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Supply–demand balancing for power management in smart grid: A Stackelberg game approach

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HIGHLIGHTS

- Proposal of a price-based demand-response model to balance supply and demand.
- Formulation of the electricity trading process into a Stackelberg game.
- Introduction of a pricing function as the coordinator during the trading process.
- Proposal of an iterative algorithm to determine optimal generation and demands.
- Performances of flattening peak demands and reducing supply–demand mismatch.

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ABSTRACT

Demand-response (DR) is regarded as a promising solution for future power grids. Here we use a Stackelberg game approach, and describe a novel DR model for electricity trading between one utility company and multiple users, which is aimed at balancing supply and demand, as well as smoothing the aggregated load in the system. The interactions between the utility company (leader) and users (followers) are formulated into a 1-leader, *N*-follower Stackelberg game, where optimization problems are formed for each player to help select the optimal strategy. A pricing function is adopted for regulating real-time prices (RTP), which then act as a coordinator, inducing users to join the game. An iterative algorithm is proposed to derive the Stackelberg equilibrium, through which optimal power generation and power demands are determined for the utility company and users respectively. Numerical results indicate that the proposed method can efficiently reshape users' demands, including flattening peak demands and filling the vacancy of valley demands, and significantly reduce the mismatch between supply and demand.

1. Introduction

Traditional power grids are confronting the challenges of increased demand, and grid stability and environmental pollution [1,2]. Smart grids are envisioned as novel power-grid systems incorporating a smart metering infrastructure capable of sensing and measuring the power consumption of users [3–5], along with demand-response (DR) programs that promise solutions for enhancing the efficiency of future power girds [6–8]. DR considers energy usage changes of users in response to varying electricity prices or to incentive payments with the aim of balancing supply and demand and reducing power generation costs through alleviation of the peak load and shifting demand from on-peak to off-peak times [9–11]. Hence, it hopes to achieve better utilization of generated power and to bring economic benefits for both the utility supplier and users. Using a DR program, it becomes possible for the utility supplier to motivate users to jointly flatten the demand curve and match supply to demand [12,13], ensuring the stability of the grid [14–17].

Given the interoperation parameters among different entities in the DR program, game theory provides a naturally suitable framework for modeling interactions among different participators with various objectives [18–20]. Recently, Stackelberg games, which are used to study hierarchical decision-making processes of multiple decision makers, have attracted attention in the design of energy management schemes [21]. The Stackelberg game has been used to model electricity trading between the retailer and customers [22], with the aim of minimizing the customer's daily payments while maximizing the retailer's profit by optimizing electricity

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prices. Chen et al. [23] proposed a Stackelberg game-based power scheduling scheme between a service provider and residential consumers with similar objectives; the inconvenience cost incurred by delaying loads to a cheaper price period is also considered, alongside minimization of electricity bills. A bi-level programming technique has been used [24] to design a Stackelberg game for modeling the demand response in electricity retail markets with the aim of reducing the comfort losses of consumers as well as the costs of purchasing electricity in the lower sub-problems, which is subject to the retailer's upper sub-problem of reducing imbalances caused by deviations in wind power production from day-ahead forecasts. Kilkki et al. [25] proposed a Stackelberg game scenario for electricity markets, wherein the retailer is taken as the main perspective, with the goal of profit maximization. A simulation framework was designed involving customers' uncertainties of electricity storage space heating loads, upon which partial imbalance could be eliminated by offering additional discounts to customers. Maharjan et al. [26] presented a Stackelberg game framework involving multiple utilities and consumers aimed at maximizing each game player's revenue.

In general, the players in a game, together with their strategies and utility functions, differ from each other according to the specific system model [27]. Most DR models presented so far aim to maximize the profit of a utility/retailer/service provider without considering load fluctuations in the power system [22–24,26]. However, in practice, it is also important to flatten loads in the system in order to avoid building expensive backup generators to compensate for the peak load [15,28], and a reduced peak load is advantageous for maintaining the stability of the power grid [29].

In this paper, we present a novel demand-response model between one utility company and multiple users. Unlike previous studies, which dealt solely with profit maximization for the utility company and cost minimization for the user, this study aimed to balance supply and demand as well as flatten the aggregated loads in the system while guaranteeing the profit of the utility company and cost minimization for the user through carefully defining the objective function at each side. The main contributions from this paper are as follows:

(1) A price-based DR model is proposed for modeling the electricity trading process between the utility company and users, with the aim of balancing supply and demand, as well as smoothing the aggregated load in the system.
(2) The interactions between the utility company and users is formulated into a 1-leader, N-follower Stackelberg game, where a pricing function is adopted for regulating real-time prices (RTP) and acts as a coordinator to induce users to join the proposed game.
(3) An iterative algorithm is proposed between the utility company and users to derive the Stackelberg equilibrium, through which the optimal power generation and demands are determined for the utility company and users respectively.

The rest of the paper is organized as follows: In Section 2, the system model is presented in detail including the formulation of the Stackelberg game and description of an iterative algorithm for reaching the outcome of the game. Section 3 provides the numerical analyses of the proposed method. Conclusions and future works are presented in Section 4.

2. System model

Fig. 1 shows the system model with advanced metering infrastructures enabling two-way communication between one utility company and a set $\boldsymbol{N}$ of multiple users with the number $N = |\boldsymbol{N}|$. The utility company should provide power to users at certain prices, and each user $n \in \boldsymbol{N}$ will manage their energy consumption when receiving the announced prices from the utility company.

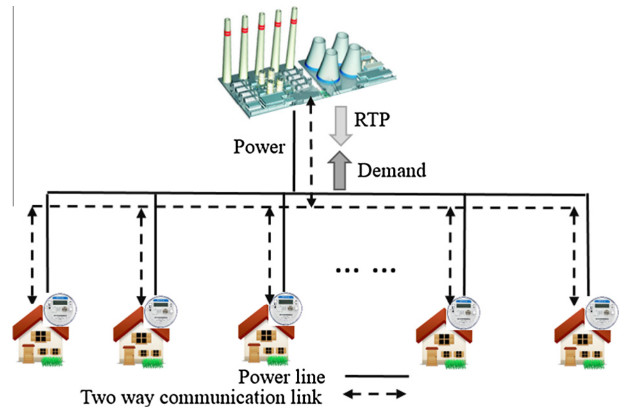


Fig. 1. System model between the utility company and multiple users.

