COMBINATIONS OF PROPORTIONATE ADAPTIVE FILTERS IN ACOUSTICS: AN APPLICATION TO ACTIVE NOISE CONTROL

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ABSTRACT

Proportionate adaptive schemes have been proposed to exploit sparsity and accelerate filter convergence in acoustic echo cancellation. Recently, combinations of adaptive filters have been extended to operate with proportionate schemes, in order to achieve more robust operation when the actual degree of sparsity of the optimal solution is unknown. Furthermore, it is possible to exploit the asymmetric distribution of adaptation energy in proportionate schemes to reduce the overall steady-state misadjustment. In this contribution, we explain how these novel adaptive filtering structures, which have been proposed and tested mainly for echo cancellation, can also be effectively extended to improve the usual performance trade-offs that appear in active noise control scenarios by introducing some minor modifications. Experimental results in realistic scenarios show that the proposed schemes provide an interesting alternative to the traditional use of a single adaptive filter.

1. INTRODUCTION

Active noise control (ANC) is a field of growing interest that combines digital signal processing techniques with traditional acoustics. The use of adaptive algorithms for ANC [1] has been subject of continuous study and research since the 1980s. ANC systems attempt to reduce the noise by generating an antinoise that cancels out the primary noise [2]. Fig. 1 shows a typical configuration of an ANC system. The signal produced by the noise source propagates through a primary echo path towards the point where noise is to be cancelled, producing, after the addition of noise $e_0(n)$, the disturbance signal $d(n)$. The input to the noise control system, $x(n)$, is correlated with the noise source, and thus can be used to generate a signal $y(n)$ which, after propagating through an unavoidable secondary path with impulse response $\mathbf{h}$, is added to the disturbance signal, producing an error signal $e(n)$. The objective of the adaptive algorithm is to iteratively estimate the filter weights in such a way that a function of the error signal $e(n)$ is minimized.

There are some fundamental differences between an ANC configuration and the standard setup for channel identification [2]. First, the disturbance signal can never be accessed directly, as it is only possible to measure the error after noise cancellation. Such error is obtained as the acoustical combination of $d(n)$ and the adaptive filter output filtered by $\mathbf{h}$, whereas in a standard identification scenario, a subtraction is typically considered. Second, and more importantly, the presence of a secondary path between the adaptive filter output [$y(n)$] and the noise sensor [where $e(n)$ is measured] makes necessary the introduction of *ad hoc* configurations for ANC. The usual way to take into account this response $\mathbf{h}$ consists of filtering the input signal $x(n)$ through a previous estimation of this response ($\hat{\mathbf{h}}$), providing the conventional filtered-x (FX) scheme depicted in Fig. 2 [3]. As an alternative to the FX scheme,

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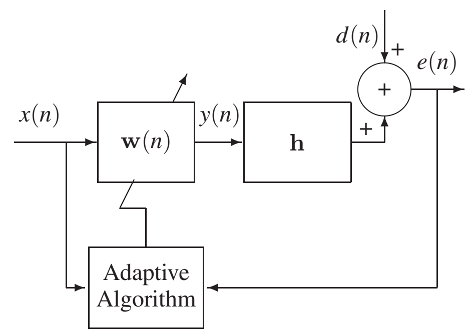

Figure 1: Block diagram of an active noise control system.

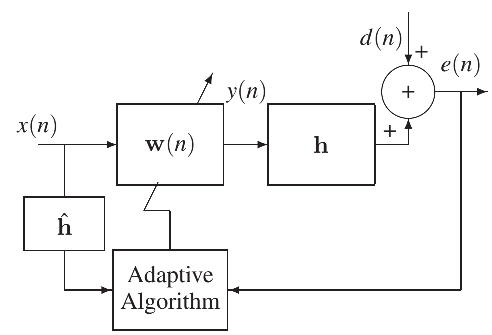

Figure 2: Conventional filtered-x (FX) structure for ANC.

the modified filtered-x (MFX) structure allows recovery of the disturbance signal, $d'(n)$ in Fig. 3, and provides the best convergence performance compared to other filtering structures for ANC [4, 5].

