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Scaling laws and the modern city

Antonio Isalgue*, Helena Coch, Rafael Serra

AiEM, Barcelona School of Architecture, UPC, Diagonal 649, 08028 Barcelona, Spain

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Abstract

The inter-relations and the complexity of modern urban spaces are difficult to analyse in a way that allows improving living conditions or help to ascertain optimal decisions for saving energy or improving sustainability. Carefully designed decisions and guidelines might produce unexpected results because of particularities, or complex sets of reactions from residents or economic counterparts. Complexity tends to increase with size, such as when, for instance, services tend to concentrate in large agglomerations, and transportation needs take on critical importance. Complex systems such as living organisms are known to follow approximate relationships as scaling laws between the variables that describe them. Some of these kinds of relationships are tested in relation to modern developed urban spaces, in which it is possible to find a reasonable continuity with the types of scales seen in living organisms, and some preliminary conclusions are drawn.

Keywords: Scaling laws; Cities; Complex systems; Transportation

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1. Introduction

Large cities have a high degree of complexity and strong inter-relations among their inhabitants and surroundings. The degree of complexity and these inter-relations nowadays seem to produce favourable conditions, or better living conditions, as the size and number of large cities appear to increase. Urban planning in large cities is a formidable challenge, because of the range of previsions that must be taken into account. It would be very attractive if there were natural laws that guided growth, or at least yielded desirable conditions. However, detailed physical relationships between the variables that determine these conditions are very complex due to the complexity of the system and the multiple sets of variables needed to describe it, which are together considered as a physical system. Nevertheless, some ideas and tendencies seem to be shared by many modern cities: one should optimise such things as energy expenses, the transport of goods (including food, water, …), public and private transport, and pollution issues. It then seems reasonable to expect some general trends (if not "exact" laws) that modern cities should follow, and it should be acceptable to think about it as complex, physical systems.

Then, we should learn from phenomenological relations in existing complex systems that have evolved over hundreds of millennia. As living organisms represent one of these systems, which have been largely studied,

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*Corresponding author. Tel.: +34 934010868; fax: +34 934016426.
E-mail address: antonio.isalgue@upc.edu (A. Isalgue).

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and are indeed a paradigm of complexity, studying general laws or general relationships in biology might give us an insight into our problem. In fact, it seems reasonable to conjecture that the coarse-grained behaviour of living systems might obey quantifiable universal laws that capture the systems' essential features, as Ref. [1] does.

Scaling, as a manifestation of the underlying general dynamics and geometry, is familiar throughout physics. It has helped scientists gain deeper insights into problems ranging across the entire spectrum of science and technology, as scaling laws typically reflect generic features and physical principles that are independent of the detailed dynamics or specific characteristics of particular models. Fluid mechanics and phase transitions are significant examples in which scaling has illuminated important universal principles or structures, and has provided responses to practical problems.

It seems reasonable to conjecture that the coarse-grained behaviour of living systems might obey quantifiable laws that capture these systems' essential features, if these do in fact exist. In contrast to the large diversity and complexity of living organisms, one finds the simplicity of the scaling behaviour of biological processes that holds true in a wide range of phenomena and a large range of energy and mass.

As pointed out by Ref. [1], in biology the scaling observed is typically a simple power law: $Y = Y_0 M^b$, where Y is some observable magnitude, Y_0 a constant, and M the mass of the organism. The exponent b usually approximates a simple multiple of $\frac{1}{4}$. Among the many fundamental variables that obey such scaling laws are metabolic rate, life span, heart rate, lengths of aortas and genomes, tree height, mass of cerebral grey matter, and others [see Ref. [1] and references therein].

The metabolic rate for mammals and birds was shown to scale as $M^{3/4}$ many years ago for about four orders of magnitude in mass [1,2]. This work was generalised by subsequent researchers to other systems, and, with some particularities, the metabolic exponent $b \approx \frac{3}{4}$ is found across nearly 27 orders of magnitude in life [1,3]. Other examples include heart rate ($b \approx -\frac{1}{4}$), and the radii of aortas and tree trunks ($b \approx \frac{3}{8}$) [1].

