a_l) = \frac{\hat{\beta}_{\Omega,b(i)}(a_l)}{\sum_{n=1}^{N}\hat{\beta}_{n,b(i)}(a_l) + \hat{\beta}_{\Omega,b(i)}(a_l) + \hat{\beta}_{P(\Omega),b(i)}(a_l)}, \tag{20}$$

$$\vec{\beta}_{P(\Omega),b(i)}(a_l) = \frac{\hat{\beta}_{P(\Omega),b(i)}(a_l)}{\sum_{n=1}^{N}\hat{\beta}_{n,b(i)}(a_l) + \hat{\beta}_{\Omega,b(i)}(a_l) + \hat{\beta}_{P(\Omega),b(i)}(a_l)}, \tag{21}$$

$$\begin{aligned}\hat{\beta}_{n,b(i)}(a_l) =& \left[\left(1-r_{i,l}w_i\right)\vec{\beta}_{n,b(i-1)}(a_l) + \vec{\beta}_{P(\Omega),b(i-1)}(a_l)r_{i,l}w_i\beta_{n,i}(a_l)\right] \\ &+ \vec{\beta}_{n,b(i-1)}(a_l)\cdot r_{i,l}w_i\beta_{n,i}(a_l) + \vec{\beta}_{n,b(i-1)}(a_l)\cdot r_{i,l}w_i\beta_{\Omega,i}(a_l) \\ &+ \vec{\beta}_{\Omega,b(i-1)}(a_l)\cdot r_{i,l}w_i\beta_{n,i}(a_l)\end{aligned}, \tag{22}$$

$$\begin{aligned}\hat{\beta}_{\Omega,b(i)}(a_l) =& \left[\left(1-r_{i,l}w_i\right)\vec{\beta}_{\Omega,b(i-1)}(a_l) + \vec{\beta}_{P(\Omega),b(i-1)}(a_l)r_{i,l}w_i\beta_{\Omega,i}(a_l)\right] \\ &+ \vec{\beta}_{\Omega,b(i-1)}(a_l)\cdot r_{i,l}w_i\beta_{\Omega,i}(a_l)\end{aligned}, \tag{23}$$

and

$$\hat{\beta}_{P(\Omega),b(i)}(a_l) = \left(1-r_{i,l}w_i\right)\vec{\beta}_{P(\Omega),b(i-1)}(a_l). \tag{24}$$

In Definition 3, it is clear that $0 \le \beta_{n,b(L)}(a_l)$, $\beta_{\Omega,b(i)}(a_l)$, $\vec{\beta}_{n,b(i)}(a_l)$, $\vec{\beta}_{\Omega,b(i)}(a_l) \le 1$, $0 \le \vec{\beta}_{P(\Omega),b(i)}(a_l) \le 1$, and $\sum_{n=1}^{N}\vec{\beta}_{n,b(i)}(a_l) + \vec{\beta}_{\Omega,b(i)}(a_l) + \vec{\beta}_{P(\Omega),b(i)}(a_l) = 1$ for $i = 2, \ldots, L$ recursively. When L individual assessments are combined using Definition 3, the combined reliability of the L individual assessments is defined as

$$r_{b(L),l} = \sum_{i=1}^{L} r_{i,l}\cdot w_i. \tag{25}$$

To focus on the correctness of the combination result shown in Definition 3, two extreme situations presented in Propositions 1 and 2 are discussed.

Theorem 1 Suppose that the L individual assessments $B(e_i(a_l))$ ($i = 1, \ldots, L$) are combined using Definition 3 with the help of their reliabilities $r_{i,l}$ and their weights w_i to generate the overall assessment $B(a_l) = \{(H_n, \beta_{n,b(L)}(a_l)), n = 1, \ldots, N; (\Omega, \beta_{\Omega,b(L)}(a_l))\}$. Then, the overall assessment satisfies the two principles presented in Prepositions 1 and 2, respectively.

Proof of Theorem 1 is presented in Appendix. Theorem 1 indicates that the combination of individual assessments presented in Definition 3 can be correctly applied in the MCDM in two extreme situations presented in Propositions 1 and 2.

In general situations, to examine the correctness of the combination result presented in Definition 3, the combination of the five individual assessments shown in Example 1 is conducted using Definition 3.

Example 2 Given the five individual assessments and the five sets of $r_{i,l}$ ($i = 1, \ldots, 5$) used in Example 1, 10 sets of combination results are generated using Definition 3, as presented in Table 2.

As observed in Table 2, $\beta_{\Omega}^{1}(a_l)$ and $\beta_{\Omega}^{2}(a_l)$ are always far less than 1 with the decrease in $r_{i,l}$ although they have become larger. Meanwhile, $\Delta B^{12}(a_l)$ has increased with the increase in $r_{i,l}$, which is consistent with the assumption that the individual assessments on all criteria are fully reliable when only criterion weights are used to make trade-offs among all criteria. The deficiencies of Jiao et al.'s method and Yang and Xu's method in general situations are effectively overcome. This means that Definition 3 can be correctly applied in the MCDM in general situations.

To carry out the combination presented in Definition 3, the reliabilities of individual assessments should be determined. Next, the obtainment of the reliabilities from relevant data and the preferences of a decision maker will be discussed.

3.2 Obtainment of the reliabilities of individual assessments

The prerequisite of implementing the combination of individual assessments presented in Definition 3 is to determine the reliabilities of the individual assessments. To facilitate obtaining the reliability of an individual assessment, its concept is qualitatively defined.

Definition 4 The reliability of an individual assessment on a criterion for an alternative is defined as the degree to which the assessment can characterize the real performance of the alternative on the criterion.

Determining the reliability of an individual assessment described in Definition 4 is closely related to the way in which the assessment is obtained. For this reason, three ways to generate individual assessments are introduced and then the corresponding obtainment of the reliabilities of individual assessments is discussed.

(1) A decision maker provides an individual assessment by considering the information and data from different information sources, such as relevant reports, online collected data, and domain experts.
(2) A decision maker provides an individual assessment by combining his or her opinion and experts' opinions.
(3) A decision maker combines the opinions of experts to produce an individual assessment.

Table 2 Combination results generated using the proposed method with the consideration of different reliabilities of individual assessments

$r_{i,l}$ ($i = 1, \ldots, 5$)	Weights	Combination results	$\Delta B^{12}(a_l)$
0.1	w^1	$B^1(a_l) = \{(H_1, 0.0836), (H_2, 0.121), (H_3, 0.4), (H_4, 0.1231), (H_5, 0.0985), (\Omega, 0.1738)\}$	0.1656
0.1	w^2	$B^2(a_l) = \{(H_1, 0.104), (H_2, 0.0898), (H_3, 0.3639), (H_4, 0.1238), (H_5, 0.1603), (\Omega, 0.1582)\}$	
0.3	w^1	$B^1(a_l) = \{(H_1, 0.0805), (H_2, 0.123), (H_3, 0.4108), (H_4, 0.119), (H_5, 0.0953), (\Omega, 0.1714)\}$	0.1778
0.3	w^2	$B^2(a_l) = \{(H_1, 0.1019), (H_2, 0.0894), (H_3, 0.3723), (H_4, 0.121), (H_5, 0.1608), (\Omega, 0.1546)\}$	
0.5	w^1	$B^1(a_l) = \{(H_1, 0.0771), (H_2, 0.1253), (H_3, 0.4227), (H_4, 0.1143), (H_5, 0.0916), (\Omega, 0.1689)\}$	0.1916
0.5	w^2	$B^2(a_l) = \{(H_1, 0.097), (H_2, 0.0884), (H_3, 0.3915), (H_4, 0.1147), (H_5, 0.1619), (\Omega, 0.1465)\}$	
0.7	w^1	$B^1(a_l) = \{(H_1, 0.0733), (H_2, 0.1279), (H_3, 0.4359), (H_4, 0.1091), (H_5, 0.0875), (\Omega, 0.1662)\}$	0.2073
0.7	w^2	$B^2(a_l) = \{(H_1, 0.097), (H_2, 0.0884), (H_3, 0.3915), (H_4, 0.1147), (H_5, 0.1619), (\Omega, 0.1465)\}$	
0.9	w^1	$B^1(a_l) = \{(H_1, 0.0691), (H_2, 0.1308), (H_3, 0.4508), (H_4, 0.1031), (H_5, 0.0828), (\Omega, 0.1634)\}$	0.2252
0.9	w^2	$B^2(a_l) = \{(H_1, 0.0941), (H_2, 0.0877), (H_3, 0.4027), (H_4, 0.1112), (H_5, 0.1623), (\Omega, 0.142)\}$	

In these three ways, the degree to which the decision maker is sure about the performance of an alternative on a criterion is gradually decreasing. When the decision maker is very sure about the performance of the alternative on the criterion, he or she is willing to provide the individual assessment. When the decision maker is partially sure about the performance of the alternative, he or she wishes to evaluate the performance of the alternative with the help of experts. If the decision maker is completely unsure about the criterion, he or she expects to depend on experts to provide the individual assessment. The obtainment of the reliability of an individual assessment conforms with the generation of the individual assessment.

(1) The reliability of a decision maker on a criterion is considered as the reliability of the individual assessment, which is evaluated by considering the reliabilities and sufficiency of information sources.
(2) The combined reliability of a decision maker's opinion and experts' opinions is considered as the reliability of the individual assessment.
(3) The combined reliability of experts' opinions is regarded as the reliability of the individual assessment.

For an MCDM problem, a decision maker can flexibly select different ways to provide individual assessments on different criteria and generate the reliabilities of the assessments. This is beneficial for generating solutions that are consistent with what is anticipated by the decision maker. As to whether the solutions are correct, it can only be verified by the practice. Meanwhile, to facilitate generating the reliabilities of the individual assessments, the meanings and functions of the weights and reliabilities are differentiated. From the perspective of meaning, the weight of a criterion in the MCDM characterizes how the individual assessment on the criterion affects the overall assessment in comparison with the assessments on the other criteria (Butler et al., 1997). However, the reliability of the individual assessment of an alternative on a criterion reflects the degree to which the individual assessment characterizes the real performance of the alternative on the criterion, as described in Definition 4. From the perspective of function, the weight of the individual assessment on a criterion directly controls the contribution of the individual assessment to the overall assessment, but the reliability of the individual assessment influences the overall assessment in an indirect way.

As a traditional concept, the weights of a decision maker and experts on a criterion are easy to be provided by the decision maker. The weights are associated with the positions, roles, and responsibility of the decision maker and the experts. Differently, the decision maker evaluates his or her reliabilities and the experts' reliabilities by considering the degree to which the decision maker and the experts characterize real situations. Evaluating the reliabilities of the experts is not difficult for the decision maker in some situations. For example, the decision maker can easily provide the experts' reliabilities when he or she has cooperated with the experts for a long period of time, such as one year or several years.

