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Innovative Applications of O.R.

Dynamic investment strategies and leadership in product innovation[☆]

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We study the inter-temporally optimal innovation strategies of incumbent manufacturing firms that compete in an established market and can extend their product line through product innovation. Firms invest in production capacity and R&D knowledge stock, where the R&D knowledge stock and the current R&D investment determine the hazard rate of innovation. Our findings show that the firms' optimal R&D strategies are driven by a subtle interplay between the relative positions of their R&D knowledge stocks and their current relative positions on the established market. First, we find that under symmetric investment costs the knowledge leader should spend more on R&D than the knowledge laggard only if it has a substantially smaller market share on the established market. If the knowledge leader's market share is sufficiently large, its optimal investment in R&D is so small that its innovation rate is lower than the knowledge laggard's. Second, optimal investment in R&D knowledge is negatively affected by the opponent's production capacity on the established market if the competitor has not innovated yet. However, we find that this effect is reversed after the competitor has successfully introduced the new product on the submarket. Third, the manufacturing firm with higher costs of adjusting production capacity for the established product has a higher incentive to engage in product innovation and might even achieve a higher total discounted profit than its more efficient competitor.

1. Introduction

A considerable fraction of product innovations in related submarkets is accomplished by established incumbents (e.g. Buensdorf, 2016; Chandy & Tellis, 2000; Franco, Sarkar, Agarwal, & Echambadi, 2009; King & Tucci, 2002; Sood & Tellis, 2011). This observation raises the question how an incumbent's optimal strategy in an R&D race depends on its strength on its established market. To illustrate, in 2010 when Apple and Samsung introduced Tablet PCs and thus created a new submarket that coexisted with the established market of portable computers, they had relatively small market shares on the Laptop market (3.4% and 2.8% respectively) compared to Hewlett Packard and Dell (18% and 12% respectively). Interestingly, HP and Dell entered the Tablet market much later in 2013.[1] In the market for smartphones, Nokia, as the clear market leader in the early 2000s (market share 2005: 32.5%), introduced its first touchscreen phone in 2011, while its initially smaller competitor Samsung (market share 2005: 12.7%) introduced its first smartphone with a touchscreen in 2008. As a result, Samsung achieved a higher market share in 2012 and also exhibited strongly positive dynamics of profit in the smartphone market compared to Nokia.

Why do dominant incumbent firms leave emerging innovative submarkets to their competitors? The operations management literature provides several explanations for an observed pattern of *action-reaction* where big incumbents are often slower than smaller rivals (or entrants) in pioneering newly created submarkets. First, Christensen (1997) argues with his Innovator's Dilemma theory that big incumbent firms rely on a dominant managerial logic, organizational capabilities and cognitive frames, which lead these firms to miss new opportunities sometimes even after investing

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[1] HP made a short - but unsuccessful - premature attempt of offering Tablet PCs in 2010.

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large sums (see also, e.g., Henderson, 1993; Henderson, 2006; Tripsas, 1997; Tripsas & Gavetti, 2000).

Second, incumbents underinvest in the development of new technologies or products due to the fear of cannibalizing their existing businesses. However, Nault & Vandenbosch (1996) argue that incumbent firms have to 'eat their own lunches before somebody else does'. Third, large firms lack the potential of small firms to motivate their engineering staff. The associated bureaucracy and incentive effects lead to lower innovation intensity (e.g. Zenger, 1994). Fourth and finally, incumbent firms are less successful in innovation since there are diseconomies of scope which arise from key assets that have to be shared across businesses (Bresnahan, Greenstein, & Henderson, 2012).

In contrast to these explanations that focus on the suboptimality of firms' decisions, our paper provides a rational argument for why the innovator's dilemma happens in certain markets. We in fact show that large incumbents might enter a new market relatively late as a result of intertemporally optimal behavior of all firms in the market (see also Pacheco-de Almeida, 2010; Swinney, Cachon, & Netessine, 2011).

However, smaller incumbent firms do not always leapfrog their dominant rivals. Frequently, *increasing dominance* emerges and dominant firms introduce their new products earlier than their competitors. For example, as noted by Chandy & Tellis (2000), Hattori-Seiko – which was a dominant producer of mechanical watches at the time – was the first firm to introduce the analog quartz watch. Likewise the Hamilton Company, another incumbent firm, was an early mover in the digital quartz watch submarket. So, what drives dominant manufacturing firms to stay in the leadership position? As an explanation why some dominant firms invest aggressively in innovation while others do not, Chandy, Prabhu, & Antia (2003) argue that some managers might have different expectations about a new technology's impact on existing products than other managers. In contrast, our paper also provides a rational explanation for the outcome of increasing dominance.

In particular, we employ a dynamic duopoly model in which manufacturing firms compete in an established market and can extend their product line through product innovation by investing in R&D. Firms invest in production capacity and R&D knowledge stock, where the R&D knowledge stock jointly with the firm's current R&D investment determines the hazard rate of innovation. After a firm has successfully innovated, it is active on the established and the new market. We capture the strength of a firm on the established market by the firm's production capacity for the established product and its capability to successfully develop a new product by its R&D knowledge stock.[2] We find that the occurrence of an action-reaction pattern or an increasing dominance pattern not only depends on the strength on the established product market, but that instead it is driven by an interplay between the firms' relative positions of their production capacities for the established product and their relative positions of their current R&D knowledge stocks.

The main contribution of this paper is that it provides new insights into the interplay between firms' production capacities and their R&D investments under dynamic duopoly competition. First, we consider a market where manufacturing firms are symmetric except for their production capacities on the established market and for their knowledge stocks. We find that the knowledge leader invests more in R&D than its competitor only if it has a substantially smaller market share on the established market. In contrast, if the knowledge leader's market share is sufficiently large, it invests so little in R&D that its innovation rate is lower than the knowledge laggard's. Intuitively, the fear of cannibalization arising after the new product introduction reduces the R&D incentives of a knowledge leader with a large market share. Hence, whether the knowledge leader should invest more or less in R&D than its competitor depends on the relative strengths of the two firms on the established market. In our setting, both patterns of R&D behavior, increasing dominance of the knowledge leader or leapfrogging by the knowledge laggard, can occur under the same mode of competition. This is in contrast to the existing literature, which highlights that the particular pattern strongly depends on whether firms compete in quantities or prices (see Vickers, 1986).

Second, we consider a market where one manufacturing firm has higher investment costs than its rival. We find that the firm with the cost *disadvantage* on the established market invests more in building up knowledge for product innovation than its competitor and, therefore, expects to innovate faster. It might even turn out that this firm can actually have a higher expected discounted profit stream than its more cost-efficient opponent. Intuitively, the cost disadvantage acts as a commitment device for the less efficient firm to innovate faster.

Third, we exploit the opportunity offered by our dynamic setup to study a manufacturing firm's optimal product innovation strategy in the period *before* the innovation, where competition only occurs on the established market, and the period *after* the competitor has created a new submarket with its product innovation. We find that there are significant qualitative differences in the optimal innovation strategy. Innovation activity is negatively affected by the opponent's production capacity on the established market if the opponent has not innovated yet. However, this influence is reversed *after* the opponent has successfully innovated. Intuitively, as long as a firm has a chance to become the sole manufacturer of the new product, a large production capacity of the competitor on the established market reduces the profitability of the innovation. As soon as the competitor is already active on the new market, an increase of its production capacity on the established market reduces its activity on the new market, which makes this market more profitable for the focal firm. Furthermore, the optimal level of a firm's investment in R&D exhibits a downward jump at the time when the opponent creates the new submarket, since this precludes the firm from ever becoming the sole producer of the new product. Hence, our paper extends the literature by showing that the properties of a firm's optimal R&D strategy are crucially affected by the competitor's product range.

A methodological contribution of our paper is that it develops a modeling approach which captures dynamic strategic interaction in a market environment characterized by the interplay of continuously evolving state variables (production capacities, R&D stocks) and endogenous discrete changes which are governed by arrival rates depending on knowledge stocks (i.e. state variables) and R&D investments (i.e. control variables). Such situations occur frequently in real-world markets, for example, if suppliers invest in capabilities to subsequently encroach in the downstream manufacturer's retail market or if manufacturing firms invest in process R&D and in case of success have access to better technologies. Characterizing Markov Perfect Equilibria in such games requires the solution of a system of coupled Hamilton-Jacobi-Bellman equations. We put forward a numerical procedure based on sparse grid methods, which allows us to obtain (approximate) solutions to such a model even for relatively high dimensions of the state space. The application of such a method to managerial strategy analysis is new to the literature and highlights the potential of our approach for applications in the field of production and operations management.

[2] The monopoly version of this model has been analyzed in Dawid, Keoula, Kopel, & Kort (2015), however the agenda of this paper requires the consideration of strategic competition. Dawid, Kopel, & Kort (2013) explore the effect of differences in production capacity on incentives to introduce a new product in a static duopoly model. Our dynamic setting here allows us to focus on the R&D investments and the evolution and influence of firms' knowledge stocks.

The paper is organized as follows. The next section gives an overview of the related literature. Section 3 introduces the model and details its assumptions. Section 4 provides a formal characterization of the equilibrium investment strategies. Section 5 presents our main insights about the properties of firms' optimal R&D strategies. In Section 6 we analyze the effect of competition on R&D investments. In Section 7 we investigate the robustness of our results and show that our qualitative insights remain valid for a large range of all our model parameter values. Section 8 concludes and discusses some possible extensions of our analysis. Appendix A contains the proof of Proposition 1 and Online Appendices provide a detailed description of our numerical method (Appendix B) and additional illustrations of model dynamics (Appendix C).

2. Literature

A rich stream of literature has addressed the question why leading incumbents often are slower to enter newly created submarkets than their smaller rivals or entrants. Game-theoretic analyses of this issue, however, have to a large extent relied on static models (see e.g. Druehl & Schmidt, 2008; Huang & Sosic, 2010; Schmidt & Porteus, 2000a; Schmidt & Porteus, 2000b). We complement this literature by considering a dynamic game theoretic setting in which firms' strengths on the existing market as well as the hazard rates of the innovations are endogenously determined by the firms' investment strategies.

A paper close to ours is Dawid et al. (2013). They study a static game-theoretic model, where firms, which have already developed a new product, decide on whether to add this new differentiated product to their product portfolio. Their main insight is that the larger incumbent has a lower incentive to introduce the new product and they identify four main effects whose interplay is responsible for this result. Our stochastic and genuinely dynamic setting additionally incorporates the firms' decisions about their R&D investment. We extend the analysis in Dawid et al. (2013) by focusing on the innovation race between the competing firms and explore how the interplay between the firms' capacities and knowledge stocks influence their R&D incentives. Our finding that in a situation where the firms have identical knowledge stocks it is the larger incumbent which invests less in R&D resembles the main insight in Dawid et al. (2013). We also find that the different effects identified in their setting are also the main drivers of these results in our model. However, the main contribution of this paper lies in the analysis of situations where firms differ in their knowledge stocks and additional effects become relevant. In particular, in our first main finding, we identify situations where, depending on the firms' knowledge stocks, the firm with the larger capacity invests either less or more in R&D compared to its smaller competitor, and where it has a lower or higher probability to be the first innovator. Jointly considering the firms' production capacities on the established market and their relative positions in R&D knowledge stock enables us to give a more refined picture of conditions for action-reaction or increasing dominance. Also our other two main findings substantially extend Dawid et al. (2013). By investigating the effect of asymmetric investment costs, which are not considered in Dawid et al. (2013), we characterize conditions under which the less efficient firm has a higher expected discounted profit stream. Moreover, our inter-temporal framework allows us to investigate situations before and after the competitor has innovated and identify managerial implications for a firm's innovation strategy contingent on the competitor's product range.

The literature so far has not considered the determinants of dynamic *product innovation strategies* of incumbent firms which intend to extend their product range.[3] For example, work on dynamic R&D competition between incumbents has focused on the dependence of a firm's R&D investment on its (relative) level of technology (e.g. Aghion, Harris, Howitt, & Vickers, 2001; Canbolat, Golany, Mund, & Rothblum, 2012; Grishagin, Sergeyev, & Silipo, 2001). In these models, a firm typically carries out R&D in order to improve its technology, which determines its current profit and also the expected return from innovation.

Additionally, patent race models with *exogenously* given value of innovation (see Reinganum, 1989) have studied the dependence of R&D investment on different factors. Within a framework of an n-firm race, Canbolat et al. (2012) show how the set of firms active in the race and their (one-time) spending on the innovation project depends on their technological and marketing efficiency. Extending memoryless patent race models, a variety of contributions have analyzed models with multiple innovation stages (e.g. Fudenberg, Gilbert, Stiglitz, & Tirole, 1983; Grossman & Shapiro, 1987; Harris & Vickers, 1985). Concerning the R&D leader's and the R&D follower's innovation incentives, it has been analyzed if the current R&D leader will become increasingly dominant or if R&D leadership will change. A general insight from these contributions is that firms invest most in R&D when they are neck-and-neck with their competitors and that R&D laggards trailing the R&D leader by too many steps have incentives to drop out of the race. Hence, multi-stage patent race models tend to predict increasing dominance of R&D leaders.

