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Inhibitory control and visuospatial working memory contribute to 5-year-old children's use of quantitative inversion☆

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ABSTRACT

This study examined the hypothesis that general cognitive resources moderated 5-year-old children's performance differences between the Concrete Identical and the Pure Quantity conditions on inversion problems (a + b – b) but not on standard problems (a + b – c). Study 1 ($N = 104$) showed that children who experienced higher visuospatial working memory burden performed significantly poorer in solving the inversion problems in the Pure Quantity condition than in the Concrete Identical condition, whereas those who experienced lower working memory burden showed no such difference. Study 2 ($N = 194$) demonstrated that children with lower levels of inhibitory control solved significantly fewer inversion problems in the Pure Quantity condition than in the Concrete Identical condition, whereas no such difference was found in children with higher levels of inhibitory control. These findings suggest that inhibitory control and visuospatial working memory may support children's use of quantitative inversion.

1. Introduction

Understanding quantitative and numerical relations is a key to children's success in mathematics learning (Bryant, 1995; Ching et al., 2020; Ching & Kong, 2022b; Nunes & Bryant, 1996; Nunes et al., 2007; Piaget, 1952; Thompson, 1993; Vergnaud, 2009). One such relation is the inverse relation between addition and subtraction (Bisanz et al., 2009; Bryant et al., 1999; Canobi et al., 2003; Eaves et al., 2019; Gilmore & Papadatou-Pastou, 2009; Greer, 2012; Nunes et al., 2015; Robinson, 2017; Robinson & Dubé, 2009, 2013; Torberyns et al., 2016; Verschaffel et al., 2012). The inversion principle refers to the fact that a + b − b must equal to a. Piaget (1952) argued that people cannot be said to understand addition and subtraction if they fail to coordinate these operations (Baroody & Lai, 2007; Ching & Nunes, 2017a; 2017b; Gilmore & Bryant, 2008; Nunes et al., 2007; 2012a; Nunes & Bryant, 2015; Stern, 1992; Vergnaud, 1997).

Knowledge of inversion enables individuals to use a shortcut procedure to solve certain arithmetic problems more efficiently (Bisanz et al., 2009; Greer, 2012; Siegler & Stern, 1998). For example, when people are presented with a three-term arithmetic problem "23 + 57–57", those who do not recognize and use inversion may add and subtract the numbers from left to right. By contrast, those who know the inversion principle should realize that successive computation is not necessary for solving this problem. If children use a shortcut strategy based on their understanding of inversion, they will perform better on inversion problems than standard problems. Most research on inversion examines whether and when children understand and use the principle (e.g., Baroody & Lai, 2007; Bryant et al., 1999; Klein & Bisanz, 2000; Rasmussen et al., 2003; Sherman & Bisanz, 2007), less is known about what factors may contribute to individual differences in inversion use. The current study aimed to examine whether general cognitive skills (inhibitory control and visuospatial working memory) facilitate the use and, possibly, the development of inversion in 5-year-old children.

1.1. Assessing qualitative and quantitative inversion

One way to assess whether children use the inversion principle to solve problems is to examine how they respond to (a) *inversion* problems ($a + b - b$) and (b) *standard* or *control* problems ($a + b - c$). If they understand the mathematical structure of inversion problems, they will solve these problems more accurately and quickly than standard problems in which the inversion logic does not apply. In a meta-analysis, Gilmore and Papadatou-Pastou (2009) found that children on average scored higher on inversion than standard problems and the difference was not moderated by age (5–13 years). These findings suggest that children can recognize and use the inversion principle for problem

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solving and the inversion effect does not vary with age or school year. Inversion and standard problems can be presented using digits, word problems, pictures, and concrete materials (Bryant et al., 1999; Canobi et al., 2003; Canobi, 2005; Ching & Wu, 2019; Gilmore, 2006; Gilmore & Bryant, 2006). Gilmore and Papadatou-Pastou (2009) also found that different ways of presentation influenced the size of the inversion effect, which was greater when the problems were shown with pictures describing actions and smaller when the problems were presented with numerical symbols. The effect of problem type suggests that providing more context may help children identify and use conceptual knowledge to solve mathematical problems.

The manner inversion problems are presented does not only influence children's accuracy, but it also affects how they solve the problems. For example, in a study (Bryant et al., 1999), five- and six-year-old children were presented with inversion and standard problems in different nonsymbolic contexts, namely the "Concrete Identical" and "Concrete Nonidentical" conditions (Figs. 1 and 2). For the inversion problems (a + b − b) in the Concrete Identical condition, an experimenter added "b" blocks to an array of blocks, and removed the same set of "b" blocks from it in front of a child. Then the child was asked how many blocks were left. If children recognize that the effects of addition and subtraction cancel out each other (i.e., using a *quantitative form of inversion*), they will realize that the answer must be "a" without counting or calculation. Alternatively, some children may base their answers on a *qualitative form of inversion*, i.e., the physical status quo is restored if the same matter is added and then removed. Because the blocks being added and subtracted are the same in the Concrete Identical condition, children can solve the problem correctly with qualitative inversion without considering the quantities involved in the transformation. As for the inversion problems in the Concrete Nonidentical condition, the experimenter added "b" blocks to one side and then removed a different set of "b" blocks from the other side of the initial array. Presenting the problem in this way affords a quantitative solution only – children must consider the *number* of blocks being added and subtracted when solving this problem. Bryant et al. showed that 5- and 6-year-old children performed significantly better on the inversion problems in the Concrete Identical condition than those in the Concrete Nonidentical condition.

In a subsequent study, Rasmussen et al. (2003) argued that the children in Bryant et al. study might not solve the problems based on inversion knowledge, but length differences. In Bryant et al.'s study, all the blocks had the same size, so the lengths of the original and final arrays were the same in all inversion trials. It was possible that the children solved the problems successfully based on length cues (i.e., same length = same quantity). Thus, Rasmussen et al. (2003) incorporated an additional experimental condition, namely the "Pure Quantity" condition (Fig. 3), in which children could not solve the problems by using either length cues or qualitative inversion. The design of this condition was similar to the Concrete Nonidentical condition – blocks were added to one end of an array and different blocks were taken away from the other end, but because the blocks were different in length, adding and subtracting the same number of blocks would result in a final row that is either longer or shorter than the original one. Therefore, children will make mistakes if they do not consider the changes in quantity (i.e., using quantitative inversion). Rasmussen et al. (2003) found that preschool and Grade 1 children performed similarly across the three experimental conditions, which suggest that children within this age range, on average, can solve inversion problems in a quantitative manner.