With the aid of the above analysis, the quantitative obtainment of the reliability of an individual assessment is discussed. Suppose that the reliability of the individual assessment $B(e_i(a_l))$ is represented by $r_{i,l}$. In the second and third ways to generate the individual assessment, assume that the K experts denoted by t_k ($k = 1, \ldots, K$) provide opinions and the weights and reliabilities of the decision maker and the experts are denoted by $\lambda_{i,l}^D$, $\lambda_{i,l}^k$, $r_{i,l}^D$, and $r_{i,l}^k$ such that $0 \leq \lambda_{i,l}^D, \lambda_{i,l}^k, r_{i,l}^D, r_{i,l}^k \leq 1$. Specifically, it can be obtained that $\lambda_{i,l}^D + \sum_{k=1}^K \lambda_{i,l}^k = 1$ in the second way and $\sum_{k=1}^K \lambda_{i,l}^k = 1$ in the third way. Under the conditions, the reliability of an individual assessment is quantitatively obtained.

(1) The reliability of the individual assessment $B(e_i(a_l))$, $r_{i,l}$, is equal to the reliability of the decision maker $r_{i,l}^D$.
(2) The reliability of the individual assessment $B(e_i(a_l))$, $r_{i,l}$, is equal to $r_{i,l}^D \cdot \lambda_{i,l}^D + \sum_{k=1}^K r_{i,l}^k \cdot \lambda_{i,l}^k$.
(3) The reliability of the individual assessment $B(e_i(a_l))$, $r_{i,l}$, is equal to $\sum_{k=1}^K r_{i,l}^k \cdot \lambda_{i,l}^k$.

3.3 Generation of a solution

Based on Sects. 3.1 and 3.2, the process of generating a solution by using the proposed method is presented in Fig. 1.

To generate the solution that is satisfactory or acceptable to the decision maker, a common law is to guarantee that the solution is consistent with what is anticipated by the decision maker. In detail, all information related to the MCDM should be controlled by the decision maker and his or her preferences should be reflected in each step of the MCDM process. According to this law, the formation of a belief decision matrix, the determination of decision

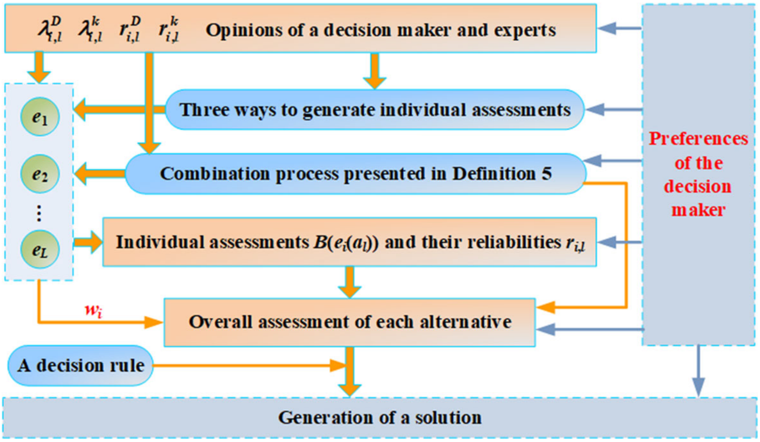

Fig. 1 Process of generating a solution by using the proposed method

parameters, the aggregation of individual assessments on different criteria, and the comparison between alternatives in the proposed method should reflect the preferences of the decision maker and (or) be controlled by the decision maker. With the aid of Fig. 1, whether the process of generating a solution by using the proposed method conforms to the law is analyzed.

Figure 1 shows that the weights and reliabilities of a decision maker and experts, $\lambda^D_{i,l}$, $\lambda^k_{i,l}$, $r^D_{i,l}$, and $r^k_{i,l}$, and the opinions of the decision maker and the experts are the important foundations for generating a solution. The determination of $\lambda^D_{i,l}$, $\lambda^k_{i,l}$, $r^D_{i,l}$, and $r^k_{i,l}$, and the decision maker's opinions clearly reflect the preferences of the decision maker. Although the decision maker cannot change the opinions of the experts, he or she can control the contributions of the experts' opinions to individual assessments through adjusting $\lambda^D_{i,l}$, $\lambda^k_{i,l}$, $r^D_{i,l}$, and $r^k_{i,l}$. In terms of the different degrees to which the decision maker is sure about different criteria, three ways are provided to help the decision maker flexibly generate the individual assessments and the corresponding reliabilities from the opinions of the decision maker and the experts. The choice of appropriate ways also reflects the preferences of the decision maker. Meanwhile, because $r^D_{i,l}$ and $r^k_{i,l}$ are included in the combination process presented in Definition 3, the process reflects the preferences of the decision maker.

After the individual assessments $B(e_i(a_l))$ and their reliabilities $r_{i,l}$ are obtained, the overall assessment of each alternative $B(a_l)$ is generated from the individual assessments through using the combination process presented in Definition 3, w_i, and $r_{i,l}$. Because $B(e_i(a_l))$, $r_{i,l}$, and the combination of $B(e_i(a_l))$ are closely associated with the preferences of the decision maker, $B(a_l)$ reflects the preferences of the decision maker. Specifically, the combination presented in Definition 3 can effectively overcome the deficiencies of Jiao et al.'s method and Yang and Xu's method. This is beneficial for reflecting the decision maker's preferences about $r_{i,l}$ and w_i in $B(a_l)$.

To facilitate the comparison between alternatives, the overall assessment $B(a_l)$ is combined with the utilities of the grades $u(H_n)$ to produce the minimum and maximum expected utilities of alternative a_l, which are $u^-(a_l) = \sum_{n=2}^{N} \beta_n(a_l)u(H_n) + (\beta_1(a_l) + \beta_\Omega(a_l))u(H_1)$ and $u^+(a_l) = \sum_{n=1}^{N-1} \beta_n(a_l)u(H_n) + (\beta_N(a_l) + \beta_\Omega(a_l))u(H_N)$, respectively. With $[u^-(a_l), u^+(a_l)]$, the Hurwicz rule (Corrente et al., 2017; Jiang et al., 2015) is adopted to compare the alternatives and generate a solution. In detail, the expected utility of the alternative a_l is calculated as

$$E(a_l) = \gamma \cdot u^{+}(a_l) + (1 - \gamma) \cdot u^{-}(a_l), \tag{26}$$

where γ represents the optimism degree that is limited to [0,1]. The larger the $E(a_l)$, the better the alternative a_l. Focusing on the real situation of the MCDM problem, the decision maker usually controls the value of γ in terms of his or her preferences. This indicates that the comparison between the alternatives and the further generation of a solution reflect the preferences of the decision maker.

4 Case study

In this section, a strategic project evaluation (SPE) problem is analyzed by the proposed method to demonstrate its validity and applicability.

4.1 Description of the SPE problem

In the past ten years, high-speed trains have gradually become a strategic emerging industry. As an important member in the industry, an enterprise located in Changzhou, Jiangsu, China has developed more rapidly than ever before. With the development, the abundant knowledge and experience for the R&D, production, and maintenance of gear boxes and their relevant components have been accumulated. To achieve more profits from the accumulated knowledge and experience, the board of directors in the enterprise plans to develop new businesses, which is a strategic challenge. To select the most appropriate business from potential ones, the director of the operations department is appointed by the general manager of the enterprise to be the decision maker. Six experts are invited to help the decision maker handle the challenge, and their details are presented in Table 3.

Through a detailed discussion among the decision maker and the six experts shown in Table 3 from the perspectives of markets, techniques, and operations, four strategic projects are screened from potential businesses. They are the driving system of new energy automobiles (P_1), the RV reducer of industrial robots (P_2), the reducer of shield-driven machines (P_3), and the cleaning machine with high pressure water jets (P_4). The four projects own competitive market prospects. Although developing the four projects simultaneously may gain more profits, it needs a large amount of resource inputs. The dispersed use of resources may increase the risk of causing some failed projects and the risk of weakening the successful potential of each project. To avoid these risks, the decision maker decides to select the most appropriate one from the four projects to implement. For this purpose, eight criteria are identified to evaluate the four projects, which are the capability of conforming with the policies of the nation and the industry of high-speed trains (e_1), the capability of being in line with the development strategy of the enterprise (e_2), the market potential (e_3), the profitability (e_4), the maturity of design and materials technology (e_5), the maturity of experimental testing and manufacturing technology (e_6), the basic resources (e_7), and the resources from supply chain (e_8).

Note that the decision maker and the six experts have served the enterprise for more than 5 years at least. Among them, t_1 has provided service for the enterprise beyond 15 years. This indicates that the decision maker has cooperated with the six experts for more than 5 years at least. As a result, he is very sure about who should provide opinions on which criterion and can specify the reasonable weights and reliabilities of people providing opinions

Table 3 Details of the six experts

Experts	Duty	Title	Responsibility of post
t_1	Head of the department of integrated technology and equipment	Senior engineer with professorship	He is responsible for the whole R&D of products
t_2	R&D engineer in the department of integrated technology and equipment	Senior engineer with professorship	He focuses on the study on integrated technologies and R&D of products
t_3	Technological engineer in the department of integrated technology and equipment	Senior engineer	He focuses on the study on the technologies of transmission and integration
t_4	Director of scientific plan and development	Engineer	He is responsible for the organization, analysis and evaluation of scientific and technological activities in relevant departments
t_5	Technological engineer in the department of integrated technology and equipment	Engineer	He focuses on the study on the technologies of transmission and integration
t_6	Director of new businesses	Senior engineer	He is responsible for the selection and cultivation of new businesses

on different criteria. The relevant details are presented in Table 4. Meanwhile, the decision maker and the six experts are very sure about research & development, manufacturing, and other perspectives of the enterprise. These facts indicate that the six experts can be responsible

Table 4 Details about people providing opinions on each criterion and their settings

Criteria	Familiarity of the decision maker	People providing opinions	People's weights	People's reliabilities
e_1	Partially familiar	t_1, DM	0.6, 0.4	0.7, 0.8
e_2	Completely familiar	DM	1	0.95
e_3	Completely unfamiliar	t_2, t_6	0.5, 0.5	0.8, 0.9
e_4	Partially familiar	t_2, t_6, DM	0.1, 0.1, 0.8	0.6, 0.8, 0.7
e_5	Completely unfamiliar	t_3	1	0.95
e_6	Completely unfamiliar	t_3, t_5	0.6, 0.4	0.85, 0.7
e_7	Completely familiar	DM	1	0.85
e_8	Partially familiar	t_4, DM	0.5, 0.5	0.9, 0.8

for their assessments for the four strategic projects on each criterion and the decision maker can verify the rigorousness of his assessments and experts' assessments.