Another paper close to ours is Doraszelski (2003). Like us, he considers a single-step race between two firms, in which the innovation rate of a firm depends both on current R&D effort and its R&D knowledge stock accumulated through past R&D activities. A main difference however is that Doraszelski (2003) analyzes a patent race with *exogenously* given innovation value, whereas in our paper the innovation payoff is *endogenously* determined. Intuitively, in our setting the innovator's payoff depends negatively on the R&D knowledge stock of the R&D laggard since the larger the R&D laggard's knowledge stock, the sooner the laggard will catch up with the innovator. This reduces the innovator's value. The implication is that our results crucially differ from Doraszelski (2003) in two important ways. *First*, due to the negative dependence of the innovator's value on the R&D laggard's knowledge stock, we find a negative relationship between R&D investment and the competitor's R&D knowledge stock. In contrast, Doraszelski (2003) obtains that the effect of an increase of the opponent's knowledge stock on a firm's R&D investments is positive if either the own knowledge stock or the payoff of winning the race is sufficiently large. As a *second* important difference, in our dynamic model both patterns – catch-up by the R&D laggard *and* increasing dominance by the R&D leader – can result from optimal R&D strategies. Consequently, we are able to identify both patterns of innovation behavior observed in real-world markets within the same model setting by highlighting the crucial role of the interplay of firms' relative positions in terms of their R&D knowledge stocks and their relative strengths on the established product market. Quite in contrast, Doraszelski (2003) finds that as long as the hazard rate depends in a (weakly) concave way on the knowledge stock (which also holds in our case), the laggard always invests more in R&D than the knowledge leader.

Dynamic innovation competition models, in which the value of innovation is endogenously determined by some form of market competition have been considered e.g. by Hörner (2004), Ludkovski & Sircar (2016) and Dawid, Keoula, & Kort (2017).[4] In these models,

[3] An exception is Igami (2017), who considers a dynamic industry model in which incumbents take binary decisions to extend their product range. Differently from our setup, this model captures neither the uncertainty of the innovation process nor the dynamic adjustment of R&D knowledge and production capacity levels.

the flow profits of firms depend on their (relative) number of successful innovations, where innovation probabilities are determined by current R&D effort. Hörner (2004) analyzes a setting in which only the sign but not the size of the difference in the number of successful innovation steps of the two competitors determines their profits. He demonstrates that investment is highest in situations in which the gap between firms is large rather than when competitors are neck-and-neck. Ludkovski & Sircar (2016) consider a setting with Cournot competition where a successful innovator improves on its own previous technology. They show that in this setting increasing dominance of the technology leader occurs. Dawid et al. (2017) consider a dynamic Cournot duopoly model in which both competitors sell an established product and invest in R&D in order to develop a new product. Differently from our paper, in their setting, the innovation rate of a firm depends on a combination of its current R&D investment and the level of a *public* knowledge stock, the accumulation of which is driven by both firms' total R&D investment. Additionally, firms can boost the demand for the established as well as the new product by investing in public relations measures improving the acceptance of the corresponding product. A key finding of their analysis is that in equilibrium two locally stable steady states might co-exist, such that it depends on the initial strength of the demand for the established product whether the firms engage in R&D investment or not. Our setup shares with this literature that the value of innovation for the firms is determined endogenously. However, differently from all these papers, we incorporate manufacturing firms' investments in production capacities into a race for obtaining a product innovation. This allows us to disentangle the effect of the relative position on the established market and the relative position in the innovation race and enables us to characterize the occurrence of increasing dominance versus action-reaction in terms of the interplay of these forces.

Our paper also builds on the literature on capital accumulation games (e.g. Jun & Vives, 2004; Reynolds, 1991), in which capacity investments of single-product firms engaged in oligopolistic competition have been characterized both in the framework of Open-Loop and Markov Perfect Equilibria. Besanko & Doraszelski (2004) introduce a capacity accumulation game in discrete time to characterize an evolving industry structure and to explain the persistence of differences in firm size (cf. also Doraszelski & Pakes, 2007; Escobar, 2013). Besanko, Doraszelski, Lu, & Satterthwaite (2010) study capacity accumulation patterns in a discrete-time dynamic duopoly game with strategic uncertainty (about the rival's cost). This literature, however, has not dealt with investments in capacity of multi-product firms. Furthermore, we extend this literature by analyzing the interplay of dynamic investments in production capacities with the firms' incentives to invest in R&D to enlarge their product range.[5]

The main focus of this paper is on the strategic competition between firms, and in particular on the innovation race. It builds on work exploring the effect of the interplay of a firms' production capacity and knowledge stock in the absence of strategic competition. Dawid et al. (2015) analyze the incentives of a monopolist to invest in R&D for product innovation in a dynamic setting similar to the one considered in this paper. They show that a firms' incentive to invest in R&D is negatively affected by the size of its production capacity on the established market. This insight also carries over to our setting with duopolistic competition. Indeed, the main novelty of this paper relative to Dawid et al. (2015) is in the incorporation of competition. This allows us to study the impact of production capacities on the innovation race between two incumbents and also to examine the effects of cost asymmetries between firms on innovation speed and (relative) profits. These issues could not be addressed in a monopoly model.

With respect to the numerical method, our paper builds on work using a Chebyshev collocation method for the determination of Markov Perfect Equilibria of differential games (e.g. Dawid et al., 2017; Doraszelski, 2003; Vedenov & Miranda, 2001). Sparse-grids (Smolyak bases) have been employed for the solution of large dynamic equilibrium models in macroeconomics (see Maliar & Maliar, 2014 and the survey in Fernandez-Villaverde, Rubio-Ramirez, & Schorfheide, 2016). To our knowledge, the present paper is the first application of a sparse grid method to an Operations Management problem with dynamic strategic interaction. In particular, we are the first to provide a numerical characterization of a Markov Perfect Equilibrium in a multi-mode differential game with a high-dimensional state space. Since differential games with evolving structure are capable of capturing a variety of applications in management science and operations management, we believe that our paper is of interest to scholars beyond the area of firm strategy and innovation.

3. The model

We consider a dynamic duopoly in continuous time with evolving market structure.[6] Initially, two incumbent manufacturing firms A and B produce a homogeneous product called product 1. We refer to product 1 as the established product.[7] Both firms invest in the accumulation of an R&D knowledge stock which is essential to develop a new and differentiated product, called product 2. Let τ_A and τ_B be the stochastic completion times of the firms' R&D projects. Once one of the firms has innovated, a new submarket can be created.[8] At this stage, only the innovator can sell the established product and the new product. The innovation laggard is selling the established product while simultaneously investing in R&D knowledge stock to eventually enter the new submarket as well. Once both firms have innovated, the established product 1 and the new product 2 are supplied by both firms. In the sequel, we refer to both firms A and B by subscript $f (f = A, B)$ but keep the distinction whenever it is necessary.

To enable production, firms A and B need production capacities. These capacities are denoted by $K_{if}(t), f = A, B$, where subscript $i = 1, 2$ refers to product *i*. In the period prior to the creation of the new submarket, called mode m_1, both firms invest in their production capacity $K_{1f}(t)$ for product 1. Additionally, they also invest in the accumulation of their firm-specific R&D knowledge stock $K_{Rf}(t)$. In the period after the new submarket has been created, the innovator (i.e. the firm that has first completed the R&D project) can invest in production capacities for both products. The innovation laggard which has not innovated yet continues to invest in production capacity for product 1 and in its R&D knowledge stock. We will denote the scenario where firm A (firm B) innovates first mode m_2 (mode m_3). In the last phase or mode m_4, which emerges after the new product has also been launched by the in-

[4] See Vickers (1986) for a seminal contribution to this stream of literature.

[5] Dynamic models of product introduction strategies of capacity-constrained firms have for example been analyzed by Bilginer & Erhun (2015).

[6] Formulating the game in continuous time implies that no scenarios with simultaneous innovations by both firms within one time period have to be considered, thereby reducing the computational burden of the analysis, see also Doraszelski & Judd (2012).

[7] The assumption that both firms sell a homogeneous established product is made for analytical convenience, but is not essential for our results. Our findings would not change qualitatively, if we would consider a setting where the established products of the two firms are horizontally differentiated substitutes.

[8] The submarket is actually created once the innovator invests in production capacity for the new product and hence this product is manufactured and sold on the market.

novation laggard, firms compete on both markets and (dis)invest in both production capacities.

At each point in time t, the status of firm $f = A, B$ is characterized by production capacities and the R&D knowledge stock $(K_{1f}(t), K_{2f}(t), K_{Rf}(t))$ and the mode of the game $m(t) \in M = \{m_1, m_2, m_3, m_4\}$, that captures which of the firms has already innovated. The state of the game consists of the three stock variables of both firms and is therefore six-dimensional. Production capacities and R&D knowledge stocks accumulate according to

$$\dot{K}_{if}(t) = I_{if}(t) - \delta_i K_{if}(t) \qquad i = 1, 2, R, \quad f = A, B, \tag{1}$$

where $I_{if}(t)$ is the investment of firm f in stock K_{if} at time t. These standard dynamics account for the fact that accumulation of production capacities and stock of knowledge take time, but also that depreciation of production capacities takes place where $\delta_i > 0, i = 1, 2$ denote the (symmetric across firms) depreciation rates (see, e.g., Besanko et al., 2010 and Dierickx & Cool, 1989). With regard to the depreciation of R&D knowledge stock, organizational forgetting (see Doraszelski, 2003 and references therein) is captured by $\delta_R > 0$.[9]

Concerning production capacities, we allow the firms to intentionally disinvest, i.e. $I_{if} \in \mathbb{R}$. With regard to the R&D knowledge stocks, we make the sensible assumption that knowledge investments are non-negative, i.e. $I_{Rf} \geq 0$. The firms cannot invest in production capacity of the second product before the R&D project has been completed, which implies that $I_{2f}(t) = 0$ for all $t < \tau_f$.

Furthermore, all stocks have to be non-negative:

$$K_{if}(t) \geq 0 \quad \forall t \geq 0 \quad i = 1, 2, R, f = A, B. \tag{2}$$

In the following analysis we assume that both firms have perfect information about the current levels of all stock variables, including production capacities and knowledge stock of the competitor. This assumption seems innocuous with respect to production capacities, which relate to observable physical capital. However it seems less obvious with respect to the size of the knowledge stock. Note however, that although the competitor's knowledge stock is not directly observable, firms obtain signals about this stock through communication with industry experts or competitor's employees, labor poaching or industrial espionage. For reasons of analytical tractability we assume here that the signal obtained through these channels is perfect. Making such an assumption is very common in the related literature. In particular, all the papers on innovation races reviewed in Section 2 (Dawid et al., 2017; Doraszelski, 2003; Hörner, 2004; Ludkovski & Sircar, 2016) also assume perfect information about the competitors' knowledge stocks and technological states.

The probability that firm f successfully innovates is determined by a firm's hazard rate. The hazard rate in mode m_1 is positively affected by the current investment $I_{Rf}(t)$ in the R&D knowledge stock, as well as by accumulated knowledge through past R&D investments captured by the firm's R&D knowledge stock $K_{Rf}(t)$ itself. We employ an additive form of the hazard rate (see Doraszelski, 2003) given by

$$\lambda(I_{Rf}(t), K_{Rf}(t)) = \alpha I_{Rf}(t) + \beta (K_{Rf}(t))^{\psi}, \; \alpha \geq 0, \; \beta \geq 0, \; \psi > 0, \quad f = (A, B), \tag{3}$$

which captures the described determinants of the hazard rate in the most simple way. The parameters α and β determine, respectively, the marginal impact of the current investment and the accumulated knowledge on the hazard rate. In what follows, we will focus on scenarios where $\psi = 1$, i.e. where the firm's hazard rate is linear in its R&D knowledge stock. In Section 7 we check the robustness of our findings also for concave ($\psi < 1$) and convex ($\psi > 1$) hazard rates. The hazard rate in modes m_2 or m_3, where the competitor has already innovated, is given by $\omega\lambda(I_{Rf}(t), K_{Rf}(t))$ with $\omega > 0$. In the main part of our analysis, we assume that the hazard rate does not change between modes and set $\omega = 1$. However, in our robustness section we also consider a scenario where innovation by the laggard firm is impeded by patents granted (or other forms of intellectual property protection) to the innovator ($\omega < 1$). Additionally, we study the case where due to imitation effects the product innovation of a laggard firm becomes easier once the new product has already been introduced by the competitor ($\omega > 1$).

