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Correspondence to:
Shilong Wang
wang.sl@cqu.edu.cn

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Precision allocation optimization modeling of large-scale CNC hobbing machine based on precision reliability

Zongyan Hu, Shilong Wang and Chi Ma

The State Key Laboratory of Mechanical Transmission, Chongqing University, Chongqing 400044, China

Abstract Reducing manufacturing cost is the main goal of machine tool precision allocation under the condition of meeting precision design requirements. The previous studies generally took "ensuring machining errors completely within the allowable range of design precision" as constraint. Due to the strict constraint, the cost reduction effect is limited. This paper proposes a new method of precision allocation based on precision reliability. The probability that the machining errors are within the design precision allowable range is taken as the measurement index of precision reliability, and the optimization model constraint is relaxed to "that the machining errors are within the allowable range of design precision with predefined precision reliability", so as to obtain lower manufacturing cost under tolerable precision loss risk. The case study of a large-scale gear hobbing machine shows that this method can effectively reduce the manufacturing cost, and the precision allocation is more economical and reasonable.

1. Introduction

Due to the fierce competition in the machinery industry, higher requirements are put forward for the design of machine tools. The traditional empirical design method cannot meet the design requirements of balancing the precision, cost and reliability of machine tools. Optimization design is a kind of modern design method, which is to optimize the design parameters under various specified constraints to obtain the optimal value of one or more design indicators. It aims at improving product quality and reducing product cost.

The machine tool design mainly includes the structural design, electrical design, precision design, and thermal design. Precision design, also known as precision allocation, is to reasonably allocate the precision of each part of the machine tool according to the machining performance requirements of the machine tool. Due to the complex impact on product quality and cost, the precision allocation is a highly responsible task [1]. Precision allocation generally includes three steps: precision prediction, precision allocation optimization modeling and model solving.

The first step of machine tool precision optimization is precision prediction. The influence of transmission chain errors on machine tool precision is analyzed by using machine tool comprehensive error model. There are methods of matrix transformation [2], error matrix [3], D-H [4], rigid body kinematics [5], screw theory [6, 7], differential transform [8] and multi-body system (MBS) theory modeling [9-12]. Among them, MBS modeling method is a highly formal modeling method with less modeling assumptions, which is very suitable for computer automatic modeling. In recent years, it has been used by many scholars for machine tool error modeling.

Zhu et al. established a geometric error model of machine tool based on MBS [13]. Aiming at the randomness of volume error of multi-axis machine tools, Cheng presented the mean value analysis model of volume error considering geometric error based on MBS [14]. Shi et al. established a compensation model considering comprehensive error of three-axis CNC machine tools with MBS and homogeneous transformation matrix [15]. Shi et al. substituted the deformation stiffness coefficient equation of each part into the whole machine deformation model based on MBS to obtain a 3D stiffness model [16]. Tao analyzed the influence of tool, workpiece position error, angle error and its change speed on gear machining precision, and established an error model of six-axis CNC hobbing machine [17]. Guo deeply analyzed the errors of moving pair, rotating pair and perpendicularity of each moving axis of YK3610 gear hobbing machine (GHM), established a motion error model of the GHM [18]. Zhao established a linear error model for the transmission chain system of a hobbing machine rest, and analyzed contributions of transmission part errors to transmission errors of the whole transmission chain [19]. Sun established a gear geometric error mapping model under the coupling effect of hob geometric error and machine tool geometric error [20]. Ren analyzed 57 geometric errors of six motion axes, hobs and workpieces of a GHM, presented a mapping model of geometric errors for the GHM [21].

The second step of precision optimization is precision allocation optimization modeling. In the early days, the widely used precision allocation method was based on principles, such as principle of similarity, equal, tolerance, equal influence, equal precision, etc. Based on these allocation principles, the mathematical relationship model of precision allocation was established to meet the machine design precision [22]. On the premise of meeting the design precision, reducing manufacturing costs and improving product quality is the goal of enterprises. For this reason, scholars put forward methods of machine tool precision allocation with manufacturing cost and product quality as the optimization objectives.

The precision of machine tool and manufacturing cost are contradictory indicators. It has become a valid method to resolve the precision allocation problem in machine tool design by transforming the precision allocation into an optimization problem, aiming at minimizing the manufacturing cost and satisfying the design precision as the constraint condition. At present, common models are exponential model, power exponential model, negative square model, cubic or quartic polynomial model, as well as composite model composed of two kind of models, such as exponential and power exponent, linear and exponential models.

Cheng proposed an allocation method considering geometric accuracy using sensitivity analysis and reliability theory for multi-axis CNC machines [23]. Sheng et al. comprehensively considered the relationship among the three factors of function, cost and value, and proposed a value analysis method of mechanism precision allocation [24]. Based on the empirical data of typical production process, Dong established a cost error model [25]. Feng proposes a design method of error allocation based on stochastic integer programming [26]. Ashagbor et al. applied simulated annealing algorithm to continuous error optimization problem with the goal of minimum cost and a reciprocal cost function [27]. According to the interval analysis, Rao et al. proposed a precision allocation optimization method, which can minimize the given objective function on the premise of meeting required function and constraints [28]. Huang et al. proposed a global precision allocation optimization method for machine tool parts by combining BPNNs with GA [29]. Kang et al. optimized the error parameters using genetic algorithm and established a precision allocation model of machine tool with the goal of minimum manufacturing cost [30]. Sarina aimed to minimize manufacturing cost and motion error, and used the multi-objective nonlinear optimization method to optimize the design [31]. Using interval analytic hierarchy process (AHP), Xu proposed a comprehensive weight distribution method based on machine geometric accuracy [32]. According to reliability theory, Yu et al. established the reliability limit state machining precision function to meet design requirements, and proposed a precision allocation method for large-scale CNC machine tools [33]. A reverse CNC machine precision design method was presented by Xing et al. Based on the MBS theory, an approximate model of machining quality under the comprehensive effect of machine tool's geometric error was established, which was used to reversely deduce the geometric precision of each motion axis of machine tools [34]. Guo et al. established a state space model considering error transfers and geometric errors of each part in the assembly process, and realized the optimization of precision allocation [35]. Guo proposed a precision distribution scheme of transmission chain optimization design with comprehensive cost, robustness and balance index [36]. Liu et al. established the machine tool assembly precision allocation model using state space model, considering the adjustment control quantity in the assembly process [37]. Ma proposed an initial allocation method of machine tool geometric error precision based on sensitivity analysis [38]. Cheng et al. established an optimization model of assembly error allocation under actual working conditions, based on the modified Jacobian screw model [39]. In order to optimize the total cost and reliability, Zhang proposed a precision allocation method under the geometric and operational constraints of machine tools [40]. Based on the interval optimization theory, Liu established the interval optimization model of geometric precision allocation [41]. Wu abstracted the precision allocation problem into a constrained minimization problem, and proposed a machine tool precision allocation method based on contribution analysis [42].

Zhang et al. established an error identification and optimization model for geometric error parameters that have a great impact on reliability, and used the model to determine the allowable level of each error and optimize the processing cost [43]. Tlija et al. proposed a tolerance allocation method to maximize the interval width of geometric error sources by using difficulty coefficient evaluation and Lagrange multiplier [44]. Wang et al. established a static volume error kinematic model of heavy machine tools and proposed a tolerance analysis method based on interval theory [45]. He et al. took the target function requirements, consistency, cost of precision maintenance and processing into consideration, proposed a statistical tolerance allocation method using deep Q-learning with reward function [46]. Fan et al. presented a tolerance allocation optimal method based on finite element analysis. The manufacturing cost is reduced about 11.5 % [47].

Taking improving product quality as the optimization objective is another effective method to solve the problem of machine tool precision allocation [48]. Taguchi proposed a quality loss model which is the most commonly used to describe the degree of product quality deviation from the expected goal [48-51]. Aiming at the limitation of traditional Taguchi square mass loss function, Cao et al. proposed the concepts of fuzzy loss of mass and mass cost. According to the fuzzy mass loss function, a robust tolerance design model was established [52]. Qiang et al. proposed a robust precision allocation method with trade-off between multi-objective cost and quality for multi-axis machines. The manufacturing cost model of machine tool parts which have a significant impact on geometric error is established, based on processing characteristics of machine parts. It integrates the quality loss of machine tools into a single optimization objective [53]. Liu et al. set goals as both quality loss and manufacturing cost, considered assembly constraints and machining precision constraints. A tolerance optimization model was established. A closed solution of optimal tolerance was obtained by Lagrange multiplier method [54].

The last step of precision allocation is to solve the optimization model. Traditional optimization algorithms are generally deterministic algorithms, such as linear programming, nonlinear programming, integer programming, mixed programming, etc. these algorithms are generally aimed at structured problems, with relatively clear problem and condition descriptions, and their solution results are also determined. However, the traditional optimization algorithm must know the mathematical characteristics of the optimal solution, which has great limitations. Modern intelligent optimization algorithm is a heuristic algorithm rising in the early 1980s. Typical algorithms include tabu search, simulated annealing, genetic algorithm, particle swarm optimization algorithm, artificial neural network and so on. The intelligent algorithm does not need to know the mathematical characteristics of the optimal solution. It is simple, universal and convenient for parallel processing. It can solve the complexity, nonlinearity, constraints and modeling difficulties of engineering problems. It has been widely used in machine tool structure design [55], machining parameter optimization [56, 57], machining error compensation [58, 59], machining quality evaluation [60].

For the basic precision allocation problem, the traditional deterministic optimization algorithm can generally be used. However, when the precision-cost optimization becomes more complex, the intelligent algorithm needs to be used [61]. For example, Ashagbor et al. used simulated annealing algorithm [27], Huang et al. used neural network algorithm [29], Kang et al. [30], Guo [36], Zhao et al. [62], and Liu [41], respectively, used genetic algorithm to solve precision allocation problem, Sarina [31], Xu [32], Ma [38], and Zhang et al. [40] applied non dominated sorting genetic algorithm (NSGA-II) to solve precision allocation problem.

In the above studies, the objective of optimization is to minimize the manufacturing cost, with the overall precision fully meeting the design requirements as the optimization constraints. This condition is too harsh, and the optimization results are not the most economical and reasonable. The research of introducing quality loss cost into cost calculation also takes the overall precision fully meeting the design requirements as the optimization constraint condition. The machining reliability is only considered in the precision quality loss cost model, which is complex and difficult to model and solve.