To facilitate providing opinions, a set of grades is determined by the decision maker and the six experts, which is $\Omega = \{H_n, n = 1, \ldots, 5\} = \{VeryLow, Low, Average, High, VeryHigh\} = \{VL, L, A, H, VH\}$. After the decision maker and the six experts unify the understanding of the five grades in Ω, a probability assignment approach (Winston, 2011) is used to set $u(H_n)$ ($n = 1, \ldots, 5$) to be (0, 0.25, 0.5, 0.75, 1). Meanwhile, through a discussion among the decision maker and the six experts, a method of determining criterion weights developed by Ölçer and Odabaşi (2005) is applied to set that w_i ($i = 1, \ldots, 8$) = (0.05, 0.1, 0.2, 0.15, 0.15, 0.1, 0.15, 0.1). To avoid the negative influence of the failed project on the production and quality assurance of gear boxes and the reputation of the enterprise in the industry, the decision maker specifies that $\gamma = 0.3$.

4.2 Generation of the solution to the SPE problem

As shown in Table 4, there are different people who provide opinions on different criteria. By using the above set of grades Ω, the decision maker and the six experts provide their assessments on different criteria, as presented in Table 5.

Through using Definition 3 and the weights and reliabilities of people providing opinions that are presented in Table 4, the assessment data presented in Table 5 are aggregated to form the belief decision matrix $S_{8\times4}$, which is presented in Table 6. As indicated in Table 4, the decision maker specifies the weights and reliabilities of the people who provide opinions without the consideration of the four strategic projects. This indicates that the combined reliabilities of the assessments on the eight criteria are independent of the four strategic projects. In accordance with Eq. (25), the combined reliabilities are obtained as $r_b(e_i)$ ($i = 1, \ldots, 8$) = (0.74, 0.95, 0.85, 0.7, 0.95, 0.79, 0.85, 0.85). From the belief decision matrix shown in Table 6, w_i, and $r_b(e_i)$, the aggregated assessment of each strategic project can be obtained by using the combination process presented in Definition 3, which is presented in Table 7.

The aggregated assessments of the four strategic projects can be combined with the utilities of grades $u(H_n)$ ($n = 1, \ldots, 5$) to generate the minimum and maximum expected utilities of the four projects, which are $[u^-(P_l), u^+(P_l)]$ ($l = 1, \ldots, 4$) = {[0.77, 0.7908], [0.7676, 0.8083], [0.6842, 0.7465], [0.5872, 0.7159]}. The $[u^-(P_l), u^+(P_l)]$ is then used to calculate the expected utilities of the four strategic projects through Eq. (26) and the value of γ specified in Sect. 4.1, which are $E(P_l)$ ($l = 1, \ldots, 4$) = (0.7762, 0.7798, 0.7029, 0.6258). From the $E(P_l)$, a ranking order of the four strategic projects can be obtained as $P_2 \succ P_1 \succ P_3 \succ P_4$, where the notation '$\succ$' denotes 'is superior to'. The ranking order indicates that the RV reducer of industrial robots (P_2) is the most appropriate project to be implemented. This is the solution to the SPE problem under the current conditions.

4.3 Comparison with existing methods

The proposed method is compared with the method of Jiao et al. (2016) and the method of Yang and Xu (2013) based on the SPE problem.

Table 5 Assessment data for the four strategic projects in the SPE problem

Criteria	P_1	P_2	P_3	P_4
e_1	t_1: $\{(H, 0.2), (VH, 0.8)\}$; DM: $\{(H, 0.1), (VH, 0.9)\}$	t_1: $\{(A, 0.2), (VH, 0.8)\}$; DM: $\{(H, 0.1), (VH, 0.9)\}$	t_1: $\{(A, 0.4), (H, 0.6)\}$; DM: $\{(A, 0.3), (H, 0.6), (\Omega, 0.1)\}$	t_1: $\{(A, 0.4), (VH, 0.6)\}$; DM: $\{(A, 0.5), (H, 0.5)\}$
e_2	DM: $\{(H, 0.1), (VH, 0.9)\}$	DM: $\{(L, 0.1), (A, 0.3), (H, 0.6)\}$	DM: $\{(VL, 0.2), (A, 0.4), (\Omega, 0.4)\}$	DM: $\{(VL, 0.2), (A, 0.5), (\Omega, 0.3)\}$
e_3	t_2: $\{(L, 0.2), (H, 0.7), (VH, 0.1)\}$; t_6: $\{(H, 0.8), (VH, 0.2)\}$	t_2: $\{(VH, 0.9), (\Omega, 0.1)\}$; t_6: $\{(VH, 1)\}$	t_2: $\{(A, 0.7), (H, 0.3)\}$; t_6: $\{(A, 0.8), (\Omega, 0.2)\}$	t_2: $\{(H, 0.2), (VH, 0.8)\}$; t_6: $\{(L, 0.3), (A, 0.6), (\Omega, 0.1)\}$
e_4	t_2: $\{(L, 0.4), (H, 0.6)\}$; t_6: $\{(H, 0.7), (VH, 0.3)\}$; DM: $\{(A, 0.3), (H, 0.5), (\varOmega, 0.2)\}$	t_2: $\{(L, 0.35), (A, 0.5), (H, 0.15)\}$; t_6: $\{(L, 0.1), (A, 0.8), (H, 0.1)\}$; DM: $\{(H, 0.5), (VH, 0.35), (\varOmega, 0.15)\}$	t_2: $\{(L, 0.2), (A, 0.7), (H, 0.1)\}$; t_6: $\{(A, 0.1), (H, 0.6), (VH, 0.3)\}$; DM: $\{(H, 0.4), (VH, 0.6)\}$	t_2: $\{(H, 0.1), (VH, 0.9)\}$; t_6: $\{(H, 0.3), (VH, 0.7)\}$; DM: $\{(A, 0.4), (H, 0.3), (\varOmega, 0.3)\}$
e_5	t_3: $\{(H, 0.4), (VH, 0.6)\}$	t_3: $\{(A, 0.2), (H, 0.8)\}$	t_3: $\{(VH, 1)\}$	t_3: $\{(H, 0.8), (\varOmega, 0.2)\}$
e_6	t_3: $\{(A, 0.3), (H, 0.7)\}$; t_5: $\{(H, 0.8), (\varOmega, 0.2)\}$	t_3: $\{(L, 0.4), (H, 0.6)\}$; t_5: $\{(H, 0.7), (\varOmega, 0.3)\}$	t_3: $\{(H, 0.1), (VH, 0.9)\}$; t_5: $\{(H, 0.7), (\varOmega, 0.3)\}$	t_3: $\{(A, 0.6), (H, 0.4)\}$; t_5: $\{(A, 0.3), (H, 0.5), (\varOmega, 0.2)\}$
e_7	DM: $\{(A, 0.3), (H, 0.5), (VH, 0.2)\}$	DM: $\{(A, 0.4), (H, 0.6)\}$	DM: $\{(L, 0.2), (A, 0.3), (H, 0.5)\}$	DM: $\{(A, 0.2), (H, 0.6), (\varOmega, 0.2)\}$
e_8	t_4: $\{(A, 0.2), (H, 0.5), (VH, 0.3)\}$; DM: $\{(L, 0.3), (A, 0.4), (H, 0.3)\}$	t_4: $\{(H, 0.5), (VH, 0.4), (\Omega, 0.1)\}$; DM: $\{(VL, 0.2), (A, 0.5), (\varOmega, 0.3)\}$	t_4: $\{(A, 0.1), (H, 0.7), (VH, 0.2)\}$; DM: $\{(A, 0.2), (H, 0.5), (\varOmega, 0.3)\}$	t_4: $\{(H, 0.4), (VH, 0.6)\}$; DM: $\{(A, 0.2), (H, 0.7), (\varOmega, 0.1)\}$

Table 6 Belief decision matrix derived from the assessment data shown in Table 5

Criteria	P_1	P_2	P_3	P_4
e_1	$\{(H, 0.1373), (VH, 0.8627)\}$	$\{(A, 0.1006), (H, 0.0327), (VH, 0.8668)\}$	$\{(A, 0.3486), (H, 0.6176), (\Omega, 0.0338)\}$	$\{(A, 0.4696), (H, 0.1863), (VH, 0.344)\}$
e_2	$\{(H, 0.1), (VH, 0.9)\}$	$\{(L, 0.1), (VH, 0.9)\}$	$\{(VL, 0.2), (A, 0.4), (\Omega, 0.4)\}$	$\{(VL, 0.2), (A, 0.5), (\Omega, 0.3)\}$
e_3	$\{(L, 0.074), (H, 0.7921), (VH, 0.1339)\}$	$\{(VH, 0.9672), (\Omega, 0.0328)\}$	$\{(A, 0.7913), (H, 0.1225), (\Omega, 0.0862)\}$	$\{(L, 0.1594), (A, 0.3189), (H, 0.0937), (VH, 0.3748)\ (\Omega, 0.0531)\}$
e_4	$\{(L, 0.0209), (A, 0.2497), (H, 0.5416), (VH, 0.0214), (\Omega, 0.1664)\}$	$\{(L, 0.0254), (A, 0.0842), (H, 0.4567), (VH, 0.3037), (\Omega, 0.1301)\}$	$\{(L, 0.0087), (A, 0.0365), (H, 0.4056), (VH, 0.5492)\}$	$\{(A, 0.3397), (H, 0.2932), (VH, 0.1123)\ (\Omega, 0.2548)\}$
e_5	$\{(H, 0.4), (VH, 0.6)\}$	$\{(A, 0.2), (H, 0.8)\}$	$\{(VH, 1)\}$	$\{(H, 0.8)\ (\Omega, 0.2)\}$
e_6	$\{(A, 0.1937), (H, 0.7615), (\Omega, 0.0448)\}$	$\{(L, 0.2701), (H, 0.6621), (\Omega, 0.0678)\}$	$\{(H, 0.2639), (VH, 0.6623), (\Omega, 0.0739)\}$	$\{(A, 0.5182), (H, 0.435), (\Omega, 0.0467)\}$
e_7	$\{(A, 0.3), (H, 0.5), (VH, 0.2)\}$	$\{(A, 0.4), (H, 0.6)\}$	$\{(L, 0.2), (A, 0.3), (H, 0.5)\}$	$\{(A, 0.2), (H, 0.6), (\Omega, 0.2)\}$
e_8	$\{(L, 0.1242), (A, 0.2943), (H, 0.4291), (VH, 0.1524)\}$	$\{(VL, 0.0855), (A, 0.2138), (H, 0.2911), (VH, 0.2328), (\Omega, 0.1768)\}$	$\{(A, 0.1315), (H, 0.638), (VH, 0.2305)\}$	$\{(A, 0.0788), (H, 0.5723), (VH, 0.3095), (\Omega, 0.0394)\}$

Table 7 Aggregated assessments of the four strategic projects derived from the belief decision matrix shown in Table 6

Projects	Aggregated assessments
P_1	$\{(L, 0.0267), (A, 0.1104), (H, 0.5359), (VH, 0.3062), (\Omega, 0.0208)\}$
P_2	$\{(VL, 0.0074), (L, 0.0348), (A, 0.1231), (H, 0.3866), (VH, 0.4074), (\Omega, 0.0407)\}$
P_3	$\{(VL, 0.0196), (L, 0.0294), (A, 0.2887), (H, 0.2696), (VH, 0.3303), (\Omega, 0.0624)\}$
P_4	$\{(VL, 0.02), (L, 0.0317), (A, 0.2749), (H, 0.4113), (VH, 0.1334), (\Omega, 0.1287)\}$

4.3.1 Comparison of solutions

Through iteratively using Jiao et al.'s method and Yang and Xu's method, the corresponding belief decision matrices can be obtained. In the same way, the assessments on each criterion for each strategic project can be combined to generate the aggregated assessments of the four projects, which are presented in Table 8.

