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Research on four kinds of uncertain preference information aggregation approach in group decision making

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Abstract

Purpose – The purpose of this paper is to aggregate different preference information in group decision-making process such as interval preference order, interval utility value, interval number reciprocal comparison matrix, and interval number complementary comparison matrix.

Design/methodology/approach – First, the consistency definitions of four kinds of uncertain preference information are defined. Then, the upper- and low errors are introduced to solve the inconsistent decision-making case. Following that, the weight model for each uncertain preference is proposed, respectively.

Findings – The aggregation approach based on minimal group deviation errors is suggested in order to obtain the utmost consistent opinion. In addition, the consistency judgment level and consistency extent are defined owing to the aggregation result.

Research limitations/implications – The calculation scale is large, if many decision makers will attend group decision-making process.

Practical implications – A very useful approach for aggregation of the different preference in group decision-making case.

Originality/value – Because of differences in knowledge structure, judgment level, and individual preference, decision makers express their judgment preferences via differently structured decision-making processes. Owing to the complexity and uncertainty of decision-making problems and the fuzziness of human thought, it is unrealistic to depict complex problems in the certain preference style. For decision-making preference structures, group decision-making aggregation approaches include the aggregation on the same kind of preference structure and the different kinds of preference structures. The study on the aggregation of the same kind of preference structure has received a deal of attention, but study into the aggregation of the different kinds of uncertainty preference structures is still a new field.

Keywords Cybernetics, Group theory, Decision making

Paper type Technical paper

1. Introduction

In group decision making, the different preference information structures of the comparison matrix, utility value, and preference order are probably adopted owing to the decision-making difference on knowledge structures, individual preference, and judgment level of the decision maker. In this case, the aggregation approach of how

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to aggregate different structure preference from single decision maker into group preference should be studied (Yao and Yue, 2006; Hu *et al.*, 2005; Zhang *et al.*, 2004). According to the preference structure, the aggregation approaches are divided into the same structure aggregation approach and the different structure aggregation approach. The same preference structure aggregation method has already obtained many research results (Ray and Triantaphyllou, 1998; Beynon *et al.*, 2000), while the different preference structure aggregation research is still a new research topic (Chiclana and Herrera, 1998; Delgado *et al.*, 1998; Xiao *et al.*, 2001). Since the complexity and uncertainty of decision problem and fuzziness of human thinking, it is not realistic to use the certain preference information to portray the complex question. In fact, the uncertainty is absolute while the certainty is relative. The literature (Yager, 2004) summarized the progress in field of uncertainty decision making, which only limited to single preference information. Based on four kinds of uncertain preference, the aggregation approach is put forward and consistency level of the group is defined.

2. Major results

2.1 Basic concepts

Definition 1. Based on the alternative set X, the decision maker adopts the interval number reciprocal comparison matrix $\bar{A} = (\overline{a_{ij}})_{n\times n}$ to express his/her preference (Xiao *et al.*, 2001):

$$\overline{a_{ij}} = \left[a_{ij}^L, a_{ij}^U\right], \quad a_{ij}^L \le a_{ij}^U,$$

a_{ij}^U is the upper value of the judgment, a_{ij}^L is the lower value:

$$\overline{a_{ji}} = \left[\frac{1}{a_{ij}^U}, \frac{1}{a_{ij}^L}\right], \quad \overline{a_{ii}} = [1, 1].$$

Definition 2. The decision maker adopts the interval number complementary comparison matrix $\overline{B} = (\overline{b_{ij}})_{n\times n}$ to express his/her preference (Xiao *et al.*, 2001):

$$\overline{b_{ij}} = \left[b_{ij}^L, b_{ij}^U\right], \ b_{ij}^L \le b_{ij}^U, \ \overline{b_{ji}} = \left[1 - b_{ij}^U, 1 - b_{ij}^L\right], \quad \overline{b_{ij}} = [0.5, 0.5].$$

Definition 3. The decision maker adopts preference ordering $\overline{F_i} = \left[f_i^L, f_i^U\right]$ to express his/her preference. $f_i^L, f_i^U \ge 0$. f_i means the alternative x_i is ranked f_i in all alternatives. Generally, the less value f_i is, the better corresponding alternative is.