1.2. Factors that facilitate the use of quantitative inversion

Knowledge of inversion may stem from quantitative skills, such as counting and simple calculation (Bisanz et al., 2009). However, evidence showed that counting skills were not associated with the performance on inversion problems among 3-year-old children (Sherman & Bisanz, 2007). Similarly, another study (Rasmussen et al., 2003) indicated that 4-year-old children's performance on inversion problems did not relate to their abilities to count and calculate. Gilmore and Bryant (2008) also found that calculation skills were not necessarily associated with inversion knowledge among 8- to 9-year-old children. These studies suggest that quantitative skills may not be the basis for the development of inversion understanding in children. Alternatively, Bisanz et al. (2009) proposes a "representational account of inversion" (p. 21). This perspective suggests that non-numerical cognitive processes may facilitate children to use quantitative inversion to solve problems. Based on this theory, the current research explored whether

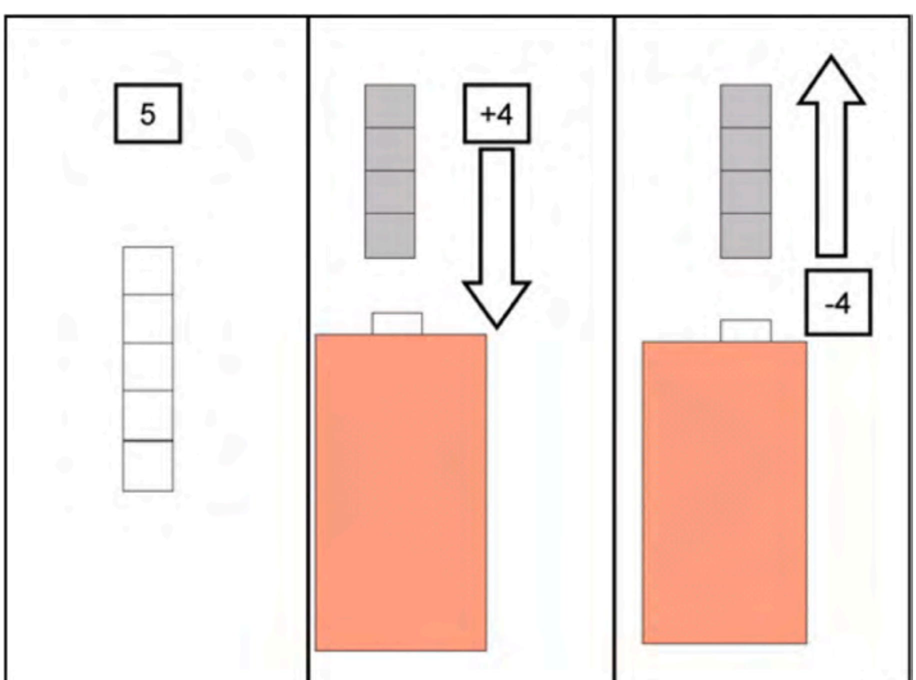

Fig. 1. A graphical depiction of inversion problems under the "Concrete Identical" condition.

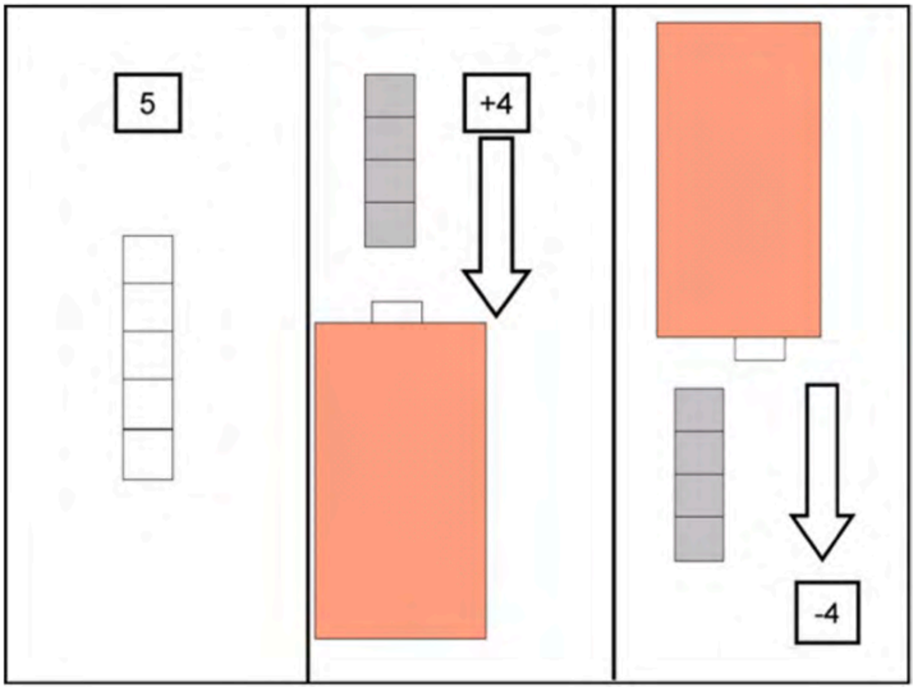

Fig. 2. A graphical depiction of inversion problems under the "Concrete Nonidentical" condition.