As it occurs in other acoustic applications, there is a requirement for long adaptive filters in ANC. It is a well-known result in the adaptive filtering literature that stochastic gradient algorithms, such as least-mean-square (LMS) or normalized LMS (NLMS), suffer from a slow convergence in such situations [6]. In order to speed up filter convergence, proportionate adaptation [7, 8] has been proposed for the identification of sparse or quasi-sparse systems, i.e., systems where only a few so-called active coefficients are significant. The operating principle of proportionate adaptation is rather simple: to distribute the adaptation energy of the filter unevenly among the coefficients, adapting the active coefficients faster.

Proportionate schemes were primarily proposed for acoustic and network echo cancellation and, to the best of our knowledge, no work has been reported in the literature in the context of ANC. Thus, a first objective of this paper is to study the behavior of pro-

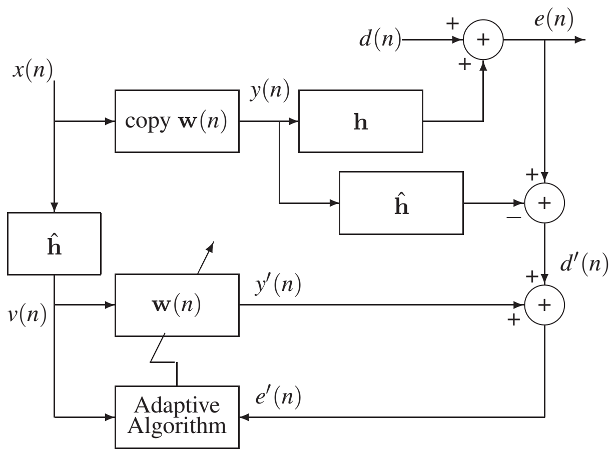

Figure 3: Modified filtered-x (MFX) structure for ANC.

portionate schemes in FX and MFX configurations for ANC. This extension is motivated by the fact that the optimal solutions encountered in echo cancellation and ANC share similar properties, particularly a high (but unknown) degree of sparsity. Among the different available schemes, we will consider the improved proportionate NLMS (IPNLMS) filter of [8], which offers a more robust behavior than other similar schemes.

The application of proportionate filters is subject to different kinds of compromises. For instance, as with any other kind of adaptive filter, the selection of the step size introduces a trade-off involving speed of convergence, steady-state misadjustment, and tracking capability. Furthermore, the IPNLMS filter introduces an additional *asymmetry parameter*, whose optimal selection is dependent on the degree of sparsity of the optimal solution, which is rarely known in practice. In [9], it was shown that combination approaches, where filters with complementary capabilities are adaptively fused to produce an output of improved quality (see also [10]), can be successfully applied to alleviate the two aforementioned IPNLMS compromises.

Extending the combination of adaptive filters scheme to ANC is not straightforward, and requires the introduction of some changes (see as an example the adaptive algorithms introduced in [11]). Thus, a second goal of this paper is to present IPNLMS combination schemes that can satisfactorily work in an ANC setup, both using the FX and MFX configurations. We will show that the derived schemes improve convergence, steady-state error, and robustness to unknown or time-varying degrees of sparsity, with respect to standard IPNLMS filters.

The rest of the paper is organized as follows: The next two sections are devoted to the generalization of IPNLMS and combination schemes, respectively, for ANC configurations. Then, the performance of the new schemes, and their different benefits, are discussed together with experimental results in Section 4. The paper finishes presenting the main conclusions of our work.

2. PROPORTIONATE ADAPTIVE FILTERS FOR ANC

In this section, we extend the IPNLMS algorithm from [8] to ANC. In our presentation, we will consider both conventional and modified filtered-x structures, resulting in two schemes that we will call throughout this paper IPNLMS-FX and IPNLMS-MFX, respectively. According to the notation in Table 1, both algorithms are described by the following equations:

$$e(n) = d(n) + y(n) * \mathbf{h}, \tag{1}$$

$$y(n) = \mathbf{w}^T(n)\mathbf{x}(n), \tag{2}$$

$x(n)$	Reference signal at time n
$y(n)$	Output signal of the adaptive filter at time n
$e(n)$	Error signal at time n
$\mathbf{w}(n)$	Weight vector of the adaptive filter (length L)
$\hat{\mathbf{h}}$	Estimated impulse response (length M) of the FIR filter modelling the secondary path $\mathbf{h}$
$w_l(n)$	lth coefficient of the adaptive filter
$\mathbf{x}(n)$	$[x(n)\, x(n-1) \cdots x(n-L+1)]^T$
$\mathbf{x}_M(n)$	$[x(n)\, x(n-1) \cdots x(n-M+1)]^T$
$v(n)$	Reference signal $x(n)$ filtered by the plant model $\hat{\mathbf{h}}$ at time n
$\mathbf{v}(n)$	$[v(n)\, v(n-1) \cdots v(n-L+1)]^T$
$\mathbf{y}(n)$	$[y(n)\, y(n-1) \cdots y(n-M+1)]^T$

