Modeling exchange rates using ARCH family of models

MODELING EXCHANGE RATES USING ARCH FAMILY OF MODELS

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Abstract

In this study, after a brief literature review, the RON / EURO exchange rate time series over the 03.01.2005 - 05.02.2015 time period is analyzed.

After checking the stationarity of the data - ARCH, GARCH, EGARCH and TARCH models will be developed and compared. Next the best model is chosen and the serial correlation and the Jarque-Bera test are further analyzed with various conclusions being drawn.

Keywords: *ARCH, GARCH, GARCH, exchange rate, RON / EURO.*

Introduction

The analysis of the evolution of exchange rates is an important topic for economic researchers because it affects a whole range of economic actors like banks, governmental agencies, companies and households.

In the first section of the article (the brief literature review) it is shown the usage of ARCH family models in scientific literature and some of these models applications on exchange rate analysis.

Autoregressive conditional heteroskedasticity (ARCH) models are used for modelling observed time series. Also, they are used in order to characterize various observed time series. An ARCH(q) model is estimated using ordinary least squares.

In this study, it is analyzed the evolution of daily RON / EURO exchange rates over the 03.01.2005 - 05.02.2015 time period using specialized IT software for data analysis.

Appling the methodology presented in the second section of this article - ARCH, GARCH, EGARCH and TARCH models are developed for the data chosen for this study.

As in many research studies that use ARCH family models for exchange rate analysis, we test in this article different models in order to find the best one for our data series. After choosing the best model for our data series (by comparing and analyzing the models) different tests are applied on it.

We check the ARCH model for serial correlation, we also check the heteroskedasticity of the model and we check if the residuals are normally distributed or not.

Finally we take a look at the volatility, which is measured by the conditional standard deviation.

Literature review

ARCH family models are frequently used for exchange rate time series. Most of the articles in this area of the literature deal with the analysis of the exchange rate volatility or with the forecast of the exchange rates.

The autoregressive conditional heteroskedasticity (ARCH) method for modeling volatility has been introduced by (Engle, 1982), which modeled the heteroskedasticity by relating the conditional variance of the disturbance term to the linear combination of the squared disturbances in the recent past.

The GARCH model is a generalized ARCH model which was obtained by (Bollerslev, 1986) by modeling the conditional variance to depend on its lagged values as well as squared lagged values of disturbance.

Other models from the ARCH family are: the EGARCH model which was proposed by Nelson (1991) or the TARCH model introduced independently by Glosten, Jaganathan, and Runkle (1993) and Zakoïan (1994).

Various ARCH models have been applied by researchers to analyze the volatility of exchange rates in different countries. For example some studies are: (Benavides, 2006) in which the author analyses the volatility forecast for the Mexican Peso – U.S. Dollar exchange rate, (Trenca et. al., 2011) which analyzes the evolution of the exchange rate for: Euro / RON, dollar / RON, yen / RON, British pound / RON, Swiss franc / RON for a period of five years from 2005 until 2011, (Alam et. al., 2012) in which the authors analyze exchange rates of Bangladeshi Taka (BDT) against the U.S. Dollar (USD) for the period of July 03, 2006 to April 30, 2012, (Musa et. al, 2014) forecast the exchange rate volatility between Naira and US Dollar using GARCH models.

Other interesting articles are:

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- (Spulbar et. al., 2012) in which the impact of political news and economic news from euro area on the exchange rate between Romanian currency and EURO is analyzed using a GARCH model,
- (Hartwell, 2014) in which is analyzed the impact of institutional volatility on financial volatility in transition economies using a GARCH family approach (the paper posits that institutional changes, and in particular the volatility of various crucial institutions, have been the major causes of financial volatility in transition and the researcher examines 20 transition economies over various time-frames within the period 1993 - 2012),
- (Teyssiere, 1998), a vast and very good written study, in which two classes of multivariate longmemory ARCH models are considered.