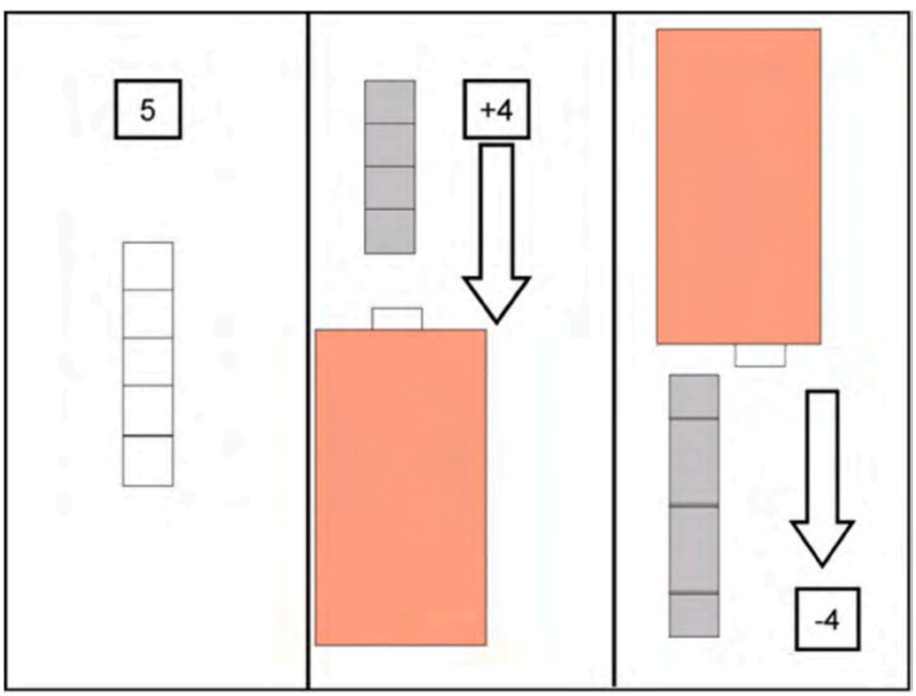

Fig. 3. A graphical depiction of inversion problems under the "Pure Quantity" condition.

inhibitory control and visuospatial working memory may contribute to children's use of quantitative inversion to solve arithmetic problems.

1.2.1. *Contributions of inhibitory control*

To solve a mathematical task successfully, individuals must ascertain what information is relevant to the task and what is irrelevant (Macleod, 2007; Miyake et al., 2000; Van Dooren & Inglis, 2015). The ability to suppress competing responses refers to *inhibitory control*, which was defined as "the stopping or overriding of a mental process, in whole or in part, with or without instruction" (Macleod, 2007, p.5). Some correlational research has shown that inhibitory control is associated with individuals' ability to solve inversion problems. Gilmore et al. (2015) demonstrated that a stronger ability to recognize conceptual relations between arithmetic problems was related to better performance on a numerical Stroop task among young adults. Among children from Grades 6 to 8, Dubé and Robinson (2010) found that those who used more inversion shortcut to solve arithmetic problems performed better on a digit ordering task that demanded inhibitory and attentional abilities (participants had to recall lists of digits in an ascending numerical order). Similarly, younger children (Grades 3 to 5) who used more inversion and associativity shortcut performed better on a Go/no-Go task that required response inhibition than those who used neither (Robinson & Dubé, 2013).

To solve inversion problems in the Pure Quantity condition (Rasmussen et al., 2003), children should ignore the length differences between the initial and final arrays of blocks and focus on the changes in quantity. Thus, children who have a weaker ability to suppress goal-irrelevant information (i.e., the length differences) may perform significantly worse for inversion problems in the Pure Quantity condition than those in the Concrete Identical condition in which length differences are not present. By contrast, children with stronger inhibitory control may perform similarly for inversion problems in the Pure Quantity condition and the Concrete Identical condition because they are less vulnerable to the distraction by length cues. Because the inversion principle does not apply to standard problems, there should not be significant differences in individuals' performance between the two conditions for standard problems. Taken together, I hypothesized that individual differences in inhibitory control would moderate the performance differences between the Concrete Identical condition and the Pure Quantity condition on inversion problems but not on standard problems.

1.2.2. Contributions of visuospatial working memory

Inhibitory control is often considered as "central executive" in the working memory literature. Based on the classic working memory model (Baddeley & Hitch, 1974), there are two storage systems distinct from central executive, namely the phonological loop and the visuospatial sketchpad. The phonological loop serves to store verbal information, whereas the visuospatial sketchpad is responsible for holding visuospatial information. In the present research, I proposed that *visuospatial sketchpad/visuospatial working memory* might be related to young children's performance in solving inversion problems in the Pure Quantity condition.

Huttenlocher et al. (1994) contended that young children may construct a mental model to solve mathematical problems. For instance, to solve a three-term arithmetic problem (a + b – b), children would represent (a) the quantity of the initial set in visuospatial working memory as separate units (e.g., 3 entities that map one-to-one with external objects to represent a set size of three rather than one single unit for "3"), (b) the quantitative transformation applied to that set, and (c) the quantity of the set after the transformation (Klein & Bisanz, 2000). In support of this hypothesis, Rasmussen and Bisanz (2005) showed that Corsi span (a standard measure of visuospatial working memory) was a better predictor for preschool children's success in solving nonverbal arithmetic problems, compared with Digit span (a measure of the phonological loop). However, the association between the scores on Corsi span and nonverbal problems was not significant among Grade 1 children. These findings suggest that younger children tend to use a mental model based on visuospatial working memory to solve arithmetic problems, whereas school-aged children may rely on other strategies, such as phonological-based strategies, as they gain more experience with formal, symbol-based arithmetic in school. Similarly, Rasmussen et al. (2003) also demonstrated that Corsi span was positively linked to better performance in inversion problems among preschool children (4 years of age), but the association was not significant among Grade1 children (6 years of age).

The problem-solving approach of 5-year-old children may be more similar to that of 4-year-olds than 6-year-olds because most of them do not have formal schooling experience. Thus, I expected that 5-year-old children would use a visually-based mental model to solve inversion problems. Further, I proposed that visuospatial working memory would moderate the performance differences between the Concrete Identical condition and the Pure Quantity condition on inversion problems but not on standard problems. There is a close association between working memory and inhibitory control. Based on the load theory of selective attention (Lavie et al., 2004), working memory is important for maintaining stimulus priorities. It has been argued that working memory enables individuals to override automatic stimulus selection and response execution, thereby engaging in goal-directed cognition and behaviour (Friedman & Miyake, 2004; Kane & Engle, 2002; Kane et al., 2007).