Tables 7 and 8 show that the degrees of global ignorance in the aggregated assessments from Jiao et al.'s method are quite larger than those from the proposed method and those from Yang and Xu's method. This is because $(1-r_{i,l})\cdot w_i$ is assigned to the degree of global ignorance in Jiao et al.'s method. The aggregated assessments shown in Table 8 are combined with $u(H_n)$ ($n = 1, \ldots, 5$) to generate two sets of $[u^-(P_l), u^+(P_l)]$ ($l = 1, \ldots, 4$). One set of $[u^-(P_l), u^+(P_l)]$ from Jiao et al.'s method is {[0.6069, 0.8408], [0.5992, 0.8506], [0.5267, 0.8024], [0.4564, 0.7776]} and the other from Yang and Xu's method is {[0.8377, 0.8397], [0.8203, 0.828], [0.7567, 0.7935], [0.6411, 0.698]}. The two groups of $E(P_l)$ ($l = 1, \ldots, 4$) are then obtained using Eq. (26), which are (0.6771, 0.6746, 0.6094, 0.5527) and (0.8383,

Table 8 Aggregated assessments of the four strategic projects derived from the assessment data shown in Table 5 by using the method of Jiao et al. and the method of Yang and Xu

Projects	Methods	Aggregated assessments
P_1	Jiao et al	$\{(L, 0.021), (A, 0.0824), (H, 0.4091), (VH, 0.2537), (\Omega, 0.2338)\}$
P_2	Jiao et al	$\{(VL, 0.0059), (L, 0.0274), (A, 0.0965), (H, 0.2987), (VH, 0.3201), (\Omega, 0.2514)\}$
P_3	Jiao et al	$\{(VL, 0.0171), (L, 0.0225), (A, 0.2283), (H, 0.198), (VH, 0.2584), (\Omega, 0.2757)\}$
P_4	Jiao et al	$\{(VL, 0.0177), (L, 0.0249), (A, 0.21), (H, 0.3244), (VH, 0.1019), (\Omega, 0.3212)\}$
P_1	Yang and Xu	$\{(L, 0.0033), (A, 0.0239), (H, 0.5834), (VH, 0.3874), (\Omega, 0.002)\}$
P_2	Yang and Xu	$\{(VL, 0.0018), (L, 0.0177), (A, 0.0828), (H, 0.462), (VH, 0.428), (\Omega, 0.0076)\}$
P_3	Yang and Xu	$\{(VL, 0.0163), (L, 0.0152), (A, 0.2564), (H, 0.202), (VH, 0.4732), (\Omega, 0.0368)\}$
P_4	Yang and Xu	$\{(VL, 0.0195), (L, 0.0165), (A, 0.2528), (H, 0.575), (VH, 0.0793), (\Omega, 0.0569)\}$

0.8226, 0.7678, 0.6581). The same ranking order of the four strategic projects is generated from the two groups of $E(P_l)$, which is $P_1 \succ P_2 \succ P_3 \succ P_4$. It can be found that the driving system of new energy automobiles (P_1) replaces the RV reducer of industrial robots (P_2) to become the most appropriate project to be implemented. When it is confirmed by the decision maker that the two principles presented in Preposition 1 and 2 should be satisfied to guarantee the correctness of the aggregated assessments, P_2 is accepted by the decision maker as the solution to the SPE problem. Otherwise, P_1 may be accepted by the decision maker.

4.3.2 Comparison based on sensitivity analysis

To examine the influence of the optimism degree and criterion weights on the solution to the SPE problem, the proposed method is compared with Jiao et al.'s method and Yang and Xu's method from the perspective of sensitivity analysis.

First, suppose that the value of the optimism degree γ is changed from 0 to 1 with a step of 0.1. The movement of the expected utilities of the four strategic projects derived from the three methods is plotted in Fig. 2. As indicated in Fig. 2, the expected utilities of the four strategic projects derived from the proposed method and Jiao et al.'s method are dependent of the value of γ, but those derived from Yang and Xu's method are independent of the value of γ. Figure 2 also demonstrates the reason why the solution generated using the proposed method is different from those generated using Jiao et al.'s method and Yang and Xu's method.

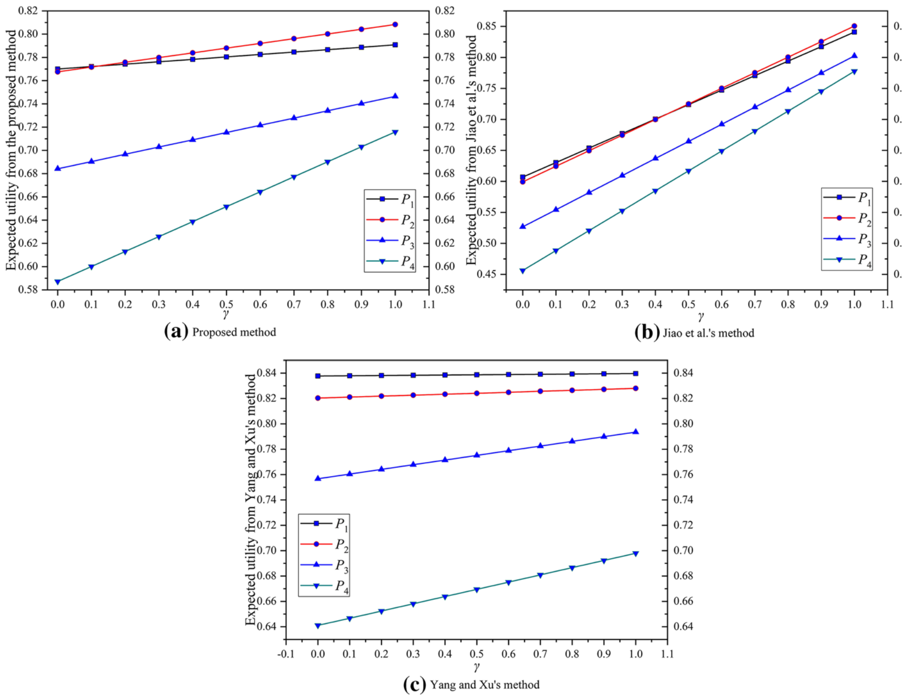

Fig. 2 Movement of the expected utilities of the four strategic projects derived from the three methods with the variation in the value of γ

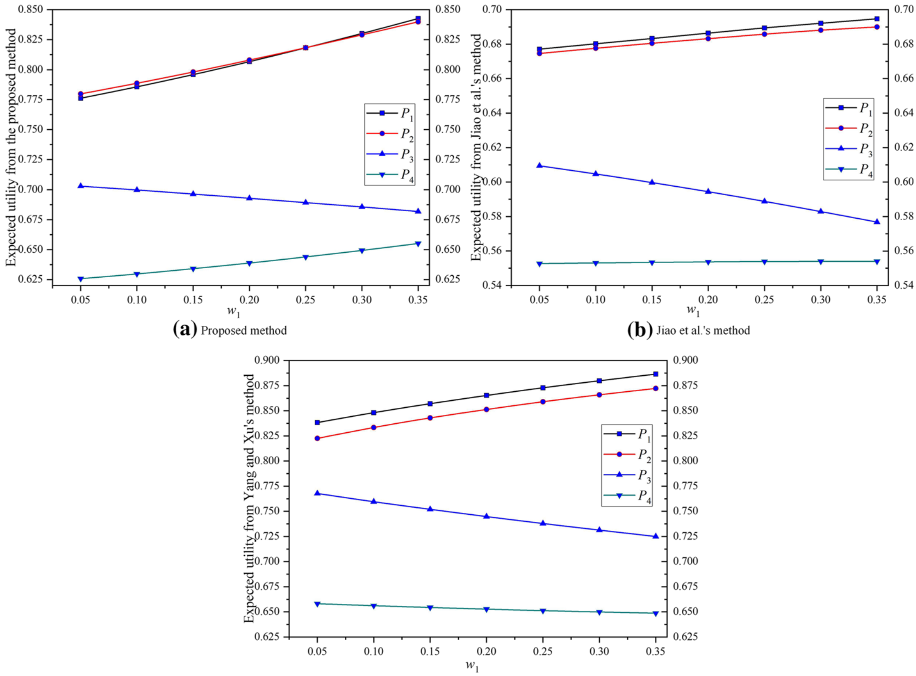

Fig. 3 Movement of the expected utilities of the four strategic projects derived from the three methods with the variation in the value of w_1

Second, suppose that $\gamma = 0.3$ and w_1 is changed from 0.05 to 0.35 with a step of 0.05. Under the conditions, the movement of the expected utilities of the four strategic projects is plotted in Fig. 3. As indicated in Fig. 3, the expected utilities of the four strategic projects derived from the proposed method are dependent of the value of w_1, but those derived from Jiao et al.'s method and Yang and Xu's method are independent of the value of w_1. This means that the expected utilities of the four strategic projects generated using the proposed method rather than the other two methods can reflect the influence of w_1 on the solution to the SPE problem.

Figures 2 and 3 also indicate that with the variation in w_1, the changes in the differences between the expected utilities of any two strategic projects derived from Yang and Xu's method are clearly less than those derived from the proposed method and those derived from Jiao et al.'s method. This implies that the reliability of the individual assessment contributes more than its weight to the overall assessment in Yang and Xu's method rather than the other two methods.

The above analyses reveal that the proposed method is more beneficial than Jiao et al.'s method and Yang and Xu's method for generating solutions to the SPE problem that are consistent with what is anticipated by the decision maker.

4.3.3 Comparison based on effectiveness and compatibility

Based on the analysis of Jiao et al.'s method and Yang and Xu's method presented in Sect. 2.2, the proposed method is compared with the two methods from the perspectives of effectiveness and compatibility.