Methodology

First the stationarity of the data will be checked using the ADF test (Augmented Dickey-Fuller test) (Dickey & Fuller, 1981).

The ADF test estimates the equation:

$$\Delta y_t = a_0 + \gamma y_{t-1} + \sum_{i=2}^{p} \beta_i \Delta y_{t-i+1} + \epsilon_t$$

The time series y_t is stationary if for every $h \epsilon Z$, the y_{t+h} series has the same distribution as the y_t series for any $t = 1, 2, \ldots, n$ (Enders, 1995).

Then the models:

- ARCH - autoregressive conditional heteroskedastic model,
- GARCH - generalized autoregressive conditional heteroskedastic model,
- EGARCH - exponential generalized autoregressive conditional heteroskedastic model,
- TARCH - threshold GARCH model are developed.

After choosing the best model, serial correlation and heteroscedasticity is checked. Also, by using the Jarque-Bera test it can be stated if the residuals are normally distributed or not. At last the conditional standard deviation is plotted.

Autoregressive conditional heteroskedasticity (ARCH) models are used for modelling observed time series. Also they are used in order to characterize various observed time series. An ARCH(q) model is estimated using ordinary least squares.

Data analysis

For this article, data has been collected from the National Bank of Romania website (http://bnr.ro). The data consists of daily RON / EURO exchange rates over the 03.01.2005 - 05.02.2015 time period.

First we run, in EViews software, a descripting statistics on our data in order to observe the mean, median, maximum, minimum, standard deviation, skewness, kurtosis, Jaque-Bera, Probability, Sum, sum sq. dev. and the number of observations. Interesting to see is that the difference between the minimum and the maximum values is rather significant (from a minimum of 3.111200 to a maximum of 4.648100).

	CURS
Mean	4.020349
Median	4.213900
Maximum	4.648100
Minimum	3.111200
Std. Dev.	0.413540
Skewness	-0.442294
Kurtosis	1.663810
Jarque-Bera	274.4443
Probability	0.000000
Sum	10312.20
Sum Sq. Dev.	438.4839
Observations	2565

Table 1. RON / EURO exchange rates – descriptive statistics

Data plot:

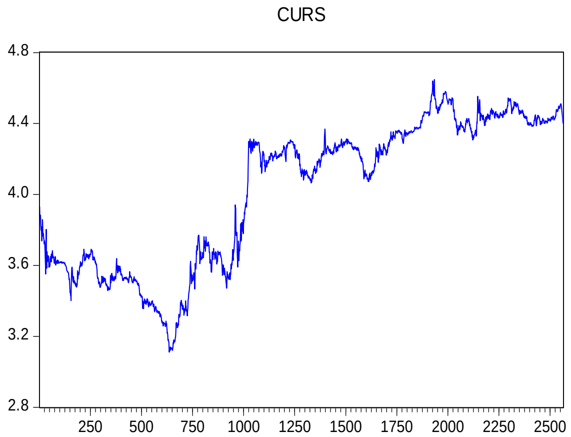


Figure 1. RON / EURO exchange rates over the 03.01.2005 - 05.02.2015 time period

From figure 1 it can be seen that the data is non-stationary, but we will run the Augmented Dickey-Fuller test to make sure. Also, from figure 1, an increasing trend of the time series can be noticed.

Null Hypothesis: CURS has a unit root
Exogenous: Constant, Linear Trend
Lag Length: 3 (Automatic - based on SIC, maxlag=27)

		t - Statistic	Prob.*
Augmented Dickey - Fuller test statistic		-2.591988	0.2841
Test critical values:	1% level	-3.961627	
	5% level	-3.411562	
	10% level	-3.127647	

*MacKinnon (1996) one-sided p-values.