2.1. Utility company model

Let $C_t(g_t)$ be the cost function for the utility company generating a quantity of power g_t during slot t ($t \in \boldsymbol{T}$, $T = |\boldsymbol{T}|$), which is a monotonically increasing function of the generation quantity and is strictly convex [10]; the most commonly used cost function is as follows [9,22,30].

$$C_t(g_t) = \frac{a_t}{2} g_t^2 + b_t g_t + c_t \tag{1}$$

where a_t, b_t and c_t are the generation coefficients, which are predetermined and may vary between different slots of the day.

The marginal cost function (where the marginal cost is defined as the change in the cost when the produced quantity changes by one unit) can be defined as follows

$$C_t'(g_t) = a_t g_t + b_t \tag{2}$$

With price-based DR programs, the utility company is responsible for regulating real-time prices to induce users to participate in the DR program, such that the utility company and users can jointly help calculate the quantity of generated power, as well as the demand, so as to reduce the difference between supply and demand.

To guarantee profit for the utility company, it is clear that real-time prices used to bill users should not be lower than the marginal cost. An efficient pricing function has been proposed [15], whereby the utility company regulates the price $p_t(g_t)$ for slot t by multiplying a time-dependent profit coefficient λ_t ($\lambda_t \geqslant 1$) with the marginal cost, i.e.,

$$p_t(g_t) = \lambda_t C_t'(g_t) = \lambda_t (a_t g_t + b_t) \quad \lambda_t \geqslant 1 \tag{3}$$

The effectiveness of the pricing function in (3) has been validated [15], and coordinates the interactions between the utility and users, and helps to minimize the generation cost of the utility company.

According to (3), the daily prices can be expressed as $\boldsymbol{p}(\boldsymbol{g}) = [p_1(g_1),\ p_2(g_2), \ldots,\ p_T(g_T)]$ or $[p_t(g_t)]_{t=1}^{T}$ where $\boldsymbol{g} = [g_1,\ g_2, \ldots, g_T]$ denotes the power generation vector across a day. These prices are then used to encourage users to shift demand to off-peak times.

From the utility company's perspective, besides considering a reduction in the generation cost, it is also desirable to smooth the hourly generation [15,31], so as to avoid building expensive backup generators to compensate for peak load. A reduced peak load is thus beneficial for maintaining the stability of the power grid [15,29]. Accordingly, we assume that the objective of the utility company is to determine the optimal generation vector through minimizing variations in generation [16,29,32], while meeting the requirements of the user [16], through which supply and demand can be matched. To this end, the optimization problem is formulated as follows

$$\min_{g_t} \quad \boldsymbol{U}_{UC}(\boldsymbol{g}) = \sum_{t \in \boldsymbol{T}} (g_t - \bar{g})^2 \tag{4a}$$

$$s.t.\ L_t \leqslant g_t \leqslant \min\ (g_t^+, L_t^{\max}) \tag{4b}$$

where $\boldsymbol{U}_{UC}$ denotes the utility function of the utility company, $\bar{g}$ represents the average power generation during the day; i.e., $\bar{g} = \sum_{t \in \boldsymbol{T}} g_t / T$, and L_t is the sum of power demands of all users at slot t, i.e., $L_t = \sum_{n \in \boldsymbol{N}} l_{n,t}$, and $l_{n,t}$ denotes the demand of user n during slot t. Note that (4b) regulates g_t such that it will always be equal to or greater than L_t; the primary reason for adopting this constraint is to guarantee that generation can meet users' requirements at all times [16]. $L_t^{\max}$ denotes the maximum power demand of all users at slot t, and g_t^+ is the maximum generation capacity of the utility company during slot t.

Note that the objective in (4a) differs from profit maximization as defined in [22–24,26]; however, the proposed model indirectly accounts for profit because the pricing function in (3) has been validated to guarantee low generation costs [15]. To some extent, reducing costs is equivalent to increasing profits. Moreover, the objective defined in (4a) brings additional advantages besides smoothing the hourly generation (or minimizing the generation variance). In power systems, the load factor (LF) is utilized as a measure of efficiency for electrical energy usage, which is defined as the ratio of the average energy demand to the maximum demand during a period. A greater value of LF indicates higher energy usage efficiency. As proven in previous studies [29,33], minimizing the variance of the generation in (4a) is practically equivalent to maximizing the load factor, which is defined as follows:

$$LF = \frac{L_{avg}}{L_{\max}} \tag{5}$$

where $L_{avg} = \sum_{t \in \boldsymbol{T}} L_t / T$ denotes the average load in the system, and $L_{\max} = \max L_t$ ($\forall t \in \boldsymbol{T}$) represents the maximum load during a single slot.

2.2. User model

The utility function for each user n is defined as

$$\boldsymbol{U}_n(\boldsymbol{l}_n) = \sum_{t \in \boldsymbol{T}} \varphi_{n,t}(l_{n,t}) - \sum_{t \in \boldsymbol{T}} p_t(g_t) \cdot l_{n,t} \tag{6}$$

where $\boldsymbol{l}_n = [l_{n,1}, l_{n,2}, \ldots, l_{n,T}]$ represents the power demand vector of user n, and $p_t(g_t) \cdot l_{n,t}$ represents the payment of user n for consuming power $l_{n,t}$ during slot t, where $p_t(g_t)$ ($\forall t \in \boldsymbol{T}$) are received from the utility company. $\varphi_{n,t}(l_{n,t})$ denotes the satisfaction gain of user n as a function of its consumed power $l_{n,t}$ at slot t. Without losing generality, $\varphi_{n,t}(l_{n,t})$ adopts the quadratic function form defined as follows [30,34]

$$\varphi_{n,t}(l_{n,t}) = \omega_{n,t} l_{n,t} - \frac{\theta_n}{2} l_{n,t}^2, \quad \omega_{n,t} > 0 \quad \theta_n > 0 \tag{7}$$

where $\omega_{n,t}$ is a user preference parameter characterizing user types, which varies between users and may also vary along different time slots [34], and θ_n is a predetermined constant [14]. As indicated by (7), a user with a greater $\omega_{n,t}$ prefers to consume more $l_{n,t}$ in order to improve his/her satisfaction level.