Assume that $r_b(e_i)$ ($i = 1, \ldots, 8$) is changed from 0.1 to 0.9 with a step of 0.2. Suppose that $w^1 = (0.05, 0.1, 0.2, 0.15, 0.15, 0.1, 0.15, 0.1)$, as presented in Sect. 4.1, and $w^2 = (0.05, 0.05, 0.5, 0.05, 0.05, 0.1, 0.1, 0.1)$. The aggregated assessments of the four strategic projects generated using the two sets of criterion weights and the proposed method, Jiao et al.'s method, and Yang and Xu's method are denoted by $B_m^1(P_l) = \{(H_n, \beta_{m,n}^1(P_l)), n = 1, \ldots, 5; (\Omega, \beta_{m,\Omega}^1(P_l))\}$ ($m = 1, 2, 3, l = 1, \ldots, 4$) and $B_m^2(P_l) = \{(H_n, \beta_{m,n}^2(P_l)), n = 1, \ldots, 5; (\Omega, \beta_{m,\Omega}^2(P_l))\}$. The difference between $B_m^1(P_l)$ and $B_m^2(P_l)$ is defined as.

$$\Delta B_m^{12}(P_l) = \sum_{n=1}^{5} \left|\beta_{m,n}^1(P_l) - \beta_{m,n}^2(P_l)\right| + \left|\beta_{m,\Omega}^1(P_l) - \beta_{m,\Omega}^2(P_l)\right|.$$

All relevant results are calculated and presented in Tables 9–12.

First, the proposed method is compared with the two methods from the perspective of effectiveness. It is observed from Tables 9–12 that $\beta_{2,\Omega}^1(P_l)$ ($l = 1, \ldots, 4$) and $\beta_{2,\Omega}^2(P_l)$ have increased and become close to 0.9 with the decrease in $r_b(e_i)$ ($i = 1, \ldots, 8$). This means that a large amount of ineffective information may be included in the aggregated assessments of the four strategic projects generated using Jiao et al.'s method, which prevents the correct application of Jiao et al.'s method in the MCDM. Meanwhile, $\beta_{1,\Omega}^1(P_l)$ ($l = 1, \ldots, 4$), $\beta_{1,\Omega}^2(P_l)$, $\beta_{3,\Omega}^1(P_l)$, and $\beta_{3,\Omega}^2(P_l)$ have slightly increased and are always far less than 0.9 with the decrease in $r_b(e_i)$ ($i = 1, \ldots, 8$). This indicates that the proposed method and Yang and Xu's method are correct from the perspective of effectiveness.

Second, from the perspective of the compatibility between criterion weights and the aggregated assessment, the proposed method is compared with the two methods. It is found from Tables 9–12 that $\Delta B_3^{12}(P_1)$ and $\Delta B_3^{12}(P_4)$ have decreased with the increase in $r_b(e_i)$ ($i = 1, \ldots, 8$). This finding violates the assumption that the individual assessments on all criteria are fully reliable when only criterion weights are used to make trade-offs among all criteria. Meanwhile, although $\Delta B_3^{12}(P_2)$ has increased with the increase in $r_b(e_i)$ ($i = 1, \ldots, 8$), the changes in $\Delta B_3^{12}(P_2)$ are less than those in $\Delta B_1^{12}(P_2)$ in most cases. As a result, Yang and Xu's method cannot guarantee the compatibility between criterion weights and the aggregated assessments of the four strategic projects. Tables 9–12 also show that $\Delta B_1^{12}(P_l)$ and $\Delta B_2^{12}(P_l)$ have increased with the increase in $r_b(e_i)$ ($i = 1, \ldots, 8$), which indicates that the proposed method and Jiao et al.'s method can guarantee the compatibility.

As a whole, the above analyses indicate that only the proposed method can guarantee the effectiveness and the compatibility simultaneously.

Through comprehensively considering the comparison of solutions, the comparison based on sensitivity analysis, and the comparison based on effectiveness and compatibility, the advantages of the proposed method are summarized in the following.

(1) The proposed method can generate a solution in which the two principles presented in Propositions 1 and 2 are satisfied.
(2) The proposed method can effectively avoid an ineffective solution caused by large global ignorance in the aggregated assessments of alternatives.
(3) The proposed method can guarantee the compatibility between the preferences of a decision maker and the solution generated, and the compatibility between the solution and what is anticipated by the decision maker.

Table 9 Aggregated assessments of the first strategic project by using the three methods with the consideration of different reliabilities of individual assessments

$r_b(e_i)$ $(i = 1, \ldots, 8)$	Weights	Methods	Aggregated assessments	$\Delta B_m^{12}(P_1)$
0.1	w^1	Proposed method	$B_1^1(P_1) = \{(H_2, 0.03), (H_3, 0.1298), (H_4, 0.5143), (H_5, 0.297), (\Omega, 0.0289)\}$	0.2733
0.1	w^2	Proposed method	$B_1^2(P_1) = \{(H_2, 0.0504), (H_3, 0.0885), (H_4, 0.6306), (H_5, 0.2182), (\Omega, 0.0123\}$	
0.3	w^1	Proposed method	$B_1^1(P_1) = \{(H_2, 0.0292), (H_3, 0.1267), (H_4, 0.5221), (H_5, 0.2941), (\Omega, 0.028)\}$	0.2866
0.3	w^2	Proposed method	$B_1^2(P_1) = \{(H_2, 0.0502), (H_3, 0.0826), (H_4, 0.6445), (H_5, 0.2115), (\Omega, 0.0113\}$	
0.5	w^1	Proposed method	$B_1^1(P_1) = \{(H_2, 0.0284), (H_3, 0.1234), (H_4, 0.5304), (H_5, 0.2909), (\Omega, 0.0269)\}$	0.3014
0.5	w^2	Proposed method	$B_1^2(P_1) = \{(H_2, 0.0499), (H_3, 0.0761), (H_4, 0.6596), (H_5, 0.2041), (\Omega, 0.0103\}$	
0.7	w^1	Proposed method	$B_1^1(P_1) = \{(H_2, 0.0276), (H_3, 0.12), (H_4, 0.5392), (H_5, 0.2874), (\Omega, 0.0258)\}$	0.3179
0.7	w^2	Proposed method	$B_1^2(P_1) = \{(H_2, 0.0497), (H_3, 0.069), (H_4, 0.676), (H_5, 0.196), (\Omega, 0.0092\}$	
0.9	w^1	Proposed method	$B_1^1(P_1) = \{(H_2, 0.0267), (H_3, 0.1164), (H_4, 0.5486), (H_5, 0.2836), (\Omega, 0.0247)\}$	0.3364
0.9	w^2	Proposed method	$B_1^2(P_1) = \{(H_2, 0.0495), (H_3, 0.0613), (H_4, 0.694), (H_5, 0.1872), (\Omega, 0.0081\}$	
0.1	w^1	Jiao et al.	$B_2^1(P_1) = \{(H_2, 0.0037), (H_3, 0.0152), (H_4, 0.0628), (H_5, 0.0385), (\Omega, 0.8798)\}$	0.0349

Table 9 (continued)

$r_b(e_i)$ $(i=1, \ldots, 8)$	Weights	Methods	Aggregated assessments	$\Delta B_m^{12}(P_1)$
0.1	w^2	Jiao et al.	$B_2^2(P_1) = \{(H_2, 0.006), (H_3, 0.0101), (H_4, 0.0742), (H_5, 0.0262), (\Omega, 0.8835)\}$	
0.3	w^1	Jiao et al.	$B_2^1(P_1) = \{(H_2, 0.0098), (H_3, 0.0404), (H_4, 0.1727), (H_5, 0.1039), (\Omega, 0.6732)\}$	0.0954
0.3	w^2	Jiao et al.	$B_2^2(P_1) = \{(H_2, 0.0165), (H_3, 0.0263), (H_4, 0.2089), (H_5, 0.0703), (\Omega, 0.678)\}$	
0.5	w^1	Jiao et al	$B_2^1(P_1) = \{(H_2, 0.0147), (H_3, 0.0605), (H_4, 0.2683), (H_5, 0.1582), (\Omega, 0.4983)\}$	0.1491
0.5	w^2	Jiao et al.	$B_2^2(P_1) = \{(H_2, 0.0255), (H_3, 0.0381), (H_4, 0.3313), (H_5, 0.1061), (\Omega, 0.499\}$	
0.7	w^1	Jiao et al.	$B_2^1(P_1) = \{(H_2, 0.0188), (H_3, 0.0771), (H_4, 0.3554), (H_5, 0.2051), (\Omega, 0.3436)\}$	0.2126
0.7	w^2	Jiao et al.	$B_2^2(P_1) = \{(H_2, 0.0336), (H_3, 0.0464), (H_4, 0.4468), (H_5, 0.1355), (\Omega, 0.3377\}$	
0.9	w^1	Jiao et al.	$B_2^1(P_1) = \{(H_2, 0.0222), (H_3, 0.0913), (H_4, 0.4384), (H_5, 0.2474), (\Omega, 0.2006)\}$	0.2814
0.9	w^2	Jiao et al.	$B_2^2(P_1) = \{(H_2, 0.0411), (H_3, 0.0517), (H_4, 0.5603), (H_5, 0.156), (\Omega, 0.1869\}$	
0.1	w^1	Yang and Xu	$B_3^1(P_1) = \{(H_2, 0.0159), (H_3, 0.1011), (H_4, 0.5839), (H_5, 0.2826), (\Omega, 0.0165)\}$	0.2515
0.1	w^2	Yang and Xu	$B_3^2(P_1) = \{(H_2, 0.0221), (H_3, 0.0734), (H_4, 0.7035), (H_5, 0.1939), (\Omega, 0.0072\}$	
0.3	w^1	Yang and Xu	$B_3^1(P_1) = \{(H_2, 0.0159), (H_3, 0.1011), (H_4, 0.5839), (H_5, 0.2826), (\Omega, 0.0165)\}$	0.2468

Table 9 (continued)

$r_b(e_i)$ $(i = 1, \ldots, 8)$	Weights	Methods	Aggregated assessments	$\Delta B_m^{12}(P_1)$
0.3	w^2	Yang and Xu	$B_3^2(P_1) = \{(H_2, 0.0211), (H_3, 0.0706), (H_4, 0.7114), (H_5, 0.19), (\Omega, 0.0069\}$	
0.5	w^1	Yang and Xu	$B_3^1(P_1) = \{(H_2, 0.0138), (H_3, 0.0902), (H_4, 0.6118), (H_5, 0.2702), (\Omega, 0.014)\}$	0.2379
0.5	w^2	Yang and Xu	$B_3^2(P_1) = \{(H_2, 0.0195), (H_3, 0.0659), (H_4, 0.725), (H_5, 0.1832), (\Omega, 0.0063\}$	
0.7	w^1	Yang and Xu	$B_3^1(P_1) = \{(H_2, 0.011), (H_3, 0.0757), (H_4, 0.651), (H_5, 0.2513), (\Omega, 0.011)\}$	0.2162
0.7	w^2	Yang and Xu	$B_3^2(P_1) = \{(H_2, 0.0161), (H_3, 0.0562), (H_4, 0.754), (H_5, 0.1685), (\Omega, 0.0051\}$	
0.9	w^1	Yang and Xu	$B_3^1(P_1) = \{(H_2, 0.0037), (H_3, 0.0313), (H_4, 0.7907), (H_5, 0.1711), (\Omega, 0.0033)\}$	0.1251
0.9	w^2	Yang and Xu	$B_3^2(P_1) = \{(H_2, 0.0065), (H_3, 0.0263), (H_4, 0.8504), (H_5, 0.1148), (\Omega, 0.0019\}$	

5 Conclusions

To focus on MCDM problems modeled by belief functions with the consideration of the weights and reliabilities of individual assessments, the deficiencies of existing studies in two extreme situations and general situations are analyzed. To overcome the deficiencies, a new MCDM method with belief functions is proposed. In the method, a new process is designed to combine individual assessments with their weights and reliabilities. The process is theoretically proven to be correct in two extreme situations and is numerically demonstrated to be correct in general situations. To make the new process applicable, three ways are designed to help a decision maker flexibly provide individual assessments and correspondingly obtain their reliabilities. The generation of a solution is then presented. The proposed method is applied to solve the SPE problem for an enterprise that primarily provides high-quality gear boxes for high-speed trains. Through the data in the SPE problem, the main contributions of the proposed method are highlighted by its comparison with existing studies.