Variable	Coefficient	Std. Error	t - Statistic	Prob.
D(CURS(-1))	-1.025038	0.030925	-33.14621	0.0000
D(CURS(-1),2)	0.217311	0.024944	8.711893	0.0000
D(CURS(-2),2)	0.100922	0.019652	5.135481	0.0000
C	-8.76E-05	0.000619	-0.141621	0.8874
@TREND(1)	2.33E-07	4.18E-07	0.557143	0.5775

Table. 2. Augmented Dickey - Fuller Test – trend and intercept

In table 2 we have the output of the Augmented Dickey - Fuller test. From it, it can be stated that our time series in level is non-stationary (it has a unit root) – the t-stats -2.591988 in absolute value is lower than the critical values.

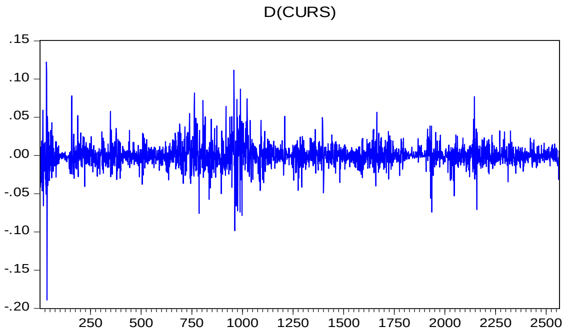


Figure 2. RON / EURO exchange rates over the 03.01.2005 - 05.02.2015 time period in 1st difference

From figure 2 it can be seen that the data is stationary in 1st difference. The 1st difference $(x_t - x_{t-1})$ is generally used in order to transform non-stationary data into stationary data.

Null Hypothesis: D(CURS) has a unit root
Exogenous: Constant, Linear Trend
Lag Length: 2 (Automatic - based on SIC, maxlag=27)

		t - Statistic	Prob.*
Augmented Dickey - Fuller test statistic		-33.14621	0.0000
Test critical values:	1% level	-3.961627	
	5% level	-3.411562	
	10% level	-3.127647	

*MacKinnon (1996) one-sided p-values.

Variable	Coefficient	Std. Error	t - Statistic	Prob.
CURS(-1)	-0.004111	0.001586	-2.591988	0.0096
D(CURS(-1))	0.193646	0.019663	9.847995	0.0000
D(CURS(-2))	-0.114693	0.019895	-5.764962	0.0000
D(CURS(-3))	-0.099430	0.019638	-5.063053	0.0000
C	0.013836	0.005407	2.558782	0.0106
@TREND(1)	2.26E-06	8.87E-07	2.549576	0.0108

Table. 3. Augmented Dickey - Fuller Test – trend and intercept – the 1st difference of the time series

From table 3 it can be observed that in first difference the time series becomes stationary, so further in our analysis we will use the data in 1st difference. We have included in both tests trend and intercept.

ARCH family models analysis and comparison

We shall be developing four types of models (ARCH - autoregressive conditional heteroskedastic model, GARCH - generalized autoregressive conditional heteroskedastic model, EGARCH - exponential generalized autoregressive conditional heteroskedastic model and TARCH - threshold generalized autoregressive conditional heteroskedastic model) and compare them in order to find the best model by looking at the AIC - Akaike information criterion and SIC - Schwarz information criterion. Lower the value of Akaike information criterion and Schwarz information criterion, better fitted is the model.

First we run a model using the least squares method:

Dependent Variable: D(CURS)
Method: Least Squares
Date: 14/02/15 Time: 19:25
Sample (adjusted): 2 2565
Included observations: 2564 after adjustments

Variable	Coefficient	Std. Error	t - Statistic	Prob.
C	0.000186	0.000319	0.584928	0.5586

R - squared	0.000000	Mean dependent var	0.000186
Adjusted R - squared	0.000000	S. D. dependent var	0.016132
S. E. of regression	0.016132	Akaike info criterion	-5.415650
Sum squared resid.	0.666988	Schwarz criterion	-5.413368
Log likelihood	6943.863	Hannan - Quinn criter.	-5.414822
Durbin - Watson stat	1.633308		