Each user should obtain its optimal power demand vector by maximizing its utility function.

$$\max \quad \boldsymbol{U}_n(\boldsymbol{l}_n) = \sum_{t \in \boldsymbol{T}} \varphi_{n,t}(l_{n,t}) - \sum_{t \in \boldsymbol{T}} p_t(g_t) \cdot l_{n,t} \tag{8a}$$

$$s.t.\ l_{n,t}^- \leqslant l_{n,t} \leqslant l_{n,t}^+ \tag{8b}$$

where $l_{n,t}^-$ ($l_{n,t}^+$) represents the minimum (maximum) power demand of user n at slot t.

In addition, a user may not wish to reduce daily power consumption but may be willing to shift the consumption from peak to off-peak time [10,30,35]; thus, a temporally-coupled constraint in (8c) can be included to couple the power consumption across the time horizon so as to constrain the cumulative consumption at a user designated value (e.g., daily target power consumption, denoted as L_n).

$$\sum_{t \in \boldsymbol{T}} l_{n,t} = L_n \tag{8c}$$

2.3. Problem formulation between the utility company and users

In a realistic power system, it is expected that generation always matches demand; smart metering and two-way communications enable the supply and demand sides to interact by exchanging price and demand information. For instance, the price vector announced by the utility company will affect how the users determine their optimal power demands; in contrast, the adjusted power demands of the users will inversely impact on the utility company's generation plan, as the utility company would like to adjust generation in order to balance demand and supply, which thus pushes the utility company to regulate the new price vector. As a consequence, the adjusted power demands of a user will inherently affect how other users determine their power demands, due to the new price vector. Thus, these factors naturally lead to interactions between the utility company and users.

The Stackelberg game is suitable means to illustrate the concept behind the presented system model, where the utility company acts as the leader announcing prices to followers, which are the N users. Given those prices, users will react by playing a non-cooperative game, as each user's decision will inherently affect how other users make decisions.

The formal definition of the 1-leader, N-follower Stackelberg game is the following:

$$\xi = \langle UtilityCompany \ \cup \boldsymbol{N}, \{\Omega_{UC}\}, \{\Omega_n\}_{n \in \boldsymbol{N}}, \boldsymbol{U}_{UC}, \boldsymbol{U}_n \rangle \tag{9}$$

- Player set $UtilityCompany \ \cup \boldsymbol{N}$:

The utility company acts as the leader and the users in set $\boldsymbol{N}$ take the roles of followers in response to the utility company's strategy.

- Strategy set Ω_{UC} and Ω_n:

$\Omega_{UC} = \{\boldsymbol{g} | \boldsymbol{g} \in R^T, L_t \leqslant g_t \leqslant \min \ (g_t^+, L_t^{\max})\}$ denotes the feasible strategy set of the utility company referring to (4b), from which the utility company chooses its strategy $\boldsymbol{g}$ (the daily power generation vector). And each user will select its strategy $\boldsymbol{l}_n$ (daily power demands) from its feasible strategy set $\Omega_n = \{\boldsymbol{l}_n | \boldsymbol{l}_n \in R^T, l_{n,t}^- \leqslant l_{n,t} \leqslant l_{n,t}^+\}$ which is defined based on (8b).

- Utility functions $\boldsymbol{U}_{UC}$ and $\boldsymbol{U}_n$:

The utility function evaluates the selected strategy of a player in the game. $\boldsymbol{U}_{UC}$ denotes the utility function of the utility company which is defined in (4a) and (6) defines the utility function of each user n, i.e., $\boldsymbol{U}_n$.

The desired outcome of a given hierarchical decision-making game takes the form of the Stackelberg equilibrium (SE). The definition of a Stackelberg equilibrium strategy (SES) together with an SE for a two-person game is given in Appendix A. As an extension, the SE of a 1-leader, N-follower game corresponds to the status at which the leader maximizes its utility given the reaction set of the followers while the followers respond to the leader's announced strategy by playing according to a specific equilibrium concept [36]. Following Definition 1 in Appendix A, an SES for the leader (utility company) in the game ξ should satisfy

$$\max_{\boldsymbol{L}\in R_{\boldsymbol{N}}(\boldsymbol{g}^*)} U_{UC}(\boldsymbol{g}^*,\boldsymbol{L}) = \min_{\boldsymbol{g}\in\Omega_{UC}} \max_{\boldsymbol{L}\in R_{\boldsymbol{N}}(\boldsymbol{g})} U_{UC}(\boldsymbol{g},\boldsymbol{L}) = U^*_{UC} \tag{10}$$

where $\boldsymbol{L} = [\boldsymbol{l}_1, \boldsymbol{l}_2, \ldots, \boldsymbol{l}_N]$ represents the strategy profile of all the users, and $R_{\boldsymbol{N}}(\boldsymbol{g})$ denotes the *best response set* of N users to the strategy $\boldsymbol{g} \in \Omega_{UC}$ of utility company, clearly, $R_{\boldsymbol{N}}(\boldsymbol{g})$ is included in the joint strategy sets of all the users, i.e., $R_{\boldsymbol{N}}(\boldsymbol{g}) \subseteq \Omega_1 \times \Omega_2, \ldots, \Omega_N$. The latter two terms in (10) imply that, depending on the status of SE, the utility company minimizes the variation in the generated power in response to the set of all the users, wherein the reaction set contains all the users' optimal power demand vectors as responses to the utility company's strategic choices.

Furthermore, if the quantity U^*_{UC} in (10) admits a unique value, it means the utility company will not accept a utility value that is higher than U^*_{UC}, which thus constitutes a secured utility level for the utility company.