Table 1: Notation of the IPNLMS algorithms.

$$\mu_l(n) = \frac{\mu g_l(n)}{\delta + \sum_{k=0}^{L-1} g_k(n) v^2(n-k)}, \tag{3}$$

where $\mathbf{w}(n)$ is the length-L IPNLMS weight vector at iteration n,

$$v(n) = \mathbf{x}_M^T(n)\hat{\mathbf{h}} \tag{4}$$

is a filtered version of the input signal through the estimated secondary path, δ is a small parameter to avoid division by zero, and $\mu_l(n)$, $l = 0, \cdots, L-1$, determines the adaptation speed for each filter weight, with μ being the step size for the IPNLMS filter, and

$$g_l(n) = (1-\kappa)\frac{1}{2L} + (1+\kappa)\frac{|w_l(n)|}{\varepsilon + 2\sum_k |w_k(n)|} \tag{5}$$

being adaptation gain factors. In the expression for $g_l(n)$, ε is a small constant that avoids division by zero, and $\kappa \in [-1, 1]$ is an *asymmetry factor*. If $\kappa = -1$, the filter reduces to the standard NLMS and all filter weights are updated with the same energy. On the contrary, for $\kappa = 1$, adaptation is proportional to the absolute value of each filter weight, speeding up filter convergence for sparse solutions.

The IPNLMS-FX weights are updated at each iteration according to

$$w_l(n) = w_l(n-1) - \mu_l(n) e(n) v(n-l), \quad l = 0, ..., L-1. \tag{6}$$

Alternatively, the IPNLMS-MFX follows the update rule

$$w_l(n) = w_l(n-1) - \mu_l(n) e'(n) v(n-l), \quad l = 0, ..., L-1. \tag{7}$$

where

$$e'(n) = d'(n) + y'(n), \tag{8}$$

$$d'(n) = e(n) - \mathbf{y}^T(n)\hat{\mathbf{h}}, \tag{9}$$

and

$$y'(n) = \mathbf{v}^T(n)\mathbf{w}(n). \tag{10}$$

As discussed in [9], the IPNLMS filter is subject to the two following compromises:

- Selection of the step size μ imposes a trade-off regarding speed of convergence (faster for large μ) and steady-state misadjustment (which is reduced for small μ).
- An asymmetry factor $\kappa \approx 1$ provides the maximum convergence gain for very sparse systems. However, if the solution of the filter is not so sparse, such a setting can in fact degrade the performance. The opposite situation is observed for $\kappa = -1$. Since the actual degree of sparsity of the solution is rarely known *a priori*, or may even be time-varying, intermediate values of $\kappa = 0$ or $\kappa = -0.5$ are usually indicated [8].

In the next section, we propose combination approaches to alleviate both kinds of compromises.

3. CONVEX COMBINATION OF IPNLMS ALGORITHMS

Adaptive combinations of filters is a simple, yet effective, way to improve the performance of adaptive algorithms [10, 12]. Recently, there has been an increasing interest on these algorithms, both on theoretical aspects, and also on the proposal and study of practical rules for implementing the combinations. Here, we will consider a combination of two independent IPNLMS filters that differ on their step sizes (μ_1 and μ_2) or their asymmetry factors (κ_1 and κ_2):

- Setting $\mu_1 > \mu_2$ and $\kappa_1 = \kappa_2$, the overall filter will enjoy faster convergence of the filter with μ_1, together with the smaller steady-state error associated with step size μ_2.
- Selection of $\mu_1 = \mu_2$, $\kappa_1 \lessapprox 1$ and $\kappa_2 = -0.5$ will result in an overall filter able to exploit a high degree of sparsity, but showing robust behavior to situations with dispersive optimal solutions.