Table. 4. Model - least squares method

Next we check the residuals of this model:

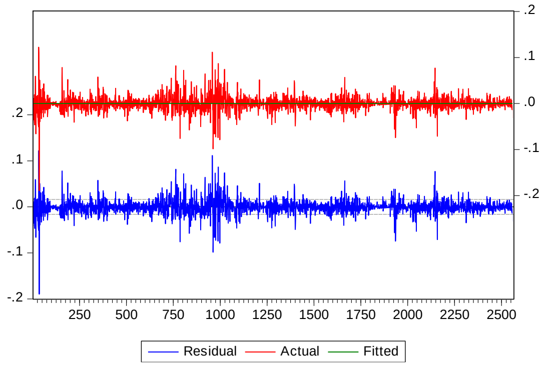


Figure 3. Residuals of the model

Looking at the figure above, at the residuals plot, we can observe that there are **long** periods with low fluctuations and also **long** periods with high fluctuations, meaning that periods of low volatility tend to be followed by periods of low volatility for a prolonged period and periods of high volatility are followed by periods of high volatility for a prolonged period. When these things happen for residuals, we have all justification to run ARCH family models.

In order to be sure, we will run a heteroskedasticity test.

Heteroskedasticity Test: ARCH			p-value
F - statistic	122.8733	Prob. F (1,2561)	0.0000
Obs * R - squared	117.3394	Prob. Chi - Square (1)	0.0000

Table. 5. ARCH test

From the heteroskedasticity test, we can see that the p value is less than 5%, meaning that there is an ARCH effect.

Next we developed the four models. All of them can be seen in the appendix. The Akaike info criterion and the Schwarz criterion for each model:

	ARCH	GARCH	TARCH	EGARCH
AIC	-5.890104	-5.895482	-5.894853	-5.895363
SIC	-5.874134	-5.886357	-5.883446	-5.883957

Table. 6. AIC & SIC values for the four models (ARCH; GARCH; TARCH; EGARCH)

From the values obtained and presented in table 6 it can be stated that the best model is the ARCH model.

ARCH model analysis

Next we will check the ARCH model for serial correlation, we will check if it has an arch effect and if the residuals are normally distributed or not.

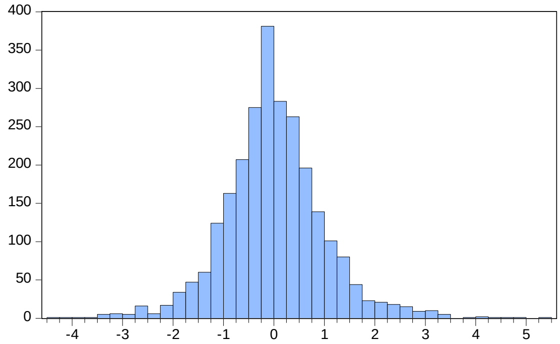
The output for **serial correlation** is also in the appendix. From it, having in mind that almost all the p-values are more than 5%, we can conclude that there is no serials correlation in the residuals.

In order to test the remaining ARCH effect in the residuals we perform a Heteroskedasticity test (table 6).

F - statistic	0.079006	Prob. F (1,2561)	0.7787
Obs * R - squared	0.079065	Prob. Chi - Square (1)	0.7786

Table. 7. Heteroskedasticity Test: ARCH

The test p-values from tables 7 (shown in the second column) are more than 5%, so in this model there is no ARCH effect, meaning that we have a good model.



Series: Standardized Residuals
Sample 2 2565
Observations 2564

Mean	0.001323
Median	-0.035984
Maximum	5.320488
Minimum	-4.499429
Std. Dev.	1.000191
Skewness	0.265227
Kurtosis	5.163102
Jarque-Bera	529.9350
Probability	0.000000

Figure 4. Residuals of the model

From the p-value of the Jarque - Bera test we can state that the residuals are not normally distributed, which is not desirable.

So, the only problem of the model is that the residuals are not normally distributed.