The main contributions of this paper include the following. (1) A new MCDM method with belief distributions is proposed to help generate solutions by considering the weights and reliabilities of individual assessments, which are consistent with what is anticipated by

Table 10 Aggregated assessments of the second strategic project by using the three methods with the consideration of different reliabilities of individual assessments

$r_b(e_i)$ $(i = 1, \ldots, 8)$	Weights	Methods	Aggregated assessments	$\Delta B_m^{12}(P_2)$
0.1	w^1	Proposed method	$B_1^1(P_2) = \{(H_1, 0.0084), (H_2, 0.0402), (H_3, 0.1281), (H_4, 0.3766), (H_5, 0.3968), (\Omega, 0.0498)\}$	0.4444
0.1	w^2	Proposed method	$B_1^2(P_2) = \{(H_1, 0.0083), (H_2, 0.0323), (H_3, 0.0785), (H_4, 0.2152), (H_5, 0.619), (\Omega, 0.0466)\}$	
0.3	w^1	Proposed method	$B_1^1(P_2) = (H_1, 0.0081), (H_2, 0.0391), (H_3, 0.1263), (H_4, 0.3789), (H_5, 0.3994), (\Omega, 0.0482)\}$	0.4767
0.3	w^2	Proposed method	$B_1^2(P_2) = \{(H_1, 0.0078), (H_2, 0.0304), (H_3, 0.074), (H_4, 0.2052), (H_5, 0.6378), (\Omega, 0.0449)\}$	
0.5	w^1	Proposed method	$B_1^1(P_2) = \{(H_1, 0.0078), (H_2, 0.0379), (H_3, 0.1243), (H_4, 0.3811), (H_5, 0.4021), (\Omega, 0.0467)\}$	0.5132
0.5	w^2	Proposed method	$B_1^2(P_2) = \{(H_1, 0.0072), (H_2, 0.0282), (H_3, 0.069), (H_4, 0.1937), (H_5, 0.6587), (\Omega, 0.0432)\}$	
0.7	w^1	Proposed method	$B_1^1(P_2) = \{(H_1, 0.0075), (H_2, 0.0367), (H_3, 0.1223), (H_4, 0.3835), (H_5, 0.4048), (\Omega, 0.0451)\}$	0.5546
0.7	w^2	Proposed method	$B_1^2(P_2) = \{(H_1, 0.0066), (H_2, 0.0259), (H_3, 0.0636), (H_4, 0.1805), (H_5, 0.6821), (\Omega, 0.0413)\}$	
0.9	w^1	Proposed method	$B_1^1(P_2) = \{(H_1, 0.0072), (H_2, 0.0354), (H_3, 0.1202), (H_4, 0.3859), (H_5, 0.4077), (\Omega, 0.0435)\}$	0.6019
0.9	w^2	Proposed method	$B_1^2(P_2) = \{(H_1, 0.0059), (H_2, 0.0233), (H_3, 0.0575), (H_4, 0.1653), (H_5, 0.7087), (\Omega, 0.0393)\}$	
0.1	w^1	Jiao et al.	$B_2^1(P_2) = \{(H_1, 0.0011), (H_2, 0.0049), (H_3, 0.0158), (H_4, 0.0462), (H_5, 0.0496), (\Omega, 0.8824)\}$	0.0583

Table 10 (continued)

$r_b(e_i)$ $(i = 1, \ldots, 8)$	Weights	Methods	Aggregated assessments	$\Delta B_m^{12}(P_2)$
0.1	w^2	Jiao et al.	$B_2^2(P_2) = \{(H_1, 0.001), (H_2, 0.0038), (H_3, 0.0091), (H_4, 0.025), (H_5, 0.0741), (\Omega, 0.887)\}$	
0.3	w^1	Jiao et al.	$B_2^1(P_2) = \{(H_1, 0.0028), (H_2, 0.0131), (H_3, 0.0423), (H_4, 0.1258), (H_5, 0.1351), (\Omega, 0.6809)\}$	0.1637
0.3	w^2	Jiao et al.	$B_2^2(P_2) = \{(H_1, 0.0026), (H_2, 0.0098), (H_3, 0.0239), (H_4, 0.0659), (H_5, 0.2095), (\Omega, 0.6883)\}$	
0.5	w^1	Jiao et al.	$B_2^1(P_2) = \{(H_1, 0.0041), (H_2, 0.0195), (H_3, 0.064), (H_4, 0.1937), (H_5, 0.2083), (\Omega, 0.5103)\}$	0.2625
0.5	w^2	Jiao et al.	$B_2^2(P_2) = \{(H_1, 0.0037), (H_2, 0.0142), (H_3, 0.0349), (H_4, 0.0972), (H_5, 0.3344), (\Omega, 0.5455)\}$	
0.7	w^1	Jiao et al.	$B_2^1(P_2) = \{(H_1, 0.0052), (H_2, 0.0248), (H_3, 0.0825), (H_4, 0.2545), (H_5, 0.2739), (\Omega, 0.3591)\}$	0.3624
0.7	w^2	Jiao et al.	$B_2^2(P_2) = \{(H_1, 0.0046), (H_2, 0.0174), (H_3, 0.043), (H_4, 0.1209), (H_5, 0.4552), (\Omega, 0.3589)\}$	
0.9	w^1	Jiao et al.	$B_2^1(P_2) = \{(H_1, 0.0061), (H_2, 0.0293), (H_3, 0.099), (H_4, 0.3114), (H_5, 0.3358), (\Omega, 0.2183)\}$	0.4835
0.9	w^2	Jiao et al.	$B_2^2(P_2) = \{(H_1, 0.0051), (H_2, 0.0196), (H_3, 0.0486), (H_4, 0.1381), (H_5, 0.5775), (\Omega, 0.211)\}$	
0.1	w^1	Yang and Xu	$B_3^1(P_2) = \{(H_1, 0.0054), (H_2, 0.0374), (H_3, 0.1292), (H_4, 0.3918), (H_5, 0.4056), (\Omega, 0.0306)\}$	0.4656
0.1	w^2	Yang and Xu	$B_3^2(P_2) = \{(H_1, 0.0055), (H_2, 0.0291), (H_3, 0.0732), (H_4, 0.228), (H_5, 0.6382), (\Omega, 0.0259)\}$	

Table 10 (continued)

$r_b(e_i)$ $(i = 1, \ldots, 8)$	Weights	Methods	Aggregated assessments	$\Delta B_m^{12}(P_2)$
0.3	w^1	Yang and Xu	$B_3^1(P_2) = \{(H_1, 0.0052), (H_2, 0.0362), (H_3, 0.1264), (H_4, 0.3953), (H_5, 0.4076), (\Omega, 0.0293)\}$	0.4687
0.3	w^2	Yang and Xu	$B_3^2(P_2) = \{(H_1, 0.0054), (H_2, 0.0283), (H_3, 0.0718), (H_4, 0.2278), (H_5, 0.6417), (\Omega, 0.025)\}$	
0.5	w^1	Yang and Xu	$B_3^1(P_2) = \{(H_1, 0.00482), (H_2, 0.0341), (H_3, 0.1216), (H_4, 0.4014), (H_5, 0.4109), (\Omega, 0.0272)\}$	0.4741
0.5	w^2	Yang and Xu	$B_3^2(P_2) = \{(H_1, 0.0051), (H_2, 0.027), (H_3, 0.0692), (H_4, 0.2274), (H_5, 0.6477), (\Omega, 0.0236)\}$	
0.7	w^1	Yang and Xu	$B_3^1(P_2) = \{(H_1, 0.0041), (H_2, 0.0297), (H_3, 0.1111), (H_4, 0.415), (H_5, 0.4173), (\Omega, 0.0228)\}$	0.4868
0.7	w^2	Yang and Xu	$B_3^2(P_2) = \{(H_1, 0.0045), (H_2, 0.0242), (H_3, 0.0638), (H_4, 0.2266), (H_5, 0.6604), (\Omega, 0.0206)\}$	
0.9	w^1	Yang and Xu	$B_3^1(P_2) = \{(H_1, 0.0019), (H_2, 0.0157), (H_3, 0.073), (H_4, 0.4676), (H_5, 0.4317), (\Omega, 0.01)\}$	0.5459
0.9	w^2	Yang and Xu	$B_3^2(P_2) = \{(H_1, 0.0025), (H_2, 0.0147), (H_3, 0.0439), (H_4, 0.2248), (H_5, 0.7032), (\Omega, 0.0109)\}$	

a decision maker. (2) Three ways are designed to flexibly determine the reliabilities of individual assessments according to the preferences of the decision maker. (3) A new process is designed to combine individual assessments by considering their weights and reliabilities, which is theoretically proven to be correct in two extreme situations and numerically demonstrated to be correct in general situations. (4) The proposed method is applied to help an enterprise select an appropriate strategic project, in which its advantages are demonstrated by its comparison with two existing methods.

When all historical data are available and the preferences of a decision maker are not necessarily required, how to objectively estimate the reliabilities of individual assessments and use them in MCDM is an interesting issue. It will be studied in our future research.