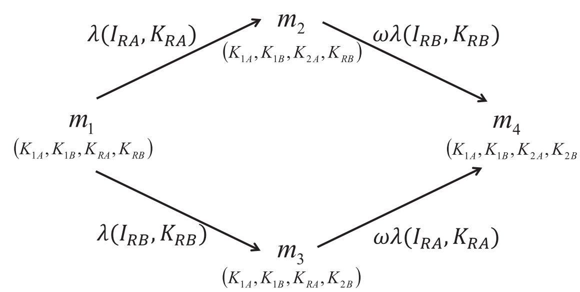


Fig. 1. The transition rates between the different modes of the game. Below each mode we indicate in brackets the vector of relevant state variables in that mode.

Formally, the changes between the modes of the game are described by a Markov process $m(t)$ on the set of modes M where the transition rates are given by

$$\lim_{\Delta \to 0} \frac{1}{\Delta} Prob\big\{m(t+\Delta) = m_j \mid m(t) = m_i\big\} = \begin{cases} \lambda(I_{RA}, K_{RA}) & (i, j) = (1, 2), \\ \lambda(I_{RB}, K_{RB}) & (i, j) = (1, 3), \\ \omega\lambda(I_{RA}, K_{RA}) & (i, j) = (3, 4), \\ \omega\lambda(I_{RB}, K_{RB}) & (i, j) = (2, 4), \\ 0 & \text{else}. \end{cases} \tag{4}$$

This formulation embodies the idea that before the creation of the new submarket (i.e. in mode m_1), there are positive probabilities of transition either to mode m_2 with firm A as innovator or to mode m_3 with firm B as innovator. From modes m_2 or m_3, the transition results in a switch to mode m_4, where both firms offer both products. The expected time of this final transition depends on the hazard rate of the innovation laggard. Once both firms offer both products, no more transitions are possible. We illustrate the possible transitions between the different modes as well as the state variables that are relevant in each mode in Fig. 1. In each mode only four out of the six states are relevant. The reason is that before a firm has innovated, it cannot invest in production capacity of the new product and therefore $K_{2f} = 0$. After firm f has innovated, both production capacities (K_{1f}, K_{2f}) are relevant, but the R&D knowledge stock K_{Rf} is not since no further innovation is possible.

At any point in time t, firms compete in quantities, where it is assumed that current production capacities for the two products are always fully used. This assumption is commonly made in the literature on capacity-constrained oligopoly competition (e.g. Anand & Girotra, 2007; Goyal & Netessine, 2007 and Huisman & Kort, 2015). For example, Goyal & Netessine (2007) argue that firms may find it difficult to produce below capacity due to fixed costs associated with, for example, labor inputs, commitments to suppliers, or production ramp-up. Given this assumption, prices are

[9] These standard dynamics are also used in empirical work to obtain estimates for the depreciation rates of physical capital stock, e.g. Nadir & Prucha (1996), as well as of R&D capital stock, e.g. Li & Hall (2020).

given by the linear inverse demand system (see e.g. Lus & Muriel, 2009):[10]

$$p_1(t) = 1 - (K_{1A}(t) + K_{1B}(t)) - \eta(K_{2A}(t) + K_{2B}(t)) \tag{5}$$

$$p_2(t) = 1 + \theta - \eta(K_{1A}(t) + K_{1B}(t)) - (K_{2A}(t) + K_{2B}(t)). \tag{6}$$

In this setting, the parameter η $(-1 < \eta < 1)$ determines the degree of horizontal differentiation between the established product and the new product. In line with our main research questions we restrict attention to scenarios where the new product is a partial substitute of the established product, corresponding to $0 < \eta < 1$. Accordingly, a firm's dynamic strategy of building up capacities for the established product and the new product is inextricably linked through its impact on the prices of the products on the two markets. The parameter θ determines the degree of vertical differentiation and measures the difference in quality between the new product and the established product. The assumption that product 2 is at least of the same quality as product 1 translates into $\theta \geq 0$.

Investment costs are assumed to have the linear-quadratic form

$$\Gamma_{if}(I_i(t)) = \mu_i I_{if}(t) + \frac{\gamma_{if}}{2} I_{if}(t)^2 \quad i = 1, 2, R. \tag{7}$$

For products $i = 1, 2$ the parameter μ_i represents the unit price of capacity and $\gamma_{if} > 0$ the adjustment cost parameter for firm $f = A, B$.[11] Increasing R&D knowledge stock is associated with a convex cost function, i.e. $\gamma_{Rf} > 0$, in line with standard arguments that building up knowledge takes time and therefore fast R&D stock accumulation is more costly compared to slower accumulation. In our default setting, all cost parameters are symmetric across firms. Asymmetry between manufacturing firms arises due to heterogeneity of the initial production capacity on the established market. However, in order to be able to study the impact of structural (dis-)advantages of a firm on a certain market, in Section 5.2 we further consider asymmetric adjustment costs between firms. For simplicity, marginal production costs are normalized to zero.

Firms choose their investment strategies in order to maximize their expected infinite horizon discounted profit stream. Formally, we have

$$\begin{aligned} J_f = \mathbb{E}\bigg\{ \int_0^\infty e^{-rt} \Big[&(1 - (K_{1A} + K_{1B}) - \eta(K_{2A} + K_{2B}))K_{1f} \\ &+ (1 + \theta - \eta(K_{1A} + K_{1B}) - (K_{2A} + K_{2B}))K_{2f} \\ &- \mu_1 I_{1f} - \frac{\gamma_{1f}}{2} I_{1f}^2 - \mu_2 I_{2f} - \frac{\gamma_{2f}}{2} I_{2f}^2 - \mu_R I_{Rf} - \frac{\gamma_{Rf}}{2} I_{Rf}^2 \Big] dt \bigg\}, \end{aligned} \tag{8}$$

where the expectation is taken with respect to the mode dynamics. The first two lines capture the instantaneous sales revenue for the established product and the new product. The third line contains the current costs of investment in production capacity and R&D knowledge stock. It should be noted that in modes where a firm has not introduced the new product yet, both investment and production capacity for that product are zero, such that the corresponding terms vanish in the instantaneous profit function.[12]

This formulation of the interplay between the two firms gives rise to a piecewise deterministic differential game with objective functions (8), state dynamics (1) and mode dynamics (4). Since a firm can build up production capacity for the new product only after the new submarket has been created and it has added the new product to its product line, the following constraints hold in the different modes:

$$I_{2f}(t) = 0, \ \forall t \text{ s.t. } m(t) = m_1, \quad f = A, B$$

$$I_{2B}(t) = 0, \ \forall t \text{ s.t. } m(t) = m_2,$$

$$I_{2A}(t) = 0, \ \forall t \text{ s.t. } m(t) = m_3.$$

Non-negativity constraints for production capacities, the R&D knowledge stock, and the investments in R&D knowledge stock have to be satisfied. To study how the anticipation of the emergence of a new submarket impacts the firms' current R&D investments, we assume that the game starts before the new submarket has been created. That is, $m(0) = m_1$ and the initial values of production capacities and the R&D knowledge stock are given by $K_{1f}(0) = K_{1f}^{ini}, \ K_{2f}(0) = 0, \ K_{Rf}(0) = K_{Rf}^{ini}$.

4. Dynamic investment strategies

In order to analyze optimal strategies for investing in production capacities and R&D knowledge stocks, we consider stationary Markov Perfect Equilibria (MPE) of the game described in the previous section.[13] A stationary Markovian strategy of firm f is given by a triple $(\phi_{1f}, \phi_{Rf}, \phi_{2f})$ such that each of the feedback strategies ϕ_{if} and ϕ_{Rf} describe the optimal dynamic investment for accumulating production capacity and R&D knowledge, respectively, as a function of the states and the current mode of the game. Stationary strategies however do not explicitly depend on time.[14] More precisely, each of these feedback strategies has the form $\phi_{if} : [0, 1]^2 \times [0, \bar{K}_R]^2 \times [0, 1 + \theta]^2 \times M \mapsto \mathbb{R}$ for $i \in \{1, 2\}$ and $\phi_{Rf} : [0, 1]^2 \times [0, \bar{K}_R]^2 \times [0, 1 + \theta]^2 \times M \mapsto \mathbb{R}_0^+$. The upper bound $\bar{K}_R$ of the R&D knowledge stock is assumed to be sufficiently large to ensure that the stable steady states characterized in the following analysis are interior. Using strategies ϕ_{if}, $i = 1, 2, R$, firm $f = A, B$ at each point in time invests $I_{if}(t) = \phi_{if}(K_{1A}(t), K_{1B}(t), K_{2A}(t), K_{2B}(t), K_{RA}(t), K_{RB}(t), m(t))$.

Although formally the feedback strategies have the general form with six states and one mode as arguments, some arguments are irrelevant in some modes (see Fig. 1). To ease notation, in what follows we therefore drop all irrelevant arguments in the corresponding modes and write the feedback strategies in each mode only as

[10] This demand system does not take into account product diffusion as in, e.g., Bilginer & Erhun (2015) and Balakrishnan & Pathak (2014). This means that in our setting, willingness to pay for the new product is already high from the moment the product is launched. There are examples for that, like tablets and smartphones. Still, in our model, the speed of market development is restricted by the capacity build-up.

[11] We abstract from potential differences between sale and resale price of capital. However, due to adjustment costs returns from selling are lower than costs of buying. Furthermore, we assume adjustment costs to be the same for increasing or decreasing capacity, whereas they could differ substantially in reality. However, it should be noted that disinvestment hardly plays any role in our analysis; see, e.g., Fig. 2(b) in Section 5.

[12] In our treatment of the investment decisions of the firms we abstract from potential financial constraints the firms might face. This means that in line with the vast majority of analyses of firms' investment in the industrial organization as well as in the patent race literature, we assume that firms have free access to perfect capital markets. Studying the implications of limited access to credit for the relative size of investments by large and small incumbents in our setting is an interesting and promising research topic, but beyond the scope of this paper.

[13] For other applications of this equilibrium concept for the analysis of optimal firm strategies in Operations Management problems, see Dockner & Fruchter (2014), Chevalier-Roignant, Flath, & Trigeorgis (2019) or Huberts, Dawid, Huisman, & Kort (2019).

[14] Since under the considered stationary Markovian strategies investment at t depends on the mode $m(t)$, which follows a stochastic process, the actual investment at time t for a given state (K_{1f}, K_{Rf}, K_{2f}), $f = A, B$ is stochastic. Since the probability that $m(t) = m_i, i = 1, 2, 3$ changes with t, from an ex-ante perspective, i.e. based on the information available at $t = 0$, the distribution of investment at t therefore changes with time t although the considered strategies are stationary.

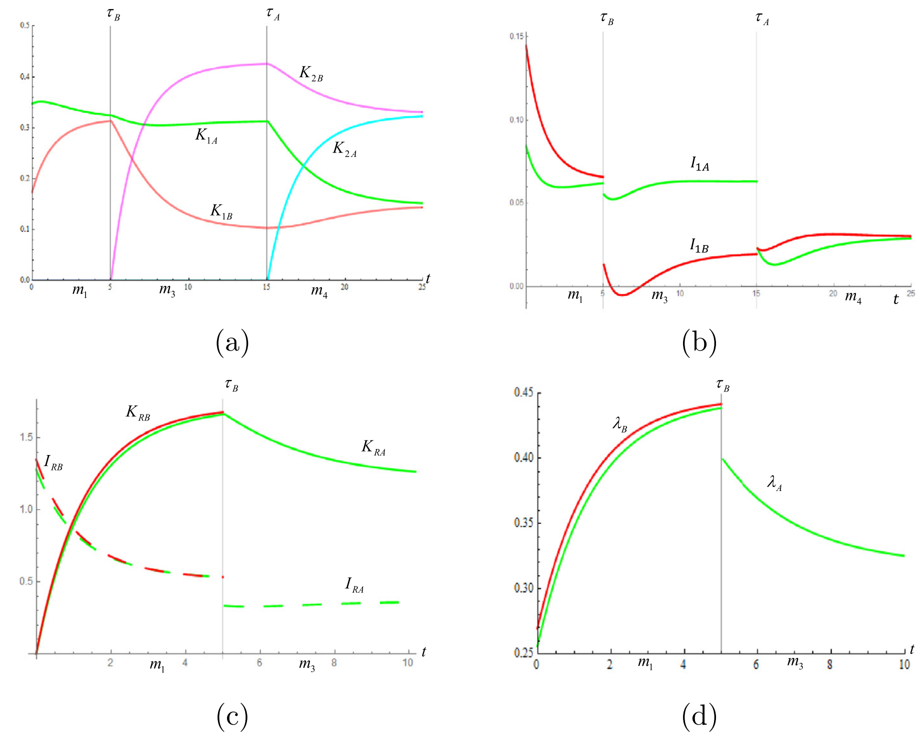


Fig. 2. Equilibrium dynamics under the baseline parameter setting for asymmetric initial production capacities on the established market. (a) Production capacities. (b) Investments in production capacities for the established product. (c) R&D knowledge stocks and R&D investments. (d) Hazard rates. It is assumed that the smaller firm B (shown in red) innovates earlier than firm A (shown in green). The innovation time of firm B is indicated as τ_B. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

functions of the four relevant states. As a notational convention, the first two arguments in the feedback functions of both firms always refer to the production capacities on the established market, followed by the second relevant state variables of firm A and firm B, which differ across modes. The vector of the relevant states in mode m is denoted by $\vec{K}^m$. Furthermore, due to the investment constraints in the different modes, $\phi_{2f} = 0$ has to hold in mode m_1 for both firms $f = A, B$, since no investment in production capacity of the new product is possible before the product innovation is successful. For the non-innovator the same holds in modes m_2 or m_3. Additionally, we have $\phi_{RA} = 0 (\phi_{RB} = 0)$ in mode m_2 (m_3) and $\phi_{Rf} = 0, f = A, B$, in m_4 since no more innovations are possible. In accordance with the literature (see Dockner, Jorgensen, Van Long, & Sorger, 2000) we only consider non-anticipating strategies, i.e. strategies where firms cannot condition their action on realizations of the time of mode transitions which lie in the future.