Gear machining precision is mainly affected by the geometric errors of transmission parts, structural stiffness, dynamic response and other factors. The research shows that manufacturing errors of transmission parts are random variables and so are the geometric errors caused by these errors. According to the modern mechanical precision design theory, the overall precision is also random variable with normal distribution. Based on the properties of normal distribution random variables, the probability of reaching maximum is very low. Therefore, if the optimization constraints are relaxed so that the overall precision of the machine tool meets the design requirements with a certain probability, a more economical and reasonable optimization result is expected.

Based on this idea, this paper presents a method of machine tool precision allocation based on precision reliability. Its goal is to reduce the manufacturing cost of machine tools more effectively and obtain more economical and reasonable allocation results. A definition of precision reliability of transmission chain is given, and an optimization model of precision allocation with the constraint of precision reliability is established. The model aims at minimizing the manufacturing cost, and adds the machining reliability index to the constraint conditions without establishing a complex cost model considering quality loss. Results in case study show that the optimization model can get more economical and reasonable results of precision allocation optimization.

Large-scale CNC gear hobbing machine (CNC-LGHM) is widely used in machinery manufacturing industry, mainly used to process gears with high requirements for motion stability and service life. Gear machining precision is a very significant performance index for CNC-LGHM. On the premise of meeting the design objectives of precision, quality, reliability and stability, how to allocate precision of each part of transmission chain is an important issue. As the key to optimize technical and economic comprehensive performance index of CNC-LGHM, it must be considered in design and manufacture. So this paper takes the CNC-LGHM as studying case to prove the proposed precision allocation method.

The paper structure is arranged as follows. The basic idea and overall flow of the proposed precision allocation optimization method is introduced in Sec. 2. Based on the MBS theory, the comprehensive error model of CNC-LGHM is established in Sec. 3. Based on the constraint of transmission chain precision reliability, an optimization model of precision allocation is established aiming at minimizing manufacturing cost in Sec. 4. Taking a large-scale gear hobbing machine as an example, the precision allocation optimization model is applied to optimize the precision allocation, and the results are discussed in Sec. 5.

Finally, a summary of the paper is given in Sec. 6.

2. Basic idea and overall flow

This paper presents an optimization method of precision allocation based on precision reliability. Its core idea is to relax the constraint of the precision allocation optimization model to the fact that the comprehensive error of the machine tool meets the design requirements with a certain probability, rather than fully meeting the design requirements. In this way, the minimum risk of precision loss is exchanged for a lower cost, making the precision allocation optimization result more economical and reasonable. In order to establish such a model, this paper defines a precision reliability index based on the normal distribution characteristics of machine tool errors.

The precision allocation optimization method based on precision reliability proposed in this paper includes three parts. Firstly, based on the theory of multi-body system, the geometric error model of hobbing machine is established, including the manufacturing errors and assembly errors of moving axes. The second step is to establish a precision-cost optimization model based on the precision reliability constraints. The model adopts the power exponent model, and in the modeling, according to the characteristics of large-scale hobbing machine, the influence of the machine bed and column size on the precision is considered. Finally, the gray wolf optimization (GWO) algorithm is used to solve the model, and precision allocation scheme with the lowest cost is obtained. The overall flow of the proposed precision allocation optimization method is shown in Fig. 1.

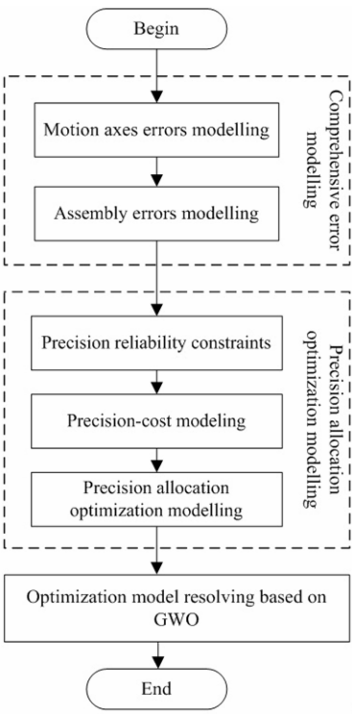

Fig. 1. Overall flow of the proposed precision allocation optimization method.

3. Comprehensive error modelling

3.1 Topological structure description of CNC-LGHM

The structure of typical CNC-LGHM shows in Fig. 2. The main components include worktable, bed, column, longitudinal supporting plate, rotating supporting plate, tangential supporting plate and hob.

In this paper, the topological CNC-LGHM structure is described by using the theory of MBS, as shown in Fig. 3. Considering that the workpiece should be fixed on worktable by the gear processing fixture during the gear processing of large-scale hobbing machine, there is no relative motion between them in the machining process, which can be regarded as a rigid body connection, so the worktable and the workpiece are defined as a whole with an individual number. Thus, define the worktable-workpiece as B_1, the bed as B_2, the column as B_3, the longitudinal support plate as B_4, the rotary support plate as B_5, the tangential support plate as B_6, and the hob as B_7, and a multi-body system is established. Here, the global reference coordinate system of the machine tool and the local coordinate system of each component are defined according to ISO 230-1 (Test code for machine tools - part 1: geometric accuracy of

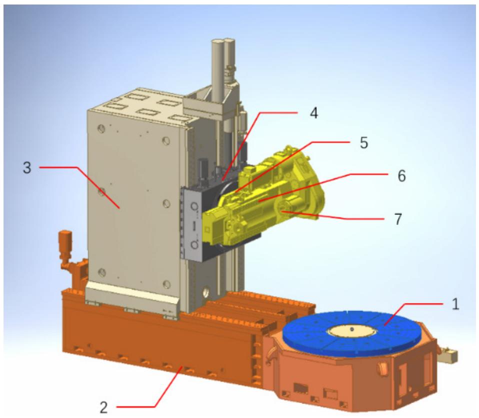

Fig. 2. Structure of typical large-scale CNC gear hobbing machine (1: worktable; 2: bed; 3: column; 4: longitudinal pallet; 5: rotary pallet; 6: tangential pallet; 7: hob).

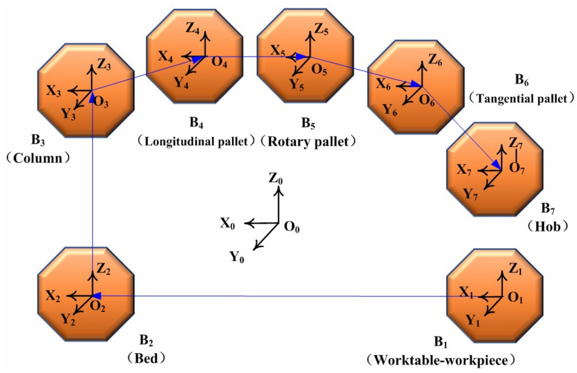

Fig. 3. Definition of multi-body system of large gear hobbing machine.

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machines operating under no-load or quasi-static conditions). The global reference coordinate system is defined as $O_0X_0Y_0Z_0$, and its coordinate origin O_0 is located at the gear hobbing machine C and X-axis intersection. The directions of coordinate axes X_0, Y_0 and Z_0 are the same as X, Y and Z-axis of gear hobbing machine, respectively. $O_iX_iY_iZ_i$ is built on body B_i (i = 1, 2, ..., 7). The initial position of the coordinate origin is O_0, and the direction of the coordinate axis is the same as $O_0X_0Y_0Z_0$.

3.2 Motion axis errors

The motion errors of transmission chain of CNC-LGHM are aroused by manufacturing errors of moving parts and positioning errors of motion axis servo control. In the process of motion, there will be six degrees of freedom error which changes with the motion state.

1) Motion errors of linear axis

$O_3X_3Y_3Z_3$ is the ideal motion coordinate system of the column moving along the X-axis, and $O_3^eX_3^eY_3^eZ_3^e$ the actual one, as illustrated in Fig. 4.

In the ideal condition, when the column moves S_x along X-axis, $O_3X_3Y_3Z_3$ moves S_x along coordinate axis X_2. The ideal motion transformation matrix from bed to column is as Eq. (1).

$$M_{2,3} = T_x(S_x) = \begin{bmatrix} 1 & 0 & 0 & S_x \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}. \tag{1}$$

When there are motion errors in X-axis, the actual motion coordinate system $O_3^eX_3^eY_3^eZ_3^e$ deviates from the ideal motion coordinate system $O_3X_3Y_3Z_3$, resulting in the relative position and posture errors between them. The errors are related to the motion displacement S_x of X-axis and can be decomposed into 3 basic position errors ${}_X\delta_x(S_x)$, ${}_X\delta_y(S_x)$, ${}_X\delta_z(S_x)$ and 3 angular errors ${}_X\varepsilon_x(S_x)$, ${}_X\varepsilon_y(S_x)$, ${}_X\varepsilon_z(S_x)$, and the error transformation matrix is as Eq. (2).

$$E_{2,3}^m = \begin{bmatrix} 1 & -{}_X\varepsilon_z(S_x) & {}_X\varepsilon_y(S_x) & {}_X\delta_x(S_x) \\ {}_X\varepsilon_z(S_x) & 1 & -{}_X\varepsilon_x(S_x) & {}_X\delta_y(S_x) \\ -{}_X\varepsilon_y(S_x) & {}_X\varepsilon_x(S_x) & 1 & {}_X\delta_z(S_x) \\ 0 & 0 & 0 & 1 \end{bmatrix}. \tag{2}$$

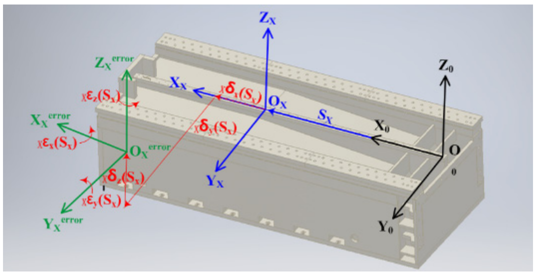


Fig. 4. Motion errors of linear X-axis.