Table 11 Aggregated assessments of the third strategic project by using the three methods with the consideration of different reliabilities of individual assessments

$r_b(e_i)$ $(i = 1, \ldots, 8)$	Weights	Methods	Aggregated assessments	$\Delta B_m^{12}(P_3)$
0.1	w^1	Proposed method	$B_1^1(P_3) = \{(H_1, 0.0197), (H_2, 0.0311), (H_3, 0.2797), (H_4, 0.2819), (H_5, 0.3223), (\Omega, 0.0653)\}$	0.422
0.1	w^2	Proposed method	$B_1^2(P_3) = \{(H_1, 0.0097), (H_2, 0.0199), (H_3, 0.4843), (H_4, 0.2511), (H_5, 0.1633), (\Omega, 0.0717)\}$	
0.3	w^1	Proposed method	$B_1^1(P_3) = \{(H_1, 0.019), (H_2, 0.0306), (H_3, 0.2804), (H_4, 0.2829), (H_5, 0.3237), (\Omega, 0.0634)\}$	0.4497
0.3	w^2	Proposed method	$B_1^2(P_3) = \{(H_1, 0.0091), (H_2, 0.0189), (H_3, 0.4979), (H_4, 0.2476), (H_5, 0.1557), (\Omega, 0.0708)\}$	
0.5	w^1	Proposed method	$B_1^1(P_3) = \{(H_1, 0.0184), (H_2, 0.0301), (H_3, 0.2812), (H_4, 0.2838), (H_5, 0.325), (\Omega, 0.0614)\}$	0.4807
0.5	w^2	Proposed method	$B_1^2(P_3) = \{(H_1, 0.0084), (H_2, 0.0177), (H_3, 0.5131), (H_4, 0.2436), (H_5, 0.1473), (\Omega, 0.0599)\}$	
0.7	w^1	Proposed method	$B_1^1(P_3) = \{(H_1, 0.0177), (H_2, 0.0296), (H_3, 0.282), (H_4, 0.2849), (H_5, 0.3264), (\Omega, 0.0594)\}$	0.5153
0.7	w^2	Proposed method	$B_1^2(P_3) = \{(H_1, 0.0077), (H_2, 0.0165), (H_3, 0.5301), (H_4, 0.239), (H_5, 0.1378), (\Omega, 0.069)\}$	
0.9	w^1	Proposed method	$B_1^1(P_3) = \{(H_1, 0.017), (H_2, 0.0291), (H_3, 0.2829), (H_4, 0.286), (H_5, 0.3277), (\Omega, 0.0573)\}$	0.5545
0.9	w^2	Proposed method	$B_1^2(P_3) = \{(H_1, 0.0069), (H_2, 0.0151), (H_3, 0.5494), (H_4, 0.2335), (H_5, 0.127), (\Omega, 0.0681)\}$	

Table 11 (continued)

$r_b(e_i)$ $(i = 1, \ldots, 8)$	Weights	Methods	Aggregated assessments	$\Delta B_m^{12}(P_3)$
0.1	w^1	Jiao et al.	$B_2^1(P_3) = \{(H_1, 0.0027), (H_2, 0.0038), (H_3, 0.0352), (H_4, 0.0331), (H_5, 0.0397), (\Omega, 0.8856)\}$	0.0559
0.1	w^2	Jiao et al.	$B_2^2(P_3) = \{(H_1, 0.0013), (H_2, 0.0023), (H_3, 0.0578), (H_4, 0.0287), (H_5, 0.0189), (\Omega, 0.8909)\}$	
0.3	w^1	Jiao et al.	$B_2^1(P_3) = \{(H_1, 0.0071), (H_2, 0.0101), (H_3, 0.0957), (H_4, 0.0898), (H_5, 0.108), (\Omega, 0.6894)\}$	0.1547
0.3	w^2	Jiao et al.	$B_2^2(P_3) = \{(H_1, 0.0033), (H_2, 0.006), (H_3, 0.1631), (H_4, 0.0782), (H_5, 0.05), (\Omega, 0.6993)\}$	
0.5	w^1	Jiao et al.	$B_2^1(P_3) = \{(H_1, 0.0105), (H_2, 0.0152), (H_3, 0.1471), (H_4, 0.1378), (H_5, 0.1661), (\Omega, 0.5233)\}$	0.2446
0.5	w^2	Jiao et al.	$B_2^2(P_3) = \{(H_1, 0.0047), (H_2, 0.0089), (H_3, 0.2599), (H_4, 0.1197), (H_5, 0.074), (\Omega, 0.5329)\}$	
0.7	w^1	Jiao et al.	$B_2^1(P_3) = \{(H_1, 0.0133), (H_2, 0.0196), (H_3, 0.1831), (H_4, 0.1804), (H_5, 0.2179), (\Omega, 0.3757)\}$	0.3331
0.7	w^2	Jiao et al.	$B_2^2(P_3) = \{(H_1, 0.0058), (H_2, 0.011), (H_3, 0.353), (H_4, 0.1555), (H_5, 0.0924), (\Omega, 0.3823)\}$	
0.9	w^1	Jiao et al.	$B_2^1(P_3) = \{(H_1, 0.0158), (H_2, 0.0125), (H_3, 0.2362), (H_4, 0.2201), (H_5, 0.2665), (\Omega, 0.238)\}$	0.4263
0.9	w^2	Jiao et al	$B_2^2(P_3) = \{(H_1, 0.0065), (H_2, 0.0125), (H_3, 0.447), (H_4, 0.1874), (H_5, 0.1061), (\Omega, 0.2403)\}$	

Table 11 (continued)

$r_b(e_i)$ ($i = 1, \ldots, 8$)	Weights	Methods	Aggregated assessments	$\Delta B_m^{12}(P_3)$
0.1	w^1	Yang and Xu	$B_3^1(P_3) = \{(H_1, 0.0175), (H_2, 0.0291), (H_3, 0.2873), (H_4, 0.2962), (H_5, 0.3188), (\Omega, 0.0511)\}$	0.4276
0.1	w^2	Yang and Xu	$B_3^2(P_3) = \{(H_1, 0.0092), (H_2, 0.0195), (H_3, 0.5011), (H_4, 0.2557), (H_5, 0.1634), (\Omega, 0.051)\}$	
0.3	w^1	Yang and Xu	$B_3^1(P_3) = \{(H_1, 0.0168), (H_2, 0.0284), (H_3, 0.2863), (H_4, 0.299), (H_5, 0.3204), (\Omega, 0.0491)\}$	0.4279
0.3	w^2	Yang and Xu	$B_3^2(P_3) = \{(H_1, 0.009), (H_2, 0.0192), (H_3, 0.4998), (H_4, 0.259), (H_5, 0.1635), (\Omega, 0.0496)\}$	
0.5	w^1	Yang and Xu	$B_3^1(P_3) = \{(H_1, 0.0157), (H_2, 0.0271), (H_3, 0.2844), (H_4, 0.3039), (H_5, 0.3231), (\Omega, 0.0458)\}$	0.4284
0.5	w^2	Yang and Xu	$B_3^2(P_3) = \{(H_1, 0.0087), (H_2, 0.0186), (H_3, 0.4973), (H_4, 0.2648), (H_5, 0.1635), (\Omega, 0.0471)\}$	
0.7	w^1	Yang and Xu	$B_3^1(P_3) = \{(H_1, 0.0134), (H_2, 0.0243), (H_3, 0.2793), (H_4, 0.3157), (H_5, 0.3285), (\Omega, 0.0389)\}$	0.4289
0.7	w^2	Yang and Xu	$B_3^2(P_3) = \{(H_1, 0.0079), (H_2, 0.0174), (H_3, 0.4907), (H_4, 0.2786), (H_5, 0.1635), (\Omega, 0.0419)\}$	
0.9	w^1	Yang and Xu	$B_3^1(P_3) = \{(H_1, 0.0064), (H_2, 0.0145), (H_3, 0.2465), (H_4, 0.3745), (H_5, 0.34), (\Omega, 0.0181)\}$	0.4163
0.9	w^2	Yang and Xu	$B_3^2(P_3) = \{(H_1, 0.005), (H_2, 0.0124), (H_3, 0.4489), (H_4, 0.3491), (H_5, 0.1608), (\Omega, 0.0238)\}$	

Table 12 Aggregated assessments of the fourth strategic project by using the three methods with the consideration of different reliabilities of individual assessments

$r_b(e_i)$ $(i = 1, \ldots, 8)$	Weights	Methods	Aggregated assessments	$\Delta B_m^{12}(P_4)$
0.1	w^1	Proposed method	$B_1^1(P_4) = \{(H_1, 0.0197), (H_2, 0.0318), (H_3, 0.2785), (H_4, 0.3851), (H_5, 0.1397), (\Omega, 0.1452)\}$	0.3601
0.1	w^2	Proposed method	$B_1^2(P_4) = \{(H_1, 0.0097), (H_2, 0.0808), (H_3, 0.3059), (H_4, 0.2692), (H_5, 0.2434), (\Omega, 0.091)\}$	
0.3	w^1	Proposed method	$B_1^1(P_4) = \{(H_1, 0.0192), (H_2, 0.0317), (H_3, 0.2798), (H_4, 0.3898), (H_5, 0.1391), (\Omega, 0.1405)\}$	0.379
0.3	w^2	Proposed method	$B_1^2(P_4) = \{(H_1, 0.009), (H_2, 0.0832), (H_3, 0.3088), (H_4, 0.2638), (H_5, 0.2481), (\Omega, 0.087)\}$	
0.5	w^1	Proposed method	$B_1^1(P_4) = \{(H_1, 0.0186), (H_2, 0.0316), (H_3, 0.2811), (H_4, 0.3947), (H_5, 0.1384), (\Omega, 0.1357)\}$	0.4004
0.5	w^2	Proposed method	$B_1^2(P_4) = \{(H_1, 0.0084), (H_2, 0.0858), (H_3, 0.3196), (H_4, 0.2574), (H_5, 0.2535), (\Omega, 0.083)\}$	
0.7	w^1	Proposed method	$B_1^1(P_4) = \{(H_1, 0.018), (H_2, 0.0314), (H_3, 0.2823), (H_4, 0.3997), (H_5, 0.1377), (\Omega, 0.1308)\}$	0.4246
0.7	w^2	Proposed method	$B_1^2(P_4) = \{(H_1, 0.0076), (H_2, 0.0887), (H_3, 0.3155), (H_4, 0.2499), (H_5, 0.2596), (\Omega, 0.0786)\}$	
0.9	w^1	Proposed method	$B_1^1(P_4) = \{(H_1, 0.0141), (H_2, 0.0313), (H_3, 0.2836), (H_4, 0.4049), (H_5, 0.137), (\Omega, 0.1258)\}$	0.4523
0.9	w^2	Proposed method	$B_1^2(P_4) = \{(H_1, 0.0068), (H_2, 0.092), (H_3, 0.3195), (H_4, 0.241), (H_5, 0.2665), (\Omega, 0.0741)\}$	