The starting point to understanding the origin of this scaling is recognising that highly complex living structures require a close integration of massive numbers of localised microscopic units that must be serviced in an approximately "democratic" and efficient fashion [1]. To meet that challenge, natural selection has evolved hierarchical fractal-like branching networks that distribute energy, goods, or metabolites, and information from macroscopic reservoirs to microscopic sites. Examples include animal circulatory systems, plant vascular systems, and intracellular networks.

The formidable quest for optimisation of transport (of matter, energy, and information) might be considered as giving rise to scaling laws in the actual level.

It can be proposed that scaling laws and the generic coarse-grained dynamical behaviour of biological systems reflect the constraints inherent in the universal properties of such networks. According to Ref. [1], these constraints can be approached as follows:

- Networks service all local biologically active regions in both mature and growing biological systems. Such networks are called space-filling (no place without service).
- The networks' terminal units are invariant within a class (microscopic localised units do not on average show any appreciable change).
- Organisms evolve towards an optimal state in which the energy required for resource distribution is minimised (optimisation of the network).

These properties, which characterise an idealised biological organism, are presumed to be consequences of natural selection: all forms of life function by transforming energy from physical or chemical sources into organic molecules that are then metabolised to build, maintain, and reproduce complex, highly organised systems.

In fact, the constructal theory states that every flow system evolves in time so that it develops the flow architecture that maximises flow access under the constraints posed to the flow. It has been quite successful in justifying allometric scaling laws [4], global circulation and climate characteristics [5], and even scaling effects in running, swimming, and flying [6]. However, as the boundaries and the conditions in complex systems might not be so evident, as Ref. [7] points for the case of traffic organisation in ant societies, care should be exercised in interpreting. We maintain the main idea that very complex systems, transports conditioned, such as living organisms, should follow some allometric scaling laws.

Even though cities seem quite far from living organisms, some general activities take place in both of them: among these activities one finds transforming energy, transporting and transmitting energy, information and goods, and repairing damage to some amount. Furthermore, if biological scaling laws are the consequence of the above constraints, modern, developed cities are also subject to some of these kinds of network-related constraints, even though the classes of complex networks of cities might differ from those in living organisms, giving rise to distinct patterns of connections among nodes. Even if it is the case, Ref. [8] indicates that these structural features cannot be captured by means of studying global properties. Thus, it seems reasonable to try to compare the gross behaviour of modern cities with the biological systems, even if this is only a first approach to more detailed considerations as an open way for considerations as in Ref. [9]. A first approach was intended in Ref. [10].

The general scaling laws followed by living organisms are herein explored to be extrapolated to the dimensions of the modern city, even though looser relations are to be expected, as there is a large difference in history (cities are much more recent than living organisms) and statistics: living organisms as animals are usually composed of a very large number of elemental blocks (cellules), compared with the number of inhabitants in a city.

Research on scaling in cities has also been done for the structure of the city. The aim in Ref. [11] is on the structure of the plan of cities, and the scaling of lengths. Ref. [12] is also concerned with the near-fractal characteristics of cities and transport in them.

Furthermore, urban supply networks have been shown to follow scaling laws [13]. In Fig. 3 of Ref. [13] it is shown that, for German cities, electric energy delivered to households grows nearly proportional to the population. Other magnitudes have been shown there to scale with the population in a different way. However, no trial to extend these data to allometric scaling laws has been done.