Accordingly, the SE for the proposed game can be defined as a strategy profile $(\boldsymbol{g}^*, \boldsymbol{L}^*)$ [36], where $\boldsymbol{g}^*$ is an SES for the utility company satisfying (10), and $\boldsymbol{L}^* \in R_{\boldsymbol{N}}(\boldsymbol{g}^*)$ denotes the strategy profile that is in equilibrium with $\boldsymbol{g}^*$, and that provides optimal strategies for all the users.

In conventional game theory, a player's utility is a function of both players' strategies (e.g., in a two-person game) [37]. Accordingly, in the remainder of the paper, we write $\boldsymbol{U}_{UC}$ and $\boldsymbol{U}_n$ as a function of both the utility company's and users' strategies because the decision made by either side will affect how the other side chooses the strategy, as mentioned above. However, it should be noted that even as we write $\boldsymbol{U}_n$ in the form $\boldsymbol{U}_n(\boldsymbol{g}, \boldsymbol{l}_n, \boldsymbol{l}_{-n})$ (where $\boldsymbol{l}_{-n}$ denotes all other N-1 users' strategies except user n), $\boldsymbol{U}_n$ is not *directly* affected by the utility company's strategy $\boldsymbol{g}$ or $\boldsymbol{l}_{-n}$, but directly related to the utility company's price vector $\boldsymbol{p}(\boldsymbol{g})$ (i.e., a function of the utility company's strategy $\boldsymbol{g}$ as indicated in (3)), which actually acts as the coordinator between the utility company and users. Moreover, as described earlier, the strategy chosen by user n will also affect how the other N-1 users choose their strategies, due to the inherence among them. For consistency, we apply the $\boldsymbol{U}_n(\boldsymbol{g}, \boldsymbol{l}_n, \boldsymbol{l}_{-n})$ form and declare that $\boldsymbol{U}_n(\boldsymbol{g}, \boldsymbol{l}_n, \boldsymbol{l}_{-n})$ is affected by the utility company's strategy $\mathbf{g}$ and all other N-1 users' strategies $\boldsymbol{l}_{-n}$.

2.4. The existence of the Stackelberg equilibrium

In this subsection, the existence of SE is discussed. As mentioned in Section 2.3, when provided with the utility company's prices, users will play a non-cooperative game in reaction to these prices. It has been shown that a unique NE exists in a strictly concave N-player game [38]. In the following, we show that a non-cooperative game among users is equivalent to a strictly concave N-player game.

First, by observing (6), it is straightforward that $\boldsymbol{U}_n$ is continuous and differentiable in Ω_n such that $\boldsymbol{U}_n$ can be found analytically. Taking user n as an example, when receiving the price vector $\boldsymbol{p}(\boldsymbol{g})$ from the utility company, the best-response function can be obtained by taking the first derivative of $\boldsymbol{U}_n$ with respect to $l_{n,t}$; i.e.,

$$\frac{\partial \boldsymbol{U}_n}{\partial l_{n,t}} = \omega_{n,t} - \theta_n l_{n,t} - p_t(g_t) \tag{11}$$

By setting (11) to zero, the best-response function is obtained as follows:

$$l_{n,t}(p_t(g_t)) = \frac{\omega_{n,t} - p_t(g_t)}{\theta_n} \tag{12}$$

Furthermore, if the Hessian matrix $\mathbf{H}(\boldsymbol{U}_n)$ is definite negative, then $\boldsymbol{U}_n$ is strictly concave. By taking the second derivative of $\boldsymbol{U}_n$ with respect to $\boldsymbol{l}_n$, we obtain $\mathbf{H}(\boldsymbol{U}_n)$ as follows

$$\frac{\partial^2 \boldsymbol{U}_n}{\partial l_{n,t} \partial l_{n,s}} = \begin{cases} -\theta_n & \text{when } t = s \\ 0 & \text{when } t \neq s \end{cases} \tag{13}$$

where s denotes any slot in the time horizon $\mathbf{T}$. From (13), we may observe that all the diagonal elements of $\mathbf{H}(\boldsymbol{U}_n)$ are negative due to (7), and the off-diagonal elements are zero. Therefore, $\mathbf{H}(\boldsymbol{U}_n)$ is negative definite.

Second, one may observe that the user strategy set Ω_n $(\forall n \in \boldsymbol{N})$ is convex, closed and bounded, since the set Ω_n is already defined as a convex constraint (see Section 2.3).

From the above, we may conclude that a non-cooperative game among users is equivalent to a strictly concave N-player game and it follows that, a unique Nash equilibrium (NE) exists among N users [38].

As discussed above, each time the utility company's strategy is revealed, there exists a unique NE among users, which provides the best response strategy profile for users. In the presence of such a strategy profile, the utility company will adjust its strategy in order to minimize (4a). Note that if the users' group response (i.e., the NE) to the utility company's announced strategy is not unique, then it will result in ambiguity for the utility company when choosing its strategy [39], which forms the basis of an analysis of the existence of the SE.

In the presence of the strategy profile containing the best response strategies of all the users, the utility company chooses a strategy $\boldsymbol{g} \in \Omega_{UC}$ aiming to minimize (4a), where the result of (4a) – i.e., the variation in the generated power – either decreases or remains unchanged each time a new strategy is selected. Moreover, note the utility company's utility value in the form of (4a) has a lower bound (since the minimum "variance" is zero). Therefore, there exists a secured utility value U^*_{UC} for the utility company, which satisfies (10). Following the definition of the SE in Section 2.3, we conclude that an SE exists for the proposed 1-leader, N-follower Stackelberg game.

2.5. An iterative DR algorithm for SE

In Section 2.4, the NE was utilized to emphasize the existence of the SE analytically, where users should react to the utility company's strategy at the same time. However, in practice, it is not appropriate for users to respond to the utility company simultaneously, as they may neutralize each other's impact on the aggregated demands. Instead, we aim to design an iterative DR algorithm for reaching the SE in an asynchronous manner; i.e., supposing no two users adjust their power demands at the same time on receipt of the utility company's prices and, more importantly, information exchange between the utility company and a user is executed by hiding private information (e.g., user preference parameter $\omega_{n,t}$).