In one study (De Fockert, Rees, Frith, & Lavie, 2001), participants were asked to perform a selective attention task that required them to ignore distractor faces while holding in working memory a list of digits that were visually presented in the same order (low memory load) or a different order (high memory load). The researchers found that participants were better able to block out the interfering effects by distractors when concurrent memory load was low, whereas when memory load was high, there was more extensive processing of the distractors resulting in more interference. Other studies also showed that the efficiency of visual search and selective attention, as well as the suppression of task-irrelevant responses, are associated with individual differences in working memory capacity (Burnham et al., 2014; Kane et al., 2001; Lavie, 2005; Roberts et al., 1994; Unsworth et al., 2004). These findings suggest that the availability of working memory is important for directing attention to relevant rather than irrelevant stimuli, and thus minimizing the intrusion of irrelevant distractors.

Based on the assumptions that 5-year-old children use a visually-based mental model to solve inversion problems as well as the close association between working memory and inhibitory control, my second hypothesis was that visuospatial working memory would moderate the performance differences between the Concrete Identical condition and the Pure Quantity condition on inversion problems but not on standard problems. I proposed that when concurrent memory load was low, children would be better able to suppress the interference by the length cues when solving the inversion problems in the Pure Quantity condition. Thus, there should not be significant performance difference for the inversion problems between the Concrete Identical condition and the Pure Quantity condition. By contrast, high memory load would result in a lower ability to withstand the interfering effects by the length cues, thereby leading to poorer performance for inversion problems in the Pure Quantity condition. Thus, these children would perform significantly poorer for the inversion problems in the Pure Quantity condition than the same problems in the Concrete Identical condition. Because the inversion principle does not apply to standard problems, there should not be significant differences in children's performance between the two conditions for standard problems.

1.3. Overview of the present studies

The current research involved two studies that examined the role of executive functions in solving inversion problems among 5-year-old children. Following Rasmussen et al. (2003) study, inversion (a + b – b) and standard problems (a + b – c) were presented in two conditions: Concrete Identical and Pure Quantity conditions in both studies (Table 1). In Study 1, children's visuospatial working memory was experimentally manipulated. This study aimed to investigate the causal effects of visuospatial working memory on children's performance in solving inversion and standard problems in different conditions.

Table 1
A summary of problem types in different conditions.

Problem Types	Conditions	Characteristics
Inversion a + b – b	Concrete Identical	Both the quantity and length of the initial and final arrays are the same.
	Pure Quantity	The final array has the same quantity, but not the same length, with the initial array.
Standard a + b – c	Concrete Identical	Both the quantity and length of the initial and final arrays are different.
	Pure Quantity	The final array has the same length, but not the same quantity, with the initial array.

Previous research (Rasmussen et al., 2003) showed that individual differences in visuospatial working memory were positively associated with better performance in inversion problems among 4-year-old children, but the results do not imply causality. Thus, the first study aimed to supplement their findings by elucidating the causal roles of visuospatial working memory. I hypothesized that cognitive burden on visuospatial working memory would moderate the performance differences between the Concrete Identical condition and the Pure Quantity condition on inversion problems but not on standard problems. Specifically, I predicted that children who experienced higher memory load would perform significantly better in the Concrete Identical condition than the Pure Quantity condition for inversion problems, but not for standard problems. By contrast, for children who experienced lower memory load, the performance differences between the two conditions would be negligible for both inversion and standard problems.

As I assumed that visuospatial working memory would affect performance in solving inversion problems via its influence on children's ability to suppress goal-irrelevant information, the second study aimed to examine the role of inhibitory control directly. In Study 2, individual differences in inhibitory control were measured. I employed a non-experimental approach to test the moderating effects of inhibitory control because this variable is difficult to manipulate. The hypothesis in Study 2 was that individual differences in inhibitory control would moderate the performance differences between the Concrete Identical condition and the Pure Quantity condition on inversion problems but not on standard problems. In particular, I predicted that children with lower inhibitory control performed significantly better in the Concrete Identical condition than the Pure Quantity condition for inversion problems, but not for standard problems. By contrast, for children with higher inhibitory control, the performance differences between the two conditions would not be significant for both inversion and standard problems.

2. Study 1: Method

The first study aimed to examine whether experimentally burdening visuospatial working memory resources would adversely affect children's use of quantitative inversion. Specifically, they were asked to do a secondary task that demanded visuospatial working memory (Bethell-Fox & Shepard, 1988; De Neys, 2006; Gillard et al., 2009; Miyake et al., 2001; Verschueren et al., 2004) while solving the three-term arithmetic problems simultaneously.

2.1. Participants

One hundred and four children (51 boys, 53 girls) studying in three kindergartens in cities located at the Pearl River Delta by the South China Sea participated in this study. The mean age of the children was 64.24 months (SD = 2.25 months, ranging from 62.15 to 66.47 months). All children had normal intelligence without any learning difficulties based on parental reports. The highest educational levels attained by the mothers of the children in the sample were as follows: No schooling/pre-primary school level – 8.4%, primary school graduates – 25.8%, secondary school graduates – 46.3%, and university graduates 19.5%. The relative distribution of educational levels was comparable to that of the overall Hong Kong population according to Hong Kong Population Census (2016) in which the majority of the population was secondary school graduates, whereas a small proportion received no schooling or had pre-primary educational level. All children studied in the second year of kindergartens and had not learned addition and subtraction formally.

2.2. Materials

2.2.1. Three-term problems

Following previous studies (Bryant et al., 1999; Rasmussen et al., 2003), children were presented with a set of connectable blocks assembled in a row and they were asked to solve three-term arithmetic problems. Following Bryant et al. (1999) procedure, children were prevented from counting as I used a cloth to cover the initial row of block so that only the ends were exposed. First, each child was told that there was a particular number of blocks in the initial array. After that, the experimenter added several blocks and told the child how many blocks he was adding to the original set. Then, he took away some blocks and told the child how many blocks he was removing from the set. Finally, the child was asked how many blocks were left in the final array. During the transformations, the cloth was shifted (but not removed) to keep it in the center of the row so that the middle portion of the row of blocks was not visible. The children were not allowed to move the blocks or the cloth.