The volatility is measured by the conditional standard deviation and it is presented in figure 5.

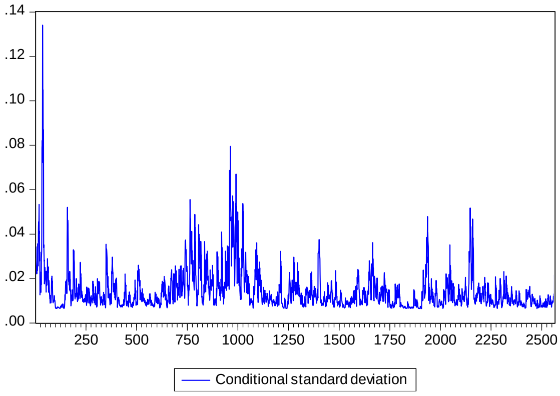


Figure 5. Conditional standard deviation plot

Conclusions

Computer software EViews is a powerful tool for data analysis that can be used on various time series from the social sciences area.

In this paper, using EViews software, ARCH, GARCH, EGARCH and TARCH models are developed and analyzed on the RON / EUR exchange rate time series over the 03.01.2005 - 05.02.2015 time period.

As seen in the data analysis section of the article, the best model is the ARCH (5) model, which has been further analyzed in the article, various results being obtained.

As future development of the paper, the models could be further commented and tested.

Appendix

ARCH model

Dependent Variable: D (CURS)
Method: ML - ARCH (Marquardt) - Normal distribution
Date: 14/02/15 Time: 19:48
Sample (adjusted): 2 2565
Included observations: 2564 after adjustments
Convergence achieved after 15 iterations
Presample variance: backcast (parameter = 0.7)
GARCH = C(2) + C(3)*RESID(-1)^2 + C(4)*RESID(-2)^2 + C(5)*RESID(-3)^2 + C(6)*RESID(-4)^2 + C(7)*RESID(-5)^2

Variable	Coefficient	Std. Error	z - Statistic	Prob.
C	-1.34E-05	0.000205	-0.065120	0.9481
	Variance Equation			
C	4.08E-05	2.49E-06	16.41387	0.0000
RESID(-1)^2	0.353942	0.027159	13.03202	0.0000
RESID(-2)^2	0.088498	0.018935	4.673898	0.0000
RESID(-3)^2	0.219024	0.022003	9.954237	0.0000
RESID(-4)^2	0.142763	0.018895	7.555415	0.0000
RESID(-5)^2	0.171213	0.021143	8.097701	0.0000

R - squared	-0.000153	Mean dependent var	0.000186
Adjusted R - squared	-0.000153	S.D. dependent var	0.016132
S. E. of regression	0.016133	Akaike info criterion	-5.890104
Sum squared resid	0.667090	Schwarz criterion	-5.874134
Log likelihood	7558.113	Hannan - Quinn criter.	-5.884313
Durbin - Watson stat	1.633057		

GARCH model

Dependent Variable: D(CURS)
Method: ML - ARCH (Marquardt) - Normal distribution
Date: 14/02/15 Time: 19:56
Sample (adjusted): 2 2565
Included observations: 2564 after adjustments

Convergence achieved after 14 iterations
Presample variance: backcast (parameter = 0.7)
GARCH = C(2) + C(3)*RESID(-1)^2 + C(4)*GARCH(-1)

Variable	Coefficient	Std. Error	z - Statistic	Prob.
C	-9.62E-05	0.000214	-0.450589	0.6523
	Variance Equation			
C	6.90E-06	6.77E-07	10.18504	0.0000
RESID(-1)^2	0.224656	0.015009	14.96804	0.0000
GARCH(-1)	0.768696	0.012495	61.51978	0.0000