2. Allometric scaling of power in cities

The trial to check for allometric scaling laws in cities starts looking for the definition of a city. Then, one should obtain the “mass” and the “metabolism” or energy consumption of a city to compare it with the scaling law for living organisms. However, cities have no clear boundaries or skin as living organisms have. Furthermore, neither is the mass of a city as well defined as the mass of an animal. Larger cities should have more mass as a consequence of having more inhabitants and more buildings, but also as a consequence of more public services and transport. A minimum population is needed for a conurbation to be considered as a city; essentially, we consider that a city must have hospitals, schools at all levels (including universities, as people should be able to grant for the most qualified working posts), libraries, public transport, and the presence of relatively large service centres. Also, we consider dense metropolitan areas to be cities, though they may or may not coincide with any given municipality. Thus, we consider the effective mass of a city to be the mass of its built elements: their constituent concrete, steel, and masonry; its public facilities (street surface, squares, underground, etc.); the machines and vehicles normally operating in the city; and its inhabitants—that is, the parts that are most susceptible to being repaired or replaced in case of damage or malfunction. With this kind of definition, many points are to be considered as gross approaches, for instance, about the mass of the incoming highways and railways, which we do not consider outside of the metropolitan area, as a first approach. We consider also that the important thing in scaling laws should be mass and not number of people, as mass is what is used in living organisms and not the number of cells.

For the case of Barcelona (Spain), we have approached its mass from the maps given in the web of the city council [14] (section of urbanism), where the number of built floors (over and under the street level) might be found in the map. Even if it is known that some works are always under way, and some irregularities might exist, the data are assumed to be correct to a few per cent of margin. Then, the estimation of built floor area for the municipality (about 1.6 million people, from Ref. [10]) is near $2.6 \times 10^8\,\mathrm{m}^2$. The surface of streets is $1.7 \times 10^7\,\mathrm{m}^2$ (from Ref. [14]). To obtain the data for the whole metropolitan area, an approximate scaling with the number of people is assumed, as constructive typologies are not too different in the metropolitan area, some three million people. Then, from the building characteristics and typologies, an approximate built mass can be obtained (average building mass of 350 kg/m^2 of floor, this is a quite strong source of imprecision). The estimated mass of other built elements, sewage, tunnels of the train and subway, port, airport (these are also sources of imprecision), and other services should also be included, together with machinery, cars, animals, and people, giving an estimate of around 2×10^{11} kg. Due to the latter points, this figure is estimated to be correct to a margin narrower than between 1.5×10^{11} and 4×10^{11} kg.

It might be suggested that the tendency towards larger agglomerations would mean that a greater efficiency (including advantages as shelter, availability of services, and some optimisation of transport) is achieved. For a modern city, metabolism or power consumption might be estimated from residential (including offices and schools), transport (which is intense in cities), and industrial use, including such sources as electricity, petrol, and natural gas. Food represents a relatively small value (around 10 MJ/person/day) compared with the former, which is estimated to be around one order of magnitude (a factor of 10) more in medium modern cities in temperate climates. Small cities mean usually an increase of transport cost per person and somewhat less shelter, so more power per inhabitant would be needed; large cities mean usually more infrastructure, so the ratio of mass to power might change with size. The total power consumption of the metropolitan area of Barcelona may be obtained from the figures given in Ref. [15], to be a mean of near 3×10^9 W for the year 1999. However, the low growth of power (energy) consumption with time should be taken cautiously, as the original data refer to the municipality, and the growth is stronger in the surroundings (metropolitan area), and then any extrapolations are to be considered gross approaches. We think the actual value (2007) should be between 2×10^9 and near 4×10^9 W. Also, it should be noted that we do not distinguish between energy uses in replacement, functioning, and growth, and we can just indicate a global value.

Similar considerations have been also done for some cities of different size. Data for the smaller city of Vic (near 40 000 inhabitants, half-way between Barcelona and the French border) have been processed, from the maps in Ref. [16]. The estimated mass for Vic is 2.5×10^9 kg, and the estimated average power is 6.3×10^7 W. In this case, the maps denote larger uncertainties (height "?") in many places, so the reliability of the final figure is lower. Also, the size of Vic is relatively small, and it is a fact that some services are obtained by displacement to Barcelona, at a distance of 70 km. This produces the data for Vic to be regarded cautiously when compared to data for other cities, as Vic might be in the limit consideration for being a city from our view, even if it has public transport and a university.