The children solved the problems in two conditions, namely the Concrete Identical condition and the Pure Quantity condition. In each condition, there were six inversion problems ($a + b - b$) and six standard problems ($a + b - c$) (Table 1). As for the inversion problems in the Concrete Identical condition, both the quantity and length of the initial and final arrays are the same, and the blocks were added and removed from the same end of the array. As for the inversion problems in the Pure Quantity condition, the total length of the blocks being added always differed (at least 6 cm) from the total length of the blocks being subtracted. Children observed "b" blocks being added to one end of the array and then "b" different blocks being removed from the other end of the array. Thus, the final array had the same quantity, but not the same length, with the initial array. A pilot test was conducted with 10 preschool children prior to the study, which showed that children at this age could perceive the length differences.

For the standard problems in the Concrete Identical condition, both the quantity and length of the initial and final arrays are different. By contrast, for those standard problems in the Pure Quantity condition, the total length of the blocks being added was the same as the total length of the blocks being subtracted. There was no length difference between the initial array and the final array even different numbers of blocks were added and taken away. Operands ranged from 1 to 5 with the constraints that $a + b \leq 9$ and correct answers ranged from 1 to 6 so that the problems would be difficult for kindergartners to solve with successive addition and subtraction. For all problems, $a \neq b$ and $a \neq c$. For half the standard problems $b < c$ and for the other half $c < b$. The order of the problems and conditions was counterbalanced. Each child was given 10 s to answer each problem. Answers were marked as either correct (1 score) or not correct (0 score). No feedback was given.

2.3. Procedure

This study was approved by a research ethics committee of the university. Parental consent and children's verbal assent were obtained before the experiment started. Each child participated individually with an experimenter in a quiet location, which was separated from other children in the kindergartens. Children first completed the Dot Memory Task (Miyake et al., 2001), which was a three by three matrix comprising three to four dots. The matrix was presented briefly to the children, and they were asked to memorize and reproduce the dot patterns afterwards. The children were randomly assigned into either the "burden condition" ($n = 52$) or the "control condition" ($n = 52$). In the burden condition, the matrix contained a complex four-dot pattern (a "two-piece" or "three-piece" pattern according to Bethell-Fox & Shepard, 1988; Verschueren et al., 2004), which impaired individuals' working memory significantly (Miyake et al., 2001). In the control condition, the patterns included three dots in one line (a "one-piece" pattern according to De Neys, 2006), which only loaded individuals' working memory minimally.

The experimenter first introduced the dot memory task and conducted a practice trial with the children. It was emphasized that a correct dot pattern reproduction was important so that the participants would actively attend to the matrix. Following previous research (e.g., Gillard et al., 2009), the children were presented with the dot pattern for 1 s on a computer screen. Then, an experimenter presented the three-term problems with blocks in front of the child and asked the child to solve the problems. Each child was given 10 s to answer each three-term problem. After the children had solved the three-term problem, they were presented with an empty matrix on a paper and asked to reproduce the dot pattern. Then, the next dot pattern was shown and the same procedure repeated. The order of the problems and conditions was counterbalanced. No feedback was given.

3. Study 1: Results

3.1. Preliminary analyses

Children in the low burden condition ($M = 23.14$, $SD = 1.22$) recalled the dot patterns more correctly compared with the children in the high burden condition ($M = 22.35$, $SD = 1.58$), $t(102) = 2.84$, $p = .005$. Thus, the children in the high burden condition experienced greater cognitive difficulty compared with the children in the low burden condition. Because the high levels of accuracy in reproducing the dot patterns in both conditions (>90% in both conditions) suggest that the children on average completed the tasks attentively, subsequent analyses were conducted based on the answers from all three-term problems regardless of whether children recalled the dot pattern correctly. Independent *t*-tests were performed separately for each variable to assess whether there were gender differences in the accuracy rates for inversion and standard problems in each condition. Bivariate correlation was employed to test whether these variables were associated with age. A one-way analysis of variance (ANOVA) was used to evaluate whether each variable would differ by school. No significant age, gender, or school differences were observed in any tasks (all $ps > .05$).

Previous research (e.g., Bryant et al., 1999; Rasmussen et al., 2003) showed that if children used a shortcut strategy based on their understanding of inversion, they would perform significantly better on inversion problems than standard problems. Consistent with these findings, the present study showed that children performed significantly better on inversion problems ($M = 3.19$, $SD = 1.78$) than standard problems ($M = 1.20$, $SD = 0.96$), $F(1, 102) = 74.66$, $p < .001$, partial $\eta^2 = 0.423$. A majority of children (Concrete Identical condition: $n = 78$, 75%; Pure Quantity condition: $n = 66$, 63.5%) answered more inversion problems correctly than the standard problems – both conditions showed a clear majority.

In response to any three-term problems, some children may simply repeat the first term and say that the first term is the answer (i.e., s/he always answers “a” in both a + b – b and a + b –c problems). If the children's responses were based on this bias, the number of *a* responses should be similarly high for both inversion problems and standard problems. However, results showed that the number of *a* responses was greater for the inversion ($M = 3.16$, $SD = 1.80$) than for the standard problems ($M = 1.12$, $SD = 1.03$) in the Concrete Identical condition: $t(103) = 10.17$, $p < .001$. The number of *a* responses was also greater for the inversion ($M = 2.69$, $SD = 1.95$) than for the standard problems ($M = 1.42$, $SD = 1.33$) in the Pure Quantity condition: $t(103) = 5.48$, $p < .001$. Thus, the children did not show the *a* term bias in Study 1.