R - squared	-0.000307	Mean dependent var	0.000186
Adjusted R - squared	-0.000307	S.D. dependent var	0.016132
S. E. of regression	0.016134	Akaike info criterion	-5.895482
Sum squared resid	0.667193	Schwarz criterion	-5.886357
Log likelihood	7562.008	Hannan - Quinn criter.	-5.892174
Durbin - Watson stat	1.632807		

TARCH model
Dependent Variable: D(CURS)
Method: ML - ARCH (Marquardt) - Normal distribution
Date: 14/02/15 Time: 20:03
Sample (adjusted): 2 2565
Included observations: 2564 after adjustments
Convergence achieved after 16 iterations
Presample variance: backcast (parameter = 0.7)
GARCH = C(2) + C(3)*RESID(-1)^2 + C(4)*RESID(-1)^2*(RESID(-1)<0) + C(5)*GARCH(-1)

Variable	Coefficient	Std. Error	z - Statistic	Prob.
C	-0.000125	0.000223	-0.561744	0.5743
	Variance Equation			
C	6.78E-06	6.67E-07	10.17308	0.0000
RESID(-1)^2	0.215377	0.016100	13.37758	0.0000
RESID(-1)^2*(RESID(-1)<0)	0.018095	0.020042	0.902829	0.3666
GARCH(-1)	0.769988	0.012544	61.38494	0.0000

R - squared	-0.000373	Mean dependent var	0.000186
Adjusted R - squared	-0.000373	S.D. dependent var	0.016132
S. E. of regression	0.016135	Akaike info criterion	-5.894853
Sum squared resid	0.667237	Schwarz criterion	-5.883446
Log likelihood	7562.202	Hannan - Quinn criter.	-5.890717
Durbin - Watson stat	1.632698		

EGARCH model
Dependent Variable: D(CURS)
Method: ML - ARCH (Marquardt) - Normal distribution
Date: 14/02/15 Time: 20:11
Sample (adjusted): 2 2565
Included observations: 2564 after adjustments
Convergence achieved after 16 iterations
Presample variance: backcast (parameter = 0.7)

LOG(GARCH) = C(2) + C(3)*ABS(RESID(-1) / @SQRT(GARCH(-1))) + C(4)
*RESID(-1) / @SQRT(GARCH(-1)) + C(5)*LOG(GARCH(-1))

Variable	Coefficient	Std. Error	z - Statistic	Prob.
C	-0.000322	0.000141	-2.278700	0.0227
	Variance Equation			
C(2)	-0.947555	0.061312	-15.45464	0.0000
C(3)	0.431769	0.021422	20.15562	0.0000
C(4)	0.013049	0.010643	1.226088	0.2202
C(5)	0.927585	0.005577	166.3138	0.0000

R - squared	-0.000992	Mean dependent var	0.000186
Adjusted R - squared	-0.000992	S.D. dependent var	0.016132
S. E. of regression	0.016140	Akaike info criterion	-5.895363
Sum squared resid	0.667650	Schwarz criterion	-5.883957
Log likelihood	7562.856	Hannan - Quinn criter.	-5.891228
Durbin - Watson stat	1.631690		

ARCH model – checking serial correlation

Date: 15/02/15 Time: 19:14
Sample: 2 2565
Included observations: 2564

	AC	PAC	Q-Stat	Prob
1	-0.006	-0.006	0.0792	0.778
2	-0.031	-0.031	2.5603	0.278
3	-0.018	-0.018	3.3618	0.339
4	-0.019	-0.020	4.2673	0.371
5	-0.019	-0.020	5.1628	0.396
6	0.013	0.011	5.5686	0.473
7	0.021	0.019	6.6534	0.466
8	-0.001	-0.001	6.6553	0.574
9	-0.027	-0.026	8.5655	0.478
10	0.005	0.005	8.6203	0.568
11	0.030	0.029	10.884	0.453
12	0.024	0.024	12.366	0.417
13	-0.000	0.001	12.366	0.498
14	0.010	0.012	12.636	0.555
15	-0.008	-0.005	12.815	0.617
16	0.004	0.007	12.851	0.684
17	0.015	0.015	13.434	0.707
18	0.015	0.014	14.026	0.727
19	0.059	0.060	22.928	0.241
20	0.026	0.030	24.637	0.216
21	0.014	0.020	25.111	0.242
22	0.016	0.020	25.742	0.263
23	0.038	0.042	29.430	0.166
24	0.003	0.007	29.449	0.204
25	0.017	0.020	30.183	0.217
26	-0.005	-0.004	30.260	0.257
27	0.020	0.024	31.340	0.257
28	0.048	0.052	37.343	0.111
29	-0.031	-0.030	39.846	0.086
30	0.062	0.062	49.941	0.013
31	0.023	0.021	51.259	0.012