A Markov Perfect Equilibrium of the game is a profile of stationary Markovian strategies, where each manufacturing firm uses a strategy that maximizes expected profit given the strategy of the opponent. The following proposition characterizes the firms' optimal investment strategies. Appendix A contains the proof of the proposition and provides the explicit formulations of the Hamilton-Jacobi-Bellman (HJB) equations in the different modes that characterize the equilibrium value functions.

Proposition 1. *Denote by $V_f(\vec{K}^m, m)$ the value function of firm $f = A, B$ in mode $m \in M$ satisfying the Hamilton-Jacobi-Bellman Eqs. (A.7)–(A.10) given in Appendix A. Then, the feedback strategies in a Markov Perfect Equilibrium of the game are given by*

$$\phi_{if}^*(\vec{K}^m, m) = \frac{1}{\gamma_{if}} \left(\frac{\partial V_f(\vec{K}^m, m)}{\partial K_{if}} - \mu_i \right), \quad i = 1, 2, f = A, B, m \in \{m_1, \dots, m_4\}, \tag{9}$$

$$\phi_{RA}^*(\vec{K}^{m_1}, m_1) = \frac{1}{\gamma_{RA}} \left(\frac{\partial V_A(\vec{K}^{m_1}, m_1)}{\partial K_{RA}} - \mu_R + \alpha \left(V_A(K_{1A}, 0, K_{1B}, K_{RB}, m_2) - V_A(\vec{K}^{m_1}, m_1) \right) \right), \tag{10}$$

$$\phi_{RB}^*(\vec{K}^{m_1}, m_1) = \frac{1}{\gamma_{RB}} \left(\frac{\partial V_B(\vec{K}^{m_1}, m_1)}{\partial K_{RB}} - \mu_R + \alpha \left(V_B(K_{1A}, K_{RA}, K_{1B}, 0, m_3) - V_B(\vec{K}^{m_1}, m_1) \right) \right), \tag{11}$$

$$\phi_{Rf}^*(\vec{K}^m, m) = \frac{1}{\gamma_{Rf}} \left(\frac{\partial V_f(\vec{K}^m, m)}{\partial K_{Rf}} - \mu_R + \alpha \left(V_f(K_{1A}, K_{2A}, K_{1f}, 0, m_4) - V_f(\vec{K}^m, m) \right) \right) \begin{array}{c} f = A, B, \\ m = m_2, m_3 \end{array}. \tag{12}$$

Optimal investment in production capacity of the established product is proportional to the difference between the marginal effect of an increase in capacity on the firm's value function and the unit price of capacity (see (9)). Concerning a firm's investment in R&D knowledge stock, an additional effect arises because such an investment increases the hazard rate of making the transition to a different mode where the firm is active on both markets. This mode transition induces a jump in the value function and the corresponding effect on R&D incentives is captured by the last terms in Eqs. (10) to (12).

The main technical and computational challenge for analyzing the firms' optimal investment strategies is to determine the value functions $(V_A(\vec{K}^m, m), V_B(\vec{K}^m, m))$ which satisfy the system of Hamilton-Jacobi-Bellman Eqs. (A.7)–(A.10) containing one equation for each firm in each mode. To illustrate the technical challenges associated with finding the solutions of the system of HJB equations we show in Eq. (13) the schematic form of the HJB equation for firm A in mode m_1,[15]

$$\begin{aligned} & rV_A(\vec{K}^{m_1}, m_1) \\ & = max_{I_{1A}, I_{RA}}\Bigg[\pi_A(\vec{K}^{m_1}, m_1) \\ & \quad + \frac{\partial V_A(\vec{K}^{m_1})}{\partial K_{1A}}\dot{K}_{1A} + \frac{\partial V_A(\vec{K}^{m_1})}{\partial K_{RA}}\dot{K}_{RA} + \frac{\partial V_A(\vec{K}^{m_1})}{\partial K_{1B}}\dot{K}_{1B} + \frac{\partial V_A(\vec{K}^{m_1})}{\partial K_{2R}}\dot{K}_{2R} \\ & \quad + \lambda(I_{RA}, K_{RA})(V_A(\vec{K}^{m_2}, m_2) - V_A(\vec{K}^{m_1}, m_1)) \\ & \quad + \lambda(\phi_{RB}(\vec{K}^{m_1}, m_1), K_{RB})(V_A(\vec{K}^{m_3}, m_3) - V_A(\vec{K}^{m_1}, m_1))\Bigg], \end{aligned} \tag{13}$$

where $\pi_A(\vec{K}^{m_1}, m_1)$ denotes the instantaneous profit of firm A in mode m_1 given by the market profit minus firm A's costs of investing in production capacity for the established product and in R&D. Hence, the first two lines of the right hand side of (13) are standard expressions capturing the current flow of profits and investment costs for firm A as well as the effect of the state dynamics on firm A's value function. Since in our model specification profit and cost functions are quadratic functions of the state and control variables and all state dynamics are linear, the form of this part of the HJB equation corresponds to that in a linear-quadratic game. From a technical perspective, the terms in the last two lines, which capture the implications of a possible change of the mode to m_2 or m_3 for the value function of firm A, make the task of finding a solution more challenging. First, the value functions for firm A in modes m_2 and m_3 appear in these terms, implying that the HJB equations of both players are linked across modes and hence cannot be solved separately. Second, due to the fact that in these terms the value function $V_A(\vec{K}^{m_1}, m_1)$ is multiplied by the hazard rates of the transition to modes m_2 and m_3, which are both functions of the state vector, further implies that (13) does not allow for a closed-form (polynomial) solution. Similar statements apply for the HJB equations in modes m_2 and m_3. Only mode m_4 allows for quadratic solutions of the HJB equations of the two firms, since no further mode transition is possible from mode m_4. The solutions for mode m_4 can be found using the standard approach of comparison of coefficients (see e.g. Dockner et al., 2000).

For the computation of the value functions in modes m_1 - m_3 we rely on numerical collocation methods. The general approach underlying this method is to determine polynomial approximations of the value functions of both firms in these modes with the property that, after inserting these approximate value functions and the corresponding feedback functions into the HJB equation of the corresponding mode, the (absolute) value of the difference between the left and the right hand side of the HJB equation is sufficiently small on an appropriate grid of points in the state space (see Dawid et al., 2017; Vedenov & Miranda, 2001). The large dimension of the state space in our model requires the application of sparse grid methods for constructing the considered grid of points in the state space and the set of basis functions to be used in the collocation. For all results presented in the following sections this approach has been used and it has then been checked that the HJB equations hold up to a numerical error below a given error bound on the entire state space and that the transversality conditions are satisfied in all modes. Details of the numerical method underlying our analysis are given in Appendix B. The application of a sparse grid collocation approach for calculating optimal firm strategies in a dynamic Operations Management problem with strategic interactions is an innovative methodological contribution of our paper.

[15] The complete expression of this equation is given in (A.7) in Appendix A.

Table 1
Baseline Parameter Setting of the Model.

Symbol	Definition	Constraint	Baseline
α	Effectiveness of current R&D	≥ 0	0.2
β	Effectiveness of knowledge stock	≥ 0	0.2
ψ	Exponent of know. stock in innov. rate	≥ 0	1
ω	Coeff. of innovation rate in mode m_2/m_3	≥ 0	1
η	Horizontal differentiation	$\lvert\eta\rvert < 1$	0.65
θ	Vertical differentiation	≥ 0	0.2
μ_1, μ_2	Unit costs of prod. capacity $i = 1, 2$	≥ 0	0
μ_R	Unit costs of R&D investment	≥ 0	0.1
γ_{1A}, γ_{1B}	Adjustment costs for product 1	> 0	3
γ_{2A}, γ_{2B}	Adjustment costs for product 2	> 0	3
γ_{RA}, γ_{RB}	Adjustment costs for knowledge stock	> 0	0.1
δ_1, δ_2	Depreciation rates for capacities K_1, K_2	> 0	0.2
δ_R	Depreciation rate of knowledge stock	> 0	0.3
r	Discount rate	$0 < r \leq 1$	0.04

In what follows, we present numerical results obtained by this approach for the baseline parameter setting given in Table 1. The unit prices of production capacity, μ_i, are normalized to zero. The values of the adjustment cost parameters, γ_{if}, are positive. To capture the interplay between the markets, the degree of horizontal differentiation is assumed to be $\eta = 0.65$. We deal with a scenario where the difference in qualities between the new product and the established product is moderate, which is reflected by $\theta = 0.2$. Depreciation rates and discount rate are set to values which are in line with empirical estimates (see, e.g., Nadir & Prucha, 1996 and Li & Hall, 2020). As discussed above, we assume that the hazard rate is a linear function of the firm's R&D knowledge stock ($\psi = 1$). The coefficients α and β of the hazard rate as well as the parameters μ_R, γ_{Rf} of the R&D investment costs have been chosen such that expected innovation times in equilibrium are less than 1.5 years. This is in accordance with innovation clockspeed in high-technology industries, like for instance the computer industry (see also Pacheco-de Almeida, 2010). In Section 7 we provide an extensive robustness analysis in which we demonstrate that the main qualitative findings presented in this paper continue to hold for a large range of parameter values around our baseline setting.

5. Strategy analysis

In this section we first briefly illustrate the dynamics generated by our model in a setting where firms are symmetric with respect to their cost parameters and their initial knowledge stock, but differ with respect to their initial production capacities on the established product market. Subsequently, we discuss in several subsections the key strategic effects determining the shape of the optimal R&D investment strategies and how these effects influence market dynamics and firm values. Figure 2 shows the evolution of production capacities, investments in capacities for the established product, R&D investments and R&D knowledge stocks, and the innovation hazard rate for each firm if both firms follow their optimal investment strategies.[16] Both firms start with zero R&D knowledge stocks. Firm A's initial production capacity for the established product corresponds to the steady state level in a scenario where firms do not account for the option to develop a new product, i.e. a setting where the firms just invest in capacity for the established

[16] Although, in general, we cannot expect uniqueness of Markov Perfect Equilibria of the considered game, we have always found only a single MPE in our numerical explorations. To ensure robustness, we have carried out the analysis for a wide range of initializations of the collocation algorithm without finding other MPEs. All figures show the trajectories resulting from this single MPE.

product. Firm B starts with half of firm A's production capacity, where the lower initial production capacity of firm B might be due to the fact that it has entered the established market later than firm A. For the purpose of illustration, in Fig. 2 it is assumed that firm B innovates first.[17]

At first sight it might be surprising to see that the production capacities of both firms in mode m_1 stay below the steady state level of the corresponding game without innovation option (i.e. the initial capacity of firm A).[18] Intuitively, this is because firms in equilibrium anticipate the introduction of the new product at a future point in time, which then reduces the value of their production capacity on the established market.[19] The inter-temporally optimal reactions of both firms to this reduction in the future expected value of the production capacity of the established product is to reduce the investment in this capacity. As both firms accumulate more R&D knowledge over time (see panel (c)), their hazard rates increase (panel (d)). This reduces the expected time until the introduction of the new product. Accordingly, the anticipation effect becomes stronger during mode m_1 which leads to a decreasing pattern of investments in production capacities on the established market (see panel (b)). Furthermore, panel (b) of the figure shows that the optimal reactions of both firms to the introduction of the new product at time τ_B at the beginning of mode m_3, is to cut their investments in the production capacities of the established product. For firm A the reason is that the launch of the new product causes additional competition to its established product. Firm B produces both products in mode m_3 and even dis-invests to reduce its production capacity for the established product. The reason for firm B's disinvestment is that besides the intensified competition for the established product, sales of the established product reduce the price of the new product for which firm B is now the sole manufacturer. As soon as mode m_4 is reached, firm A has also innovated and launched the new product. To reduce the competitive effect of the established submarket on the new product, it is now optimal for firm A to severely reduce the investment in its capacity for the established product. This, in turn, triggers a slight increase of firm B's investment in its established product 1. It should be noted that, whereas total production capacity for the established product decreases after each innovation, this is not necessarily true for each firm. More precisely, firms might actually expand their capacity on the established market in the aftermath of an innovation by their competitor. This can be seen in Fig. 2(a), where K_{1A} increases in the second half of mode m_3 (where firm B has already innovated), and K_{1B} increases in mode m_4 (after firm A has innovated). The corresponding expansions are a strategic reaction of the firm to the strong reduction of the innovator's production capacity on the established market.