Definition 4. The decision maker adopts the value of utility $\overline{E_i} = \left[e_i^L, e_i^U\right]$ to express his/her preference. The more value e_i is, the better corresponding alternative is. e_i can be regarded as the weight of x_i.

2.2 Aggregation model

The literatures (Xiao *et al.*, 2001; Yager, 2004) study on the weight solving approach of interval number reciprocal judgment matrix and interval number complementary judgment matrix. Based on existing research, the decision-making characteristic of uncertain preference information is analyzed, and then the unified model is established to derive the weight from each kind of uncertain preference information. At last, the aggregation approach for the multiple uncertain preference information is proposed.

(1) Weight modeling for interval number reciprocal judgment matrix. For the matrix $\bar{A} = (\overline{a_{ij}})_{n\times n}$, the $wa_i, i = 1, \ldots, n$ is denoted as the weight from interval number

comparison matrix. Generally, if the formula (1) is satisfied, the matrix $\bar{A}$ is holding the complete consistency:

$$a_{ij}^L \leq \frac{wa_i}{wa_j} \leq a_{ij}^U, \quad \forall i,j \tag{1}$$

If $\overline{A}$ does not have the complete consistency, the deviation variable ap_{ij}, ad_{ij} is introduced, that is, the formula (2) is satisfied:

$$\begin{cases} a_{ij}^L wa_j \leq wa_i + ap_{ij}, & i,j = 1, \ldots, n, \ \ i \neq j \\ wa_i \leq a_{ij}^U wa_j + ad_{ij}, & i,j = 1, \ldots, n, \ \ i \neq j \end{cases} \tag{2}$$

The less deviation of ap_{ij}, ad_{ij} is, the better consistency of $\overline{A}$ is. If $ap_{ij} = 0, ad_{ij} = 0, \forall i,j$, $\overline{A}$ has complete consistency. The P_1 is suggested to derive the weight from interval number reciprocal judgment matrix:

$$\min \sum_{i,j} ap_{ij} + ad_{ij} \tag{3}$$

$$\text{s.t.} \begin{cases} a_{ij}^L wa_j \leq wa_i + ap_{ij}, \qquad i,j = 1, \ldots, n, \ \ i \neq j & (4) \\ wa_i \leq a_{ij}^U wa_j + ad_{ij}, \qquad i,j = 1, \ldots, n, \ \ i \neq j & (5) \\ \sum_{i=1}^{n} wa_i = 1 & (6) \\ wa_i \geq 0, ap_{ij}, ad_{ij} \geq 0, \quad i = 1, \ldots, n & (7) \end{cases}$$

(2) Weight modeling for interval number complementary judgment matrix. For the interval number complementary judgment matrix $B = (b_{ij})_{n \times n}$, the wb_i' is denoted as the weight derived from the interval number complementary judgment matrix. If it has the complete consistency, the formula:

$$b_{ij} = \frac{wb_i'}{wb_i' + wb_{i,j}'}$$

is satisfied from the literature (Xiao *et al.*, 2001). According to the interval number complementary consistence formula, the formula (8) should satisfy, that is:

$$b_{ij}^L \leq \frac{wb_i}{wb_i + wb_j} \leq b_{ij}^U. \tag{8}$$

If $\overline{B}$ does not have the complete consistency, the deviation bp_{ij}, bd_{ij}, is introduced so formula (9) is satisfied, that is:

$$\begin{cases} b_{ij}^L (wb_i + wb_j) \leq wb_i + bp_{ij} \\ wb_i \leq b_{ij}^U (wb_i + wb_j) + bd_{ij} \end{cases}. \tag{9}$$

The less value of bp_{ij}, bd_{ij} is, the better consistency of $\overline{B}$ is. If $bp_{ij} = 0, bd_{ij} = 0, \quad \forall i,j$, $\overline{B}$ has the complete consistency. P_2 is suggested to estimate the weight for interval complementary judgment matrix.