In the same way, the ideal motion transformation matrix $M_{3,4}$, $M_{5,6}$ and their error transformation matrix $E_{3,4}^m$, $E_{5,6}^m$ of Z and Y-axis can be obtained, as shown in Eqs. (3)-(6).

$$M_{5,6} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & S_y \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \tag{3}$$

$$E_{5,6}^m = \begin{bmatrix} 1 & -{}_Y\varepsilon_z(S_y) & {}_Y\varepsilon_y(S_y) & {}_Y\delta_x(S_y) \\ {}_Y\varepsilon_z(S_y) & 1 & -{}_Y\varepsilon_x(S_y) & {}_Y\delta_y(S_y) \\ -{}_Y\varepsilon_y(S_y) & {}_Y\varepsilon_x(S_y) & 1 & {}_Y\delta_z(S_y) \\ 0 & 0 & 0 & 1 \end{bmatrix}, \tag{4}$$

$$M_{3,4} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & S_z \\ 0 & 0 & 0 & 1 \end{bmatrix}, \tag{5}$$

$$E_{3,4}^m = \begin{bmatrix} 1 & -{}_Z\varepsilon_z(S_z) & {}_Z\varepsilon_y(S_z) & {}_Z\delta_x(S_z) \\ {}_Z\varepsilon_z(S_z) & 1 & -{}_Z\varepsilon_x(S_z) & {}_Z\delta_y(S_z) \\ -{}_Z\varepsilon_y(S_z) & {}_Z\varepsilon_x(S_z) & 1 & {}_Z\delta_z(S_z) \\ 0 & 0 & 0 & 1 \end{bmatrix}. \tag{6}$$

2) Motion errors of rotating axis

$O_1X_1Y_1Z_1$ is the ideal motion coordinate system of the worktable rotating around the C-axis, and $O_1^eX_1^eY_1^eZ_1^e$ the actual one, as illustrated in Fig. 5.

In the ideal condition, when the worktable rotates θ_C around the C-axis, $O_1X_1Y_1Z_1$ rotates θ_C around the coordinate axis Z_2. The ideal motion transformation matrix from worktable- workpiece to bed is as Eq. (7).

$$M_{1,2} = R_Z(\theta_C) = \begin{bmatrix} \cos(\theta_C) & -\sin(\theta_C) & 0 & 0 \\ \sin(\theta_C) & \cos(\theta_C) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}. \tag{7}$$

When there is motion error in C-axis, the actual motion coordinate system $O_1^eX_1^eY_1^eZ_1^e$ deviates from the ideal motion coordinate system $O_1X_1Y_1Z_1$, resulting in the relative position and posture errors between them. The errors are related to the

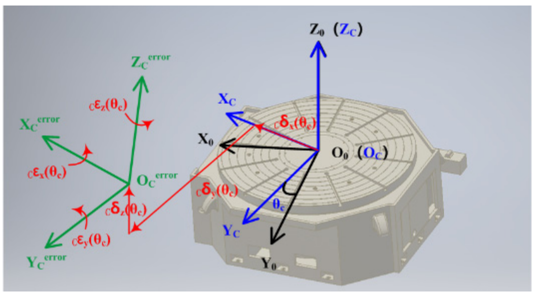


Fig. 5. Motion error of rotating C-axis.

rotating angle θ_C of C-axis and can be decomposed into 3 basic position errors ${}_C\delta_x(\theta_C)$, ${}_C\delta_y(\theta_C)$, ${}_C\delta_z(\theta_C)$ and 3 angular errors ${}_C\varepsilon_x(\theta_C)$, ${}_C\varepsilon_y(\theta_C)$, ${}_C\varepsilon_z(\theta_C)$, and the error transformation matrix is as Eq. (8).

In the same way, the ideal motion transformation matrix $M_{4,5}$, $M_{6,7}$ and their error transformation matrix $E_{4,5}^m$, $E_{6,7}^m$ of A-axis and M-axis can be obtained, as shown in Eqs. (9)-(12).

$$E_{1,2}^m = \begin{bmatrix} 1 & -{}_C\varepsilon_z(\theta_C) & {}_C\varepsilon_y(\theta_C) & {}_C\delta_x(\theta_C) \\ {}_C\varepsilon_z(\theta_C) & 1 & -{}_C\varepsilon_x(\theta_C) & {}_C\delta_y(\theta_C) \\ -{}_C\varepsilon_y(\theta_C) & {}_C\varepsilon_x(\theta_C) & 1 & {}_C\delta_z(\theta_C) \\ 0 & 0 & 0 & 1 \end{bmatrix}, \tag{8}$$

$$M_{4,5} = R_x(\theta_A) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(\theta_A) & -\sin(\theta_A) & 0 \\ 0 & \sin(\theta_A) & \cos(\theta_A) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \tag{9}$$

$$E_{4,5}^m = \begin{bmatrix} 1 & -{}_A\varepsilon_z(\theta_A) & {}_A\varepsilon_y(\theta_A) & {}_A\delta_x(\theta_A) \\ {}_A\varepsilon_z(\theta_A) & 1 & -{}_A\varepsilon_x(\theta_A) & {}_A\delta_y(\theta_A) \\ -{}_A\varepsilon_y(\theta_A) & {}_A\varepsilon_x(\theta_A) & 1 & {}_A\delta_z(\theta_A) \\ 0 & 0 & 0 & 1 \end{bmatrix}, \tag{10}$$

$$M_{6,7} = R_y(\theta_M) = \begin{bmatrix} \cos(\theta_M) & 0 & \sin(\theta_M) & 0 \\ 0 & 1 & 0 & 0 \\ -\sin(\theta_M) & 0 & \cos(\theta_M) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \tag{11}$$

$$E_{6,7}^m = \begin{bmatrix} 1 & -{}_M\varepsilon_z(\theta_M) & {}_M\varepsilon_y(\theta_M) & {}_M\delta_x(\theta_M) \\ {}_M\varepsilon_z(\theta_M) & 1 & -{}_M\varepsilon_x(\theta_M) & {}_M\delta_y(\theta_M) \\ -{}_M\varepsilon_y(\theta_M) & {}_M\varepsilon_x(\theta_M) & 1 & {}_M\delta_z(\theta_M) \\ 0 & 0 & 0 & 1 \end{bmatrix}. \tag{12}$$

3.3 Assembly errors between axes

On the foundation of the spatial CNC-LGHM transmission chain structure and the distribution of the motion axes, the motion axes are connected in sequence. In the ideal condition, two adjacent axes are perpendicular or parallel to each other in space. In the actual assembly process of gear hobbing machine, there will be certain assembly errors, which will lead to two mutually perpendicular or parallel axes no longer present mutually perpendicular or parallel attitude in space, so that the movement of transmission parts deviates from the ideal trajectory. They are called inter-axis assembly errors.

For the convenience of modeling and description, this paper gives the number definition of each axis of hobbing machine. Generally speaking, a motion axis is fixed on one part, and the other part moves along (around) the axis. For example, the X-axis connects the large column and the bed, the X-axis is fixed on the bed B_2, and the large column B_3 moves along the X-axis. The number of a motion axis is defined as the same as that of the part on which it fixed, such as the number of C-axis is 1, the number of X-axis is 2, the number of Z-axis is 3, the number of Y-axis is 4, the number of A-axis is 5, and the number of M-axis is 6.

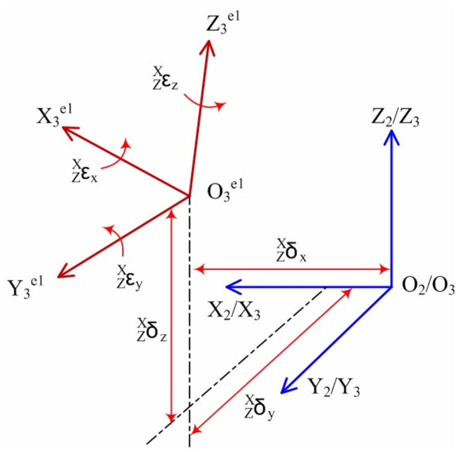


Fig. 6. Assembly errors between axes.

As illustrated in Fig. 6, according to the definition of the body coordinate systems in Sec. 3.1, $O_2X_2Y_2Z_2$ is the X-axis coordinate system and $O_3X_3Y_3Z_3$ is the Z-axis coordinate system. When there is no assembly error between the column and the bed, the origin of the two coordinate systems coincides and the direction of each axis is consistent. Due to assembly errors in the machine assembling process, the Z-axis coordinate system $O_3X_3Y_3Z_3$ deviates from its ideal position and becomes coordinate system $O_3^{e1}X_3^{e1}Y_3^{e1}Z_3^{e1}$. The assembly errors between X and Z-axis can be expressed by a spatial relative pose relationship between the two coordinate systems, where position errors are ${}_Z^X\delta_x$, ${}_Z^X\delta_y$ and ${}_Z^X\delta_z$, and angle errors are ${}_Z^X\varepsilon_x$, ${}_Z^X\varepsilon_y$ and ${}_Z^X\varepsilon_z$, and the transformation matrix of the assembly errors is shown in Eq. (13).

$$E_{2,3}^P = \begin{bmatrix} 1 & -{}_Z^X\varepsilon_z & {}_Z^X\varepsilon_y & {}_Z^X\delta_x \\ {}_Z^X\varepsilon_z & 1 & -{}_Z^X\varepsilon_x & {}_Z^X\delta_y \\ -{}_Z^X\varepsilon_y & {}_Z^X\varepsilon_x & 1 & {}_Z^X\delta_z \\ 0 & 0 & 0 & 1 \end{bmatrix}. \tag{13}$$

In the same way, the assembly error matrix $E_{2,3}^P$, $E_{3,4}^P$, $E_{4,5}^P$, $E_{5,6}^P$ of the other four axes can be derived. In general, it can be expressed as shown in Eq. (14).

$$E_{m,n}^P = \begin{bmatrix} 1 & -{}_N^M\varepsilon_z & {}_N^M\varepsilon_y & {}_N^M\delta_x \\ {}_N^M\varepsilon_z & 1 & -{}_N^M\varepsilon_x & {}_N^M\delta_y \\ -{}_N^M\varepsilon_y & {}_N^M\varepsilon_x & 1 & {}_N^M\delta_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{14}$$

where *m* and *n* are the axis numbers, and *M* and *N* are the labels of the axes.

3.4 Comprehensive error model

Every two components of a gear hobbing machine are connected by a motion axis. When the motion axis has no error, the two components make ideal relative motion, and the whole transmission chain has no transmission error. The transformation matrix of coordinate system between workpiece (B_1) and hob (B_7) is shown in Eq. (15).