Concerning firms' R&D investments, Fig. 2(c) shows that during the innovation race in mode m_1, the smaller firm B should invest more and accumulate a larger knowledge stock than what is optimal for firm A. Hence, this firm also has a higher hazard rate than its competitor and is more likely to win the innovation race. The assumption underlying Fig. 2 that both firms start from a symmetric knowledge stock is key for these observations, as will become clear from our detailed discussion of the interplay of the effects of the relative sizes of knowledge stocks and capacities on R&D investment in Section 5.1. Furthermore, at the moment of the introduction of the new product by firm B (mode m_3), the optimal R&D investment, and therefore also the hazard rate, of firm A exhibits a downward jump. This points to a qualitative difference of the optimal R&D investment strategy in the modes before and after the competitor's innovation. A more detailed discussion of the reasons for this difference is postponed until Section 5.3.

5.1. R&D incentives and the interplay of knowledge leadership and market position

In an R&D race with knowledge accumulation and exogenously given fixed profits for the innovator and the losing firm, in equilibrium the knowledge laggard, i.e. the firm with the smaller R&D knowledge stock, invests more in R&D than the knowledge leader, see Doraszelski (2003).[20] As Fig. 3(a) shows, this observation does not necessarily carry over to our setting of an R&D race with *endogenously* determined flow profits. In the green area the optimal R&D investment of firm A is higher than that of firm B. In the yellow area, the opposite holds. It is evident from the figure that the boundary between these two areas does not coincide with the line $K_{RA} - K_{RB} = 0$. Therefore, for the part of the green area where $K_{RA} < K_{RB}$, the optimal R&D investment of the firm with the smaller knowledge stock is higher than the investment of the knowledge leader. Hence, we find an action-reaction pattern. In the part with $K_{RA} > K_{RB}$, the knowledge leader invests more. Hence, we observe an increasing dominance pattern. Analogous conclusions can be drawn for the yellow area in Fig. 3(a). From a managerial point of view, these arguments highlight that in the common situation where the firms' profits are endogenously determined by the firms' decisions, the optimal R&D strategy crucially depends on the firms' relative positions on the established market **and** on the firms' relative positions with regard to their accumulated R&D knowledge. In what follows we discuss the economic mechanisms underlying these observations.

If the knowledge leader has a substantially smaller capacity on the established market than its competitor, then it is optimal for this firm to invest more in R&D than the knowledge laggard (see region I in Fig. 3(b)). As shown in Fig. 4(a), the optimal R&D investment of a firm negatively depends on both firms' production capacities on the established market, where the negative dependence is much stronger for a firm's own production capacity.[21] Therefore, a smaller production capacity on the established market makes a firm more aggressive with respect to R&D effort.

This observation is driven by two effects. First, the introduction of the new product in the new (but linked) submarket results in a decrease of the price for the established product. The larger a firm's production capacity on the established market, the more strongly it is affected by this decrease (*cannibalization effect*).[22] Second, the incentive to invest in the R&D knowledge stock is influenced by a *size effect*. A larger total production capacity on

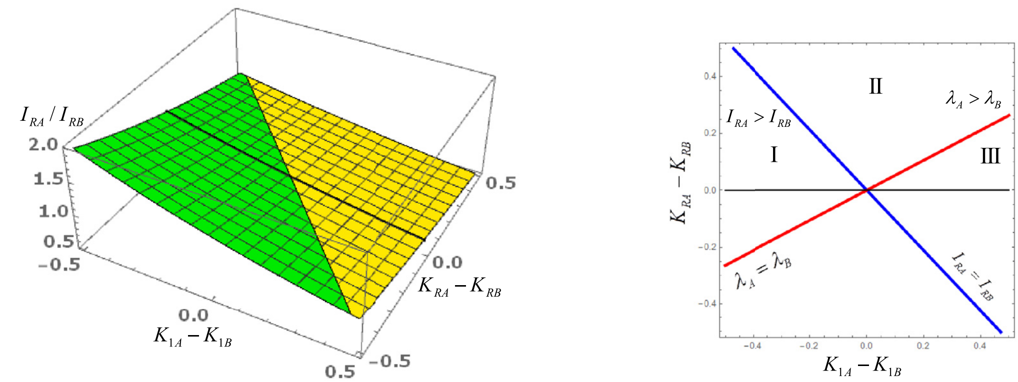
[17] The comparison of hazard rates in panel (d) of Fig. 2 confirms that firm B entering the new market first has higher probability than firm A being the first innovator. To check the qualitative robustness of our findings reported below, Fig. 9 in Appendix C shows the dynamics of stocks and investments for the case where firm A enters the new market first. Comparison of Fig. 9 with Fig. 2 ensures that our observations about the change in the optimal investment strategies of the innovator and innovation laggard after the first introduction of the new product do not depend on whether the larger or the smaller firm on the established market is the first innovator.

[18] Since both firms are symmetric with respect to their cost parameters, the steady state level of production capacities in mode m_1 is symmetric and this explains why the production capacities of the two firms approach each other in mode m_1 before the innovation occurs and the game jumps to mode m_3.

[19] Although we assume that firms use non-anticipating strategies, i.e. they do not know at which exact time in the future the new product will be introduced, they know the arrival rate of the new product under the equilibrium investments. Therefore, they know the distribution of the time left until the introduction of the new product happens. In this sense, the firms anticipate that the new product will eventually be launched in the market.

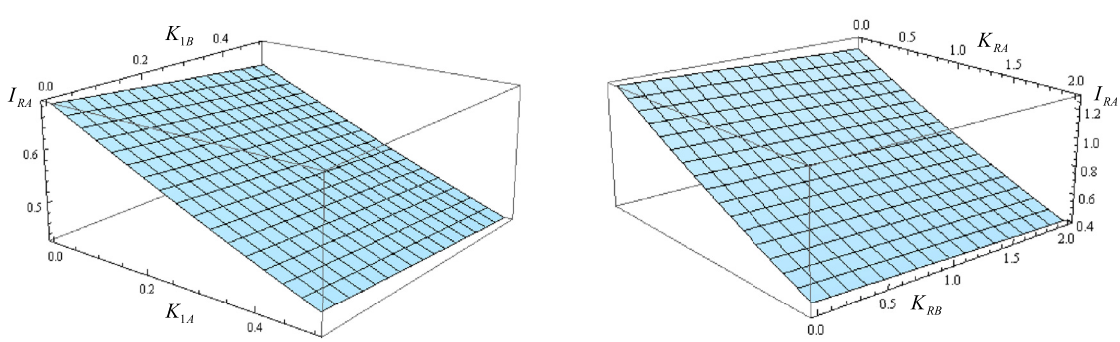
[20] To be precise, this relationship is shown if a firm's hazard rate is linear (or concave) with respect to the firm's knowledge stock, which is the baseline case considered also in this paper.

[21] The range of capacities K_{1A}, K_{1B} considered in the figure covers the whole interval between 0 and the monopoly output level on the established market.



(a) (b)

Fig. 3. (a) The ratio of equilibrium R&D investments of firms A and B depending on the difference in capacities on the established market and on the difference in knowledge stocks. The green (yellow) region indicates where firm A (firm B) invests more in R&D than its competitor. (b) Regions in the state space in which firm A has higher R&D investment than firm B (below the blue line) and in which firm A has a higher hazard rate than firm B (above the red line). In both panels, the black line indicates the boundary between the regions in which firm A is the knowledge leader or knowledge laggard. The center point (i.e. the point for which $K_{1A}-K_{1B}=K_{RA}-K_{RB}=0$) corresponds to the steady state values of the corresponding state variables in mode m_1. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)



(a) (b)

Fig. 4. R&D investment of firm A depending on (a) production capacities on the established market, and (b) R&D knowledge stocks of the two firms. The values of the arguments of the feedback functions not varied are set at the steady state levels in mode m_1.

the established market induces a smaller price for the new product after it has been launched. This reduces the profitability of the new product and thereby the incentive to speed up the introduction of the new product. Note that, in contrast to the cannibalization effect, for the size effect it is irrelevant whether the capacity expansion on the established market is due to an increase of the competitor's production capacity or the firm's own production capacity. The joint influence of the cannibalization effect and the size effect explains why an increase of the firm's own production capacity decreases the firm's own incentives to invest in R&D more strongly than an increase in the competitor's production capacity.[23]

If the R&D knowledge leader is the much smaller firm on the established market, it should optimally invest more in R&D than its competitor, and therefore in equilibrium a pattern of increasing dominance of the knowledge leader might arise. In this case, the gap in R&D knowledge stock grows over time (region I in Fig. 3(b)). However, if the R&D knowledge gap becomes sufficiently large, the negative effect of R&D knowledge leadership on R&D investment, starts to dominate and the R&D knowledge leader then invests less than the R&D knowledge laggard (transition from region I to region II in Fig. 3(b)). To understand this transition, we observe in Fig. 4(b) that the firm's optimal R&D investment depends negatively on its own R&D knowledge stock as well as on the R&D knowledge stock of its competitor, where the negative dependence of the firm's own knowledge stock is much more pronounced. Hence, the larger the size of the R&D knowledge gap between the two firms the lower is the R&D investment of the knowledge leader relative to the

[22] Formally, the difference in value functions $V_A(\vec{K}^{m_2},m_2)-V_A(\vec{K}^{m_1},m_1)$ decreases with respect to K_{1A} and hence the gains from innovation are weaker for the larger firm with a higher current profit. In this sense, this observation is related to the well-known Arrow replacement effect (Arrow, 1962).

[23] The insight that the interplay of the cannibalization and the size effect induces that a firm's excess profit from innovation is more strongly influenced by a change in the firm's own production capacity compared to a change in the competitor's capacity is also obtained in Dawid et al. (2013) in a static setting which does not incorporate firms' knowledge stocks. In such a setting the firm with the smaller capacity on the established market *always* has a larger incentive to innovate. In our dynamic setting with endogenous evolution of knowledge stocks this statement is no longer true and the interplay of the effects emerging from differences in knowledge stocks and capacities determines which firm invests more in R&D and which firm has a higher hazard rate.

knowledge laggard. The intuition for the negative relationship between R&D investment and both knowledge stocks is that larger R&D knowledge stocks reduce the expected duration of mode m_1 and any transition to another mode (m_3 or m_2) reduces the value of the knowledge stock.[24] A transition to mode m_3, where firm B has innovated, reduces the value of firm A's R&D knowledge stock. In fact, the increase in the value function generated by a potential innovation of firm A is substantially larger as long as firm B has not introduced the new product yet since the price of the new product is lower if the opponent is already active on the new market. If the transition is to mode m_2 and firm A itself is the innovator, the value of the firm's R&D knowledge stock completely vanishes because no further innovation is possible by the firm.[25]

The discussion above shows that both R&D knowledge leadership and dominance on the established market have a negative impact on the relative incentive of a firm to invest in product innovation. In situations in which the same firm dominates its competitor both in terms of its knowledge stock and its production capacity on the established market, this has the clear-cut implication that the strategic interaction pattern of action-reaction emerges. In this case, it is optimal for the smaller firm, which is also the R&D knowledge laggard, to invest more in R&D. This effect can be so strong, that the hazard rate of the knowledge leader falls below the hazard rate of the R&D knowledge laggard (region III in Fig. 3(b)). In other words, our model highlights that R&D knowledge leadership does not coincide with innovation leadership. Indeed, the knowledge leader's probability of winning the innovation race might be smaller than the R&D knowledge laggard's probability of winning.

5.2. Competitive disadvantage turns into innovation leadership

Our analysis so far has assumed that the two competitors (only) possess different initial production capacities on the established market. In this context, we have been studying the strategic implications of transitory differences for the R&D investments of otherwise symmetric firms. This section turns to the case where firms differ in their (long term) competitiveness on the established market. In particular, we impose that investment costs of firm B on the established market are higher than the investment costs of firm A. We adjust our baseline parameter setting given in Table 1 by increasing the adjustment cost parameter of firm B on the established market to $\gamma_{1B} = 9$, but keep firm A's adjustment cost parameter at the baseline value $\gamma_{1A} = 3$.