Algorithm 1: An iterative DR algorithm for the SE

1: The utility company arbitrarily initializes $\boldsymbol{g}^0 = [g_1^0, g_2^0, \ldots, g_T^0]$ and calculates the initial $\boldsymbol{p}^0 = [p_1^0, p_2^0, \ldots, p_T^0]$ according to (3), denote $\boldsymbol{g}^* = \boldsymbol{g}^0$.

2: The utility company sends $\boldsymbol{p}^0$ to all the users, and each user updates its demand vector $\boldsymbol{l}_n^*$ according to

$$\boldsymbol{l}_n^* = \arg \max_{\boldsymbol{l}_n} \ \{\boldsymbol{U}_n(\boldsymbol{g}, \boldsymbol{l}_n, \boldsymbol{l}_{-n}) \quad s.t. \quad l_{n,t}^- \leqslant l_{n,t} \leqslant l_{n,t}^+\}$$

3: Each user n sends $\boldsymbol{l}_n^*$ back to the utility company.
Start iteration with index k for convergence to SE:

4: Upon the received $\boldsymbol{l}_n^*$ from each user, the utility company updates $\boldsymbol{g}^{*,k}$ by solving

$$\boldsymbol{g}^{*,k} = \arg\ \min\ \boldsymbol{U}_{UC}(\boldsymbol{g}^k, \boldsymbol{L}^k) = \sum_{t \in \boldsymbol{T}} (g_t - \bar{g})^2$$
$$s.t.\ L_t^* \leqslant g_t^k \leqslant \min \quad (g_t^+, L_t^{\max})$$
$$\text{where } L_t^* = \sum_{n \in \boldsymbol{N}} l_{n,t}^*$$

5: Based on $\boldsymbol{g}^{*,k}$, the utility company updates $\boldsymbol{p}^k$ according to (3) and triggers iteration k:
Sequential polling of one user at each time:

6: Sequentially select a user n to send $\boldsymbol{p}^k$ at each time.

7: Upon the received $\boldsymbol{p}^k$, user n updates $\boldsymbol{l}_n^{*,k}$ according to

$$\boldsymbol{l}_n^{*,k} = \arg\ \max_{\boldsymbol{l}_n}\ \{\boldsymbol{U}_n(\boldsymbol{g}, \boldsymbol{l}_n, \boldsymbol{l}_{-n})\ s.t. \quad l_{n,t}^- \leqslant l_{n,t} \leqslant l_{n,t}^+\}$$

8: User n sends $\boldsymbol{l}_n^{*,k}$ back to the utility company in case $\boldsymbol{l}_n^{*,k}$ is updated, and then the utility company updates $\boldsymbol{g}^{*,k}$ by solving

$$\boldsymbol{g}^{*,k} = \arg\ \min \quad \boldsymbol{U}_{UC}(\boldsymbol{g}^k, \boldsymbol{L}^k) = \sum_{t \in \boldsymbol{T}} (g_t^k - \bar{g}^k)^2$$
$$s.t.\ L_t^* \leqslant g_t^k \leqslant \min \quad (g_t^+, L_t^{\max})$$
$$\text{where } L_t^* = \sum_{\substack{m=1 \\ m \neq n}}^{N-1} l_{m,t}^* + l_{n,t}^{*,k}$$

9: The utility company calculates new $\boldsymbol{p}^k$ accordingly and polls next user.
If the polling is not finished,
Go to line 6.
else
The utility company evaluates the SE and triggers the next iteration k+1 (go to line 5) in case the SE has not arrived.
end if

10: Repeat 5 to 9 until no player deviates from the current strategy, indicating the SE has arrived.

11: The utility company announces to the users that the SE has arrived.

Algorithm 1 begins with the utility company arbitrarily initializing the generation vector $\boldsymbol{g}^0 = [g_1^0, g_2^0, \ldots, g_T^0]$, and calculating the initial price vector $\boldsymbol{p}^0 = [p_1^0, p_2^0, \ldots, p_T^0]$ accordingly. Regard $\boldsymbol{g}^0$ as the optimal generation vector $\boldsymbol{g}^*$ temporarily, see line 1.

During the initialization, the utility company broadcasts $\boldsymbol{p}^0$ to all the users through the two-way communication link, upon which each user will update its demand vector $\boldsymbol{l}_n^*$ by solving its optimization problem (8); afterwards, each user sends $\boldsymbol{l}_n^*$ back to the utility company, see line 2 to 3.

In line 4, upon the received $\boldsymbol{l}_n^*$ from each user, the utility company will update its generation vector $\boldsymbol{g}^{*,k}$ (k denotes the index of iterations), by solving its optimization problem (4), wherein $\boldsymbol{L}_t$ is updated based on the newly received $\boldsymbol{l}_n^*$ from users.

In line 5, based on the $\boldsymbol{g}^{*,k}$ obtained in line 4, the utility company will update the price vector $\boldsymbol{p}^k$; next, the utility company triggers the kth iteration to interact with users, i.e., the utility company polls each user during iteration k. Fig. 2 depicts the interactions between the utility company and users during one iteration, where the utility company sequentially selects one non-repetitive user (e.g., user n) to send $\boldsymbol{p}^k$ at each time. On the receipt of $\boldsymbol{p}^k$, user n will update $\boldsymbol{l}_n^{*,k}$ by solving (8), see line 7.

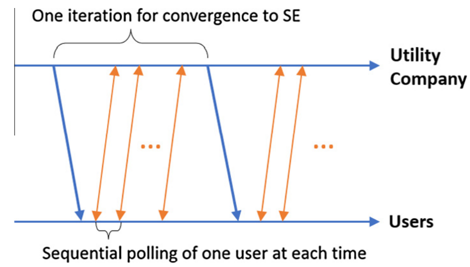


Fig. 2. Interaction between the utility company and users.