The relative motion of the two components deviates from the ideal motion path due to motion-axis errors and inter-axis errors between two axes, which eventually leads to the position and posture errors of the workpiece and the hob at the cutting point, as shown in Eq. (16).

$$M_{1,7} = M_{1,2}M_{2,3}M_{3,4}M_{4,5}M_{5,6}M_{6,7}\ , \tag{15}$$

$$\begin{aligned} M_{1,7}^{e} = {} & E_{1,2}^{m}M_{1,2}E_{1,2}^{P}E_{2,3}^{m}M_{2,3}E_{2,3}^{P}E_{3,4}^{m}M_{3,4}E_{3,4}^{P} \\ & E_{4,5}^{m}M_{4,5}E_{4,5}^{P}E_{5,6}^{m}M_{5,6}E_{5,6}^{P}E_{6,7}^{m}M_{6,7}\ . \end{aligned} \tag{16}$$

If the comprehensive pose error between workpiece and hob of CNC-LGHM is *E*, then

$$M_{1,7}^{e} = M_{1,7}E\ . \tag{17}$$

It can be concluded that:

$$E = \left(M_{1,7}\right)^{-1} M_{1,7}^{e}\ . \tag{18}$$

4. Precision allocation optimization

4.1 Precision reliability of CNC-LGHM

A CNC-LGHM comprehensive error model is established in Sec. 3. It is a function of 36 axis geometric motion errors and 30 axis assembly errors. Here, the 66 errors are expressed as $\mathbf{Error} = \left(X_1, X_2, \cdots, X_{66}\right)$, which are normal distribution and independent random variables, $X_i \sim N\left(\mu_{X_i}, \sigma_{X_i}\right)$. In this paper, the first order second moment method is adopted to establish the precision reliability model.

Set $I_{\delta x}$ as the design precision of δx, and construct the function as

$$Z_{\delta x} = I_{\delta x} - \left|\delta x\right| = g_{\delta x}\left(X_1, X_2, \cdots, X_{66}\right)\ . \tag{19}$$

When $I_{\delta x} > \left|\delta x\right|$, the comprehensive error δx meets the design requirements, that is, $S = \left\{\mathbf{Error} : g_{\delta x}\left(\mathbf{Error}\right) > 0\right\}$ is the area where the precision meets the design requirements.

Then

$$\begin{cases} \mu_{g_{\delta x}} = g_{\delta x}\left(\mu_{X_1}, \mu_{X_2}, \cdots, \mu_{X_{66}}\right) \\ \sigma_{g_{\delta x}}^2 = E\left[g_{\delta x} - E\left(g_{\delta x}\right)\right]^2 = \sum_{i=1}^{66}\left(\dfrac{\partial g_{\delta x}}{\partial X_i}\right)_{\mu}^2 \sigma_{x_i}^2 \end{cases}. \tag{20}$$

Precision reliability index is

$$\beta_{\delta x} = \frac{\mu_{g_{\delta x}}}{\sigma_{g_{\delta x}}}\ . \tag{21}$$

Precision reliability probability is

$$P_{\delta x} = \Phi\left(\beta_{\delta x}\right)\ , \tag{22}$$

expressed as

$$P_{\delta x}\left(I_{\delta x} > \left|\delta x\right|\right) = \Phi\left(\beta_{\delta x}\right)\ . \tag{23}$$

$P_{\delta y}, P_{\delta z}, P_{\varepsilon x}, P_{\varepsilon y}, P_{\varepsilon z}$ can be obtained in the same way.

Suppose that the design precision requirements of 6 basic terms $(\delta x, \delta y, \delta z, \varepsilon x, \varepsilon y, \varepsilon z)$ of CNC-LGHM comprehensive error are $I_{\delta x}, I_{\delta y}, I_{\delta z}, I_{\varepsilon x}, I_{\varepsilon y}, I_{\varepsilon z}$, and the precision reliability probability requirements are $R_{\delta x}, R_{\delta y}, R_{\delta z}, R_{\varepsilon x}, R_{\varepsilon y}, R_{\varepsilon z}$, then the precision reliability constraints are shown in Eq. (24).

$$\begin{cases} P_{\delta x}\left(I_{\delta x} > \left|\delta x\right|\right) \ge R_{\delta x} \\ P_{\delta y}\left(I_{\delta y} > \left|\delta y\right|\right) \ge R_{\delta y} \\ P_{\delta z}\left(I_{\delta z} > \left|\delta z\right|\right) \ge R_{\delta z} \\ P_{\varepsilon x}\left(I_{\varepsilon x} > \left|\varepsilon x\right|\right) \ge R_{\varepsilon x} \\ P_{\varepsilon y}\left(I_{\varepsilon y} > \left|\varepsilon y\right|\right) \ge R_{\varepsilon y} \\ P_{\varepsilon z}\left(I_{\varepsilon z} > \left|\varepsilon z\right|\right) \ge R_{\varepsilon z} \end{cases}. \tag{24}$$

4.2 Precision-cost modeling

Based on the power exponent model, the precision cost function is established for minimizing manufacturing costs. Considering that the constant term in the cost function does not change with the precision, this paper ignores the constant term when calculating the cost with the same structure. The motion error symbol of each axis is expressed in a simplified form in order to simplify the expression. For example, ${}_X\delta_x\left(S_x\right)$ is simplified to ${}_X\delta_x$.

1) Precision-cost functions of linear axes

The length and width of linear guide affect the manufacturing difficulty. The longer the guide is, the more difficult it is to control the precision. Motion errors of the guide rail are mainly come from the control system and the machining precision. Other errors are mainly produced in the manufacturing process and have relation to guide rail length. The cost functions of linear pairs are shown in Eq. (25).

$$\begin{cases} F_X = \dfrac{a}{{}_X\delta_x^{\,2}} + bL_X\left(\dfrac{1}{{}_X\delta_y^{\,2}} + \dfrac{1}{{}_X\delta_z^{\,2}} + \dfrac{1}{{}_X\varepsilon_x^{\,2}} + \dfrac{1}{{}_X\varepsilon_y^{\,2}} + \dfrac{1}{{}_X\varepsilon_z^{\,2}}\right) \\ F_Y = \dfrac{a}{{}_Y\delta_y^{\,2}} + bL_Y\left(\dfrac{1}{{}_Y\delta_x^{\,2}} + \dfrac{1}{{}_Y\delta_z^{\,2}} + \dfrac{1}{{}_Y\varepsilon_x^{\,2}} + \dfrac{1}{{}_Y\varepsilon_y^{\,2}} + \dfrac{1}{{}_Y\varepsilon_z^{\,2}}\right) \\ F_Z = \dfrac{a}{{}_Y\delta_z^{\,2}} + bL_Y\left(\dfrac{1}{{}_Z\delta_x^{\,2}} + \dfrac{1}{{}_Z\delta_y^{\,2}} + \dfrac{1}{{}_Z\varepsilon_x^{\,2}} + \dfrac{1}{{}_Z\varepsilon_y^{\,2}} + \dfrac{1}{{}_Z\varepsilon_z^{\,2}}\right) \end{cases} \tag{25}$$

where, *a* is the unit length fuzzy cost coefficient related to the linear positioning precision of the control system, and *b* is the unit length fuzzy cost coefficient related to the linear guide design precision. L_X, L_Y and L_Z are the length of X, Y and Z-axis guide rails (unit: m).

2) Precision-cost functions of rotation axes

In the rotary pair, rotary errors mainly come from the rotary axis control, and other errors mainly come from the manufacturing error, which is related to its rotary diameter. The cost functions of the rotating pairs are shown in Eq. (26).

$$\begin{cases} F_C = \dfrac{c}{{}_C\varepsilon_z^{\,2}} + dD_C\left(\dfrac{1}{{}_C\delta_x^{\,2}} + \dfrac{1}{{}_C\delta_y^{\,2}} + \dfrac{1}{{}_C\delta_z^{\,2}} + \dfrac{1}{{}_C\varepsilon_x^{\,2}} + \dfrac{1}{{}_C\varepsilon_y^{\,2}}\right) \\ F_A = \dfrac{c}{{}_A\varepsilon_x^{\,2}} + dD_A\left(\dfrac{1}{{}_A\delta_x^{\,2}} + \dfrac{1}{{}_A\delta_y^{\,2}} + \dfrac{1}{{}_A\delta_z^{\,2}} + \dfrac{1}{{}_A\varepsilon_y^{\,2}} + \dfrac{1}{{}_A\varepsilon_z^{\,2}}\right) \\ F_M = \dfrac{c}{{}_M\varepsilon_y^{\,2}} + dD_M\left(\dfrac{1}{{}_M\delta_x^{\,2}} + \dfrac{1}{{}_M\delta_y^{\,2}} + \dfrac{1}{{}_M\delta_z^{\,2}} + \dfrac{1}{{}_M\varepsilon_x^{\,2}} + \dfrac{1}{{}_M\varepsilon_z^{\,2}}\right) \end{cases} \tag{26}$$

where, c is the unit diameter fuzzy cost coefficient related to the rotary control system positioning precision, and d is the unit diameter fuzzy cost coefficient related to the rotary axis design precision. D_C, D_A and D_M are the rotation diameters of C, A and M-axes (unit: m).

3) Precision-cost functions of assembly between axes

The assembly errors in the process of installation and debugging mainly come from the manufacturing precision and difficulty of the mating surface. The fuzzy cost functions of assembly error adjustment are shown in Eq. (27).

$$FA_{ij} = m_{ij}\left(\frac{1}{{}_j^i\delta_x^{\,2}} + \frac{1}{{}_j^i\delta_y^{\,2}} + \frac{1}{{}_j^i\delta_z^{\,2}} + \frac{1}{{}_j^i\varepsilon_x^{\,2}} + \frac{1}{{}_j^i\varepsilon_y^{\,2}} + \frac{1}{{}_j^i\varepsilon_z^{\,2}}\right) \tag{27}$$

where m_{ij} is the fuzzy cost coefficient related to the assembly precision of two adjacent axes.

By combining the cost functions Eqs. (25)-(27), the overall precision- cost function of the machine tool can be obtained, as shown in Eq. (28).

$$F(Error) = k_1(F_X + F_Y + F_Z) + k_2(F_C + F_A + F_M) + k_3\sum FA_{ij} \tag{28}$$

where *Error* represents all 66 geometric and kinematic errors, and k_1, k_2 and k_3 are the weight coefficients of various costs.

4) Model coefficients determination

In Eqs. (25)-(27), the fuzzy cost coefficients *a, b, c, d* and m_{ij} reflect the influence of different error terms on the manufacturing cost of machine tools, which need to be reasonably determined according to the design needs and expert experience.