32	-0.009	-0.004	51.490	0.016
33	-0.021	-0.020	52.644	0.016
34	-0.012	-0.015	53.020	0.020
35	0.025	0.022	54.620	0.018
36	0.008	0.005	54.801	0.023

References:

- Alam, Z., Rahman, A., "Modelling Volatility of the BDT / USD Exchange Rate with GARCH Model", *International Journal of Economics and Finance* 4, 11 (2012): 193 – 204;
- Benavides, Guillermo, "Volatility Forecasts for the Mexican Peso - U.S. Dollar Exchange Rate: An Empirical Analysis of GARCH", *Option Implied and Composite Forecast Models, Banco de Mexico, Research Document* 2006 - 04 (2006): 1 – 39;
- Bollerslev, T., "Generalized Autoregressive Conditional Hetroscedasticity", *Journal of Econometrics* 31 (1986): 307 – 327;
- Dickey, D., Fuller, W., "Distribution of the estimators for autoregressive time series with a unit root", *Econometrica Journal* 49 (1981): 1057 – 72;
- Durnel, J., "Applied Econometric Time Series, Arch Garch Modelling", Workshop, Lund University (2012), accesed 11 January 2016, http://www.academia.edu/1506454/ Arch_Garch_Modelling;
- Enders, W., *Applied Econometric Time Series* (New York: John Wiley and Sons, 1995);
- Engle, R. F., "Autoregressive Conditional Heteroscedasticity with Estimates of the Variance of United Kingdom Inflation", *Econometrica* 50, 4 (1982): 987 – 1008;
- Glosten, L, Jagannathan, R., Runkle, D., "On the relationship between the Expected Value and the Volatility of the Nominal Excess Return on Stocks", *Journal of Finance* 48 (1993): 1779 – 1802;
- Musa, Y., Tasi'u, M., Abubakar, B., "Forecasting of Exchange Rate Volatility between Naira and US Dollar Using GARCH Models", *International Journal of Academic Research in Business and Social Sciences*, ISSN: 2222 - 6990, 4, 7 (2014): 369 – 381;
- Nelson, Daniel B., "Conditional Heteroskedasticity in Asset Returns: A New Approach", *Econometrica* 59, 2 (1991): 347 – 370;
- Sayed Hossain, Hossain Academy, accesed 11 January 2016, http://www.sayedhossain.com;
- Spulbar, C., Nitoi, M., "The Impact Of Political And Economic News On The Euro / Ron Exchange Rate: A Garch Approach", *Annals of the „Constantin Brâncuşi" University of Târgu Jiu, Economy Series*, 4 (2012): 52 – 58;
- Teyssiere, G., "Modelling Exchange Rates Volatility with Multivariate Long-Memory ARCH Processes", *G.R.E.Q.A.M., Universite Aix-Marseille* III (1995): 1 – 16;
- Trenca, I., Cociuba, M., "Modeling Romanian Exchange Rate Evolution With Garch, Tgarch, Garch- In Mean Models", *The Annals of the University of Oradea. Economic Sciences*, 1, July (2011): 299 – 305;
- Zakoïan, J. M., "Threshold Heteroskedastic Models", *Journal of Economic Dynamics and control*, 18 (1994): 931 - 944.