The dynamics emerging in equilibrium in such a setting is depicted in Fig. 5. Like in Fig. 2, initial R&D knowledge stocks of both firms are assumed to be zero. The initial production capacities on the established market correspond to the steady state values of this game without product innovation option. Due to its competitive disadvantage with respect to capacity adjustment costs, the initial production capacity of firm B on the established market is, therefore, lower than firm A's initial production capacity. Figure 5 highlights two important implications of this asymmetry between firms. First, panel (a) reveals that throughout mode m_1, firm B has a higher probability of winning the innovation race than its competitor, which can be explained by its higher R&D investment and subsequently its higher hazard rate.

Second, under the assumption that firm B innovates first, it has a strictly smaller production capacity on the established market, but a strictly larger production capacity on the new submarket throughout all modes where these markets exist (see panel (b)). Consequently, the disadvantage of higher capacity adjustment costs on the established market acts as a commitment device for firm B to be more aggressive during the innovation race and also to be a tougher competitor on the new submarket. Innovation leadership then follows from the negative relationship between the firm's own production capacity on the established market and its incentive to invest in R&D knowledge stock, as discussed in Section 5.1.

These arguments show that the disadvantage of firm B with respect to investment costs on the established market has two counteracting implications for the firm's profit. First, the profit of firm B on the established market is negatively affected by its larger capacity adjustment costs. Second, the competitor, firm A, invests less in its R&D knowledge stock because firm A takes into account that firm B has a stronger incentive to invest in R&D. Lower R&D investment of firm A increases the probability for firm B to win the innovation race and raises its expected profit. The second effect becomes more important if the expected time until the creation of the new submarket is shorter and depends more strongly on firms' R&D investment. In the framework of our model, this aspect is closely related to the parameter α, which measures the impact of current R&D investment on the hazard rate. Figure 6 illustrates that, for sufficiently large values of α, the firm with a cost disadvantage on the established market can indeed have a larger expected discounted payoff than its more efficient competitor.[26] The conclusion is that a firm's burden of having higher capacity adjustment costs in an established market might eventually turn out to be a blessing since this firm has to focus its attention on innovative submarkets and, surprisingly, might even turn into the innovation leader achieving a higher expected profit.

5.3. Impact of competitor's innovation on a firm's R&D strategy

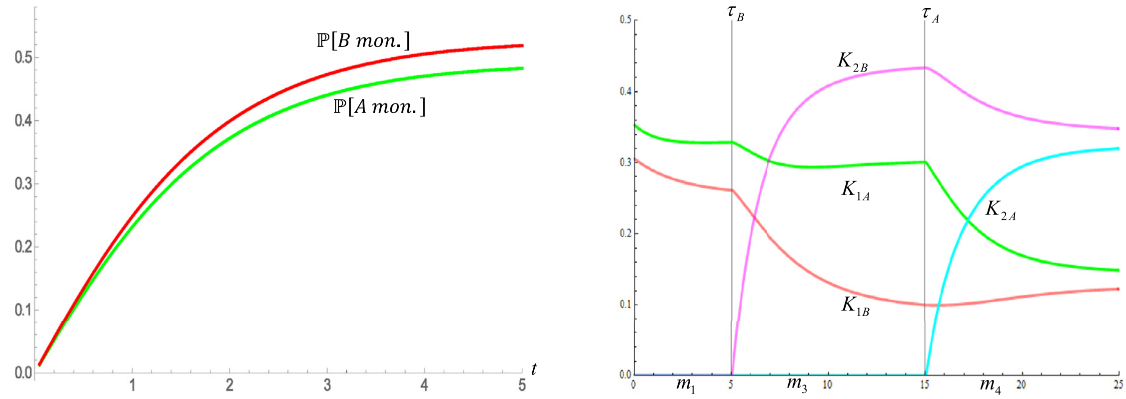
The creation of the new submarket by a competitor has substantial implications for the optimal dynamic innovation strategy of the firm that has not innovated yet. Figure 2(b) demonstrates that the R&D investment of firm A exhibits a downward jump at $t = \tau_B$ when firm B's innovation project is successful and the new product is launched in the submarket. Correspondingly, firm A's hazard rate exhibits a downward jump at $t = \tau_B$ as well. A direct implication of this observation is that the expected waiting time until firm *A*'s project is successful exhibits an upward jump after the opponent's innovation. The reason for the jump in firm A's R&D investment is that at the time firm B creates the new submarket the possibility to become temporarily the sole manufacturer of the new product and reap the benefits, vanishes. This considerably decreases the value of introducing the new product for firm A.

The discussion above highlights that the innovation laggard's (firm A) optimal level of R&D investment differs between modes m_1 and m_3. However, there are also qualitative changes in the properties of the firm's innovation strategy. In particular, the transition from mode m_1 to m_3 implies that the sign of the relation-

[24] Doraszelski (2003) discusses the negative effect of the own knowledge stock on a firm's R&D incentives in his patent race setting and denotes it as the '*pure knowledge effect*'.

[25] In a setting with repeated innovations of each firm, which is beyond the scope of this paper, the knowledge stock would not completely lose its value upon successful innovation. However, also in such a setting, due to the discounting of profits from future innovation events, the value of the knowledge stock would exhibit a downward jump at the time of the innovation breakthrough.

[26] In order to isolate the effect of an asymmetry in capacity adjustment costs (represented by γ_{1f}) on the value functions of firms *A* and *B*, the difference in value functions in Fig. 6 is calculated for symmetric initial conditions $K_{1A}^{ini} = K_{1B}^{ini} = 0.353$. This value corresponds to the steady-state capacity of firm *A* in the game without innovation option.



(a) (b)

Fig. 5. Probabilities to become temporary monopolist on the new market (a) and equilibrium dynamics of production capacities (b) in a scenario with asymmetric capacity adjustment costs on the established market ($\gamma_{1A} = 3$, $\gamma_{1B} = 9$).

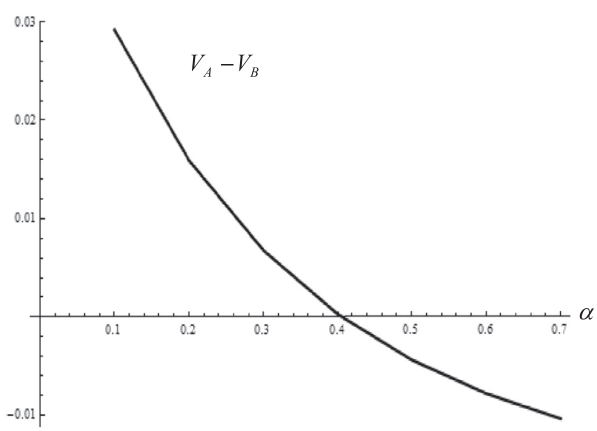


Fig. 6. Difference in the value functions of firms A and B for different values of the parameter α and symmetric initial conditions.

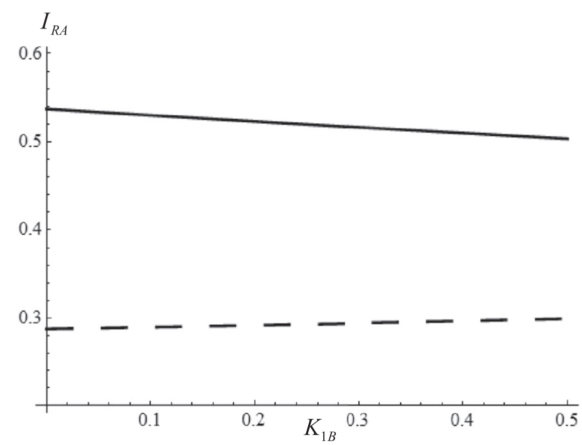


Fig. 7. Investment in R&D knowledge of firm A depending on production capacity of firm B on the established market before (solid line) and after (dashed line) firm B has innovated. The figure is based on the baseline parameter setting with symmetric investment costs on the established market. The values of the variables apart from K_{1B} are given by their steady-state value in the respective mode.

ship between investment in R&D knowledge stock and the opponent's production capacity for the established market changes (see Fig. 7). In mode m_1, an increase in incumbent B's production capacity K_{1B} decreases firm A's incentive to invest in R&D (bold line, see also Fig. 4(a)), whereas in mode m_3 it implies an increase in firm A's R&D activities (dashed line). The reason for this qualitative change is that once firm B is active on the new submarket, an increase of its production capacity on the established market induces a reduction of firm B's future investment on the new market. This, in turn, makes the new submarket more attractive for firm A and, therefore, results in an increase of firm A's optimal investment in the R&D knowledge stock.[27] In the static setting considered in Dawid et al. (2013), this effect is labeled as the *indirect effect*. Although this effect is already present in mode m_1, there the effect is weighted with the probability that the opponent wins the race and is discounted according to the expected innovation time. Furthermore, from the perspective of mode m_1 there is also a positive probability that firm A innovates first. In this case, a large production capacity K_{1B} on the established market reduces the value of innovation (size effect). The size effect is stronger and dominates in mode m_1, which yields the negative dependence of R&D investment on the opponent's capacity, as discussed above, whereas the indirect effect dominates in mode m_3. In other words, the opponent's *current* product range determines whether firm A should increase or decrease its R&D investment as a response to the competitor's capacity expansion on the established market. Hence, the direction of the optimal response changes over time. These insights extend in several ways the results obtained in a static setting in Dawid et al. (2013). There it has been shown that the established market capacity of a competitor who introduces the new product does not influence a firm's incentive to put the new product on the market (i.e. size effect and indirect effect cancel each other out). Exploiting the dynamic structure of our model allows us to demonstrate how, in a setting where the competitor does eventually introduce the new product, the interplay of the two effects and the dependence of the R&D investment from the competitor's capacities on the established market change over time. Also, contrary to Dawid et al. (2013), in the

[27] Our robustness analysis in Section 7 shows that this property no longer holds if the adjustment speed of the knowledge stock is too large relative to that of the production capacities, i.e. for very large values of γ_{1f}, γ_{2f} or very small values of γ_{Rf}. Intuitively in such a scenario the expected effect of an increase in K_{1A} on the future values of I_{2A} becomes less important than the instantaneous negative impact of such an increase on the price of the new product.

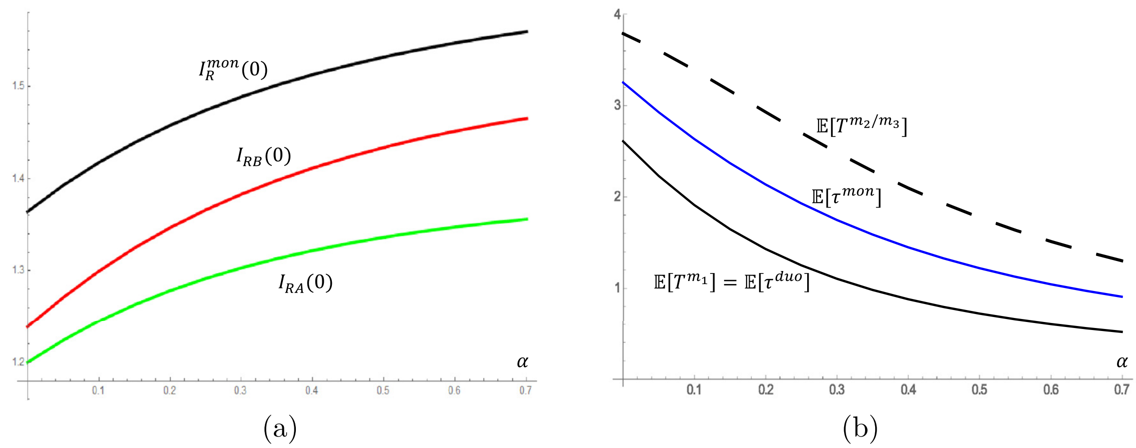


Fig. 8. (a) R&D investments at time zero of firms A and B in duopoly and in monopoly for different values of the parameter α. (b) Expected innovation time in duopoly (black solid) and in monopoly (blue) as well as expected duration of the innovator's monopoly on the new market in the duopoly model (black dashed) for different values of the parameter α. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

dynamic setting the indirect effect dominates after the product innovation of the competitor such that a positive relationship between the competitor's capacity on the established market and a firm's R&D incentive might arise. In our discussion we have assumed that the smaller firm B innovates first. However, in Appendix C we show that also if the larger firm innovates first, the slope of the innovation laggard's optimal R&D investment function with respect to the competitor's capacity on the established market changes sign after the competitor's innovation (see Fig. 10). Consequently, the property that a firm's optimal R&D strategy crucially depends on the rival's product range is robust in this respect.

6. Effect of competition on R&D investment

Our discussion in Section 5 has highlighted that each firm can induce a sudden drop in the optimal R&D activities of its incumbent competitor in the established market by launching a new differentiated product. Even before the product is launched, a firm can reduce the competitor's R&D investment by increasing its own R&D knowledge stock. Hence, compared to a single-firm setting, in a duopoly market competition induces additional (strategic) incentives to undertake investments in R&D. Therefore, from a managerial perspective the question arises how R&D investments should be adjusted in the face of changing intensity of competition.