Fuzzy comprehensive evaluation method (FCEM) is usually used for reasonable evaluation and comparison of preset conditions. It is derived from the fuzzy set theory put forward by American scientist L. A. Zadeh and has been successfully applied in many fields. The FCEM main steps are as follows:

(1) Various factors which affect the evaluation object compose the evaluation factor set $U=\{u_1,u_2,\cdots,u_m\}$. The element u_i represents the i-th factor which affecting the evaluation object and usually has different degrees of fuzziness.

(2) The evaluation grade set $V=\{v_1,v_2,\cdots,v_n\}$ is composed of all possible evaluation results of objects, where v_j represents the evaluation result of the *j-th* evaluation level.

(3) Each evaluation grade in the evaluation grade set V is given a score to form a score vector $S=(s_1,s_2,\cdots,s_n)$, where s_i is the weight of evaluation grade v_i in V.

(4) According to the domain knowledge and expert experience, the fuzzy comprehensive evaluation matrix $R_{m\times n}=\{r_{ij}\}$ is constructed, where r_{ij} is the degree of membership of the factor u_i in the evaluation factor set U to the evaluation grade v_j in the evaluation set V.

(5) According to the domain knowledge and expert experience, the weight vector $A=(a_1,a_2,\cdots,a_m)$ of evaluation factors is constructed according to the importance of each factor from evaluation factor set U, where a_i is the weight of the factor u_i from evaluation factor set U.

(6) The fuzzy comprehensive evaluation model is obtained by Eq. (29).

$$B = A\cdot R\,. \tag{29}$$

(7) The total score of fuzzy comprehensive evaluation is calculated by Eq. (30).

$$F = B\cdot S^T\,. \tag{30}$$

In this paper, FCEM is used to determine the undetermined coefficients in precision-cost models. The corresponding evaluation factors are mainly selected from two aspects: one is the influence factors on the cost to improve the precision requirements of CNC-LGHM in manufacturing process, and the other is the influence factors on the cost to maintain the design precision of CNC-LGHM in using process. The evaluation factor set U is as follow:

$$U=\{u_1,u_2,u_3,u_4\} \tag{31}$$

where u_1 represents the impact of improving the precision of machine parts on the cost, u_2 represents the impact of improving the assembly precision of machine parts on the cost, u_3 represents the probability that the machining precision of the machine tool is lower than the design precision, and u_4 represents the difficulty of solving the problem of precision decline.

For each evaluation factor, five levels of evaluation are set and corresponding scores are given. The evaluation level set V and score vector S are constructed, as shown in Eqs. (32) and (33).

$$V=\{v_1,v_2,v_3,v_4,v_5\}\,, \tag{32}$$

$$S=(5,4,3,2,1)\,. \tag{33}$$

The evaluation grade and score of each factor are shown in

Table 1. Factor evaluation grade and score.

Factor	Evaluation grade	Score
u_1 u_2	Most cost increase	5
	More cost increase	4
	Moderate cost increase	3
	Less cost increase	2
	Little cost increase	1
u_3	Very high failure probability	5
	High failure probability	4
	General failure probability	3
	Low failure probability	2
	Very low failure probability	1
u_4	Very difficult	5
	Difficult	4
	Moderate	3
	Easy	2
	Very easy	1

Table 1.

For each evaluation object, i.e., the coefficient to be determined, a fuzzy comprehensive evaluation matrix *R* is set according to the experience of experts in the field. The total score of fuzzy comprehensive evaluation, i.e., the coefficient value to be determined, can be obtained from Eq. (30).

4.3 Optimization model of precision allocation

The optimization model of precision allocation with the objective of minimizing manufacturing cost and the constraint of precision reliability is shown in Eq. (34).

$$\begin{aligned} & \min\left(F\left(Error\right)\right) \\ & Subject\ to: \\ & \qquad \begin{cases} P_{\delta x}\left(I_{\delta x} > \left|\delta x\right|\right) \geq R_{\delta x} \\ P_{\delta y}\left(I_{\delta y} > \left|\delta y\right|\right) \geq R_{\delta y} \\ P_{\delta z}\left(I_{\delta z} > \left|\delta z\right|\right) \geq R_{\delta z} \\ P_{\varepsilon x}\left(I_{\varepsilon x} > \left|\varepsilon x\right|\right) \geq R_{\varepsilon x} \\ P_{\varepsilon y}\left(I_{\varepsilon y} > \left|\varepsilon y\right|\right) \geq R_{\varepsilon y} \\ P_{\varepsilon z}\left(I_{\varepsilon z} > \left|\varepsilon z\right|\right) \geq R_{\varepsilon z} \\ S_X \in \left(S_{XMin}, S_{XMax}\right) \\ S_Y \in \left(S_{YMin}, S_{YMax}\right) \\ S_Z \in \left(S_{ZMin}, S_{ZMax}\right) \\ S_A \in \left(-\frac{\pi}{4}, \frac{\pi}{4}\right) \end{cases} \end{aligned} \tag{34}$$

where S_{XMin}, S_{XMax}, S_{YMin}, S_{YMax}, S_{ZMin} and S_{ZMax} represent the minimum and maximum stroke position of X, Y and Z-axis, respectively.

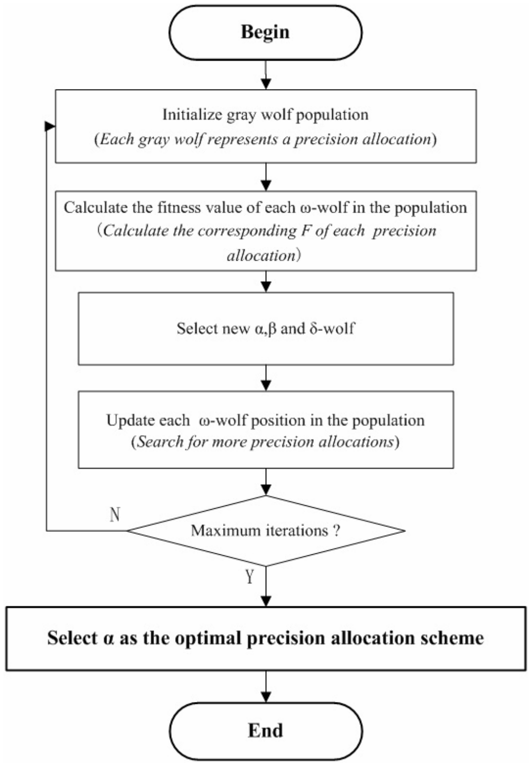


Fig. 7. The precision allocation optimization algorithm.

4.4 Model resolution

The grey wolf optimization (GWO) algorithm is adopted in this paper to optimize precision allocation of CNC-LGHM transmission chain.

Mirjalili proposed GWO which is an intelligent optimization algorithm in 2014 [63]. There is a cooperative mechanism in the predation of gray wolves in nature. GWO simulates this behavior for optimization purposes. GWO algorithm is simple in structure and easy to implement, with less parameter adjustment. In order to achieve the balance between local optimization and global search, it adopts adaptive convergence factor and information feedback mechanism and sets adaptive convergence factor, which makes it have good accuracy and convergence speed in solving problems.

In the algorithm, the current three optimal individuals are called *α-wolf*, *β-wolf* and *δ-wolf* who leads the pack to track, surround, chase and attack. The main flow of precision allocation optimization algorithm is shown in Fig. 7.

5. Case study and discussion

5.1 Case description

The design precision and reliability requirements of a CNC-LGHM are shown in Table 2. Its structure coefficients are shown in Table 3.

Generally, the design experience is used in the factory to

Table 2. Design precision and reliability requirements.

Error term	Design precision	Precision and reliability
δx	0.02	99 %
δy	0.02	99 %
δz	0.05	99 %
εx	4.8e-5	99 %
εy	4.8e-5	99 %
εz	4.8e-5	99 %

Table 3. Machine structure parameters coefficients.

Parameter	Value	Parameter	Value
L_X	4	D_A	1.5
L_Y	2	D_C	3
L_Z	5	D_M	1
S_{XMin}	0	S_{XMax}	400
S_{YMin}	0	S_{YMax}	200
S_{ZMin}	100	S_{ZMax}	500

Table 4. Preliminary precision allocation results.

Error term	Precision
${}_X\delta_x$, ${}_X\delta_y$, ${}_X\delta_z$, ${}_Y\delta_x$, ${}_Y\delta_y$, ${}_Y\delta_z$, ${}_Z\delta_x$, ${}_Z\delta_y$, ${}_Z\delta_z$, ${}_A\delta_x$, ${}_A\delta_y$, ${}_A\delta_z$, ${}_C\delta_x$, ${}_C\delta_y$, ${}_C\delta_z$, ${}_M\delta_x$, ${}_M\delta_y$, ${}_M\delta_z$, ${}_X^C\delta_x$, ${}_X^C\delta_y$, ${}_X^C\delta_z$, ${}_Z^X\delta_x$, ${}_Z^X\delta_y$, ${}_Z^X\delta_z$, ${}_A^Z\delta_x$, ${}_A^Z\delta_y$, ${}_A^Z\delta_z$, ${}_Y^A\delta_x$, ${}_Y^A\delta_y$, ${}_Y^A\delta_z$, ${}_M^Y\delta_x$, ${}_M^Y\delta_y$, ${}_M^Y\delta_z$	0.003
${}_X\varepsilon_x$, ${}_X\varepsilon_y$, ${}_X\varepsilon_z$, ${}_Y\varepsilon_x$, ${}_Y\varepsilon_y$, ${}_Y\varepsilon_z$, ${}_Z\varepsilon_x$, ${}_Z\varepsilon_y$, ${}_Z\varepsilon_z$, ${}_A\varepsilon_x$, ${}_A\varepsilon_y$, ${}_A\varepsilon_z$, ${}_C\varepsilon_x$, ${}_C\varepsilon_y$, ${}_C\varepsilon_z$, ${}_M\varepsilon_x$, ${}_M\varepsilon_y$, ${}_M\varepsilon_z$, ${}_X^C\varepsilon_x$, ${}_X^C\varepsilon_y$, ${}_X^C\varepsilon_z$, ${}_Z^X\varepsilon_x$, ${}_Z^X\varepsilon_y$, ${}_Z^X\varepsilon_z$, ${}_A^Z\varepsilon_x$, ${}_A^Z\varepsilon_y$, ${}_A^Z\varepsilon_z$, ${}_Y^A\varepsilon_x$, ${}_Y^A\varepsilon_y$, ${}_Y^A\varepsilon_z$, ${}_M^Y\varepsilon_x$, ${}_M^Y\varepsilon_y$, ${}_M^Y\varepsilon_z$	1.45E-5

guide the precision allocation. In this case, the initial allocations are obtained by using the equal precision design method commonly used in practical applications. First, a group of position and angle precision values are approximately set according to the experience, and then adjusted gradually by trial and error until the overall design precision requirements are met. The initial allocation results are shown in Table 4.