Next, in line 8, user n sends $\boldsymbol{l}_n^{*,k}$ back to the utility company, and the utility company updates $\boldsymbol{g}^{*,k}$ (by solving (4)). Here, it deserves notice that the lower constraint is updated to $L_t^* = \sum_{\substack{m=1 \\ m \neq n}}^{N-1} l_{m,t}^* + l_{n,t}^{*,k}$, wherein only $l_{n,t}^{*,k}$ is newly received from user n, and all other N-1 users' hourly aggregated loads remain the same as when interacting with the last user.

In line 9, the utility company calculates the new $\boldsymbol{p}^k$ according to the updated $\boldsymbol{g}^{*,k}$ obtained in line 8 and goes to line 6 to poll the next user. In the case where all users have been polled, the utility company will evaluate the SE for the kth iteration, and trigger the next iteration $k + 1$ if the SE has not been obtained (the algorithm then goes to line 5).

In this way, line 5 to line 9 will be repeated until the SE is obtained, where the utility company cannot further reduce the generation variation by updating the generation vector, indicating that it has obtained its secured utility value. Accordingly, the utility company announces to the users that the SE has arrived and each user chooses an optimal strategy obtained by playing with the utility company.

In the proposed algorithm, the utility company selects users in an asynchronous fashion, i.e., no two users update their strategies simultaneously. This can be realized by supposing that the utility company can determine a time when each user should update its strategy. Note that each time new price information is received from the utility company, a user will respond by reducing demand during high-price periods, while increasing demand during low-price periods, resulting in flattened demands. Such "flattened demands" sent from the user to the utility company will naturally contribute to the lowering of the generation variance from the perspective of the utility company, because constraint (4b) couples users' aggregated demands with generation, and the utility company will adjust generation to meet users' flattened demands. Furthermore, as the objective of the utility company is to minimize the generation variance (equivalent to acquiring flattened generation), through a number of iterations the generation variance will gradually decrease and the algorithm will eventually converge to a fixed point, i.e., either to zero or a lower bound of variance.

3. Numerical analyses

This section presents the numerical analyses and assesses the performance of the proposed algorithm. For ease of illustration, simulations are conducted based on one utility company and three users. The entire time cycle is divided into 24 time slots representing the 24 h of a day. For the generation cost, the cost of the same load can be different at different times of day. In particular, the cost may be less at night compared to the day time [10]. For simplicity, we set the parameters in (1) to $a_t = 0.02$ during daytime, i.e., from 8:00 to 24:00 and $a_t = 0.01$ in the remaining hours, $b_t = 0.2$ and $c_t = 0$, and the price coefficient λ_t was selected to be 1.2 [15]. For the user utility function, the parameter θ_n was selected as 0.1 for all users, and $\omega_{n,t}$ was set to different values of 5.0, 5.5, 6.0; the effect of these differing values will be discussed later in the simulation results. The target power demand of user 1, 2 and 3 is shown as a dotted line in Fig. 4(a)–(c), respectively, where the target power demand is defined as the power demand of a user without the adoption of demand response management. In this study, we obtained target power demands from an existing electric power market, which provided daily loads for certain local regions [40]; however, we changed the order of magnitude from GW to kW to account for the limited number of users in the sample. The minimum and maximum of each user's demands are set to certain percentages of the target demands, as given in Table 1 [16]. For simplicity, the maximum generation capacity was assumed to be equal to the maximum total power demands of all the users [34]; therefore, we have $g_t^+ = \sum_{n\in \boldsymbol{N}} l_{n,t}^+$, for all $t \in \boldsymbol{T}$. In the case whereby the temporally-coupled constraint in (8c) is included, we suppose that a user would like to consume a fixed amount of power equal to the sum of the target demands.

Fig. 3 shows the real-time prices obtained from Algorithm 1 by distinguishing two cases: with and without the temporally-coupled constraint of (8c). In the following, the performance will be analyzed with various scenarios.

3.1. Optimal power demands of users

Fig. 4 shows each user's power demands with and without constraint (8c) as shown by the blue and orange line, respectively, and also in comparison to each user's target demands. In general, for either case (with or without (8c)), a user demanded more power than the target amount during off-peak times, and curtailed their demand during peak times, indicating a large amount of demand was shifted from on-peak to off-peak slots.

Specifically, when (8c) was not applied, by comparing users' power demand results in Fig. 4, it can be observed that user 1 ($\omega_{n,t} = 5.0$) would be more willing to participate in the demand response process, as it reduced larger amounts of demand during the high price period compared with the other two users. This phenomenon coincides with the physical meaning of $\omega_{n,t}$ declared in Section 2, i.e., a user with a greater $\omega_{n,t}$ preferred to consume more $l_{n,t}$ in order to reach a higher satisfaction level and vice versa.

In the case where (8c) was applied, we found that each user demanded more power than without that constraint in order to complete the target daily consumption, whereas the extra demands were increased during lower price slots.

3.2. The comparison of supply and demand

Figs. 5 and 6 show the resulting hourly power generation (supply) together with users' hourly aggregated demands compared to