To address this issue, in panel (a) of Fig. 8 we vary the marginal impact of current R&D investment on the hazard rate, captured by α, and compare the optimal initial R&D investments of firms A and B in our standard duopoly setting with the scenario in which one of the firms is a monopolist.[28] R&D investments are increasing in α, because a larger value of α implies that a given R&D investment results in a higher innovation probability. The figure shows that the monopolist always invests more in R&D. The main driving force of this finding is that in a monopoly the innovator does not need to take into account the later entry of a competitor into the new submarket. Consequently, the expected intertemporal rent is larger than in the duopoly. Put more formally, in a duopoly the value function of the innovator is negatively affected if eventually the other firm also launches the new product. In contrast, under monopoly the innovator can extract the monopoly rent for the new product indefinitely.

Although the incentive to invest in R&D for the individual firm is larger in a monopoly market compared to duopoly, panel (b) of Fig. 8 shows that nevertheless innovation occurs faster in duopoly. Despite the lower hazard rate for each individual firm, the expected time until the creation of the new submarket is smaller in duopoly since both firms are working independently to achieve the innovation breakthrough. Figure 8(b) also demonstrates that the expected time the innovation laggard needs to enter the new market after the competitor has successfully innovated is larger than the expected innovation time in monopoly. This holds true although the innovation laggard has already accumulated some R&D knowledge at the time of the competitor's innovation, whereas we assume that the initial knowledge stock of the firm under monopoly is zero. However, compared to a monopolist, the innovation laggard's advantage of accumulated R&D knowledge is outweighed by the smaller expected return from introducing the new product to the market where the competitor is already active.

7. Robustness

In this section, to attest the robustness of our findings, we identify intervals around the baseline values of the model parameters for which all the qualitative properties of the R&D investment functions identified in Sections 5.1 to 5.3 hold. In order to carry out this robustness check, each considered parameter is varied in the given interval and the value functions for these parameter variations are calculated. The analysis is then based on the consideration of the properties of the Markov-perfect equilibrium strategies associated with these value functions at the steady states in the corresponding modes. In Table 2 we give a description of the properties that we have checked. To account for potential numerical errors stemming from our method, a margin of $5 \cdot 10^{-4}$ has been used to check for the signs of the involved expressions.

Table 3 gives the intervals of the model parameters for which we have checked and found that the key properties listed in Table 2 hold. All properties (i)–(v) are robust and, consequently, we can be confident that the insights reported in Section 5 are robust with respect to changes in the parameter setting.

[28] More formally, we consider the dynamic optimization problem of firm A if firm B is absent from the established market and also cannot innovate; see Dawid et al. (2015) for a formal definition of the monopoly problem. Furthermore, to make the monopoly scenario comparable to the duopoly, we assume that the initial production capacity of the monopolist on the established market is given by the sum of the initial production capacities of the two firms in the duopoly scenario.

Table 2
Qualitative properties checked in the robustness analysis.

	Section	Mode	Description	Formal Requirement
(i)	5.1	m_1	R&D investment decreases w.r.t. own old market capacity	$\frac{\partial \phi_{RA}(.;m_1)}{\partial K_{1A}} < -5.10^{-4}$
(ii)	5.1	m_1	R&D investment decreases w.r.t. competitor's old market capacity	$\frac{\partial \phi_{RA}(.;m_1)}{\partial K_{1B}} < -5.10^{-4}$
(iii)	5.1	m_1	Negative effect of own old market capacity on R&D is stronger than that of competitor's capacity (⇒ larger firm invests less in R&D)	$\lvert \frac{\partial \phi_{RA}(.;m_1)}{\partial K_{1A}} \rvert / \lvert \frac{\partial \phi_{RA}(.;m_1)}{\partial K_{1B}} \rvert > 1 + 5.10^{-4}$
(iv)	5.3	m_1, m_3	R&D investment exhibits a downward jump when competitor innovates	$\phi_{RA}(.; m_1) > \phi_{RA}(.; m_3)$
(v)	5.3	m_3	R&D investment increases w.r.t. competitor's old market capacity after opponent's innovation	$\frac{\partial \phi_{RA}(.;m_3)}{\partial K_{1B}} > 5.10^{-4}$

Table 3
Range of parameter values for which the different qualitative properties listed in Table 2 are satisfied. A checkmark indicates that these properties are satisfied on the entire tested interval.

	Description	Baseline	Tested interval	Robust (i) - (v)
α	Effectiveness of current R&D	0.2	[0.1, 0.3]	✓
β	Effectiveness of knowledge stock	0.2	[0.1, 0.3]	✓
ψ	Exponent of know. stock in innov. rate	1	[0.5, 2]	✓
ω	Coeff. of innovation rate in mode m_2/m_3	1	[0.5, 1.5]	✓
η	Horizontal differentiation	0.65	[0.3, 0.8]	✓
θ	Vertical differentiation	0.2	[0, 0.4]	✓
μ_R	Unit costs of R&D investment	0.1	[0.05, 0.3]	✓
γ_{1A}, γ_{1B}	Adjustment costs for product 1	3	[1, 5]	✓
γ_{2A}, γ_{2B}	Adjustment costs for product 2	3	[1, 5]	✓
γ_{RA}, γ_{RB}	Adjustment costs for knowledge stock	0.1	[0.05, 0.5]	✓
δ_1	Depreciation rate for capacity K_1	0.2	[0.1, 0.3]	✓
δ_2	Depreciation rate for capacity K_2	0.2	[0.1, 0.3]	✓
δ_r	Depreciation rate of knowledge stock	0.3	[0.1, 0.5]	✓
r	Discount rate	0.04	[0.02, 0.06]	✓

8. Conclusions

In real-world markets, action-reaction or increasing dominance patterns of innovation can be observed. Under action-reaction, the smaller incumbent manufacturing firm in the established market becomes the innovation leader in the new market. Under increasing dominance, the dominant incumbent manufacturing firm in the established market also dominates the new market. In this paper, we develop a stochastic duopoly framework in order to highlight some of the drivers that endogenously lead to one of these patterns of innovation in equilibrium. In particular, we study the factors that determine the incentives of incumbent firms to invest in the development of a new product which extends their product range. Our dynamic setting particularly emphasizes the interplay between the firms' relative positions in terms of their R&D knowledge stocks and their relative strengths on the market for the established product. We explicitly take into account that the adjustment of production capacities as well as the build-up of R&D knowledge are costly and take time.

The first main insight from our analysis is that the interplay between the firms' R&D knowledge stocks and their positions on the established market allows a more fine-grained analysis than existing work and can provide additional answers concerning the question how an incumbent firm should invest in R&D and which firm will dominate innovative submarkets. In particular, we show that the knowledge leader might have a smaller innovation rate than its competitor if it has a sufficiently large market share on the established market. Furthermore, for a firm with a sufficiently small capacity on the established market it is optimal to invest more in R&D than its competitor even if it already has a knowledge advantage. These insights have managerial implications for optimal R&D investment and also provide a theoretical explanation for the empirically observed pattern that larger incumbents are often late to enter emerging new submarkets. It particularly highlights that the "innovator's dilemma", i.e. that dominant incumbents are frequently late in moving into newly emerging submarkets, might not be due to myopic decisions of firm management, but instead might be fully in line with intertemporally optimal firm behavior.

Our second main finding addresses the impact of a firm's structural disadvantage, captured by higher costs of adjusting production capacity for the established product, on the firm's incentive to invest in R&D knowledge. What we find is that the burden of having higher adjustment costs can actually be a blessing in the innovation race in the long run, as it acts as a commitment device for the disadvantaged firm to invest more aggressively in product innovation. We identify scenarios where this effect can be so substantial that despite the higher capacity adjustment costs on the established market, the cost follower can end up with a higher overall expected discounted profit than the more efficient cost leader.

A third innovative contribution that our dynamic approach allows is the characterization of the firms' optimal innovation strategies before and after a product innovation of the competitor. We show that it is optimal for an incumbent to reduce its investment in product innovation once the opponent has successfully launched its new product in the submarket. We also demonstrate that the relationship between the optimal innovation effort and the capacities on the established market crucially depends on whether the competitor is already active on the new submarket or not. The main take-away for a firm's optimal R&D strategy is that the rival's current product range matters when determining the optimal reaction to a change in capacities on the established market.

A limitation we share with most of the literature on innovation incentives under oligopolistic competition is that firms do not face any financial constraints. As a consequence, firms are always able to fully implement their planned investment strategies. However, a rich empirical literature indicates that many firms encounter difficulties in obtaining external funding for investments in R&D, and, therefore, have to rely on internal sources for financing their R&D activities. This opens up an additional channel which influences the interaction between a firm's accumulated profits resulting from established products and a firm's incentive to invest in product innovation. Examining the impact of financial constraints and internal funding of R&D activities on the firm's optimal dynamic innovation strategy however requires to consider a model with a richer representation of the firms' financial side, taking into account the firms' dividend policy and the dynamics of firm liquidity. Developing our analysis further in this direction is a promising avenue for future research.

Appendix A. Proof of Proposition 1

Adopting the notation developed in Dockner et al. (2000)[p. 209] we write the objective function (8) of firm $f = A, B$ as

$$J_f = \mathrm{IE}\left\{\int_0^\infty e^{-rt} F_f(m(t), \vec{K}(t), \vec{I}(t))dt\right\} \tag{A.1}$$

with $\vec{K}(t) = (K_{1A}(t), K_{1B}(t), K_{RA}(t), K_{RB}(t), K_{2A}(t), K_{2B}(t))$, $\vec{I}(t) = (I_{1A}(t), I_{1B}(t), I_{RA}(t), I_{RB}(t), I_{2A}(t), I_{2B}(t))$ denoting state and control vectors[29] and

$$F_A(m,\vec{K},\vec{I}) = \begin{cases} (1-(K_{1A}+K_{1B}))K_{1A} - \mu_1 I_{1A} - \frac{\gamma_{1f}}{2}I_{1A}^2 - \mu_R I_{RA} - \frac{\gamma_{RA}}{2}I_{RA}^2 & m = m_1 \\ (1-(K_{1A}+K_{1B}) - \eta K_{2A})K_{1A} + (1+\theta-\eta(K_{1A}+K_{1B}) - K_{2A})K_{2A} & \\ -\mu_1 I_{1A} - \frac{\gamma_{1A}}{2}I_{1A}^2 - \mu_2 I_{2A} - \frac{\gamma_{2A}}{2}I_{2A}^2 & m = m_2 \\ (1-(K_{1A}+K_{1B}) - \eta K_{2B})K_{1A} - \mu_1 I_{1A} - \frac{\gamma_{1A}}{2}I_{1A}^2 - \mu_R I_{RA} - \frac{\gamma_{RA}}{2}I_{RA}^2 & m = m_3 \\ (1-(K_{1A}+K_{1B}) - \eta(K_{2A}+K_{2B}))K_{1A} & \\ +(1+\theta-\eta(K_{1A}+K_{1B}) - (K_{2A}+K_{2B}))K_{2A} - \mu_1 I_{1A} - \frac{\gamma_{1A}}{2}I_{1A}^2 - \mu_2 I_{2A} - \frac{\gamma_{2A}}{2}I_{2A}^2 & m = m_4 \end{cases} \tag{A.2}$$

$$F_B(m,\vec{K},\vec{I}) = \begin{cases} (1-(K_{1A}+K_{1B}))K_{1B} - \mu_1 I_{1B} - \frac{\gamma_{1B}}{2}I_{1B}^2 - \mu_R I_{RB} - \frac{\gamma_{RB}}{2}I_{RB}^2 & m = m_1 \\ (1-(K_{1A}+K_{1B}) - \eta K_{2A})K_{1B} - \mu_1 I_{1B} - \frac{\gamma_{1B}}{2}I_{1B}^2 - \mu_R I_{RB} - \frac{\gamma_{RB}}{2}I_{RB}^2 & m = m_2 \\ (1-(K_{1A}+K_{1B}) - \eta K_{2B})K_{1B} + (1+\theta-\eta(K_{1A}+K_{1B}) - K_{2B})K_{2B} & \\ -\mu_1 I_{1B} - \frac{\gamma_{1B}}{2}I_{1B}^2 - \mu_2 I_{2B} - \frac{\gamma_{2B}}{2}I_{2B}^2 & m = m_3 \\ (1-(K_{1A}+K_{1B}) - \eta(K_{2A}+K_{2B}))K_{1B} & \\ +(1+\theta-\eta(K_{1A}+K_{1B}) - (K_{2A}+K_{2B}))K_{2B} - \mu_1 I_{1A} - \frac{\gamma_{1B}}{2}I_{1B}^2 - \mu_2 I_{2B} - \frac{\gamma_{2B}}{2}I_{2B}^2 & m = m_4 \end{cases} \tag{A.3}$$

the mode depending instantaneous profit functions of the two firms. Denoting the transition rates between modes $m \in M$ and $\tilde{m} \in M$ by $q_{m,\tilde{m}}(\vec{K},\vec{I})$ we get in light of (4)

$$q_{m,\tilde{m}}(\vec{K},\vec{I}) = \begin{cases} \lambda(I_{RA}, K_{RA}) & (m,\tilde{m}) = (m_1, m_2), \\ \lambda(I_{RB}, K_{RB}) & (m,\tilde{m}) = (m_1, m_3), \\ \omega\lambda(I_{RA}, K_{RA}) & (m,\tilde{m}) = (m_3, m_4), \\ \omega\lambda(I_{RB}, K_{RB}) & (m,\tilde{m}) = (m_2, m_4), \\ 0 & \text{else.} \end{cases} \tag{A.4}$$

Finally, we denote the right hand side of the state dynamics for the six states by

$$\begin{aligned} & f_{1f}(m,\vec{K},\vec{I}) = I_{1f} - \delta K_{1f} m \in M, f = A, B \\ & f_{2A}(m,\vec{K},\vec{I}) \begin{cases} 0 & m = m_1, m_3 \\ I_{2A} - \delta K_{2A} & m = m_2, m_4 \end{cases} \\ & f_{2B}(m,\vec{K},\vec{I}) \begin{cases} 0 & m = m_1, m_2 \\ I_{2B} - \delta K_{2B} & m = m_3, m_4 \end{cases} \\ & f_{RA}(m,\vec{K},\vec{I}) \begin{cases} I_{RA} - \delta K_{RA} & m = m_1, m_3 \\ 0 & m = m_2, m_4 \end{cases} \\ & f_{RB}(m,\vec{K},\vec{I}) \begin{cases} I_{RB} - \delta K_{RB} & m = m_1, m_2 \\ 0 & m = m_3, m_4. \end{cases} \end{aligned} \tag{A.5}$$

For simplicity we assume here that the knowledge stock of a firm stays constant after its innovation (rather than decreasing with rate δ), which however is irrelevant since the knowledge stock is of no use for the firm after its innovation.