5.2 Precision allocation modeling and solution

First, the optimization model is established according to the design requirements and hobbing machine parameters in Sec. 5.1. The fuzzy cost function $F(Error)$ of the hobbing machine can be obtained by Eq. (28). The constraints on the optimization model include two groups. The constraints about the precision reliability of the hobbing machine can be constructed by Eq. (24) according to data in Table 2. The constraints about the working scope of X, Y, Z, and A-axis can be constructed according to data in Table 3. Thus, the precision allocation optimization model of the study case can be established, as shown in Eq. (35).

In Eq. (35), the target fuzzy cost function $F(Error)$ contains undetermined constant coefficients *a*, *b*, *c*, *d* and m_{ij}. Taking the coefficient *a* as an example, the weight vector *A* and the comprehensive evaluation matrix *R* is determined according to the experience of domain experts as Eqs. (36) and (37).

$$\min\left(F\left(Error\right)\right)$$
$$Subject\ to:\quad \begin{cases} P_{\delta x}\left(0.02>\left|\delta x\right|\right)\geq 0.99\\ P_{\delta y}\left(0.02>\left|\delta y\right|\right)\geq 0.99\\ P_{\delta z}\left(0.02>\left|\delta z\right|\right)\geq 0.99\\ P_{\varepsilon x}\left(4.8e-5>\left|\varepsilon x\right|\right)\geq 0.99\\ P_{\varepsilon y}\left(4.8e-5>\left|\varepsilon y\right|\right)\geq 0.99\\ P_{\varepsilon z}\left(4.8e-5>\left|\varepsilon z\right|\right)\geq 0.99\\ S_X\in\left(0,400\right)\\ S_Y\in\left(0,200\right)\\ S_Z\in\left(100,500\right)\\ S_A\in\left(-\frac{\pi}{4},\frac{\pi}{4}\right)\end{cases},\tag{35}$$

$$A=\left(0.4,0.2,0.1,0.3\right),\tag{36}$$

$$R=\begin{bmatrix}0&0&0.2&0.2&0.6\\0&0&0.2&0.4&0.4\\0&0&0.6&0.4&0\\0&0.2&0.2&0.4&0.2\end{bmatrix}.\tag{37}$$

From Eq. (29), it can be obtained

$$B=A\cdot R=\left(0,0.06,0.24,0.32,0.38\right).\tag{38}$$

From Eq. (30), it can be obtained

$$F=B\cdot S^T=1.98.\tag{39}$$

That is, *a* = 1.98. In the same way, other constant coefficients in the precision cost model can be obtained. All the results are shown in Table 5.

Table 5. Precision-cost function coefficients.

Coefficient	Value
a	1.98
b	1.5
c	1.83
d	1.4
m_{12}	1.3
m_{23}	1.7
m_{34}	1.1
m_{45}	1.34
m_{56}	1.07

Table 6. Optimization results of precision allocation.

Error term	Allocated value	Optimization rate	Error term	Allocated value	Optimization rate
${}_{X}\delta_x$	0.0036	20.00 %	${}_{M}\varepsilon_x$	1.58E-05	8.97 %
${}_{X}\delta_y$	0.0036	20.00 %	${}_{M}\varepsilon_y$	1.11E-05	-23.45 %
${}_{X}\delta_z$	0.0033	10.00 %	${}_{M}\varepsilon_z$	1.57E-05	8.28 %
${}_{X}\varepsilon_x$	1.54E-05	6.21 %	${}^{C}_{X}\delta_x$	0.0031	3.33 %
${}_{X}\varepsilon_y$	1.45E-05	0.00 %	${}^{C}_{X}\delta_y$	0.0031	3.33 %
${}_{A}\varepsilon_z$	1.51E-05	4.14 %	${}^{C}_{X}\delta_z$	0.0028	-6.67 %
${}_{Y}\delta_x$	0.0031	3.33 %	${}^{C}_{X}\varepsilon_x$	1.29E-05	-11.03 %
${}_{Y}\delta_y$	0.0031	3.33 %	${}^{C}_{X}\varepsilon_y$	1.22E-05	-15.86 %
${}_{Y}\delta_z$	0.0028	-6.67 %	${}^{C}_{X}\varepsilon_z$	1.27E-05	-12.41 %
${}_{Y}\varepsilon_x$	1.29E-05	-11.03 %	${}^{A}_{Y}\delta_x$	0.0031	3.33 %
${}_{Y}\varepsilon_y$	1.28E-05	-11.72 %	${}^{A}_{Y}\delta_y$	0.0028	-6.67 %
${}_{Y}\varepsilon_z$	1.33E-05	-8.28 %	${}^{A}_{Y}\delta_z$	0.0031	3.33 %
${}_{Z}\delta_x$	0.0038	26.67 %	${}^{A}_{Y}\varepsilon_x$	1.39E-05	-4.14 %
${}_{Z}\delta_y$	0.0039	30.00 %	${}^{A}_{Y}\varepsilon_y$	1.33E-05	-8.28 %
${}_{Z}\delta_z$	0.0035	16.67 %	${}^{A}_{Y}\varepsilon_z$	1.38E-05	-4.83 %
${}_{Z}\varepsilon_x$	1.75E-05	20.69 %	${}^{X}_{Z}\delta_x$	0.0031	3.33 %
${}_{Z}\varepsilon_y$	1.74E-05	20.00 %	${}^{X}_{Z}\delta_y$	0.0031	3.33 %
${}_{Z}\varepsilon_z$	1.67E-05	15.17 %	${}^{X}_{Z}\delta_z$	0.0028	-6.67 %
${}_{A}\delta_x$	0.0045	50.00 %	${}^{X}_{Z}\varepsilon_x$	1.29E-05	-11.03 %
${}_{A}\delta_y$	0.0041	36.67 %	${}^{X}_{Z}\varepsilon_y$	1.28E-05	-11.72 %
${}_{A}\delta_z$	0.0045	50.00 %	${}^{X}_{Z}\varepsilon_z$	1.33E-05	-8.28 %
${}_{A}\varepsilon_x$	1.17E-05	-19.31 %	${}^{Y}_{M}\delta_x$	0.0026	-13.33 %
${}_{A}\varepsilon_y$	1.95E-05	34.48 %	${}^{Y}_{M}\delta_y$	0.0023	-23.33 %
${}_{A}\varepsilon_z$	2.04E-05	40.69 %	${}^{Y}_{M}\delta_z$	0.0026	-13.33 %
${}_{C}\delta_x$	0.0041	36.67 %	${}^{Y}_{M}\varepsilon_x$	1.19E-05	-17.93 %
${}_{C}\delta_y$	0.0037	23.33 %	${}^{Y}_{M}\varepsilon_y$	1.11E-05	-23.45 %
${}_{C}\delta_z$	0.0041	36.67 %	${}^{Y}_{M}\varepsilon_z$	1.18E-05	-18.62 %
${}_{C}\varepsilon_x$	1.88E-05	29.66 %	${}^{Z}_{A}\delta_x$	0.0031	3.33 %
${}_{C}\varepsilon_y$	1.77E-05	22.07 %	${}^{Z}_{A}\delta_y$	0.0031	3.33 %
${}_{C}\varepsilon_z$	1.18E-05	-18.62 %	${}^{Z}_{A}\delta_z$	0.0028	-6.67 %
${}_{M}\delta_x$	0.0034	13.33 %	${}^{Z}_{A}\varepsilon_x$	1.39E-05	-4.14 %
${}^{Y}_{M}\delta_y$	0.0031	3.33 %	${}^{Z}_{A}\varepsilon_y$	1.38E-05	-4.83 %
${}_{M}\delta_z$	0.0034	13.33 %	${}^{Z}_{A}\varepsilon_z$	1.33E-05	-8.28 %

GWO algorithm is used to solve the optimization model. The basic algorithm parameters are as follows: the iteration number is 500, the size of gray wolf population is 100. The results of precision allocation are shown in Table 6. Discussions about the results are made in the next section.

5.3 Results discussion

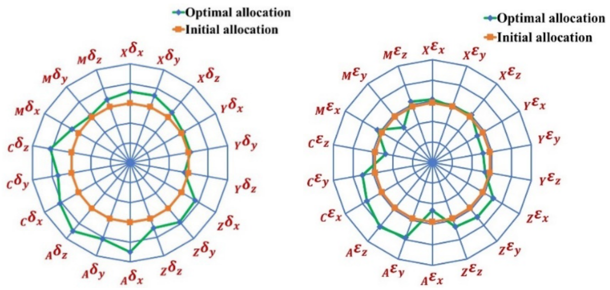
1) Discussion about precision allocation

The comparison before and after the optimization of the precision allocation of 36 motion axis error items is illustrated in Fig. 8. Compared with the initial precision allocation, among the 36 items of motion axis errors, the precision requirements of 1 item (${}_{X}\varepsilon_y$) have not changed, the precision requirements of 7 items (${}_{Y}\delta_z$, ${}_{Y}\varepsilon_x$, ${}_{Y}\varepsilon_y$, ${}_{Y}\varepsilon_z$, ${}_{A}\varepsilon_x$, ${}_{C}\varepsilon_z$, ${}_{M}\varepsilon_y$) have been increased by 6.67 %-23.45 %, and the precision requirements of the remaining 28 items have been reduced, with the average reduction rate (optimization rate) 21.56 %, the maximum reduction rate being 50 % (${}_{A}\delta_z$), and the minimum reduction rate being 3.33 % (${}_{Y}\delta_x$, ${}_{Y}\delta_y$, ${}^{Y}_{M}\delta_y$). The precision requirements of almost all position error terms are reduced except for ${}_{Y}\delta_z$. Among the angular error items of the rotating axes A, C and M, only the error terms in the direction parallel to the axis have higher precision requirements.