Table 12 (continued)

$r_b(e_i)$ ($i = 1, \ldots, 8$)	Weights	Methods	Aggregated assessments	$\Delta B_m^{12}(P_4)$
0.1	w^1	Jiao et al.	$B_2^1(P_4) = \{(H_1, 0.0027), (H_2, 0.004), (H_3, 0.0336), (H_4, 0.048), (H_5, 0.0168), (\Omega, 0.895)\}$	0.0373
0.1	w^2	Jiao et al.	$B_2^2(P_4) = \{(H_1, 0.0013), (H_2, 0.0095), (H_3, 0.0353), (H_4, 0.0315), (H_5, 0.0282), (\Omega, 0.8943)\}$	
0.3	w^1	Jiao et al.	$B_2^1(P_4) = \{(H_1, 0.0072), (H_2, 0.0108), (H_3, 0.0919), (H_4, 0.1324), (H_5, 0.0457), (\Omega, 0.712)\}$	0.112
0.3	w^2	Jiao et al.	$B_2^2(P_4) = \{(H_1, 0.0033), (H_2, 0.0268), (H_3, 0.0983), (H_4, 0.0855), (H_5, 0.0793), (\Omega, 0.7069)\}$	
0.5	w^1	Jiao et al.	$B_2^1(P_4) = \{(H_1, 0.0108), (H_2, 0.0165), (H_3, 0.1417), (H_4, 0.2058), (H_5, 0.0699), (\Omega, 0.5553)\}$	0.1883
0.5	w^2	Jiao et al.	$B_2^2(P_4) = \{(H_1, 0.0048), (H_2, 0.0427), (H_3, 0.1541), (H_4, 0.1302), (H_5, 0.1254), (\Omega, 0.5428)\}$	
0.7	w^1	Jiao et al.	$B_2^1(P_4) = \{(H_1, 0.0138), (H_2, 0.0215), (H_3, 0.1857), (H_4, 0.272), (H_5, 0.091), (\Omega, 0.416)\}$	0.2684
0.7	w^2	Jiao et al.	$B_2^2(P_4) = \{(H_1, 0.0058), (H_2, 0.058), (H_3, 0.2054), (H_4, 0.1679), (H_5, 0.169), (\Omega, 0.3938)\}$	
0.9	w^1	Jiao et al.	$B_2^1(P_4) = \{(H_1, 0.0162), (H_2, 0.026), (H_3, 0.2261), (H_4, 0.334), (H_5, 0.1099), (\Omega, 0.2877)\}$	0.3552
0.9	w^2	Jiao et al.	$B_2^2(P_4) = \{(H_1, 0.0066), (H_2, 0.0733), (H_3, 0.2545), (H_4, 0.2002), (H_5, 0.2118), (\Omega, 0.2536)\}$	

Table 12 (continued)

$r_b(e_i)$ $(i = 1, \ldots, 8)$	Weights	Methods	Aggregated assessments	$\Delta B_m^{12}(P_4)$
0.1	w^1	Yang and Xu	$B_3^1(P_4) = \{(H_1, 0.0178), (H_2, 0.0312), (H_3, 0.2749), (H_4, 0.4157), (H_5, 0.1468), (\Omega, 0.1136)\}$	0.3371
0.1	w^2	Yang and Xu	$B_3^2(P_4) = \{(H_1, 0.0093), (H_2, 0.0783), (H_3, 0.3221), (H_4, 0.291), (H_5, 0.221), (\Omega, 0.0783)\}$	
0.3	w^1	Yang and Xu	$B_3^1(P_4) = \{(H_1, 0.0173), (H_2, 0.0304), (H_3, 0.2763), (H_4, 0.4228), (H_5, 0.145), (\Omega, 0.1082)\}$	0.3365
0.3	w^2	Yang and Xu	$B_3^2(P_4) = \{(H_1, 0.0091), (H_2, 0.0763), (H_3, 0.3247), (H_4, 0.2953), (H_5, 0.2189), (\Omega, 0.07575)\}$	
0.5	w^1	Yang and Xu	$B_3^1(P_4) = \{(H_1, 0.0163), (H_2, 0.0289), (H_3, 0.2785), (H_4, 0.4353), (H_5, 0.1418), (\Omega, 0.0993)\}$	0.3358
0.5	w^2	Yang and Xu	$B_3^2(P_4) = \{(H_1, 0.0087), (H_2, 0.0728), (H_3, 0.3291), (H_4, 0.3029), (H_5, 0.2151), (\Omega, 0.0713)\}$	
0.7	w^1	Yang and Xu	$B_3^1(P_4) = \{(H_1, 0.0141), (H_2, 0.0257), (H_3, 0.2824), (H_4, 0.4623), (H_5, 0.1341), (\Omega, 0.0814)\}$	0.3351
0.7	w^2	Yang and Xu	$B_3^2(P_4) = \{(H_1, 0.0079), (H_2, 0.0653), (H_3, 0.3383), (H_4, 0.3203), (H_5, 0.2061), (\Omega, 0.0621)\}$	
0.9	w^1	Yang and Xu	$B_3^1(P_4) = \{(H_1, 0.0072), (H_2, 0.0145), (H_3, 0.2852), (H_4, 0.5608), (H_5, 0.0996), (\Omega, 0.0327)\}$	0.3324
0.9	w^2	Yang and Xu	$B_3^2(P_4) = \{(H_1, 0.005), (H_2, 0.0383), (H_3, 0.3642), (H_4, 0.3977), (H_5, 0.1629), (\Omega, 0.0318)\}$	

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Appendix Proof of Theorem 1

Without the loss of generality, suppose that $r_{i,l} = 0$ and $r_{j,l} > 0$ for $j \neq i$. Under this assumption, it can be derived from Eqs. (22)–(24) that

$$\hat{\beta}_{n,b(i)}(a_l) = \vec{\beta}_{n,b(i-1)}(a_l), \tag{27}$$

$$\hat{\beta}_{\Omega,b(i)}(a_l) = \vec{\beta}_{\Omega,b(i-1)}(a_l), \tag{28}$$

and

$$\hat{\beta}_{P(\Omega),b(i)}(a_l) = \vec{\beta}_{P(\Omega),b(i-1)}(a_l). \tag{29}$$

This indicates that the individual assessment $B(e_i(a_l))$ has no influence on the combination result of the first i-1 assessments and further on the overall assessment.

When there are multiple individual assessments with zero-valued reliabilities, it can be similarly known that these assessments contribute nothing to the overall assessment. As a result, the principle presented in Proposition 1 is satisfied by the combination presented in Definition 3.

To focus on the principle presented in Proposition 2, the process of iteratively combining individual assessments developed by Yang and Xu (2013) is presented.

Definition A.1 (Yang & Xu, 2013) Given the individual assessments $B(e_i(a_l))$ $(i = 1, \ldots, L)$ and their weights w_i, the combination result of the first i assessments is defined as.

$$\left\{\left(H_n,\ \beta_{n,b(i)}(a_l)\right),\ n = 1, \ldots, N;\ \left(\Omega,\ \beta_{\Omega,b(i)}(a_l)\right)\right\}, \tag{30}$$

where

$$\beta_{n,b(i)}(a_l) = \frac{\hat{\beta}_{n,b(i)}(a_l)}{\sum_{n=1}^{N}\hat{\beta}_{n,b(i)}(a_l) + \hat{\beta}_{\Omega,b(i)}(a_l)}, \tag{31}$$

$$\beta_{\Omega,b(i)}(a_l) = \frac{\hat{\beta}_{\Omega,b(i)}(a_l)}{\sum_{n=1}^{N}\hat{\beta}_{n,b(i)}(a_l) + \hat{\beta}_{\Omega,b(i)}(a_l)}, \tag{32}$$

$$\vec{\beta}_{n,b(i)}(a_l) = \frac{\hat{\beta}_{n,b(i)}(a_l)}{\sum_{n=1}^{N}\hat{\beta}_{n,b(i)}(a_l) + \hat{\beta}_{\Omega,b(i)}(a_l) + \hat{\beta}_{P(\Omega),b(i)}(a_l)}, \tag{33}$$

$$\vec{\beta}_{\Omega,b(i)}(a_l) = \frac{\hat{\beta}_{\Omega,b(i)}(a_l)}{\sum_{n=1}^{N}\hat{\beta}_{n,b(i)}(a_l) + \hat{\beta}_{\Omega,b(i)}(a_l) + \hat{\beta}_{P(\Omega),b(i)}(a_l)}, \tag{34}$$

$$\vec{\beta}_{P(\Omega),b(i)}(a_l) = \frac{\hat{\beta}_{P(\Omega),b(i)}(a_l)}{\sum_{n=1}^{N}\hat{\beta}_{n,b(i)}(a_l) + \hat{\beta}_{\Omega,b(i)}(a_l) + \hat{\beta}_{P(\Omega),b(i)}(a_l)}, \tag{35}$$

$$\begin{aligned}\hat{\beta}_{n,b(i)}(a_l) = &\left[(1 - w_i)\vec{\beta}_{n,b(i-1)}(a_l) + \vec{\beta}_{P(\Omega),b(i-1)}(a_l)w_i \cdot \beta_{n,i}(a_l)\right] \\ &+ \vec{\beta}_{n,b(i-1)}(a_l) \cdot w_i\beta_{n,i}(a_l) \\ &+ \vec{\beta}_{n,b(i-1)}(a_l) \cdot w_i\beta_{\Omega,i}(a_l) + \vec{\beta}_{\Omega,b(i-1)}(a_l) \cdot w_i\beta_{n,i}(a_l)\end{aligned}, \tag{36}$$

$$\begin{aligned}\hat{\beta}_{\Omega,b(i)}(a_l) = &\left[(1 - w_i)\vec{\beta}_{\Omega,b(i-1)}(a_l) + \vec{\beta}_{P(\Omega),b(i-1)}(a_l)w_i \cdot \beta_{\Omega,i}(a_l)\right] \\ &+ \vec{\beta}_{\Omega,b(i-1)}(a_l) \cdot w_i\beta_{\Omega,i}(a_l)\end{aligned}, \tag{37}$$

and

$$\hat{\beta}_{P(\Omega),b(i)}(a_l) = (1 - w_i)\vec{\beta}_{P(\Omega),b(i-1)}(a_l). \tag{38}$$

Suppose that $r_{i,l} = 1$ for $i = 1, \ldots, L$. Then, it can be found that Eqs. (22)-(24) reduce to Eqs. (36)–(38). In this situation, the overall assessment generated using Definition 3 is the same as the one generated using Definition A.1.

In other situations, where the above assumption is not satisfied, one can find that the combination result derived from Definition 3 is different from the result derived from Definition A.1 through comparing Eqs. (22)–(24) with Eqs. (36)–(38).

As a whole, this theorem is verified. □

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