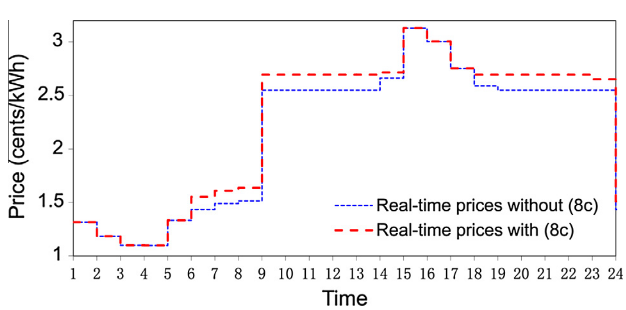
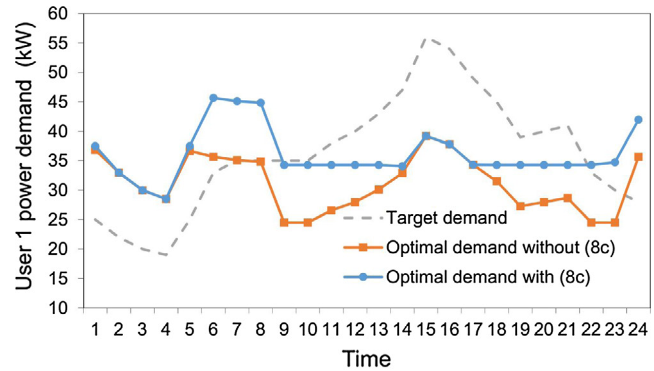
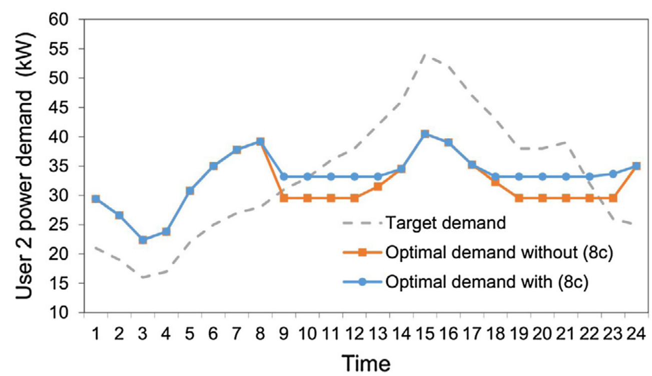


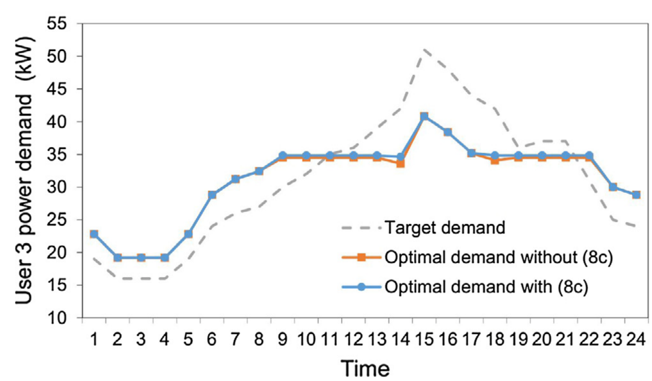
Fig. 3. Real-time prices with and without (8c).



(a) User 1 with w=5.0



(b) User 2 with w=5.5



(c) User 3 with w=6.0

Fig. 4. Users' optimal power demands with real-time prices. (a) User 1. (b) User 2. (c) User 3.

Table 1
User power demand ranges.

	User 1 (%)	User 2 (%)	User 3 (%)
Min demand	70	75	80
Max demand	150	140	120

the case when there was no demand response scheme applied, i.e., supposing hourly generation was chosen at the median between the minimum aggregated demands of all users ($\sum_{n\in \boldsymbol{N}} l_{n,t}^-$) and the maximum generation capacity (g_t^+); while each user demanded the target power amount regardless of energy cost. Thus, in Figs. 5 and 6, "users' aggregated demands without DR" were the sum of each user's target demands as illustrated in Fig. 4.

Clearly, when no demand response was applied, there existed a large gap between supply and demand. In the case where the demand response scheme was applied (without (8c) and with

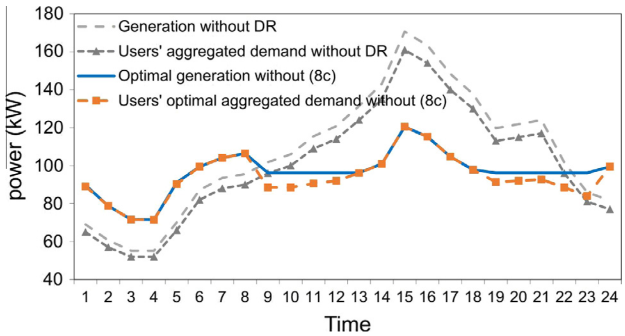


Fig. 5. The supply and aggregated demand without (8c).

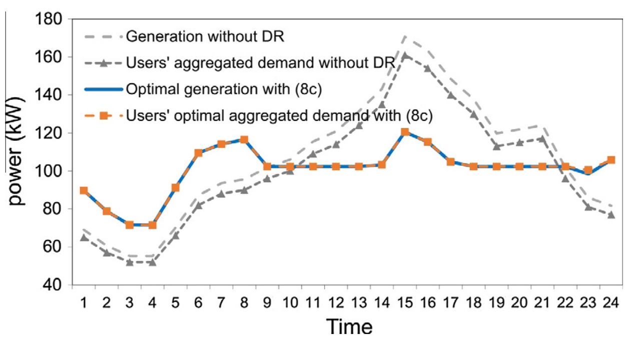


Fig. 6. The supply and aggregated demand with (8c).

(8c)), it efficiently reshaped the generation and users' demands including reducing the peak demand and filling the vacancy of valley demands. As shown in Figs. 5 and 6, the gap between supply and demand was reduced significantly. Moreover, without (8c), the mismatch cannot be eliminated completely as illustrated in Fig. 5. For comparison, the generation in Fig. 6 matched well with users' demands when (8c) was implemented. The numerical of analyses of the supply–demand mismatch will be discussed in the next part.

3.3. The performance evaluation

We evaluated the performance of three cases (No DR, DR without (8c) and DR with (8c)) from various aspects. The numerical comparison results are listed in Table 2. The load factor (LF) [33] is defined as the average to peak load ratio (see (5)), which is expected to be as large as possible.

From Table 2, it is observed that the peak demand apparently decreased from 161 kW h (per hour) to 121 kW h (per hour) with the help of the demand response scheme. As (8c) was not considered in Case 2, total demand is reduced by 160 kW h (per 24 h) compared to Case 1 and Case 3, however, Case 3 was able to achieve the lowest PAR and highest LF, which are advantageous for the utility company in balancing loads in the power system.

When comparing the generation amount and the total demand, it is clear that supply and demand were generally matched under the demand response scheme but that a large gap exists in Case 1, which can be seen in Figs. 5 and 6. Specifically, supply and demand were matched appropriately in Case 3.