Maps for Grenoble, a French city near the Alps, with near 400 000 inhabitants in the metropolitan area, have been used, together with aerial views, to have a rough estimation of the mass of a medium-sized city. Also, an estimation of power consumption has been done, even though larger uncertainties are present, because of the particularities of the city, with many large research sites (ESRF, ILL, CENG) and a very large university compared to the size of the city, which might produce some underestimation of power consumption. The estimated mass for Grenoble is then 1.3×10^{10} kg, and the estimated average power consumption 4×10^8 W.

Data for Paris have been retrieved from the plans in Ref. [17], where building height is indicated by colours (not the number of floors, but an estimate can be done assuming a floor for 3 m height). Then, the value for the Paris mass has a much larger uncertainty than the data for Barcelona (further, built underground is not given for Paris). The same methodology as that for Barcelona is applied, as the data in the maps refer to central Paris (around 2.2 million people), and not to the whole metropolitan area (around 10 million people). Then, much larger uncertainties are expected. The estimated mass is 6.3×10^{11} kg. Also, an estimation of the power consumption for Paris has been done, as 9×10^9 W.

A rough estimation for a large modern city such as New York has also been done, from maps, aerial photographs, and population statistics, to compare with the other cities. The estimated mass is 1.3×10^{12} kg, and the average power 1.6×10^{10} W. Then, the position of cities can be indicated in Fig. 1, where data for animals are also represented. The size of the dots for cities corresponds to uncertainties in mass or power by a factor of nearly 10 (i.e., multiplying or dividing by 3), as this is an estimate for the worst case.

The position of the cities in Fig. 1 is quite close to the extension of the straight line traced by the data correlating to living organisms, with slope $\frac{3}{4}$. It should also be noted that the metabolism or power used in the city lies somewhat above the straight line. Also, the slope of the data for cities (slope 0.86 when only the points for cities are considered) is somewhat different from the slope for animals, and it should be remembered that not only do large uncertainties exist for the cities, but also the listed cities have no strictly equal climatic

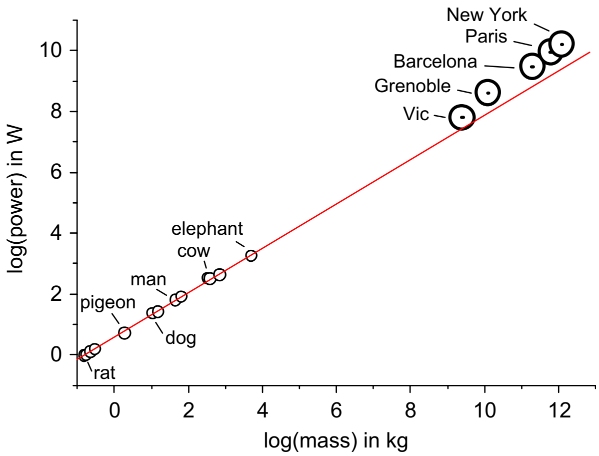

Fig. 1. Representation of the logarithm of the metabolic power versus the logarithm of the mass, for some animals and for some cities (see the text).

conditions, and they cover a reduced span in logarithmic scale. Also, smaller cities tend to drive people to larger places for some services. Further on, the mass of the cities is more difficult to define, for instance, it is not clear if the exterior water conducts and sewage, roads and railroads connecting cities, have to be included; we have not included these factors here.

Also, it has to be noted that the cities presented in the graph are amongst what is considered the "developed" or "western" world. It comes to be evident that less "developed" cities usually have lower power consumption (even though not very low, because of usually less efficient systems operating in the developing world), but also less infrastructure, and in many cases lower-quality buildings. Estimates for developing cities would be rougher that the ones we have selected, which have relatively reliable known characteristics.