(3) Weight modeling for utility value of interval number. If the utility value:

$$\overline{E_i} = \left[e_i^L, e_i^U\right]$$

is adopted to express decision maker preference, it can be considered as the weight of alternative. Let we_i denote as the alternative weight, and the formula $e_i^L \leq we_i \leq e_i^U$ is satisfied. The deviation ep_i, ed_i is introduced. And then, there is:

$$e_i^L \leq we_i + ep_i, we_i \leq e_i^U + ed_i \tag{10}$$

$$\min \sum_{i,j} bp_{ij} + bd_{ij} \tag{11}$$

$$\mathrm{P}_2: \ \text{s.t.} \begin{cases} b_{ij}^L(wb_i + wb_j) \leq wb_i + bp_{ij}, \quad i \neq j & (12) \\ wb_i \leq b_{ij}^U(wb_i + wb_j) + bd_{ij}, \quad i \neq j & (13) \\ \sum_{i=1}^{n} wb_i = 1 & (14) \\ wb_i \geq 0, bp_{ij}, bd_{ij} \geq 0, \qquad i = 1, \ldots, n & (15) \end{cases}$$

The less deviation ep_i, ed_i in formula (10) is, the better consistency is. Then the P_3 is suggested to derive the weight from this preference:

$$\min \sum_{i} ep_i + ed_i \tag{16}$$

$$\text{s.t.} \begin{cases} e_i^L \leq we_i + ep_i, \quad i = 1, \ldots, n & (17) \\ we_i \leq e_i^U + ed_i, \quad i = 1, \ldots, n & (18) \\ \sum_i we_i = 1, \qquad we_i \geq 0 & (19) \end{cases}$$

(4) Weight modeling for interval number preference order. If the interval number preference order:

$$\overline{F_i} = \left[f_i^{\prime L}, f_i^{\prime (U}\right]$$

is adopted to express the decision maker preference, set the weight of the last alternative be a positive parameter. The interval number preference order is translated into the utility value $\left[f_i^L, f_i^U\right]$:

$$f_i^L = \frac{\left(\left(n - f_i^{\prime U}\right)/\left(n-1\right)\right) + \varepsilon}{\sum\left(\left(n - f_i^{\prime U}\right)/\left(n-1\right)\right) + \varepsilon} \Rightarrow \frac{\left(\left(n - f_i^{\prime U}\right)/\left(n-1\right)\right) + \varepsilon}{(n/2) + n\varepsilon},$$
$$f_i^U = \frac{\left(\left(n - f_i^{\prime L}\right)/\left(n-1\right)\right) + \varepsilon}{(n/2) + n\varepsilon} \tag{20}$$

In equation (20), the ε is a little positive number. If the weight of the last alternative is x, one can get $\varepsilon = nx/(2(1-nx))$ from equation (20). Therefore, we can use method of utility value of interval number to solve the problem of interval order, which is expressed as P_4, and its parameter implication is as the same as P_3:

$$P_4: \min \sum_i fp_i + fd_i$$

$$s.t.\begin{cases} \frac{\left(\left(n-f_i^U\right)/(n-1)\right)+\varepsilon}{(n/2)+n\varepsilon} \leq wf_i + fp_i, wf_i \leq \frac{\left(\left(n-f_i^L\right)/(n-1)\right)+\varepsilon}{(n/2)+n\varepsilon} + fd_i, & i=1,\ldots,n \\ \sum wf_i = 1, & wf_i \geq 0 \end{cases}$$

(5) Aggregation model for general idea of group decision making. Generally speaking, decision makers should reach a coincident opinion, which is expressed by $w_i, i = 1, \ldots, n$. Set the expert weight of interval number reciprocal judgment matrix be p_1, the expert weight of interval number complementary judgment matrix be p_2, the expert weight of the utility value of interval number be p_3, and the expert weight of interval number preference order be p_4. As a result, P_5 is suggested:

$$P_5: \min \sum_{i,j} p_1(ap_{ij} + ad_{ij}) + p_2(bp_{ij} + bd_{ij}) + \sum_i p_3(ep_i + ed_i) + p_4(fp_i + fd_i) \quad (21)$$