3.2. Main analyses – hypothesis testing

The hypothesis of Study 1 was tested with a 2 (burden: control vs. burden) x 2 (problem type: inversion vs. standard) x 2 (condition: Concrete Identical vs. Pure Quantity) analysis of variance with repeated measures on the latter two variables. On average, children performed significantly better in the Concrete Identical condition ($M = 2.14$, $SD = 1.08$) than in the Pure Quantity condition ($M = 1.96$, $SD = 1.18$), $F(1, 102) = 8.91$, $p = .004$, partial $\eta^2 = 0.08$. The effect of condition interacted with problem type, $F(1, 102) = 21.64$, $p < .001$, partial $\eta^2 = 0.175$, and with burden, $F(1, 102) = 15.24$, $p < .001$, partial $\eta^2 = 0.130$. Both of these interactions were qualified by the three-way interaction, $F(1, 102) = 7.62$, $p = .007$, partial $\eta^2 = 0.069$. The overall results are graphically illustrated in Fig. 4. Consistent with my predictions, children who experienced higher memory load performed significantly better in the Concrete Identical condition than the Pure Quantity condition for inversion problems, $t(51) = 6.36$, $p < .001$, but not for standard problems ($p = .888$). For children who experienced lower memory load, the performance differences between the two conditions were negligible for both inversion and standard problems ($ps > .162$). In support of the hypothesis, cognitive burden on visuospatial working memory moderated the performance differences between the Concrete Identical and the Pure Quantity on inversion problems but not on standard problems.

4. Study 2: Method

4.1. Participants

One hundred and ninety-four children (93 boys, 101 girls) studying in six kindergartens in cities located at the Pearl River Delta by the South China Sea participated in this study. The mean age of the children was 63.61 months ($SD = 2.41$ months, ranging from 62.21 to 66.52 months). All of these children did not participate in Study 1. All children had normal intelligence without any learning difficulties based on parental reports. The highest educational levels attained by the mothers of the children in the sample were as follows: No schooling/pre-primary school level – 7.2%, primary school graduates – 26.7%, secondary school graduates – 48.2%, and university graduates 17.9%. The relative distribution of educational levels was comparable to that of the overall Hong Kong population according to Hong Kong Population Census (2016) in which the majority of the population was secondary school graduates, whereas a small proportion received no schooling or had pre-primary educational level. All children studied in the second year of kindergartens and had not learned addition and subtraction formally.

4.2. Materials

4.2.1. Inhibitory control

The Head, Toes, Knees and Shoulders task (HTKS; Ponitz et al., 2008) involved two parts: Part 1 (“touch your head/toes”; 10 trials) and Part 2 (“touch your head/toes” and “shoulders/knees”; 10 trials). In each trial, the experimenter mentioned an action (e.g., touch your head), and the children were required to do the opposite action (i.e., touch their toes). For each part, the children completed four practice trials with feedback before doing the test trials. For each trial, two points were given for a correct response, one point for self-corrected trials (e.g., make an initial incorrect move and then self-correct), and zero for incorrect responses. The maximum possible score for the task was 40. In the current study, this measure demonstrated good internal consistency ($\alpha = 0.86$).

4.2.2. Three-term problems

The same set of three-term inversion and standard problems in Study 1 was used in Study 2.

4.3. Procedure

Parental consent and children's verbal assent were obtained before the experiment started. Each child participated individually with an experimenter in a quiet location, which was separate from other children in the kindergartens. The children were tested in one approximately 30-min session, in which the inhibitory control task was completed before the three-term problems. Each child was given 10 s to answer each three-term problem. Answers were marked as either correct (1 score) or not correct (0 score). No feedback was given.

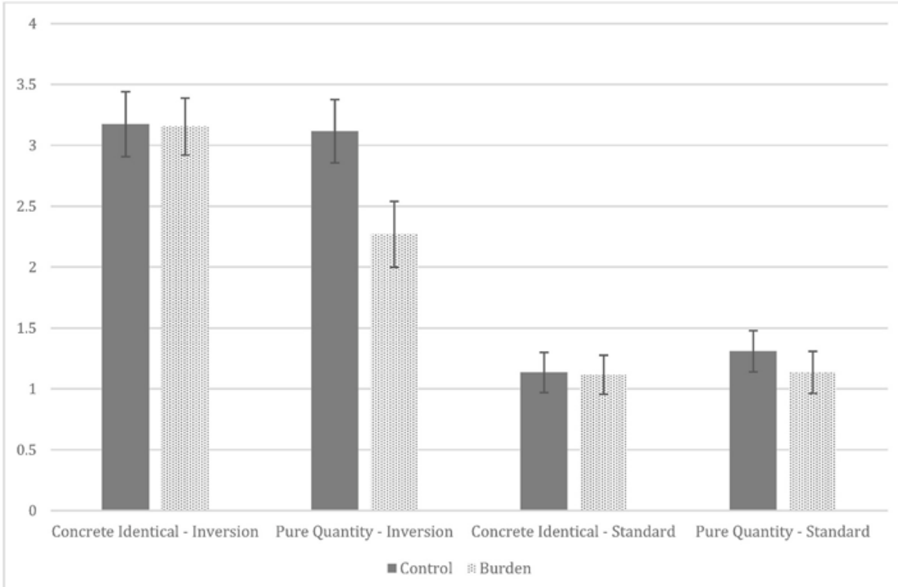

Fig. 4. Number of inversion and standard problems in the Concrete Identical condition and Pure Quantity condition solved correctly by children with high versus low burden on visuospatial working memory in Study 1.

5. Study 2: Results

5.1. Preliminary analyses

Independent *t*-tests were performed separately for each variable to assess whether there were gender differences in inhibitory control and the accuracy rates for inversion and standard problems in each condition. Bivariate correlation was employed to test whether these variables were associated with age. A one-way analysis of variance (ANOVA) was used to evaluate whether each variable would differ by school. Results showed that no significant age, gender, or school differences were observed in any tasks (all $ps > .05$). Children were divided into two groups on the basis of their scores on the inhibitory control (IC) measure: the mean of the current sample was computed ($M = 26.41$, $SD = 5.69$), then individuals who scored higher than the mean were categorized as the "higher IC group" ($n = 97$), whereas those with scores lower than the mean were categorized as the "lower IC group" ($n = 97$).