The comparison before and after the optimization of the precision allocation of 30 items of inter axis assembly errors is illustrated in Fig. 9. There are 8 items of assembly errors between axes (${}^{C}_{X}\delta_x$, ${}^{C}_{X}\delta_y$, ${}^{A}_{Y}\delta_x$, ${}^{A}_{Y}\delta_z$, ${}^{A}_{Y}\delta_x$, ${}^{X}_{Z}\delta_y$, ${}^{Z}_{A}\delta_x$, ${}^{Z}_{A}\delta_y$)

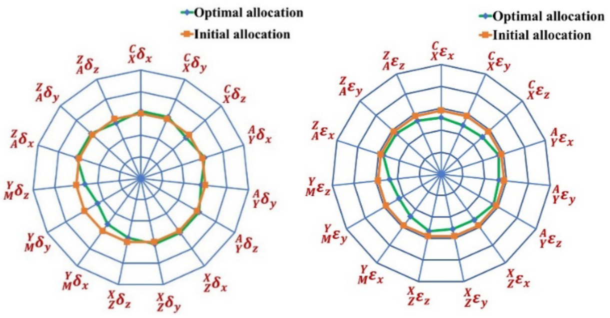
Table 7. Fuzzy manufacturing cost.

Precision allocation	Fuzzy manufacturing cost
Initial precision allocation	1.0065e+10
Optimal precision allocation	9.0886e+09



(a) Position error (b) Angle error

Fig. 8. Comparison of precision allocation of motion axis errors before and after optimization.



(a) Position error (b) Angle error

Fig. 9. Comparison of precision allocation of assembly errors between axes before and after optimization.

reduced precision requirements by 3.33 %, the other 22 items have increased precision requirements, the average increase rate is 10.98 %, the maximum increase rate is 23.45 % ($^{Y}_{M}\varepsilon_y$), the minimum is 4.14 % ($^{A}_{Y}\varepsilon_x$, $^{Z}_{A}\varepsilon_x$). Although the precision requirements of most assembly error terms are increased, the increasement is relatively small.

2) Discussion about optimal fuzzy cost

The initial and optimized fuzzy manufacturing costs are calculated. The data in Tables 3 and 5 are substituted into Eqs. (25)-(27) to establish the fuzzy cost models of each moving axis and the fuzzy cost models of axes assembly, and then combine the three models into Eq. (28) to establish the overall fuzzy cost model of the hobbing machine. The initial cost can be obtained by substituting the initial precision allocation data in Table 4 into the overall fuzzy cost model. The optimized cost can be obtained by substituting the optimized precision allocation data in Table 6 into the overall fuzzy cost model. The results are shown in Table 7.

The optimized fuzzy manufacturing cost is 9.7 % lower than the initial cost. The results proved that reducing precision requirement based on precision reliability is an effective way to reduce manufacturing cost.

The precisions of machine tool components have an important impact on the manufacturing cost of machine tools. The influence of the precisions of different components on the overall precision of machine tool is different, and the influence on the manufacturing cost is also different. On the premise of ensuring that the overall precision of the machine tool meets the design requirements, the purpose of reducing manufacturing costs can be achieved by optimizing the precision allocation of machine tool components.

From the practical experience of CNC-LGHM manufacturing, it is more difficult to improve the precisions of large-size parts. Reducing their precision requirements will help reducing the manufacturing cost greatly. In this case, the bed (X-axis), column (Z-axis) and worktable (C-axis) are large-size components. According to the previous discussion on the precision allocation, the precision requirements of X, Z and C axes are greatly reduced. That played a major role in reducing manufacturing costs.

The assembly precision also affects the manufacturing cost of machine tools. During parts assembling, improving the assembly precision with the help of high-precision measuring instruments is not difficult and has less impact on the manufacturing cost. Therefore, improving the assembly precision can improve the overall precision of machine tool with a small increase in cost. According to the previous discussion on the precision allocation, the assembly precision requirements between axes are generally increased, especially the assembly precision requirements of tool spindle (Y-M axis) are greatly increased, and the assembly precision requirements of worktable (C-X axis) and column (X-Z axis) are also increased.

In summary, the fuzzy manufacturing cost is effectively reduced by optimizing the precision allocation of gear hobbing machine using the proposed method. The optimization results are consistent with the practical experience of machine tool manufacturing.

3) Discussion about precision reliability

The results of the above example show that by optimizing the precision allocation, the manufacturing cost is effectively reduced on meeting the design precision requirements, and the expected purpose of precision optimization allocation is achieved. Based on the optimization results of the above examples, this section verifies the reliability of the optimization results through simulation experiments.

Firstly, according to the precision requirements given by the precision allocation results, the simulation data of motion axis errors and assembly errors between axes are generated. According to ISO 230-1, uncertainty of measurement should be taken into account when specifying tolerances and when evaluation conformance with specified tolerances. So the simulation data are generated randomly in the workspace of the whole machine. In the stroke of each axis, five points are equidistant, and six position and posture errors of each point are randomly generated according to the normal distribution probability within the allowable range of the optimized precision

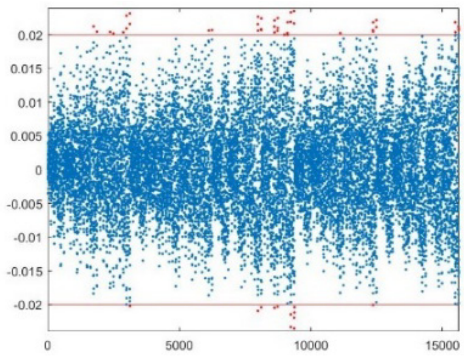


Fig. 10. Scatter diagram of tool-workpiece comprehensive error δx .

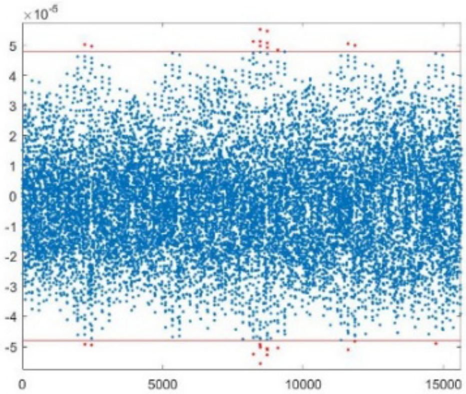


Fig. 11. Scatter diagram of tool-workpiece comprehensive error εx .

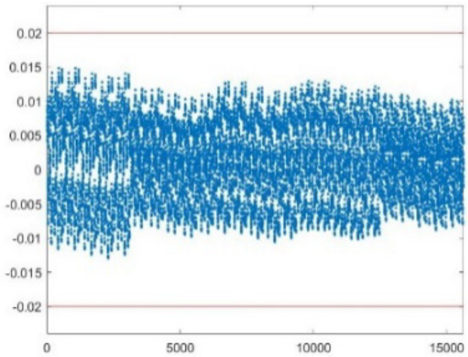


Fig. 12. Scatter diagram of tool-workpiece comprehensive error δy .

Table 8. Precision reliability of 6 basic tool-workpiece comprehensive errors.

Error	Precision requirement	The number meeting the requirement	The number out of the requirement	Precision reliability
δx	0.02	15568	57	99.64 %
δy	0.02	15625	0	100.00 %
δz	0.02	15625	0	100.00 %
εx	4.8e-5	15599	26	99.83 %
εy	4.8e-5	15585	40	98.74 %
εz	4.8e-5	15528	97	99.38 %

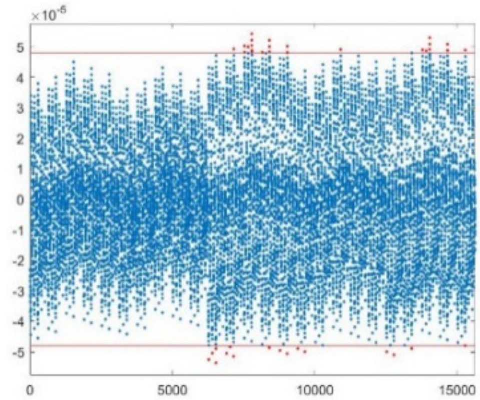


Fig. 13. Scatter diagram of tool-workpiece comprehensive error εy .

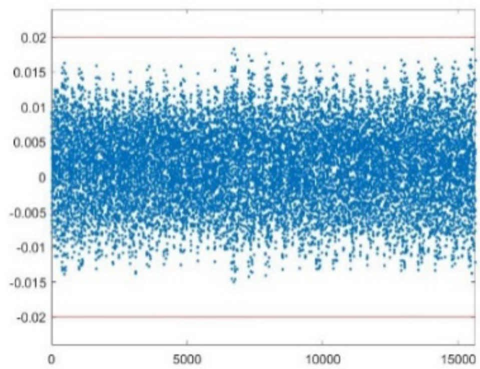


Fig. 14. Scatter diagram of tool-workpiece comprehensive error δz .

distribution. A total of 15625 groups of error values are obtained by combining the error values of six axes, and each group represents the geometric errors of a point in the 6-dimensional machining space composed of six axes. Six position and posture errors between two axes are generated randomly according to normal distribution probability within the allowable range of precision distribution. Each group of errors and the assembly errors between axes are substituted into the comprehensive error model of CNC-LGHM, and the tool-workpiece comprehensive errors of each point are calculated, including six basic error terms $\delta x, \delta y, \delta z, \varepsilon x, \varepsilon y, \varepsilon z$.

Figs. 10-15 show the simulation results. Red points indicate that the error value exceeds the precision requirements. Blue points represent that the error value meets the precision requirements.

The error value exceeds the precision requirement only at a few points. The percentage of the total number of points that meet the precision requirements of the six basic f tool-workpiece comprehensive errors is counted, and it is approximately regarded as the precision reliability, as shown in Table 8. It can be seen that in 15625 points of hobbing machine processing space, the precision reliability of the six tool-workpiece comprehensive errors exceeds 99 %, with an average of 99.60 %.

Table 9. Results of 20 simulation experiments.