Using this notation we can apply Theorem 8.2 in Dockner et al. (2000), which establishes that if there exists a pair of bounded differentiable functions $V_A(\vec{K},m), V_B(\vec{K},m)$ and Markovian strategies $\vec{\phi}_A(\vec{K},m), \vec{\phi}_B(\vec{K},m)$ such that V_A, V_B satisfies the Hamilton-Jacobi-Bellman (HJB) equations

$$\begin{aligned} & rV_f(\vec{K},m) \\ & = \max_{I_{1f},I_{Rf},I_{2f}} \Big[F_f(m,\vec{K},(\vec{I}_f,\vec{\phi}_{(-f)})) \\ & + \sum_{\tilde{m}\in M\setminus\{m\}} q_{m,\tilde{m}}(\vec{K},(\vec{I}_f,\vec{\phi}_{(-f)}))(V_f(\vec{K},\tilde{m}) - V_f(\vec{K},m)) \\ & + \frac{\partial V_f}{\partial K_{1f}} f_{1f}(m,\vec{K},(\vec{I}_f,\vec{\phi}_{(-f)})) + \frac{\partial V_f}{\partial K_{2f}} f_{2f}(m,\vec{K},(\vec{I}_f,\vec{\phi}_{(-f)})) \\ & + \frac{\partial V_f}{\partial K_{Rf}} f_{Rf}(m,\vec{K},(\vec{I}_f,\vec{\phi}_{(-f)})) \\ & + \frac{\partial V_f}{\partial K_{1(-f)}} f_{1(-f)}(m,\vec{K},(\vec{I}_f,\vec{\phi}_{(-f)})) + \frac{\partial V_f}{\partial K_{2(-f)}} f_{2(-f)}(m,\vec{K},(\vec{I}_f,\vec{\phi}_{(-f)})) \\ & + \frac{\partial V_f}{\partial K_{R(-f)}} f_{R(-f)}(m,\vec{K},(\vec{I}_f,\vec{\phi}_{(-f)}))|I_{1f}, I_{2f} \in \mathbb{R}, I_{Rf} \in \mathbb{R}_0\Big], \end{aligned} \tag{A.6}$$

[29] In what follows we denote by $\vec{I}_f$ the three dimensional vector (I_{1f}, I_{Rf}, I_{2f}) of controls of firm f. Similarly, $\vec{\phi}_f = ((\phi_{1f}, \phi_{Rf}, \phi_{2f}))$ denotes the vector of feedback functions of firm f.

and $\vec{\phi}_f(\vec{K}, m)$ maximizes the right hand side of this equation for all $\vec{K} \in [0,1]^2 \times [0, \bar{K}_R]^2 \times [0, 1+\theta]^2$ and all $m \in M$, then the strategy profile $(\vec{\phi}_A(\vec{K}, m), \vec{\phi}_B(\vec{K}, m))$ is a stationary Markov perfect equilibrium.[30]

Inserting (A.2)–(A.5) into (A.6) and adopting the simplified notation, which we also use in the paper, that in each mode only the relevant state and control variables are listed, yields the following HJB equations in the different modes.

Mode m$_1$: the HJB equations of both firms are symmetric and are given by

$$\begin{aligned}
&r\, V_f(K_{1f}, K_{Rf}, K_{1(-f)}, K_{R(-f)}, m_1)\\
&= \max_{I_{1f}, I_{Rf}} \Big[(1-(K_{1A}+K_{1B}))K_{1f} - \mu_{1f} I_{1f} - \frac{\gamma_1}{2} I_{1f}^2 - \mu_{Rf} I_{Rf} - \frac{\gamma_R}{2} I_{Rf}^2\\
&+ \frac{\partial V_f(\vec{K}^{m_1}, m_1)}{\partial K_{1f}}(I_{1f} - \delta K_{1f}) + \frac{\partial V_f(\vec{K}^{m_1}, m_1)}{\partial K_{Rf}}(I_{Rf} - \delta_R K_{Rf})\\
&+ \frac{\partial V_f(\vec{K}^{m_1}, m_1)}{\partial K_{1(-f)}}(\phi_{1(-f)} - \delta K_{1(-f)}) + \frac{\partial V_f(\cdot, m_1)}{\partial K_{R(-f)}}(\phi_{R(-f)} - \delta_R K_{R(-f)})\\
&+ (\alpha I_{Rf} + \beta K_{Rf}^{\psi})\big(V_f(K_{1f}, 0, K_{1(-f)}, K_{2(-f)}), m^{in}) - V_f(\vec{K}^{m_1}, m_1)\big)\\
&+ (\alpha \phi_{R(-f)} + \beta K_{R(-f)}^{\psi})\big(V_f(K_{1f}, K_{2f}, K_{1(-f)}, 0), m^{lag}) - V_f(\vec{K}^{m_1}, m_1)\big) \Big],
\end{aligned} \tag{A.7}$$

where $m^{in} = m_2, m^{lag} = m_3$ for $f = A$ and $m^{in} = m_3, m^{lag} = m_2$ for $f = B$. The right hand side (RHS) of equation (A.7) is strictly concave in (I_{1f}, I_{Rf}). Consequently, the first order conditions for the maximization of the RHS are necessary and sufficient and yield expressions (9), (10) and (11) for I_{1f} and I_{Rf}.

Mode m$_2$: in modes m_2 (and m_3) the HJB equations of the innovator and the laggard differ substantially. In mode m_2, the HJB equation of the innovator firm A reads

$$\begin{aligned}
&r\, V_A(\vec{K}^{m_2}, m_2)\\
&= \max_{I_{1A}, I_{2A}} \Big[(1-(K_{1A}+K_{1B}) - \eta K_{2A})K_{1A} - \mu_1 I_{1A} - \frac{\gamma_{1A}}{2} I_{1A}^2\\
&\quad + (1+\theta - K_{2A} - \eta(K_{1A}+K_{1B}))K_{2A} - \mu_2 I_{2A} - \frac{\gamma_{2A}}{2} I_{2A}^2\\
&\quad + \frac{\partial V_A(\vec{K}^{m_2}, m_2)}{\partial K_{1A}}(I_{1A} - \delta_1 K_{1A})\\
&\quad + \frac{\partial V_A(\vec{K}^{m_2}, m_2)}{\partial K_{1B}}(\phi_{1B}(\vec{K}^{m_2}, m_2) - \delta_1 K_{1B})\\
&\quad + \frac{\partial V_A(\vec{K}^{m_2}, m_2)}{\partial K_{2A}}(I_{2A} - \delta_2 K_{2A})\\
&\quad + \frac{\partial V_A(\vec{K}^{m_2}, m_2)}{\partial K_{RB}}(\phi_{RB}(\vec{K}^{m_2}, m_2) - \delta_R K_{RB}) + (\alpha \phi_{RB}(\vec{K}^{m_2}, m_2)\\
&\quad + \beta K_{RB}^{\psi})(V_A(K_{1A}, K_{2A}, K_{1B}, 0, m_4) - V_A(\vec{K}^{m_2}, m_2)) \Big].
\end{aligned} \tag{A.8}$$

For the laggard firm B we obtain

$$\begin{aligned}
&r\, V_B(\vec{K}^{m_2}, m_2)\\
&= \max_{I_{1B}, I_{RB}} \Big[(1-(K_{1A}+K_{1B}) - \eta K_{2A})K_{1B} - \mu_1 I_{1B} - \frac{\gamma_{1B}}{2} I_{1B}^2 - \mu_R I_{RB}\\
&- \frac{\gamma_{RB}}{2} I_{RB}^2 + \frac{\partial V_B(\vec{K}^{m_2}, m_2)}{\partial K_{1B}}(I_{1B} - \delta_1 K_{1B}) + \frac{\partial V_B(\vec{K}^{m_2}, m_2)}{\partial K_{RB}}(I_{RB} - \delta_R K_{RB})\\
&+ \frac{\partial V_B(\vec{K}^{m_2}, m_2)}{\partial K_{1A}}(\phi_{1A}(\vec{K}^{m_2}, m_2) - \delta_1 K_{1A})\\
&+ \frac{\partial V_B(\vec{K}^{m_2}, m_2)}{\partial K_{2A}}(\phi_{2A}(\vec{K}^{m_2}, m_2) - \delta_2 K_{2A})\\
&+ (\alpha I_{RB} + \beta K_{RB}^{\psi})(V_B(K_{1A}, K_{2A}, K_{1B}, 0, m_4) - V_B(\vec{K}^{m_2}, m_2)) \Big].
\end{aligned} \tag{A.9}$$

Like in mode m_1, the derivation of the expressions of the investment functions (9) and (12) by the first order conditions is straightforward.

Symmetric equations are obtained for mode m_3, where firm B is the innovator and firm A is the laggard.

Mode m$_4$: in this mode the HJB equations are again symmetric across firms and read

$$\begin{aligned}
&r\, V_f(\vec{K}^{m_4}, m_4)\\
&= \max_{I_{1f}, I_{2f}} \Big[K_{1f}(1-(K_{1A}+K_{1B}) - \eta(K_{2A}+K_{2B})) - \mu_1 I_{1f} - \frac{1}{2}\gamma_{1f} I_{1f}^2\\
&\quad + K_{2f}(1+\theta - \eta(K_{1A}+K_{1B}) - (K_{2A}(t)+K_{2B})) - \mu_2 I_{2f} - \frac{1}{2}\gamma_{2f} I_{2f}^2\\
&\quad + \frac{\partial V_f(\vec{K}^{m_4}, m_4)}{\partial K_{1f}}(I_{1f} - \delta_1 K_{1f}) + \frac{\partial V_f(\vec{K}^{m_4}, m_4)}{\partial K_{2f}}(I_{2f} - \delta_2 K_{2f})\\
&\quad + \frac{\partial V_f(\vec{K}^{m_4}, m_4)}{\partial K_{1(-f)}}(\phi_{1(-f)}(\vec{K}^{m_4}) - \delta_1 K_{1(-f)})\\
&\quad + \frac{\partial V_f(\vec{K}^{m_4}, m_4)}{\partial K_{2(-f)}}(\phi_{2(-f)}(\vec{K}^{m_4}) - \delta_2 K_{2(-f)}) \Big].
\end{aligned} \tag{A.10}$$

By the strict concavity of the RHS in (I_{1f}, I_{2f}), the first order conditions are necessary and sufficient and again yield the expressions (9) for the investment functions.

Supplementary material

Supplementary material associated with this article can be found, in the online version, at doi:10.1016/j.ejor.2022.06.046.

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[30] Theorem 8.2 in Dockner et al. (2000) holds under the condition that the piecewise deterministic state-mode process given by (A.4) and (A.5) is well defined for all $(\vec{K}(0), m(0)) \in [0,1]^2 \times [0, \bar{K}_R]^2 \times [0, 1+\theta]^2 \times M$, which is satisfied in our setting. Also, strictly speaking the theorem states that $\vec{\phi}_A(\vec{K}, m), \vec{\phi}_B(\vec{K}, m)$ is a subgame perfect Markovian Nash equilibrium, which is their notation for a Markov perfect equilibrium.

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