Consistent with the findings in Study 1 and previous research (Bryant et al., 1999; Rasmussen et al., 2003), on average, children performed significantly better for the inversion problems ($M = 6.47$, $SD = 3.64$) than for the standard problems ($M = 2.40$, $SD = 1.93$), F (1, 192) = 214.47, $p < .001$, partial $\eta^2 = 0.528$. A majority of children (Concrete Identical condition: $n = 148$, 76.3%; Pure Quantity condition: $n = 132$, 68%) fit this pattern. The number of *a* responses was greater for the inversion ($M = 3.38$, $SD = 1.77$) than for standard problems ($M = 0.83$, $SD = 0.84$) in the Concrete Identical condition: t (193) = 18.61, $p < .001$. The number of *a* responses was also greater for the inversion ($M = 3.10$, $SD = 1.93$) than for the standard problems ($M = 1.30$, $SD = 1.30$) in the Pure Quantity condition: t (193) = 9.93, $p < .001$. Thus, the children did not show the *a* term bias in Study 2.

5.2. Main analyses – hypothesis testing

The hypothesis of Study 2 was tested with a 2 (problem type: inversion vs. standard) x 2 (condition: Concrete Identical vs. Pure Quantity) x 2 (inhibitory control: higher IC vs. lower IC) analysis of variance with repeated measures on the first two variables. On average, children performed significantly better in the Concrete Identical condition ($M = 4.54$, $SD = 2.16$) than in the Pure Quantity condition ($M = 4.34$, $SD = 2.31$), F (1, 192) = 7.26, $p = .008$, partial $\eta^2 = 0.528$. The effect of condition interacted with problem type, $F\,(1, 192) = 23.12$, $p < .001$, partial $\eta^2 = 0.107$, and with inhibitory control, $F\,(1, 192) = 16.63$, $p < .001$, partial $\eta^2 = 0.08$. Both of these interactions were qualified by the three-way interaction, F (1, 192) = 8.163, $p = .005$, partial $\eta^2 = 0.041$. The overall results are illustrated in Fig. 5. For children with lower inhibitory control, performance was better in the Concrete Identical condition than the Pure Quantity condition for inversion problems, t (96) = 6.18, $p < .001$, but not for standard problems ($p = .698$). For children with higher inhibitory control, the performance differences between the two conditions were negligible for both inversion and standard problems ($ps > .152$). Consistent with the hypothesis, individual differences in inhibitory control moderated the performance differences between the Concrete Identical and the Pure Quantity on inversion problems but not on standard problems.

6. Discussion

The ability to reason about quantitative and numerical relations is important for children's mathematics learning (Bryant, 1995; Ching & Kong, 2022a; Ching & Wu, 2021; Nunes et al., 2012; Nunes & Bryant, 2015; Piaget, 1952; Thompson, 1993; Vergnaud, 2009). The current study used both experimental and non-experimental approaches to explore whether visuospatial working memory and inhibitory control affect 5-year-old children's use of quantitative inversion to solve three-term arithmetic problems in a nonsymbolic context. In two studies, five-year-old children showed some sensitivity to the inversion principle, but not all of them applied quantitative inversion to solve the problems consistently. Some children performed significantly worse in the inversion problems when they were presented in the Pure Quantity (versus Concrete Identical) condition. It appears that inhibitory control and visuospatial working memory underlie the individual differences in the use of quantitative inversion to solve three-term arithmetic problems. The results are discussed in detail in the following.

Consistent with previous research (Baroody & Lai, 2007; Bryant et al., 1999; Klein & Bisanz, 2000; Rasmussen et al., 2003; Sherman & Bisanz, 2007), this study indicated that children as young as 5 years of age could use inversion appropriately to solve three-term arithmetic problems. This conclusion is based on the results that 5-year-old children, on average, performed better for the inversion than the standard problems across the two studies. The findings also suggest that the

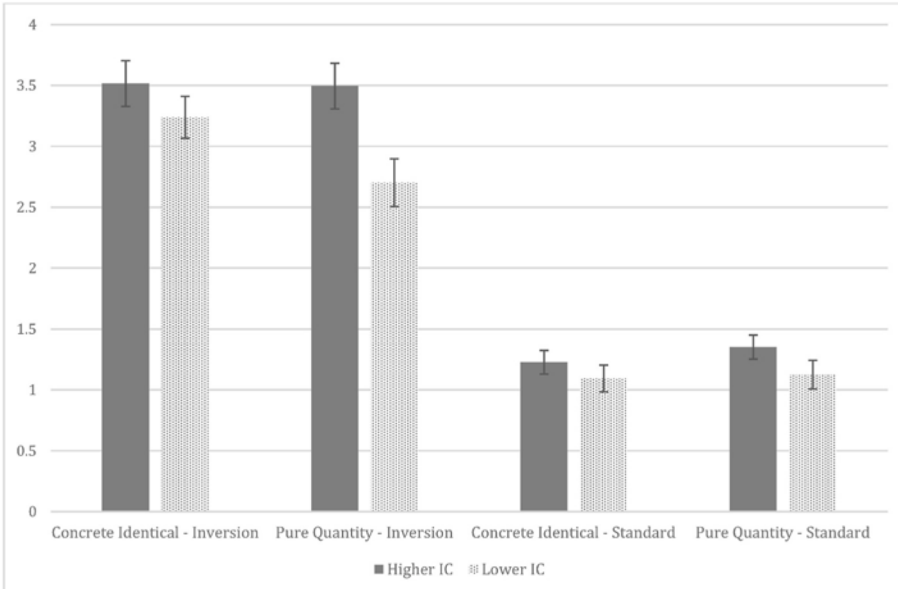

Fig. 5. Number of inversion and standard problems in the Concrete Identical condition and the Pure Quantity condition solved correctly by children with higher versus lower inhibitory control (IC) in Study 2.

differences could not be attributed to response bias (the *a* term bias) or to the use of inversion by only a subset of children. Thus, although formal education on arithmetic may enhance children's consistency with the use of inversion to solve three-term arithmetic problems (Watchorn et al., 2014), it is not a necessary condition for most of them. Indeed, previous research showed that preschool children who were more proficient in counting or simple arithmetic were not necessarily more likely to use inversion (Rasmussen et al., 2003; Sherman & Bisanz, 2007).