In the rest of the section, we will briefly review the principles of combination schemes, paying special attention to some *ad hoc* modifications that are necessary for their extension to ANC. Such modifications are justified by the distinguishing characteristics of this application, namely the presence of the secondary path, and the unavailability of $d(n)$ for the FX configuration.

3.1 General principles of combination schemes

We will consider a convex combination of two filters characterized by

$$y(n) = \lambda(n)y_1(n) + [1-\lambda(n)]y_2(n), \tag{11}$$

where $y_1(n)$ and $y_2(n)$ are the outputs of the IPNLMS filters, and where $\lambda \in [0,1]$ is the mixing parameter characterizing the convex combination.

For an appropriate behavior of the combination filter, both IPNLMS components have to be updated independently of each other using their own adaptation rules (which were reviewed in the previous section) and parameters, whereas the combination parameter needs to be updated in order to minimize the power of the overall error signal. We will use the combination scheme from [13], which ensures that $\lambda(n)$ lies in the range by defining it as the output of a sigmoidal activation function,

$$\lambda(n) = \text{sigmoid}[a(n)] = \{1+\exp[-a(n)]\}^{-1}. \tag{12}$$

Then, at each iteration $a(n)$ is updated using a gradient descent scheme

$$a(n+1) = a(n) - \frac{\mu_a}{p(n)} \cdot \frac{\partial e^2(n)}{\partial a(n)}, \tag{13}$$

where μ_a is the step size for the combination and $p(n)$ is a normalizing factor. Then, $\lambda(n)$ is recovered according to (12). The particular analytical expression for the derivative in (13) depends on which ANC configuration is implemented, and will be presented next for the FX and MFX structures.

3.2 Conventional filtered-x structure

The combination of IPNLMS filters using the FX structure provides an algorithm that we will refer to as CIPNLMS-FX. Taking the derivative of $e^2(n)$ with respect to $a(n)$ gives

$$a(n+1) = a(n) - \frac{\mu_a}{p(n)} e(n)[e_1(n) - e_2(n)]\lambda(n)[1-\lambda(n)], \tag{14}$$

where $p(n)$ is given by [13]

$$p(n) = \beta p(n-1) + (1-\beta)[e_1(n) - e_2(n)]^2 \tag{15}$$

β being a constant close to 1. However, the FX structure only provides the error signal $e(n)$, and it is not straightforward to obtain error signals $e_1(n)$ and $e_2(n)$, in order to update $a(n)$. Thus, we will have to rely on some estimation, which can be obtained as follows. Using $\hat{\mathbf{h}}$, the estimated, disturbance signal is given by,

$$\hat{d}(n) = e(n) - \mathbf{y}^T(n)\,\hat{\mathbf{h}}, \tag{16}$$

and the error signals of the component adaptive filters can now be approximated by

$$\hat{e}_i(n) = \hat{d}(n) + \mathbf{y}_i^T(n)\hat{\mathbf{h}}, \quad i = 1,2. \tag{17}$$

These estimations can be used in (14) and (15) for the update of the mixing parameter. The CIPNLMS-FX algorithm for ANC is described in **Algorithm 1**.

Algorithm 1 CIPNLMS-FX algorithm.

Require: Reference signal $x(n)$ and error signal $e(n)$
Ensure: Output of the parallel filter $y(n)$
Update the vectors $\mathbf{x}_M(n)$ and $\mathbf{x}(n)$
$y_i(n) = \mathbf{w}_i^T(n)\mathbf{x}(n), \quad i = 1,2$
$y(n) = \lambda(n)y_1(n) + [1-\lambda(n)]y_2(n)$
Update vectors $\mathbf{y}(n)$ and $\mathbf{y}_i(n), \quad i = 1,2$
$\hat{d}(n) = e(n) - \mathbf{y}^T(n)\,\hat{\mathbf{h}}$
$\hat{e}_i(n) = \hat{d}(n) + \mathbf{y}_i^T(n)\hat{\mathbf{h}}, \qquad i = 1,2$
$p(n) = \beta p(n-1) + (1-\beta)[\hat{e}_1(n) - \hat{e}_2(n)]^2$
$a(n+1) = a(n) + \frac{\mu_a}{p(n)} e(n)[\hat{e}_1(n) - \hat{e}_2(n)]\lambda(n)[1-\lambda(n)]$
$\lambda(n+1) = \text{sigmoid}[a(n+1)]$
$v(n) = \mathbf{x}_M^T(n)\hat{\mathbf{h}}$
$g_{l,i}(n) = (1-\kappa)\frac{1}{2L} + (1+\kappa)\frac{|w_{l,i}(n)|}{\varepsilon + 2\|\mathbf{w}_i(n)\|}, \quad i = 1,2$
$\mu_{l,i}(n) = \frac{\mu_i g_{l,i}(n)}{\delta + \sum_{k=0}^{L-1} g_{k,i}(n) v(n-k)^2}, \quad i = 1,2$
$w_{l,i}(n+1) = w_{l,i}(n) - \mu_{l,i}(n) v(n-l)\hat{e}_i(n), \quad i = 1,2$