	δx	δy	δz
1	100.00 %	100.00 %	100.00 %
2	99.99 %	100.00 %	100.00 %
3	100.00 %	100.00 %	100.00 %
4	99.64 %	100.00 %	100.00 %
5	100.00 %	100.00 %	99.91 %
6	100.00 %	100.00 %	100.00 %
7	100.00 %	99.99 %	99.93 %
8	100.00 %	100.00 %	100.00 %
9	100.00 %	100.00 %	100.00 %
10	100.00 %	100.00 %	100.00 %
11	100.00 %	100.00 %	100.00 %
12	100.00 %	100.00 %	100.00 %
13	99.97 %	100.00 %	99.67 %
14	100.00 %	100.00 %	100.00 %
15	99.69 %	100.00 %	100.00 %
16	100.00 %	100.00 %	100.00 %
17	99.99 %	100.00 %	100.00 %
18	100.00 %	100.00 %	100.00 %
19	99.96 %	100.00 %	100.00 %
20	99.99 %	100.00 %	100.00 %
Avg	99.96 %	100.00 %	99.98 %
Min	99.64 %	99.99 %	99.67 %
Max	100.00 %	100.00 %	100.00 %
	εx	εy	εz
1	99.98 %	100.00 %	100.00 %
2	100.00 %	99.33 %	99.99 %
3	99.94 %	99.89 %	100.00 %
4	99.83 %	99.74 %	99.38 %
5	99.95 %	100.00 %	100.00 %
6	100.00 %	100.00 %	100.00 %
7	99.64 %	100.00 %	99.59 %
8	100.00 %	99.97 %	99.14 %
9	100.00 %	100.00 %	99.35 %
10	100.00 %	100.00 %	100.00 %
11	99.97 %	99.96 %	99.88 %
12	100.00 %	100.00 %	99.97 %
13	99.98 %	99.53 %	99.96 %
14	99.90 %	100.00 %	99.99 %
15	100.00 %	100.00 %	100.00 %
16	99.96 %	100.00 %	99.87 %
17	100.00 %	100.00 %	100.00 %
18	99.92 %	99.96 %	99.99 %
19	100.00 %	100.00 %	99.73 %
20	99.96 %	99.92 %	99.73 %
Avg	99.95 %	99.92 %	99.83 %
Min	99.64 %	99.33 %	99.14 %
Max	100.00 %	100.00 %	100.00 %

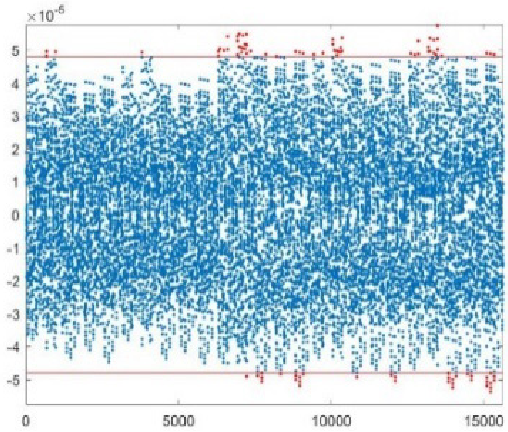


Fig. 15. Scatter diagram of tool-workpiece comprehensive error εz .

Repeat the above simulation experiment for 20 times, and calculate the precision reliability of 6 basic gear hobbing machine comprehensive errors. As shown in Table 9, the precision reliability of six basic error items of CNC-LGHM is above 99 % by using the optimized precision allocation results, and the expected goal is achieved.

6. Conclusions

Precision allocation is an important work in machine tool design, which is of great significance to ensure the machining precision, improve product quality and reduce manufacturing cost. The precision allocation method mainly includes the basic principle method, the method with the optimization objective of minimizing manufacturing cost, and the method with the optimization objective of minimizing quality loss. The precision allocation of motion parts is mostly based on the sensitivity (or contribution) of their errors to the manufacturing cost.

According to the modern mechanical precision design theory, the overall precision of machine tool transmission chain is a random variable with normal distribution. Taking the normal distribution random variables properties into consideration, the probability is very low with variable reaching the maximum. Obviously, it is too harsh for the overall precision to fully meet the requirements of design precision, and the precision allocation scheme based on this constraint is not economical and reasonable. In the design of precision allocation, it does not seek meeting 100 % design requirements. Based on the characteristics of normal distribution of the comprehensive machining precision, it takes the probability of the machining precision meeting the design requirements to reach a certain threshold as the constraint condition for precision allocation, which can ensure the machining precision of the CNC-LGHM and effectively reduce the manufacturing cost.

This paper proposes a precision allocation method based on precision reliability. Based on the characteristic of error normal distribution, the precision reliability index is defined, and a precision allocation optimization model aiming at minimizing manufacturing cost is established. The influence of the size of large parts on the manufacturing cost is considered in the manufacturing cost calculation. Taking the CNC-LGHM as an example, the model solution and simulation results show that the precision allocation method considering precision reliability reduces the manufacturing cost of CNC-LGHM effectively, and the precision reliability reaches more than 99 %.

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Nomenclature

S_x, S_y, S_z : Displacements of X, Y and Z-axis

$\theta_A, \theta_C, \theta_M$: Rotation angles of A, C and M-axis

$\left.\begin{matrix} {}_X\delta_x(S_x), {}_X\delta_y(S_x), {}_X\delta_z(S_x) \\ {}_X\varepsilon_x(S_x), {}_X\varepsilon_y(S_x), {}_X\varepsilon_z(S_x) \end{matrix}\right\}$: Errors of X-axis at S_x

$\left.\begin{matrix} {}_Y\delta_x(S_y), {}_Y\delta_y(S_y), {}_Y\delta_z(S_y) \\ {}_Y\varepsilon_x(S_y), {}_Y\varepsilon_y(S_y), {}_Y\varepsilon_z(S_y) \end{matrix}\right\}$: Errors of Y-axis at S_y

$\left.\begin{matrix} {}_Z\delta_x(S_z), {}_Z\delta_y(S_z), {}_Z\delta_z(S_z) \\ {}_Z\varepsilon_x(S_z), {}_Z\varepsilon_y(S_z), {}_Z\varepsilon_z(S_z) \end{matrix}\right\}$: Errors of Z-axis at S_z

$\left.\begin{matrix} {}_A\delta_x(\theta_A), {}_A\delta_y(\theta_A), {}_A\delta_z(\theta_A) \\ {}_A\varepsilon_x(\theta_A), {}_A\varepsilon_y(\theta_A), {}_A\varepsilon_z(\theta_A) \end{matrix}\right\}$: Errors of A-axis at θ_A

$\left.\begin{matrix} {}_C\delta_x(\theta_C), {}_C\delta_y(\theta_C), {}_C\delta_z(\theta_C) \\ {}_C\varepsilon_x(\theta_C), {}_C\varepsilon_y(\theta_C), {}_C\varepsilon_z(\theta_C) \end{matrix}\right\}$: Errors of C-axis at θ_C

$\left.\begin{matrix} {}_M\delta_x(\theta_M), {}_M\delta_y(\theta_M), {}_M\delta_z(\theta_M) \\ {}_M\varepsilon_x(\theta_M), {}_M\varepsilon_y(\theta_M), {}_M\varepsilon_z(\theta_M) \end{matrix}\right\}$: Errors of M-axis at θ_M

${}^C_X\delta_x, {}^C_X\delta_y, {}^C_X\delta_z, {}^C_X\varepsilon_x, {}^C_X\varepsilon_y, {}^C_X\varepsilon_z$: Errors between X-C axes

${}^X_Z\delta_x, {}^X_Z\delta_y, {}^X_Z\delta_z, {}^X_Z\varepsilon_x, {}^X_Z\varepsilon_y, {}^X_Z\varepsilon_z$: Errors between Z-X axes

${}^Z_A\delta_x, {}^Z_A\delta_y, {}^Z_A\delta_z, {}^Z_A\varepsilon_x, {}^Z_A\varepsilon_y, {}^Z_A\varepsilon_z$: Errors between A-Z axes

${}^A_Y\delta_x, {}^A_Y\delta_y, {}^A_Y\delta_z, {}^A_Y\varepsilon_x, {}^A_Y\varepsilon_y, {}^A_Y\varepsilon_z$: Errors between Y-A axes

${}^Y_M\delta_x, {}^Y_M\delta_y, {}^Y_M\delta_z, {}^Y_M\varepsilon_x, {}^Y_M\varepsilon_y, {}^Y_M\varepsilon_z$: Errors between M-Y axes

$\left.\begin{matrix} M_{1,2}, M_{2,3}, M_{3,4} \\ M_{4,5}, M_{5,6}, M_{6,7} \end{matrix}\right\}$: Motion transformation matrices

$\left.\begin{matrix} E^m_{1,2}, E^m_{2,3}, E^m_{3,4} \\ E^m_{4,5}, E^m_{5,6}, E^m_{6,7} \end{matrix}\right\}$: Motion axis error transformation matrices

$\left.\begin{matrix} E^P_{1,2}, E^P_{2,3}, E^P_{3,4} \\ E^P_{4,5}, E^P_{5,6} \end{matrix}\right\}$: Inter axis error transformation matrices

$M_{1,7}$: Ideal motion transformation matrix

$M^e_{1,7}$: Motion transformation matrix with errors

E : Comprehensive error matrix of machine

$\delta x, \delta y, \delta z, \varepsilon x, \varepsilon y, \varepsilon z$: Comprehensive errors of machine

$I_{\delta x}, I_{\delta y}, I_{\delta z}, I_{\varepsilon x}, I_{\varepsilon y}, I_{\varepsilon z}$: Precisions design requirements

$F_X, F_Y, F_Z, F_A, F_C, F_M, FA_{ij}$: Fuzzy manufacturing costs

a, b, c, d, m_{ij} : Precision-cost function coefficients

$F(Error)$: Fuzzy cost optimization objective

Motion axis errors : Errors caused by manufacturing and servo positioning control of motion axes

Assembly errors : Errors caused by motion axes assembly

Comprehensive errors : Relative pose errors between tool and workpiece caused by motion axis errors and assembly errors

Precision allocation : Allocate the precision of each moving part according to the performance design requirements of the machine tool

Precision-cost function : Relationship function between machine tool precision and manufacturing cost

Precision reliability : Probability of machine tool machining precision meeting design requirements

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Zongyan Hu is a doctoral candidate of the College of Mechanical Engineering, Chongqing University, Chongqing, China. His research interests include modeling, prediction, traceability and compensation of comprehensive error of gear hobbing machine.

Shilong Wang is a Professor of the College of Mechanical Engineering, Chongqing University, Chongqing, China. His research interests include precision control of gear machine tools, advanced manufacturing technology and structure design.

Chi Ma is an Associate Professor of the College of Mechanical Engineering, Chongqing University, Chongqing, China. His research interests include the precision and the thermal deformation of gear machine tools.