The current research showed that cognitive burden on visuospatial working memory and individual differences in inhibitory control moderated the performance differences between the Concrete Identical and the Pure Quantity on inversion problems but not on standard problems. Study 1 provides evidence for the causal role of visuospatial working memory in 5-year-old children's use of quantitative inversion. Based on the load theory of selective attention (Lavie et al., 2004), working memory is important for maintaining stimulus priorities. The availability of working memory helps individuals direct attention to goal-relevant rather than goal-irrelevant information, thereby minimizing the influence of irrelevant distractors. Consistent with this theory and the first hypothesis of the present research, Study 1 showed that children who experienced higher memory load performed significantly better in the Concrete Identical condition than the Pure Quantity condition for inversion problems, but not for standard problems. By contrast, for children who experienced lower memory load, the differences between the two conditions were negligible for both inversion and standard problems. Prior correlational research (e.g., Rasmussen et al., 2003) showed that individual differences in Corsi span were associated with accuracy on inversion problems among preschool children. Study 1 extended their findings by providing experimental evidence that visuospatial working memory has a causal relation to 5-year-old children's use of quantitative inversion.

As I speculated that visuospatial working memory affected performance in solving inversion problems via its influence on children's ability to suppress goal-irrelevant information, the second study aimed to examine the role of inhibitory control directly. Consistent with my hypothesis, Study 2 showed that children with lower inhibitory control performed significantly better in the Concrete Identical condition than the Pure Quantity condition for inversion problems, but not for standard problems. By contrast, for children with higher inhibitory control, the performance differences between the two conditions were not significant for both inversion and standard problems. These findings concur with the view that children with a stronger ability to suppress goal-irrelevant information are less prone to the distraction by length cues, so they can focus on the quantitative transformation to solve the three-term arithmetic problems. Previous research showed that young children (Klein & Bisanz, 2000; Rasmussen et al., 2003; Sherman & Bisanz, 2007) understood the inverse relation between addition and subtraction, but some of them might not apply their knowledge consistently across situations (Bryant et al., 1999; Schneider & Stern, 2012). The results from Study 2 contribute to this literature by showing that individual differences in inhibitory control are related to children's use of quantitative inversion to solve arithmetic problems in a nonsymbolic context.

Altogether, these studies entail several theoretical and educational implications. When discussing the development of inversion knowledge in children, Bisanz et al. (2009) proposed a "representational account of inversion" (Bisanz et al., 2009, p. 21), which states that quantitative inversion emerges from "non-numerical attentional processes". The current work showed some evidence to this hypothesis, suggesting that whether young children can apply the inversion knowledge consistently depends on general cognitive resources. Past research showed that stronger attentional skills were associated with more use of inversion in a previous study (Watchorn et al., 2014), the current work suggests that inhibitory control and visuospatial working memory may also be important for children's use of quantitative inversion. A growing number of studies have examined how young children differ in the extent to which they spontaneously attend to and use numerical information in their environments. This attention to numbers is often referred to as spontaneous focusing on numerosity (SFON), with some children tending to focus on precise numbers and quantities without explicit prompting and other children not doing so (Hannula & Lehtinen, 2005). It has been argued that children who are more sensitive to numerical information in their environment have more opportunity to learn from this information (Hannula & Lehtinen, 2005; McMullen et al., 2015). Evidence showed that SFON task performance during early childhood predicted children's concurrent and later mathematical skills (Hannula-Sormunen et al., 2015; Lepola & Hannula-Sormunen, 2019; McMullen et al., 2015). It is possible that children's ability to spontaneously focus on numerosity, as opposed to non-numerical properties, may contribute to children's success with the use of quantitative inversion to solve problems. This is a hypothesis worth testing in future research.

Regarding the assessment of inversion knowledge in young children, the current research suggests that how the three-term inversion problems are presented should be considered seriously. It appears that children's use of quantitative inversion can be more reliably assessed by comparing the performance in solving inversion and standard problems in both the Concrete Identical condition and the Pure Quantity condition. As for teaching mathematical concepts, previous studies have shown that it is not easy to help children use inversion spontaneously (Ching & Wu, 2019; Nunes et al., 2009). The present research suggests that inhibitory control and visuospatial working memory may support children's application of quantitative inversion to solve problems. Thus, interventions that purport to enhance inversion understanding could consider incorporating training for general cognitive resources at the same time.

The current research has limitations. For example, only accuracy measures were used. We can also rely on solution latencies to test whether children use an inversion-based shortcut to solve problems. If people use inversion knowledge to respond, their latencies should be shorter for inversion problems compared with standard problems. By contrast, if they use computation to solve inversion and standard problems, the response latencies should be similar for both types of problems. However, these measures may work better with older children because the reaction times of younger children "can be very long, wildly variable, and interrupted by attentional side trips" (Rasmussen et al., 2003, p.90). Similar issues may also arise if we ask children of such a young age to justify their answers. They may not be able to give an adequate explanation because of various reasons, such as lack of vocabulary. However, future research may consider incorporating multi-faceted assessments on inversion use (Bisanz et al., 2009; Gilmore & Papadatou-Pastou, 2009; Schneider, 2009; Schneider & Stern, 2012; Wong et al., 2021) among school-aged children who are older.

In conclusion, the current research presents some evidence for the moderating roles of visuospatial working memory and inhibitory control in young children's use of quantitative inversion. In particular, children with lower levels of inhibitory control were less likely to use quantitative inversion to solve problems when the tasks contained distractors such as irrelevant length cues. Similarly, when children were asked to do a secondary task that demanded visuospatial working memory at the same time, their performance dropped significantly. The results of the present research contribute to the understanding of the use of quantitative inversion as well as the assessment and teaching of inversion knowledge in young children.

Author statement and declarations

Boby Ho-Hong Ching: Conceptualization, Methodology, Formal analysis, Writing - original draft, Writing - review & editing, Project administration.

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Consent to participate

Informed consent was obtained from each participant included in the study.

Data availability statement

The data that support the findings of this study are available from the corresponding author upon reasonable request.

Declaration of competing interest

The author declares that there is no conflict of interest.

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