3.3 Modified filtered-x structure

The combination of IPNLMS filters with the MFX structure provides the CIPNLMS-MFX algorithm. This algorithm estimates the disturbance signal, and the errors for each of the components according to (8)–(10).

In this case, the derivative of the squared error signal with respect to $a(n)$ results in

$$a(n+1) = a(n) - \frac{\mu_a}{p(n)} e(n)[y_1(n) - y_2(n)]\lambda(n)[1-\lambda(n)], \tag{18}$$

where $p(n) = \beta p(n-1) + (1-\beta)[y_1(n) - y_2(n)]^2$.

4. SIMULATION RESULTS

In this section, we carry out a series of experiments in an ANC setup to illustrate the performance of the adaptive algorithms that have been introduced. We start by showing the potential advantages of proportionate schemes over standard NLMS for realistic room responses. Then, we turn our attention to the problem of boosting the performance of IPNLMS even further by means of combinations. Both the conventional and modified filtered-x structures for ANC will be considered.

To analyze robustness of the algorithms when identifying paths with different degrees of sparsity, all our simulations will initially use a non-so-sparse primary path [Fig. 4(a)], that will then be changed to a sparse one [Fig. 4(b)] provoking a re-convergence of the adaptive filter. Both primary paths are realistic impulse responses with 350 taps. The sparse primary path has been obtained by taking the first 105 samples of the corresponding impulse response and zero-padding to length 350. The secondary path between the loudspeaker and the error sensor is also depicted in Fig. 5 and is assumed to be known by the adaptive controller (e.g., via

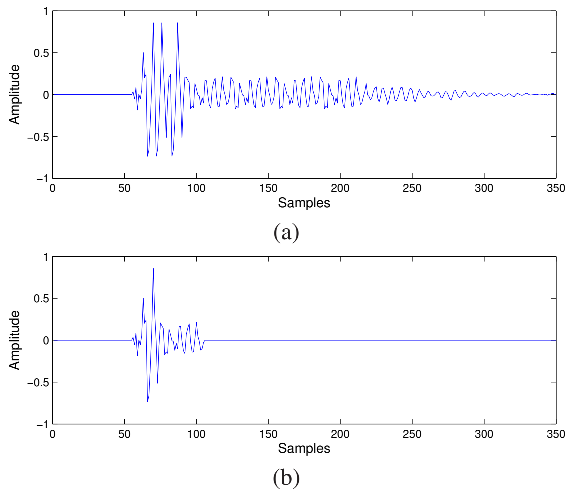

Figure 4: Primary paths with 350 coefficients used in the simulations: (a) dispersive, and (b) sparse.

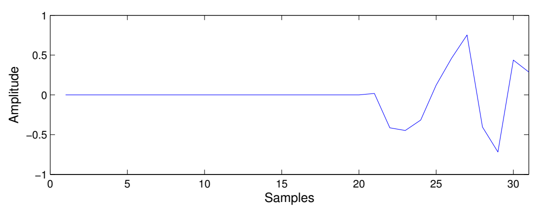

Figure 5: Sparse secondary path.

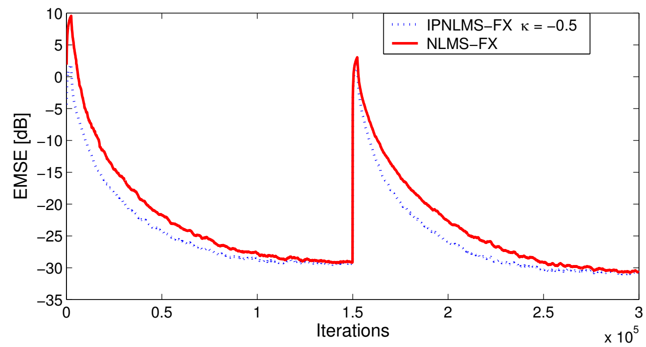

Figure 6: EMSE evolution for the NLMS-FX ($\mu = 0.2$) and IPNLMS-FX ($\mu = 0.2$ and $\kappa = -0.5$) filters.