In addition, both generation costs and user payments were much lower in Case 2 and Case 3 than in Case 1. Case 2 reduced payments more than Case 3; however, this was achieved at the expense of missing the users' target demands, meaning that some daily tasks may not be completed. Lastly, it can be seen that the generation variance in Case 2 and Case 3 (to meet users' target demands, Case 3 resulted in slightly higher variance than Case 2) was significantly reduced compared with Case 1, which is desirable for the utility company to maintain the stability of the power grid.

3.4. Scalability

For the three users above, the algorithm took seven iterations to converge to the SE. To examine the scalability of the algorithm, we also increased the user number from 20 to 200, wherein $\omega_{n,t}$ were randomly selected between [5.0, 6.0], and users' target hourly demands were randomly set from 14 kW h to 56 kW h (i.e., the min and max target demand of the three example users). Fig. 7 shows the number of iterations needed with increasing user number, and shows a linear rather than exponential increase in iterations, which is desirable for the proposed algorithm to be practically implemented in a smart grid application.

To get insight into the effectiveness of the proposed DR algorithm in presence of considerable number of users, we also present the resulted optimal supply and aggregated demand under the extreme case of 200 users. As shown in Fig. 8, by deploying the algorithm, the generation and users' aggregated demand were

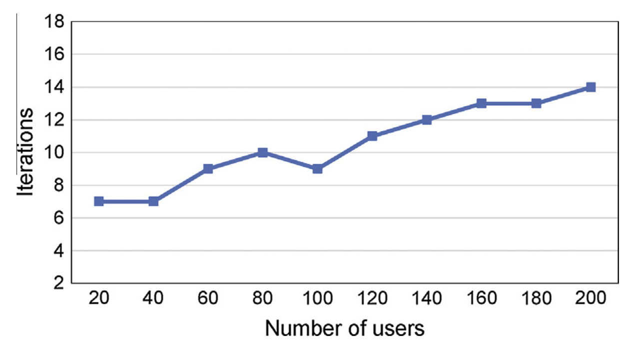


Fig. 7. The number of iterations toward the increasing user number.

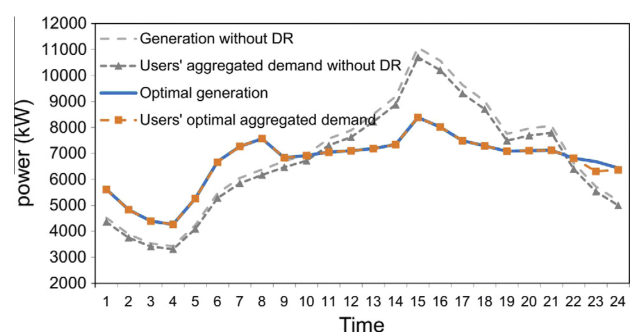


Fig. 8. The supply and aggregated demand under 200 users.

Table 2
Performance evaluation.

Cases	Peak demand (kW h)	Total demand (kW h)	LF	Generation amount (per 24 h) (kW h)	Generation cost ($)	Generation variance	Total payments ($)
Case 1: No DR	161	2414	0.625	2560	32.3	1080	76.4
Case 2: DR Without (8c)	121	2254	0.775	2314	23.5	141	49.3
Case 3: DR With (8c)	121	2414	0.833	2416	25.4	175	55.1

rescheduled and matched generally, resulting in smoothed overall loads in the system. In specific, the load factor was increased from 0.62 (without DR) to 0.8 (with DR), indicating that the presented algorithm is able to handle the power management problem between one utility and multiple users.

4. Conclusion and future work

We have described a Stackelberg game based demand response model between one utility company and multiple users, aimed at balancing supply and demand as well as flattening the aggregated load in the system. The game formulation process is described in detail together with an analysis of the existence of the Stackelberg equilibrium. An iterative algorithm between the utility company and users was proposed to derive the Stackelberg equilibrium, which provides the optimal power generation and demand for the utility company and users. The numerical results show that the proposed method can help flatten aggregated loads in the system and significantly reduce the mismatch between supply and demand. As an extension of the current work, intermittent power resources (e.g., photovoltaic cells and wind turbines) may be taken into account, so as to make the existing model accommodate dynamic ambient changes. Also, as a future study, the proposed algorithm can be evaluated in a distribution network with nodal pricing approaches and power flow analyses.

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Appendix A

A.1. Definition of a SES for the leader

Definition 1. In a two-person finite game with player 1 as the leader (player 2 as the follower), a strategy $s_1^* \in S_1$ is called a *Stackelberg equilibrium strategy* (*SES*) for the leader [36], if

$$\max_{s_2 \in R_2(s_1^*)} u_1(s_1^*, s_2) = \min_{s_1 \in S_1} \max_{s_2 \in R_2(s_1)} u_1(s_1, s_2) = u_1^* \tag{A.1}$$

where u_i is the utility function of player i, S_i is the strategy set of player i. $R_2(s_1)$ represents the *best response set* of player 2 to the strategy $s_1 \in S_1$ of player 1 defined as follows

$$R_2(s_1) = \{s_2' \in S_2 : s_2' = \arg \max_{s_2 \in S_2} u_2(s_1, s_2)\} \tag{A.2}$$

The quantity u_1^* in (A.1) is the *Stackelberg utility* for the leader, which admits a unique value in the given hierarchical decision-making game referring to Theorem 3.9 in [36]. Moreover, the SES s_1^* in (A.1) ensures that the leader does not receive a utility that is lower than u_1^*, which thus constitutes a secured utility level for the leader. Accordingly, the *Stackelberg equilibrium* is defined as follows

Definition 2. Let $s_1^* \in S_1$ be an SES for the leader, i.e., player 1. Then, for any strategy $s_2^* \in R_2(s_1^*)$ that is in equilibrium with s_1^* (satisfying (A.1)) is an optimal strategy for the follower of player 2. Thus, the pair (s_1^*, s_2^*) is a *Stackelberg equilibrium* for the two-person game [36].

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