The case of Japan might be considered independently. A rough analysis shows that the living built area per person is lower than in the "western" cases, because of cultural traditions, but also seismic regulations impose a higher stiffness for the buildings and public constructions. Then, a large deviation from the points presented is not expected.

3. Pulsatory transport in cities

We think that the way in which the position of a developed modern city matches approximately the trend concerning the relationship between power and mass for living organisms means that the dynamical characteristics of these systems are not too far distant from one another. Of course, detailed configurations can be very different from one system to the other (coherently with Ref. [8]), and only gross characteristics or structures are in common. Furthermore, from our point of view, this reflects the dominance of transport problems (as far as concerns mass, energy, and information) in these systems. If this is the case, other approximate relationships between mass and diverse properties of cities could exist, even if cities have not reached as fully evolved a system as living organisms seem to have (mostly because evolution has acted on living organisms for millions of years, and only for a few millennia on cities), and deviations could be also found because of the comparative lack of history and evolution of cities, and lack of statistics and influence of particularities in each city. Some of these relationships could even prove to be meaningless, as cities are comparatively not as evolved as living organisms.

Fig. 2 shows the relationship between heart rate and mass, for animals, and locates the position of some cities. Extrapolation of the straight line to the mass of a developed large city gives nearly a 4-min period for the beats (rate) of "medium" cities as Barcelona. Even if cities do not have an "organic" heart, like animals, a city's heartbeat could be identified with a pulsating transport activity, which might be either public transport events (the passing of buses, trains, …), or the regular changes of traffic lights. While the former could have a

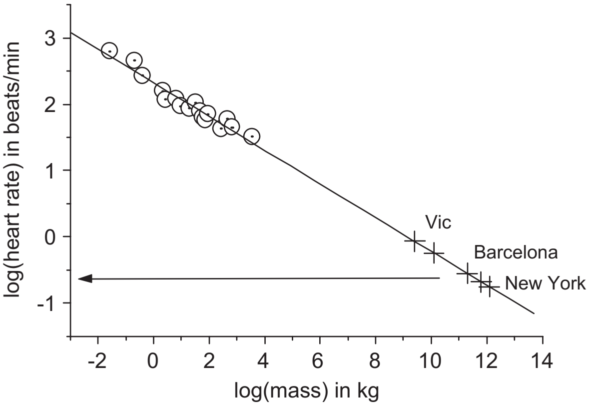

Fig. 2. Extended heart rate (beating or rhythm of pulsatory transport) versus mass, including the position of modern cities. From the graph, which is an extension of a graph in Ref. [1] to the effective masses of cities, an "extended equivalent" heart rate for the cities may be deduced.

passing period of really near 4 or 5 min, the latter usually have a period of nearly 1.5 min (100 s in Barcelona). If the figure really indicates an optimisation, then the actual frequency of change of the traffic lights seems to be too high for cities like Barcelona or larger. Even if lowering the frequency of traffic light changes might require solutions for pedestrians (such as elevated walkways), a lower frequency of change would increase the relative time at which the traffic moves at a constant speed, which would then enhance the optimisation of movement. It might be recalled that after an energy crisis, it was recommended in some countries such as Switzerland that cars stop their engines while waiting at traffic lights (the lights had an advice some seconds before the changes of state in cities like Geneva). The effects of these kinds of measures would be favoured by longer traffic light periods.

It should be also noted that for small towns, there might be no traffic lights at all. Then, the adoption of traffic lights might be regarded as an optimisation in transport—this means not only decrease in waiting time but also decrease in accidents and injuries—arising when size, and then complexity, increases.

4. Conclusions

A rough estimation shows that modern cities approach some of the scaling behaviour followed by living organisms. This might be interpreted as due to that both systems are essentially transport conditioned, and optimisation of systems is under way. The scaling might be useful to take into account when looking for optimisation of some problems, transport related. An application to timing of traffic lights is suggested.

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