$$\text{s.t.} \begin{cases} a_{ij}^L w_j \leq w_i + ap_{ij}, w_i \leq a_{ij}^U w_j + ad_{ij}, & i \neq j = 1,\ldots,n \quad (22) \\ b_{ij}^L(w_i + w_j) \leq w_i + bp_{ij}, w_i \leq b_{ij}^U(w_i + w_j) + bd_{ij}, & i \neq j = 1,\ldots,n \quad (23) \\ e_i^L \leq w_i + ep_i, w_i \leq e_i^U + ed_i, & i = 1,\ldots,n \quad (24) \\ \frac{\left(\left(n-f_i^U\right)/\left(n-1\right)\right)+\varepsilon}{(n/2)+n\varepsilon} \leq w_i + fp_i, w_i \leq \frac{\left(\left(n-f_i^L\right)/\left(n-1\right)\right)+\varepsilon}{(n/2)+n\varepsilon} + fd_i, & i = 1,\ldots,n \quad (25) \\ \sum_{i=1}^n w_i = 1, w_i \geq 0 & (26) \\ \varepsilon, ap_{ij}, ad_{ij}, bp_{ij}, bd_{ij}, ep_i, ed_i, fp_i, fd_i \geq 0, & i,j = 1,\ldots,n \quad (27) \end{cases}$$

Theorem 1. P_5 must have the optimal solution.

Theorem 2. Let θ^* be the optimal solution of P_5. If the $\theta^* = 0$, the decision-making group's opinion is completely consistent. If decision-making group's opinion is incompletely consistent, there must be $\theta^* > 0$, and the bigger value θ^* is, the more dispersible of the group opinion is.

Proof. θ^* is composed of the non-negative deviation variable. If $\theta^* = 0$ and $\exists i, j, ap_{ij}, ad_{ij}, bp_{ij}, bd_{ij}, ep_i, ed_i, fp_i, fd_i = 0$, then equations (22)-(25) are tenable, therefore, each expert's opinion is completely consistent. If $\theta^* > 0$, ap_{ij}, ad_{ij}, $bp_{ij}, bd_{ij}, ep_i, ed_i, fp_i, fd_i$ must have one value not equal to zero, therefore decision-making group's opinion is incompletely consistent. The bigger value is, the more dispersive the group opinion is.

According to the Theorem 1, through solving the P_5, the preference information can be aggregated. While the following questions still need to be further considered:

(1) Decision-making group's synthesis preference always can be obtained according to P_5. But is the numerous position experts' opinion consistent? If consistent, how to express the uniform degree? If inconsistent, how is the divergent degree?

(2) How to determine the experts' weight in P_5? Usually, the value of experts' weight is difficult to determine, if making various experts' weight to be equal, then the obtained result is the expert advice compromise, does not have the consideration commonly used "most" principle, whether expert's weight carries on the suitable evaluation according to the decision-making community's synthesis by chance?

In question (1), θ^* can be used to determine uniform degree of the decision-making group, however, what value of θ^* can express the decision-making group's uniform degree is good, and the expert opinion is consistent? The inconsistent degree size of the expert advice is analyzed through the vector method.

Theorem 3. When $p_1 = 1$, p_2, p_3, $p_4 = 0$ the decision-making group's synthesis preference which obtained through P_5, $w_i, i = 1, \ldots, n$ completely obeys Expert's 1 opinion, when $p_2 = 1$, p_1, p_3, $p_4 = 0$, the results of P_5 obeys the Expert 2 opinions completely, analogized in turn has the similar conclusion.

Proof. When $p_1 = 1$, p_2, p_3, $p_4 = 0$, according to P_5, the value of synthesis preference was restricted by the constraint condition formula (22) deviation variable, it is always established by adjusting other constraint condition in the formula the deviation variable, therefore, the decision-making group's synthesis preference is constructed by formula (22). Other situation's proof is similar.

Based on P_5, set p_1, p_2, p_3, $p_4 = 0$ be equal to 1 separately, and others are 0, according to Theorem 3, it obeys completely in this expert opinion, recording $w_i^k, i = 1, \ldots, n$. According to the vector inner-product formula, the angle of the vector has reflected two vector uniform degree, therefore, basing on $w_{i,} w_i^k$ calculated separately, the expert k and the synthesis opinion uniformity conforms to the consistency degree. If the expert k and the synthesis opinion uniformity conforms to the degree is good, then this expert's weight should also be supposed to be big, otherwise supposed to be small.