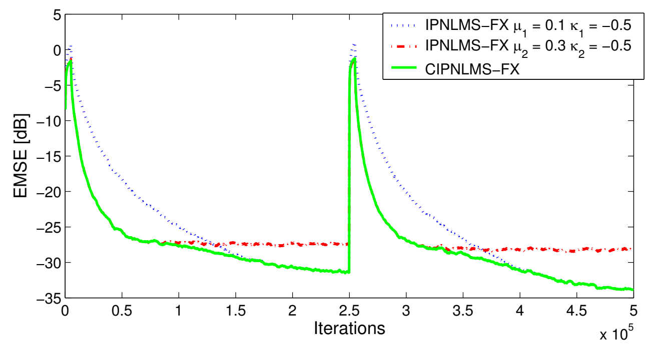

Figure 7: EMSE performance for the adaptive combination of two IPNLMS algorithms in a conventional filtered-x ANC configuration (CIPNLMS-FX). Component settings are shown in the legend.

prior offline estimation). This secondary path, with only 11 active coefficients, has been generated by truncating a real impulse response. The rest of the settings for the ANC scenario are as follows: The input signal is a zero-mean Gaussian random variable with unit variance, whereas the disturbance signal $d(n)$ is obtained as the output of the primary path, contaminated by zero-mean Gaussian additive noise, $e_0(n)$, with variance adjusted to get SNR $= 30$ dB.

Common settings for the NLMS and IPNLMS adaptive filters are $L = 320$, $\delta = 10^{-7}$, and $\varepsilon = 10^{-6}$, whereas the step sizes and asymmetry parameters (κ) are adjusted to different values depending on the purpose of the experiment. Regarding the adaptation of the combination parameter, we use $\mu_a = 0.1$ and $\beta = 0.9$ in all cases.

The figure of merit to evaluate the performance of the different methods will be the excess mean-square error, $\mathrm{EMSE}(n) = E\{[e(n) - e_0(n)]^2\}$, which will be estimated by averaging 100 independent runs of the algorithms.

4.1 Comparison of the IPNLMS-FX and NLMS-FX filters

To start with, we illustrate the performance gain that can be achieved by using proportionate schemes in ANC filtering structures. To this end, Fig. 6 represents the EMSE of the NLMS and IPNLMS filters in a FX structure (NLMS-FX and IPNLMS-FX algorithms, respectively) when using a common step size ($\mu = 0.2$) and $\kappa = -0.5$ for IPNLMS-FX, as recommended in [8]. As can be observed, IPNLMS-FX exhibits faster convergence both initially, and especially after the change in the primary path that increases the degree of sparsity of the path to identify. Similar results have been obtained for the MFX structure (not shown here).

4.2 Combination of IPNLMS filters with different step sizes

In this subsection, we consider the combination of two IPNLMS filters with different adaptation speeds to improve the well-known convergence *vs* steady-state misadjustment tradeoff that is inherent to adaptive filters. We have studied the performance of a combination of two IPNLMS algorithms with settings $\mu_1 = 0.1$, $\mu_2 = 0.3$, and $\kappa_1 = \kappa_2 = -0.5$. Fig. 7 shows the EMSE evolution for the two component filters as well as for their combination using a FX structure (CIPNLMS-FX). Again, similar behavior would be obtained for the MFX configuration.

As expected, the filter with large step size shows faster convergence, whereas the component with $\mu_1 = 0.1$ achieves smaller error in steady-state. The CIPNLMS-FX method inherits the best properties of each component, thus combining fast convergence and small steady-state misadjustment. This capability has been well-studied in the adaptive filtering literature, mostly in an identification configuration. Here, we observe that such an improvement can also be achieved in ANC setups, where the output of the filter passes through a secondary path before being added to the disturbance signal.