Definition 5. Set:

$$\eta^k = \frac{w \bullet w^k}{|w||w^k|}$$

be called expert's judgment uniform level, in which $w = [w_1, \ldots, w_n], w^k = [w_1, \ldots, w_n]$ was obtained separately from P_5.

According to Definition 5, $1 \geq \eta^k \geq 0$. The $\eta^k = 1$ indicated the expert k's opinion is completely consistent. The little η^k is, the bigger the expert's opinion. Based on Definition 5, defining expert k's weight is:

$$p_k = \frac{\eta^k}{\sum_k \eta^k}.$$

Based on p_k, re-computing P_5, the new result $w_i, i = 1, \ldots, n$ will be obtained to be the synthesis preference of decision-making group.

Definition 6. Set:

$$\eta = \frac{\sum_{k=1}^{k=K} \eta^k}{K}$$

be called the average uniform degree of the decision-making group's opinion, K is the experts' number.

3. Example analyses

A risk investment company wants to have the optimal investment. There are four alternatives, which is a bio-pharmacy company, a food company, a fashion company, and a computer software company. The company employs m experts to give decision ($m \geq 2$), give interval number reciprocal judgment matrix, interval number complementary judgment matrix, and interval number preference and interval number utility values:

$$A = \begin{bmatrix} [1,1] & [1,2] & [2,3] & [1,2] \\ [1/2,1] & [1,1] & [3,5] & [1/2,1] \\ [1/3,1/2] & [1/5,1/3] & [1,1] & [1,2] \\ [1/2,1] & [1,2] & [1/2,1] & [1,1] \end{bmatrix},$$

$$B = \begin{bmatrix} [0.5,0.5] & [0.5,0.6] & [0.6,0.7] & [0.5,0.6] \\ [0.4,0.5] & [0.5,0.5] & [0.5,0.6] & [0.3,0.4] \\ [0.3,0.4] & [0.4,0.5] & [0.5,0.5] & [0.2,0.3] \\ [0.4,0.5] & [0.6,0.7] & [0.7,0.8] & [0.5,0.5] \end{bmatrix}$$

$$f_1 = [0.4, 0.5],\ \ f_2 = [0.1, 0.2],\ \ f_3 = [0.1, 0.2],\ \ f_4 = [0.2, 0.4],$$

$$e_1 = [1, 2],\ \ e_2 = [2, 3],\ e_3 = [2, 4],\ e_4 = [1, 2].$$

Set $\varepsilon = 0.05$, solve P_5 and get weight of these four alternatives. They are: $w_1 = 0.375$, $w_2 = 0.25$, $w_3 = 0.125$, $w_4 = 0.25$. Set the k expert's weight p_k for 1, the others are 0. Solve P_5, and list the result in the Table I. Alternative 3 is the worst alternative that four experts think, and there are three experts think the best alternative is alternative 1.

Table I.
The preference of the experts and the synthesis preference of the decision-making group

Alternative	Expert 1 Weight	Expert 1 Order	Expert 2 Weight	Expert 2 Order	Expert 3 Weight	Expert 3 Order	Expert 4 Weight	Expert 4 Order	Synthesis preference Weight	Synthesis preference Order
1	0.333	1	0.323	1	0.4	1	0.326	2	0.375	1
2	0.333	1	0.215	2	0.1	2	0.174	3	0.250	2
3	0.111	3	0.138	3	0.1	2	0.023	4	0.125	3
4	0.222	2	0.323	1	0.4	1	0.477	1	0.250	2

According Table I and Definition 5 calculating the consistency level of the experts, those are: 0.9829, 0.9831, 0.9237, 0.8990. Re-calculating P_5, get the same result as the aggregation result, the experts' average degree of consistency is 0.947, that is to say, the experts' suggestion is consistent, and the sequence from the worst to the best is: alternatives 1, 2, 4, and 3.

4. Conclusions

This paper researched on four kinds of uncertain preference information aggregation approach, and proposed a new method for determine the experts' weights in group decision making which is based on much structural uncertain preference information. The model is clear and simple to use. It has considerable reference value for the weight solving of relative preference information to introduce the deviation variables to solve interval number complementary judgment matrix, interval number utility values and interval number preference. The next step of research aggregate more kinds of uncertain preference information, and develop practical group decision-making support soft system in the environment of web based on corresponding models algorithms.

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