4.3 Combination of IPNLMS filters with different asymmetry factors

In Subsec. 4.1 we showed how proportionate schemes can accelerate the convergence with respect to standard NLMS filters, benefiting from sparseness in typical room impulse responses. When the degree of sparsity of the optimal solution is very large, IPNLMS performance can be further improved by selecting κ very close to 1. However, when doing so, filter performance can be seriously degraded if the path is not as sparse as expected. In order to benefit from very sparse paths whenever possible, while maintaining an appropriate response to more dispersive solutions, we consider in this section a combination of two IPNLMS algorithms just differing in

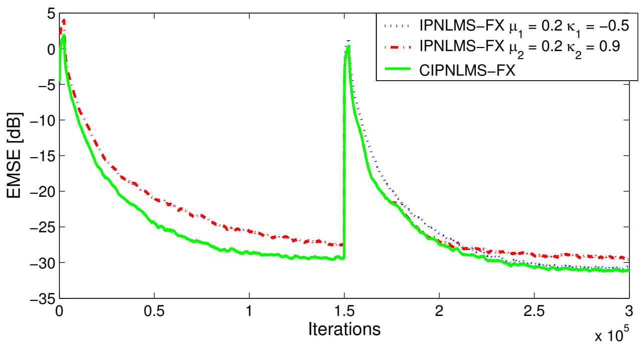

Figure 8: EMSE performance for the adaptive combination of two IPNLMS algorithms in a conventional filtered-x ANC configuration (CIPNLMS-FX). Component settings are shown in the legend.

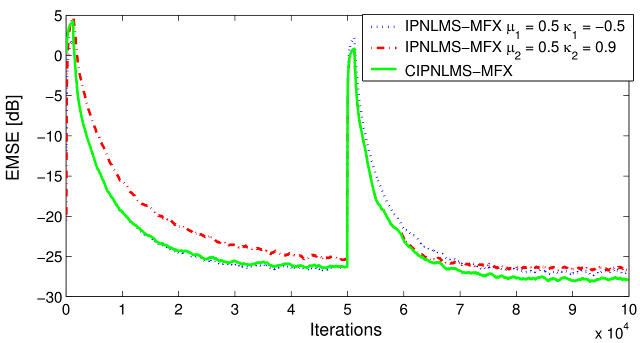

Figure 9: EMSE performance for the adaptive combination of two IPNLMS algorithms in a modified filtered-x ANC configuration (CIPNLMS-MFX). Component settings are shown in the legend.

their asymmetry factors. Thus, we use $\mu_1 = \mu_2 = 0.2$, $\kappa_1 = -0.5$ and $\kappa_2 = 0.9$. Fig. 8 presents the EMSE of CIPNLMS-FX with the FX scheme embedded, and of its two component filters (IPNLMS-FX). We see that the combination scheme retains the best characteristics of each component. During the first part of the experiment, the IPNLMS-FX filter with $\kappa_2 = 0.9$ performs poorly. However, this is precisely the filter that shows faster convergence when the acoustic path is sparse (after the change at iteration 150,000), and the combination benefits from this.

Simulation results using the IPNLMS filters based on the MFX structure are displayed in Fig. 9. Results can be discussed in a very similar way to the FX case. Interestingly, using the MFX configuration, the CIPNLMS-MFX filter is able to outperform both component filters with respect to steady-state EMSE. This is due to low correlation between component filter errors and the variance reduction that results from their average (see, e.g., [9]).

5. CONCLUSIONS

In this paper, we have proposed novel adaptive algorithms for ANC applications both using conventional and modified filtered-x structures. First, proportionate adaptation implemented with the IPNLMS algorithm and based on one of the previous filtering structures has been shown to provide improved convergence with respect to an adaptive filter based on the standard NLMS algorithm. To improve the convergence *vs* steady-state error, as well as to make the algorithms more robust to unknown or time-varying degrees of sparsity, combinations of IPNLMS filters have been presented, introducing several *ad hoc* modifications motivated by the characteristics of ANC systems. Simulation results in non-stationary conditions support the advantages of the presented schemes. The performance of such combination schemes, the CIPNLMS-FX (based on the conventional filtered-x scheme) and the CIPNLMS-MFX (based on the modified filtered-x scheme) has been illustrated when combining IPNLMS filters with different parameter settings, showing that the combination filter inherits the best properties of each component filter, and in some cases it can outperform both filters.

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